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INFLUENCE OF NUMBER AND SPACING
OF SENSORS ON THE EFFECTIVENESS OF SEISMIC ARRAYS

6 April 1970

Prepared For
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ARPA Order No. 624

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INFLUENCE OF NUMBER AND SPACING
OF SENSORS ON THE EFFECTIVENESS OF SEISMIC ARRAYS

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ABSTRACT

Ideally, geophones would be placed in a noiseless environment, in which case there would be no reason to resort to arrays of geophones. If the noise is such that an array is required, the objective of the array is to enhance the signal-to-noise ratio and thus maximize the intelligence that can be derived from a given signal. The design of the array will be a function of the signal characteristics of the direction and velocity of the noise in the bandpass of the signal, and of the site geology.

It has been demonstrated that in a practical sense the optimum array processing is represented by precise beam forming by which we mean simple delay and summing. Increasing the number N of sensors within a given area decreases the inter-element spacing and may increase the coherency between noise samples at adjacent sensors, thus yielding poorer results compared to $\sqrt{N}$ improvement one expects to get. Increasing the number of sensors by proportionately increasing the area is liable to result in signal deterioration also yielding an unfavorable comparison to $\sqrt{N}$ improvement in signal-to-noise. These two effects, together with economical factors, combine to limit the number of sensors that can be used.

Although the data on which our conclusions are reached were drawn from earthquake seismology, the principles involved are equally applicable to exploration seismology and to other geophysical measurements in which arrays of sensors are required.
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INTRODUCTION

This report has been written to integrate our work in the analysis of seismic array, which has in turn led to a scheme for the rational design of an array. We are concerned with noise reduction, signal reduction, and gain in the signal-to-noise ratio as functions of the array parameters.

Various arrays were simulated by combining selected sensors from the Large Aperture Seismic Array in Montana. The signals used in the whole study consisted of night-time recordings of teleseismic earthquakes which occurred over the two-month period from January to March 1967.

After defining some of the terms used, we develop the subject by first considering briefly some of the statistical aspects of the problem. We next demonstrate practical results with one subarray. In order to increase the range of consideration by an order of magnitude, we then demonstrate the results using the whole of LASA.

Our report is essentially a condensation of three reports of the Seismic Data Laboratory, indicated by asterisks in the list of references; and, indeed, we must give credit to the whole staff for contributions to this work in one way or another. We would particularly like to acknowledge the ideas of Dr. Robert Shumway who contributed the statistical thinking; of Drs. E. Chiburis and W. C. Dean who made the original studies of signal and noise coherency as a function of sensor spacing, and of Dr. E. A. Flinn who gives us advice on filtering problems and seismological support in all of our efforts.
The work on which this report is based was done to relate the efficiency of a seismic array to the distance between array sensors, to the total surface over which the array is spread, and to the number of sensors. It is obvious that only two of these three parameters can be considered to be independent variables.

The short period seismometers in the Large Aperture Seismic Array (LASA) in Montana provide sufficient flexibility and coverage to make such a comprehensive study simply by selecting a variety of combinations of the individual sensors. The total array extends over an area approximately 200 km in diameter (Figure 1) and consists of 21 subarrays. Each subarray in turn consists of 16 individual sensors spread over a circular area 7 km in diameter (Figure 2). One of these subarrays labeled E3 was reconfigured to spread the normal 25 seismometers over a circular area 19 km in diameter (Figure 3). It is obvious that a wide variety of arrays can be simulated from the total LASA by selecting only desired combinations of sensors. The sensor spacing can be changed by dropping sensors within the array and the area covered by the simulated array can be varied by including or by not including outer rings of sensors. For example, the manner in which the minimum sensor spacing was made 3 or 6 km was to include only 7 or only 3 seismometers, respectively, from each subarray (Figure 4). In some of the results reported below, limited numbers of sensors were chosen from each subarray, and limited numbers of subarrays were
used; in some cases, only combinations of sensors from the extended E3 subarray were used. Since uniform distribution of sensors was not achieved, only the minimum spacing is a meaningful parameter.

The data used in this study consisted of nighttime recordings of teleseismic earthquakes which occurred over the two-month period from January to March 1967. The data were reduced by detrending all seismograms and by correcting for system magnification at 1 cps to convert digital counts to millimicrons (mm) ground displacement. The data were further prepared for beamforming by prefiltering using one of three recursive bandpass filters, independent of one another, to eliminate noise frequencies lying well outside the passband of the signal, i.e., long-period microseisms and frequencies greater than 2-3 cps. In any given set of data, the same bandpass was used so that the comparisons are valid. Outputs from each filter were beam-steered within subarrays automatically by computer, using the assumed apparent phase velocity and back azimuth derived from known or computed epicenters. The subarray beams were aligned to form the final beam using computed travel times adjusted for average predetermined travel time residuals.
DEFINITIONS

Since we had no way to distribute sensors uniformly over a given array, our references to sensor spacing \( \Delta \) are to minimum spacing between nearest neighbors. In general, the greatest number of adjacent pairs of sensors would display a spacing near the stated value. The number of sensors in a given array is represented by \( N \).

We define signal amplitude as one-half the maximum peak-to-trough excursion in \( m_m \) occurring in the first eight seconds of the P signature. Noise is considered to be either the rms value in \( m_m \) obtained in a 50-second interval ahead of P, or noise power at 1 cps computed from a 60-second sample ahead of the P arrival. Signal-to-noise ratios are based on signal amplitude and rms noise values. The signal loss, rms noise reduction, and signal-to-noise improvement were each computed as:

\[
db = 20 \log \left( \frac{\text{value on the beamformed output trace}}{\text{average value from traces in the beam}} \right) \tag{1}
\]

Noise power reduction at 1 cps was determined by:

\[
db = 10 \log \left( \frac{\text{noise power on the beamformed output trace}}{\text{average noise power on input traces}} \right) \tag{2}
\]

Finally, in those cases where noise reduction was computed in terms of the zero-lag autocorrelation and cross-correlations, we used the following formula:

\[
db = -10 \log \left[ \log N - \log\{1 + (N-1)\rho\} \right] \tag{3}
\]
It is important to emphasize that the principal source of seismic noise is due to microseisms with six-second periods and the principal energy in seismic body waves is peaked at or near a period of one second. Thus seismograms can be considerably improved simply by bandpass filtering. In order to isolate the effects of bandpass filtering so that the results of array beam forming can be properly evaluated, all this work was performed on noise restricted to the bandpass at the signal.

Zero-lag correlations and noise reduction.

The reduction in noise due to simple delay and summing is based on the assumption that each original data trace from a given array consists of a zero-mean time, stationary-noise process with a cross-correlation given by

$$E[n_k(t)n_\ell(t')] = R_{k\ell}(t-t')$$  (4)

Under these conditions it can be shown that the noise reduction due to summing can be expressed as

$$R = -10 \left[ \log N - \log(1 + (N-1)\bar{\rho}) \right]$$  (5)

where $\bar{\rho} = \bar{M}/\bar{N}^2$ is the average normalized zero-lag cross-correlation. Hence

$$\bar{M} = \frac{\sum_{k \neq \ell} R_{k\ell}(0)}{N(N-1)} , \quad \bar{N}^2 = \frac{\sum_k R_{kk}(0)}{N} , \quad \bar{\rho} = \bar{N}^2 \frac{R_{kk}(0)}{(N-1) \left[ \sum R_{kk}(0) \right]^2}$$  (6)
This equation is the direct time domain equivalent of that used by Capcn et al. (1967) in the frequency domain, and can be interpreted as the reduction over the entire band of interest. In this report, the band of interest is either (0.4-3.0 cps) or (0.6-2.0 cps), since the data is prefiltered to either one of these two bands. Now, examining equation (2) shows that if $\bar{\bar{p}} = 0$ the reduction is $-10 \log N$ which is the $(N)^{1/2}$ value expected with uncorrelated noise. However, if $\bar{\bar{p}}$ is negative, one may expect on certain occasions to have noise reductions exceeding $(N)^{1/2}$.

In the computational procedure for an array of $N$ elements, we shall present the sample estimates for the reduction which are calculated from the estimated zero-lag auto-correlation and cross-correlation functions

$$\hat{R}_{k\ell}(0) = \frac{1}{T} \int_{0}^{T} n_k(t)n_\ell(t) \, dt, \quad k, \ell = 1, \ldots, N$$

(7)

If the estimated reduction is $\hat{R}$ and the estimated value of the parameter $\bar{\bar{p}}$ is $\hat{p}$, the sample reduction is written as

$$\hat{R} = -10[\log N - \log \{1+(N-1)\hat{p}\}]$$

(8)

It is observed experimentally that the cross-correlations between sensors tend to decrease proportionally to the spacing and that sets of seismometers at the same spacing tend to produce uniform sample reductions in noise. This suggests that $\bar{\bar{p}}$ is approximately constant for a given spacing. We assume that the normalized zero-lag cross-correlation is constant for each pair in a given array, i.e.,
\[ \rho = \frac{R_{k\ell}(0)}{R_{k\ell}(0)} \quad k, \ell = 1, \ldots, N; \]  \hspace{1cm} (9)

then, using an argument similar to that used in deriving Fisher's asymptotic z approximation (Anderson, 1958, pp. 74-5) we may show that the distribution of the sample reduction approaches a normal distribution with mean

\[ \mu R = -10[\log N - \log(1+N-1)\rho] \]  \hspace{1cm} (10)

and standard error

\[ \sigma_R = \frac{20(1-\rho)}{\sqrt{2NBT}} \]  \hspace{1cm} (11)

where B is the bandwidth over which the zero-lag correlations are computed and T is the sample length in seconds. Figure 5 shows the expected reduction for each value of the common theoretical noise correlation for N=3. The two reduction points of interest on the curve are for the values of \( \rho \) (as indicated experimentally) corresponding to 3-km and greater than or equal to 6-kilometer spacings. The vertical deviations are 95% confidence limits for the SDL filter with the parameters specified in the following table.

<table>
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<th>Lincoln Lab Filter (.6-2.0)</th>
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<td>50 Sec</td>
<td>2.6 ops</td>
<td>1.4 cps</td>
</tr>
<tr>
<td>780</td>
<td>.715(1-\rho)</td>
<td>.985(1-\rho)</td>
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The variability of the correlation of both signal and noise between pairs of sensors is demonstrated by the variation of coherency as a function of sensor separation (Figure 6). Each curve in this figure represents an energy for a narrow bandpass peaked sharply at the frequency indicated for the given curve. Unfortunately, there are erratic changes in slope in the curves due to the fact that sensors were available only at certain fixed locations. Although this data was taken from the extended array at Tonto Forest in Arizona, the principles are thought to be general.

The most striking feature of this relationship is that the coherency of signals is greater at all distances than that of the noise; this fact of course is simply verification of what we would intuitively expect to be true. A second important feature is that noise coherency falls faster than signal coherency over the range up to 30 km.

A feature that is rather surprising is the manner in which both sets of curves level off, each for its own reason. The curves for noise are asymptotic to the coherency that one gets from completely random noise. The curves for signals on the other hand are asymptotic to the coherency that is determined by a complicated mixture of a completely coherent signal component, a partially coherent mixture of several different arrivals slightly out of phase, and signal generated noise rendered only partially coherent by varying local geology.
RESULTS

We first present results pertinent to the effectiveness of beamsteering outputs from the extended E3 subarray (Figures 7, 8, 9, and 10), and then extend the discussion to consider the effect of inter-sensor spacing on short-period beamforming results (Figures 11 and 12).

Figure 7 is a plot of noise reduction, either rms or power at 1 cps, as a function of \( N \). The figure illustrates four significant points: first, \( N^{\frac{1}{2}} \) reduction is obtained for noise power at 1 cps only in the case of \( N=6 \) (the outer ring); second, the reduction of rms noise levels never quite reached \( N^{\frac{1}{2}} \); third, noise reduction is less favorable, relative to \( N^{\frac{1}{2}} \), for greater \( N \); and fourth, beams made of outputs from the outer ring(s) yield more noise reduction than those consisting of traces recorded in the inner ring(s). The last result is explained by the fact that inter-sensor spacing tends to be greater on the outside rings, and the noise is therefore less correlated between adjacent sensors.

Figure 8 shows average S/N gain as a function of \( N \). Here we see immediately that \( N^{\frac{1}{2}} \) enhancement is never achieved, due largely to the fact the rms noise reduction falls short of \( N^{\frac{1}{2}} \) as shown by Figure 3, and partly because 1-2 db of signal is lost in the beamforming process. We further note that enhancement is less favorable relative to \( N^{\frac{1}{2}} \) for larger \( N \), and that the outer ring(s) yield better results than the inner ring(s).

Figures 9 and 10 show noise reduction and S/N enhancement versus sensor spacing for \( N=6 \). In this case, beams were formed using outputs from individual rings so that
values plotted at \( \Delta = 3 \) km correspond to data recorded on the inside ring, \( \Delta = 6 \) the second ring, \( \Delta = 8 \) the third ring, and \( \Delta = 9.5 \) km the outside ring; these spacings could more appropriately be called "minimum" intervals. As shown in Figure 9, noise power at 1 cps is reduced by \( N^{\frac{3}{2}} \) in the \( \Delta \) interval 6-8 kilometers, and rms noise is reduced to within 1 db of \( N^{\frac{3}{2}} \) at \( \Delta = 6 \) and remains reasonably constant thereafter. On the other hand, S/N enhancement (Figure 10) reaches a maximum of +5 db at \( \Delta = 6 \) and remains essentially constant beyond. Once again we are reminded that imprecision in the beamforming process accounts for 1-2 db signal loss.

We turn now to examples of beamforming in which \( N \) has been held constant and spacing between adjacent sensors has been changed from a minimum of 3 km to a maximum of 16 km (Figures 11 and 12). Data plotted on Figure 11 were prefiltered to 0.4-3.0 cps, while those shown in Figure 12 were bandlimited in the range 0.6-2.0 cps. In both figures the dashed curves represent results for noise reduction based on the average normalized cross correlation (equation 5), whereas the plotted points are based on the average rms value input to the beam. Referring to Figure 11, we note that the minimum sensor spacing indicated by either experimental method for \( N=3 \) or \( N=7 \) is about 6 km, if \( N^{\frac{3}{2}} \) noise reduction is desired. Actually, values based on average rms reach \( N^{\frac{3}{2}} \) reduction at 8 or 9 km. It is important to remember that the plotted data for \( N=3 \) are really averages of either two or six beams, whereas, each plot for \( N=7 \) was taken from a single beam. As shown in Figure 11, the minimum spacing indicated for data prefiltered 0.6-2 cps is about 5 km, and rms values reach

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at about 8 km spacing.

The results are comparable when the same processing and comparisons are extended to all of LASA. When traces from 8 teleseismic events are beam formed (Figure 13), the noise is reduced by a factor nearly $N^{1/2}$ when the number of sensors is such that the minimum spacing is 6 km or more. For configurations with smaller and smaller sensor spacings, the noise reduction decreases correspondingly as compared to $N^{1/2}$. Because the signal is not perfectly coherent over the area of the array, there is also significant signal loss in the beamforming process.

Another informative way to look at the results is to examine the equivalent ground motion (Figure 14). It is seen that the actual noise is reduced until the number of sensors in the array has been increased to 119. Beyond that number, there is very little improvement if any in the noise reduction; and the use of all 525 sensors even causes the noise to deteriorate. The net result is that the gain in signal to noise ratio is significantly less than a factor of $N^{1/2}$ for all sensor combinations (Figure 15). On the basis of this demonstration, it is concluded that 119 sensors is the most efficient if cost is not a factor; if cost is considered, it is probably unjustified to increase the sensors beyond 51 which corresponds to the same minimum spacing of 6 km mentioned above.

The degree to which noise suppression approaches a factor of $N^{1/2}$ for given values of the separation appears to depend on the band-width chosen. In the results described above, the band-widths were either 0.4-3.0 cps or 0.6-2.0 cps. When the band-width was restricted even
further to 0.7-2.0 cps, suppression by a factor of $N^{1/2}$ was obtained for separations down to 3 km (Figure 16). Although this result is based only on data from one event, work on other events tends to substantiate the conclusions. However, further work will be required to learn precisely the relationship between bandwidth and noise suppression at various sensor separations.

The data also shows that a given sensor separation yields practically a maximum noise suppression when the number of sensors is far less than the total number available in the whole of LASA.

The data also implies that, for a given sensor separation, the improvement in signal-to-noise ratio increases asymptotically to some practical maximum as the number of sensors is increased (Figure 17).
CONCLUSIONS

In an SDL study LASA prefiltered short-period recordings of teleseismic events were beamsteered on the P arrival using a variable number of beam inputs. Our objective was to determine the efficiency of the beams with respect to the number of inputs and the spacing of sensors contributing to the beams. The following conclusions are based on that analysis:

1. The net effect of increasing the number of beam inputs while simultaneously decreasing sensor spacing is to produce progressively less rms noise reduction and S/N gain, relative to $N^k$.

2. Average signal loss amounts to 4 db. We attribute part of the loss (1 db?) to misalignment of P waves within subarrays. The remaining signal loss is due either to inaccurate array alignment or to small differences in wave form across the LASA.

3. Beams composed of 51 traces reduce rms noise and improve the signal-to-noise ratio within 1 db of that produced by 525-element beams. The 51 elements were selected to have minimum sensor spacing greater than 6 km.

The following conclusion is based on an event prefiltered 0.7–2.0 cps and beamformed not only to determine beam efficiency in terms of number and spacing of elements contributing to the beams, but also to learn the effect of deriving N inputs from progressively smaller areas.
4. The data implies that, for a given sensor spacing, the improvement in signal-to-noise ratio due to beam-forming increases asymptotically to some practical maximum as the number of sensors is increased. For the case of 3 km spacing the data imply that a maximum of 16 db improvement in signal-to-noise ratio can be obtained by inputing 80 traces derived from subarrays A0 through D4.
REFERENCES


Figure 1. Original LASA Configuration for N=525. Each circle represents one subarray of 25 sensors.
Figure 2. LASA subarray configuration of 25 seismometers before sensors marked "X" were removed.
Figure 3. LASA Extended E3 Subarray of 25 Seismometers.
Figure 4. LASA Subarray Configuration for $N=51$. 

Figure 4. LASA Subarray Configuration for $N=119$. 
Figure 5. Noise Reduction for N=3.
Figure 6. Average Signal and Noise Correlation vs. Distance and Frequency at TFSO Extended Array for 10 Teleseismic Events.
Figure 7. Average Noise Reduction by Beamforming Outputs of the Extended E3 Subarray.

Data are SPZ recordings of 9 earthquakes & associated noise prefiltered 0.4-3.0 cps.

- rms noise
- noise power at 1 cps
- inside ring
- inside 2 rings plus center
- inside 3 rings plus center
- all
- outside ring
- outside 2 rings
- outside 3 rings
Data are prefiltered (0.4-3.0 cps) SPZ recordings of P-waves from 9 earthquakes and associated noise.

1. inside ring
2. inside 2 rings and center
3. inside 3 rings and center
4. all
5. outside ring
6. outside 2 rings
7. outside 3 rings

Figure 8. Average S/N Gain by Beamforming Outputs from the Extended E3 Subarray.
Data are SPZ recordings of 7 earthquakes & associated noise prefiltered 0.4-3.0 cps.

N=6
- rms noise
- noise power at 1 cps
1 inside ring
2 second ring
3 third ring
4 outside ring

Figure 9. Average Noise Reduction by Beamforming Six Outputs of the Extended E3 Subarray.
Data are SPZ recordings of 7 earthquakes & associated noise prefiltered 0.4-3.0 cps.

* N = number of outputs summed = 6
  1 inside ring
  2 second ring
  3 third ring
  4 outside ring

Figure 10. Average S/N Gain by Beamforming Six Outputs of the Extended E3 Subarray
Figure 11. Average Noise Reduction in the Extended E3 Subarray for Two Experimental Methods

Data are SPZ noise recorded during night of 17 March 67 & prefiltered 0.6-2.0 cps.

RMS noise for N=3

RMS noise for N=7

Sensor Spacing (km)

RMS Noise Reduction (dB)
Figure 12. Average Noise Reduction in the Extended E3 Subarray for Two Experimental Methods.
Figure 13. Average Signal Loss and RMS Noise Reduction Produced by Beamforming LASA Traces.
AVERAGE S/N GAIN PRODUCED BY BEAMFORMING LASA TRACES.
DATA ARE PREFILTERED (0.4 - 3.0 cps)
SPZ RECORDINGS OF P-WAVES FROM 8 TELESEISMIC EVENTS.

Figure 15. Average S/N Gain Produced by Beamforming LASA Traces.
Minimum inter-sensor spacing and aperture are parameters. Data prefiltered 0.7-2 cps.

Figure 16. Noise Reduction and Signal Loss as Functions of Number of Inputs to Beams for Ryukyu Event.
Ideally, geophones would be placed in a noiseless environment, in which case there would be no reason to resort to arrays of geophones. If the noise is such that an array is required, the objective of the array is to enhance the signal-to-noise ratio and thus to maximize the intelligence that can be derived from a given signal. The design of the array will be a function of the signal characteristics of the direction and velocity of the noise in the bandpass of the signal, and of the site geology.

It has been demonstrated that in a practical sense the optimum array processing is represented by beam forming by which we mean simple delay and summing. Increasing the number N of sensors within a given area decreases the inter-element spacing and may increase the coherence between noise samples at adjacent sensors, thus yielding poorer results compared to N improvement one expects to get. Increasing the number of sensors by proportionately increasing the area is liable to result in signal deterioration also yielding an unfavorable comparison to N improvement in signal-to-noise. These two effects, together with economical factors, combine to limit the number of sensors that can be used.

Although the data on which our conclusions are reached were drawn from earthquake seismology, the principles involved are equally applicable to exploration seismology and to other geophysical measurements in which arrays of sensors are required.

**Key Words:**
- Beam efficiency
- Noise reduction