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A MODEL STUDY OF CHANGES IN WIND WITH ALTITUDE IN A PLANETARY BOUNDARY LAYER

By

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EDITED TRANSLATION

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This paper presents a solution from which the majority of the known solutions are derived as partial cases. These equations and models are used in studying the time-wise changes in wind with height in the planetary boundary layer. Here $K(z)$ is the kinematic coefficient of turbulent exchange along the $z$ axis and liter is the Coriolis force. Orig. art. has: 9 formulas.
A MODEL STUDY OF CHANGES IN WIND WITH ALTITUDE IN A PLANETARY BOUNDARY LAYER

N. Godev, D. Iordanov

(Presented by Academician L. Krystanov on 25 May 1967)

The study of the time-steady variation in wind with altitude in the planetary boundary layer is associated with the solution of the following system of differential equations:

\[
\begin{align*}
\frac{d}{dz} K(z) + \frac{d}{dz} (\nu - \xi) &= \nu - \xi \\
\frac{d}{dz} K(z) - \frac{d}{dz} \xi &= -\xi 
\end{align*}
\]

where \( u, v, u_\theta, v_\theta \) are the components of the wind and of the geostrophic wind, respectively, along the \( x \)- and \( y \)-axes, \( K(z) \) is the kinematic coefficient of turbulent exchange along the \( z \)-axis and \( \xi \) is the Coriolis parameter.

The following are the boundary conditions at which System (1) is solved:

\[
\begin{align*}
&u = v = 0 \text{ when } z = z_0, \\
&u, v \text{ limited as } z \to \infty,
\end{align*}
\]

where \( z_0 \) is the roughness factor assumed to be constant.

From (1) we easily obtain:

\[
\frac{d}{dz} K(z) \frac{d}{dz} M = -\xi M, \tag{3}
\]

while from (2)

\[
M = 0 \text{ when } z = z_0 \text{ and } M \text{ limited as } z \to \infty, \tag{4}
\]

where \( M = u + iv; M_\theta = u_\theta + iv_\theta \).

A number of the works examined in the exhaustive review of Reference [1] yield the solution for Eq. (3) for various models of \( K(z) \). An attempt is made in the present paper to provide a solution from which a large number of the known solutions will be derived as special cases. With this purpose in mind we will seek out solutions to Eq. (3) for the following model of \( K(z) \):

\[
K(z) = \begin{cases} 
K_{z>h} & \text{when } z < h, \\
K_{z=h} \text{ or } K_{z=h-1} & \text{when } z \geq h
\end{cases}
\]

1
for the boundary conditions of (4) and the condition when \( z = h \):

\[
M(z)_{z=h-0} = M(z)_{z=h+0}
\]

(6)

\[
K(z) \frac{dM}{dz} igg|_{z=h-0} = K(z) \frac{dM}{dz} igg|_{z=h+0}
\]

Solution (3) for Conditions (4), (5) and (6) is given by the expression

\[
M(z) = M_0 \left\{ -I \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z) - \psi_{n-1}^{(1)} (z) \psi_m^{(2)} \left( k_r \gamma h \right)}{\left( \psi_{n-1}^{(1)} (z) \right) \left( \psi_{n-1}^{(1)} (z) \right)} \right\} (7)
\]

when \( z_0 < z < h \)

\[
M(z) = M_0 \left( 1 - \frac{1-I}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)} \right) \text{ when } z > h
\]

(8)

where

\[
a_1 = k \frac{\psi_m^{(2)} \left( k_r \gamma h \right)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)}; \quad b_1 = h \frac{\psi_m^{(2)} \left( k_r \gamma h \right)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)};
\]

\[
a_2 = k \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)}; \quad b_2 = h \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)};
\]

\[
a_3 (z) = z \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)}; \quad b_3 (z) = z \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)};
\]

\[
a_4 = z_0 \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)}; \quad a_4 = \frac{(a_3 (z) z_0 - \psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z))}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)};
\]

\[
- \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)} - \frac{\psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)} + \frac{(a_3 (z) z_0 - \psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z))}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)}
\]

\[
2 \psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z) + \psi_m^{(2)} \left( k_r \gamma h \right) \psi_{n-1}^{(1)} (z)
\]

in the case \( K(z) = K_0 z^a; a = g; k = 1; x_0 = h \)

in the case \( K(z) = K_0 x^a; a = 2; g = 3; k = x_0 = \mu \).

It is not difficult from Expressions (7) and (8) to derive certain of the known solutions. For example, from (7), for the condition \( h = \infty \), we obtain the solution

\[
M(z) = M_0 \left\{ -I \frac{\psi_m^{(2)} \left( k_r \gamma h \right)}{\psi_{n-1}^{(1)} (z) \psi_{n-1}^{(1)} (z)} \right\}
\]

(9)

which was considered in the work by Köhler [2]. From Eq. (9) when \( p = 1 \) we obtain the Blinov-Kibel' [3] solution and when \( p = 0 \) we obtain the Ekman [sic] spiral. When \( \frac{1-p}{2} = r + \frac{1}{2} (r = 0, 1, 2 \ldots) \) we obtain
the solution considered by Takev [4]. When \( p = 2 \) we obtain the solution considered by Takaya [5]. From Eq. (8) when \( h + a \), we can obtain: Expression (9) corresponding to the power model of \( K(s) \) for \( a=q; x=z; b=1 \) or a known solution [6, 7] for a single-layer exponential model of \( K(s) \). From (7) and (8) we can derive known two-layer models. Thus, for example, when \( a=q=0; b=1; x=z; K = K_0 h \) we obtain the Shvets and Yudin [8] model. When \( p=p; a=q=0; x=z; b=1 \) and \( K_0 = K_0 h^{\alpha} \) we obtain the Berlyand [9] model. When \( \alpha=0; a=q=0; x=z; b=1 \) we obtain the Ariyel [10] model. When \( p=p; K = K_0; K_0 = K_0 h^{\alpha}; b=-\frac{e}{K} \); \( \alpha=0; q=0; x=z; b=1 \) we obtain the model developed by Klyuchnikova, Laykhtman and Tseytin [11].

REFERENCES

Geophysical Institute
Bulgarian Academy of Sciences