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THEORETICAL INVESTIGATION OF ACOUSTIC SURFACE WAVES
ON PIEZOELECTRIC CRYSTALS

by

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P.O. Box 3310
Fullerton, California 92634

Contract No. F19628-69-C-0132
Project No. 5635
Task No. 563503
Work Unit No. 56350301

FINAL REPORT
Period Covered: 1 December 1968 through 30 November 1969
4 December 1969

Contract Monitor: Andrew J. Slobodnik, Jr., 1/1t, USAF
Microwave Physics Laboratory

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Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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ABSTRACT

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.
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I. INTRODUCTION

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.

The physical configurations of the propagation problems considered in the sequel are shown in Figure 1 and are enumerated below:

(1) Surface wave propagation on a piezoelectric half space in the presence of an infinitesimal electric or "magnetic" conductor located at an arbitrary but fixed distance h above the crystal surface.

(2) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to a perfect isotropic elastic conductor (e.g. gold or aluminum) of arbitrary thickness h.

(3) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to a perfect fluid half space.

(4) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to an isotropic elastic layer of arbitrary thickness h.

The following section contains the details of the analyses of the propagation problems described above including special degenerate cases which are encountered. These cases correspond to conditions of surface wave propagation wherein one or more components of displacement vanish or the electric and mechanical fields become decoupled (in the general piezoelectric case the surface wave contains all components of displacement and is coupled via the piezoelectric constants to the electric field). In practice degenerate cases have been found to occur when the sagittal plane lies either in a plane of symmetry of the crystal under consideration, in the basal plane of crystals of class 6 mm, or in the principal plane(s) of cubic crystals.
2

infinitesimally thin perfect electric or "magnetic" conductor

(h vacuum

piezoelectric crystal half-space

perfectly conducting isotropic elastic medium

piezoelectric or pure elastic crystal half-space

ideal fluid half-space

piezoelectric or pure elastic crystal half-space

vacuum

isotropic elastic medium with isotropic dielectric property

piezoelectric or pure elastic crystal half-space

Figure 1
The third section presents an outline of the analysis pertaining to the excitation of surface and bulk wave by an electric current line source located above a piezoelectric half space. This problem was originally undertaken as an approach to the study of interdigital electric transducers but was ultimately abandoned due to lack of funds and the fact that inordinate amounts of computation would be required to extract useful data regarding the efficiency of excitation of surface waves.

Finally, Section II provides detailed descriptions of the computer programs written to implement the numerical analyses of the surface wave propagation problems described in Section II. The material presented in this section provides the reader with sufficient information for the use of the computer programs and the comprehension of the programming methods employed therein. Source deck listings for the various programs are also provided.
II. PROPAGATION OF SURFACE WAVES ON PIEZOELECTRIC SUBSTRATES

1. Surface Waves on Piezoelectric Crystals in the Presence of Infinitesimally Thin Electric and "Magnetic" Conductors

In this section the formal analysis is presented for the propagation characteristics of surface waves on a general piezoelectric crystal surface in the presence of perfect electric and "magnetic" conductors. The geometries under consideration are depicted in Figure 2.

A rectangular coordinate system is chosen with the $x_3$ axis normal to the crystal surface and the $x_1$ axis in the direction of propagation. Arbitrary orientations of the crystal surface with respect to the crystal axes are considered. This is accomplished by means of a coordinate rotation through the Euler angles from the crystal axes to the desired $x'_1, x'_2, x'_3$ coordinate system. The matrix defining such a rotation is given by

$$V = \begin{pmatrix}
\cos \alpha \cos \gamma - \sin \alpha \cos \beta \sin \gamma & \sin \alpha \cos \gamma + \cos \alpha \cos \beta \sin \gamma & \sin \beta \sin \gamma \\
\cos \alpha \sin \gamma - \sin \alpha \cos \beta \cos \gamma & \sin \alpha \sin \gamma + \cos \alpha \cos \beta \cos \gamma & \sin \beta \cos \gamma \\
\sin \alpha \sin \beta & -\cos \alpha \sin \beta & \cos \beta
\end{pmatrix}$$

where $\alpha$, $\beta$, and $\gamma$ are the Euler angles. Since the $x'_1, x'_2, x'_3$ coordinate system is relative to the crystal surface and direction of propagation, the form of the differential equations for the mechanical displacements and electric potentials in this coordinate system is independent of the surface under consideration.

Only the values of the coefficients change with the surface orientation relative to the crystal axes. This is also true of the boundary conditions. Different cuts are thus distinguished only through the transformed tensor quantities involved in coefficients of the differential operators.

In terms of the Euler transformation matrix $V$, the tensor quantities of interest; viz., the elastic constants $(C'_{ijkl})$, the piezoelectric constants $(e'_{ijk})$, and the dielectric constants $(\varepsilon'_{ij})$ are transformed as follows:

$$C'_{ijkt} = \sum_{r,s,t,u=1}^{3} V_{ir} V_{js} V_{kt} V_{tu} C_{rstu}$$

$$e'_{ijk} = \sum_{r,s,t=1}^{3} V_{ir} V_{js} V_{kt} e_{rst}$$
Figure 2

(a) Perfect electric conductor

(b) Perfect magnetic conductor
\[ \varepsilon'_{ij} = \sum_{r,s=1}^{3} V_{ir} V_{js} \varepsilon_{rs} \]  

where the primed quantities refer to a rotated coordinate system and the unprimed correspond to the coincidence of the \(x_1x_2x_3\) coordinate system and the crystal axes, i.e., when \(V\) is the identity matrix. The elastic and piezoelectric constants can be reduced to 2 index symbols in the usual fashion, viz. \(C_{ijk\ell} \rightarrow C_{pq}\) and \(e'_{ij\ell} \rightarrow e_{ip}\), where \(p\) or \(q = 1, 2, 3, 4, 5, 6\) are equivalent to \(11, 22, 33, 23, 32, 13\) or \(12, 21\) respectively.

The differential equations for the components \(U_i\), \(i = 1, 2, 3\), of the mechanical displacement and electric potential \(\varphi\) are given by

\[
\begin{align*}
C_{ijk\ell} U_{k, \ell i} + e'_{kij, \ell i} \varphi_{ki} &= \rho \dddot{U}_j \\
e'_{ik\ell} U_{k, \ell i} - e'_{ik, \ell i} \varphi_{ki} &= 0
\end{align*}
\]

\[ x > 0 \]  

\[ \varphi = 0 \quad \text{for} \quad -h < x_3 < \rho \]  

In the above equations, indices preceded by a comma denote differentiation with respect to space coordinates. The summation convention for repeated indices is employed as is the dot notation for differentiation with respect to time.

As indicated above, the surface waves under consideration are assumed to be traveling in the \(x_1\) direction along a surface whose normal is in the \(x_3\) direction. The displacements and potentials are considered to be independent of the \(x_2\) coordinate. Consequently, traveling wave solutions of the form

\[ U_i = \beta_i e^{-\alpha x_3/V_s} e^{i\omega(t-x_1/V_s)} \]

and

\[ \varphi = \beta_4 e^{-\alpha x_3/V_s} e^{i\omega(t-x_1/V_s)} \]

are sought. When these surface waveforms (as identified by an exponential decay into the crystal) are substituted into the differential equations for \(x_3 > 0\) a linear homogeneous system of four equations in the unknowns \(\beta_1, \beta_2, \beta_3, \beta_4\)
results. The determinant of the coefficients of the unknowns in these equations must be zero in order that a non-trivial solution exist, that is

\[
\begin{vmatrix}
C_{55} \alpha^2 + 2C_{15} \alpha & C_{45} \alpha^2 + [C_{14} + C_{56}] \alpha \\
- C_{11} + \rho \nu s^2 & - C_{16} \\
- C_{16} & - C_{66} + \rho \nu s^2 \\
C_{33} \alpha^2 + [C_{13} + C_{55}] \alpha & C_{34} \alpha^2 + [C_{36} + C_{45}] \alpha \\
- C_{15} & - C_{56} \\
e_{35} \alpha^2 + [e_{15} + e_{31}] \alpha & e_{34} \alpha^2 + [e_{14} + e_{36}] \alpha \\
e_{11} & - e_{16}
\end{vmatrix} = 0
\]
Evaluation of the above determinant results in an eighth order polynomial in \( a \) of the form

\[
A_8 \alpha^8 + jA_7 \alpha^7 + A_6 \alpha^6 + jA_5 \alpha^5 + A_4 \alpha^4 + jA_3 \alpha^3 + A_2 \alpha^2 + jA_1 \alpha + A_0 = 0 ,
\]

with the coefficients \( A_n, \ n = 0, 1, \ldots, 8 \), purely real. Since the fields must be bounded, or go to zero as \( x_3 \to -\infty \), only the roots with non-negative real parts are allowed. If the unknown in equation (7) is considered to be \( ja \) instead of \( a \) then the polynomial in \( ja \) has purely real coefficients. Thus, either the roots \( ja \) are real or occur in conjugate pairs, e.g.

\[
ja_1 = a + jb
\]

\[
ja_2 = a - jb
\]

whence

\[
\alpha_1 = b - ja
\]

\[
\alpha_2 = -b - ja
\]

Therefore, the roots \( a \) are either pure imaginary or occur in pairs with positive and negative real parts.

In the range of velocities where generally surface waves can exist (i.e. velocities below the lowest bulk wave velocity in the direction of propagation under consideration) the roots occur such that four with positive real parts can be selected. However, if for a given velocity four such roots are not found the possibility of the existence of a degenerate surface wave remains and must be considered. These special waves are discussed in detail in the section on degenerate cases. Upon obtaining the admissible values of \( a \), corresponding values of \( \beta_1 \) (to within a constant factor) can be found for each \( a \).

In addition to the equations for \( x_3 > 0 \), the differential equation (5) for \(-h \leq x_3 \leq 0 \) must be satisfied together with appropriate boundary conditions at \( x_3 = 0 \) and \( x_3 = -h \). Assuming that the crystal surface is stress free \((T_{3j} = 0 \text{ at } x_3 = 0)\), the mechanical boundary conditions at each point of the surface of the crystal are

*The term degenerate is used to signify that certain components of displacement and/or the electric potential vanish identically.
For the electric wall case (Figure 2a) the boundary conditions on the electric potential are the continuity of \( \varphi \) at \( x_3 = 0 \) and, without loss of generality, \( \varphi(-h) = 0 \). Also the normal component of electrical displacement must be continuous across the surface of the crystal.

The total fields (mechanical displacement and potential) may be expressed as a linear combination of the fields associated with the admissible values of \( \alpha \) for \( x_3 > 0 \), namely,

\[
U_j = \sum_{i=1}^{4} B^{(i)} \beta^{(i)} \left( \alpha^{(i)} \right) e^{-\alpha^{(i)} x_3 / v_s} e^{j \omega (t-x_3 / v_s)}, \quad i = 1, 2, 3
\]

\[
\varphi = \sum_{i=1}^{4} B^{(i)} \beta^{(i)} \left( \alpha^{(i)} \right) e^{-\alpha^{(i)} x_3 / v_s} e^{j \omega (t-x_3 / v_s)},
\]

In the region \(-h < x_3 < 0\) the potential is a solution of Laplace's equation (5). A solution satisfying the continuity condition at \( x_3 = 0 \) and vanishing at \( x_3 = -h \) is

\[
\varphi = \sum_{i=1}^{4} B^{(i)} \beta^{(i)} \left( \alpha^{(i)} \right) \text{csch} \left( \frac{|h|}{v_s} \right) \sin h \left( \frac{w}{v_s} (x_3 + h) \right) e^{j \omega (t-x_3 / v_s)},
\]

Finally, the normal component of \( \vec{D} \) (viz. \( D_3 \)) must be continuous across the surface \( x_3 = 0 \). Inside the crystal the electrical displacement is given by

\[
D_i = \varepsilon_{ikl} U_{k_i}, \quad i = 1, 2, 3
\]

while in the region \(-n < x_3 < 0\), \( \vec{D} = -\varepsilon_0 \varphi \).

Substituting the waveforms (9), (10) in equation (8) and expressing the continuity of \( D_3 \) at \( x_3 = 0 \) in terms of equations (9), (10), and (11) yields the following set of homogeneous equations for the amplitudes \( B^{(i)} \), namely,
The transcendental equation obtained by setting the determinant of the matrix \( \hat{L} \) of coefficients of this system equal to zero determines the surface wave velocities.

In the limiting case \( h \to 0 \) the region \(-h < x_3 < 0\) disappears and the boundary conditions on the electric potential and normal component of displacement in the crystal are replaced by \( \phi(0) = 0 \). In this case equation (15) above reduces to

\[
\sum_{i=1}^{4} \left[ \beta_{4}^{(i)} \left[ j e_{15} + \alpha^{(t)} e_{35} \right] \right] B^{(i)}(t) = 0.
\]  

In addition to the electric wall problem, the magnetic wall case (Figure 2b) also has been considered. The only change in the formulation of this problem is the boundary condition at \( x_3 = -h \). In this case the solution for the potential in the region \(-h < x_3 < 0\) assumes the form

\[
\sum_{i=1}^{4} \left[ \beta_{4}^{(i)} \left[ j e_{15} + \alpha^{(t)} e_{35} \right] \right] B^{(i)}(t) = 0.
\]
The function satisfies the condition that the normal component of electrical displacement \( D_3 \) vanish at the magnetic wall \( x_3 = -h \).

Equation (15), the continuity of \( D_3 \) at \( x_3 = 0 \), is modified for the magnetic wall case by replacing \( \coth \left( \frac{uh}{v_s} \right) \) with \( \tanh \left( \frac{uh}{v_s} \right) \). Otherwise the equations (12)-(15) remain unchanged. The limiting case \( h \to 0 \) requires no special change as it did in the electric wall case since the continuity condition on \( D_3 \) at \( x_3 = 0 \) now simply becomes \( D_3 = 0 \) and \( D_3 \big|_{x_3=0} \to \tanh \left( \frac{uh}{v_s} \right) \) (outside the crystal), which goes to zero as \( h \to 0 \). Thus equation (15) with the \( \tanh \left( \frac{uh}{v_s} \right) \) term needs no modification for the limiting case.

Once a surface wave velocity has been found the partial field amplitudes \( b_{ij}^{(l)} \), \( j=1, 2, 3, 4 \) may be calculated to within a constant factor. Consequently, \( b_{4}^{(4)} \) is chosen as unity (except for certain degenerate cases described later) and \( b_{1}^{(1)} \), \( b_{2}^{(2)} \), \( b_{3}^{(3)} \) are found from equations (12)-(14). These amplitudes are used to evaluate the displacement components (eq. (9)), electric potential (eq. (10)), the components of stress, strain, electric displacement, electric field, and the time average power flow as functions of \( \omega x_3 \). The explicit forms of the components of the aforementioned physical quantities are:

**Stress**

\[
\begin{align*}
\frac{T_{11}}{w} & = \sum_{l=1}^{4} b_{l}^{(l)} e^{-\alpha_{wx_{3}}/v_s} e^{i\omega(t-x_{1}/v_s)} \left\{ b_{1}^{(l)} \left[ \frac{-1}{v_s} C_{11} - \frac{\alpha}{v_s} \frac{C}{C_{15}} \right] \\
& \quad + b_{2}^{(l)} \left[ \frac{-1}{v_s} C_{16} - \frac{\alpha}{v_s} \frac{C}{C_{14}} \right] + b_{3}^{(l)} \left[ \frac{-1}{v_s} C_{15} - \frac{\alpha}{v_s} \frac{C}{C_{13}} \right] + b_{4}^{(l)} \left[ \frac{-1}{v_s} e_{11} - \frac{\alpha}{v_s} e_{31} \right] \right\} \\
\frac{T_{12}}{w} & = \frac{T_{21}}{w} = \sum_{l=1}^{4} b_{l}^{(l)} e^{-\alpha_{wx_{3}}/v_s} e^{i\omega(t-x_{1}/v_s)} \left\{ b_{1}^{(l)} \left[ \frac{-1}{v_s} C_{16} - \frac{\alpha}{v_s} \frac{C}{C_{56}} \right] \\
& \quad + b_{2}^{(l)} \left[ \frac{-1}{v_s} C_{66} - \frac{\alpha}{v_s} \frac{C}{C_{46}} \right] + b_{3}^{(l)} \left[ \frac{-1}{v_s} C_{56} - \frac{\alpha}{v_s} \frac{C}{C_{36}} \right] + b_{4}^{(l)} \left[ \frac{-1}{v_s} e_{16} - \frac{\alpha}{v_s} e_{36} \right] \right\}
\end{align*}
\]
\[
\frac{T_{13}}{\omega} = \frac{T_{31}}{\omega} = \sum_{l=1}^{4} B(l) e^{-\alpha(t)u_{x3}/v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1(t) \left[ \frac{1}{v_s} C_{15} - \frac{\alpha(t)}{v_s} C_{55} \right] + \beta_2(t) \left[ \frac{1}{v_s} C_{56} - \frac{\alpha(t)}{v_s} C_{45} \right] + \beta_3(t) \left[ \frac{1}{v_s} C_{35} - \frac{\alpha(t)}{v_s} C_{33} \right] + \beta_4(t) \left[ \frac{1}{v_s} e_{15} - \frac{\alpha(t)}{v_s} e_{35} \right] \right\}
\]

\[
\frac{T_{22}}{\omega} = \sum_{l=1}^{4} B(l) e^{-\alpha(t)u_{x3}/v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1(t) \left[ \frac{1}{v_s} C_{12} - \frac{\alpha(t)}{v_s} C_{25} \right] + \beta_2(t) \left[ \frac{1}{v_s} C_{26} - \frac{\alpha(t)}{v_s} C_{24} \right] + \beta_3(t) \left[ \frac{1}{v_s} C_{23} - \frac{\alpha(t)}{v_s} C_{32} \right] + \beta_4(t) \left[ \frac{1}{v_s} e_{12} - \frac{\alpha(t)}{v_s} e_{32} \right] \right\}
\]

\[
\frac{T_{23}}{\omega} = \frac{T_{32}}{\omega} = \sum_{l=1}^{4} B(l) e^{-\alpha(t)u_{x3}/v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1(t) \left[ \frac{1}{v_s} C_{14} - \frac{\alpha(t)}{v_s} C_{45} \right] + \beta_2(t) \left[ \frac{1}{v_s} C_{46} - \frac{\alpha(t)}{v_s} C_{44} \right] + \beta_3(t) \left[ \frac{1}{v_s} C_{45} - \frac{\alpha(t)}{v_s} C_{34} \right] + \beta_4(t) \left[ \frac{1}{v_s} e_{14} - \frac{\alpha(t)}{v_s} e_{34} \right] \right\}
\]

\[
\frac{T_{33}}{\omega} = \sum_{l=1}^{4} B(l) e^{-\alpha(t)u_{x3}/v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1(t) \left[ \frac{1}{v_s} C_{13} - \frac{\alpha(t)}{v_s} C_{35} \right] + \beta_2(t) \left[ \frac{1}{v_s} C_{36} - \frac{\alpha(t)}{v_s} C_{34} \right] + \beta_3(t) \left[ \frac{1}{v_s} C_{35} - \frac{\alpha(t)}{v_s} C_{33} \right] + \beta_4(t) \left[ \frac{1}{v_s} e_{13} - \frac{\alpha(t)}{v_s} e_{33} \right] \right\}
\]

**Strain**

\[
\frac{S_{11}}{\omega} = \sum_{l=1}^{4} B(l) \beta_1(t) e^{-\alpha(t)u_{x3}/v_s} e^{j\omega(t-x_1/v_s)}
\]

\[
\frac{S_{22}}{\omega} = 0
\]

\[
\frac{S_{33}}{\omega} = \sum_{l=1}^{4} \frac{\alpha(t)}{v_s} B(l) \beta_3(t) e^{-\alpha(t)u_{x3}/v_s} e^{j\omega(t-x_1/v_s)}
\]
\[ S_{12} = \frac{S_{21}}{w} = \frac{1}{2} \sum_{\ell=1}^{4} \frac{-i}{v_s} B(t) \beta_2(\ell) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \]

\[ S_{13} = \frac{S_{31}}{w} = \frac{1}{2} \sum_{\ell=1}^{4} B(t) \left[ -\frac{\alpha(t)}{v_s} \beta_1(\ell) - \frac{i}{v_s} \beta_3(\ell) \right] e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \]

\[ S_{23} = \frac{S_{32}}{w} = \frac{1}{2} \sum_{\ell=1}^{4} \frac{-\alpha(t)}{v_s} B(t) \beta_2(\ell) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \]

**Electric Field**

\[ E_1 = \frac{1}{w} \frac{4}{v_s} \sum_{\ell=1}^{4} B(t) \beta_4(\ell) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} = \frac{i}{v_s} \phi \]

\[ E_2 = \frac{1}{w} \frac{4}{v_s} \sum_{\ell=1}^{4} \frac{\alpha(t)}{v_s} B(t) \beta_4(\ell) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \]

**Electric Displacement**

\[ D_1 = \sum_{\ell=1}^{4} B(t) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \left\{ \beta_1(t) \left[ \frac{-i}{v_s} e_11 \cdot \frac{\alpha(t)}{v_s} e_15 \right] + \frac{\alpha(t)}{v_s} e_16 \right\} \]

\[ + \beta_2 \left[ \frac{-i}{v_s} e_14 \cdot \frac{\alpha(t)}{v_s} e_15 \right] + \beta_3 \left[ \frac{-i}{v_s} e_11 \cdot \frac{\alpha(t)}{v_s} e_14 \right] + \beta_4 \left[ \frac{-i}{v_s} e_11 \cdot \frac{\alpha(t)}{v_s} e_13 \right] \]

\[ D_2 = \sum_{\ell=1}^{4} B(t) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \left\{ \beta_1(t) \left[ \frac{-i}{v_s} e_21 \cdot \frac{\alpha(t)}{v_s} e_25 \right] + \frac{\alpha(t)}{v_s} e_26 \right\} \]

\[ + \beta_2 \left[ \frac{-i}{v_s} e_24 \cdot \frac{\alpha(t)}{v_s} e_25 \right] + \beta_3 \left[ \frac{-i}{v_s} e_25 \cdot \frac{\alpha(t)}{v_s} e_23 \right] + \beta_4 \left[ \frac{-i}{v_s} e_21 \cdot \frac{\alpha(t)}{v_s} e_23 \right] \]

\[ D_3 = \sum_{\ell=1}^{4} B(t) e^{-\alpha(t)ux_3/v_s} e^{jut-x_1/v_s} \left\{ \beta_1(t) \left[ \frac{-i}{v_s} e_31 \cdot \frac{\alpha(t)}{v_s} e_35 \right] + \frac{\alpha(t)}{v_s} e_36 \right\} \]

\[ + \beta_2 \left[ \frac{-i}{v_s} e_34 \cdot \frac{\alpha(t)}{v_s} e_35 \right] + \beta_3 \left[ \frac{-i}{v_s} e_35 \cdot \frac{\alpha(t)}{v_s} e_33 \right] + \beta_4 \left[ \frac{-i}{v_s} e_13 \cdot \frac{\alpha(t)}{v_s} e_33 \right] \]
Power Flow

The flow of complex mechanical power at any point in the piezoelectric medium is given in component form as follows:

\[ P_i = \frac{1}{2} \sum_{l=1}^{3} T_{i l} \cdot \overrightarrow{U}_l \]

The real part of this expression represents the time average power flow at a point.

Since all fields decay exponentially in the \( x_3 \) direction there is no net flow of real power in this direction. Thus, only \( P_1 \) and \( P_2 \) need be considered.

The components of the total time-average power flow are as follows:

\[ P_1^t = \int_0^\infty \text{Re} [P_1] \, dx_3 = \text{Re} [P_1^t] \]

\[ P_2^t = \int_0^\infty \text{Re} [P_2] \, dx_3 = \text{Re} [P_2^t] \]

where \( \text{Re} [P_1, 2] \) means the real part of \( P_1, 2 \). The final expressions for the complex mechanical power flow \( P_1^t \) and \( P_2^t \) are

\[ P_1^t = \frac{1}{2} \sum_{k=1}^{4} \sum_{l=1}^{4} \left\{ \frac{1}{\alpha(t) + \alpha(k)} \right\} \mathbf{B}(t) \mathbf{B}(k) \left( \beta_1^{(t)} \alpha^{(t)} \mathbf{C}_{11} \right) + \beta_2^{(t)} \alpha^{(t)} \mathbf{C}_{14} \right) + \beta_3^{(t)} \alpha^{(t)} \mathbf{C}_{12} \right) + \beta_4^{(t)} \alpha^{(t)} \mathbf{C}_{14} \right) \]

\[ + \beta_2^{(t)} \left( e_{16} \alpha^{(t)} \mathbf{e}_{36} \right) + \beta_3^{(t)} \left( e_{15} \alpha^{(t)} \mathbf{e}_{35} \right) + \beta_4^{(t)} \left( e_{15} \alpha^{(t)} \mathbf{e}_{35} \right) \]}
The flow of electromagnetic power (Poynting Vector) in a piezoelectric medium requires a knowledge of the magnetic field as well as the electric field. It is a common belief (although an incorrect one) that the complex Poynting vector $E \times H^*$ reduces to $\varphi \vec{D}^*$ when the electric field is approximately derivable from a scalar potential function $\varphi$. This mistaken notion is based upon the following derivation for energy flow out of a closed surface.

Maxwell's equations are as follows for fields derivable approximately from a scalar potential function (i.e. where $\vec{E} = -\nabla \varphi$) in a non-conducting medium.

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \approx 0 \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \vec{D} \quad \nabla \cdot \vec{D} = 0$$

Now the flow of power out of any closed surface with a surface normal element $\mathrm{d}s$ is

$$P = \frac{1}{2} \int_S (\vec{E} \times \vec{H}^*) \cdot \mathrm{d}s$$

but

$$\int_S (\vec{E} \times \vec{H}^*) \cdot \mathrm{d}s = \iiint_V \nabla \cdot (\vec{E} \times \vec{H}^*) \, \mathrm{d}v$$
where the second integral is over the volume enclosed by the surface $S$. Using

$$\nabla \cdot (E \times H^*) = H^* \cdot \nabla E - E \cdot \nabla H^* \cong -E \cdot \nabla H^*$$

it follows that

$$\frac{1}{2} \iiint_V \nabla \cdot (E \times H^*) \, dv \cong \frac{1}{2} \iiint_V E \cdot \nabla H^* \, dv = \frac{1}{2} \iiint_V (\nabla \varphi \cdot \vec{D}^*) \, dv.$$

Furthermore

$$\nabla \cdot \varphi \vec{D}^* = \nabla \varphi \cdot \vec{D}^* + \varphi \nabla \cdot \vec{D}^* = \nabla \varphi \cdot \vec{D}^*$$

may be used to infer that

$$\frac{1}{2} \iiint_V \nabla \cdot (E \times H^*) \, dv \cong \frac{1}{2} \iiint_V \nabla \cdot (\varphi \vec{D}^*) \, dv$$

$$= \frac{1}{2} \iiint_S \varphi \vec{D}^* \cdot da$$

Therefore

$$P = \frac{1}{2} \iiint_S (E \times H^*) \cdot \vec{ds} \cong \frac{1}{2} \iiint_S \varphi \vec{D}^* \cdot \vec{da}.$$

This equation states that the surface integral over a closed surface of $E \times H^*$ is equal to that of $\varphi \vec{D}^*$ or alternately that $\nabla \cdot (E \times H^*) = \nabla \cdot (\varphi \vec{D}^*)$, consequently

$$E \times H^* = \varphi \vec{D}^* + \vec{A}$$

where $\vec{A}$ is some vector whose divergence is zero ($\nabla \cdot \vec{A} = 0$).

It seems a natural assumption to assume that $\vec{A} = 0$ and that $E \times H^* = \varphi \vec{D}^*$. However, this is not necessarily so as an illustrative example from electromagnetic theory shows.

Consider a surface wave propagating in the free space region

\[
\begin{array}{c}
\hat{x} \\
\hat{y}
\end{array}
\]

Impedance Sheet
A solution of the following form will exist.

\[ H_z = e^{-j\beta x} e^{-\sqrt{\beta^2 - k_0^2} y} \]

\[ E_x = \frac{-\sqrt{\beta^2 - k_0^2}}{j\omega\varepsilon} H_z \]

\[ E_y = \frac{\beta}{\omega\varepsilon} H_z \]

The wave impedance of this wave is

\[ Z = \frac{E_x}{H_z} = \frac{+j\sqrt{\beta^2 - k_0^2}}{\omega\varepsilon} \]

Assume an impedance sheet with surface impedance \( jX_s \). Then we find

\[ \frac{\sqrt{\beta^2 - k_0^2}}{\omega\varepsilon} = X_s \]

determining the value of \( \beta \). If \( k_0 << \beta \) (low frequency limit) then we have

\[ \beta = \omega\varepsilon X_s \]

and the field quantities assume the form

\[ H_z = e^{-j\beta x} e^{-\beta y} \]

\[ E_x \approx j\frac{\beta}{\omega\varepsilon} H_z \quad \text{and} \quad E_y = \frac{\beta}{\omega\varepsilon} H_z \]

This is the quasi-static case (\( k_0 << \beta \)) and an approximate expression for the electric field is derivable from a scalar potential. Indeed, the scalar potential may be taken to be

\[ \phi = \frac{H_z}{\omega\varepsilon} = \frac{e^{-j\beta x} e^{-\beta y}}{\omega\varepsilon} \]

with the result \( \vec{E} \approx -\nabla\phi \).

For this approximation the x component of the Poynting vector (\( E \times H^* \))
reduces to \( N_x = E_y H^*_z = \beta / \omega \varepsilon e^{-2\beta y} \) while

\[
\begin{align*}
\varepsilon D^*_x &= \frac{H_z}{\omega \varepsilon} e\ B^*_x \\
&= -\frac{\beta}{\omega \varepsilon} e^{-2\beta y}
\end{align*}
\]

and clearly \( \vec{N} \neq \varepsilon D^* \).

Summing up, the electromagnetic power flow \((E \times H^*)\) at any point requires a full solution of Maxwell's equations and cannot be obtained from the scalar potential method. Only the total power flow out of a closed surface can be obtained from the scalar potential method since

\[
\int_S E \times H^* \cdot \text{da} = \int_S \varepsilon D^* \cdot \text{da}
\]

where \( S \) is closed. A more thorough investigation into the electromagnetic power flow will be taken up during the transducer study part of the study. For now it can be said that the electromagnetic power flow is expected to be much smaller than the mechanical power flow based on computer runs where \( \varepsilon D^* \) was used as an order of magnitude estimate of the Poynting vector.

2. **Surface Waves on Non-Piezoelectric (Pure Elastic) Media**

Surface wave propagation on a free surface of a non-piezoelectric elastic medium can be accounted for by appropriate modifications of the foregoing analysis. In this case the piezoelectric constants \( e_{ijk} \) are identically zero and the electric field and mechanical displacements are decoupled. Consequently, the fourth order matrix appearing in equation (6) reduces to the third order matrix obtained by deleting the last row and column. Subject to this modification equation (7) reduces to a sixth order equation in \( \alpha \) which in general will have three roots with positive real parts. The boundary conditions at the free surface are given by equation (8) with the coefficients \( \varepsilon_{ki}^{\prime} = 0 \). Inasmuch as there is no coupling to an electric field the additional boundary conditions applicable thereto are unnecessary.
The relative amplitudes of the component of displacement $\beta_k^{(l)}$, $k = 1, 2, 3$ are evaluated for each $\alpha^{(l)}$ with positive real part in a manner identical to that used in the general piezoelectric case. Upon evaluation of these quantities the boundary conditions are invoked and the equations (12-14) result with the piezoelectric constants equal to zero and the sums only over the indices 1, 2, and 3. The characteristic equation for the determination of surface wave velocities again obtains from the condition that the determinant of the matrix coefficients associated with the linear system (12-14) vanishes.

The stresses, strains, power flow, etc., may be calculated as before by using the appropriate evaluations of $\beta_k^{(l)}$, $k = 1, 2, 3$, $\beta_k^{(1)}$, $k = 1, 2, 3$ and setting the $\beta_s^{(1)}$, $A^{(1)}$, and $e_{t_p}$ equal to zero in the equations for these quantities given previously.

3. **Degenerate Cases – Piezoelectric Medium**

The modes of surface wave propagation described as degenerate cases arise when the four coupled partial differential equations (4) which govern the mechanical components $u_1$, $u_2$, and $u_3$ and the electric field (via a scalar potential $\phi$) reduce to two independent sets of coupled equations, assuming traveling wave motions independent of the coordinate normal to the sagital plane.

With the coordinate system chosen assuming no variation in the $x_2$ direction, the equations of motion (4) for the displacement components and potential may be written in the operator form (assuming $e^{jut}$ time dependence)

\[
\begin{align*}
L_{11}U_1 + L_{12}U_2 + L_{13}U_3 + L_{14}\phi &= 0 \\
L_{21}U_1 + L_{22}U_2 + L_{23}U_3 + L_{24}\phi &= 0 \\
L_{31}U_1 + L_{32}U_2 + L_{33}U_3 + L_{34}\phi &= 0 \\
L_{41}U_1 + L_{42}U_2 + L_{43}U_3 + L_{44}\phi &= 0
\end{align*}
\]

where .
\[ L_{11} = C_{55} \frac{\partial^2}{\partial x_3^2} + 2C_{15} \frac{\partial^2}{\partial x_3 \partial x_1} + C_{11} \frac{\partial^2}{\partial x_1^2} + \omega^2 \rho \]

\[ L_{12} = L_{21} = C_{45} \frac{\partial^2}{\partial x_3^2} + (C_{14} + C_{56}) \frac{\partial^2}{\partial x_3 \partial x_1} + C_{16} \frac{\partial^2}{\partial x_1^2} \]

\[ L_{13} = L_{31} = C_{35} \frac{\partial^2}{\partial x_3^2} + (C_{13} + C_{55}) \frac{\partial^2}{\partial x_3 \partial x_1} + C_{15} \frac{\partial^2}{\partial x_1^2} \]

\[ L_{14} = L_{41} = e_{35} \frac{\partial^2}{\partial x_3^2} + (e_{15} + e_{31}) \frac{\partial^2}{\partial x_3 \partial x_1} + e_{11} \frac{\partial^2}{\partial x_1^2} \]

\[ L_{22} = C_{44} \frac{\partial^2}{\partial x_3^2} + 2C_{46} \frac{\partial^2}{\partial x_3 \partial x_1} + C_{66} \frac{\partial^2}{\partial x_1^2} + \omega^2 \rho \]

\[ L_{23} = L_{32} = C_{34} \frac{\partial^2}{\partial x_3^2} + (C_{36} + C_{45}) \frac{\partial^2}{\partial x_3 \partial x_1} + C_{56} \frac{\partial^2}{\partial x_1^2} \]

\[ L_{24} = L_{42} = e_{34} \frac{\partial^2}{\partial x_3^2} + (e_{14} + e_{36}) \frac{\partial^2}{\partial x_3 \partial x_1} + e_{16} \frac{\partial^2}{\partial x_1^2} \]

\[ L_{33} = C_{33} \frac{\partial^2}{\partial x_3^2} + 2C_{35} \frac{\partial^2}{\partial x_3 \partial x_1} + C_{55} \frac{\partial^2}{\partial x_1^2} + \omega^2 \rho \]

\[ L_{34} = L_{43} = e_{33} \frac{\partial^2}{\partial x_3^2} + (e_{13} + e_{35}) \frac{\partial^2}{\partial x_3 \partial x_1} + e_{15} \frac{\partial^2}{\partial x_1^2} \]

\[ L_{44} = -e_{33} \frac{\partial^2}{\partial x_3^2} - 2e_{13} \frac{\partial^2}{\partial x_3 \partial x_1} - e_{11} \frac{\partial^2}{\partial x_1^2} \]
The elastic $c_{ij}$, piezoelectric $e_{ij}$, and dielectric $\varepsilon_{ij}$ constants refer to the transformed (from the crystal coordinate system) quantities and are represented in terms of the abbreviated double subscript notation.

If the elastic and piezoelectric constants are such that the operator matrix $[L_{ij}]_{i, j=1, 2, 3, 4}$ is appropriately sparse, the equations (1) decouple and the possibility of degenerate cases is encountered.

For example, in the case reported by Bleustein (4) the elastic and piezoelectric constants are such that $L_{12} = L_{14} = L_{23} = L_{34} \equiv 0$ and the equations of motion decouple into two independent systems; one system governing $u_2$ and $\varphi$ and the other characterizing the behaviors of $u_1$ and $u_3$. This is an example of one of the two general degenerate cases which has been found to exist for a number of crystals on particular cuts and directions of propagation.

A second degenerate case which also has been reported and which appears with some regularity is manifest by the conditions $L_{12} = L_{23} = L_{24} \equiv 0$. In this case the equations of motion decouple into a coupled system of partial differential equations for the displacement components $u_1$, $u_3$ and the potential $\varphi$, and a single partial differential equation for the displacement component $u_2$. This particular degenerate case has been studied extensively for surface wave propagation on the basal plane of hexagonal crystals (7) and it has been shown that a surface wave solution with the displacement component $u_2$ alone cannot exist. It should be noted that the latter observation carries over to the general case, independent of the crystal class, surface cut, and direction of propagation.

There are other degenerate cases that can be considered. For example,

$$L_{12} = L_{13} = L_{14} \equiv 0$$

or

$$L_{31} = L_{32} = L_{34} \equiv 0$$

These cases were considered in the analysis leading to the present study but numerical examples of these cases have not been found.

The conditions $L_{41} = L_{42} = L_{43} \equiv 0$ lead to a complete decoupling of the electric and mechanical fields and has not been found to occur for surface wave propagation on specific surfaces of any piezoelectric crystal considered thus far.
The occurrence of degenerate cases can also be described in terms of the linear equations for the relative amplitudes, \( \beta_t \), \( t = 1, 2, 3, 4 \), of the mechanical displacement and electric potential. The determinant of the coefficients of this system of equations is given by equation (6), wherein the elements of the determinant correspond to evaluations of the operators \( L_{ij} \) with

\[
\frac{\partial}{\partial x_1} = -j \frac{\omega}{v_s} \quad \text{and} \quad \frac{\partial}{\partial x_3} = - \frac{\alpha \omega}{v_s}.
\]

For the sake of the following discussions let the linear system of equations for the determination of the \( \beta_t \)s be denoted

\[
\sum_{t=1}^{4} A_{it} \beta_t = 0, \quad i = 1, 2, 3, 4. \tag{18}
\]

The possible combinations of the elastic and piezoelectric constants which caused the equations of motion to decouple lead to the decoupling of the linear equations in the exact same fashion. Inasmuch as the linear equations (18) are employed in the numerical analyses, the various cases of decoupling are discussed again in more detail below.

The following degenerate cases have been found to exist and are accounted for in the computer program which implements the numerical analysis. They are denoted by representing the matrix \( \hat{A} = [L_{ij}], t=1, 2, 3, 4 \) with its appropriate zeros displayed, viz.,

Case (1)  
\[
\begin{bmatrix}
A_{11} & 0 & A_{13} & A_{14} \\
0 & A_{22} & 0 & 0 \\
A_{13} & 0 & A_{33} & A_{34} \\
A_{14} & 0 & A_{34} & A_{44}
\end{bmatrix}
\]

Case (2)  
\[
\begin{bmatrix}
A_{11} & 0 & A_{13} & 0 \\
0 & A_{22} & 0 & A_{24} \\
A_{13} & 0 & A_{33} & 0 \\
0 & A_{24} & 0 & A_{44}
\end{bmatrix}
\]

In Case (1) we note that \( \beta_2 \) decouples from \( \beta_1, \beta_3, \) and \( \beta_4 \). The determinant of \( \hat{A} \) is zero if \( A_{22} = 0 \) (as a function of \( \alpha \)) or if the determinant

\[
A_1 = \begin{vmatrix}
A_{11} & A_{13} & A_{14} \\
A_{13} & A_{33} & A_{34} \\
A_{14} & A_{34} & A_{44}
\end{vmatrix} = 0.
\]
The condition $A_{22} = 0$ leads to a quadratic equation in $\alpha$. If $A_{22} = 0$ the system of equations yields a solution $\beta_1 = \beta_3 = \beta_4 = 0$ while $\beta_2$ can be chosen as an arbitrary constant.

If $A_1 = 0$, giving a sixth order equation in $\alpha$, the system of equations requires a solution $\beta_2 = 0$ while either $\beta_1$, $\beta_3$, or $\beta_4$ may be chosen arbitrarily and the remaining two $\beta$ is calculated from any two of the three equations not involving $A_{22}$.

In case (2) we note that $\beta_1$ and $\beta_3$ decouple from $\beta_2$ and $\beta_4$. The determinant of $A$ goes to zero if the determinant

$$A_2 = \begin{vmatrix} A_{11} & A_{13} \\ A_{13} & A_{33} \end{vmatrix} = 0$$

or the determinant

$$A_3 = \begin{vmatrix} A_{22} & A_{24} \\ A_{24} & A_{44} \end{vmatrix} = 0.$$ 

Both the equations $A_2 = 0$ and $A_3 = 0$ lead to quartic equations in $\alpha$. If $A_2 = 0$ the system yields the solution $\beta_2 = \beta_4 = 0$ while $\beta_1$ or $\beta_3$ may be arbitrarily chosen and the remaining $\beta$ calculated from either the first or third equation of the system. If $A_3 = 0$ the system yields the solution $\beta_1 = \beta_3 = 0$ while $\beta_2$ or $\beta_4$ may be arbitrarily chosen and the remaining $\beta$ calculated from either the second or fourth equation of the system.

Let the coefficients of the amplitudes $B^{(q)}$ (equations (12)-(16)) be considered elements of the matrix $\hat{L}$. In case (1) $\hat{L}$ takes the form

$$\begin{vmatrix} 0 & L_{12} & L_{13} & L_{14} \\ L_{21} & 0 & 0 & 0 \\ 0 & L_{32} & L_{33} & L_{34} \\ 0 & L_{42} & L_{43} & L_{44} \end{vmatrix}.$$
(considered as a function of the velocity \( v_s \)) then the following solution is found: \( B^{(1)} = 0 \) while \( B^{(2)}, B^{(3)}, \) or \( B^{(4)} \) can be chosen arbitrarily and the remaining \( B \)'s calculated from any two of the three equations not involving \( L_{21} \). This situation corresponds to a wave with displacement components \( U_1, U_3 \) and potential \( \phi \) and \( U_2 \) is identically zero.

If \( L_{21} \) is zero we would be led to a solution where \( U_1, U_3, \) and \( \phi \) are zero while only \( U_2 \) would be present in the wave. However, it can be shown that \( L_{21} \) can not be equal to zero and therefore such a mode does not exist.*

In case (2) \( \hat{L} \) takes the form

\[
\begin{vmatrix}
L_{11} & L_{12} & 0 & 0 \\
0 & 0 & L_{23} & L_{24} \\
L_{31} & L_{32} & 0 & 0 \\
0 & 0 & L_{43} & L_{44}
\end{vmatrix}
\]

If the determinant

\[
L_2 = \begin{vmatrix} L_{11} & L_{12} \\ L_{31} & L_{32} \end{vmatrix} = 0
\]

then the following solution is found: \( B^{(3)} = B^{(4)} = 0 \) while \( B^{(1)} \) or \( B^{(2)} \) can be arbitrarily chosen and the remaining \( B \) can be calculated from the first or third equation of the system (equations (12) or (14)). This situation corresponds to a wave with displacement components \( U_1 \) and \( U_3 \) while \( U_2 \) and \( \phi \) are identically zero.

If the determinant

\[
L_3 = \begin{vmatrix} L_{23} & L_{24} \\ L_{43} & L_{44} \end{vmatrix} = 0
\]

*This case was considered in detail for surface wave propagation in the basal plane of hexagonal piezoelectric crystals by Tseng and White(7).
then the following solution is found: $B^{(1)} = B^{(2)} = 0$ while $B^{(3)}$ or $B^{(4)}$ can be arbitrarily chosen and the remaining $B$ can be calculated from the second or fourth equation of the system (equations (13) or (15)). This situation corresponds to a wave with displacement component $U_2$ and potential $\phi$ while $U_1$ and $U_3$ are identically zero.

It should be noted that in paragraph 1.1 it was stated that four $\alpha$'s with positive real parts can be found when in the range of velocities below the slowest bulk wave velocity in the direction of propagation being considered. However, in the degenerate cases surface waves may exist when there are less than four such $\alpha$'s provided the appropriate $\alpha$'s have positive real parts. For example, in case (1) only three roots of $A_1$ must have positive real parts for the existence of a surface wave. The roots of $A_{22} = 0$ are not required to assume any particular form. That is, both roots corresponding to $A_{22} = 0$ may be purely imaginary but if three of the six roots corresponding to $A_1 = 0$ have positive real parts a surface wave may still exist. Similarly, for case (2) it is possible to have a solution with only two $\alpha$'s with positive real parts provided that both of the $\alpha$'s come from the same equation (i.e. both come from $A_2 = 0$ or both from $A_3 = 0$). An example of the latter situation has been reported in the literature and corresponds to a wave with displacement component $U_2$ and potential $\phi$. It may be noted that the (necessarily) degenerate waveforms that occur with higher velocities than the bulk waves alluded to above appear to be the non-attenuated limits of leaky or pseudo-waves and have been described for non-piezoelectric crystals by Lim and Farnell.

One further case of a peculiar nature will be mentioned here although it does not fit into the category of degenerate cases. On a surface of a hexagonal crystal, solutions of the algebraic equations for the decay coefficients $\alpha^{(k)}$ exist which cause the minors of the last row and column of $\hat{\alpha}$ to vanish. Since all the minors of the elements of $\hat{\alpha}$ are not new the rank of the determinant is not reduced but the component $\beta_4$ is forced to be zero. Thus only $\beta_1$, $\beta_2$, and $\beta_3$ exist for such an $\alpha$. The total wave however is not an example of a degenerate case since the other $\alpha$'s are needed for a surface wave solution. This behavior is a manifestation of the fact that one of the general bulk wave solutions is decoupled from the electric field for all directions of propagation in a piezoelectric hexagonal crystal.
4. **Degenerate Cases – Non-piezoelectric (Pure Elastic) Medium**

For surface wave propagation in a pure elastic medium the system of equations discussed in the preceding paragraph takes the form

\[ \sum_{i=1}^{3} A_{i} \beta_{i} = 0 \quad i = 1, 2, 3 \]  

(19)

where the \( A_{i} \) are the same as in the piezoelectric case but involve only the elastic constants (the piezoelectric constants are zero and the dielectric constants do not enter the problem). The equation for \( \alpha \) is now sixth order. Generally three roots with positive real parts are required for a surface wave solution.

Only one degenerate case occurs for a pure elastic medium. In this case the \( \hat{A} \) matrix assumes the form

\[
\begin{pmatrix}
A_{11} & 0 & A_{13} \\
0 & A_{22} & 0 \\
A_{13} & 0 & A_{33}
\end{pmatrix}
\]

As before the condition \( A_{22} = 0 \) leads to a quadratic equation in \( \alpha \) but this case is of no interest inasmuch as it would lead to a solution with \( U_{2} \) the only component of displacement and this is impossible for the same reason as in the piezoelectric case (viz. \( L_{21} \) cannot equal zero for the form of the decay coefficient \( \alpha \) required for a surface wave solution). If

\[
A_{1} = \begin{vmatrix} A_{11} & A_{13} \\ A_{13} & A_{33} \end{vmatrix} = 0
\]

we obtain the solution \( \beta_{2} = 0, \beta_{1}, \beta_{3} \neq 0 \), and either \( \beta_{1} \) or \( \beta_{3} \) can be chosen arbitrarily with the other \( \beta \) being calculated from one of the two remaining equations of the system. In this case the \( \hat{L} \) matrix assumes the form
and the boundary conditions can be satisfied if

\[
L_1 = \begin{vmatrix}
L_{12} & L_{13} \\
L_{32} & L_{33}
\end{vmatrix} = 0.
\]

The above form of the linear system requires that \(B^{(1)} = 0\) while either \(B^{(2)}\) or \(B^{(3)}\) can be chosen arbitrarily and the remaining amplitude calculated from either of the equations

\[
L_{12}B^{(2)} + L_{13}B^{(3)} = 0,
\]

or

\[
L_{32}B^{(2)} + L_{33}B^{(3)} = 0.
\]

This solution corresponds to a wave with displacement components \(U_1\) and \(U_3\) while \(U_2\) is identically zero, i.e., a Rayleigh wave.

5. Surface Wave Propagation in an Isotropic Elastic, Perfectly Conducting Film on a Piezoelectric Substrate

An additional problem that has been considered is that of a finite thickness layer of isotropic elastic conductor on a piezoelectric substrate. When the displacement component waveforms

\[
U_i = \beta_i e^{-\alpha x_j / v_j} e^{j\left(t-x_j / v_j\right)}, \quad i = 1, 2, 3,
\]

are substituted into the equations of motion for an isotropic elastic medium, a linear system of equations for the relative amplitudes \(\beta_1, \beta_2, \beta_3\) of the displacement components is obtained. The determinant of the system, set equal to zero, yields the equation for the determination of the exponents \(\alpha\), namely
where \( \lambda, \mu \) are the Lamé constants of the medium. The polynomial form of (18) is of order six and, inasmuch as the medium is of finite thickness, the solution corresponding to all six roots is needed to satisfy the boundary conditions.

The assumed forms of the solutions in the piezoelectric medium with those employed in Section II.1, namely,

\[
\text{In the elastic conductor the total displacements assume the form}
\]

\[
\begin{align*}
U_1^P &= \sum_{l=1}^{4} A^{(l)} P_l^{(l)} e^{-\alpha^{(l)}_p \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \\
q^P &= \sum_{l=1}^{4} A^{(l)} P_l^{(l)} e^{-\alpha^{(l)}_p \omega x_3/v_s} e^{j\omega(t-x_1/v_s)}.
\end{align*}
\]

and the potential function is identically zero. Thus there are 10 unknown amplitude coefficients \( A^{(l)}, B^{(k)}, l = 1, 2, 3, 4; k = 1, \ldots, 6 \) to be determined.

The boundary conditions applicable in this problem are as follows:
Continuity of displacement at $x_3 = 0$

$$U_i^P(x_1, 0) = U_i^C(x_1, 0) \quad i = 1, 2, 3$$

Continuity of stress components at $x_3 = 0$

$$T_{3j}^P(x_1, 0) = T_{3j}^C(x_1, 0) \quad j = 1, 2, 3$$

Vanishing of stress components at the free surface at $x_3 = -h$

$$T_{3j}^C(x_1, -h) = 0 \quad j = 1, 2, 3$$

Vanishing of potential at $x_3 = 0$

$$\phi^P(x_1, 0) = 0$$

Applying these ten conditions to the above solutions yields a system of ten homogeneous algebraic equations in the ten unknown amplitude coefficients $A^{(i)}$, $B^{(i)}$. From this point the solution for surface wave velocities and field distributions proceeds as before except that there are now ten equations instead of (4). The explicit form of the determinant of the system stemming from the boundary conditions is given in Appendix I.

6. Surface Wave Propagation at the Interface between a Piezoelectric Substrate and a Semi-Infinite Fluid Medium

The physical problem considered in this section is that of a surface wave propagating along the interface between a piezoelectric crystal and a semi-infinite fluid. Again a rectangular coordinate system is chosen with the $x_3$ axis normal to the crystal surface and the $x_1$ axis in the direction of propagation as in the preceding problems. The fields in the crystal and fluid media are assumed to be independent of the $x_2$ direction. Arbitrary orientations of the crystal surface with respect to the crystal axes are handled by means of an Euler Transformation as before and the differential equations in the crystal medium are the same.

The elastic properties of the fluid medium are described in terms of a single elastic constant $\lambda$ (modulus of compression); the effect of viscosity is ignored. Consequently, the differential equations in the fluid are

$$U_{1,11} + U_{3,13} = \rho_1 \ddot{U}_1 / \lambda$$

$$U_{1,13} + U_{3,33} = \rho_1 \ddot{U}_3 / \lambda \quad , \quad x_3 < 0 ,$$

$$\nabla^2 \phi = 0 \quad ,$$

$$\nabla^2 \psi = 0$$
wherein the derivatives with respect to $x_2$ are taken to be zero in keeping with the uniformity of the field in this direction and $\rho_f$ is the density of the fluid medium.

In the crystal medium ($x_3 > 0$) traveling wave solutions of the form

$$U_i = \beta_i e^{-\alpha c \omega x_3 / v_s} e^{jw(t-x_1 / v_s)}, \quad i = 1, 2, 3,$$

$$\varphi = \beta 4 e^{-\alpha c \omega x_3 / v_s} e^{jw(t-x_1 / v_s)}$$

are sought and all analytical considerations pertaining to the crystal medium are identical with those discussed in Sections II.1 and II.2.

In the fluid ($x_3 < 0$) the particle displacements and electric potential are decoupled and are assumed to have the forms

$$U_1 = \gamma_1 e^{-\alpha f \omega x_3 / v_s} e^{jw(t-x_1 / v_s)}$$

$$U_3 = \gamma_3 e^{-\alpha f \omega x_3 / v_s} e^{jw(t-x_1 / v_s)}$$

$$\varphi = C e^{-\alpha f \omega x_3 / v_s} e^{jw(t-x_1 / v_s)}$$

Substitution of these displacement and potential waveforms into the differential equations (24) leads to the following equation for $\alpha_f$ in terms of the velocity $v_s$, namely

$$\det \begin{pmatrix} \rho_f v_s^2 - \lambda & j\alpha_f \lambda \\ j\alpha_f \lambda & \lambda^2 + \rho_f v_s^2 \end{pmatrix} = 0,$$

from which it follows that

$$\alpha_f = \pm \sqrt{\frac{\lambda \rho_f v_s^2}{\lambda^2}}.$$

The relative amplitudes $\gamma_1$ and $\gamma_3$ are obtained from the homogeneous linear system of equations whose coefficient matrix appears in equation (7), viz.,
\[ \gamma_1 = \frac{j\alpha^*_f}{\lambda - \rho_f v_s^2} \gamma_3 \quad . \]

(29)

The sign of \( \alpha_f \) in equation (28) is determined by the condition that the surface wave is bounded as \( x_1 \to +\infty \).

The total field in the crystal is expressed as a linear combination of the "partial" fields associated with the allowed values of \( \alpha_c \), namely

\[ U_i = \sum_{\ell=1}^{4} B(\ell) \frac{\beta_1(\ell)}{\alpha_c} e^{-\alpha_c j\omega x_3/v_s} e^{j\omega (t-x_1/v_s)} \]

(30)

\[ \varphi = \sum_{\ell=1}^{4} B(\ell) \frac{\beta_4(\ell)}{\alpha_c} e^{-\alpha_c j\omega x_3/v_s} e^{j\omega (t-x_1/v_s)} \]

In the fluid medium the total displacements and potential are given by equation (26).

The amplitude coefficients \( B^{(1)} \), \( B^{(2)} \), \( B^{(3)} \), \( B^{(4)} \), \( C \) and \( \gamma_3 \) are determined by the boundary conditions:

- \( U_3 \) continuous at \( x_3 = 0 \)
- \( \varphi \) continuous at \( x_3 = 0 \)
- \( D_3 \) continuous at \( x_3 = 0 \) (electric displacement)
- \( T_{33} \) continuous at \( x_3 = 0 \)
- \( T_{31} = 0 \) at \( x_3 = 0 \)
- \( T_{32} = 0 \) at \( x_3 = 0 \)

The components of the electric displacement vector \( \vec{D} \) in the crystal are given by

\[ D_i = e_{ik} \frac{U_k}{\omega} - \frac{\varepsilon_i}{\omega} \rho, \quad i = 1, 2, 3 \]

in the fluid medium, \( \vec{D} = -\varepsilon_f \varphi \), where \( \varepsilon_f \) is the dielectric constant of the fluid.

Application of the boundary conditions to the total field solutions leads to a set of six homogeneous equations in the unknown amplitudes \( B^{(t)} \), \( t = 1, 2, 3, 4 \), \( C \) and \( \gamma_3 \). The coefficient matrix of this system of equations, \( M = [M_{ik}]_{i,k=1, \ldots, 6} \), assumes the form*

*The explicit equations for the elements of \( M \) are given in Appendix II.
The characteristic equation for the surface wave velocity $v_s$ is obtained from the condition for the existence of a non-trivial solution of the aforementioned homogeneous system, namely, $\det M = 0$.

The complex solutions to the equation $\det M = 0$ can be obtained in a straightforward fashion using the iterative scheme described in the programming sections and such a procedure has been built into the computer program for the fluid problems. However such a procedure is time consuming and does not fully exploit prior work on piezoelectric surface wave propagation problems. Inasmuch as a computer program exists to calculate piezoelectric surface wave characteristics under a variety of conditions, in particular, when the surface of the crystal is traction free and the adjacent half space is a massless, non elastic dielectric, it is desirable to make maximum use of this program. This can be done for a wide range of parameter values for the fluid medium by making use of a perturbation scheme for obtaining the roots of $\det M = 0$ which utilizes the results of this program and requires, in addition, only the evaluation of a few determinants at specified velocities.

The implementation of the perturbation procedure is based on the fact that a particular sub-matrix $N$ of the matrix $M$ (as indicated by the partitioned matrix in equation (31)) is the coefficient matrix of the linear system corresponding to the boundary conditions at the surface of a crystal in contact with a medium whose elastic properties are those of a vacuum but whose dielectric constant is that of the fluid medium. Consequently, the equation $\det N = 0$ is the characteristic equation for the velocity of the surface waves.
which can propagate in this configuration, and the roots of this equation can be found using the computer program for the first problem with \( \nu h = 0 \) and the dielectric constant of a vacuum \( \varepsilon_0 \) replaced by the dielectric constant of the fluid \( \varepsilon' \).

The perturbation procedure is based on the assumption that the complex velocity which satisfies \( \det M = 0 \) corresponds to a small perturbation on the real velocity solution of \( \det N = 0 \), i.e. that the mechanical loading of the substrate by the fluid medium is quite small.

Formally the perturbation scheme is derived as follows. Let \( v_{so} \) be the velocity such that \( \det N(v_{so}) = 0 \) and assume that there exists a complex perturbation \( \Delta v_s \) such that the \( \det M(v_s) = \det M(v_{so} + \Delta v_s) = 0 \) and \( |\Delta v_s/v_{so}| \ll 1 \).

For \( |\Delta v_s/v_{so}| \ll 1 \)

\[
\det M(v_s) = \det M(v_{so}) + \frac{d}{dv_s} \left[ \det M(v_s) \right]_{v_s = v_{so}} \cdot \Delta v_s
\]

\[+ O((\Delta v_s)^2) = 0,
\]

whereupon neglecting terms \( O((\Delta v_s)^2) \) yields

\[
\Delta v_s = -\frac{\det M(v_{so})}{\frac{d}{dv_s} (\det M(v_{so}))}
\]  

(33)

Expanding \( \det M(v_{so}) \) about the last column (which has only two elements) gives

\[
\Delta v_s = \frac{M_{46} K}{M_{16}(\det N)' - M_{46} K' - M_{46} K' K}
\]  

(34)

where the quantity \( K \) in (34) is the minor of the element \( M_{46} \) in the matrix \( M \), the primes denote differentiation with respect to \( v_s \), and all quantities are evaluated at \( v_{so} \). In obtaining (34) explicit use has been made of the fact that \( \det N(v_{so}) = 0 \).

The derivatives of the determinants employed in the perturbation procedure are calculated numerically. The derivative of the matrix element \( M_{46} \) was obtained analytically.
In both methods of obtaining the complex velocity $v_s$ the functions involved (matrix elements and minors of the matrix $M$) contain $\alpha_f$ as an independent variable which in turn is a dependent variable with argument $v_s$. Equation (28) shows that there is an ambiguity in the sign of $\alpha_f$. The resolution of this ambiguity leads to the particular character of the piezoelectric surface wave.

The variation of the surface wave in the direction of propagation is assumed to be bounded in the positive ($x_1 \rightarrow +\infty$) direction of propagation. This assumption imposes the requirement that $\text{Im} [v_s] \geq 0$. Consequently, the sign of $\alpha_f$ must be chosen such that this condition is satisfied.

Since $\lambda$ and $\rho_f$ are positive real it can be shown that

$$\text{Re} \left[ \frac{\alpha_f^{(4)}}{v_s} \right] \leq 0,$$

where $\alpha_f^{(4)}$ denotes to the values of $\alpha_f$ from equation (28) corresponding to the positive and negative signs of the radical. Consequently, if $\alpha_f^{(4)}$ is required to obtain a root $v_s$ such that $\text{Im} [v_s] \geq 0$ the corresponding surface wave is of the leaky type. On the other hand, if $\alpha_f^{(4)}$ is required to obtain a solution of the determinantal equation the surface wave is evanescent in character. In all numerical cases considered the surface wave was a leaky wave.

In the program described in the following section, two values of input velocity are required depending on the program option used. If the perturbation scheme is used, a very accurate value (at least 6 place accuracy) of velocity must be input. This value is to be computed from the existing surface wave program wherein the dielectric constant of the fluid medium is substituted for that of free space (outside the crystal medium). On the other hand, if the root finding scheme is employed only a reasonable estimate of the complex velocity is required.

A final word of caution is in order regarding the use of the computer program. In checking out the various options available with the program, it was found that if the leaky wave velocity is a small perturbation on the surface wave velocity in the absence of the fluid medium, then the use of the perturbation scheme led to more reliable results than the root finding option. On the other hand, if the fluid medium significantly loaded the substrate material the root

finding scheme gave good results whereas the perturbation scheme (as would be expected) gave erratic results in some cases. The former differences stem from the fact that the change in the velocity due to the air loading is on the order of the errors incurred in the root finding scheme while the latter are due to the approximations inherent to the perturbation procedure.

7. Surface Wave Propagation in an Isotropic Elastic Film on a Piezoelectric Substrate

This section gives a brief description of the theoretical analysis of surface wave propagation on a semi-infinite piezoelectric substrate with a contiguous isotropic dielectric-elastic layer, as shown in Figure 3.

The substrate is assumed to be a completely general piezoelectric (or non-piezoelectric) crystal medium with arbitrary surface normal direction relative to the crystal axes of the medium. The material layer adjacent to the substrate is assumed to be a general isotropic elastic medium with isotropic dielectric properties. Only pure modes of propagation are considered, that is, leaky surface waves or evanescent (or cut off) modes of propagation have not been accounted for in the computer program.

Figure 3. Semi-infinite Piezoelectric Substrate with a Contiguous Isotropic Elastic Layer.
The coordinate system employed in the analysis is illustrated in Figure 3. The piezoelectric crystal medium occupies the region \( x_3 > 0 \) and the direction of propagation is assumed to be in the \( x_1 \) direction. The fields in both the crystal and layer are assumed to be independent of the \( x_2 \) coordinate. Arbitrary orientations of the crystal surface with respect to the crystal axes are considered as before by an Euler Transformation. Inasmuch as the dielectric layer is isotropic, both from an elastic and an electromagnetic point of view, the quantities characterizing the medium are invariant under coordinate transformations.

The analysis pertaining to the crystal or "substrate" medium is identical to that described in Sections II.1 and II.2.

The elastic properties of the dielectric layer are described in terms of two elastic constants, \( \lambda_d \), the modulus of compression or Lame's constant, and \( \mu_d \) the shear modulus. Inasmuch as the layer is non-piezoelectric the differential equations for the mechanical displacements and electric potential decouple and assume the form

\[
\mu_d v^2 \ddot{U} + (\lambda_d + \mu_d) \nabla (\nabla \cdot \ddot{U}) = \rho_d \ddot{U}, \quad -h < x_3 < 0, \quad (35)
\]

and

\[
v^2 \phi = 0, \quad (36)
\]

where \( \ddot{U} = (U_1, U_2, U_3) \) and \( \rho_d \) is the density of the dielectric medium.

In the dielectric medium the assumed displacement waveforms may be expressed as

\[
U_i = \beta_d e^{-\alpha_d u x_3 / v_s} e^{jw(t-x_1/v_s)}, \quad i = 1, 2, 3. \quad (37)
\]

Substitution of these waveforms into the differential equations (35) yields a linear system of homogeneous equations in the unknowns \( \beta_{d1}, \beta_{d2} \) and \( \beta_{d3} \). The existence of non-trivial solutions requires that the determinant of the coefficients of the system vanish thus leading to the following classical equations for normalized transverse wave numbers \( \alpha_d \) in terms of the velocity \( v_s \), namely,
The terms \( \alpha_d^{(1, 2)} \) and \( \alpha_d^{(3, 4)} \) are defined as,

\[
\alpha_d^{(1, 2)} = \alpha_d^\pm (\text{shear}) = \pm \sqrt{\frac{\mu_d - \rho_d v_s^2}{\mu_d}} \\
\alpha_d^{(3, 4)} = \alpha_d^\pm (\text{compressional}) = \pm \sqrt{\frac{\lambda_d + 2\mu_d - \rho_d v_s^2}{\lambda_d + 2\mu_d}}
\]

and

\[
\alpha_d^{(5, 6)} = \alpha_d^{(1, 2)}
\]

since the shear mode is degenerate for an isotropic elastic medium.*

Finally, in the dielectric medium, the two independent solutions of (36), assuming \( e^{-j\omega x/v_s} \) variation in the \( x_1 \) direction, are

\[
\varphi_{1, 2} = C_{1, 2} e^{\pm jwx_3/v_s} e^{j\omega(t-x_1/v_s)}
\]

In the "free space" region \(-\infty < x_3 < -h\) there are no mechanical displacements but a potential function exists and must satisfy the differential equation (36). In addition, the requirement that the potential be bounded as \( x \to -\infty \) is imposed. Therefore, the form of the potential is taken to be

\[
\varphi_s = C_3 e^{jwx_3/v_3} e^{j\omega(t-x_1/v_s)}
\]

The total displacement and potential waveforms in the piezoelectric crystal are expressed as linear combinations of the "partial" fields associated with the allowed values of \( \alpha_c \). Denoting these values \( \alpha_c^{(\ell)} \), \( \ell = 1, 2, 3, 4 \), the displacement components and potential may be expressed as

---

*The term degenerate is used here in the sense that in the characteristic equation for the normalized transverse wave numbers \( \alpha \) (for example, corresponding to the determinantal equation (3) for the general piezoelectric crystal) the roots \( \alpha_c^{(1)} \) and \( \alpha_c^{(2)} \) are double roots and hence two linearly independent eigenvectors can be defined for each distinct value.
In the dielectric medium the total displacement components assume the form

\[
U_1^{(c)} = \sum_{\ell=1}^{4} B^{(\ell)} \beta^{(\ell)}_{Cl} e^{-\alpha^{(\ell)}_{C} x_3/v_s} e^{j\omega (t-x_1/v_s)}, \quad i = 1, 2, 3, \quad (42)
\]

\[
\phi = \sum_{\ell=1}^{4} B^{(\ell)} \beta^{(\ell)}_{Cl} e^{-\alpha^{(\ell)}_{C} x_3/v_s} e^{j\omega (t-x_1/v_s)}.
\]

while the total potential is given by

\[
\phi^d = \left( C_1 e^{ux_3/v_s} + C_2 e^{-ux_3/v_s} \right) e^{j\omega (t-x_1/v_s)} \quad (44)
\]

In the free space region the total potential is given by equation (41).

The as yet unspecified amplitude coefficients \( B^{(\ell)} \), \( \beta^{(\ell)}_{Cl} \), \( \alpha^{(\ell)}_{C} \), \( C_1 \), \( C_2 \), and \( C_3 \) are determined, to within a constant, together with the surface wave velocity \( v_s \), by the following continuity and boundary conditions:

(i) \( U_1, U_2, \) and \( U_3 \) continuous at \( x_3 = 0 \)

(ii) \( \phi \) continuous at \( x_3 = 0 \) and \( x_3 = -h \)

(iii) Continuity of the normal component of electric displacement at \( x_3 = 0 \) and \( x_3 = -h \)

(iv) Continuity of shear and normal stresses \( (T_{31}, T_{32}, T_{33}) \) at \( x_3 = 0 \)

(v) The surface \( x_3 = -h \) is stress free \( (T_{31} = T_{32} = T_{33} = 0) \).

The components of the electric displacement vector \( \mathbf{D} \) are given by

\[
D_i = \varepsilon_{ik\ell} U_k, \quad i = 1, 2, 3, \quad x_3 > 0,
\]

\[
\mathbf{D} = \varepsilon_{d\ell} \phi \quad -h < x_3 < 0.
\]
and

$$\mathbf{D} = \varepsilon_0 \varphi \mathbf{v} x_3 < -h$$

The components of stress $T_{31}$, $T_{32}$, and $T_{33}$ are given by

$$T_{3j} = C_{3jk} U_k_{,t} + \varepsilon_{3jk} \varphi_{,k} \mathbf{v} x_3 > 0 \quad j = 1, 2, 3 \quad x_3 > 0$$

$$T_{31} = \mu_d (U_{1,3} + U_{3,1})$$

$$T_{32} = \mu_d U_{2,3}$$

$$T_{33} = (\lambda_d + 2\mu_d) U_{3,3} + \lambda_d U_{1,1}$$

Application of the continuity and boundary conditions (i) . . . (v) to the total displacements and potentials (40), (41), (42), (43), and (44) leads to a system of thirteen linear homogeneous equations in the thirteen unknown amplitudes $B_i (z)$, $i = 1, 2, 3, 4$, D and $C_1$, $C_2$ and $C_3$. The equation for the surface wave velocity $v_s$ is obtained from the condition for the existence of a non-trivial solution of this system of equations, namely, that the determinant of the system vanish. The explicit forms of the coefficients $L_{ij}$, $i, j = 1, \ldots, 13$, of this system are contained in Appendix III where the appropriate boundary conditions represented by each row of the matrix are indicated.

If the substrate is non-piezoelectric, modifications of the foregoing analyses identical to those described in Section II.1 are required. In this case, the characteristic equation for the surface wave velocity is the determinant of a $(9 \times 9)$ matrix comprised of the coefficients of the amplitudes $D_{(t)}^{(l)}$, $t = 1, \ldots, 6$, and $B_{(l)}^{(t)}$, $t = 1, 2, 3$ in the homogeneous system of 9 equations in 9 unknowns derived from the boundary conditions given above upon neglecting the electric field and setting the piezoelectric constants equal to zero.

**Degenerate Cases (Piezoelectric Substrate)**

The same degenerate cases arise as those considered in the preceding sections and the selection of $D_i$'s proceeds as before.

For case (i) (Section II.2) solutions are sought wherein $U_1$, $U_3$ and $\varphi$ only exist in the crystal. This type of solution uses only the $a$ values which
lead to $\beta_2 = 0$ and $\beta_1$, $\beta_3$, and $\beta_4$ non-zero. Also in this problem solutions
where $U_2$ only exists in the crystal must be considered (e.g. Love waves or
the piezoelectric perturbations thereof). This solution stems from the root
$\alpha$ which leads to non-zero $\beta_2$ and zero $\beta_1$, $\beta_3$, and $\beta_4$.

As in the previously considered degenerate cases the determinant of
the boundary condition matrix $L$ factors into the product of two determinants.
The determinant which corresponds to the $U_1$, $U_3$, $\varphi$ solutions is denoted $M$
and assumes the form,

\[
M = \begin{vmatrix}
L_{11} & L_{12} & L_{13} & L_{14} & L_{18} & L_{19} & L_{1,10} & L_{1,11} & L_{1,12} & L_{1,13} \\
L_{41} & - & - & - & - & - & - & - & - & - \\
L_{91} & - & - & - & - & - & - & - & - & - \\
L_{10,1} & - & - & - & - & - & - & - & - & - \\
L_{11,1} & - & - & - & - & - & - & - & - & - \\
L_{12,1} & - & - & - & - & - & - & - & - & - \\
L_{13,1} & - & - & - & - & - & - & - & - & L_{13,13}
\end{vmatrix}
\]

The solution for the $U_2$ case depends upon existence of zeros of a determinant
$N$ where $N$ is a $(3 \times 3)$ determinant.

\[
N = \begin{vmatrix}
L_{25} & L_{26} & L_{27} \\
L_{55} & L_{56} & L_{57} \\
L_{85} & L_{86} & L_{87}
\end{vmatrix}
\]

For case (2) (Section II.2) solutions wherein only $U_1$ and $U_3$ exist are considered.
This type of solution uses only those $\alpha$'s which lead to zero $\beta_2$ and $\beta_4$ and non-
zero $\beta_1$ and $\beta_3$. Solutions where only $U_2$ and $\varphi$ exist also must be considered.
This solution employs the $\alpha$'s which give non-zero $\beta_2$ and $\beta_4$ but zero $\beta_1$ and $\beta_3$. 
The solution for the $U_1$, $U_3$ case depends upon existence of zeros of a determinant $P$ (6 x 6), where,

\[
P = \begin{vmatrix}
L_{11} & L_{12} & L_{13} & L_{14} & L_{17} & L_{18} \\
L_{31} & & & & & \\
L_{41} & & & & & \\
L_{61} & & & & & \\
L_{71} & & & & & \\
L_{81} & & & & & 
\end{vmatrix}
\]

The solution for the $U_2$, $U_4$ case depends upon existence of zeros of a determinant $Q$ (7 x 7), which assumes the form,

\[
Q = \begin{vmatrix}
L_{25} & L_{26} & L_{29} & L_{2,10} & L_{2,11} & L_{2,12} & L_{2,13} \\
L_{55} & & & & & & \\
L_{85} & & & & & & \\
L_{10,5} & & & & & & \\
L_{11,5} & & & & & & \\
L_{12,5} & & & & & & \\
L_{13,5} & & & & & & 
\end{vmatrix}
\]

**Degenerate Cases (Non-Piezoelectric Substrate)**

The degenerate cases $U_1$, $U_3$, only or $U_2$ only involve one $\alpha$ with zero $\beta_1$, $\beta_3$, and non-zero $\beta_2$ and two $\alpha$'s with zero $\beta_2$ and non-zero $\beta_1$, $\beta_3$. The $U_1$, $U_3$ case requires the investigation of the roots of a determinant of the same form as $P$ except for relabeling the columns due to a relabeling of the $\alpha$'s. This determinant is designated $R$ and has the form,
Solutions with $U_2$ only (Love waves) lead to the consideration of the roots of a determinant which is identical to $N$.

\[
R = \begin{vmatrix}
L_{11} & L_{12} & L_{13} & L_{14} & L_{18} & L_{19} \\
L_{31} & - & - & - & - & - \\
L_{41} & - & - & - & - & - \\
L_{61} & - & - & - & - & - \\
L_{71} & - & - & - & - & - \\
L_{91} & - & - & - & - & - \\
\end{vmatrix}
\]
III. ANALYSIS OF AN ELECTRIC CURRENT LINE SOURCE ABOVE A PIEZOELECTRIC HALF-SPACE

In this section a study is made of the excitation of piezoelectric waves by means of an interdigital electrode transducer. Arbitrary crystals and crystal orientations are considered as in the preceding chapter. The problem is treated from a field theory point of view and is case in the formalism of a Green's function solution. Due to the complexity of the problem it is expedient to make some simplifying assumptions before the analysis is attempted. Consequently, it is assumed that the coupling between the individual strips can be neglected and that the current on the strips can be approximated by an assumed current distribution. Furthermore it is assumed that only current flow normal to the array is of importance in exciting the piezoelectric waves and that the strips of the array can be considered to be of infinite extent thus reducing the problem to a two dimensional one.

With these assumptions in mind the Green's function sought is one for an infinitesimal two dimensional electric dipole above a piezoelectric substrate as illustrated in Figure 4.

\[ \nabla \times \nabla \times E = j \omega \mu_0 J + \omega^2 \mu_0 D \]  

(45)
For the two dimensional dipole \( \vec{J} = \hat{U}_x \delta(z-z_0) \delta(x-x_0) \) and equation (45) reduces to

\[
\nabla^2 \vec{E} + k_0^2 \vec{E} = -j \mu_0 \left[ 1 + \frac{\nabla^2}{k_0^2} \right] \cdot \vec{J}
\]

where \( 1 \) is the unit dyadic, \( k_0 = \frac{2\pi}{\lambda_0} \), and \( \lambda_0 \) is the free space wavelength.

Setting \( \vec{E} = (1 + \nabla^2/k_0^2) \hat{G} \) it is easily seen that

\[

\nabla^2 \hat{G} + k_0^2 \hat{G} = -j \mu_0 \hat{U}_x \delta(z-z_0) \delta(x-x_0) \hat{U}_x \tag{46}
\]

A particular solution to equation (46) is

\[
\hat{G} = -\hat{U}_x \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{j \sqrt{k_0^2 - k_x^2} |z-z_0|}{\sqrt{k_0^2 - k_x^2}} e^{jk_x(x-x_0)} dk_x \tag{47}
\]

A particular solution for \( \vec{E} \) (viz. \( \vec{E}_p \)) is derivable from \( \hat{G} \) and in the region

\( 0 < z < z_0 \) the following expression results, namely,

\[
\vec{E}_p = \frac{1}{4\pi} \sqrt{\mu_0 / \varepsilon_0} \left\{ -\hat{U}_x \int_{-\infty}^{\infty} \sqrt{1-k_x^2} e^{-j \sqrt{1-k_x^2} (z-z_0)} e^{jk_x(x-x_0)} dk_x \right. \\
+ \left. \hat{U}_z \int_{-\infty}^{\infty} k_x e^{-j \sqrt{1-k_x^2} (z-z_0)} e^{jk_x(x-x_0)} dk_x \right\} \tag{48}
\]

where \( k_x = k_x/k_0 \), \( \tilde{x} = k_0 x \), and \( \tilde{z} = k_z z \). A solution for the total electric field may be obtained as a superposition of \( \vec{E}_p \) and a general solution of the homogeneous equation \( \nabla^2 \vec{E} + k_0^2 \vec{E} = 0 \). That is, in the region \( 0 < z < z_0 \), \( \vec{E} \) may be written in the form
where \( A_0(K_x) \) and \( B_0(K_x) \) are functions of \( K_x \) which are determined through the application of the boundary conditions on the total fields at \( z = 0 \).

In the crystal medium \((z < 0)\) the mechanical displacement fields and the electric fields may be expressed as follows:

\[
\begin{align*}
U_i &= \int_{-\infty}^{\infty} U_i(K_x) e^{-jK_x(z-z_0)} e^{jk_x(x-x_0)} \, dk_x \\
E_i &= \int_{-\infty}^{\infty} E_i(K_x) e^{-jK_x(z-z_0)} e^{jk_x(x-x_0)} \, dk_x
\end{align*}
\]

When the above integral representations are substituted into the differential equations for the crystal, viz.

\[
\begin{align*}
C_{ijk} U_k, t_l &- e_{klj} E_k, i = \rho U_j \\
\nabla \times \nabla \times \vec{D} &= \omega^2 \mu_0 \vec{D}
\end{align*}
\]

there results a linear system of homogeneous equations for the amplitudes \( U_i(K_x) \) and \( E_i(K_x) \). The determinant of the coefficients of \( U_i \) and \( E_i \) must vanish for a non-trivial solution to exist, namely,
\[
\begin{array}{|c|c|c|c|c|c|}
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
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& & & & & \\
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& & & & & \\
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& & & & & \\
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& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
\end{array}
\]
In the above equation, \( C \) is the velocity of light. Expanding this determinant leads to a 10'th order algebraic equation for \( K_z \) as a function of \( K_y \).

For a given \( K_x \) in the range \(-\infty < K_x < \infty\) there will be 10 values of \( K_z \) satisfying the determinental equation but only 5 will be admissable (representing field solutions that are bounded as \( z \rightarrow -\infty \) and have the form of outgoing waves in the region \( z < 0 \)). For each usable \( K_z \), it is necessary to solve the homogeneous system for the corresponding field amplitudes \( U_1, \psi_1 \).

Thus \( U_1 \) and \( E_1 \) can be expressed as follows:

\[
U_1 = \int_{-\infty}^{\infty} \sum_{n=1}^{5} A_n(K_x) U_1^{(n)}(K_x) e^{-jK_z(z-z_0)} e^{jk_x(x-x_0)} dk_x
\]

\[
E_1 = \int_{-\infty}^{\infty} \sum_{n=1}^{5} A_n(K_x) \psi_1^{(n)}(K_x) e^{-jK_z(z-z_0)} e^{jk_x(x-x_0)} dk_x
\]

where \( A_n(K_x) \) are unknown amplitude coefficients to be determined by an application of the boundary conditions. The magnetic field in the crystal medium and in the vacuum can be written in a similar fashion and is derivable from the equation

\[
\nabla \times \mathbf{E} = j\omega \mu_0 \mathbf{H}
\]

The boundary conditions imposed on the field solutions at \( z = 0 \) are as follows:

- Continuity of \( T_{3j} \) for \( j = 1, 2, 3 \)
- Continuity of \( B_1 \) and \( E_2 \)
- Continuity of \( H_1 \) and \( H_2 \)

The imposition of these conditions leads to the following set of equations in the amplitude coefficients \( A_0, B_0, A_n \). The limiting case \( z_0 \rightarrow 0 \) has been taken in the following since the electrodes will be located on the crystal surface at \( z = 0 \).
Continuity of $T_{3j}$, $j = 1, 2, 3$

\[
\sum_{n=1}^{5} \left\{ \left[ jK_x C_{15} - jK_z C_{55} \right] U_1^{(n)} + \left[ jK_x C_{56} - jK_z C_{45} \right] U_2^{(n)} \right. \\
\left. + \left[ jK_x C_{55} - jK_z C_{35} \right] U_3^{(n)} \right\} = 0
\]

\[
\sum_{n=1}^{5} \left\{ \left[ jK_x C_{14} - jK_z C_{45} \right] U_1^{(n)} + \left[ jK_x C_{46} - jK_z C_{44} \right] U_2^{(n)} \right. \\
\left. + \left[ jK_x C_{45} - jK_z C_{34} \right] U_3^{(n)} \right\} = 0
\]  \hspace{0.5cm} (54)

\[
\sum_{n=1}^{5} \left\{ \left[ jK_x C_{13} - jK_z C_{35} \right] U_1^{(n)} + \left[ jK_x C_{36} - jK_z C_{34} \right] U_2^{(n)} \right. \\
\left. + \left[ jK_x C_{35} - jK_z C_{33} \right] U_3^{(n)} \right\} = 0
\]

Continuity of $E_1, E_2$

\[
\sum_{n=1}^{5} \psi_1^{(n)} A_n - B_0 = -\frac{1}{4\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{1 - k^2} \hspace{0.5cm} (55)
\]

\[
\sum_{n=1}^{5} \psi_2^{(n)} A_n - A_0 = 0
\]

Continuity of $H_1, H_2$

\[
\sum_{n=1}^{5} jk_z \psi_2^{(n)} A_n + j 1 - \frac{k^2}{x} A_0 = 0
\]

\[
\sum_{n=1}^{5} \left[ jk_z \psi_1^{(n)} + jk_x \psi_3^{(n)} \right] A_n + \left[ j \sqrt{1 - k^2} + j \frac{k^2}{x} \right] B_0 = \frac{1}{4\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}} \hspace{0.5cm} (56)
\]
In practice the solution of equations (54), (55), and (56) would be performed numerically on a computer as a function of $K_x$.

The integral expressions for the total mechanical displacements and electromagnetic field components follow from the solution of the system of equations described above. An asymptotic evaluation of the resulting integrals may be employed to obtain formal expressions for the physical quantities of significant interest such as the surface wave fields, power in the surface wave, total power input to the crystal by the transducer, and the bulk wave scattered amplitude pattern. The expressions for the aforementioned quantities are very formidable and would require an extensive amount of numerical computation to obtain even limited information. Consequently, it was decided to abandon this approach and the numerical implementation of the theoretical analysis was not carried out.
IV. COMPUTER PROGRAM OUTLINES

This section describes the use and programming format of the computer programs which were written to implement the numerical analysis of the various surface wave propagation problems described in Section II.

1. Surface Waves on Piezoelectric Crystals in the Presence of Infinitesimally Thin Electric and "Magnetic" Conductors

This computer program is divided into two parts: Part A is concerned with an isotropic elastic conductor (such as gold) of finite thickness above a piezoelectric substrate (such as lithium niobate); Part B is concerned with an infinitesimally thin electric or magnetic conductor above a piezoelectric substrate. All information necessary for the operation of the program is described below. For example, an initial guess of the surface wave velocity is required. From this information, the program refines the initial guess, resulting in a velocity accurate to input specifications.

The program is set up to run on an IBM 7094, and the form of input is FORTRAN Namelist input. Although it is discussed in this document, it is suggested that those not familiar with Namelist input read the appropriate sections in a Fortran manual.

Deck Setup
The $ID$ and $TCP$ control cards must be supplied by the user; the remaining control cards are already in the program deck. The data deck (i.e., the input data for the program) utilizes Namelist input. Two input sections are required: the first describes the parameters of the substrate crystal; the second provides the remainder of the information necessary for the execution of the program.

The first data set is called CONST. This set includes the piezoelectric, elastic and dielectric constants. Column 2 of the data card contains a dollar sign ($$) and columns 3-7 contain the letters CONST. The constants begin in column 9 of the first card and continue up to column 72; they then continue to columns 2-72 of each succeeding card for as many cards as needed. The general form of the data to be input is:

Variable name = 1st value, 2nd value, ..., last value.

For example, the piezoelectric constants ($e_{ip}$), called P in the program, could be input as:

\[ P = 0.,0.,0.,0.,3.7,-2.5,...,0., \]

There are 18 piezoelectric constants and they should be input in the following order:

\[ \begin{align*}
& e_{11} \\
& e_{12} \\
& e_{13} \\
& e_{14} \\
& e_{15} \\
& e_{16} \\
& e_{21} \\
& e_{22} \\
& e_{23} \\
& e_{24} \\
& e_{25} 
\end{align*} \]
\[ e_{26} \]
\[ e_{31} \]
\[ e_{32} \]
\[ e_{33} \]
\[ e_{34} \]
\[ e_{35} \]
\[ e_{36} \]

i.e. \[ P = e_{11}, e_{12}, \ldots, e_{36} \]

In the program the transformed piezoelectric constants \( e'_{ij} \) are printed out as

\[
E_1 = e'_{11} \\
E_2 = e'_{13} \\
E_3 = e'_{14} \\
E_4 = e'_{15} \\
E_5 = e'_{16} \\
E_6 = e'_{31} \\
E_7 = e'_{33} \\
E_8 = e'_{34} \\
E_9 = e'_{35} \\
E_{10} = e'_{36} \\
E_{11} = e'_{12} \\
E_{12} = e'_{32} \\
E_{13} = e'_{21} \\
E_{14} = e'_{23} \\
E_{15} = e'_{24} \\
E_{16} = e'_{25} \\
E_{17} = e'_{26} \]
The transformed constant $e_{22}'$ is never used and therefore is not printed out.

The elastic constants $(C_{pq})$, called $G$ in the program, are next. Immediately following the comma (,) behind the last piezoelectric constant (excluding blanks), print:

\[ G = \]

followed by the 21 values of the elastic constants in the following order, separating each variable by a comma:

\[ C_{11}, C_{22}, C_{33}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{23}, C_{24}, C_{25}, C_{26}, C_{34}, C_{35}, C_{36}, C_{44}, C_{45}, C_{46}, C_{55}, C_{56}, C_{66} \]
In the program the transformed elastic constants $C'_{ij}$ are printed out as:

\[
\begin{align*}
C_1 &= c'_{11} \\
C_2 &= c'_{13} \\
C_3 &= c'_{14} \\
C_4 &= c'_{15} \\
C_5 &= c'_{33} \\
C_6 &= c'_{34} \\
C_7 &= c'_{35} \\
C_8 &= c'_{36} \\
C_9 &= c'_{44} \\
C_{10} &= c'_{45} \\
C_{11} &= c'_{46} \\
C_{12} &= c'_{55} \\
C_{13} &= c'_{56} \\
C_{14} &= c'_{66} \\
C_{15} &= c'_{16} \\
C_{16} &= c'_{12} \\
C_{17} &= c'_{25} \\
C_{18} &= c'_{26} \\
C_{19} &= c'_{24} \\
C_{20} &= c'_{23}
\end{align*}
\]

The transformed constant $c'_{22}$ is not used and therefore not printed out.
The dielectric constants ($\varepsilon_{ij}$), called BPS in the program, are the last constants to be entered. They should be entered following the comma after the last value of the elastic coefficients, as

\[ \text{BPS} = \]

followed by 9 values of BPS in the following order, separating each variable by a comma:

\[
\begin{align*}
\varepsilon_{11} \\
\varepsilon_{12} \\
\varepsilon_{13} \\
\varepsilon_{21} \\
\varepsilon_{22} \\
\varepsilon_{23} \\
\varepsilon_{31} \\
\varepsilon_{32} \\
\varepsilon_{33}
\end{align*}
\]

In the program the transformed dielectric constants $\varepsilon'_{ij}$ are printed out as:

\[
\begin{align*}
T_1 &= \varepsilon'_{11} \\
T_2 &= \varepsilon'_{13} \\
T_3 &= \varepsilon'_{33} \\
T_4 &= \varepsilon'_{21} \\
T_5 &= \varepsilon'_{23}
\end{align*}
\]
The transformed constant $c'_{22}$ is never used and therefore is not printed out.

After the last value of EPS, namely $c_{33}$, print a dollar sign ($) instead of a comma. That is,

$$\text{EPS} = e_{11}, e_{12}, e_{13}, e_{21}, e_{22}, e_{23}, e_{31}, e_{32}, e_{33}.$$ 

This signals the end of the first data set.

The second data set is called "INPUT." $\$\text{INPUT}$ must be printed in columns 2-7 of the next card (following the EPS data). Then each input parameter should be entered, followed by a comma (except the last value, which should be followed by a dollar sign, $\$\$). The following is a definition of each input parameter (unless otherwise stated, the input parameters will refer to both Part A and Part B):
<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Names</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUA</td>
<td>$\mu$ (Part A)</td>
<td>Lame's constants for elastic conductor</td>
</tr>
<tr>
<td>LAMDA A</td>
<td>$\lambda$ (Part A)</td>
<td></td>
</tr>
<tr>
<td>RHOA</td>
<td>$\rho$</td>
<td>Mass density of elastic conductor</td>
</tr>
<tr>
<td>LAMDA B</td>
<td>$\lambda$ (Part B)</td>
<td>Buler Angles</td>
</tr>
<tr>
<td>MUB</td>
<td>$\mu$ (Part B)</td>
<td></td>
</tr>
<tr>
<td>NUB</td>
<td>$\nu$ (Part B)</td>
<td></td>
</tr>
<tr>
<td>RHOB</td>
<td>$\rho$ (Part B)</td>
<td>Mass density of crystal</td>
</tr>
<tr>
<td>VS</td>
<td>$v_s$</td>
<td>Initial guess to a velocity. This initial value will be used to find a final velocity, $\bar{v}_s$ such that $</td>
</tr>
<tr>
<td>KS</td>
<td>$k_s$</td>
<td>Can take on two values: $k_s = 0$ for Part B, $k_s = 1$ for Part A</td>
</tr>
<tr>
<td>EPS LON</td>
<td>$\epsilon$</td>
<td>A positive number used as a convergence criterion. When $</td>
</tr>
<tr>
<td>WH</td>
<td>$uh$</td>
<td>Normalized height of conducting wall or magnetic wall (Part B) Normalized thickness of elastic conductor (Part A). To input $uh = \infty$, set WH &gt; 10^{10}.</td>
</tr>
<tr>
<td>WXA</td>
<td>$ux_a$ (Part A)</td>
<td>Normalized distance into elastic conductor</td>
</tr>
<tr>
<td>WXB</td>
<td>$ux_b$ (Part A)</td>
<td>Normalized distance into crystal</td>
</tr>
<tr>
<td>KL</td>
<td>$K_L$ (Part B)</td>
<td>$K_L$ is normally 0. However, if the electric wall case is being run (see $K_M$) and if $wh = 0$, then $K_L$ should be set to 1.</td>
</tr>
<tr>
<td>KM</td>
<td>$K_M$ (Part B)</td>
<td>This can take on two values: $K_M = 0$ electric wall, $K_M = 1$ magnetic wall</td>
</tr>
<tr>
<td>Input Name</td>
<td>Equation Names</td>
<td>Definition</td>
</tr>
<tr>
<td>------------</td>
<td>----------------</td>
<td>------------</td>
</tr>
<tr>
<td>MAX</td>
<td>--</td>
<td>Since an iteration scheme is used for convergence for a final root $v_s$, there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 10). If MAX is set to zero (MAX = 0) the determinant $</td>
</tr>
<tr>
<td>ICHECK</td>
<td>--</td>
<td>A logical parameter which controls the use of a checkout option. If ICHECK = .FALSE., all FINAL ANSWERS* are computed in addition to the evaluation of the determinant $</td>
</tr>
<tr>
<td>DVS</td>
<td>$\Delta v_s$</td>
<td>Increment to be used for $v_s$ when ICHECK = .TRUE. (DVS $\neq 0$)</td>
</tr>
<tr>
<td>VSMAX</td>
<td>$v_{s_{\text{max}}}$</td>
<td>Maximum value of $v_s$ to be used when DVS $\neq 0$.</td>
</tr>
<tr>
<td>EPSO</td>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space</td>
</tr>
<tr>
<td>WX</td>
<td>$u_x$ (Part B)</td>
<td>Normalized distance into crystal</td>
</tr>
<tr>
<td>DNU</td>
<td>$\Delta v$</td>
<td>If the user wishes to vary $v$ (NUB) from some initial value, $v_i$ to some final value, $v_{\text{max}}$, in steps of $\Delta v$, then set DNU equal to the steps desired; also, see NUMAX.</td>
</tr>
<tr>
<td>NUMAX</td>
<td>$v_{\text{max}}$</td>
<td>The maximum value of $v$ (see DNU). $v_{\text{max}}$ is only used when DNU $\neq 0$.</td>
</tr>
<tr>
<td>DWX</td>
<td>$\Delta u_x$</td>
<td>An increment for $u_x$, similar to DNU. If DWX = 0, then $u_x$ is not incremented.</td>
</tr>
</tbody>
</table>

*The FINAL ANSWERS consist of the partial field relative amplitudes ($\Delta u$), stress components, strain components, time average power flow, electric and mechanical displacements, electric potential, and electric field.
### Input Names and Equations

<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Names</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>WXMAX</td>
<td>( w_{x\text{max}} )</td>
<td>The maximum value of ( w_x ) (see DWX). ( w_{x\text{max}} ) is only used when ( DWX \neq 0 ).</td>
</tr>
<tr>
<td>TITLE</td>
<td>--</td>
<td>An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: ( TITLE = nH ) name of crystal, where ( n ) is the number of characters following the H (including blanks). For example ( TITLE = 6HQUARTZ ).</td>
</tr>
<tr>
<td>REPEAT</td>
<td>--</td>
<td>REPEAT is a logical variable and in its usage, can take only one value: <code>.TRUE.</code>. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input as <code>.TRUE.</code>. This means that the $CONST data will have to be input again (in the other cases above, $CONST would not have to be input again).</td>
</tr>
<tr>
<td>HXAGNL</td>
<td>--</td>
<td>Parameter which controls the calculation of betas (( \beta )'s) for a hexagonal crystal (such as zinc oxide) <code>.TRUE.</code> hexagonal crystal (use special technique) <code>.FALSE.</code> non-hexagonal crystal (use normal procedure)</td>
</tr>
<tr>
<td>VSINC</td>
<td>--</td>
<td>VSINC = <code>.TRUE.</code> New estimates of initial velocity ( v_0 ) are computed using a linear fit to the two previous values. (Used when ( NUB ) varies over a range ( NUB, NUB + DNU, \ldots, NUMAX )) VSINC = <code>.FALSE.</code> The same initial estimate of velocity is used for all values over the specified range of ( NUB ).</td>
</tr>
</tbody>
</table>
The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

- **ROOTS** Print the roots of the polynomial each time they are calculated.
- **COEFF** Print the constants $E, C$, and $T$ (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants $P, G$, and $E$.
- **DETERM** Print the $L$ matrix and the value of the determinant.
- **POLY** Print the coefficients of the 8th order polynomial.
- **BETA** Print the values of $\beta_{ij}$.
- **ALPHA** Print $\alpha_A$'s, Part A.
- **ALL** Print all of the above.

The manner in which the above listed parameters in the $\$INPUT$ data set are input is best illustrated by an example (assume Part B is being run):

$\$INPUT  
MUB = 90., LAMDAB = 90., NUB = 100., RHOB = 4700.,  
VS = 3400., KS = 0., EPSLON = 1.E-11, WH = 0., KL = 1.,  
KM = 0., WX = 0., DWX = 10., WXMAX = 100.,  
title = 1SHLITHIUM NIOBATE

$uh$ is zero in the above example. To input $uh = \infty$, set $uh > 10^{10}$. Note that some of the values discussed in the list are not present in the above example. This is because either they are not required or the program assumed nominal values. A nominal value is a value that a parameter will take on if no other value is input. In the above example, $MAX, EPSO$, and $DNU$ take on their nominal values of $10, 8.85 \times 10^{-12}$, and $0$, respectively. It is not necessary to input NUMAX since $DNU = 0$; all parameters referring to Part A are not necessary since Part B is being run; and all the logical parameters take on their nominal value of false. The following is a complete list of nominal values:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUA</td>
<td>$2.85 \times 10^{10}$</td>
</tr>
<tr>
<td>LAMDA A</td>
<td>$1.5 \times 10^{11}$</td>
</tr>
<tr>
<td>RHOA</td>
<td>$1.888 \times 10^{4}$</td>
</tr>
<tr>
<td>VS</td>
<td>3000</td>
</tr>
<tr>
<td>EPSLON</td>
<td>$1 \times 10^{-11}$</td>
</tr>
<tr>
<td>KL</td>
<td>0</td>
</tr>
<tr>
<td>KM</td>
<td>0</td>
</tr>
<tr>
<td>MAX</td>
<td>10</td>
</tr>
<tr>
<td>EPSO</td>
<td>$8.85 \times 10^{-12}$</td>
</tr>
<tr>
<td>DNU</td>
<td>0</td>
</tr>
<tr>
<td>DWX</td>
<td>0</td>
</tr>
<tr>
<td>DVS</td>
<td>0</td>
</tr>
<tr>
<td>TITLE</td>
<td>Lithium Niobate</td>
</tr>
<tr>
<td>ROOTS</td>
<td></td>
</tr>
<tr>
<td>COEFF</td>
<td></td>
</tr>
<tr>
<td>DETERM</td>
<td></td>
</tr>
<tr>
<td>POLY</td>
<td></td>
</tr>
<tr>
<td>BETA</td>
<td>.FALSE.</td>
</tr>
<tr>
<td>ALL</td>
<td></td>
</tr>
<tr>
<td>REPPEAT</td>
<td></td>
</tr>
<tr>
<td>ALPHA</td>
<td></td>
</tr>
</tbody>
</table>
Sample Data Decks:

The following sample data deck, illustrated on the attached code sheet, gives an example of three data runs: the first is a 90-90-100 degree cut of lithium niobate. The second is the same, except for a new value of \( \alpha \). The third is a 0, 0, -90 cut of quartz (note that REPEAT is set to true in the second case, just prior to the case when new coefficients are to be input).
The following is a description of the program flow diagram provided at the end of this section.

(1) First, the nominal data values are set up in the program. These values are assumed by certain parameters in the program unless new values are specified. Following this, the program reads in the elastic $G$, piezoelectric $P$, and dielectric $\varepsilon$ constants (CONST DATA) of the substrate medium. Finally, the remaining input data is read in (INPUT).

Next, subroutine SETCTE is called to perform the Euler transformation to obtain the elastic $C$, piezoelectric $C$, and dielectric $C$ constants relative to the input coordinate system as specified by the constants $\lambda$, $\mu$, and $\nu$. At this point subroutine ROOT is called to perform the calculations leading to the evaluation of the determinant of the boundary condition matrix ($L$ matrix of the analysis). The determinant is referred to as $F(\text{VS})$ since it is evaluated as a function of velocity ($\text{VS}$).

There is an option in the program to use a root finding scheme to minimize $|F(\text{VS})|$ or simply to increment $\text{VS}$ in steps of $D\text{VS}$ and calculated $F(\text{VS})$ at each value. To perform these various calculations at a particular velocity ($\text{VS}$) ROOT calls subroutine $F$ which is described in detail below (in Determination of $F(\text{VS})$).

After exiting from ROOT and returning to the main program logical checks are made to establish the type of case considered in ROOT. Depending upon the results of these checks the values of the amplitudes of the partial surface wave fields are computed ($A(t)$ of analysis section). In the program these are called ETA(1), ETA(2), etc. The program now proceeds to compute the magnitude (MAG $U(I)$) and phase (PHASE $U(I)$) of the mechanical displacements ($\vec{U}(I)$, $I=1,2,3$) and electric potential ($\vec{U}(4)$). Next subroutine PIFUN is called to compute the time average flow ($P1M, P2M$) followed by the computation of the stress components ($TW31$, $TW32$, $TW33$, $TW11$, $TW12$, $TW22$) in subroutine TFUN. Subroutine SFUN then implements the calculation of the strain components ($S11$, $S33$, $S12$, $S13$, $S23$). Finally, the electric field ($E1$, $E3$) and electric displacement ($D1$, $D2$, $D3$) are computed. All the above quantities are evaluated as a function of normalized distance ($WX$) into the crystal. They can be computed at incremented values of $WX$ for any specified initial and final values.
The velocity (VS) can be incremented if desired (up to some specified maximum value, VSMAX) and the steps in ROOT and that which follows are repeated for each new velocity. Thus it is possible to plot the determinant as a function of velocity. After VSMAX is reached there is an option to increment the third Euler angle (NU) and repeat the steps from SETCTE on. When this has been completed the program returns to read in new CONST DATA if the crystal is being changed or to read in new INPUT DATA if the crystal is to remain unchanged but the orientation is to be changed. After all data from both sources has been exhausted the program stops.

**Determination of F(VS)**

Subroutine F calls subroutine STRIP to compute the coefficients of the eighth order polynomial equation in \( \alpha \).* Next subroutine CROOT calculates the 8 roots (ALFA(I), I = 1, 8) of the polynomial equation by Muller's method. If the medium is non-piezoelectric the solution for the roots involves two extraneous roots which are rendered useless by setting them equal to \(-10^{-10}j\). The roots with positive real part are chosen (ALFA(I), I=1,K).

If the medium is piezoelectric and the number of roots with positive real part (K) is equal to 4 the program proceeds to calculate the relative amplitudes (\( \beta_1^{(l)} \) of the analysis section) of the displacement and potential corresponding to each \( \alpha \). These amplitudes are referred to as BETAB(I,J) in the program. The matrix (ACAP, \( \hat{A} \) for simplicity) of coefficients of the amplitudes (\( \beta_1^{(l)} \)) is set up for each \( \alpha \). If the crystal is not hexagonal non-degenerate cases are solved by setting \( \beta_4^{(l)} = 1 \) and solving the first three equations of the system for \( \beta_1^{(l)} \) I = 1, 2, 3. If the crystal is hexagonal one of the \( \alpha \)'s naturally leads to an ill-conditioned system if the first three equations are solved for \( \beta_1^{(l)} \), \( \beta_2^{(l)} \), and \( \beta_3^{(l)} \) in terms of \( \beta_4^{(l)} \). Thus \( \beta_1^{(l)} \) is set equal to \( 10^{-10} \) and the system composed of the second, third, and fourth equation are solved for \( \beta_2^{(l)} \), \( \beta_3^{(l)} \), \( \beta_4^{(l)} \). If the case is degenerate the \( \beta_1^{(l)} \)'s are calculated in the fashion indicated in the analysis.

If \( K < 4 \) the procedure for calculating the \( \beta_1^{(l)} \)'s is dependent upon the value of \( K \). If \( K \leq 1 \) the case terminates since no solution is possible with only one available value of \( \alpha \). If \( K > 1 \), \( \hat{A}_{12} \) and \( \hat{A}_{23} \) are investigated to see if they are identically zero.

*See Appendix IV.
If either $\hat{\lambda}_{12}$ or $\hat{\lambda}_{23}$ is not identically zero the case cannot be degenerate. The program proceeds in one of two possible ways. If the crystal is non-piezoelectric and $K = 3$ the $\beta_i(t)$'s are calculated as indicated in the analysis (i.e., $\beta_1(t) = 0$, $\beta_3(t) = 1$ and the first two equations of the system are solved for $\beta_1(t)$ and $\beta_2(t)$). If either $K \neq 3$ or the crystal is piezoelectric the case terminates. This is due to the fact that if the crystal is piezoelectric and non-degenerate, four $\alpha$'s are necessary for a solution in the general case.

If both $\hat{\lambda}_{12}$ and $\hat{\lambda}_{23}$ are equal to zero, the non-piezoelectric case is degenerate and is treated as follows. $|\hat{\lambda}_{22}|$ is calculated for each value of $\alpha$.

For $K = 2$ it is necessary that $|\hat{\lambda}_{22}|$ be non-zero for both values of $\alpha$ (due to the large magnitude of the individual terms in $\hat{\lambda}_{22}$ it is sufficient to compare $|\hat{\lambda}_{22}|$ to $10^7$). If $|\hat{\lambda}_{22}| > 10^7$ for both values of $\alpha$ then we may set

$$\beta_2(t) = 0, \beta_3(t) = 10^{-10}, \text{ and } \beta_1(t) = -\frac{\lambda_1(t)}{\hat{\lambda}_{22}} \cdot 10^{-10}.$$ 

Otherwise the case is terminated. If $K = 3$ the minimum value of $|\hat{\lambda}_{22}|$ is calculated and the corresponding $\alpha$ is discarded. The $\beta$'s are then calculated for the other two $\alpha$'s from the above formulas.

If $\hat{\lambda}_{12}$ and $\hat{\lambda}_{23}$ are identically equal to zero and the crystal is piezoelectric the program proceeds as follows. $\hat{\lambda}_{24}$ is tested and if equal to zero the first degenerate case of the analysis section must be considered. The case is terminated if $K = 2$ but if $K = 3$ a check is made of $|\hat{\lambda}_{22}|$. If $|\hat{\lambda}_{22}| > 10^7$ for all three $\alpha$'s, the $\beta$'s are calculated as indicated in the analysis. If $|\hat{\lambda}_{22}| < 10^7$ for any of the $\alpha$'s the case is terminated.

If $\hat{\lambda}_{22} \neq 0$ a check of $\hat{\lambda}_{14}$ and $\hat{\lambda}_{34}$ is made. If they are not both identically equal to zero the case is terminated. If both are equal to zero the second degenerate case of the analysis is considered. In this case $|\hat{\lambda}_{22} \hat{\lambda}_{44} - \hat{\lambda}_{24}^2|$ (TERM in the program) is calculated for each $\alpha$. If TERM $> 10^{-5}$ for two of the $\alpha$'s the $\beta_1(t)$ and $\beta_3(t)$ split of the analysis arises and the $\beta$'s are appropriately calculated. If TERM $< 10^{-5}$ for two of the $\alpha$'s the $\beta_2(t)$ and $\beta_4(t)$ split arises and the $\beta$'s are appropriately calculated. Under any other conditions the case is terminated.

Now that the $\alpha$'s and $\beta$'s are known the boundary condition matrix ($\mathbf{L}$) is set up and its determinant evaluated. If the problem of a conducting elastic medium in contact with a piezoelectric or elastic medium is being considered
the $\alpha$'s and $\beta$'s appropriate to the conductor are first evaluated then the appropriate boundary condition matrix is set up and its determinant evaluated. This completes the computation of $F(VS)$ whereupon the subroutine is exited back to the main program.
MAIN PROGRAM
LINB03

SET UP NOMINAL DATA VALUES

READ $CONST DATA (PIEZOELECTRIC, ELASTIC, AND DIELECTRIC CONSTANTS P, G AND EPS)

REPEAT=.FALSE.

VS=VSAVE
KTIME=0

READ INPUT DATA

END OF FILE

(YES) → STOP

(NO)

SNU=NUB
SWX=WX

VSINC (.FALSE.)

(.TRUE.)

VS=VSAVE

KTIME=KTIME+1

KTIME:2

(≠)

COMPUTE NEW ESTIMATE OF INITIAL VELOCITY (VS) USING A LINEAR FIT TO THE TWO PREVIOUS VALUES

1.1

1.2

1.3
\textbf{COMPUTER USED IN CROOT FOR THE ORDERING OF THE ALPHAS}

\textbf{_ROOT USING AN INITIAL ESTIMATE OF VELOCITY, VSO, CALCULATE (a) THE VALUE OF THE DETERMINANT |f(VSO)| AT VSO (MAX = 0) OR (b) A NEW VELOCITY, VS, FOR WHICH |f(VS)| IS LESS THAN EPSILON (MAX > 0)}

\textbf{COMPUTE VSI = 1/VS}
\textbf{ALFAI = 1/ALFAI}

\textbf{TABULATE INPUT DATA TOGETHER WITH COMPUTED VALUES OF VS, |f(VS)|, VSI AND ALFAI}

\textbf{IALF} (.TRUE.)
\textbf{(.FALSE.)}

\textbf{ICHECK} (.TRUE.) \rightarrow 3
\textbf{(.FALSE.)}

\textbf{KS} (=0)
\textbf{(<0)}

\textbf{GOLD} \textbf{LITHIUM}
\textbf{REGULAR CRYSTAL PROBLEM}

\textbf{NON-DEGENERATE CASE FOR WHICH THERE ARE LESS THAN 4 \alpha's WITH A POSITIVE REAL PART - NO SURFACE WAVE EXISTS SO THE CASE IS TERMINATED}

\textbf{L3}
\textbf{L4}
2

COMPUTE FINAL ANSWERS
(REGULAR CRYSTAL PROBLEM)

ICASE

COMPUTE ETA'S

<table>
<thead>
<tr>
<th>ICASE</th>
<th>CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NON-DEGENERATE</td>
</tr>
<tr>
<td></td>
<td>a. PIEZOELECTRIC (NBETA=3)</td>
</tr>
<tr>
<td></td>
<td>b. ZERO-PIEZOELECTRIC (NBETA=2)</td>
</tr>
<tr>
<td>1</td>
<td>1 ROW, 3 ZERO</td>
</tr>
<tr>
<td></td>
<td>a. 4 ALPHA'S</td>
</tr>
<tr>
<td>3</td>
<td>b. 3 ALPHA'S</td>
</tr>
<tr>
<td>2</td>
<td>4 ROW, 2 ZERO</td>
</tr>
<tr>
<td></td>
<td>a. 4 ALPHA'S</td>
</tr>
<tr>
<td></td>
<td>b. LESS THAN 4 ALPHA'S</td>
</tr>
<tr>
<td>4</td>
<td>1. (1, 3) CASE</td>
</tr>
<tr>
<td>5</td>
<td>2. (2, 4) CASE</td>
</tr>
</tbody>
</table>

TABULATE ETA'S

| 2j |

COMPUTE:
MECHANICAL DISPLACEMENT
MAGNITUDE - MAGU(i)
PHASE - PHASEU(i)
(\(i = 1, 3\))

ELECTRIC POTENTIAL
MAGNITUDE - MAGU(4)
PHASE - PHASEU(4)

PEFUN: COMPUTE TIME AVERAGE
POWER FLOW
(P1M, P2M)

TFUN: COMPUTE STRESS
(TW31, TW32, TW33, TW11, TW12, TW22)
SFUN

COMPUTE STRAIN
(S11, S33, S12, S13, S23)

COMPUTE
ELECTRIC FIELD (E1, E3)
ELECTRIC DISPLACEMENT (D1, D2, D3)

TABULATE FINAL ANSWERS

DWX

WX = WX + DWX

WX = WXMAX

WX = SWX
EVALUATE THE DETERMINANT FOR A GIVEN VS NBETA=2: ZERO PIEZOELECTRIC NBETA=3: PIEZOELECTRIC NBETA IS EVALUATED IN SETCTE

IALF=.FALSE. ICASE=0 ICB=0 K1=1 K2=4 L1=1 L2=16 IB1=1 IB3=4

STRIP COMPUTE COEFFICIENTS OF POLYNOMIAL POLY(i), i=1,8

CROOT CALCULATE ROOTS OF POLYNOMIAL ALFA(i), i=1,8 USES MULLER'S METHOD (MULLER,POL)

PIEZOELECTRIC (NO)

(YES)

ELIMINATE THE ALFA CORRESPONDING TO THE POTENTIAL

ALFA(1, I=1, K)

SELECT ROOTS WITH POSITIVE REAL PART

(YES)

(?) 4 ALPHAS

(?) 1

(?) 2
LESS THAN 4 ALPHAS (K=NO. OF ALPHAS)

1 ALPHAL:

NON-DEGENERATE CASE
LESS THAN 4 ALPHAS
CASE TERMINATED

IALF=TRUE.
RETURN

(~2)

NBETA

(=3)

PIEZOLECTRIC

AC24 = TRUE.

(=TRUE)

(=FALSE)

(~2)

AC14 .AND. AC34

(=TRUE)

(=FALSE)

(1)

A12 = 0 AND A23 = 0

A14 = 0 AND A34 = 0

DEGENERATES
4 ROW, 2 ZERO CASE
DEGENERATE ZEBO-PIEZO-ELECTRIC

\[ L_4 \]

1. \( I_1 = 0 \)

2. \( |\lambda_{22}^j| \)

3. \( (\neq 2) K \rightarrow (\neq 3) \)

4. 2 ALPHAS

5. 3 ALPHAS

6. \( CA_{22} \rightarrow \lambda_{22}^j \)

7. \( I_{11} = I_1 + 1 \)

8. \( ICASE = 4 \)

9. CALCULATE 3'S BETAB(I,J)

10. \( I_4 \)
11 = 0
12 = 0

\[ \text{TERM} = | \hat{A}_{22} \hat{A}_{44} \hat{A}_{24} | \]

11 = 11 + 1
\( 1 \leq j \leq k \)

12 = 12 + 1

12:2

1, 3 CASE

ICASE = 4

CALCULATE BETA'S

2, 4 CASE

ICASE = 5

CALCULATE BETA'S
DIGERIVATE
1 ROW, 3 ZERO

LESS THAN 3 ALPHAS
TERM COASE

ICASE=3
N1=0

PRINT ERROR MESSAGE

\[|A_{22}| < 10\] (\(j=1,3\))

\[N1=N1+1\]

CASE TERMINATED

CALCULATE BETA'S
\( \lambda_{12} \neq 0 \) AND/OR \( \lambda_{23} \neq 0 \)

\[ \text{NBETA=2 AND } K=3 \]

\( .FALSE. \)

\( \text{PIEZOLECTRIC NON-DEGENERATE LESS THAN 4 } \alpha \)'S

\( .TRUE. \)

\( \text{ZERO-PIEZOLECTRIC 3 ALPHAS} \)
2

\text{NBETA:3}

(\neq)

\text{PIEZOELECTRIC (4 ALPHAS)}

\text{ROOTS .OR. ALL}

(.\text{TRUE.})

\text{ZERO-PIEZOELECTRIC (3 ALPHAS)}

\text{SET 4thROOT TO ZERO}

\text{ALFAB}_4=0

\text{CALCULATE B}_j\text{'S (BETA}_{1,j})

3
SET UP \( A \) MATRIX

\( \text{NBETA:2} \)

ZERO-PIEZOELECTRIC

COMPUTE \( \varepsilon_{11}^2, \varepsilon_{22}^2 \) FROM \( \tilde{A}_{ij} \), \((i=1, 2, \ldots, 3)\)

SET \( \varepsilon_1^2 = 1, \varepsilon_2^2 = 0 \).

\( k : > \)

SIGMA = 1

ROW, 4 ZEROS

SCALE \( A \) MATRIX

\( \text{ICASE=1} \)

ICASE=2

DEGENERATE (NO)

\( \begin{cases} 1 \text{ ROW, 3 ZEROS} \\ 4 \text{ ROW, 2 ZERO} \end{cases} \)

CALCULATE BETA'S

\( \begin{cases} \text{SOLVE FOR } \varepsilon_{11}' (i=1, 3) \\ \text{USING 1st 3 ROWS OF } A \text{ MATRIX} \end{cases} \)

SET \( \varepsilon_{11}' = 1 \).

\( \begin{cases} \text{SOLVE FOR } \varepsilon_{14}' (i=2, 4) \\ \text{USING LAST 3 ROWS OF } A \text{ MATRIX} \end{cases} \)

SET \( \varepsilon_{14}' = 1 \).

\( \varepsilon_{1}=\varepsilon_{1}^{(1)} \cdot 10^{-10} \) \((i=1, 4)\)
2. **Surface Waves at the Boundary Between a Fluid Medium and Piezoelectric Crystal – Program Description**

The purpose of this program is to determine the complex velocity of propagation of surface waves at the interface between a semi-infinite fluid and a piezoelectric substrate. Input parameters which define the fluid, the piezoelectric medium, and control the use of the program are described on the following pages.

The program is set up to run on the IBM 7094, using FORTRAN IV and Namelist input and the deck set up is identical with that given for the preceding program.

As in the preceding program, two input sections are required: the first describes the material constants of the piezoelectric crystal and the second describes the orientation of the crystal as well as other information pertinent to the execution of the program. The first data set is called CONST and is identical with that presented in Section IV.1. The following is a definition of each input parameter in the "INPUT" data set. Medium A refers to the dielectric (elastic) layer and medium B, to the piezoelectric substrate.
<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Name</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAMDAB</td>
<td>( \lambda_B )</td>
<td>real</td>
<td>Lame constants for Medium B.</td>
</tr>
<tr>
<td>MUB</td>
<td>( \nu_B )</td>
<td>real</td>
<td>Euler angles for Medium B.</td>
</tr>
<tr>
<td>NUB</td>
<td>( \gamma_B )</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>DNU</td>
<td>( \Delta \nu )</td>
<td>real</td>
<td>If the user wishes to vary ( \nu ) (NUB) from some initial value, ( \nu_i ) to some final value, ( \nu_{\text{max}} ), in steps of ( \Delta \nu ), then set DNU equal to the step size desired; also, see NUMAX. (See VSINC)</td>
</tr>
<tr>
<td>NUMAX</td>
<td>( \nu_{\text{max}} )</td>
<td>real</td>
<td>The maximum value of ( \nu ) (see DNU). ( \nu_{\text{max}} ) is only used when DNU ( \neq 0 ).</td>
</tr>
<tr>
<td>VS</td>
<td>( v_s )</td>
<td>real</td>
<td>Initial estimate of velocity. This initial value will be used to find a final velocity, ( v_s ), such that (</td>
</tr>
<tr>
<td>DVS</td>
<td>( \Delta v_s )</td>
<td>real</td>
<td>If the user does not care to use the root-finding scheme in determining a final value for ( v_s ), but wishes, instead, to evaluate the determinant (</td>
</tr>
<tr>
<td>VSMA ( \text{X} )</td>
<td>( v_{s,\text{max}} )</td>
<td>real</td>
<td>Maximum value of ( v_s ) to be used when DVS ( \neq 0 ).</td>
</tr>
<tr>
<td>LAMDA ( \text{A} )</td>
<td>( \lambda_A )</td>
<td>real</td>
<td>Lame constants for Medium A.</td>
</tr>
<tr>
<td>MUA</td>
<td>( \mu_A )</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>RHOA</td>
<td>( \rho_A )</td>
<td>real</td>
<td>Mass density of Medium A.</td>
</tr>
<tr>
<td>RHOB</td>
<td>( \rho_B )</td>
<td>real</td>
<td>Mass density of Medium B.</td>
</tr>
<tr>
<td>EPSLON</td>
<td>( \varepsilon )</td>
<td>real</td>
<td>A positive number used as a convergence criterion by the root-finding scheme (( \text{MAX} &gt; 0 )). If (</td>
</tr>
<tr>
<td>EPSO</td>
<td>( \varepsilon_0 )</td>
<td>real</td>
<td>Permittivity of free space.</td>
</tr>
<tr>
<td>Input Name</td>
<td>Equation Name</td>
<td>Type</td>
<td>Definition</td>
</tr>
<tr>
<td>------------</td>
<td>---------------</td>
<td>------</td>
<td>------------</td>
</tr>
<tr>
<td>EPSA</td>
<td>( \varepsilon_A )</td>
<td>real</td>
<td>Dielectric constant for Medium A.</td>
</tr>
<tr>
<td>WH</td>
<td>( \omega_h )</td>
<td>real</td>
<td>Frequency thickness product.</td>
</tr>
<tr>
<td>WXA</td>
<td>( \omega x_A )</td>
<td>real</td>
<td>Normalized distance into Medium A.</td>
</tr>
<tr>
<td>DWXA</td>
<td>( \Delta \omega x_A )</td>
<td>real</td>
<td>In order to vary ( \omega x_A ) (WXA) from an initial value, ( \omega x_A ), to a final value ( \omega x_{A\text{max}} ), DWXA must be set equal to the desired step size. See WXAMAX.</td>
</tr>
<tr>
<td>WXAMAX</td>
<td>( \omega x_{A\text{max}} )</td>
<td>real</td>
<td>The maximum value of ( \omega x_A ) to be used when DWXA ( \neq 0 ).</td>
</tr>
<tr>
<td>WXB</td>
<td>( \omega x_B )</td>
<td>real</td>
<td>Normalized distance into Medium B.</td>
</tr>
<tr>
<td>DWXB</td>
<td>( \Delta \omega x_B )</td>
<td>real</td>
<td>In order to vary ( \omega x_B ) from an initial value, ( \omega x_B ), to a final value, ( \omega x_{B\text{max}} ), DWXB must be set equal to the step size desired. See WXBMAX.</td>
</tr>
<tr>
<td>WXBMAX</td>
<td>( \omega x_{B\text{max}} )</td>
<td>real</td>
<td>The maximum value of ( \omega x_B ) to be used when DWXB ( \neq 0 ).</td>
</tr>
</tbody>
</table>
| ICHECK     | logical | | ICHECK = .TRUE. – All FINAL ANSWERS* are computed in addition to the evaluation of the determinant \( |f(v_s)| \).
| MAX        | integer | | MAX = 0 the determinant \( |f(v_s)| \) will be evaluated for the particular \( v_s \) value input – the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which \( v_s \) lies. |

*The FINAL RESULTS, which are computed for all values of WXA (dielectric layer) and WXB (piezoelectric layer), include the following:

- Stress Components
- Strain Components
- Time Average Power Flow
- Electric Displacement
- Mechanical Displacement
- Electric Potential Magnitude
- Electric Field
<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Name</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td></td>
<td>BCD</td>
<td>An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: ( \text{TITLE} = nH ) name of crystal, where ( n ) is the number of characters following the ( H ) (including blanks). For example ( \text{TITLE} = 6HQUARTZ )</td>
</tr>
<tr>
<td>HXAGNL</td>
<td></td>
<td>logical</td>
<td>Parameter which controls the calculation of betas ($\beta$s) for a hexagonal crystal (such as zinc oxide)</td>
</tr>
<tr>
<td>VSINC</td>
<td></td>
<td>logical</td>
<td>VSINC = .TRUE. - New estimates of initial velocity ($v_s$) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ..., NUMAX)</td>
</tr>
<tr>
<td>VSINC</td>
<td></td>
<td>logical</td>
<td>VSINC = .FALSE. - The same initial estimate of velocity is used for all values over the specified range of NUB.</td>
</tr>
<tr>
<td>IOP</td>
<td></td>
<td>integer</td>
<td>Degenerate case options (used when exactly four ( \alpha )’s with positive real part occur). IOP = 1 - seek modes of propagation of the Quasi-Rayleigh or Sesawa type. IOP = 2 - seek modes of propagation of Love type.</td>
</tr>
<tr>
<td>REPEAT</td>
<td></td>
<td>logical</td>
<td>REPEAT is a logical variable and in its usage, can take only one value: .TRUE.</td>
</tr>
</tbody>
</table>

If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input.
The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Name</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABCTE</td>
<td></td>
<td></td>
<td>Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.</td>
</tr>
<tr>
<td>ROOTS</td>
<td></td>
<td></td>
<td>Print the roots of the polynomial each time they are calculated.</td>
</tr>
<tr>
<td>BETA</td>
<td></td>
<td></td>
<td>Print the values of $\beta_{ij}$.</td>
</tr>
<tr>
<td>DETERM</td>
<td></td>
<td></td>
<td>Print the value of the determinant.</td>
</tr>
<tr>
<td>COEFF</td>
<td></td>
<td></td>
<td>Print the coefficients of the 8th order polynomial.</td>
</tr>
<tr>
<td>TABL</td>
<td></td>
<td></td>
<td>Print the L matrix (or $\beta$, $\theta$, $\lambda$, etc., when used).</td>
</tr>
<tr>
<td>ALPHA</td>
<td></td>
<td></td>
<td>Print the roots of the polynomial ($\alpha_J$'s) and the re-ordered roots for degenerate cases.</td>
</tr>
<tr>
<td>ALL</td>
<td></td>
<td></td>
<td>Print all of the above.</td>
</tr>
</tbody>
</table>

Data items may be excluded from the input stream at the discretion of the user. Items omitted from the first data set will take on nominal values (i.e.: values assigned within the program). Items omitted from succeeding data sets will take on previously assigned values. The following is a complete list of nominal values:*

*All logical parameters have a nominal value of .FALSE.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAMDAB</td>
<td>0.</td>
</tr>
<tr>
<td>MUB</td>
<td>0.</td>
</tr>
<tr>
<td>NUB</td>
<td>0.</td>
</tr>
<tr>
<td>DNU</td>
<td>0.</td>
</tr>
<tr>
<td>NUMAX</td>
<td>0.</td>
</tr>
<tr>
<td>VS</td>
<td>3000.</td>
</tr>
<tr>
<td>DVS</td>
<td>0.</td>
</tr>
<tr>
<td>VSMAX</td>
<td>0.</td>
</tr>
<tr>
<td>LAMDAA</td>
<td>1.5 \times 10^{11}</td>
</tr>
<tr>
<td>MUA</td>
<td>2.85 \times 10^{10}</td>
</tr>
<tr>
<td>RHOA</td>
<td>1.888 \times 10^{4}</td>
</tr>
<tr>
<td>RHOB</td>
<td>4700.</td>
</tr>
<tr>
<td>EPSLON</td>
<td>1. \times 10^{-11}</td>
</tr>
<tr>
<td>EPSO</td>
<td>8.85 \times 10^{-12}</td>
</tr>
<tr>
<td>EPSA</td>
<td>44.25 \times 10^{-12}</td>
</tr>
<tr>
<td>WH</td>
<td>0.</td>
</tr>
<tr>
<td>WXA</td>
<td>0.</td>
</tr>
<tr>
<td>DWXA</td>
<td>0.</td>
</tr>
<tr>
<td>WXAMAX</td>
<td>0.</td>
</tr>
<tr>
<td>WXB</td>
<td>0.</td>
</tr>
<tr>
<td>DWXB</td>
<td>0.</td>
</tr>
<tr>
<td>WXBMAX</td>
<td>0.</td>
</tr>
<tr>
<td>MAX</td>
<td>15.</td>
</tr>
<tr>
<td>TITLE</td>
<td>LITHIUM NIOBATE</td>
</tr>
<tr>
<td>IOP</td>
<td>1</td>
</tr>
</tbody>
</table>
The computer program flow described below shares a good many features of the program described in Section IV.1. As in the preceding programs the nominal data values are set up first. Next the piezoelectric (P), elastic (G), and dielectric (EPS) constants are read in (CONST DATA). Following this the rest of the input data is entered (INPUT DATA).

At this point subroutine SETCTE is called to compute the transformed piezoelectric (CE), elastic (CC), and dielectric (CT) constants. Next subroutine ROOT is called. ROOT performs the calculations and calls the subroutines necessary to perform the following tasks:

(a) Compute $F(VS)$, the boundary condition determinant,

(b) implement a complex root finding scheme to minimize $|F(VS)|$ as a function of complex velocity $(VS)$,

(c) perform the perturbation analysis.

ROOT calls subroutine F to perform the manipulations necessary to compute the boundary condition determinant. F will be discussed in some detail below.

After exiting from ROOT and returning to the main program the trial velocity $(VS)$ can be incremented if it is desired only to compute $F(VS)$ at specified velocities rather than implement the root finding scheme or the perturbation scheme. When this has been completed the third Euler angle $(NU)$ can be incremented and all of the steps, from the point where SETCTE is called to calculate the transformed piezoelectric, elastic, and dielectric constants, are repeated for each value of $NU$.

Following this the program returns to read in new data in either of the following fashions:

(a) If the crystal is to remain the same but the orientation of the crystal face is changed (new Euler angles) the new data comes from INPUT DATA.

(b) If the crystal itself is changed as well as the Euler angles the data comes from CONST DATA and INPUT DATA in that order.

When all the data has been exhausted the program stops.
Subroutine F

Upon entering subroutine F a check is made to determine whether the perturbation scheme is to be used. If it is not to be used, VS is set equal to VSO (the input value). If the perturbation scheme is to be used a loop is begun which will allow the numerical derivatives of the various determinants to be taken by setting VS = VSO + DEL, VS = VSO - DEL and VS = VSO, in that order. For each of these velocities the necessary determinants needed to evaluate $\Delta V_{S}$ (the perturbed velocity) are computed. The convergence of the numerical derivatives is checked by noting the differences in the computed determinants and $\Delta V_{S}$ as DEL is allowed to take on subsequently smaller values.

The loop is begun by setting counters INIT and IDX both equal to 1. DEL = EPS(INIT) • VSO is calculated and VS is set equal to VSO + DEL. EPS(INIT) is a small number depending upon the value of INIT. Three values EPS(1), EPS(2), and EPS(3) will be used eventually as a convergence test on the numerical derivatives. The program now proceeds to set up the M, N, and P matrices as a function of VS (the P matrix is the cofactor of $M_{46}$ and its determinant has been referred to as K in the analysis section, K = det(P)).

When the determinant of the P matrix (DP(IDX)) has been evaluated for the first time (IDX = 1) a logical check notes that IDX $\neq$ 3 and proceeds to evaluate the determinant of the N matrix (DN(IDX)). At this point another logical check notes that IDX = 1 and proceeds to set VS = VSO - DEL and IDX = 2. The program then returns to set up the new M, N, and P matrices and evaluate DP(IDX) = DP(2). The logical check following the evaluation of DP(IDX) again notes that IDX $\neq$ 3 and therefore evaluates DN(IDX) = DN(2). The logical check following evaluation of DN(IDX) now notes that IDX $\neq$ 1 as it was before. The program therefore proceeds to calculate the numerical derivatives ((det N)’ and (det P)’) by the approximate formulas

$$\frac{\Delta N(INIT)}{(det N)'} = \frac{DN(1)-DN(2)}{2 \cdot DEL} \quad \text{and} \quad \frac{\Delta P(INIT)}{(det P)'} = \frac{DP(1)-DP(2)}{2 \cdot DEL}.$$

Next a new logical check notes that INIT $\neq$ 3 (it is still 1) and returns to the point where INIT and IDX were originally initialized. It now increments INIT by one (i.e. INIT = 2 now) and resets IDX = 1. DEL = EPS(2) • VSO and VS = VSO + DEL are evaluated and all the steps from this point are repeated until finally the numerical derivatives are again taken with the new value of DEL.
Again the logical check at this point notes \( \text{INIT} \neq 3 \) (\( \text{INIT} = 2 \)) and again the point of initialization of \( \text{INIT} \) and \( \text{IDX} \) is entered. \( \text{INIT} \) is again incremented by one (\( \text{INIT} = 3 \) now) and \( \text{IDX} \) is reset to 1. \( \text{DEL} = \text{EPS}(3) \cdot \text{VSO} \) and \( \text{VS} = \text{VSO} + \text{DEL} \) are evaluated again and the subsequent steps again taken until the numerical derivatives are taken again at this third value of \( \text{DEL} \). The logical check following the evaluation of the numerical derivatives now notes that \( \text{INIT} = 3 \) and thus sets \( \text{IDX} = 3 \) and returns to the point where \( \text{VS} \) was first set equal to \( \text{VSO} \). The perturbation loop has been bypassed and \( \text{VS} = \text{VSO} \) is now used to evaluate \( M, N, P \) matrix elements. When the determinant of the \( P \) matrix (\( \text{DP}(\text{IDX}) \)) has been evaluated the subsequent logical check notes that \( \text{IDX} = 3 \) and all the quantities needed have been evaluated. It therefore proceeds to print out the results. After this the main program is re-entered to look for new cases.

The steps for any particular velocity \( (\text{VS}) \) taken to evaluate the elements of the \( M, N, P \) matrices will now be discussed. First quantities \( \text{VSQ} = (\text{VS})^2 \) and \( \{ \text{RVS} = \text{RHOB} \cdot \text{VSQ} \} \) are set up. \( \text{RHOL} \) is the mass density of the liquid while \( \text{RHOB} \) is the mass density of the crystal. Next subroutine \text{STRIP} is called to compute the coefficients of the eighth order polynomial in \( \alpha \) just as was done in the first program. Subroutine \text{CROOT} is now called to calculate the roots \( (\alpha) \) of the polynomial. If a non-piezoelectric case is being considered the two extraneous roots are eliminated as in the first program. The roots with positive real part are now selected (\( \text{ALFAB}(l) = 1, K \)).

If \( K \leq 1 \) the case terminates. If \( K = 2 \) or 3 checks are made of various elements of the matrix of coefficients \( (\hat{A}) \) of the relative field amplitudes \( (\hat{\beta}^{(l)}) \), \( \hat{A}_{12} \) and \( \hat{A}_{23} \) are tested to see if they are identically zero. If they are not both identically zero a degenerate case cannot exist. If the crystal is piezoelectric, then, there are insufficient \( \alpha \)'s and the case terminates. If the crystal is non-piezoelectric and \( K = 2 \) the case terminates also (insufficient \( \alpha \)'s). If the crystal is non-piezoelectric and \( K = 3 \) the appropriate \( \beta \)'s are calculated as indicated in the analysis of the first problem. If, however, \( \hat{A}_{12} \) and \( \hat{A}_{23} \) are both identically zero and a non-piezoelectric case is being considered, it is a degenerate case and is so treated. If the crystal is piezoelectric a check of \( \hat{A}_{24} \) is made. If \( \hat{A}_{24} \) is zero and \( K = 2 \) the program terminates but if \( K = 3 \) the first degenerate case of the analysis section of the first problem has arisen \( (\hat{\beta}_1, \hat{\beta}_3, \hat{\beta}_4 \neq 0, \hat{\beta}_2 = 0) \) and is treated appropriately.
If $\hat{A}_{24}$ is not identically zero a check of $\hat{A}_{14}$ and $\hat{A}_{34}$ is made. If they are not both equal to zero the case terminates since no degenerate case has arisen. If both are identically zero the second degenerate case of the analysis section of the first problem has arisen and is treated accordingly.

If $K = 4$ and a piezoelectric case is being considered or $K = 3$ and a non-piezoelectric case is being considered the $\beta$'s are derived in the fashion indicated in the first program. After the $\beta$'s have been computed the quantity $\text{ARAD} = 1 - \frac{\text{RVSQ}}{\text{LAMDAL}}$ is computed where LAMDAL ($\lambda_d$) is the modulus of compression of the fluid. If no perturbation scheme is to be used the program computes $\text{ALFAL} = \sqrt{\text{ARAD}}$ and tests to see if the imaginary part of $\text{ALFAL}$ ($\text{Im}(\text{ALFAL})$) is equal to zero. If $\text{Im}(\text{ALFAL}) = 0$ the negative square root is taken; otherwise the root is taken so that $\text{Im}(\text{ALFAL}) < 0$ (this was necessary in order that a velocity with positive imaginary part result as a solution). The elements of the $M$ matrix are set up next and the determinant evaluated ($\text{det}(M) = F(VS)$). If the perturbation scheme is to be used the various matrices indicated are set up as indicated earlier and the velocity perturbation $\Delta v_s$ is calculated.
LIQUID

SET UP NOMINAL DATA VALUES

1

READ $CONST DATA PIEZOELECTRIC (E), ELASTIC (G), AND DIELECTRIC (EPS) CONSTANTS

REPEAT=.FALSE.

L

VS=VSAVE

READ $INPUT DATA LAMBDA, MU, NU, VS, ETC.

P

IBOF=0

(f)

END OF FILE ENCOUNTERED READING $INPUT DATA EXECUTION TERMINATED

CVMAX=IVSMAX I EEUINTRIA

SNU=NUEXCTO

T

AD

I6

STOP

1.2

VSAVE=VS

EL=ECON+EPSO

BC=-1/ECON

FIRST=.TRUE.

VSAVE=VS

CALL SUBROUTINE TO COMPUTE TRANSFORMED PIEZOELECTRIC (CE), ELASTIC (CC), AND DIELECTRIC (CT) CONSTANTS

2
CALL SUBROUTINE TO CALCULATE (a) \(|f(v_0)|\) FOR MAX=0 OR (b) VS AT WHICH 
\(|f(v_s)| < \text{EPSLON FOR MAX} > 0\)

\[ \text{PRINT INPUT DATA, TOGETHER WITH } |f(v_0)|, \text{ VS, ALFA, AND ALFAX} \]

\(\text{IF}(\alpha) \text{ (TRUE)}\)
\(\text{(.FALSE.)}\)

NON-DEGENERATE CASE
FOR WHICH THERE ARE LESS THAN 4 ALPHAS WITH A
POSITIVE REAL PART –
NO SURFACE WAVE EXISTS,
SO THE CASE IS TERMINATED
IALF=.FALSE.
ICASE=0
ICB=0
K1=1
K2=4
L1=1
L2=16
IB1=1
IB2=4
INIT=0

PERTURB (.TRUE.)
(.FALSE.)

12

VS=VSO

13

VSQ=VS
RVS=RHS-VSQ
RVSO=RHS-VSQ

STRUC

COMPUTE

COEFFICIENTS

OF

POLYNOMIAL

POLY(I), I=1, 3

SRCH

CALCULATE

ROOTS OF

POLYNOMIAL

ALFA(I), I=1, 3

NBETA:3 > (?)

PIEZOELECTRIC

NBETA=2

ZERO-PIEZOELECTRIC

ELIMINATE THE ALPHA

CORRESPONDING TO THE

POTENTIAL

SET α=(-10, -10)
ROOTS OR ALL (.FALSE.)

PRINT OUT INTERMEDIATE ROOTS OF POLYNOMIAL ALFA(i), i=1, 8

PRE-SET ORDER OF ALPHAS

IAK(k)=k
K=1, 4

SELECT ROOTS WITH POSITIVE REAL PART ALFA(i), i=1, K

(<1) K (=4)

PIEZOELECTRIC 4 ALPHAS

NON-DEGENERATE, LESS THAN 4 CASE TERMINATE CASE

PRINT OUT K

(2)

|ALF=.TRUE.. | RETURN |

(24)

AC12 (.FALSE.)

AC12 .AND. AC25

(.TRUE.) A_{12} # A_{25} = 0

NBETA2 (.TRUE.)

NBETA2 .AND. AC25

ZERO-PIEZOELECTRIC 3 ALPHAS

AC24 (.FALSE.)

AC24 .AND. AC25

(.TRUE.) A_{24} = 0

IC=1

(2)

K=2

DEGENERATE 4 ROW, 2 ZERO CASE - 3 ALPHAS

(33)

AC14 (.FALSE.)

AC14 .AND. AC34

(.TRUE.) A_{14} AND A_{34} = 0

DEGENERATE 4 ROW, 2 ZERO CASE - 3 ALPHAS

(33)
ROOTS .OR. ALL

(PRINT OUT INTERMEDIATE POSITIVE ROOTS)

ALFA(I), I=1, 4

K4=NBETA+1

SET UP A MATRIX

A(i,j), I=1,3

J=1,4

NBETA2

(=)

PENOELECTRIC

SET UP A(4,i):

I=1,4

k1

(N)

CHECK FOR DEGENERATE CASE

1 ROW, 3 ZERO

4 ROW, 2 ZERO

HEXAGONAL CRYSTAL

SOLVE FOR B1K (B1K=1.)

S2K A22 + S3K A23 + S4K A24 = B1K A21

S2K A32 + S3K A33 + S4K A34 = B1K A31

S2K A42 + S3K A43 + S4K A44 = B1K A41

B1K A11 + B2K A12 + B3K A13 = B4K A14

B1K A21 + B2K A22 + B3K A23 = B4K A24

B1K A31 + B2K A32 + B3K A33 = B4K A34

SOLVE FOR B4K (B4K=1.)

SET B1K = B1K \cdot 10^{-10}, I=1,3

B1K = B1K

BETA(I), I=1,4
DEGENERATE DEGENERATE
I ROW, 3 ZERO
4 α's

ICASE=1

RE-ORDER α's
AND CALCULATE β's

3.9

ROOTS
. OR.
ALL
(.TRUE.)

PRINT OUT
RE-ORDERED α's

4

DEGENERATE DEGENERATE
1 ROW, 3 ZERO
3 α's

ICASE=3
K=3

PRINT OUT TYPE
OF CASE f α's

CALCULATE β's

4

DEGENERATE DEGENERATE
4 ROW, 2 ZERO
4 α's

ICASE=2

RE-ORDER α's
AND CALCULATE β's

3.9

4.4 CASE
(NO) → 3.2

(YES)

ICASE=4
K=2

RE-ORDER α's
AND CALCULATE β's

4
DEGENERATE ZERO-PIEZOELECTRIC

ICASE=6

RE-ORDER a's AND CALCULATE b's

SOLVE FOR $\beta_1^k, \beta_2^k$

$\beta_1^kA_{11} + \beta_2^kA_{12} = \beta_2^kA_{13}$

$\beta_1^kA_{21} + \beta_2^kA_{22} = \beta_3^kA_{23}$

SET $\beta_3^k=1, \beta_4^k=0$. 

ZERO-PIEZOELECTRIC
4

BETA .OR. ALL .FALSE.

PRINT OUT BETADQ I=1, 4

ARAD = 1 - RVSO LAMDA

PERTURB .TRUE.

(ALFAL = VRAD

IDX: 3

ALFAL = ARAD

ALFAL = ALFAL

IM(ALFAL) > (ε)

ALFAL = -ALFAL

EM(4, 6) = (RVSO/ALFAL) * 10^-10

CALCULATE EMI(J)

EM(1, 6)

EM(2, 6)

EM(3, 6)

EM(4, 6)

EM(5, 6)

EM(6, 6)
CASE
SET UP M-MATRIX BASED ON ICASE
FR TURB (.FALSE.)

CASE TYPE (.TRUE.)

<table>
<thead>
<tr>
<th>ICASE</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Non-degen piezo (4 a's)</td>
</tr>
<tr>
<td>1</td>
<td>degen(1, 3) 4 a's</td>
</tr>
<tr>
<td>2</td>
<td>degen(2, 4) 4 a's</td>
</tr>
<tr>
<td>3</td>
<td>degen(1, 3) &lt; 4 a's</td>
</tr>
<tr>
<td>4</td>
<td>degen(2, 4) &lt; 4 a's</td>
</tr>
<tr>
<td>5</td>
<td>non-degen 0-piezezo (3 a's)</td>
</tr>
<tr>
<td>6</td>
<td>degen 0-piezezo (&lt;3 a's)</td>
</tr>
</tbody>
</table>

EVALUATE DETR M-MATRIX BASED ON ICASE
F VS
F = F VS
J-1

(.FALSE.)

DETERM
.OF.
.ALL.

(.TRUE.)

RETURN

PRINT OUT
FVS, FVS', VS
PERTURBATION SCHEME

EVALUATE DETERM OF P-MATRIX DP(IDX)

IDX:3

(PRINT OUT M-MATRIX VSO, DP(3), DPP(3))

EVALUATE DETERM OF N-MATRIX DN(IDX)

IDX:1

(PRINT OUT RESULTS EPS, DNP, DPP, ETC.)

F=FVS

RETURN

VS=VSO-DEL
IDX=2

DNP(INIT)=\frac{DN(1)-DN(2)}{2 \times \text{DEL}}
DPP(INIT)=\frac{DP(1)-DP(2)}{2 \times \text{DEL}}

IDX=3

(PRINT OUT M-MATRIX)

F=FVS

RETURN

IDX=3

(PRINT OUT M-MATRIX)

F=FVS

RETURN
3. **Isotropic, Elastic, Dielectric Layer on Piezoelectric Substrate—Program Description**

The purpose of this program is to determine the complex velocity of propagation of surface waves at the interface between a semi-infinite fluid and a piezoelectric substrate. The necessary input and control parameters are described on the following pages.

As with the proceeding program, this program is set up to run on the IBM 7094, using FORTRAN IV and Namelist input and the deck set up is again identical with that described in Section IV.1. Again, two input sections are required: the first describes the material constants of the piezoelectric crystal and the second describes the orientation of the crystal as well as other information pertinent to the execution of the program. The first data set is called CONST and is identical with that described in Section IV.1. The second data set is called "INPUT," and the following is a definition of each input parameter. Medium A refers to the dielectric (elastic) layer and medium B, to the piezoelectric substrate.
<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Name</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAMDAB</td>
<td>$\lambda_B$</td>
<td>real</td>
<td>Euler angles for Medium B.</td>
</tr>
<tr>
<td>MUB</td>
<td>$\mu_B$</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>NUB</td>
<td>$\nu_B$</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>DNU</td>
<td>$\Delta \nu$</td>
<td>real</td>
<td>If the user wishes to vary $\nu$ (NUB) from some initial value, $\nu_i$ to some final value, $\nu_{i+\Delta\nu}$, in steps of $\Delta\nu$, then set DNU equal to the steps desired; also, see NUMAX. (See VSINC)</td>
</tr>
<tr>
<td>NUMAX</td>
<td>$\nu_{\text{max}}$</td>
<td>real</td>
<td>The maximum value of $\nu$ (see DNU). $\nu_{\text{max}}$ is only used when DNU $\neq 0$.</td>
</tr>
<tr>
<td>VS</td>
<td>$\nu_s$</td>
<td>real</td>
<td>Initial estimate of velocity. This initial value will be used to find a final velocity, $\nu_s$, such that $</td>
</tr>
<tr>
<td>DVS</td>
<td>$\Delta \nu_s$</td>
<td>real</td>
<td>If the user does not care to use the root-finding scheme in determining a final value for $\nu_s$, but wishes, instead, to evaluate the determinant $</td>
</tr>
<tr>
<td>VSMAX</td>
<td>$\nu_{s_{\text{max}}}$</td>
<td>real</td>
<td>Maximum value of $\nu_s$ to be used when DVS $\neq 0$.</td>
</tr>
<tr>
<td>LAMDA</td>
<td>$\lambda_A$</td>
<td>real</td>
<td>Lame constants for Medium A.</td>
</tr>
<tr>
<td>MUA</td>
<td>$\mu_A$</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>RHOA</td>
<td>$\rho_A$</td>
<td>real</td>
<td>Mass density of Medium A.</td>
</tr>
<tr>
<td>RHOB</td>
<td>$\rho_B$</td>
<td>real</td>
<td>Mass density of Medium B.</td>
</tr>
<tr>
<td>EPSLON</td>
<td>$\varepsilon$</td>
<td>real</td>
<td>A positive number used as a convergence criterion by the root-finding scheme (MAX $&gt; 0$). If $</td>
</tr>
<tr>
<td>EPSO</td>
<td>$\varepsilon_0$</td>
<td>real</td>
<td>Permittivity of free space.</td>
</tr>
<tr>
<td>Input Name</td>
<td>Equation Name</td>
<td>Type</td>
<td>Definition</td>
</tr>
<tr>
<td>------------</td>
<td>--------------</td>
<td>------</td>
<td>------------</td>
</tr>
<tr>
<td>EPSA</td>
<td>$\varepsilon_A$</td>
<td>real</td>
<td>Dielectric constant for Medium A.</td>
</tr>
<tr>
<td>WH</td>
<td>$u_h$</td>
<td>real</td>
<td>Frequency thickness product.</td>
</tr>
<tr>
<td>WXA</td>
<td>$u_{x_A}$</td>
<td>real</td>
<td>Normalized distance into Medium A.</td>
</tr>
<tr>
<td>DWXA</td>
<td>$\Delta u_{x_A}$</td>
<td>real</td>
<td>In order to vary $u_{x_A}$ (WXA) from an initial value, $u_{x_A}$, to a final value $u_{x_A}^{\text{max}}$, DWXA must be set equal to the desired step size. See WXAMAX.</td>
</tr>
<tr>
<td>WXAMAX</td>
<td>$u_{x_A}^{\text{max}}$</td>
<td>real</td>
<td>The maximum value of $u_{x_A}$ to be used when DWXA $\neq 0$.</td>
</tr>
<tr>
<td>WXB</td>
<td>$u_{x_B}$</td>
<td>real</td>
<td>Normalized distance into Medium B.</td>
</tr>
<tr>
<td>DWXB</td>
<td>$\Delta u_{x_B}$</td>
<td>real</td>
<td>In order to vary $u_{x_B}$ from an initial value, $u_{x_B}$, to a final value, $u_{x_B}^{\text{max}}$, DWXB must be set equal to the step size desired. See WXBMAX.</td>
</tr>
<tr>
<td>WXBMAX</td>
<td>$u_{x_B}^{\text{max}}$</td>
<td>real</td>
<td>The maximum value of $u_{x_B}$ to be used when DWXB $\neq 0$.</td>
</tr>
</tbody>
</table>
| ICHECK     | -----        | logical | ICHECK = .TRUE. - All FINAL ANSWERS* are computed in addition to the evaluation of the determinant $|f(v_s)|$.
ICHECK = .FALSE. - FINAL ANSWERS are not computed; evaluate determinant only. |
| MAX        | -----        | integer | Since an iteration scheme is used for convergence for a final root $v_s$, there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 15). If MAX is set to zero (MAX = 0) the determinant $|f(v_s)|$ will be evaluated for the particular $v_s$ value input - the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which $v_s$ lies. |

*The FINAL RESULTS, which are computed for all values of WXA (dielectric layer) and WXB (piezoelectric layer), include the following:

- Stress Components
- Strain Components
- Time Average Power Flow
- Electric Displacement
- Mechanical Displacement
- Electric Potential Magnitude
- Electric Field
<table>
<thead>
<tr>
<th>Input Name</th>
<th>Equation Name</th>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td></td>
<td>BCD</td>
<td>An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example</td>
</tr>
<tr>
<td>HXAGNL</td>
<td></td>
<td>logical</td>
<td>Parameter which controls the calculation of betas ($\beta$'s) for a hexagonal crystal (such as zinc oxide)</td>
</tr>
<tr>
<td>VSINC</td>
<td></td>
<td>logical</td>
<td>Parameter which controls the calculation of initial velocity ($v_0$) for a hexagonal crystal (such as zinc oxide)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.TRUE. hexagonal crystal (use special technique)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.FALSE. non-hexagonal crystal (use normal procedure)</td>
</tr>
<tr>
<td>IOP</td>
<td></td>
<td>integer</td>
<td>Degenerate case options (used when exactly four a's with positive real part occur).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IOP = 1 - seek modes of propagation of the Quasi-Rayleigh or Sesawa type.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IOP = 2 - seek modes of propagation of Love type.</td>
</tr>
<tr>
<td>REPEAT</td>
<td></td>
<td>logical</td>
<td>REPEAT is a logical variable and in its usage, can take only one value:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.TRUE.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input</td>
</tr>
</tbody>
</table>
The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

<table>
<thead>
<tr>
<th>Input Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABCTE</td>
<td>Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.</td>
</tr>
<tr>
<td>ROOTS</td>
<td>Print the roots of the polynomial each time they are calculated.</td>
</tr>
<tr>
<td>BETA</td>
<td>Print the values of $\delta_{ij}$.</td>
</tr>
<tr>
<td>DETERM</td>
<td>Print the value of the determinant.</td>
</tr>
<tr>
<td>COEFF</td>
<td>Print the coefficients of the 8th order polynomial.</td>
</tr>
<tr>
<td>TABL</td>
<td>Print the L matrix (or $\tilde{P}$, $\tilde{Q}$, $\tilde{R}$, etc., when used).</td>
</tr>
<tr>
<td>ALPHA</td>
<td>Print the roots of the polynomial ($c_B^{j}$) and the re-ordered roots for degenerate cases.</td>
</tr>
<tr>
<td>ALL</td>
<td>Print all of the above.</td>
</tr>
</tbody>
</table>

Data items may be excluded from the input stream at the discretion of the user. Items omitted from the first data set will take on nominal values (i.e.: values assigned within the program). Items omitted from succeeding data sets will take on previously assigned values. The following is a complete list of nominal values:

*All logical parameters have a nominal value of .FALSE.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAMDAB</td>
<td>0.</td>
</tr>
<tr>
<td>MUB</td>
<td>0.</td>
</tr>
<tr>
<td>NUB</td>
<td>0.</td>
</tr>
<tr>
<td>DNU</td>
<td>0.</td>
</tr>
<tr>
<td>NUMAX</td>
<td>0.</td>
</tr>
<tr>
<td>VS</td>
<td>3000.</td>
</tr>
<tr>
<td>DVS</td>
<td>0.</td>
</tr>
<tr>
<td>VSMAX</td>
<td>0.</td>
</tr>
<tr>
<td>LAMDAA</td>
<td>$1.5 \times 10^{11}$</td>
</tr>
<tr>
<td>MUA</td>
<td>$2.85 \times 10^{10}$</td>
</tr>
<tr>
<td>RHOA</td>
<td>$1.888 \times 10^4$</td>
</tr>
<tr>
<td>RHOB</td>
<td>4700.</td>
</tr>
<tr>
<td>EPSILON</td>
<td>$1. \times 10^{-11}$</td>
</tr>
<tr>
<td>EPSO</td>
<td>$8.85 \times 10^{-12}$</td>
</tr>
<tr>
<td>EPSA</td>
<td>$44.25 \times 10^{-12}$</td>
</tr>
<tr>
<td>WH</td>
<td>0.</td>
</tr>
<tr>
<td>WXA</td>
<td>0.</td>
</tr>
<tr>
<td>DWXA</td>
<td>0.</td>
</tr>
<tr>
<td>WXAMAX</td>
<td>0.</td>
</tr>
<tr>
<td>WXB</td>
<td>0.</td>
</tr>
<tr>
<td>DWXB</td>
<td>0.</td>
</tr>
<tr>
<td>WXBMAX</td>
<td>0.</td>
</tr>
<tr>
<td>MAX</td>
<td>15.</td>
</tr>
<tr>
<td>TITLE</td>
<td>LITHIUM NIOBATE</td>
</tr>
<tr>
<td>IOP</td>
<td>1</td>
</tr>
</tbody>
</table>
The following is a description of the computer program flow diagram provided at the end of this section.

The computer program for this problem shares many common features with the programs described in the preceding sections. Again the nominal data values are set up first. The piezoelectric (P), elastic (G), and dielectric (EPS) constants are read in next (CONST DATA). Following this the rest of the input data is entered (INPUT DATA).

Subroutine SETCTE computes the transformed piezoelectric (CE), elastic (CC), and dielectric (CT) constants. Next, subroutine ROOT performs the calculations and calls the subroutines necessary to compute F(νs), the boundary condition determinant. ROOT will either minimize F(νs) (root finding scheme) or simply compute it at velocity νs depending on the setting of the counter MAX. Upon returning to the main program an option to increment νs followed by an option to increment the third Euler angle (θ) is available as in the first two programs. As θ is incremented it is also possible to update the initial velocity (νs) if the root finding scheme is being employed so that a closer initial estimate will be had as θ varies. The setting of a logical variable (VSINC) dictates whether this updating scheme is to be used or not.

When the correct velocity of propagation has been found (either from the root finding scheme or from plotting F(νs) as a function of velocity) the relative amplitudes of the partial surface wave fields are calculated (ETA(1), ETA(2), etc.). Next the various quantities of interest are calculated in medium A (the dielectric) as a function of normalized distance (WX_A) into the medium. Following this the same parameters are calculated in medium B (crystal) as a function of normalized distance (WX_B) into the crystal. For both media an incrementation scheme may be used to increase WX_A or WX_B in equal increments (DWXA or DWXB) from some initial value to some final value.

The quantities of interest mentioned above are as follows:

a) Mechanical Displacement (magnitude and phase) referred to as MAGU(i) and PHASEU(i), i=1 to 3 in the program.

b) The electric potential (magnitude and phase) referred to as MAGU(4) and PHASEU(4).

c) The time average power flow computed in the subroutine P1FUN.
d) The stresses computed in the subroutine TFUN.
e) The strains computed in subroutine SFUN.
f) The electric fields (E1, E3) and electric displacement (D1, D2, and D3).

Calculation of F(VS)

Subroutine ROOT calls subroutine F to evaluate the determinant of the boundary condition matrix. F first calls subroutine STRIP to calculate the coefficients of the eighth order equation in α. Next subroutine CROOT computes the roots of the polynomial (ALFA(I), I=1 to 8). The roots with positive real parts (ALFAB(I), I=1 to K) are selected as in the other programs and the extraneous roots in the non-piezoelectric case are eliminated.

If K = 0 the case terminates since no solution is possible. If K ≠ 0 a search for degeneracies follows. K = 1 presents a possibility now which was not present in the previous problems.* First \( \hat{A}_{12} \) and \( \hat{A}_{23} \) are checked (\( \hat{A} \) is the matrix of the coefficients of the unknown amplitudes \( \beta(1) \) as in the previous problems). If \( \hat{A}_{12} \) and \( \hat{A}_{23} \) are not both equal to zero the case cannot be degenerate. A check is made to see if the following two conditions both hold:

a) The case is non-piezoelectric
b) K ≠ 3

If both of these conditions hold the case terminates. If it is not true that both hold then a test is made to see if the following two conditions hold:

a) The case is piezoelectric
b) K ≠ 4

If both of these conditions hold the case terminates. Otherwise the program continued for now we must have either a non-piezoelectric case with K = 3 or a piezoelectric case with K = 4, both of which are proper non-degenerate cases.

If \( \hat{A}_{12} \) and \( \hat{A}_{23} \) are both identically zero and the case is non-piezoelectric, the program proceeds to the section where degenerate, non-piezoelectric cases

*Page 37 of analysis.
are handled. If, however, the case is piezoelectric \( \hat{\lambda}_{24} \) is checked. If \( \hat{\lambda}_{24} \) is zero the first type of degenerate case of the analysis section arises and the program proceeds to that section which treats such cases. If \( \hat{\lambda}_{24} \) is not identically zero then \( \hat{\lambda}_{14} \) and \( \hat{\lambda}_{34} \) are checked. If they are identically zero, the second degenerate case of the analysis section arises and the program proceeds to the section that treats these cases. If they are not both zero the program goes through the test mentioned above (i.e. is it simultaneously true that (a) the case is piezoelectric and (b) \( K \neq 4 \).

If (a) and (b) are not simultaneously true then we are dealing with a non-degenerate, piezoelectric case and the program proceeds accordingly.

Non-degenerate Cases

The \( \hat{\lambda} \) matrix is set up for each value of \( c (k = 1, K4) \) where \( K4 = 4 \) for piezoelectric cases and \( K4 = 3 \) for non-piezoelectric cases. If the case is non-piezoelectric the program sets \( \beta_4 = 0, \beta_3 = 10^{-10} \) and solves the first two equations for \( \beta_1 \) and \( \beta_2 \). If the crystal is piezoelectric and not hexagonal \( \beta_4 \) is set equal to 1 and the first three equations of the set are solved for \( \beta_1, \beta_2, \) and \( \beta_3 \). If the crystal is piezoelectric and hexagonal \( \beta_1 \) is set equal to \( 10^{-10} \) and the second, third, and fourth equations of the set are solved for \( \beta_2, \beta_3, \) and \( \beta_4 \).

Degenerate Case 1

This case is characterized by a decoupling of the equations for \( \beta^{(t)}_1 \) so that three of the equations involve \( \beta_1, \beta_3, \) and \( \beta_4 \) only and one involves \( \beta_2 \) only as discussed in the analysis. If there are four \( c \)'s with positive real part (\( K = 4 \)) the program calculates \( |\hat{\lambda}_{22}| \) for each \( c \) and determines which \( c \) leads to a minimum of this function. This now becomes \( c^{(1)} \) while the other \( c \)'s become \( c^{(2)}, c^{(3)}, \) and \( c^{(4)} \). The program proceeds to work with the relabeled \( c \)'s and computes the \( \beta \)'s as follows:

\[
\beta_2^{(t)} = 10^{-10}, \quad \beta_1^{(t)} = 0, \quad i = 1, 3, 4; \quad \beta_2^{(t)} = 0, \quad \beta_4^{(t)} = 1
\]

\[
\beta_1^{(t)} = \frac{\lambda_1^{(t)} \lambda_3^{(t)} - \lambda_2^{(t)} \lambda_3^{(t)} \lambda_3^{(t)} \lambda_3^{(t)}}{\lambda_1^{(t)} \lambda_3^{(t)} - \lambda_2^{(t)} \lambda_3^{(t)}}, \quad \text{and} \quad \beta_3^{(t)} = \frac{\lambda_1^{(t)} \lambda_1^{(t)} - \lambda_2^{(t)} \lambda_2^{(t)}}{\lambda_1^{(t)} \lambda_3^{(t)} - \lambda_2^{(t)} \lambda_3^{(t)}}
\]

\( t = 2, 3, 4 \). The program now proceeds to set up either the \( M(10 \times 10) \) or
N(3 x 3) matrix discussed in the analysis and evaluates its determinant.

If there are less than four α's with positive real part (K < 4) the program calculates $|\hat{A}_{22}|$ for each α and counts the number (N1) of α's for which $|\hat{A}_{22}| < 10^{-7}$ (this is close enough to zero considering the magnitude of the individual terms in $\hat{A}_{22}$). If K = 1 and N1 = 0 the case terminates (case 1c2). If K = 1 and N1 = 1 the program sets $\beta_2^{(1)} = 10^{-10}$ $\beta_1^{(1)} = 0$ i = 1, 3, 4 and then proceeds to set up the N matrix and evaluate its determinant (case 1c1).

If K = 2 and N1 = 0 the case terminates (case 1b3). If K = 2 and N1 = 1 the α that yielded $|\hat{A}_{22}| < 10^{-7}$ becomes $\alpha^{(1)}$. The program now sets $\beta_2^{(1)} = 10^{-10}$ and $\beta_1^{(1)} = 0$ i = 1, 3, 4 and then proceeds to set up the N matrix and evaluate its determinant (case 1b1). If K = 2 and N1 = 2 this represents an impossible case and the case terminates (1b2).

If K = 3 and N1 = 0 then all three roots correspond to the $(\beta_1, \beta_3, \beta_4)$ split. The α's become $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. The β's are calculated as they are in the four α case for $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. Only the M matrix can be set up and evaluated for this case (case 1a1). If K = 3 and N1 = 1 the corresponding α becomes $\alpha^{(1)}$ and $\beta_2^{(1)} = 10^{-10}$ while $\beta_1^{(1)} = 0$ i = 1, 3, 4. Only the N matrix is set up and evaluated (case 1a2). If K = 3 and N1 = 2 or more the case terminates since this is physically impossible.

Degenerate Case 2

This case is characterized by a decoupling of the equations for $\beta_1^{(1)}$ so that two of the equations involve $\beta_1$ and $\beta_3$ only while the other two involve $\beta_2$ and $\beta_4$ only. If there are four α's with positive real part (K = 4) the program calculates $|\hat{A}_{22} \hat{A}_{44} - \hat{A}_{24}^2|$ for each α. The (2) values of $\alpha^{(1)}$ which lead to minimum values of $|\hat{A}_{22} \hat{A}_{44} - \hat{A}_{24}^2|$ are selected and become $\alpha^{(3)}$ and $\alpha^{(4)}$. The other (2) values of α become $\alpha^{(1)}$ and $\alpha^{(2)}$. The program then computes the β's as follows:
\[\begin{align*}
\beta_{3}^{(1)} &= \beta_{3}^{(2)} = 10^{-10} ; \\
\beta_{1}^{(1)} &= \frac{-\Lambda_{13}^{(1)}}{\Lambda_{11}^{(1)}} \cdot 10^{-10} ; \\
\beta_{1}^{(2)} &= \frac{-\Lambda_{13}^{(2)}}{\Lambda_{11}^{(2)}} \cdot 10^{-10} ; \\
\beta_{2}^{(1)} &= \beta_{2}^{(2)} = 0 ; \\
\beta_{4}^{(1)} &= \beta_{4}^{(2)} = 0 ; \\
\beta_{2}^{(3)} &= \beta_{4}^{(4)} = 1; \\
\beta_{2}^{(3)} &= \frac{-\Lambda_{24}^{(3)}}{\Lambda_{22}^{(3)}} ; \\
\beta_{2}^{(4)} &= \frac{-\Lambda_{24}^{(4)}}{\Lambda_{22}^{(4)}} ; \\
\gamma_{1}^{(3)} &= \gamma_{2}^{(3)} = 0 ; \\
\gamma_{3}^{(3)} &= \gamma_{3}^{(4)} = 0 .
\end{align*}\]

The program now proceeds to set up either the P or the Q matrix and evaluates its determinant.

If there are less than four \( \alpha \)'s with positive real part (\( K < 4 \)) the program proceeds as follows:

- If \( K = 1 \) the case terminates (case 2c).
- If \( K = 2 \) or 3 the program computes the quantity \( |\Lambda_{22}^{(1)} \Lambda_{44}^{(2)} - \Lambda_{24}^{(2)}| \) for each \( \alpha \) and counts the number \( (I1) \) of \( \alpha \)'s for which this quantity < \( 10^{-5} \) and the number \( (I2) \) of \( \alpha \)'s for which this quantity \( \geq 10^{-5} \). \( 10^{-5} \) is close enough to zero due to the magnitudes of the individual terms in the quantity. If \( K = 3 \) and \( I1 = 2 \) the \( \alpha \)'s become \( \alpha_{3}^{(3)} \) and \( \alpha_{4}^{(4)} \) and the \( \beta \)'s are calculated as they were above for \( \alpha_{3}^{(3)} \) and \( \alpha_{4}^{(4)} \). Only the Q matrix is set up and evaluated (case 2a1). If \( K = 3 \) and \( I2 = 2 \) the \( \alpha \)'s become \( \alpha_{1}^{(1)} \) and \( \alpha_{2}^{(2)} \) and the \( \beta \)'s are calculated as above for \( \alpha_{1}^{(1)} \) and \( \alpha_{2}^{(2)} \). Only the P matrix is set up and evaluated (case 2a2). If \( K = 3 \) while \( I1 \neq 2 \) and \( I2 \neq 2 \) the case terminates (case 2a3). If \( K = 2 \) and \( I1 = 2 \) the \( \beta \)'s are handled as above (case 2b1). If \( K = 2 \) and \( I2 = 2 \) the \( \beta \)'s are likewise handled as above (case 2b2). If \( K = 2 \) while \( I1 \neq 2 \) and \( I2 \neq 2 \) the case terminates (case 2b3).

**Degenerate Non-piezoelectric Case**

This case is characterized by a decoupling of the equations for \( \beta_{i}^{(t)} \) such that two of the equations involve \( \beta_{1} \) and \( \beta_{3} \) only and one of the equations involves
$\beta_2$ only. If there are three $\alpha$'s with positive real part $|\hat{A}_{22}|$ is evaluated for each $\alpha$. The $\alpha$ leading to the minimum value of $|\hat{A}_{22}|$ becomes $\alpha^{(1)}$ while the others become $\alpha^{(2)}$ and $\alpha^{(3)}$. The program sets $\beta_2^{(1)} = 10^{-10}$, $\beta_1^{(1)} = 0$ for $i = 1$ and 3; $\beta_2^{(2)} = 0$, $\beta_3^{(2)} = 10^{-10}$, $\beta_1^{(2)} = \hat{A}_{13}/\hat{A}_{11}$ * $10^{-10}$ for $t = 2$ and 3. Either the R or N matrix can be set up and evaluated depending on the type of wave sought.

If there are two $\alpha$'s with positive real part $|\hat{A}_{22}|$ is evaluated for both $\alpha$'s and compared with $10^{-7}$. (This is close enough to zero considering the magnitudes of the individual terms of $\hat{A}_{22}$). If $|\hat{A}_{22}| > 10^{-7}$ for both $\alpha$'s they become $\alpha^{(2)}$ and $\alpha^{(3)}$ and the $\beta$'s are calculated as above. Only the R matrix is set up and its determinant evaluated (case xa1). If $|\hat{A}_{22}| \leq 10^{-7}$ for one of the $\alpha$'s this becomes $\alpha^{(1)}$ and the $\beta$'s are the same as above. Only the N matrix is set up and its determinant evaluated (case xa2). If $|\hat{A}_{22}| \leq 10^{-7}$ for both $\alpha$'s the case is terminated (case xa3).

If there is one $\alpha$ with positive real part $|\hat{A}_{22}|$ is evaluated and compared to $10^{-7}$. If $|\hat{A}_{22}| \leq 10^{-7}$ the program sets $\beta_2^{(1)} = 10^{-10}$, $\beta_1^{(1)} = 0$ for $i = 1$ and 3. Only the N matrix is set up and its determinant evaluated (case ya1). If $|\hat{A}_{22}| > 10^{-7}$ the case is terminated (case ya2).
DIELECTRIC

MAIN

SET UP NOMINAL VALUES

READ

$CONST DATA
PIEZOELECTRIC (P),
ELASTIC (G), AND DIELECTRIC (EPS)

REPEAT=.FALSE.

1

VS=VSAVE
K TIME=0

READ INPUT DATA
LAMDA, MU, NU, ETC.

END OF FILE? (YES) STOP

(NO)

SNU=NUB
VSAVE=VS

1.2

k.1
\textbf{IF} (.FALSE.) \textbf{THEN}\textbf{GOTO} \textbf{L3}

\textbf{IF} (VSINC) \textbf{THEN} \textbf{GO TO} \textbf{L12}

\textbf{IF} (TIME = K \newline \textbf{GOTO} \textbf{L2}}

\textbf{IF} (KTIME \geq 2) \textbf{THEN} \textbf{UPDATE INITIAL VELOCITY (VS)}

\textbf{IF} (VSAVE = VS) \textbf{GOTO} \textbf{L3}

\textbf{IF} (ONCE = .TRUE.) \textbf{GOTO} \textbf{L2}
120

**SSTC**
CALCULATE
TRANSFORMED PIEZOELECTRIC (CS),
ELASTIC (CC) AND DIELECTRIC (CT)
CONSTANTS

---

**ROOT**
CALCULATE

a) $|f(vso)|$ FOR $\text{MAX}=0$

b) $\text{VS}$ AND $|f(vs)| < \epsilon$ FOR $\text{MAX} > 0$

---

VSI = 1/VS
ALFAI = ALFA/VSI = 1, 8

---

PRINT
TITLE,
INPUT DATA, VS, VSI,
$|f(vs)|, (ALFA, ALFAI, i=1, 8$

---

ICHSCX (.TRUE.)
IALF (.FALSE.)

---

**REPEAT**

---

VS = VSO + DVS

---

**STOP**
CALCULATE ETA'S

<table>
<thead>
<tr>
<th>IDET</th>
<th>MATRIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>PIEZOELECTRIC; NON-DEGENERATE</td>
</tr>
<tr>
<td>1</td>
<td>M</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
</tr>
<tr>
<td>3</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>Q</td>
</tr>
<tr>
<td>5</td>
<td>R</td>
</tr>
<tr>
<td>6</td>
<td>ZERO-PIEZOELECTRIC; NON-DEGENERATE</td>
</tr>
</tbody>
</table>

PRINT ETA_i, i=1, 13

CALCULATE FINAL RESULTS
MEDIUM A - DIELECTRIC

WX=WXA

CALCULATE MECHANICAL DISPLACEMENT
MAGNITUDE PHASE
MAGU(i), PHASEU(i), i=1, 3

ELECTRIC POTENTIAL
MAGNITUDE PHASE
MAGU(4), PHASEU(4)

P1FUN CALCULATE TIME AVERAGE POWER FLOW
P1M, P2M

TFUN CALCULATE STRESS: TW31, TW32
TW33, TW11, TW12, TW22

SFUN CALCULATE STRAIN: S11, S33, S12,
S13, S23
CALCULATE ELECTRIC FIELD E1, E3
CALCULATE ELECTRIC DISPLACEMENT D1, D2, D3

PRINT: WX, TOGETHER WITH FINAL RESULTS

IF IMED = 2
CALCULATE FINAL RESULTS, AS FOR MEDIUM A

IF IMED = 1
MEDIUM A

IF WX = WX + DWXA
WX = WX + DWXB

CALCULATE FINAL RESULTS MEDIUM B-PIEZOELECTRIC

IF WX = WX + DWXB

MEDIUM B

IF DWXB = 0

IF WX = WX + DWXB
WX = WX - WXMAX

IMED = 2
CALCULATE FINAL RESULTS, AS FOR MEDIUM A
DIELECTRIC

F

DEGEN=0
IDET=0
JP=2
LALF=.FALSE.
ICASE=0
ICB=0
K1=1
K2=4
L1=1
L2=16
IB1=1
IB2=4

L1

STRIP
CALCULATE
COEFFICIENTS OF
POLYNOMIAL
POLY(I), I=1, 2

CROOT
CALCULATE
ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8

PIEZOELECTRIC (NO)

(YES)

(YES)

ELIMINATE ROOT
CORRESPONDING TO
THE POTENTIAL

ROOTS
. OR
ALL

(FALSE)

PRINT INTERMEDIATE
ROOTS OF POLYNOMIAL
ALFA(I), I=1, 8

IA(I)=1
I=1, 4

SELECT ROOTS
WITH POSITIVE
REAL PART
ALFA5(I), I=1, K

2 4

k:0

2.8
The program now proceeds to set up either the P or the Q matrix and evaluates its determinant.

If there are less than four \( \alpha \)'s with positive real part (\( K < 4 \)) the program proceeds as follows:

If \( K = 1 \) the case terminates (case 2c). If \( K = 2 \) or 3 the program computes the quantity \( |A_{22}^2 - \frac{A_{24}^2}{A_{24}^2} \) for each \( \alpha \) and counts the number (11) of \( \alpha \)'s for which this quantity < \( 10^{-5} \) and the number (12) of \( \alpha \)'s for which this quantity \( \geq 10^{-5} \). \( 10^{-5} \) is close enough to zero due to the magnitudes of the individual terms in the quantity. If \( K = 3 \) and \( I_1 = 2 \) the \( \alpha \)'s become \( \alpha^{(3)} \) and \( \alpha^{(4)} \) and the \( \beta \)'s are calculated as they were above for \( \alpha^{(3)} \) and \( \alpha^{(4)} \). Only the Q matrix is set up and evaluated (case 2a1). If \( K = 3 \) and \( I_2 = 2 \) the \( \alpha \)'s become \( \alpha^{(1)} \) and \( \alpha^{(2)} \) and the \( \beta \)'s are calculated as above for \( \alpha^{(1)} \) and \( \alpha^{(2)} \). Only the P matrix is set up and evaluated (case 2a2). If \( K = 3 \) while \( I_1 \neq 2 \) and \( I_2 \neq 2 \) the case terminates (case 2a3). If \( K = 2 \) and \( I_1 = 2 \) the \( \beta \)'s are handled as above (case 2b1). If \( K = 2 \) and \( I_2 = 2 \) the \( \beta \)'s are likewise handled as above (case 2b2). If \( K = 2 \) while \( I_1 \neq 2 \) and \( I_2 \neq 2 \) the case terminates (case 2b3).

Degenerate Non-piezoelectric Case

This case is characterized by a decoupling of the equations for \( \beta_i^{(4)} \) such that two of the equations involve \( \beta_1 \) and \( \beta_3 \) only and one of the equations involves
PRINT:
'NUMBER OF ALPHAS=K';
CASE TERMINATED

IALF=.TRUE.

RETURN
NON-DEGENERATE

[Flowchart diagram]

PRINT: ALFAN(k)

ROOTS OR:

(YES)

ALFAN(k) = 0

(NEW)

(YES)

ALFAN(k) = 0

(NEW)

PRINT: ALFAN(k)

ROOTS WITH POSITIVE REAL PART

CALCULATE A-MATRIX

[Expressions for A-matrix scaling]

SET B < 1; SOLVE FOR B(i=1,4)

SET B < 1; SOLVE FOR B(i=1,4)

B = B ' - 10^-10; i = 1,4

SET EPS < 4
DEGENERATE CASE 1
(1 ROW, 3 ZERO)

IC = IC + 1
NDG = NDG + 1

Diagram with flowchart notation:

1. IC = 1
2. NDG = NDG + 1
3. Decision point: K
   - If K = 4, go to
   - Otherwise, go to

Node 1:

Node 2:
(4 ALPHAS)

IDST=IOP
ICASE=1
MIN=|λ_{22}(k_{1})|

(k=2, 4)

MIN=|λ_{22}(k_{2})|

IA(A)=k
JA=JA+1

(k=1, 4)

IA(A)=1
JA=2

β_{B_{1}1} = 0; i=1, 3, 4
β_{B_{2}2} = 10^{-10}

DEGEN? (0, 1)

β_{B_{2}} = 0, β_{B_{4}} = 1

CALC:
β_{B_{1}1} = 1 AND 3 = fn(α_{IA(A)})

(j=2, 4)

CASE? (3)

(5)
DEGENERATE CASE 2
(4 ROW, 2 ZERO)

IC=2
NDG=NDG+1

= 4

K

= 4

1

2
\( q^2 \)

(LESS THAN 4 alpha)

\[ \text{TERM} = |\hat{\lambda}_{22}(q) \cdot \hat{\lambda}_{44}(q) - \hat{\lambda}_{24}(q)^2| \]

\[ (\text{TERM} < 10^{-5}) \]

\[ \begin{align*}
I_{11} &= I_{11} + 1 \\
1A1(Q) &= 1
\end{align*} \]

\[ \begin{align*}
I_{12} &= I_{12} + 1 \\
1A2(I2) &= 1
\end{align*} \]

\[ \begin{align*}
I_{21} &= 2.1 \\
2.2 &
\end{align*} \]
(3 ALPHAS)

(11=2)  
DEGEN=9  
(2.A.1)  
54

(12=2)  
DEGEN=11  
(2.A.2)  
55

(OTHER)  
DEGEN=13  
(2.A.3)  
55
SET UP L-MATRIX

\[
SL(i,j); \begin{cases} 
  i=1, 9 \\
  j=1, 10 \\
  i=1, 13 \\
  j=1, 13 
\end{cases}
\begin{cases} 
  \text{IDET} > 3 \\
  \text{IDET} \leq 3 
\end{cases}
\]

IDET 

(=0) IDET

EVALUATE FVS USING APPROPRIATE MATRIX

<table>
<thead>
<tr>
<th>IDET</th>
<th>MATRIX</th>
<th>NDIM</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>\hat{M}</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>\hat{S} OR \hat{N}</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>\hat{O}</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>\hat{F}</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>\hat{O}</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0-PIEZO (NON-DEGEN)</td>
<td>9</td>
</tr>
</tbody>
</table>
FVSMAG = |f(v0)|

TABL .OR. ALL (.TRUE.)

PRINT: APPROPRIATE MATRIX AND I.D.

(DETERM .OR. ALL (.TRUE.))

PRINT: VS, FVS, FVSMAG

RETURN
APPENDIX I. ELEMENTS OF THE L MATRIX FOR THE ELASTIC-CONDUCTOR PIEZOELECTRIC SUBSTRATE PROBLEM

The subscript $p$ corresponds to the piezoelectric medium while the subscript $c$ corresponds to the conducting elastic medium. The $C_{ij}$'s and $e_{ij}$'s correspond to the piezoelectric medium while $\lambda$, $\mu$ correspond to the conductor (Lame's constants). The expressions for $a_c^{(i)}$ are easily obtainable from equation (20) and are as follows

$$a_c^{(1, 2)} = \pm \sqrt{\frac{\mu - \rho_v^2}{\mu}} = a_c^{(5, 6)}$$

$$a_c^{(3, 4)} = \pm \sqrt{\frac{2\mu + \lambda - \rho_v^2}{2\mu + \lambda}}$$

The elements of the $10 \times 10$ boundary value determinant are as follows:

$$L_{1\ell} = \beta_{c1}^{(t)} \left[ i = 1, 2, 3 \quad \ell = 1, 2, \ldots, 6 \right]$$

$$L_{1\ell} = -\beta_{p1}^{(t-6)} \left[ i = 1, 2, 3 \quad \ell = 7, 8, 9, 10 \right]$$

$$L_{4\ell} = \beta_{c1}^{(t)} a_c^{(t)} \mu + j \beta_{c3}^{(t)} \mu \quad [\ell = 1, 2, \ldots, 6]$$

$$L_{4\ell} = -\beta_{p3}^{(t-6)} \left[ i C_{45} + a_p^{(t-6)} C_{45} \right] - \beta_{p2}^{(t-6)} \left[ j C_{56} + a_p^{(t-6)} C_{45} \right]$$

$$- \beta_{p3}^{(t-6)} \left[ j C_{55} + a_p^{(t-6)} C_{45} \right] - \beta_{p4}^{(t-6)} \left[ j e_{15} + a_p^{(t-6)} e_{35} \right] \quad [\ell = 7, 8, 9, 10]$$

$$L_{5\ell} = \beta_{c2}^{(t)} a_c^{(t)} \mu \quad [\ell = 1, 2, \ldots, 6]$$

$$L_{5\ell} = -\beta_{p1}^{(t-6)} \left[ i C_{14} + a_p^{(t-6)} C_{14} \right] - \beta_{p2}^{(t-6)} \left[ j C_{46} + a_p^{(t-6)} C_{14} \right]$$

$$- \beta_{p3}^{(t-6)} \left[ j C_{45} + a_p^{(t-6)} C_{14} \right] - \beta_{p4}^{(t-6)} \left[ j e_{14} + a_p^{(t-6)} e_{34} \right] \quad [\ell = 7, 8, 9, 10]$$
\[ L_{6\ell} = j \beta_c^{(t)} \gamma + \beta_c^{(t)} \alpha_c^{(t)} (2\mu + \lambda) \quad [\ell = 1, 2, \ldots, 6] \]

\[ L_{6\ell} = -\beta_p^{(t-6)} [j C_{13} + \alpha_p^{(t-6)} C_{35}] - \beta_p^{(t-6)} [j C_{36} + \alpha_p^{(t-6)} C_{34}] \]

\[ -\beta_p^{(t-6)} [j C_{35} + \alpha_p^{(t-6)} C_{33}] - \beta_p^{(t-6)} [j e_{13} + \alpha_p^{(t-6)} e_{33}] \quad [\ell = 7, 8, 9, 10] \]

\[ L_{7\ell} = L_{4\ell} e^{\alpha_c^{(t)} \omega / \nu_s} \quad [\ell = 1, 2, \ldots, 6] \]

\[ L_{7\ell} = 0 \quad [\ell = 7, 8, 9, 10] \]

\[ L_{8\ell} = L_{5\ell} e^{\alpha_c^{(t)} \omega / \nu_s} \quad [\ell = 1, 2, \ldots, 6] \]

\[ L_{8\ell} = 0 \quad [\ell = 7, 8, 9, 10] \]

\[ L_{9\ell} = L_{6\ell} e^{\alpha_c^{(t)} \omega / \nu_s} \quad [\ell = 1, 2, \ldots, 6] \]

\[ L_{9\ell} = 0 \quad [\ell = 7, 8, 9, 10] \]

\[ L_{10\ell} = 0 \quad [\ell = 1, 2, \ldots, 6] \]

\[ L_{10\ell} = \beta_p^{(t-6)} \quad [\ell = 7, 8, 9, 10] \]
APPENDIX II. EXPLICIT FORMS OF THE ELEMENTS OF THE MATRIX M
ASSOCIATED WITH THE FLUID MEDIUM PIEZOELECTRIC
SUBSTRATE PROBLEM

\[ M_{1\ell} = \beta_3^{(\ell)} \cdot 10^{10}, \quad \ell = 1, 2, 3, 4, \]

\[ M_{15} = 0, \]

\[ M_{16} = -1; \]

\[ M_{2\ell} = \beta_4^{(\ell)}, \quad \ell = 1, 2, 3, 4 \]

\[ M_{25} = -1, \]

\[ M_{26} = 0; \]

\[ M_{3\ell} = \left[ \beta_1^{(\ell)} (j e_{31} + \alpha_c^{(\ell)} e_{33}) + \beta_2^{(\ell)} (j e_{35} + \alpha_c^{(\ell)} e_{34}) + \beta_3^{(\ell)} (j e_{35} + \alpha_c^{(\ell)} e_{33}) - \beta_4^{(\ell)} (j e_{13} + \alpha_c^{(\ell)} e_{33}) \right] \cdot 10^{10}, \]

\[ \ell = 1, 2, 3, 4 \]

\[ M_{35} = -e_\ell \cdot 10^{10}, \]

\[ M_{36} = 0; \]

\[ M_{4\ell} = \beta_1^{(\ell)} (j C_{13} + \alpha_c^{(\ell)} C_{35}) + \beta_2^{(\ell)} (j C_{36} + \alpha_c^{(\ell)} C_{34}) + \beta_3^{(\ell)} (j C_{35} + \alpha_c^{(\ell)} C_{33}) + \beta_4^{(\ell)} (j e_{13} + \alpha_c^{(\ell)} e_{33}) , \]

\[ \ell = 1, 2, 3, 4 , \]

\[ M_{45} = 0 , \]

\[ M_{46} = \frac{\rho L v_s^2}{a_L} \cdot 10^{-10} ; \]
\[ M_{5\ell} = \beta_1^{(\ell)} (jC_{15} + \alpha_c^{(\ell)} C_{55}) + \beta_2^{(\ell)} (jC_{56} + \alpha_c^{(\ell)} C_{45}) \]
\[ + \beta_3^{(\ell)} (jC_{55} + \alpha_c^{(\ell)} C_{35}) + \beta_4^{(\ell)} (j e_{15} + \alpha_c^{(\ell)} e_{35}) \]
\[ t = 1, 2, 3, 4 \]

\[ M_{55} = M_{56} = 0 \]

\[ M_{6\ell} = \beta_1^{(\ell)} (jC_{14} + \alpha_c^{(\ell)} C_{45}) + \beta_2^{(\ell)} (jC_{46} + \alpha_c^{(\ell)} C_{44}) \]
\[ + \beta_3^{(\ell)} (jC_{45} + \alpha_c^{(\ell)} C_{34}) + \beta_4^{(\ell)} (j e_{14} + \alpha_c^{(\ell)} e_{34}) \]
\[ t = 1, 2, 3, 4 \]

\[ M_{65} = M_{66} = 0 \]

The factors $10^{10}$ and $10^{-10}$ are introduced to make the real and imaginary parts of all elements of the matrix on the order of unity.
APPENDIX III. ELEMENTS OF THE $L$ MATRIX FOR THE ELASTIC LAYER PIEZOELECTRIC SUBSTRATE PROBLEM

$L_{1t} = \beta_d^{(t)} \cdot 10^{10} \quad [i=1,2,3 \quad t=1,2,\ldots,6]$

$L_{1t} = -\beta_{c1}^{(t-6)} \cdot 10^{10} \quad [i=1,2,3 \quad t=7,8,9,10]$

$L_{1t} = 0 \quad [i=1,2,3 \quad t=11,12,13]$

$L_{4t} = \mu_d (\alpha_d^{(t)} \delta_{d1}^{(t)} + j \mu_d \delta_d^{(t)}) \quad [t=1,2,\ldots,6]$

$L_{4t} = -\beta_{c1}^{(t-6)} [j c_{15} + \alpha_c^{(t-6)} c_{55}] - \beta_{c2}^{(t-6)} [j c_{56} + \alpha_c^{(t-6)} c_{45}]$

$L_{4t} = \beta_{c3}^{(t-6)} [j c_{55} + \alpha_c^{(t-6)} c_{35}] - \beta_{c4}^{(t-6)} [j e_{15} + \alpha_c^{(t-6)} e_{35}]$

$\quad [t = 7,8,9,10]$

$L_{4t} = 0 \quad [t = 11,12,13]$

$L_{5t} = \mu_d (\alpha_d^{(t)} \delta_{d2}^{(t)}) \quad [t = 1,2,\ldots,6]$

$L_{5t} = -\beta_{c1}^{(t-6)} [j c_{14} + \alpha_c^{(t-6)} c_{45}] - \beta_{c2}^{(t-6)} [j c_{46} + \alpha_c^{(t-6)} c_{44}]$

$L_{5t} = -\beta_{c3}^{(t-6)} [j c_{45} + \alpha_c^{(t-6)} c_{34}] - \beta_{c4}^{(t-6)} [j e_{14} + \alpha_c^{(t-6)} e_{34}]$

$\quad [t = 7,8,9,10]$

$L_{5t} = 0 \quad [t = 11,12,13]$

$L_{6t} = j \lambda_d \beta_d^{(t)} + (\lambda_d + 2 \mu_d) \alpha_d^{(t)} \delta_d^{(t)} \quad [t = 1,2,\ldots,6]$

$L_{6t} = \beta_{c1}^{(t-6)} [j c_{13} + \alpha_c^{(t-6)} c_{35}] - \beta_{c2}^{(t-6)} [j c_{36} + \alpha_c^{(t-6)} c_{34}]$

$L_{6t} = -\beta_{c3}^{(t-6)} [j c_{35} + \alpha_c^{(t-6)} c_{33}] - \beta_{c4}^{(t-6)} [j e_{13} + \alpha_c^{(t-6)} e_{33}]$

$\quad [t = 7,8,9,10]
\[
\begin{align*}
L_{6t} &= 0 \quad [\ell = 11, 12, 13] \\
L_{7t} &= L_{4t} e^{(4) \frac{\alpha h}{v_s}} \quad [\ell = 1, 2, \ldots, 6] \\
L_{7t} &= 0 \quad [\ell = 7, 8, \ldots, 13] \\
L_{8t} &= L_{5t} e^{(4) \frac{\alpha h}{v_s}} \quad [\ell = 1, 2, \ldots, 6] \\
L_{8t} &= 0 \quad [\ell = 7, 8, \ldots, 13] \\
L_{9t} &= L_{6t} e^{(4) \frac{\alpha h}{v_s}} \quad [\ell = 1, 2, \ldots, 6] \\
L_{9t} &= 0 \quad [\ell = 7, 8, \ldots, 13] \\
L_{10t} &= 0 \quad [\ell = 1, 2, \ldots, 6] \\
L_{10t} &= g^{(t-6)} c_4 \quad [\ell = 7, 8, 9, 10] \\
L_{10t} &= -1 \quad [\ell = 11, 12] \\
L_{10t} &= 0 \quad [\ell = 13] \\
L_{11t} &= 0 \quad [\ell = 1, 2, \ldots, 10] \\
L_{11, 11} &= -\frac{\alpha h}{v_s} e \\
L_{11, 12} &= \frac{\alpha h}{v_s} e \\
L_{11, 13} &= -e
\end{align*}
\]
\[ \begin{align*}
L_{12,t} &= 0 \quad [t = 1, 2, \ldots, 6] \\
L_{12,t} &= \left[ g^{(t-6)} [j e_{13} + \alpha_{c}^{(t-6)} e_{35}] + \beta^{(t-6)} [j e_{35} + \alpha_{c}^{(t-6)} e_{34}] \\
&\quad + \beta^{(t-6)} [j e_{35} + \alpha_{c}^{(t-6)} e_{33}] - \beta^{(t-6)} [j e_{13} + \alpha_{c}^{(t-6)} e_{33}] \right] \cdot 10^{10} \\
&\quad t = 7, 8, 9, 10 \\
L_{12,11} &= -e_d \cdot 10^{10} \\
L_{12,12} &= e_d \cdot 10^{10} \\
L_{12,13} &= 0 \\
L_{13,t} &= 0 \quad [t = 1, 2, \ldots, 10] \\
L_{13,11} &= -\frac{\omega h}{v_s} \\
L_{13,12} &= \frac{\omega h}{v_s} \\
L_{13,13} &= -\frac{e_0}{e_d} - \frac{i h}{v_s} 
\end{align*} \]
APPENDIX IV. EXPLICIT FORMS OF THE POLYNOMIAL COEFFICIENTS $A_k$

The elements of the matrix $\hat{A}$ have the general form $\hat{A}_{ik} = a_{ik}^2 + j b_{ik} + d_{ik}$ where $a_{ik}$, $b_{ik}$, and $d_{ik}$ are easily deduced from equation (6).

Therefore, the determinant of $\hat{A}$ can be expressed as the polynomial

$$A_1 a^8 + jA_2 a^7 + A_3 a^6 + jA_4 a^5 + A_5 a^4 + jA_6 a^3 + A_7 a^2 + jA_8 a + A_9$$

with coefficients

$$A_1 = \sum_{\{j, k, t, m\}} (-1)^h H_{jkt} S_m$$

$$A_2 = \sum_{\{j, k, t, m\}} (-1)^h [H_{jkt} \bar{U}_m + \bar{I}_{jkt} S_m]$$

$$A_3 = \sum_{\{j, k, t, m\}} (-1)^h [H_{jkt} \bar{V}_m - I_{jkt} \bar{U}_m + \bar{J}_{jkt} S_m]$$

$$A_4 = \sum_{\{j, k, t, m\}} (-1)^h [I_{jkt} \bar{V}_m + \bar{J}_{jkt} \bar{U}_m + K_{jkt} S_m]$$

$$A_5 = \sum_{\{j, k, t, m\}} (-1)^h [J_{jkt} \bar{V}_m - K_{jkt} \bar{U}_m + L_{jkt} S_m]$$

$$A_6 = \sum_{\{j, k, t, m\}} (-1)^h [K_{jkt} \bar{V}_m + L_{jkt} \bar{U}_m + M_{jkt} S_m]$$
\[ A_7 = \sum_{\{j, k, t, m\}} (-1)^h [L_{jk \ell} V_m - M_{jk \ell} \bar{U}_m + N_{jk \ell} S_m] \]

\[ A_8 = \sum_{\{j, k, t, m\}} (-1)^h [M_{jk \ell} V_m + N_{jk \ell} \bar{U}_m] \]

\[ A_9 = \sum_{\{j, k, t, m\}} (-1)^h N_{jk \ell} V_m \]

\[ \sum_{\{j, k, t, m\}} \] refers to a sum over all permutations of 1, 2, 3, 4. There are 24 terms in each sum. \( h \) is the number of interchanges for each term necessary to return the indices to the order 1, 2, 3, 4; and

\[ H_{jk \ell} = a_{1j} a_{2k} a_{3 \ell} \]

\[ I_{jk \ell} = a_{1j} a_{2k} b_{3 \ell} + (a_{1j} b_{2k} + b_{1j} a_{2k}) a_{3 \ell} \]

\[ J_{jk \ell} = a_{1j} a_{2k} d_{3 \ell} - (a_{1j} b_{2k} + b_{1j} a_{2k}) b_{3 \ell} + (a_{1j} d_{2k} - b_{1j} b_{2k} + d_{1j} a_{2k}) a_{3 \ell} \]

\[ K_{jk \ell} = (a_{1j} b_{2k} + b_{1j} a_{2k}) d_{3 \ell} - (a_{1j} b_{2k} + b_{1j} a_{2k}) b_{3 \ell} + (b_{1j} d_{2k} + d_{1j} b_{2k}) a_{3 \ell} \]

\[ L_{jk \ell} = (a_{1j} d_{2k} - b_{1j} b_{2k} + d_{1j} a_{2k}) d_{3 \ell} - (b_{1j} d_{2k} + d_{1j} b_{2k}) b_{3 \ell} + d_{1j} d_{2k} a_{3 \ell} \]

\[ M_{jk \ell} = (b_{1j} d_{2k} + d_{1j} b_{2k}) d_{3 \ell} + d_{1j} d_{2k} b_{3 \ell} \]

\[ N_{jk \ell} = d_{1j} d_{2k} d_{3 \ell} \]
\[ s_m = a_{4m} \]
\[ u_m = b_{4m} \]
\[ v_n = d_{4m} \]
## APPENDIX V. COMPUTER PROGRAMS

### CONWAY PHASE PAGE

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<th>SID</th>
<th>MACONWAY PHASE</th>
<th>NO.</th>
<th>12/31/65</th>
<th>000109</th>
<th>PAGE</th>
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</table>

**4**

**FILE PAGE 25/105**

**SETUP**

**CGRAPH**

**CALL**

**CONTINUE**

**SSTATUS**

**RETURNING TO ISSYS.**

**SREJOB**

**DENUG**

**SPECIFIC LINES DECK: DEBUG**

**CLINES2**

**GOLD LITHIUM AND LITHIUM NITRATE**

**continued**

### COMMON /RESTART /CHECK

```
<table>
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<tr>
<th>LIMA03</th>
<th>COMMNT</th>
<th>PHASE</th>
<th>MN</th>
<th>R4</th>
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<td></td>
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</tbody>
</table>
+--------+---------+-------+----+----+--------+--------+--------+--------+--------+--------|
| LIMA03 | RPN     | SOURCE STATEMENT | IFMSI |        |        |        |        |        |        |        |
|        |         | IS      | MA3N0730 |        |        |        |        |        |        |        |
|        |         | LIM0740 |        |        |        |        |        |        |        |        |
|        |         |        | MA0740 |        |        |        |        |        |        |        |
|        |         |        | LIM0750 |        |        |        |        |        |        |        |
|        |         |        | MA0750 |        |        |        |        |        |        |        |
|        |         |        | LIM0760 |        |        |        |        |        |        |        |
|        |         |        | MA0760 |        |        |        |        |        |        |        |
|        |         |        | LIM0770 |        |        |        |        |        |        |        |
|        |         |        | MA0770 |        |        |        |        |        |        |        |
|        |         |        | LIM0790 |        |        |        |        |        |        |        |
|        |         |        | MA0790 |        |        |        |        |        |        |        |
|        |         |        | LIM0800 |        |        |        |        |        |        |        |
|        |         |        | MA0800 |        |        |        |        |        |        |        |
|        |         |        | LIM0810 |        |        |        |        |        |        |        |
|        |         |        | MA0810 |        |        |        |        |        |        |        |
|        |         |        | LIM0820 |        |        |        |        |        |        |        |
|        |         |        | MA0820 |        |        |        |        |        |        |        |
|        |         |        | LIM0830 |        |        |        |        |        |        |        |
|        |         |        | MA0830 |        |        |        |        |        |        |        |
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|        |         |        | MA0860 |        |        |        |        |        |        |        |
|        |         |        | LIM0870 |        |        |        |        |        |        |        |
|        |         |        | MA0870 |        |        |        |        |        |        |        |
|        |         |        | LIM0900 |        |        |        |        |        |        |        |
|        |         |        | MA0900 |        |        |        |        |        |        |        |
|        |         |        | LIM0920 |        |        |        |        |        |        |        |
|        |         |        | MA0920 |        |        |        |        |        |        |        |
|        |         |        | LIM0930 |        |        |        |        |        |        |        |
|        |         |        | MA0930 |        |        |        |        |        |        |        |
|        |         |        | LIM0940 |        |        |        |        |        |        |        |
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|        |         |        | LIM0950 |        |        |        |        |        |        |        |
|        |         |        | MA0950 |        |        |        |        |        |        |        |
|        |         |        | LIM0960 |        |        |        |        |        |        |        |
|        |         |        | MA0960 |        |        |        |        |        |        |        |
|        |         |        | LIM0970 |        |        |        |        |        |        |        |
|        |         |        | MA0970 |        |        |        |        |        |        |        |
|        |         |        | LIM0980 |        |        |        |        |        |        |        |
|        |         |        | MA0980 |        |        |        |        |        |        |        |
|        |         |        | LIM0990 |        |        |        |        |        |        |        |
|        |         |        | MA0990 |        |        |        |        |        |        |        |
|        |         |        | MA1000 |        |        |        |        |        |        |        |
|        |         |        | MA1020 |        |        |        |        |        |        |        |
|        |         |        | MA1030 |        |        |        |        |        |        |        |
|        |         |        | MA1040 |        |        |        |        |        |        |        |
|        |         |        | MA1050 |        |        |        |        |        |        |        |
|        |         |        | MA1060 |        |        |        |        |        |        |        |
|        |         |        | MA1060 |        |        |        |        |        |        |        |
|        |         |        | MA1070 |        |        |        |        |        |        |        |
|        |         |        | MA1080 |        |        |        |        |        |        |        |
|        |         |        | MA1090 |        |        |        |        |        |        |        |
|        |         |        | MA1090 |        |        |        |        |        |        |        |
|        |         |        | MA1100 |        |        |        |        |        |        |        |
|        |         |        | MA1110 |        |        |        |        |        |        |        |
|        |         |        | MA1120 |        |        |        |        |        |        |        |
|        |         |        | MA1130 |        |        |        |        |        |        |        |
|        |         |        | MA1140 |        |        |        |        |        |        |        |
|        |         |        | MA1150 |        |        |        |        |        |        |        |
|        |         |        | MA1160 |        |        |        |        |        |        |        |
|        |         |        | MA1170 |        |        |        |        |        |        |        |
```

C....CALCULATE COEFFICIENTS........
CALL SETCTE (NUS, NUB, LANDAB, COEFF...) ALL)
IF (CHECK) GO TO 2
C....CALCULATE ROOT AND OUTPUT RESULTS........
CALL ROOT (VSO, VS, LAMOAB, COEFF, NUS)
IF (PLOTIT) ED. 0 1) GO TO 32
IF (VSO * GE. VS MAX) 1) GO TO 1707
IF (VSO * LE. VSNAX) 1) GO TO 660
IF (VSO * GE. VSNAX) 1) GO TO 1708
WHITE (VS, VSNAX, 1) GO TO 1708
VSO * VSO * DVS

158
C...... CALCULATE FINAL RESULTS FOR LITHIUM NITRATE AND OUTPUT RESULTS

IF (CASE .EQ. 0) GO TO 1000

GO TO 1 30. 100. 60. 109 3. ICASE

C...... Row 3 ZERO CASE......

C...... 4 ALPHAS......
50 ETA(41) = CT1
ETA(45) = CT5
X1(I,1) = EL(5)
X(I,2,1) = EL(9)
X(I,2,2) = EL(13)
X(I,3) = EL(I3)
X(I,3) = EL(13)
X(I,3) = EL(13)
CALL CMATS (XL, ETA, 2, 1, 81740)
ETA(42) = ETA(3)
ETA(43) = ETA(4)
GO TO 1100

C...... 3 ALPHAS......
60 ETA(31) = CT1
ETA(4) = CT4
X(I,1) = EL(3)
X(I,2) = EL(7)
X(I,2) = EL(11)
X(I,3) = EL(11)
CALL CMATS (XL, ETA, 2, 1, 81740)

GO TO 1100

C...... Row 2 ZERO CASE......
100 EL = CABS (EL(I4) - EL(I14) / EL(I5) )
E1 = CABS (EL(I4) / EL(I5))
C(I, EL(I4), EL(I14) ) .GE. 110

105 ETA(11) = CT0
ETA(15) = -EL(I4) / EL(I5)
ETAT = CT0
ETAT = CT0
GO TO 1100

110 ETA(I1) = CT1
ETA(I5) = CT5
ETA(I) = CT5
ETA(I4) = CT0
GO TO 1100

1000 IF (EMET + .GT. 31) GO TO 1010

C...... ZERO - PIEZOELECTRIC CASE......
1.480 CONTINUE
M042720
1490 CONTINUE
M042730
PHUB = (0,0,1)
M042740
CC 1520 J = 1, 4
M042750
WRITE (4,1500) (1, ETA(I), I = 1, 10)
M042770
* 1. UNI3, UNI3, J = 1, 3), PHUB
M042780
1500 FORMAT (10D16/18, 36M), FINAL ANSWERS ***/
M042790
* 1.10,30.1/13PARTIAL FIELD/FIELD 271/135RELATIVE AMPLITUDES/
M042800
* 1.20,41M,17A, 12, 3M 31.47, 1M, N042810
* 1.14,7, 1M, 1M, 21.2M, MAE, 32.2M/2/
M042820
* 31M, 16, 3M 1.47, 1M, 47, 1M, 47, 1M, N042830
* 1.14,7, 1M, 1M, 24A, N042840
* 1.21M, 1M, N042850
1704 WRITE (6, 1705)
M042860
1705 FORMAT (10D1)
M042870
IPEOUT) GO TO 17
M042880
IFNEGAT) NUB=NUB
M042890
IFNEGAT) NUB=NUB
M042900
IPE VSO (.GE, VMAX) ) GO TO 1706
M042910
VSO = VSQ + DVS
M042920
GC TO 550
M042930
1707 WRITE (6,1705)
M042940
1706 IF (CNDU .EQ. 0 ) GO TO 1710
M042950
IPEMUIT) GO TO 38
M042960
NUB = NUB + NW
M042970
IF (NUB .GT. NMAX ) GO TO 1709
M042980
GC TO 38
M042990
38 PLS=PLS+60011
M043000
IPE(NUB,GT,HUMAX ) GO TO 1709
M043010
39 CONTINUE
M043020
CALL CHOKE
M043030
IFNEGAT) NUB=NUB
M043040
IFNEGAT) NUB=NUB
M043050
IPE MAX (.GE, 0 ) GO TO 930
M043060
HS = VSQ
M043070
GO TO 355
M043080
1709 NUB = NWU
M043090
1710 CONTINUE
M043100
C....INCLUDE PLOT ROUTINES HERE.....
M043110
IPEPLOIT)10,21 GO TO 34
M043120
IPEPLOIT)10,10 (GO TO 34
M043130
CALL SCALELEV(10,0,10,0,10,919)
M043140
IPEPLOT, MULT
M043150
ICALL AXISS),.2,2DIRECTION OF PROPAGATION,24,30.0,0,0,0.18,0.0
M043160
1 1501
M043170
IPEMUIT) CALL AXISS),.2,2DIRECTION OF PLATE NGAIIA,25.10,
M043180
1.0,0,18,0,10,91
M043190
ICALL AXISS),.2,2DIRECTION OF PLATE NGAIIA,25.10,
M043200
1.0,0,18,0,10,91
M043210
1 LEGT
M043220
ICALL AXISS),.2,3M 1,180,5,0,0.18,0,10,91
M043230
1 LEGT
C....ERVE - L M A T R I X S I N G U L A R .......
1740 WHITE (64, 1700) (CELL), I = 1, KGO
1750 FCMPAT (4NH) L M A T R I X S I N G U L A R (O U T P U T O F C O L U M N S ) / 
- ( I N . 46158.71) 
1760 GC TO 1700
1770 GC TO 1700
C....CALCULATE ADDITIONAL PARAMETERS FOR LITHIUM NEOBATE.....
1780 WRITE (64, 1700) (CELL), I = 1, KGO
1800 IF 4946GW II . NE. 0.9 PHASEU(II ATAN2(AINAGIU),REAL(U))S57.2977994A93200
1810 CC94TIUE 94*A943210 El - U MA*393220 94AI943230 C..CCMPUTE TIME AVERAGE POWER FLOW.;b
1820 PIP - PIFUNET,A, C11, C15, C16, C14, C15, C13, E11, E15, E33
1830 C16, C15, C16, C14, C15, C13, E11, E15, E33
1840 SPECI=SORT(SORTREAL(FLOPMAGFPHIMAG)*R2))
1850 F2M = PIFUN (ETA, C11, C55, C56, C49, C36, C39, C59, E14, E15)
1860 + (C11, C55, C56, C57, C49, C36, C39, C59, E14, E15)
1870 IF IP(97) EQ 0. GC TO 57
1880 HEATNGET+ A T A N 2 ( I M A G ) , R E A L ( P ) ) 197 .2 9 9 7 M H A 1 9 3 2 0 0
1890 IFEPLOTIT12). EQ.01 GC TO 57
1900 CONUNUE
C....CALCULATE T S.....
1910 C11 = IFUN (ETA, AX, C15, C55, C56, C49, C36, C39, E14, E15)
1920 C11 = IFUN (ETA, AX, C15, C55, C56, C49, C36, C39, E14, E15)
<table>
<thead>
<tr>
<th>LINENO</th>
<th>CONVAY</th>
<th>PHASE</th>
<th>NA</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>PRINT A, AX, C10, C45, C30, C44, C63, C23, C36, C30</td>
<td>MAIN3040</td>
<td>504</td>
</tr>
<tr>
<td>2</td>
<td>PRINT A, AX, C10, C30, C45, C44, C63, C23, C36, C30</td>
<td>MAIN3040</td>
<td>504</td>
</tr>
<tr>
<td>3</td>
<td>PRINT A, AX, C10, C30, C45, C44, C63, C23, C36, C30</td>
<td>MAIN3040</td>
<td>504</td>
</tr>
<tr>
<td>4</td>
<td>PRINT A, AX, C10, C30, C45, C44, C63, C23, C36, C30</td>
<td>MAIN3040</td>
<td>504</td>
</tr>
<tr>
<td>5</td>
<td>PRINT A, AX, C10, C30, C45, C44, C63, C23, C36, C30</td>
<td>MAIN3040</td>
<td>504</td>
</tr>
<tr>
<td>6</td>
<td>PRINT A, AX, C10, C30, C45, C44, C63, C23, C36, C30</td>
<td>MAIN3040</td>
<td>504</td>
</tr>
</tbody>
</table>

**Note:** The above snippet appears to be a fragment of a program listing, possibly from a scientific or technical document. The exact context is not clear from the image provided.
166

LIM03  CONWAY  PHASE  M4  
LIM03  -  EFN  SOURCE  STATEMENT  -  IFM33  -  

CC  6  =айлر،1L1111.INC1  
VSARAYK1=1  
V1=5  
CALL  RDOT(VSO,VS,H,FY5,EPSLON,MAX)  
AAAAVZEVME=1+AMAG(EPS5)  
AAAAVZEWME=REAL(EPS5)  
CETAKY1=REAL(FY5)  
CETAKY1=REAL(FY5)  
CETAKY1=REAL(FY5)  
CETAKY1=REAL(FY5)  
631  
637  
639  
691  
693  
695  
697  
699  
701  
704
DIM INTEGER SUBDIV
CABLE SUBDIV=2,3,4,5,6,7,8,9,10,11,12,13,14,15
SAVE NAME/0/
CATA RCBC/1/ CCMP/GETOUT
COMC/MPLOTS/ICHECK
LEGJAL GETOUT
CATA PREV/ FALSE./
LEGJCAL ICOS4
COMMON /FMDOTS/ FVSNAG, NT, ICASE
- /LESS/ IALF
CIMAL F VS, F30, F31, F32, F33.
REAL T, T 1, T 2, EL
REAL IMAEA2
LOGICAL IALF
CATA INDI/2/.
NS=.*HXM.1
FACT1=.01
FACT2=1.005
IF X(N.E.0).AND.X(N.E.HXM.1) FACT1=.99
IF X(N.E.0).AND.X(N.E.HXM.1) FACT2=.995
X = 0
VS = VSO
FVS = RELNCITY(VS)
IF IALF .GT. 600 KND0000
FIF1(XEYI) GT 500
IF XEYI .GE. 0.1 GO TO 700
IF FVSNG .LT. EPS .GT. 500
FVS = FVS
V=FACT1*FVS
FVS = ROUND (F VS 1)
IF IALF .GT. 600 KND0000
FIF1(XEYI) GT 500
IF XEYI .GE. 0.1 GO TO 700
IF FVSNG .LT. EPS .GT. 500
FVS = FVS
V=FACT1*FVS
VX = 30
FVS = ROUND (F VS 1)
IF IALF .GT. 600 KND0000
FIF1(XEYI) GT 500
IF XEYI .GE. 0.1 GO TO 700
IF FVSNG .LT. EPS .GT. 500
FVS = FVS
V=FACT1*FVS
VX = 30
FVS = ROUND (F VS 1)
IF IALF .GT. 600 KND0000
FIF1(XEYI) GT 500
IF XEYI .GE. 0.1 GO TO 700
IF FVSNG .LT. EPS .GT. 500
FVS = FVS
V=FACT1*FVS
VX = 30
FVS = ROUND (F VS 1)
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<tr>
<th>LINDO3</th>
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<th>PHASE</th>
<th>N4</th>
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<td>R00T00</td>
<td>R00T00</td>
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</table>

```
C......B E G I N  I T E R A T I O N.....

330 G2 = FAM2+AM2*AM2 - 2*AM2*DELTA2 + F2*AM2*DELTA2 + DELTA2*R00T00
    T = CSQT + FAM2*AM2 - F2*AM2*DELTA2 + DELTA2*R00T00
    T1 = G2 + T
    T2 = G2 - T

300 LAMCA3 = -g. +FX2*DELTA2/F

530 CONTINUE
  IF (PPEV) PPEV = FALSE.
  N0 = 0
  EXIT
  IF (KXKK/2.2 NE. KXKK) GO TO 999

100 IF (CABS2(T) GT. CABS2(T1)) T = T1
    IF (CABS2(T) LE. 0.3) GO TO 530

C......E N D  O F  I T E R A T I O N.....
```

```
ERROR MESSAGE NUMBER 1
R00T00
```

SIMPTE SETCT. DECK

CSETCT *** SETCTE COMPUTES THE C, T, E COEFFICIENTS FROM
THE INPUT PARAMETERS C, P, EPS, MU, LAMBD, NU

SUBCUTINE SETCTE (MU, NU, LAMBD, COEFF)

CTE00010 PAGE 14

CTE00020 CTE00010

CTE00030 CTE00010

CTE00040 CTE00010

CTE00050 CTE00010

CTE00060 CTE00010

CTE00070 CTE00010

CTE00080 CTE00010

CTE00090 CTE00010

CTE00100 CTE00010

CTE00110 CTE00010

CTE00120 CTE00010

CTE00130 CTE00010

CTE00140 CTE00010

CTE00150 CTE00010

CTE00160 CTE00010

CTE00170 CTE00010

CTE00180 CTE00010

CTE00190 CTE00010

CTE00200 CTE00010

CTE00210 CTE00010

CTE00220 CTE00010

CTE00230 CTE00010

CTE00240 CTE00010

CTE00250 CTE00010

CTE00260 CTE00010

CTE00270 CTE00010

CTE00280 CTE00010

CTE00290 CTE00010

CTE00300 CTE00010

CTE00310 CTE00010

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---SET UP D AND G MATRICES---

CTE00490 CTE00010
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LIM003 CONVAY PHASE M4

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IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02290
IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02300
IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02310
IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02320
IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02330
IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02340
IFI C(10).EQ.0. AND. C(10).EQ.0. AND. C(101).EQ.0. AND., CTE02350

1600 CONTINUE
1699 FORMAT (7H0,4X,3H8, 1H (, 1H1, 1H2, 1H0 ) // 290 1H (3PE18.7F ) / CTE02310
* 1H0,4X,6H6, 1H (, 1H1, 1H2, 1H0 ) // 176 1H (3PE18.7F ) / CTE02320
* 1H0,4X,6H6, 1H (, 1H1, 1H2, 1H0 ) // 5 1H (3PE18.7F ) / CTE02330
RETURN

CTE02340

1=0

CTE02350
```
CIRCUNC

CCMplex FUNCTION ROUND (F)

REAL I, F12
P = F12
I = F12
IF (R .EQ. 0.) OR (I .EQ. 0.) GO TO 100
IF (ABS1/F12 .LT. 1.E5) GO TO 50
B = C.
GO TO 100
50 IF (ABS1/F12) .GE. 1.E5) I = 0.
100 ROUND = CMPLEX (F12, I)
RETURN
END
SUBROUTINE CSFUN (Y, RX, SX, CX)

DOUBLE PRECISION RX, SX, CX

X = Y
IF (X .LT. 0.) X = X + 360.
IF (X .EQ. 0.) OR (X .LT. 180.) GO TO 150
IF (X .EQ. 180.) OR (X .LT. 270.) GO TO 200
SX = SIN(X/RX)
CX = COS(X/RX)

100 RETURN
150 CX = 1.
SX = 0.
IF (X .EQ. 180.) CX = -1.
GO TO 100
200 SX = 1.
CX = 0.
IF (X .EQ. 270.) SX = -1.
GO TO 100
END CSFUN200
SIBTC FF.... DECK

CFF
COUPLE PRECISION FUNCTION FF (X, Y, R, L)
CCCMXY JRFY GAMMA(3.31, 0.3.3.31, 0.3.3.31)
DOUBL PRECISION GAMMA, D, Q
INTEGER X, Y, 1, U
FF= 0,
CC 50 X = 1, 3
CC 50 Y = 1, 3
CC 50 U = 1, 3
50 FF=FF + GAMMA(J1) + GAMMA(J2) + GAMMA(J3) + GAMMA(J4) + DIR(5,7,0)
RETURN
END
SIMFTC 90, DECK

C8

COUPLE PRECISION FUNCTION R (i, j, k)

COUPLE /HEF/ GAMMA (3, 3) (0, 0) (0, 0) (0, 0)

COUPLE PRECISION GAMMA (j, k)

INTEGER S, T, U

R = 0

CC 50 S = 1, 3

CC 50 T = 1, 3

CC 50 U = 1, 3

SO R = R + GAMMA (j, s) * GAMMA (2, t) * GAMMA (1, u) * Q (S, T, U)

RETURN

END
SUBTC TIJ, DEC

CTIJ

DOUBLE PRECISION FUNCTION TIJ (I, J)
COMMON /FPR/ GAMMA(1,3), JUNK(60)
COMMON /EPSPS/ E(21), PI18, EPSLGM(1,3)
COMMON /EPSPS/ E(21), PI18, EPSLGM(1,3)

DOUBLE PRECISION GAMMA, JUNK
INTEGER R, S

TIJ = 0.
CC 50 R = 1, 3
CC 50 S = 1, 3
50 TIJ = TIJ + GAMMA(R)*GAMMA(S)*EPSLGM(R,S)
RETURN
END
SIGFTC F...... DECK

CP THIS FUNCTION EVALUATES THE L DETERMINANT FOR A VALUE OF V5
C* FOR EITHER THE LITHIUM OR GOLD LITHIUM CASE
C
C***********************************************************************

COMPLEX FUNCTION F (45)

COMMEN /1232/ CC.CE.CT
COMMEN /#ROOTS/ FISHAG, NT. ICASE
COMMEN /2021/ MRTA

COMMEN /CSETIV/ ALFA2 (1), ALFA1 (1), ELC1 (1), ALFA1 (1),
COMMEN /LINK/ ALFA1 (1), BETAM1 (1), ELC1 (1), ALFA1 (1),

COMMEN /MLA/ LHMD1, MLA1, MLA2, MLA3,
COMMEN /VUL/ EPS1, EPS2, EPS3, EPS4,

COMMEN /XP/ ALL, MULT, ITER, COEFF, DETERM, POLYN.

COMMEN /ALPHA/ MLA, ELP,
COMMEN /BETA/ MLA, ELP,

COMMEN /CFL/ RAGM

CCPLEX ALFA. FVS, EL. EA133, EA134, WA133, WA134, EA135

CCPLEX ALFA. ALFA1, MLA1, MLA2, MLA3, MLA4, MLA5

CCPLEX EA1, EA2, EA3, EA4, EA5, EA6, EA7, EA8, EA9, EA10, EA11

REAL MLA, MLA1, MLA2, MLA3, MLA4, MLA5, MLA6, MLA7, MLA8, MLA9

LOGICAL ALL, ROOTS, ITER, COEFF, DETERM, POLYN, ALPHA, BETA.

LOGICAL 1, ACAP

LOGICAL 1

LOGICAL 1, ACAP

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C..SELECT
C..CALCULATE COEFFICIENTS

510
490
EJ
K2
L2
I1
I2

C..CALCULATE COEFFICIENTS OF POLYNOMIAL....

490
480
470
460
450
440
430
420
410
400

C..SELECT POSITIVE REAL ROOTS.....

789
779
769
759
749
739
729
719
709
699
689
679
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$\left( 1 + \frac{1}{2} \right)^2 = 1.5$
C.. PIEZOELECTRIC CASE...  
830 IF IACAP = WRITE6.675) THEN ( \ ( B(1,J), J=1,4 ) I=1,4 )  
B(5,J)=B(5,J)+90  
B(6,J)=B(6,J)+90  
B(7,J)=B(7,J)+90  
B(8,J)=B(8,J)+90  
B(1,J)=B(1,J)+90  
B(2,J)=B(2,J)+90  
B(3,J)=B(3,J)+90  
B(4,J)=B(4,J)+90  
RETURN  
ENDIF  
CC TO 580  
940 IF IACAP = WRITE6.675) THEN ( \ ( B(1,J), J=1,4 ) I=1,4 )  
FORMAT(100,100,100,100,100,100,100)  
CALL GMATS(100,100,100,100,100,100,100)  
RETURN  
ENDIF  
CC TO 583  
574 IF ANETA (GE. 3) GC TO 583  
C.. ZEROC - PIEZOELECTRIC CASE...  
EC 921 = 1. 6  
RETURN(4) = 10.0. 1  
EC 922 = 1. 3  
F0001750  
F0001760  
F0001770  
F0001780  
F0001790  
F0001800
LIMNO3 CONWAY PHASE %

L2 = 12
ICE = 2

160 FORMAT // 1M *** GEMERATE ,316,18M ( ) ALPHAS / )

CC TO 125

PAGE 30

C...... 26W. 2 ZERO CASE.....

C...... 4 ALPHAS ).....

200 CONTINUE

GASF = 2

CC TO 125

PAGE 30
LI403 CONWAY PHASE N4

C....( LESS THAN 4 ALPHAS ).....
280 IC = 2
11 = 0
2
CC 310 I=1..K
TERM = CASE1 - B22(ALPHAB11*441/ALPHAB11) - B24(ALPHAB11)**2
IF1 TERM .GE. 1.0 = 5 GO TO 300
11 = 11 + 1
1A11(12) = 1
GC TO 310
300 IC = 12 + 1
1A12(12) = 1
310 CONTINUE
IFI 12 .EQ. 2 ) GO TO 340
IFI 11 .EQ. 2 ) GO TO 320
GO TO 632
C....( 2.K CASE ).....
520 IA12 = I1A11)
IA12 = IA11(I)
ICASE = 5
IFI = 3
ICB = 2
LI = 4
11 = 3
GO TO 360
C....( 1.5 CASE ).....
340 IA12 = IA11(I)
IA12 = IA11(I)
ICB = 5
550 ICASE = 4
IF2 = 2
L2 = 6
R2 = 2
360 J = IA12(I)
WRITE(8)/370 ( ALABRI,1C),1=1..3, L/AB1C), L/AB2,1C1,ALBARI
GO TO 370
370 FCFM101 // SHI 4.46 DEGENERATE .456 / FOR K = 12.
4M AND 2416 (1=1..4) //
IFI ICB .EQ. 3) ICB = 1
GC TO 245, 295 1, ICB
C....( PEC - PIEZOELECTRIC CASE ).....
400 IC = 3
11 = 0
72 = 1
45 TO 470
470 CASE2 = CASE1 - B22(ALPHAB11) 1
IFI 11 .EQ. 3 ) GO TO 410
IFI CASE2 .EQ. J5. 1 = 11 + 1
GO TO 390
410 IF1 1 .EQ. 1 ) GO TO 420
**LIMNO3 MODEL**

**SOURCE STATEMENT**

* IkhISI - CANMIN - CA22

**FORMAT**

* F0001000 J= 9

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```plaintext
187

LINB03 CONWAY PHASE N4 F... EF...

**CONWAY**

**PHASE** N4

**EPM**: 12/31/69

**SOURCE**: 000109

**PAGE**: 32

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LUMPED CONWAY PHASE 3

1220 IF (WM .NE. 0. .OR. KM .NE. 1) GO TO 1250
  ST = 0.
  GC TO 1290
1250 IF (EN .NE. 0. .OR. WN .EQ. 0. .OR. WGE .GE. 1.E10) GO TO 1280
  SWHINT(W,FYS) = SINWINT(W,FYS) * EPSR
  GO TO 1290
1280 ST = TANH(W,FYS) * EPSR
1290 EL(E3) = BETAVI(1,E14) * CMPLX(0., 1.E10) + ALFAVI(1,E10)
  - BETAIV(1,E14) * CMPLX(0., 1.E10) + ALFAIV(1,E10)
  - CMPLX(0., 1.E10) + ALFAVI(1,E10) * EPSRI * BETAVI(E3)
  EL(E3) = EL(E3) + 1.E10
1300 L = L + 4.
  IF (INAHFT .GE. 3) GO TO 1340
  GC TO 1340

C........ IEPC - PIEZOELECTRIC CASE . . .
  EL(4) = (0. , 0.)
  EL(14) = (0. , 0. )
  EL(12) = (0. , 0. )
  EL(15) = (0. , 0. )
  EL(16) = (0. , 0. )

1340 IF (ETERM .OR. ALL) WRITE(A,1355) ( EL(I1), I=1,16 )
1355 FORMAT (33X34) INTERMEDIATE L MATRIX BY COLUMNS/
  . ESM ( .FPRE3X15.5 )
  . ICBI = ICBI + 1
  . GC TO (1343), 1346, 1370, 1390 !, ICBI
1343 CC 1345 P=1,16
1345 XEL(1) = E11
  CALL CEE(I , 6, FYS, KEXP)
1350 F = FVS
  FVSAG = CSE( FVS )
  IF (ETERM .OR. ALL) WRITE (A, 1355) VS, FVS, FVSAG
1355 FORMAT (5H04VS = , E15.7, 5X, TFPSFYS = , 2E15,5X,5SAMAG = E15.7 )
1357 RETURN
1360 FVS = EL(E3*EL(E3) - EL(E3)*EL(E3) )
1370 FVS = EL(I1)*EL(I1) - EL(I2)*EL(I2)
  GC TO 1390
1380 J = 1
  CC 1385 K=1,3
  XEEL(1) = EL(I1)
  XEEL(2) = EL(I2)
  XEEL(3) = EL(I3)
1385 J = J + 4.
  CALL CEE(I , XEEL, 3, FVS, KEXP )
  GC TO 1390

C........ EVALUATE GOLD LITHIUM NIOBATE EQUATIONS . . . .
1390 CC TO.WA = WAGLAMCA
  GA = WAGCC(1,E20)
  GC TO 1390

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<table>
<thead>
<tr>
<th>L1N03</th>
<th>CONWAY</th>
<th>PHASE</th>
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<th>000109</th>
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</thead>
<tbody>
<tr>
<td>F.....</td>
<td>-1PN</td>
<td>SOURCE STATEMENT</td>
<td>-1PN(S)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FVS = CSGTR(CMOD - 4,#C40C)</td>
<td>ALFAA1 = CSGTR(0B - 0BFSI/Do + D1)</td>
<td>ALFAA1 = ALFAA1</td>
<td>ALFAA3 = CSGTR(0B - FS3/Do + D3)</td>
<td>ALFAA4 = ALFAA3</td>
<td>ALFAA5 = CSGTR(CMPX(FHA - VFSI/MUA, G, F3))</td>
<td>ALFAA6 = ALFAA5</td>
</tr>
<tr>
<td>IFI ALPHA, (7. ALL) WRITE (A,L505) ( ALFAA1, I=1,6 )</td>
<td>1505 FORMAT (12)</td>
<td>1505 K = 1, 4</td>
<td>1505 R = 0, 6</td>
<td>1505 R = 0, 6</td>
<td>1505 R = 0, 6</td>
<td>1505 R = 0, 6</td>
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<tr>
<td>BETAA(I, J) = CMPX00 - LAMDA - MU(I, F5A)</td>
<td>)</td>
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<tr>
<td>1620 ELL(1, J) = BETAA1, J3</td>
<td>1620 J = 1, 3</td>
<td>1620 J = 1, 3</td>
<td>1620 J = 1, 3</td>
<td>1620 J = 1, 3</td>
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<td>CO 1650 J = 7. 10</td>
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<tr>
<td>1740 ELL(1, J) = BETAB1, J3</td>
<td>1740 J = 1, 6</td>
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<tr>
<td>1890 ELL(1, J) = BETAB1, J3</td>
<td>1890 J = 1, 6</td>
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<td>CO 1915 J = 7, 10</td>
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<tr>
<td>1910 ELL(1, J) = BETAB2, J3</td>
<td>1910 J = 1, 6</td>
<td>1910 J = 1, 6</td>
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<td>CO 1915 J = 1, 10</td>
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<td>CO 1915 J = 1, 10</td>
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</tbody>
</table>

The image contains a section of a document with mathematical and programming content, likely related to scientific or engineering computations. The text is dense and appears to be part of a larger software or data processing program.
LiMO3 CONWAY PHASE N4 12/31/69 000109 PAGE 35

1904 ELL(I,J) = ELL(I,J+1)/10
        IF I=I1, J=J1, ALL I WRITE(A,1930) ( ELL(I,J), J=1,10 ) , I=1,10 ) F0005150
        ( ELL(I,J), J = 1, 10, I = 1, 10 ) F0005170
1930 FORMAT (120I5** L MATRIX ****/120 H. LEPI15.6)
        F0005180
        DD 1940 I=1,100
        1940 CALL COST (XEL, J0, FIV, REXP)
        GO TO 1350 F0005210
        (XEL, '0' FIV, REXP)
        F0005220
        GO TO 1350 F0005230
        END F0005280

C...ERROR IN MATRIX INVERSION.....
1960 WRITE (A, 1970)
1970 FORMAT (300H***SINGULAR MATRIX IN BETA***)
        GO TO 1350
        END
        END
F0005240
F0005250
F0005260
F0005270
F0005280
SIBFIC STRIP. GICK

CSTRIP /// ** ARRAY OF CONDUCTING STRIPS OVER PIEZOELECTRIC MEDIUM *** /// STRP0200
CPV012 // G.Y. ROBERTS /// STRP0000
CP // 02.15.AH /// STRP0500
C~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~/// STRP0000

SUBROUTINE STRIP (A, VS, RHO, ALL, COEFF)

CSTRIP /ZT2Z, C1Z11/ /// STRP0100
CCOMMON /PICLC slashed 4 COPE /// STRP0110
CCOMMON /PIAIC slashed 4 COPE /// STRP0120
PREL /// C101, C1, C2, C0, C001 /// STRP0130
INTENC P /// STRP0140
LOGIC /// ALL, COEFF, COPE /// STRP0150
EQUIVALENT /// C121, C131, C141, C151, C161 /// STRP0160
// /// C121, C131, C141, C151, C161 /// STRP0170
// /// C121, C131, C141, C151, C161 /// STRP0180
// /// C121, C131, C141, C151, C161 /// STRP0190
// /// C121, C131, C141, C151, C161 /// STRP0200
// /// C121, C131, C141, C151, C161 /// STRP0210
// /// C121, C131, C141, C151, C161 /// STRP0220
// /// C121, C131, C141, C151, C161 /// STRP0230
// /// C121, C131, C141, C151, C161 /// STRP0240
// /// C121, C131, C141, C151, C161 /// STRP0250
// /// C121, C131, C141, C151, C161 /// STRP0260
// /// C121, C131, C141, C151, C161 /// STRP0270
// /// C121, C131, C141, C151, C161 /// STRP0280
// /// C121, C131, C141, C151, C161 /// STRP0290
// /// C121, C131, C141, C151, C161 /// STRP0300
// /// C121, C131, C141, C151, C161 /// STRP0310
// /// C121, C131, C141, C151, C161 /// STRP0320
// /// C121, C131, C141, C151, C161 /// STRP0330
// /// C121, C131, C141, C151, C161 /// STRP0340
// /// C121, C131, C141, C151, C161 /// STRP0350
// /// C121, C131, C141, C151, C161 /// STRP0360
// /// C121, C131, C141, C151, C161 /// STRP0370
// /// C121, C131, C141, C151, C161 /// STRP0380
// /// C121, C131, C141, C151, C161 /// STRP0390
// /// C121, C131, C141, C151, C161 /// STRP0400
// /// C121, C131, C141, C151, C161 /// STRP0410
// /// C121, C131, C141, C151, C161 /// STRP0420
// /// C121, C131, C141, C151, C161 /// STRP0430
// /// C121, C131, C141, C151, C161 /// STRP0440
// /// C121, C131, C141, C151, C161 /// STRP0450
// /// C121, C131, C141, C151, C161 /// STRP0460
// /// C121, C131, C141, C151, C161 /// STRP0500
// /// C121, C131, C141, C151, C161 /// STRP0510

CATA TP1, JINQ, CAG, CM1Q, JMAG / 6.2831853, 10.0, 0.1, 1.0, -0.1, 1.0 / /// STRP0260
// /// IN., -1.0, 1.0, 1.0, 1.0, -1.0 / /// STRP0270
C......SET UP MATRICES......

IF (.NOT. CNCE) GO TO 670 /// STRP0280

CNCE = .FALSE. /// STRP0290
C1(1,1) = C15 /// STRP0300
C1(1,2) = C15 /// STRP0310
C1(1,3) = C15 /// STRP0320
C1(1,4) = C15 /// STRP0330
C1(1,5) = C15 /// STRP0340
C1(2,1) = C15 /// STRP0350
C1(2,2) = C15 /// STRP0360
C1(2,3) = C15 /// STRP0370
C1(2,4) = C15 /// STRP0380
C1(2,5) = C15 /// STRP0390
C1(3,1) = C15 /// STRP0400
C1(3,2) = C15 /// STRP0410
C1(3,3) = C15 /// STRP0420
C1(3,4) = C15 /// STRP0430
C1(3,5) = C15 /// STRP0440
C1(4,1) = C15 /// STRP0450
C1(4,2) = C15 /// STRP0460
C1(4,3) = C15 /// STRP0500
C1(4,4) = C15 /// STRP0510
C1(4,5) = C15 /// STRP0520
C1(5,1) = C15 /// STRP0530
C1(5,2) = C15 /// STRP0540
C1(5,3) = C15 /// STRP0550
C1(5,4) = C15 /// STRP0560
C1(5,5) = C15 /// STRP0570

C~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~/// STRP0000
C......P E W......
  670 CC AEO J = 1, 9
 480 A11 = 000

C......prepare FOR GAMMA113 LOOP......
  RC = RHOMPSVS
  CD1(11) = 8GM - C11
  CD1(31) = 8GM - C36
  CD1(33) = 8GM - C55

C......compute COEFFICIENTS OF EIGHT DEGREE POLYNOMIAL......
P = 0
CC 1020 J = 1, 4
CC 1020 K = 1, 4
IF J .EQ. 0, GO TO 1020
CC 1010 L = 1, 4
IF L .EQ. 0, GO TO 1020
\( P = 10 - J - K - L \)
\( P = P + 1 \)
IF P .GT. 91 P = 2
T1 = CA11,J*M(CB12,K1) + CB11,J1*CA12,K1
T2 = CA11,J*M(CD12,K1) - CB11,J1*CB12,K1 + CD11,J1*CA12,K1
T3 = CB11,J*M(CB12,K1) + CD11,J1*CB12,K1
T4 = CA11,J*M(CD12,K1)
T5 = CA11,J*M(CD12,K1) + T1*CA13,L1 - T2*CA13,L1
T6 = CD13,L1*M(CB13,L1) + CA11,J*M(CD12,K1) + T1*CA13,L1 - T2*CA13,L1
T7 = CD13,L1*M(CB13,L1) + CA11,J*M(CD12,K1) + T1*CA13,L1 - T2*CA13,L1
T8 = CD13,L1*M(CD12,K1) + T1*CA13,L1 - T2*CA13,L1
T9 = CD13,L1*M(CD12,K1) + T1*CA13,L1 - T2*CA13,L1
Z5 = CA11,J*M(CD12,K1) + CD13,L1
Z6 = T1*M(CD12,K1)
LIMB03 CONWAY PHASE M4

STAP. = EFM SOURCE STATEMENT - IFMIS) -

$U = (U4 + U3)${
$V = (V4 + V3)${
$A11 = A11 + 0.8*U4*M2 + 0.8*V3*M2$
$A21 = A21 + 0.8*U4*M2 + 0.8*V3*M2$
$A31 = A31 + 0.8*U4*M2 + 0.8*V3*M2$
$A41 = A41 + 0.8*U4*M2 + 0.8*V3*M2$
$A51 = A51 + 0.8*U4*M2 + 0.8*V3*M2$
$A61 = A61 + 0.8*U4*M2 + 0.8*V3*M2$

1050 CONTINUE

1090 CONTINUE

1030 CONTINUE

1050 FORMAT (2THOCOEFFICIENTS OF POLYNOMIAL/T1M + ZE1B,T3)
SUBROUTINE CROOT (A, X0, CODE, XI)

EXTERNAL POL

I = 1
DC 100 I = 1, 9
100 CII = A(I)
110 IF (CABS(CII) .NE. 0.) GO TO 130
X(1) = (0., 0.)
I = I + 1
IF (I .GT. 8) GO TO 280
P = 10 - I
GO TO 10
IL = I
120 CII = CII(I)
GO TO 110

C.........COMPUTE X-ROOT......
130 X1 = (5., 5.)
IF (CODE .NE. 0) X1 = X0(K)
C.........X1 = INITIAL GUESS. USE MULLER'S METHOD TO REFINE IT......
P = 10 - I
CALL MULLER (POL, XI, 20, 1.E-8, JUNK, X1, N)
XI = (5., 5.)
IF (N .GE. 20) CALL MULLER (POL, XI, 20, 1.E-8, JUNK, X1, N)
X(1) = ROUND(X1)
X = X - 1
IF (I .GT. 8) GO TO 280

C.........REDUCE COEFFICIENTS......
DC 240 I = 2, 9
240 CII = CII + XI(I-1) + CII(I-1)
IF (I .NE. 8) GO TO 130
IF (CII .NE. 0.) GO TO 250
X(8) = CII
GC TO 250
250 X1B = CII(I)/CII
X1B = ROUND(X1B)

C.........EXIT......
280 GC 290 I = 1, 8
290 XI(I) = X1B
X(1) = 1
RETURN
END
SIRTEC MULER. DECK

CMULEX ****** MULLERS METHOD FOR RCOT FINDING ******

C* G.V. ROBERTS
C* 04.26.68
C* **********************************************

SUBROUTINE MULLER (FX, XR, MAX, EPSLON, KODE, X, NI)

CCPLEX F, X, XR, XI, X2, X3+ H2, L2, D2, G, FX, FX1, FX2, D2.

L PARA, ONE, GPO, GPO, L2, FX, HT, [DATA TWC, ONE / (0.0, 0.1), (0.0, 0.1)]

FX = FX01
X2 = XR
KODE = 0
IF (CABS(X2) .EQ. 0.) GO TO 190
FX1 = F(X1, XP03)
FX2 = F(X2, XP03)
H2 = -0.0130
G2 = 220
FX3 = F(X3)
FX4 = F(X4)
J2 = -0.01
220 L2 = -0.50
D2 = 0.50

C*****BEGIN ITERATIONS*****

CALL OVERC

CC = 540 K = 3 MAX
A = 5 XPLKPL2 = FX1*2*FX2 + FX2*FX2 + FX2*X2 + L2 + C2)
CALL OVERC
C = C0RT (C0RT = 4.*FX2*FX2 + 2*FX2*X2 + FX2 + C2)
GPO = G + C
GPO = G - C
CSPC = CABS(GPO)
CSPC = CABS(GPO)
IF (CSPC .LT. EPSLON) GO TO 390

X2 + X2 = X3 = X3 + H2

390 X2 = X2 + H2
IF (CSPC .GT. EPSLON) GO TO 390
CALL OVERC

370 X3 = X2 + L2
CALL OVERC

G2 = 220
L2 = -0.50
FX3 = F(X3)
FX4 = F(X4)
J2 = -0.01
G2 = 220
L2 = -0.50
D2 = 0.50

C*****END OF MULLER subroutine*****

MULER020 MULER030 MULER040 MULER050 MULER060 MULER070 MULER080 MULER090 MULER100 MULER110 MULER120 MULER130 MULER140 MULER150 MULER160 MULER170 MULER180 MULER190 MULER200 MULER210 MULER220 MULER230 MULER240 MULER250 MULER260 MULER270 MULER280 MULER290 MULER300 MULER310 MULER320 MULER330 MULER340 MULER350 MULER360 MULER370 MULER380 MULER390 MULER400 MULER410 MULER420 MULER430 MULER440 MULER450 MULER460 MULER470 MULER480 MULER490
C...xCT CONVERGENT YET........

450 FX = FX2
FX2 = FX3
X = X1
X1 = X2
X2 = X3
P1 = P2
P2 + P2 - X1
L2 = P2/H1
540 D2 = L1 + L2
KDE = 2
X = X2
570 RETURN
580 GOTO X2
END
$169TC POL... DECK

CPOL
COMPLEX FUNCTION POL (X)
COMPLEX ORDER/ C; X, M
COMPLEX T, C4, X
POL = (0+C4)
T = (1+0)
J = M
DO 100 J = 1, M
POL = POL + C(J)*T
CALL CHECK
T = T*T
100 J = J - 1
RETURN
END

POL00020
POL00030
POL00040
POL00050
POL00060
POL00070
POL00080
POL00090
POL00100
POL00110
POL00120
POL00130
POL00140
CTFUN

COMPLEX FUNCTION TFUN (ETA, N5, C1, C2, C3, C4, C5, C6, C7, C8)
COMMON /LINK/, JUNK1(244), ALFAB4(4), JUNK2(343), BETAB(4,4), T
      JUNK3(187), VS, JUNK4(177)
      / CIA / IA(4)
      COMMON ALFAB, BETA, ETA(4)
      CERUG ETA=N5,C1,C2,C3,C4,C5,C6,C7,C8
      TFUN = 10, 5, 1
      EC NO K = 1, 4
      I = IA(1)
      CERUG I
      CERUG TFUN
      BO TFUN = TFUN + ETA(1)*
      - BETAB5,K1*COMPLX(0.,C1) + ALFAB1(1)*C2/15
      - BETAB5,K1*COMPLX(0.,C3) + ALFAB1(1)*C4/15
      - BETAB5,K1*COMPLX(0.,C5) + ALFAB1(1)*C6/15
      - BETAB5,K1*COMPLX(0.,C7) + ALFAB1(1)*C8/15
      
      CERUG TFUN
      RETURN
      END

TFUN0050
TFUN0060
TFUN0070
TFUN0080
TFUN0090
TFUN0100
TFUN0110
TFUN120
TFUN130
TFUN140
TFUN150
TFUN160
TFUN170
TFUN180
TFUN190
LIBFIC PIFUN.  DECK

CPIFUN

COMPLEX FUNCTION PIFUN (ETA, C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, C14, C15, C16, C17, C18, C19, C20, C21, C22, C23, C24)

PIFUN020

COMMON /JUNK1(244), ALFAB(1), JUNK2(120), BETAB(44), JUNK3(120) /BETAMF, NETA

PIFUN040

* JIA / (IJA)

PIFUN060

COMPLEX ALFAB, BETAMF, ETA(4), ETA(4)

PIFUN080

DATA J / (0.,1.) /

PIFUN100

NA = NETA + 1

PIFUN120

PIFUN = (IJA, O, J)

PIFUN140

OC 230 I = 1, 4

PIFUN150

L = IAJ11

PIFUN160

CC 230 K = 1, NA

PIFUN170

P = IAJA1

PIFUN180

PIFUN = PIFUN + (ETA(I)CMJG(ETA(4)) + ALFAB(L) + CONJG(ALFAB(4))))

PIFUN190

* CMJG(ETA(4)) = CMJG(ETA(4)) + ALFAB(L) + CONJG(ALFAB(4))

PIFUN200

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN210

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN220

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN230

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN240

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN250

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN260

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN270

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN280

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN290

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN300

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN310

* BETAB(C(5), J) + ALFAB(C(5))

PIFUN320

230 CONTINUE

PIFUN = PIFUN/2.

PIFUN330

RETURN

PIFUN340

END
SUBROUTINE CCM

C ** DETERMINANT OF COMPLEX MATRIX.  DET(A) = P * 10**N

! SUBROUTINE CCM(A,N,F,M)
C DIMENSION A(1..N,1..N)
C COMPLEX A,F,M
C EQUIVALENCE (S,S+F,S+F)
C ALGOLI = MCLOGA/MCLOG(2.1)
       SDK = 1.0
       DO 10 I = 1,N
           CC 15 J = I+1
               IF (REAL(A(I,J)) .EQ.0. AND. AIMAG(A(I,J)) .EQ.0.3) GO TO 15
          10       CC 17 K = A(I,J)
               IF (REAL(A(I,J)) .EQ.0. AND. AIMAG(A(I,J)) .EQ.0.3) GO TO 15
               B = A(I,J)
               A(I,J) = K
               CC 19 L = A(I,J) - B*AIJ
               15       CONTINUE
       16       SUM = 0.0
               PHI = 0.0
               CC 20 J = 1,N
                   IF (REAL(A(I,J)) .EQ.0. AND. AIMAG(A(I,J)) .EQ.0.3) GO TO 18
                   SUM = SUM + ALOG2(REAL(CABS(A(I,J))1))
                   20       PHI = PHI + 1.5707963267948
                   PHI = PHI + PI
                   S = ABS(SUM) - 37
                   IF (S .LT. 1.0) GO TO 100
                   P = SIGMA(SUM)
                   F = PI*10.**(SUM-4.727467)
                   GC TO 200
                   100       P = 0
                   F = PI*10.**(SUM)
                   200       IF (SUM.GT.0.1) RETURN
                   F = - F
                   RETURN
                   18       P = 0
                   F = I(O,0)
                   RETURN
                   END
SUBROUTINE CHATS

REAL A(N),X(N1),X(N2)
COMPLEX A,N1,N2

IF (N1 .EQ. 0) GO TO 15
IF (N2 .EQ. 0) GO TO 15
N1 = N1 -.1
N2 = N2 -.1
IF (REAL(A(I,J)) .EQ. 0.0 .AND. AIMAG(A(I,J)) .EQ. 0.0) GO TO 15
IF (REAL(A(I,J)) .EQ. 0.0 .AND. AIMAG(A(I,J)) .EQ. 0.0) GO TO 15
RETURN

C cooperate

GO TO 25
CONTINUE
RETURN

END
SUBROUTINE OVERCK
COMMON/OPR.FOR,HI
EQUIVALENCE (JAMD, XADD)
COMMON AUVENT2
DATA MASK/CDDDDDDDDDDDDDDDD/
K=0
XAND=AND(MASK, COUNT(11))
KNTFPT=JAND+390
KNTS=KOUNT1+KNTFPT
KNT1=KNTSAV/32768
KNT2=KNTSAV-32768*KNT1
IF (KNT2.LT.1) RETURN
KCOUNT(I)FPT+1=32768*KNT1+1
K=1
RETURN
END
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| CER           | SECTION  10 | 11  |
| JFS           | SECTION  13 | 14  |
| FJPS         | SECTION  16 | 17  |
| E           | SECTION  19 | 20  |
| SYRCC       | SECTION  22 | 23  |

| SUBROUTINES CALLED | |
|--------------------| |
| CER           | SECTION  10 | 11  |
| JFS           | SECTION  13 | 14  |
| FJPS         | SECTION  16 | 17  |
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- **LENGTH**: 00352

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- **LOCATION**: 00355
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**ENTRY POINTS**
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- ** LOCATION**: 00546

*The first location not used by this program is 00546.*
FUNCTION R TYPE D

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T1J. LINNO3 CONNAY PHASE 1A STORAGE MAP 12/31/64 000109 PAGE 60

FUNCTION T1J TYPE D

COMMON VARIALES

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SYMBOL GAMA LOCATION TYPE D SYMBOL LOCATION TYPE D
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UNDIMENSIONED PROGRAM VARIABLES

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### SUBROUTINES CALLED

- .CFWP SECTION 13
- .EXRT SECTION 14
- .FAND SECTION 15
- .FAR SECTION 16
- .FARP SECTION 17
- .FBU SECTION 18
- .FINA SECTION 19
- .FISN SECTION 20
- .FINS SECTION 21
- .FINN SECTION 22
- .FOPL SECTION 23
- .FOPN SECTION 24
- .FOPR SECTION 25
- .FOPU SECTION 26
- .FOPL SECTION 27
- .FOPN SECTION 28
- .FOPR SECTION 29
- .FOPU SECTION 30
- E.1 SECTION 31
- E.2 SECTION 32
- E.3 SECTION 33
- E.4 SECTION 34
- VARGO SECTION 35
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**UNDIMENSIONED PROGRAM VARIABLES**

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**ENTRY POINTS**

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**SUBROUTINES CALLED**

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**EFN IFN CORRESPONDENCE**

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**THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00421.**
### Subroutine Muller

#### Undimensioned Program Variables

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#### Entry Points

- **PS4**

#### Subroutines Called

- **CSMP**
- **CGDP**

#### EFN IFN Correspondence

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### SUBROUTINE CHATS

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#### ENTRY POINTS

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SLROUTINE OVERCK
COMMON VARIABLES

COMMON BLOCK OVR
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LENGTH 00002

SYMBOL LOCATION TYPE
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KCOUNT 00000
COMMON BLOCK I

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL LOCATION TYPE
LUAND 00004 R
KNTFPI 00004 R
KNT2 00011 I

ENTRY POINTS

OVERCK SECTION 4

SUBRoutines CALLED

SYSLOC SECTION 5

EFN IFN LOCATION

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00103.

SCVPEE5 ERROR 13 LEVEL 1 - WARNING ONLY
THIS STATEMENT CANNOT BE REACHED.

SCVPEE5 ERROR 20 LEVEL 1 - WARNING ONLY
THE PROGRAM SHOULD END WITH A TRANSFER, A RETURN HAS BEEN GENERATED.

SCVPEE5 ERROR 13 LEVEL 1 - WARNING ONLY
THIS STATEMENT CANNOT BE REACHED.

PHASE B DIAGNOSTIC MESSAGES
UNLESS CORE
373546 THRU 77779

BEGIN EXECUTION.
TRANSFORMED ELASTIC CONSTANTS (C11, 1+1.20)

3.5C00000E1 1
1.5C00000E1 C
0
0
2.0C00000E1 1
0.0C00000E1 C9
0
7.5C00000E1 D
6.5C00000E1 C
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-0.1373496 23 0.6646706E 23
0.7340964 22 0.7146282E 12
-0.6474200 21 0.

COEFFICIENTS OF POLYNOMIAL:
-0.78262915 50 0.
-0.30900600 01 0.
0.1000000E 01 0.
0.2889467E 00 0.
0.2389463E 01 0.
0.2089463E 01 0.
0.1959463E 01 0.

INTERMEDIATE ROOTS OF POLYNOMIAL:
-1.623956E-01 1.200000E-01 1.012956E-01 1.200000E-01 7.746336E-01 3.942776E-01 1.204136E-01 -4.495000E-02
-1.746336E-01 3.942776E-01 -1.284136E-01 4.495000E-02 1.038756E 00 3.804176E-01 1.204136E-01 3.942776E-01

INTERMEDIATE POSITIVE ROOTS:
1.623956E-01 1.200000E-01 7.746336E-01 3.942776E-01 1.204136E-01 4.495000E-02 1.038756E 00 -4.804176E-01

RE-ORDERED ALPHAS'
1. ROSE CASE:
0.623956E-01 1.200000E-01 7.746336E-01 3.942776E-01 1.204136E-01 4.495000E-02 1.038756E 00 -4.804176E-01

INTERMEDIATE MTHETA:
0. 0 0 0 0 0 0 0
1.000000E-10 0 0 0 0 0 0 0
0. 1.14997E-11 1.14997E-11 1.14997E-11 1.14997E-11 1.14997E-11
0. 0 0 0 0 0 0 0
0. 0 0 0 0 0 0 0

INTERMEDIATE MATRIX BY COLUMNS:
0. 0 0 0 0 0 0 0
1.22356G 00 2.564600E G0 0 0 0 0
0.73120E 00 2.34026E 00 0 0
0. 1.41356E 01 2.46373E 01 0. 0. 2.46373E 01 2.46373E 01 2.46373E 01
0. 0 0 0 0 0 0 0

V5 = 0.1957946E 01 'V55 = -0.5993010E 02 -0.1646OS1E 02 MAG = 0.9519467E 02

COEFFICIENTS OF POLYNOMIAL:
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-0.6912392E 24 0
-0.8783368E 24 0
-0.9323264E 24 0
-0.6451815E 24 0
0. 7365290E 23 0
0.3510238E 23 0
0. 4626011E 14 0
0.9934538E 21 0

COEFFICIENTS OF POLYNOMIAL:
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### Intermediate Roots of Polynomials

**Intermediate Roots of Polynomials**

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**Intermediate Positive Roots**

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<th>Description</th>
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**Intermediate L Matrix by Columns**

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**VS = 0.34452000 E0  F -VS = 0.1975607E 03 - 0.7503187E 02  MAG = 0.2113628E 03**

**Coefficients of Polynomial**

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**Coefficients of Polynomial**

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-5.74E00E-01 3.97E39E-01 1.41E43E-01 -6.53E00E-02 1.64E05E-00 -3.89E09E-02 -1.04E08E-02 -3.80E09E-02

INTERPRETIVE POSITIVE ROOTS
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RE-ORDERED ALPHAS (1 ROW, 3 ZERO CASE)
1.95C27E-01 1.26E00E-01 7.76E19E-01 3.97E39E-01 1.41E43E-01 -6.53E00E-02 1.04E08E-02 -3.80E09E-02

INTERPRETIVE BETA B
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0.
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INTERPRETIVE L MATRIX BY COLUMNS
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0.
0.

4.23E06E 00 5.28E09E 00 0.
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V5 = 0.31426400 04

FIT5 = -0.1171828 03 -0.4815791 02

MAG = 0.1273446 03

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-6.42E07E 24 0.
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-7.45E03E-01 3.55E07E-01 1.220E-01 1.46E07E-02 1.03E02E-01 -3.80E06E-02 1.04E08E-02 -3.80E06E-02

INTERPRETATE POSITIVE ROOTS
1.528E04E-01 1.20E00E-01 7.739E03E-01 3.85E07E-01 1.720E-01 1.22E00E-01 8.45E71E-02

RE-ORDERED ALPHAS (1 ROW, 3 ZERO CASE)

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INTERMEDIATE REGS OF POLYNOMIAL

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RE-ORDERED ALPHAS 4 1 ROW 3 ZERO CASE 3

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INTERMEDIATE BETA B

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-7.73945E-01  3.95802E-01  -1.21936E-01  -8.44346E-02  1.03790E 00  -3.80157E-01  -1.03790E 00  -3.80157E-01

INTERMEDIATE POSITIVE ROOTS
1.52732E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01  -6.44346E-02  1.03790E 00  -3.80157E-01

REORDERED ALPHA'S (1 ROW, 3 ZERO CASE)
1.52732E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21936E-01  -6.44346E-02  1.03790E 00  -3.80157E-01

INTERMEDIATE BETA B
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0.0
1.00000E-10  0.0
0.0
0.0
7.18210E-11  -5.38549E-11  -2.67548E-10  5.44232E-10  -2.09077E-11  -4.05041E-11
0.0
0.0
0.0
1.00000E-09  0.0
1.00000E-09  0.0

INTERMEDIATE L MATRIX BY COLUMNS
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0.0
0.0
0.0
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0.0
0.0
0.0
0.0
2.50683E 00  -5.32845E 00  -2.21131E 00
0.0
0.0
0.0
0.0
4.34617E 00  9.59911E 20  -2.11312E 01  -1.07904E 01
4.14365E 00  -7.82393E 00  0.0
0.0
0.0
0.0
0.0
4.45308E 00  -4.48660E 00  -1.82051E 00  -1.43628E 00

V = 0.3479689E 04  F(V) = 0.8723089E-C4  0.6554125E-04  MAG = 0.1093503E-03
### Coefficients of Polynomial

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<th>Coefficients</th>
<th>Value</th>
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### Intermediate Roots of Polynomial

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### Intermediate Positive Roots

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### Recurrence Alphas (1 Root, 3 Zero Case)

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<td>0.00000E-10</td>
<td>0.00000E-10</td>
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<tr>
<td>2.01313E-11</td>
<td>5.01313E-11</td>
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<tr>
<td>6.58214E-11</td>
<td>1.26132E-11</td>
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<tr>
<td>6.40817E-12</td>
<td>1.26132E-11</td>
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### Intermediate Beta 4

<table>
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### Intermediate Matrix by Columns

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<tr>
<td>0.23081E-01</td>
<td>0.14251E-01</td>
</tr>
<tr>
<td>0.1670369E-14</td>
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<tr>
<td>0.2145483E-23</td>
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### Coefficients of Polynomial

<table>
<thead>
<tr>
<th>Coefficients</th>
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<tr>
<td>-0.6025305E-24</td>
<td>0</td>
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### Intermediate Roots of Polynomial

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### Intermediate Positive Roots

<table>
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</tr>
<tr>
<td>1.21938E-01</td>
<td>6.46346E-01</td>
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### Recurrence Alphas (1 Root, 3 Zero Case)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>0.00000E-10</td>
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### Intermediate Beta 4

<table>
<thead>
<tr>
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### Intermediate Matrix by Columns

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>0.23081E-01</td>
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<td>0</td>
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<td>0.2145483E-23</td>
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<tr>
<td>-0.1379492E-23</td>
<td>0</td>
</tr>
</tbody>
</table>
-0.799445E+0 0.

COEFFICIENTS OF POLYNOMIAL
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C. 0.0100992E 00
0.180000E+01 -0.382790E 00
-0.713777E+00 0.
0. 0.758829E-01
C. 0.171783E+01 -0.0.
0. 0.
-0.472205E+00 0.

INTERMEDIATE ROOTS OF POLYNOMIAL
-1.52723E-01 1.20000E-01 1.52723E-01 1.20000E-01 7.73945E-01 3.95828E-01 1.21936E-01 -6.46346E-02
-7.73945E-01 3.95828E-01 -1.21936E-01 -6.46346E-02 1.03790E 00 -3.80137E-01 -1.03790E 00 -3.80137E-01

INTERMEDIATE POSITIVE ROOTS
1.52723E-01 1.20000E-01 7.73945E-01 3.95828E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80137E-01

RE-OFCOREF ALPHAS (1 ROW, 3 ZERO CASE )
1.52723E-01 1.20000E-01 7.73945E-01 3.95828E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80137E-01

INTERMEDIATE BETA B
0.
-1.60000E-10 0.
0. 0.
-1.16210E-11 -5.38589E-11 -2.67548E-10 -2.93507E-11 -6.09041E-11
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### Intermediate L Matrix by Columns

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<tbody>
<tr>
<td>6.2081E 00</td>
<td>0.29728E 00</td>
<td>0.</td>
<td>2.50883E 00</td>
<td>-4.32645E 00</td>
<td>0.</td>
<td>-2.21111E 00</td>
<td>1.82993E 00</td>
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<tr>
<td>-3.2639E 01</td>
<td>1.18060E 01</td>
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<td>-1.87902E 01</td>
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<tr>
<td>9.7598E 00</td>
<td>-7.8238E 00</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>-4.95308E 00</td>
<td>-4.98646E 00</td>
<td>-1.82051E 00</td>
<td>1.43626E 00</td>
</tr>
</tbody>
</table>

\[ V = 0.3487688E 04 \quad F(V) = -0.5421336E+02 - 0.2856658E+02 \quad \text{MAG} = 0.6305444E-02 \]

### Coefficients of Polynomial

<table>
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<th>0.8646928E 24</th>
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### Intermediate Roots of Polynomial

<table>
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<tr>
<th></th>
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<th>1.20000E-01</th>
<th>1.20000E-01</th>
<th>1.20000E-01</th>
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<th>1.21936E-01</th>
<th>1.84634E-02</th>
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<td>1.20000E-01</td>
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### Intermediate Positive Roots

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### Intermediate L Matrix by Columns

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<th>1.28000E-08</th>
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<td>-1.82051E 00</td>
<td>1.43626E 00</td>
</tr>
</tbody>
</table>

\[ V = 0.3487688E 04 \quad F(V) = 0.1979980E+04 - 0.4236713E+04 \quad \text{MAG} = 0.5226283E-04 \]
**KS = 0**

**LITHIUM NIOBATE**

**EPSILON = 0.10000000-10**

CLOSENESS OF DETERMINANT TO ZERO

KL = 0

0 = COMPUTE FOURTH RCM OF L MATRIX

1 = SET FOURTH RCM = 1

RM = 0

0 = ELECTRIC FIELD (DONI)

1 = MAGNETIC FIELD (TANI)

**MAX = 20**

MAXIMUM NUMBER OF ITERATIONS

**MNU = 20**

NUMBER OF ITERATIONS ACTUALLY USED

**LAM B = 0.00000000 02**

**LAM A = 0.00000000 12**

**RHO A = 0.00000000 05**

**RHO B = 0.47000000 04**

**RM = 0.00000000 21**

**VS = 0.34000000 04**

**FINAL VELOCITY SUCH THAT F(VS) < L.T. EPSILON**

**VS = 0.26072268-03**

**INVERSE OF VS**

---

**DETERMINANT = 0.87231898-04, 0.6994072E-04**

---

**FINAL ROOTS OF POLYNOMIAL DIVIDED BY VS**

-0.35772396 00, 0.12000000 003
-0.2378922E-04, 0.34400000 003
-0.3240916E-04, 0.34400000 003
-0.3219067E-04, 0.34400000 003
-0.3219067E-04, 0.34400000 003
-0.3219067E-04, 0.34400000 003
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-0.3219067E-04, 0.34400000 003
-0.3219067E-04, 0.34400000 003
-0.3219067E-04, 0.34400000 003
-0.3219067E-04, 0.34400000 003
*** FINAL ANSWERS ***

PARTIAL FIELD

RELATIVE AMMUTRES

1  ( 0  , 0  , 0  )
2  (- 0.7021403E -11  , 0  , 0  )
3  (- 0.1152962E -12  , - 0.1377944E 00  , 0  , 0  )
4  ( 0.1005004E 01  , 0  , 0  )

\[ \mathbf{A} = 0 \]

STRESS COMPONENTS

\[ \mathbf{T} = \begin{bmatrix} 111 \\ 121 \\ 131 \\ 112 \\ 122 \end{bmatrix} \]

STRAIN COMPONENTS

\[ \mathbf{S} = \begin{bmatrix} 511 \\ 521 \\ 531 \\ 512 \\ 522 \end{bmatrix} \]

TIME AVERAGE POWER FLOW

\[ \mathbf{P} = \begin{bmatrix} 11 \\ 12 \end{bmatrix} \]

ELECTRIC DISPLACEMENT

\[ \mathbf{D} = \begin{bmatrix} 21 \\ 22 \end{bmatrix} \]

MECHANICAL DISPLACEMENT MAGNITUDE

\[ \mathbf{U} = \begin{bmatrix} 11 \\ 12 \\ 13 \end{bmatrix} \]

ELECTRIC POTENTIAL MAGNITUDE = 6.829438E -01 PHASE = 79.149

ELECTRIC FIELD

\[ \mathbf{E} = \begin{bmatrix} 11 \\ 12 \\ 13 \end{bmatrix} \]
REFERENCES


THEORETICAL INVESTIGATION OF ACOUSTIC SURFACE WAVES ON PIEZOELECTRIC CRYSTALS

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
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