<table>
<thead>
<tr>
<th>AD NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD859286</td>
</tr>
</tbody>
</table>

NEW LIMITATION CHANGE

TO
Approved for public release, distribution unlimited

FROM
Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; APR 1969. Other requests shall be referred to Naval Air Systems Command, Washington, DC.

AUTHORITY
USNASC ltr, 26 Oct 1971

THIS PAGE IS UNCLASSIFIED
COLLOCATION
FLUTTER ANALYSIS
STUDY

This document is subject to special export controls and transmittal to foreign governments or foreign nationals may be made only with prior approval of the Naval Air Systems Command (AIR-550214).

VOLUME II.
FLUENC - COMPUTER PROGRAM TO CALCULATE STRUCTURAL INFLUENCE COEFFICIENTS

APRIL 1969

MISSILE SYSTEMS DIVISION
HUGHES
HUGHES AIRCRAFT COMPANY
COFA

COLLOCATION FLUTTER ANALYSIS

STUDY

VOLUME II

FLUENC - Computer Program to Calculate
Structural Influence Coefficients

Prepared by the Dynamics and Environment
Section Personnel, Hughes Aircraft Company
Under Contract No. 0019-68-C-0247

April 1969

This document is subject to special export controls and transmittal
to foreign governments or foreign nationals may be made only with
prior approval of the Naval Air Systems Command
ABSTRACT

A displacement solution for the calculation of structural influence coefficients (SIC's) is presented. The formulation utilizes the lumped parameter approach that is consistent with collocation flutter solutions. The structure is synthesized as concentrated mass elements connected by massless elastic plates and/or beams. There are two methods of generating the mass matrix; they are: 1) lumped concentrated mass points, 2) consistent mass matrices. Along with the calculation of the SIC's, the natural vibration modes and frequencies are calculated. There are two options for punching out the flexibility matrix for use in subsequent COFA computer programs. Option 1, punches out the full flexibility matrix; Option 2, punches out the reduced flexibility matrix eliminating the rows and columns pertaining to structural attach points.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0</td>
<td>NOMENCLATURE</td>
<td>2</td>
</tr>
<tr>
<td>3.0</td>
<td>TECHNICAL DISCUSSION</td>
<td>3</td>
</tr>
<tr>
<td>3.1</td>
<td>Influence Coefficients</td>
<td>3</td>
</tr>
<tr>
<td>3.2</td>
<td>Mass Matrix</td>
<td>12</td>
</tr>
<tr>
<td>3.3</td>
<td>Modes and Frequencies</td>
<td>15</td>
</tr>
<tr>
<td>4.0</td>
<td>PROGRAM DESCRIPTION</td>
<td>17</td>
</tr>
<tr>
<td>4.1</td>
<td>Description of Program Input</td>
<td>17</td>
</tr>
<tr>
<td>4.2</td>
<td>Description of Program Output</td>
<td>22</td>
</tr>
<tr>
<td>4.3</td>
<td>Sample Problems</td>
<td>22</td>
</tr>
<tr>
<td>4.4</td>
<td>Processing Requirements</td>
<td>23</td>
</tr>
<tr>
<td>4.5</td>
<td>Program Listing and Flow Chart</td>
<td>23</td>
</tr>
<tr>
<td>4.</td>
<td>REFERENCES</td>
<td>29</td>
</tr>
<tr>
<td>A.</td>
<td>APPENDIX A Three Sample Problems - Input and Output</td>
<td>30</td>
</tr>
<tr>
<td>B.</td>
<td>APPENDIX B Program FLUENC Listing</td>
<td>81</td>
</tr>
<tr>
<td>C.</td>
<td>APPENDIX C Program FLUENC Flow Chart</td>
<td>126</td>
</tr>
<tr>
<td>D.</td>
<td>APPENDIX D Symbol List</td>
<td>189</td>
</tr>
<tr>
<td>1.</td>
<td>Beam Stiffness Matrix</td>
<td>25</td>
</tr>
<tr>
<td>2.</td>
<td>Triangular Plate Matrix</td>
<td>26</td>
</tr>
<tr>
<td>3.</td>
<td>Beam Consistent Mass Matrix</td>
<td>27</td>
</tr>
</tbody>
</table>
1.0 INTRODUCTION

In order to determine the aeroelastic behavior of a wing or control surface, it is necessary to know the aerodynamics, elastic properties and mass distributions of the structure. The overall aeroelastic analysis is usually divided into four separate parts as shown in Figure 1.

![Analysis Procedure Diagram]

Figure 1. Analysis Procedure

This portion of the report describes the computation of the mass and stiffness distribution. The geometry of a wing or tail surface is too complex for the successful use of closed form analytical techniques. Therefore, a numerical type of analysis must be used. The end product of this analysis is the generation of overall influence coefficient and mass matrices referred to a set of node points arbitrarily picked on the surface of the structure. The finite element method (see Refs. 2 and 3) was used to form the required matrices for a planar structure. This technique is especially suited to solve complex structures and as used in the analysis is general enough to handle the following:

1. Combinations of beam and plate elements
2. Arbitrary boundary conditions
3. Lumped or distributed stiffnesses and masses

A discussion of the theory and computer program which calculates the influence coefficient and the mass matrix as well as the structural modes and frequencies is given in the following sections.
2.0 NOMENCLATURE

\begin{enumerate}
\item \(C\) = Unknown Boundary Constants
\item \(D\) = Plate Rigidity Constant
\item \(E\) = Modulus of Elasticity
\item \(F\) = Force
\item \(K\) = Stiffness Coefficients
\item \(M\) = Bending/Torsional Moment
\item \(p\) = Pressure
\item \(T\) = Coordinate Transformation
\item \(t\) = Thickness
\item \(w\) = Linear Displacement in \(z\) direction
\item \(x, y, z\) = Coordinate Axes
\item \(s\) = Linear Displacement
\item \(\frac{d^2}{dz^2}\) = Curvature
\item \(\rho\) = Density
\item \(\sigma\) = Stress
\item \(\nu\) = Poisson's Ratio
\item \(\frac{\partial}{\partial x}\) = Partial Derivative
\item \([\ ]\) = Square Matrix
\item \(\{\}\) = Column Matrix
\item \(\ \) = Row Matrix
\end{enumerate}
3.0 TECHNICAL DISCUSSION

3.1 Influence Coefficients

The stiffness method approach is first used to obtain an overall stiffness matrix of the structure. This matrix is reduced by partitioning and then inverted to obtain the influence coefficients at any desired set of control points. The number of control points are denoted by $N$. At each node, three degrees of freedom are specified: two rotations and the normal displacement. Therefore, a stiffness matrix of approximately $3N$ degrees of freedom is first formed by superimposing individual plate and plane grid beam element global coordinate matrices. The matrix will be somewhat smaller than $3N$ degrees of freedom since boundary restraint conditions will reduce the size of the matrix. To illustrate the matrix condensation method used in the computer program, we will assume that we have $N$ control point normal displacements and $M$ displacements which must be eliminated. The overall stiffness matrix is given as

$$
[K] = \begin{bmatrix}
K_{NN} & K_{NM} \\
K_{MN} & K_{MM}
\end{bmatrix}
$$

(1)

The structural equilibrium matrix equation can be written as

$$
\begin{bmatrix}
K_{NN} & K_{NM} \\
K_{MN} & K_{MM}
\end{bmatrix}\begin{bmatrix}
\delta_N \\
\delta_M
\end{bmatrix} = \begin{bmatrix}
F_N \\
F_M
\end{bmatrix}
$$

(2)
We now assume that forces at the points to be eliminated are small and can be neglected. Therefore,

\[
\begin{bmatrix}
K_{NN} & K_{NM} \\
K_{MN} & K_{MM}
\end{bmatrix}
\begin{bmatrix}
\delta_N \\
\delta_M
\end{bmatrix}
= 
\begin{bmatrix}
F_N \\
0
\end{bmatrix}
\tag{3}
\]

or

\[
[K_{NN}]\{\delta_N\} + [K_{NM}]\{\delta_M\} = \{F_N\}
\]

and

\[
[K_{MN}]\{\delta_M\} + [K_{MM}]\{\delta_M\} = \{0\}
\]

Therefore

\[
\{\delta_M\} = -[K_{MM}]^{-1}[K_{MN}]\{\delta_N\}
\tag{3a}
\]

and

\[
\left(\left[K_{NN}\right] - [K_{NM}][K_{MM}]^{-1}[K_{MN}]\right)\{\delta_N\} = \{F_N\}
\]

and since

\[
[K_{MN}]^T = [K_{NM}]
\tag{4}
\]

we have

\[
\{\delta_N\} = \left(\left[K_{NN}\right] - [K_{MN}]^T[K_{MM}]^{-1}[K_{MN}]\right)^{-1}\{F_N\}
\]

If we now let

\[
[f_{NN}] = \left(\left[K_{NN}\right] - [K_{MN}]^T[K_{MM}]^{-1}[K_{MN}]\right)^{-1}
\]

then Equation (4) can be written as

\[
\{\delta_N\} = [f_{NN}]\{F_N\}
\tag{5}
\]

The matrix \([F_{NN}]\) is called the structural influence coefficient matrix.

The application of loads at the control points yield displacements at the control points by carrying out the matrix multiplication indicated in Equation (5).
Figure 2. Plane Grid Beam Local and Global Coordinate System
The computer program FLUENC carries out the required operations to obtain the influence coefficient matrix \( [f_{NN}] \). A detailed description of the program can be found in Section 4.0. The program is written to form a 50 x 50 influence coefficient matrix. The influence coefficient matrix is punched out on cards in a format compatible with the Collocation Flutter Program.

The plane grid beam global coordinate stiffness matrix used in the program was obtained from Reference 1 and is given in Table 1. The local and global coordinate systems are shown in Figure 2. The figure also contains the sign convention for the six degrees of freedom for each element.

The triangular plate stiffness matrix given in Reference 2 was used in the computer program. The plate element can be materially or geometrically orthotropic as treated in Reference 3. Stiffened plates can be considered to be geometrically orthotropic. The sign convention and nodal degrees of freedom are shown in Figure 3.

\[ z' \parallel z \]
\[ x', y' - \text{follows principal axes of material} \]
\[ \beta - \text{angle between } x' \text{ and } x \text{ axes} \]

Figure 3. Orthotropic Triangular Element
Following the analysis given in Reference 2, the deflection shape of the plate element is assumed to be of the form

\[ w = C_1 + C_2 x + C_3 y + C_4 x^2 + C_5 xy + C_6 y^2 + C_7 x^3 + C_8 (xy^2 + x^2y) + C_9 y^3 \]

or

\[ w = \begin{bmatrix} N \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \tag{6} \]

The unknown constants \( C_1, C_2, \ldots, C_9 \) can be written in terms of the nodal displacements \( \delta_1, \delta_2, \ldots, \delta_9 \) by using the boundary conditions

\[
\begin{align*}
\text{at } x = 0, y = 0 & \quad \begin{cases}
w = \delta_1 \\
\partial w / \partial y = \delta_2 \\
\partial w / \partial x = -\delta_3
\end{cases} \\
\text{at } x = 0, y = y_e & \quad \begin{cases}
w = \delta_4 \\
\partial w / \partial y = \delta_5 \\
\partial w / \partial x = -\delta_6
\end{cases} \\
\text{at } x = x_2, y = y_3 & \quad \begin{cases}
w = \delta_7 \\
\partial w / \partial y = \delta_8 \\
\partial w / \partial x = -\delta_9
\end{cases}
\end{align*} \]

Using Equation (6) in conjunction with the boundary conditions given by Equation (7) yields

\[
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\delta_4 \\
\delta_5 \\
\delta_6 \\
\delta_7 \\
\delta_8 \\
\delta_9
\end{bmatrix} =
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
C_8 \\
C_9
\end{bmatrix} \tag{8}
\]
The constant vector \( \{ c \} \) can be obtained in terms of the nodal displacements by inverting the matrix \([ C ]\). Therefore,

\[
\{ C \} = [C]^{-1}\{ \delta \}
\]  

(9)

The curvatures for a flat plate element are given by

\[
\{ \varepsilon \} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2 \partial^2 w / \partial x \partial y \end{bmatrix}
\]  

(10)

Substituting Equation (6) into Equation (10) yields

\[
\{ \varepsilon \} = [Q] \{ \varepsilon \}
\]  

(11)

where

\[
[Q] = \begin{bmatrix} 0 & 0 & -2 & 0 & 0 & -2X & -2Y & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2X & -GY \\ 0 & 0 & 0 & C & -2 & 0 & -(4X+4Y) & 0 \end{bmatrix}
\]

Substituting Equation (9) into Equation (11) yields

\[
\{ \varepsilon \} = [C]^{-1} \{ \delta \} = [B] \{ \delta \}
\]  

(12)
If initial strains are neglected then the moment-curvature relationships can be written in the form

\[
\{\sigma\} = \begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = [D] \{\epsilon\}
\]

(13)

where

\[
[D] = \begin{bmatrix}
D_x & D_y & 0 \\
V_x & D_y & 0 \\
0 & 0 & D_{xy}
\end{bmatrix}
\]

(14)

for a materially or geometrically orthotropic plate. For an isotropic plate Equation (14) reduces to

\[
[D] = \frac{E t^3}{12(1-\nu^2)} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\]

(15)

The \([D]\) matrix must undergo a transformation if the principal axes of the material do not coincide with the local coordinate axes. The components of strain in one coordinate axes system are related to the components of strain in another coordinate axes system by the matrix equation

\[
\{\epsilon'\} = [T]^T \{\epsilon\}
\]

(16)

(The prime refers to the components of strain referred to the \(x'-y'\) axes in Figure 3)

where

\[
[T]^T = \begin{bmatrix}
\cos^2 \beta & \sin \beta \sin \gamma & -2 \sin \beta \cos \beta \\
\sin \beta \sin \gamma & \cos^2 \beta & 2 \sin \beta \cos \beta \\
\sin \beta \cos \beta & -\sin \beta \cos \gamma & \cos^2 \gamma + \sin^2 \gamma
\end{bmatrix}
\]

(11)
Since the internal work is constant no matter which coordinate system is used

\[
\{\sigma^\prime\}^T \{e^\prime\} = \{\sigma\}^T \{e\} \tag{18}
\]
or by Equation (13)

\[
\{e^\prime\}^T [D^\prime] \{e^\prime\} = \{e\}^T [D] \{e\}
\]
and by using Equation (16)

\[
\{e\}^T [T][D^\prime][T]^T \{e\} = \{e\}^T [D] \{e\}
\]

Therefore

\[
[D] = [T][D^\prime][T]^T \tag{19}
\]

The stiffness matrix for a typical element (1 2 3) is given by

\[
[K] = \iiint_A [B]^T [D] [B] \, dx \, dy \tag{20}
\]
or by Equations (12) and (19)

\[
[K] = [C^{-1}]^T \left( \iiint_A [Q]^T [D^\prime][T][Q] \, dx \, dy \right) [C]^{-1} \tag{21}
\]

Now let

\[
[D^\prime] = \iiint_A [Q]^T [T][D^\prime][T]^T [Q] \, dx \, dy
\]
and carrying out the indicated matrix multiplications yields

\[
[D^\prime] = \iiint_A \text{(see Table I)} \, dx \, dy \tag{22}
\]
In order to simplify the integration required for evaluating the matrix in Equation (22), it is suggested in Reference 2 that the independent variables be changed as shown in Figure 4.

The relationships

\[ x = \xi (1 - \eta) x_3 \]
\[ y = \xi [(1 - \eta)y_3 + \eta y_2] \]  

are used for the change of variables. The terms in Equation (22) can now be evaluated by using the relationship

\[ I (x^m, y^n) = \iint x^m y^n \, dx \, dy \]

or

\[ I (x^m, y^n) = \iint x^m y^n \left| J(x, y) \right| \, d\xi \, d\eta \]  

where

\[ J(x, y) = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{vmatrix} \]  

\[ \frac{\partial y}{\partial \xi} \]
Substituting Equation (23) into Equation (25) yields

$$J(x, y) = \xi x_3 y_2$$

Substituting Equations (23) and (26) into Equation (24) yields

$$I(x^m, y^n) = \int \int \xi^m \eta^n (1 - \eta)^m [ (1 - \eta) y_3 
+ \eta y_2 ] x_3^m y_2 \, d\xi \, d\eta$$

which can easily be evaluated for any m and n.

### 3.2 Mass Matrix

D'Alenbert's principle can be used for the formulation of the mass matrix. If masses are attached to the nodes of the structure, then the nodal dynamic forces are

$$\{ F \} = -[M] \frac{d^2\{ \delta \}}{dt^2}$$

where

$$[M] = \begin{bmatrix} M_1 & 0 \\
M_2 & 0 \\n0 & \cdots & M_n \end{bmatrix}$$

is a diagonal matrix. The mass of beam and plate elements are usually distributed throughout the structure. Therefore, the distributed pressure loading can be written in the form

$$p = -\rho \frac{d^2 \omega}{dt^2}$$

Substituting Equations (6) and (9) into Equation (30) yields

$$p = -\rho \begin{bmatrix} N \end{bmatrix} [C]^{-1} \{ \ddot{s} \}$$
or

$$p = -\rho \begin{bmatrix} R \end{bmatrix} \{ \ddot{s} \}$$

where

$$[R] = [N][C]^{-1}$$
Since the equivalent element nodal forces can be computed from the equation

$$\{ P \}^e = - \int \{ R \}^T \rho \, dV$$  \hspace{1cm} (32)

then

$$\{ P \}^e = \left\{ \int \{ R \}^T \{ R \} \rho \, dV \right\} \{ \ddot{\delta} \}$$  \hspace{1cm} (33)

Therefore the elemental consistent mass matrix is given by

$$[m]^e = \int \{ R \}^T \{ R \} \rho \, dV$$  \hspace{1cm} (34)

The consistent mass matrices given in Reference 2 (see Tables 3 and 4) are used in the computer program.

Once the elemental consistent mass and/or lumped mass matrices are computed, then the overall matrix is obtained by following the same technique as used in assembling the overall stiffness matrix.

The overall mass matrix is reduced by using Equation (3a). We again assume that we have $N$ control point normal displacements and $M$ displacements which must be eliminated. The overall mass matrix can be written in the form

$$[M] = \begin{bmatrix} M_{NN} & M_{NM} \\ M_{MN} & M_{MM} \end{bmatrix}$$  \hspace{1cm} (35)

and the displacements

$$\{ \delta \} = \begin{bmatrix} \delta_N \\ \delta_M \end{bmatrix}$$  \hspace{1cm} (36)
From Equation (3a) we have

$$\{\delta_M\} = -\left[K_{MM}\right]\left[K_{MN}\right]\{\delta_N\}$$  \hspace{1cm} (37)

Since the virtual work of the reduced mass system must equal the virtual work of the true mass system

$$-\{\Delta\delta_N\}^T[M_r]\{\dot{\delta}_N\} = -\{\Delta\delta\}^T[M]\{\dot{\delta}\}$$  \hspace{1cm} (37)

where

$$\{\Delta\delta_N\} = \text{virtual displacements of control points}$$

$$\{\Delta\delta\} = \text{virtual displacements of complete system}$$

$$[M_r] = \text{overall reduced mass matrix}$$

Equation (37) can be rewritten in the form

$$\{\Delta\delta_N\}^T[M_r]\{\ddot{\delta}_N\} = \left[\Delta\delta_N^T \Delta\delta^T\right][M]\begin{bmatrix} \delta_N \\ \delta_M \end{bmatrix}$$  \hspace{1cm} (38)

Substituting Equation (3a) into Equation (38) yields

$$\{\Delta\delta_N\}^T[M_r]\{\ddot{\delta}_N\} = \{\Delta\delta_N\}^T\left[I - [K_{NN}][K_{mm}]^{-1}\right][M]\begin{bmatrix} I \\ [K_{mm}]^{-1} \end{bmatrix}\{\delta_N\}$$

which yields the result

$$[M_r] = \left[I - [K_{NM}][K_{MN}]^{-1}\right][M]$$  \hspace{1cm} (39)

The reduced mass matrix given by Equation (39) is calculated in the computer program.
3.3 Modes and Frequencies

Since the design engineer may find it useful to know the mode shapes and natural frequencies of the structure, this information can be obtained by using the NMODE option in the computer program. If no external forces are present then the reduced mass and influence coefficient matrices are related to one another by the relationship

\[
[f_{NN}]^{-1} \{ \delta_N \} = -[M_r] \{ \delta_o \}
\]  

(40)

For determining natural frequencies, the deflections \( \{ \delta_o \} \) can be written as

\[
\{ \delta_o \} = \{ \delta_o \} \sin \omega t
\]  

(41)

Substituting Equation (41) into Equation (40) yields

\[
[f_{NN}]^{-1} \{ \delta_o \} = \omega^2 [M_r] \{ \delta_o \}
\]  

(42)

The solution of Equation (42) yields the natural frequencies, \( \omega \), and the mode shapes \( \{ \delta_o \} \). Since \( [f_{NN}]^{-1} \) and \( [M_r] \) are both symmetrical matrices, the mass matrix \( [M_r] \) can be triangularized

\[
[M_r] = [L] [L]^T
\]  

(43)

where

\[
[L] = \begin{bmatrix}
\ell_{11} & \ell_{12} & \ell_{13} & \cdots & \ell_{1n} \\
\ell_{21} & \ell_{22} & \ell_{23} & \cdots & \ell_{2n} \\
\ell_{31} & \ell_{32} & \ell_{33} & \cdots & \ell_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\ell_{n1} & \ell_{n2} & \ell_{n3} & \cdots & \ell_{nn}
\end{bmatrix}
\]

Substituting Equation (43) into Equation (42) yields

\[
[f_{NN}]^{-1} \{ \delta_o \} = \omega^2 [L] [L]^T \{ \delta_o \}
\]
\[
\begin{align*}
\left( \begin{bmatrix} L \end{bmatrix}^{-1} \begin{bmatrix} f_{NN} \end{bmatrix} \right)^{\top} \{ \delta_0 \} &= \omega^2 \begin{bmatrix} L \end{bmatrix}^{\top} \{ \delta_0 \} \\
\text{Since} \\
\begin{bmatrix} L^{\top} \end{bmatrix}^{\top} \begin{bmatrix} L \end{bmatrix} &= \begin{bmatrix} I \end{bmatrix}
\end{align*}
\] (44)

Equation (44) may be written
\[
\begin{align*}
\left( \begin{bmatrix} L \end{bmatrix}^{-1} \begin{bmatrix} f_{NN} \end{bmatrix} \right)^{\top} \begin{bmatrix} L \end{bmatrix}^{\top} \begin{bmatrix} L \end{bmatrix}^{\top} \{ \delta_0 \} &= \omega^2 \begin{bmatrix} L \end{bmatrix}^{\top} \{ \delta_0 \} \\
\text{or} \\
\begin{bmatrix} A \end{bmatrix} \{ \delta_c \} &= \omega^2 \{ \delta_0 \}
\end{align*}
\] (44a)

where
\[
\begin{align*}
\begin{bmatrix} A \end{bmatrix} &= \left( \begin{bmatrix} L \end{bmatrix}^{-1} \begin{bmatrix} f_{NN} \end{bmatrix} \right)^{\top} \begin{bmatrix} L^{\top} \end{bmatrix}^{-1} \\
\{ \delta_c \} &= \begin{bmatrix} L \end{bmatrix}^{\top} \{ \delta_0 \}
\end{align*}
\]

An eigenvalue subroutine using the Givens method was used in the computer program package to solve Equation (45). The Givens method is fully described in Reference 4.

Note that the dynamical matrix \([A]\) in the form described above is real and symmetric which is required by the Givens method. Conveniently, \([L]\) and \(\begin{bmatrix} L^{\top} \end{bmatrix}\) are in triangular form which is used in the computer program package to save core storage space.
4.0 PROGRAM DESCRIPTION

Computer program FLUENC written in FORTRAN IV carries out the operations set forth in Section 3.0 for generating the structural influence coefficients and mass matrices required by the Collocation Flutter Program. Briefly, the structure is assumed to be representable by a planar network of beams and triangular plate elements connected at discrete joints. At each joint, if there are no restraints, the program assumes three degrees of freedom; that is, one displacement normal to the plane of the structure and two rotations. The program first synthesizes the stiffness and mass matrices for the entire structure including all degrees of freedom from the data input for the beam and triangular plate elements and from the restraint information input for the joints. It then reduces the stiffness and mass matrices by eliminating all the rotational degrees of freedom and leaving only the normal displacements. As a final step, the program inverts the reduced stiffness matrix to obtain the influence coefficients.

Other features of the program include the option to compute lumped masses or to compute the consistent mass matrices for the beam and triangular plate elements or both. Also, the triangular plate elements may have either isotropic or orthotropic properties. There is an additional option to expand the reduced frequency matrix to include the degrees of freedom representing the restraint joint (one joint on a movable surface; two joints on a fixed component). This is accomplished by adding one or two zero rows and columns to the reduced flexibility matrix corresponding to the mass numbers of the attach points involved.

In the sections that follow detailed instructions are given for the preparation of input data and a description is given of the output illustrated with several sample problems. Also included are listings and flow charts of the program and a discussion of the processing requirements.

4.1 Description of Program Input

The following instructions describe the input data, their physical units, and the FORTRAN format they must be punched with. The input quantities' names, all in capitals, are their FORTRAN names and, for reference, their equivalent names in Section 3.0 are listed in Appendix D.

4.1.1 Title Card, format (12A6)

Two cards; any alphanumeric statement in columns 1 to 72.
4.1.2 Problem Size and Control Information, format (715)

<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 5</th>
<th>6 - 10</th>
<th>11 - 15</th>
<th>16 - 20</th>
<th>21 - 25</th>
<th>26 - 30</th>
<th>31 - 32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>NJTS</td>
<td>NR</td>
<td>NBE</td>
<td>NPE</td>
<td>NMØDE</td>
<td>MKEY</td>
<td>NLUMP</td>
</tr>
<tr>
<td>NJTS</td>
<td>= number of joints in structure (50 maximum)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NR</td>
<td>= number of joints with one or more restraints</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NBE</td>
<td>= number of beam elements in structure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NPE</td>
<td>= number of triangular plate elements in structure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NMØDE</td>
<td>= number of eigenvalues and eigenvectors desired (9 maximum)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MKEY</td>
<td>= 1. do not compute consistent mass terms for beam and/or triangular plate elements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 2. compute consistent mass terms for beam and/or triangular plate elements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLUMP</td>
<td>= number of lumped masses input. Only lumped masses corresponding to the normal displacement at each joint may be input.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.3 Material Properties

(a) Number of Materials, format (I5)

<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>NMAT</td>
</tr>
<tr>
<td>NMAT</td>
<td>= number of materials for which properties are input (10 max.)</td>
</tr>
</tbody>
</table>

(b) Properties, format (4E10.3)

Input NMAT number of cards, one for each material.

<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 10</th>
<th>11 - 20</th>
<th>21 - 30</th>
<th>31 - 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>YM(1)</td>
<td>PR(1)</td>
<td>GE(1)</td>
<td>DENS(1)</td>
</tr>
</tbody>
</table>

YM(1) = Young's modulus of elasticity divided by $10^6$; psi

PR(1) = Poisson's ratio

GE(1) = modulus of rigidity; psi. If input as 0, it will be computed from the following formula:

$$GE(1) = \frac{YM(1)}{2 \left[1 + PR(1)\right]}$$

DENS(1) = material density; lb/in$^3$. Not required if MKEY = 1
4.1.4 **Joint Coordinate Cards**, format (10X, 2E10.3)

Input NJTS number of cards, one for each joint. Also, the structure is assumed to lie in the x-y plane.

<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 10</th>
<th>11 - 20</th>
<th>21 - 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>m</td>
<td>X(m)</td>
<td>Y(m)</td>
</tr>
</tbody>
</table>

m = joint number (must be input consecutively starting with 1). May be placed anywhere between columns 1 and 10.

X(m) = x coordinate of joint m; inches

Y(m) = y coordinate of joint m; inches

4.1.5 **Joint Restraint Information**, format (4I5)

Input NR number of cards, one for each joint with one or more restraints.

<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 5</th>
<th>6 - 10</th>
<th>11 - 15</th>
<th>16 - 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>JT</td>
<td>M1</td>
<td>M2</td>
<td>M3</td>
</tr>
</tbody>
</table>

JT = number of joint having one or more restraints

M1 = 0 free in the z direction

= 1 fixed in the z direction

M2 = 0 free to rotate about the x axis

= 1 fixed about the x axis

M3 = 0 free to rotate about the y axis

= 1 fixed about the y axis

4.1.6 **Lumped Masses**, format (I5, 5X, E10.3)

Input NLUMP number of cards, one for each lumped mass.

<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 5</th>
<th>6 - 10</th>
<th>11 - 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>JMASS</td>
<td>blank</td>
<td>RSMASS</td>
</tr>
</tbody>
</table>

JMASS = number of joint for which lumped mass is input

RSMASS = lumped mass, lb.

If more than one lumped mass is input for a particular joint, the program will sum the masses.
4.1.7 **Beam Element Properties**, format (3E10.3, 315)

Input NBE number of cards, one for each beam element.

<table>
<thead>
<tr>
<th>Column</th>
<th>Name</th>
<th>AR</th>
<th>XI</th>
<th>YJ</th>
<th>MAT</th>
<th>JTNR</th>
<th>JTFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 - 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 - 30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 - 45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36 - 40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41 - 45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

AR = area of beam cross section, in^2
XI = moment of inertia of area, in^4
YJ = effective torsional moment of inertia, in^4
MAT = material code corresponding to one of the materials input under paragraph 4.1.3.

JTNR, JTFR = joint numbers at the ends of the beam element

4.1.8 **Triangular Plate Element Properties**, format (E10.3, 515)

Input NPE number of cards, one for each triangular plate element.

<table>
<thead>
<tr>
<th>Column</th>
<th>Name</th>
<th>PTH</th>
<th>MAT</th>
<th>JTI</th>
<th>JT2</th>
<th>JT3</th>
<th>NDX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 - 15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 - 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 - 25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26 - 30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 - 35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

PTH = plate thickness, in.
MAT = material code corresponding to one of the materials input under paragraph 4.1.3

JT1, JT2, JT3 = joint numbers at the three corners of the triangular plate

Restrictions:

a) The order of the joint numbers must be given in a clockwise manner as follows:

```
   JT3
   /   \/  \
  /      \  /
JT1   JT2
```

b) The angle formed by the edges of the triangular plate at JT1 must not be 90°.

NDX = 0 the plate has isotropic properties and the flexural rigidity terms are computed from

\[
    \begin{align*}
    DX &= \frac{YM(MAT) \times PTH^3}{12 \left(1 - \left[PR(MAT)\right]^2\right)} \\
    D1 &= [PR(MAT)] \times DX
    \end{align*}
\]
the plate has orthotropic properties and the flexural rigidity terms are input by the next card [format (4E10.3)]


<table>
<thead>
<tr>
<th>Column</th>
<th>1 - 10</th>
<th>11 - 20</th>
<th>21 - 30</th>
<th>31 - 40</th>
<th>41 - 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>DX</td>
<td>DY</td>
<td>DL</td>
<td>DXY</td>
<td>BETA</td>
</tr>
</tbody>
</table>

DX, DY, DL, DXY = flexural rigidity terms, in.lb.

BETA = angle between material principal axes and the triangular plate local coordinates as shown below

\[
\begin{align*}
\text{Material Principal Axes} \\
\text{Triangular Plate Local Coordinates}
\end{align*}
\]

4.1.9 Option to Expand Reduced Flexibility Matrix

Note: The following card (NCOD) is always required at the end of all input data for any one particular case, whether or NOT the option is to be executed.

FORMAT (116)

<table>
<thead>
<tr>
<th>Column</th>
<th>1-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>NCOD</td>
</tr>
<tr>
<td>Item</td>
<td>(1)</td>
</tr>
</tbody>
</table>

NCOD = 0 Option not executed

= 1 Option executed

If NCOD = 1, the following card is required

FORMAT (318)

<table>
<thead>
<tr>
<th>Column</th>
<th>1-8</th>
<th>9-16</th>
<th>17-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>NR</td>
<td>NNE</td>
<td>NWO</td>
</tr>
<tr>
<td>Item</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

NR = Number of boundary points used (1 or 2)
NNE = Mass number of first attach point
NWO = Mass number of second attach point, if NR = 2
NWO = 0 or left blank if NR = 1
To input more than one problem, the user need only repeat the cards in paragraph 4.1.1 through 4.1.8 for each additional problem.

4.2 Description of Program Output

The program prints out all the input data for every problem followed by the solution consisting of the reduced upper right triangular stiffness (lb/in), flexibility (in/lb) and weight (lb) matrices as well as the modes and frequencies when these are requested on the card in paragraph 4.1.2. The stiffness, flexibility, and mass matrices that are printed/punched out only contain terms that are associated with the normal displacement "z". This is done so that when the flexibility matrix is used in subsequent collocation flutter analyses only the essential degrees of freedom are included in the flutter analyses. Also, the matrices are reduced to eliminate control points associated with fixed points (boundaries). If it is desirable to include the boundary points, it is only necessary to intersperse rows and columns of zero's at the proper place in the matrices. Immediately following the joint restraint information in the output, the program prints out the coordinate numbers assigned by the program to the normal displacements at each unrestrained joint. The elements in all the reduced output matrices are ordered according to these coordinate numbers.

In addition, the program punches out the entire flexibility and weight matrices row by row with the format (1P6E12.5) which is compatible with the input requirements of the Collocation Flutter Program. Each punched matrix is identified by a little card as the first card.

4.3 Sample Problems

To illustrate the use of program FLUENCE, three sample problems are included in Appendix A. Each sample problem starts with a problem statement and is followed by a listing of the input data and the output of the program. The first sample problem is a simply supported uniform beam composed of five beam segments. The second is a uniform cantilever plate divided into 72 triangular plate elements, and the third is a lumped mass and beam network simulating a missile control surface.
4.4 Processing Requirements

Program FLUENCE has been run on the GE-635 computer and it required about 31,000 cells of core storage. It is expected that the program storage requirement will be about the same on other digital computers. In addition to using the input and output files, 05 and 06, which are standard for the GE-635 computer, the program requires six other peripheral files, five of which are designated in the program by the numeric codes 07, 08, 19, 10 and 11, and the sixth is the card punch file.

There is no general formula for determining the run time required for a problem, but if a GE-635 computer is used, an estimate may be made from the times required for the three sample problems in Appendix A, which are as follows:

<table>
<thead>
<tr>
<th>Sample Problem No.</th>
<th>No. of Joints</th>
<th>No. of Beam Elements</th>
<th>No. of Plate Elements</th>
<th>Consistent Masses Computed</th>
<th>Lumped Masses Input</th>
<th>No. of Modes &amp; Freqs Computed</th>
<th>Run Time Hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>5</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
<td>4</td>
<td>0.0015</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>0</td>
<td>72</td>
<td>Yes</td>
<td>No</td>
<td>9</td>
<td>0.0691</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>45</td>
<td>0</td>
<td>No</td>
<td>Yes</td>
<td>9</td>
<td>0.0161</td>
</tr>
</tbody>
</table>

4.5 Program Listing and Flow Chart

In the event future changes are needed in the program, a listing of the program is included in Appendix B. The program consists of a MAIN deck, 24 subroutines and one function subprogram. MAIN has the function of reading in data, numbering the coordinates (subroutine CORDN), generating the codes for assembling the stiffness and weight matrices and calling the subroutines which develop the stiffness and mass terms for the beam and triangular plate elements. When the entire stiffness and weight matrices have been established for the whole structure, the MAIN program calls a subroutine which reduces these matrices as discussed before and determines the modes and frequencies as well.

The 24 subroutines and one function subprogram can be divided conveniently into five groups according to their function. The first group consists of those routines that develop the beam stiffness terms; these are TRANS and BEAMK. The second group consists of the routines which determine the beam mass terms; these are TRANS and BEAMM. The third group develops the triangular plate stiffness terms and these are PLATEK, CMAT, MINV, DINMAT, MATMPY, DMAT, DBLINT and PLYMP. The fourth group determines the triangular plate mass terms and these consist of PLATEM, CMAT, MINV, DINMAT, MATMPY, DBLINT and PLYMP. The fifth group of subroutines reduces the stiffness and
and mass matrices, finds the eigenvalues and eigenvectors and outputs the solution. This group is comprised of EIGEN, VIVID, ZROMAK, ZROMAM, SYMINV, EIGMAT, BIGMAT, LOOP1, LOOP2, LOOP3 and LOOP4.

Since the program listing is annotated extensively with comment statements, no further explanatory remarks are given here for the program. However, to facilitate the understanding of the interrelationships among the many subroutines, a flow chart of the entire FLUENC program is included in Appendix C.
\[
\begin{array}{|c|c|c|c|}
\hline
\frac{12EI}{L^3} & \frac{6EI}{L^2}m & \frac{4EI}{L}m + \frac{GTl}{L} & \text{Symmetric} \\
\hline
\frac{6EI}{L^2} & \frac{4EI}{L}m + \frac{GTl}{L} & \frac{12EI}{L^3} & \text{Symmetric} \\
\hline
-\frac{12EI}{L^3} & -\frac{6EI}{L^2}m & \frac{GTl}{L} & \frac{12EI}{L^3} \\
\hline
\frac{6EI}{L^2}m & 2\frac{EI}{L}m - \frac{GTl}{L} & -2\frac{EI}{L}m - \frac{GTl}{L} & \frac{6EI}{L^2}m + \frac{GTl}{L} \\
\hline
-\frac{6EI}{L^2} & 2\frac{EI}{L}m - \frac{GTl}{L} & 2\frac{EI}{L}m - \frac{GTl}{L} & \frac{6EI}{L^2}m + \frac{GTl}{L} \\
\hline
\end{array}
\]

\[
\begin{align*}
\lambda &= \frac{X_j - X_i}{L} \\
m &= \frac{Y_j - Y_i}{L}
\end{align*}
\]

\(X_i, Y_i, X_j, Z_j\) are the global end coordinates of the beam in Figure 2

Table 1. Plane Grid Beam Stiffness Matrix in Global Coordinates
\[ \begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4D_n & 4D_n & 4D_n & 12D_n & 12D_n \\
0 & 0 & 0 & 4D_n & 4D_n & 4D_n & 12D_n & 12D_n \\
0 & 0 & 0 & 4D_n & 4D_n & 4D_n & 12D_n & 12D_n \\
0 & 0 & 0 & 12D_n & 12D_n & 12D_n & 36D_n & 36D_n \\
0 & 0 & 0 & 4(D_n y + D_n z) + 8D_n (x+y) & 4(D_n y + D_n z) + 8D_n (x+y) & 4(D_n y + D_n z) + 8D_n (x+y) & 12(D_n y + D_n z) + 24D_n (x+y) & 12(D_n y + D_n z) + 24D_n (x+y) \\
0 & 0 & 0 & 12D_n y & 12D_n y & 12D_n y & 36D_n y & 36D_n y \\
\end{array} \]

where \( [D] = \begin{bmatrix} D_n & D_n & D_n \\
D_n & D_n & D_n \\
D_n & D_n & D_n \end{bmatrix} \)

Table 2. Integrant Appearing in Equation 22. Triangular Plate Element
Table 3. Consistent Mass Matrix for Beam
in Local Coordinates

$$
\begin{array}{ccccccc}
\frac{13}{35} + \frac{6I}{5AL} & & & & & & \\
\frac{II}{210} + \frac{I}{10AL} & & \frac{L^2}{105} + \frac{2I}{15A} & & \text{Symmetric} & & \\
0 & 0 & \frac{J}{3A} & & & & \\
\frac{9}{70} - \frac{6I}{5AL} & \frac{13I}{420} - \frac{I}{10AL} & 0 & \frac{13}{35} + \frac{6I}{5AL} & & \\
-\frac{13I}{420} + \frac{I}{10AL} & -\frac{L^2}{140} - \frac{I}{30A} & 0 & -\frac{11I}{210} - \frac{I}{10AL} & \frac{L^2}{105} + \frac{2I}{15A} & \\
0 & 0 & \frac{J}{6A} & 0 & 0 & \frac{J}{3A} \\
\end{array}
$$
\[ [m] = \rho t [\mathbf{C}^{-1}]^T \int \int d\mathbf{x} d\mathbf{y} [\mathbf{C}]^{-1} \]

Table 4. Consistent Mass Matrix for Triangular Plate Element in Local Coordinates
References


APPENDIX A

Three Sample Problems - Input and Output
Sample Problem No. 1
Simply Supported Beam

Calculate first five vibration modes and frequencies using the consistent mass matrix option.
<table>
<thead>
<tr>
<th>#</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.3</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>4.0</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>10.0</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>19.0</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>40.0</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>0.0</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

**Listing of Input Data Cards**

**Simply Supported Beam with 6 Joints**

August 1968

---

**Input Data Card Details**

<table>
<thead>
<tr>
<th>#</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.3</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>4.0</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>10.0</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>19.0</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>40.0</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>0.0</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

---

32
SIMPLY-SUPPORTED BEAM WITH 6 JOINTS
AUGUST 1968

\( \text{NJTB} = 6 \quad \text{NR} = 2 \quad \text{NBE} = 5 \quad \text{NPE} = 0 \quad \text{NMDE} = 4 \quad \text{NKEY} = 2 \quad \text{NLUMP} = 0 \)

**Material Properties**

<table>
<thead>
<tr>
<th>NO.</th>
<th>Young's Modulus</th>
<th>Poisson Ratio</th>
<th>Modulus of Rigidity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10000E+07</td>
<td>0.330000</td>
<td>0.37994E+06</td>
<td>0.15000E+01</td>
</tr>
</tbody>
</table>

**Joint Coordinates**

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>X COORD.</th>
<th>Y COORD.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-40000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>100000</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>200000</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>300000</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>400000</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>500000</td>
</tr>
</tbody>
</table>

**Joint Restraint Core**

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>Z Displacement</th>
<th>Rotation about X</th>
<th>Rotation about Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Coordinate Numbers for each Z Displacement at each unrestrained joint**

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>COORD. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

**Beam Element Properties**

<table>
<thead>
<tr>
<th>ELEMENT NO.</th>
<th>A</th>
<th>I</th>
<th>J</th>
<th>NAT</th>
<th>JOINT 1</th>
<th>JOINT 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.000</td>
<td>2.0000</td>
<td>4.0000</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>100.000</td>
<td>2.0000</td>
<td>4.0000</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>100.000</td>
<td>4.0000</td>
<td>2.0000</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>100.000</td>
<td>4.0000</td>
<td>2.0000</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>100.000</td>
<td>2.0000</td>
<td>4.0000</td>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**Reduced Upper Triangular Stiffness Matrix**

<table>
<thead>
<tr>
<th>ROW</th>
<th>0.19731E+05</th>
<th>-0.19065E+05</th>
<th>0.882979E+04</th>
<th>-0.20670E+04</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW 2</td>
<td>0.28019E+05</td>
<td>-0.21072E+05</td>
<td>0.82079E+04</td>
<td></td>
</tr>
</tbody>
</table>
### Reduced Upper Triangular Flexibility Matrix

<table>
<thead>
<tr>
<th>Row</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3333E-03, 0.7999E-03, 0.0000E+00, 0.3333E-03</td>
</tr>
<tr>
<td>2</td>
<td>0.1250E-02, 0.1133E-02, 0.0000E+00, 0.0000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.1250E-02, 0.7999E-03, 0.0000E+00, 0.0000E+00</td>
</tr>
<tr>
<td>4</td>
<td>0.3333E-03, 0.0000E+00, 0.0000E+00, 0.0000E+00</td>
</tr>
</tbody>
</table>

### Reduced Upper Triangular Weight Matrix

<table>
<thead>
<tr>
<th>Row</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1117E+02, 0.9399E-00, 0.0000E+00, 0.0000E+00</td>
</tr>
<tr>
<td>2</td>
<td>0.1250E-02, 0.1133E-02, 0.0000E+00, 0.0000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.1250E-02, 0.7999E-03, 0.0000E+00, 0.0000E+00</td>
</tr>
<tr>
<td>4</td>
<td>0.1117E+02, 0.9399E-00, 0.0000E+00, 0.0000E+00</td>
</tr>
</tbody>
</table>
HERE ARE THE EIGENVALUES AND EIGENVECTORS

**Eigenvector Number 1**
Corresponding to 1.0030593E+04
6.1093336E+01 9.9999992E+00 1.0000000E 00 6.1003418E+01

**Eigenvector Number 2**
Corresponding to 1.4125935E+05
1.0000000E 00 6.1003418E+01 -6.1003418E+01 -9.9999998E+00

**Eigenvector Number 3**
Corresponding to 6.4129359E+05
1.0000000E 00 -6.1003418E+01 -6.1003418E+01 1.0000000E 00

**Eigenvector Number 4**
Corresponding to 2.9998634E+06
-6.1003339E+01 1.0000000E 00 -9.9999993E+00 6.1003299E+01

HERE ARE THE NATURAL FREQUENCIES

<table>
<thead>
<tr>
<th>The Natural Frequency Number</th>
<th>1</th>
<th>19,940 CPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Natural Frequency Number</td>
<td>2</td>
<td>62,981 CPD</td>
</tr>
<tr>
<td>The Natural Frequency Number</td>
<td>3</td>
<td>160,933 CPD</td>
</tr>
<tr>
<td>The Natural Frequency Number</td>
<td>4</td>
<td>277,614 CPD</td>
</tr>
</tbody>
</table>
SAMPLE PROBLEM NO. 1a

Simply Supported Beam

Identical to Sample Problem 1 with the addition of lumped mass input at joint 3 and 4.

Program Output
SIMPLY SUPPORTED BEAM WITH 4 JOINTS - USING BOTH CONSISTENT MASS MATRIX
OPTION AND LUMPED MASS INPUT AT JOINTS 3 AND 4.

NJS = 6  NPE = 2  NAV = 8  NODE = 4  MKEY = 2  NLUMP = 3

MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>No.</th>
<th>Young's Modulus</th>
<th>Poisson Ratio</th>
<th>Modulus of Rigidity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1000E+07</td>
<td>0.3500</td>
<td>0.3750E+06</td>
<td>0.1200E+01</td>
</tr>
</tbody>
</table>

JOINT COORDINATES

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>X COORD.</th>
<th>Y COORD.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2</td>
<td>15.0000</td>
<td>0.</td>
</tr>
<tr>
<td>3</td>
<td>20.0000</td>
<td>0.</td>
</tr>
<tr>
<td>4</td>
<td>40.0000</td>
<td>0.</td>
</tr>
<tr>
<td>5</td>
<td>45.0000</td>
<td>0.</td>
</tr>
</tbody>
</table>

JOINT RESTRAINT CODE

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>Z DISPLACEMENT</th>
<th>ROTATION ABOUT X</th>
<th>ROTATION ABOUT Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

COORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>COORD. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

LUMPED WEIGHTS

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>20.0000</td>
</tr>
<tr>
<td>4</td>
<td>30.0000</td>
</tr>
</tbody>
</table>

ELEMENT PROPERTIES

<table>
<thead>
<tr>
<th>ELEMENT NO.</th>
<th>A</th>
<th>J</th>
<th>MA1</th>
<th>JOINT 1</th>
<th>JOINT 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.0000</td>
<td>4.0000</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>18.0000</td>
<td>2.0000</td>
<td>4.0000</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>20.0000</td>
<td>2.0000</td>
<td>4.0000</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>10.0000</td>
<td>2.0000</td>
<td>4.0000</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

STIFFNESS MATRIX
HERE ARE THE EIGENVALUES AND EIGENVECTORS

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
<th>Corresponding to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000431E+02</td>
<td>6.047614E-03</td>
<td>9.994514E-01</td>
</tr>
<tr>
<td>1.000698E+00</td>
<td>7.253952E-01</td>
<td>5.665892E-01</td>
</tr>
<tr>
<td>1.000966E+00</td>
<td>8.739331E-01</td>
<td>1.238959E-01</td>
</tr>
<tr>
<td>1.001132E+00</td>
<td>1.395983E+00</td>
<td>3.952564E-01</td>
</tr>
</tbody>
</table>

HERE ARE THE NATURAL FREQUENCIES

<table>
<thead>
<tr>
<th>Natural Frequency Number</th>
<th>Number</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1S</td>
<td>16.854 CPS</td>
</tr>
<tr>
<td>2</td>
<td>1S</td>
<td>39.347 CPS</td>
</tr>
<tr>
<td>3</td>
<td>1S</td>
<td>133.944 CPS</td>
</tr>
<tr>
<td>4</td>
<td>1S</td>
<td>183.228 CPS</td>
</tr>
</tbody>
</table>
Sample Problem No. 2
Cantilever Plate

\[ E = 3 \times 10^7 \text{ psi} \]
\[ \nu = 0.3 \]
\[ \rho = 0.283 \text{ lb/in}^3 \]
\[ t = 0.1 \text{ in.} \]
<table>
<thead>
<tr>
<th>No.</th>
<th>0.3</th>
<th>0.4</th>
<th>0.785</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.4</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>2.0</td>
<td>.15</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.3</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.4</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.5</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.6</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.7</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.8</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.9</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

NOT REPRODUCIBLE
<table>
<thead>
<tr>
<th>0.1</th>
<th>1</th>
<th>26</th>
<th>29</th>
<th>34</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1</td>
<td>29</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>26</td>
<td>38</td>
<td>34</td>
</tr>
</tbody>
</table>
Program Output
CANTILEVER PLATE WITH 40 JOINTS
AUGUST 1968

<table>
<thead>
<tr>
<th>JNTS.</th>
<th>NH</th>
<th>HNL</th>
<th>HPL</th>
<th>MNJNT</th>
<th>HREY</th>
<th>HLMNP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40</td>
<td>n</td>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>NO.</th>
<th>YOUNG'S MODULUS</th>
<th>POISSON RATIO</th>
<th>MODULUS OF RIGIDITY</th>
<th>DENSITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0E+06</td>
<td>0.16</td>
<td>1.2E+06</td>
<td>0.281E+06</td>
</tr>
</tbody>
</table>

JOINT COORDINATES

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>X COORD.</th>
<th>Y COORD.</th>
<th>X COORD.</th>
<th>Y COORD.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>0.250000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOT REPRODUCIBLE
<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>RESTRAINT</th>
<th>CODE</th>
<th>Z DISPLACEMENT</th>
<th>ROTATION ABOUT X</th>
<th>ROTATION ABOUT Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

COORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>COORD. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>19</td>
<td>17</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>21</td>
<td>19</td>
</tr>
<tr>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>23</td>
<td>21</td>
</tr>
<tr>
<td>24</td>
<td>22</td>
</tr>
<tr>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>26</td>
<td>24</td>
</tr>
<tr>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td>28</td>
<td>26</td>
</tr>
<tr>
<td>29</td>
<td>27</td>
</tr>
<tr>
<td>30</td>
<td>28</td>
</tr>
<tr>
<td>31</td>
<td>29</td>
</tr>
<tr>
<td>32</td>
<td>30</td>
</tr>
<tr>
<td>33</td>
<td>31</td>
</tr>
<tr>
<td>34</td>
<td>32</td>
</tr>
<tr>
<td>35</td>
<td>33</td>
</tr>
<tr>
<td>36</td>
<td>34</td>
</tr>
<tr>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>38</td>
<td>36</td>
</tr>
<tr>
<td>39</td>
<td>37</td>
</tr>
<tr>
<td>40</td>
<td>38</td>
</tr>
<tr>
<td>41</td>
<td>39</td>
</tr>
<tr>
<td>42</td>
<td>40</td>
</tr>
<tr>
<td>43</td>
<td>41</td>
</tr>
<tr>
<td>44</td>
<td>42</td>
</tr>
<tr>
<td>45</td>
<td>43</td>
</tr>
<tr>
<td>46</td>
<td>44</td>
</tr>
<tr>
<td>47</td>
<td>45</td>
</tr>
<tr>
<td>48</td>
<td>46</td>
</tr>
<tr>
<td>49</td>
<td>47</td>
</tr>
<tr>
<td>50</td>
<td>48</td>
</tr>
<tr>
<td>51</td>
<td>49</td>
</tr>
<tr>
<td>52</td>
<td>50</td>
</tr>
<tr>
<td>53</td>
<td>51</td>
</tr>
<tr>
<td>54</td>
<td>52</td>
</tr>
<tr>
<td>55</td>
<td>53</td>
</tr>
<tr>
<td>56</td>
<td>54</td>
</tr>
<tr>
<td>57</td>
<td>55</td>
</tr>
<tr>
<td>58</td>
<td>56</td>
</tr>
<tr>
<td>59</td>
<td>57</td>
</tr>
<tr>
<td>60</td>
<td>58</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>ELEMENT NO.</th>
<th>MAT.</th>
<th>PLATE ELEMENT</th>
<th>JOINT 1</th>
<th>JOINT 2</th>
<th>JOINT 3</th>
<th>PROPERTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.100</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>2</td>
<td>0.100</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>3</td>
<td>0.100</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>4</td>
<td>0.100</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>5</td>
<td>0.100</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>6</td>
<td>0.100</td>
<td>4</td>
<td>4</td>
<td>11</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>7</td>
<td>0.100</td>
<td>5</td>
<td>5</td>
<td>12</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>8</td>
<td>0.100</td>
<td>6</td>
<td>6</td>
<td>13</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>9</td>
<td>0.100</td>
<td>7</td>
<td>7</td>
<td>14</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>10</td>
<td>0.100</td>
<td>8</td>
<td>8</td>
<td>15</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>11</td>
<td>0.100</td>
<td>9</td>
<td>9</td>
<td>16</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>12</td>
<td>0.100</td>
<td>10</td>
<td>10</td>
<td>17</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>13</td>
<td>0.100</td>
<td>11</td>
<td>11</td>
<td>18</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>14</td>
<td>0.100</td>
<td>12</td>
<td>12</td>
<td>19</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>15</td>
<td>0.100</td>
<td>13</td>
<td>13</td>
<td>20</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>16</td>
<td>0.100</td>
<td>14</td>
<td>14</td>
<td>21</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>17</td>
<td>0.100</td>
<td>15</td>
<td>15</td>
<td>22</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>18</td>
<td>0.100</td>
<td>16</td>
<td>16</td>
<td>23</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>19</td>
<td>0.100</td>
<td>17</td>
<td>17</td>
<td>24</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>20</td>
<td>0.100</td>
<td>18</td>
<td>18</td>
<td>25</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>21</td>
<td>0.100</td>
<td>19</td>
<td>19</td>
<td>26</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>22</td>
<td>0.100</td>
<td>20</td>
<td>20</td>
<td>27</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>23</td>
<td>0.100</td>
<td>21</td>
<td>21</td>
<td>28</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>24</td>
<td>0.100</td>
<td>22</td>
<td>22</td>
<td>29</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>25</td>
<td>0.100</td>
<td>23</td>
<td>23</td>
<td>30</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>26</td>
<td>0.100</td>
<td>24</td>
<td>24</td>
<td>31</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>27</td>
<td>0.100</td>
<td>25</td>
<td>25</td>
<td>32</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>28</td>
<td>0.100</td>
<td>26</td>
<td>26</td>
<td>33</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>29</td>
<td>0.100</td>
<td>27</td>
<td>27</td>
<td>34</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>30</td>
<td>0.100</td>
<td>28</td>
<td>28</td>
<td>35</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>31</td>
<td>0.100</td>
<td>29</td>
<td>29</td>
<td>36</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>32</td>
<td>0.100</td>
<td>30</td>
<td>30</td>
<td>37</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>33</td>
<td>0.100</td>
<td>31</td>
<td>31</td>
<td>38</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>34</td>
<td>0.100</td>
<td>32</td>
<td>32</td>
<td>39</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>35</td>
<td>0.100</td>
<td>33</td>
<td>33</td>
<td>40</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>36</td>
<td>0.100</td>
<td>34</td>
<td>34</td>
<td>41</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>37</td>
<td>0.100</td>
<td>35</td>
<td>35</td>
<td>42</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>38</td>
<td>0.100</td>
<td>36</td>
<td>36</td>
<td>43</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
<tr>
<td>39</td>
<td>0.100</td>
<td>37</td>
<td>37</td>
<td>44</td>
<td>0.27473E</td>
<td>0.27473E</td>
</tr>
</tbody>
</table>
## Reduced Upper Triangular Stiffness Matrix

<table>
<thead>
<tr>
<th>Row</th>
<th>0.3144E+06</th>
<th>0.13842E+06</th>
<th>0.1747E+06</th>
<th>0.994E+05</th>
<th>0.2747E+05</th>
<th>0.2247E+05</th>
<th>0.961E+05</th>
<th>0.621E+05</th>
<th>0.09E+05</th>
<th>0.095E+05</th>
<th>0.09E+05</th>
<th>0.095E+05</th>
<th>0.09E+05</th>
<th>0.095E+05</th>
<th>0.09E+05</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3144E+06</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>2</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>3</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>4</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>5</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>6</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>7</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>8</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
<tr>
<td>9</td>
<td>0.105E+05</td>
<td>0.13842E+06</td>
<td>0.1747E+06</td>
<td>0.994E+05</td>
<td>0.2747E+05</td>
<td>0.2247E+05</td>
<td>0.961E+05</td>
<td>0.621E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
<td>0.095E+05</td>
<td>0.09E+05</td>
</tr>
</tbody>
</table>
**HERE ARE THE EIGENVALUES AND EIGENVECTORS**

<table>
<thead>
<tr>
<th>EIGENVECTOR NUMBER</th>
<th>CORRESPONDING TO</th>
<th>[2.4852237E+01]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[2.41324568E-07]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2.41324568E-07]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2.41324568E-07]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2.41324568E-07]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2.41324568E-07]</td>
</tr>
</tbody>
</table>
Here are the natural frequencies:

<table>
<thead>
<tr>
<th>Natural Frequency Number</th>
<th>15</th>
<th>764.618 CPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>9462.187 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>4456.580 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>11001.887 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>11377.692 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>20579.995 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>21183.287 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>20819.729 CPS</td>
</tr>
<tr>
<td>Natural Frequency Number</td>
<td>15</td>
<td>20374.125 CPS</td>
</tr>
</tbody>
</table>
Sample Problem No. 3

Missile Control Surface Model
(Modelled with beam elements and lumped weights)

Find first five natural modes and frequencies.

Note: Joint 11 is restrained from rotating about y
<table>
<thead>
<tr>
<th>Joint No.</th>
<th>Mass lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.050</td>
</tr>
<tr>
<td>2</td>
<td>0.110</td>
</tr>
<tr>
<td>3</td>
<td>0.115</td>
</tr>
<tr>
<td>4</td>
<td>0.125</td>
</tr>
<tr>
<td>5</td>
<td>0.196</td>
</tr>
<tr>
<td>6</td>
<td>0.155</td>
</tr>
<tr>
<td>7</td>
<td>0.305</td>
</tr>
<tr>
<td>8</td>
<td>0.305</td>
</tr>
<tr>
<td>9</td>
<td>0.305</td>
</tr>
<tr>
<td>10</td>
<td>0.165</td>
</tr>
<tr>
<td>11</td>
<td>0.060</td>
</tr>
<tr>
<td>12</td>
<td>0.165</td>
</tr>
<tr>
<td>13</td>
<td>0.005</td>
</tr>
<tr>
<td>14</td>
<td>0.183</td>
</tr>
<tr>
<td>15</td>
<td>0.325</td>
</tr>
<tr>
<td>16</td>
<td>0.310</td>
</tr>
<tr>
<td>17</td>
<td>0.280</td>
</tr>
<tr>
<td>18</td>
<td>0.140</td>
</tr>
<tr>
<td>19</td>
<td>0.062</td>
</tr>
<tr>
<td>20</td>
<td>0.078</td>
</tr>
<tr>
<td>21</td>
<td>0.078</td>
</tr>
<tr>
<td>22</td>
<td>0.078</td>
</tr>
<tr>
<td>23</td>
<td>0.080</td>
</tr>
<tr>
<td>24</td>
<td>0.033</td>
</tr>
<tr>
<td>25</td>
<td>0.051</td>
</tr>
<tr>
<td>26</td>
<td>0.051</td>
</tr>
<tr>
<td>27</td>
<td>0.051</td>
</tr>
<tr>
<td>28</td>
<td>0.042</td>
</tr>
<tr>
<td>29</td>
<td>0.050</td>
</tr>
</tbody>
</table>
### Beam Element Properties

<table>
<thead>
<tr>
<th>Member i-j</th>
<th>Moment-of-Inertia Area</th>
<th>Torsional Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0.0009</td>
<td>0.0055</td>
</tr>
<tr>
<td>2-3</td>
<td>0.0009</td>
<td>0.0055</td>
</tr>
<tr>
<td>3-4</td>
<td>0.0009</td>
<td>0.0055</td>
</tr>
<tr>
<td>4-5</td>
<td>0.0018</td>
<td>0.0055</td>
</tr>
<tr>
<td>6-7</td>
<td>0.0164</td>
<td>0.0300</td>
</tr>
<tr>
<td>7-8</td>
<td>0.0164</td>
<td>0.0300</td>
</tr>
<tr>
<td>8-9</td>
<td>0.0164</td>
<td>0.0300</td>
</tr>
<tr>
<td>9-12</td>
<td>0.0164</td>
<td>0.0300</td>
</tr>
<tr>
<td>12-13</td>
<td>0.0160</td>
<td>0.0300</td>
</tr>
<tr>
<td>14-15</td>
<td>0.0147</td>
<td>0.0280</td>
</tr>
<tr>
<td>15-16</td>
<td>0.0147</td>
<td>0.0280</td>
</tr>
<tr>
<td>16-17</td>
<td>0.0147</td>
<td>0.0280</td>
</tr>
<tr>
<td>17-18</td>
<td>0.0147</td>
<td>0.0280</td>
</tr>
<tr>
<td>19-20</td>
<td>0.0053</td>
<td>0.0010</td>
</tr>
<tr>
<td>20-21</td>
<td>0.0053</td>
<td>0.0010</td>
</tr>
<tr>
<td>21-22</td>
<td>0.0053</td>
<td>0.0010</td>
</tr>
<tr>
<td>22-23</td>
<td>0.0053</td>
<td>0.0010</td>
</tr>
<tr>
<td>24-25</td>
<td>0.0031</td>
<td>0.0006</td>
</tr>
<tr>
<td>25-26</td>
<td>0.0031</td>
<td>0.0006</td>
</tr>
<tr>
<td>26-27</td>
<td>0.0031</td>
<td>0.0006</td>
</tr>
<tr>
<td>27-28</td>
<td>0.0031</td>
<td>0.0006</td>
</tr>
<tr>
<td>1-10</td>
<td>0.0026</td>
<td>0.0029</td>
</tr>
<tr>
<td>6-12</td>
<td>0.0503</td>
<td>0.1000</td>
</tr>
<tr>
<td>12-14</td>
<td>0.0503</td>
<td>0.1000</td>
</tr>
<tr>
<td>7-13</td>
<td>0.0255</td>
<td>0.0510</td>
</tr>
<tr>
<td>13-15</td>
<td>0.0255</td>
<td>0.0510</td>
</tr>
<tr>
<td>8-16</td>
<td>0.0380</td>
<td>0.0750</td>
</tr>
<tr>
<td>9-17</td>
<td>0.0380</td>
<td>0.0750</td>
</tr>
<tr>
<td>10-18</td>
<td>0.0377</td>
<td>0.0750</td>
</tr>
<tr>
<td>14-19</td>
<td>0.0017</td>
<td>0.0034</td>
</tr>
<tr>
<td>15-20</td>
<td>0.0035</td>
<td>0.0070</td>
</tr>
<tr>
<td>16-21</td>
<td>0.0035</td>
<td>0.0070</td>
</tr>
<tr>
<td>17-22</td>
<td>0.0035</td>
<td>0.0070</td>
</tr>
<tr>
<td>18-23</td>
<td>0.0017</td>
<td>0.0029</td>
</tr>
<tr>
<td>11-12</td>
<td>100.0000</td>
<td>0.0100</td>
</tr>
<tr>
<td>11-29</td>
<td>0.3200</td>
<td>0.0790</td>
</tr>
<tr>
<td>19-24</td>
<td>0.0017</td>
<td>0.0030</td>
</tr>
<tr>
<td>20-25</td>
<td>0.0017</td>
<td>0.0030</td>
</tr>
<tr>
<td>21-26</td>
<td>0.0035</td>
<td>0.0070</td>
</tr>
<tr>
<td>22-27</td>
<td>0.0035</td>
<td>0.0070</td>
</tr>
<tr>
<td>23-28</td>
<td>0.0017</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

\[ E = 3 \times 10^7 \text{ psi} \]

\[ \nu = 0.3 \]
## Listing of Input Data Cards

**MISSILE CONTROL SURFACE MODEL WITH 29 JOINTS**

**AUGUST 1969**

<table>
<thead>
<tr>
<th>Card</th>
<th>1</th>
<th>2</th>
<th>45</th>
<th>0</th>
<th>9</th>
<th>1</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.5</td>
<td>7.0</td>
<td>1.1</td>
<td>2.0</td>
<td>6.1</td>
<td>4.2</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>7.0</td>
<td>1.3</td>
<td>6.1</td>
<td>6.0</td>
<td>6.3</td>
<td>2.5</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>7.0</td>
<td>2.4</td>
<td>6.8</td>
<td>6.3</td>
<td>6.2</td>
<td>6.9</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>7.0</td>
<td>1.3</td>
<td>6.0</td>
<td>6.9</td>
<td>6.5</td>
<td>1.2</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>6.9</td>
<td>2.9</td>
<td>6.0</td>
<td>6.5</td>
<td>1.2</td>
<td>6.9</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>6.9</td>
<td>1.2</td>
<td>6.5</td>
<td>6.9</td>
<td>6.0</td>
<td>2.9</td>
<td>0.3</td>
</tr>
<tr>
<td>7</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>8</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>9</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>10</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>11</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>12</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>13</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>14</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>15</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>16</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>17</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>18</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>19</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>20</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>21</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>22</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>23</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>24</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>25</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>26</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>27</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
<tr>
<td>28</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>0.3</td>
</tr>
<tr>
<td>29</td>
<td>6.9</td>
<td>2.9</td>
<td>6.5</td>
<td>6.9</td>
<td>1.2</td>
<td>6.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

66
<table>
<thead>
<tr>
<th>18</th>
<th>0.14</th>
<th>0.050</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>20</td>
<td>0.049</td>
<td>0.049</td>
</tr>
<tr>
<td>21</td>
<td>0.048</td>
<td>0.048</td>
</tr>
<tr>
<td>22</td>
<td>0.047</td>
<td>0.047</td>
</tr>
<tr>
<td>23</td>
<td>0.046</td>
<td>0.046</td>
</tr>
<tr>
<td>24</td>
<td>0.045</td>
<td>0.045</td>
</tr>
<tr>
<td>25</td>
<td>0.044</td>
<td>0.044</td>
</tr>
<tr>
<td>26</td>
<td>0.043</td>
<td>0.043</td>
</tr>
<tr>
<td>27</td>
<td>0.042</td>
<td>0.042</td>
</tr>
<tr>
<td>28</td>
<td>0.041</td>
<td>0.041</td>
</tr>
<tr>
<td>29</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td>30</td>
<td>0.039</td>
<td>0.039</td>
</tr>
<tr>
<td>31</td>
<td>0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>32</td>
<td>0.037</td>
<td>0.037</td>
</tr>
<tr>
<td>33</td>
<td>0.036</td>
<td>0.036</td>
</tr>
<tr>
<td>34</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>35</td>
<td>0.034</td>
<td>0.034</td>
</tr>
<tr>
<td>36</td>
<td>0.033</td>
<td>0.033</td>
</tr>
<tr>
<td>37</td>
<td>0.032</td>
<td>0.032</td>
</tr>
<tr>
<td>38</td>
<td>0.031</td>
<td>0.031</td>
</tr>
<tr>
<td>39</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>40</td>
<td>0.029</td>
<td>0.029</td>
</tr>
<tr>
<td>41</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>42</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>43</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>44</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>45</td>
<td>0.024</td>
<td>0.024</td>
</tr>
<tr>
<td>46</td>
<td>0.023</td>
<td>0.023</td>
</tr>
<tr>
<td>47</td>
<td>0.022</td>
<td>0.022</td>
</tr>
<tr>
<td>48</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>49</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>50</td>
<td>0.019</td>
<td>0.019</td>
</tr>
<tr>
<td>51</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>52</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td>53</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>54</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>55</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>56</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>57</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>58</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>59</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>60</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>61</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>62</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>63</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>64</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>65</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>66</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>67</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>68</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>69</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>70</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>71</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>72</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>73</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>74</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>75</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>76</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>77</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>78</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>79</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>80</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.0017</td>
<td>0.003</td>
<td>1</td>
</tr>
<tr>
<td>0.0017</td>
<td>0.003</td>
<td>1</td>
</tr>
<tr>
<td>0.0035</td>
<td>0.007</td>
<td>1</td>
</tr>
<tr>
<td>0.0035</td>
<td>0.007</td>
<td>1</td>
</tr>
<tr>
<td>0.0017</td>
<td>0.0029</td>
<td>1</td>
</tr>
</tbody>
</table>
Program Output

(2)

69
MISSILE CONTROL SURFACE MODEL WITH 29 JOINTS
AUGUST 1968

<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>X COORD.</th>
<th>Y COORD.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0000</td>
<td>10.0000</td>
</tr>
<tr>
<td>2</td>
<td>2.0000</td>
<td>12.0000</td>
</tr>
<tr>
<td>3</td>
<td>2.0000</td>
<td>14.0000</td>
</tr>
<tr>
<td>4</td>
<td>2.0000</td>
<td>16.0000</td>
</tr>
<tr>
<td>5</td>
<td>2.0000</td>
<td>18.0000</td>
</tr>
<tr>
<td>6</td>
<td>6.0000</td>
<td>7.0000</td>
</tr>
<tr>
<td>7</td>
<td>6.0000</td>
<td>9.0000</td>
</tr>
<tr>
<td>8</td>
<td>6.0000</td>
<td>11.0000</td>
</tr>
<tr>
<td>9</td>
<td>6.0000</td>
<td>13.0000</td>
</tr>
<tr>
<td>10</td>
<td>6.0000</td>
<td>15.0000</td>
</tr>
<tr>
<td>11</td>
<td>6.0000</td>
<td>17.0000</td>
</tr>
<tr>
<td>12</td>
<td>6.0000</td>
<td>19.0000</td>
</tr>
<tr>
<td>13</td>
<td>6.0000</td>
<td>21.0000</td>
</tr>
<tr>
<td>14</td>
<td>6.0000</td>
<td>23.0000</td>
</tr>
<tr>
<td>15</td>
<td>6.0000</td>
<td>25.0000</td>
</tr>
<tr>
<td>16</td>
<td>6.0000</td>
<td>27.0000</td>
</tr>
<tr>
<td>17</td>
<td>6.0000</td>
<td>29.0000</td>
</tr>
<tr>
<td>18</td>
<td>6.0000</td>
<td>31.0000</td>
</tr>
<tr>
<td>19</td>
<td>6.0000</td>
<td>33.0000</td>
</tr>
<tr>
<td>20</td>
<td>6.0000</td>
<td>35.0000</td>
</tr>
<tr>
<td>21</td>
<td>6.0000</td>
<td>37.0000</td>
</tr>
<tr>
<td>22</td>
<td>6.0000</td>
<td>39.0000</td>
</tr>
<tr>
<td>23</td>
<td>6.0000</td>
<td>41.0000</td>
</tr>
<tr>
<td>24</td>
<td>6.0000</td>
<td>43.0000</td>
</tr>
<tr>
<td>25</td>
<td>6.0000</td>
<td>45.0000</td>
</tr>
</tbody>
</table>

COORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT
<table>
<thead>
<tr>
<th>JOINT NO.</th>
<th>CODE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>ROW 1</td>
<td>(0.6379)</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
</tr>
<tr>
<td>ROW 2</td>
<td>0.29272</td>
</tr>
<tr>
<td>ROW 3</td>
<td>0.29272</td>
</tr>
<tr>
<td>ROW 4</td>
<td>0.3503</td>
</tr>
</tbody>
</table>

**Reduced Upper Triangular Stiffness Matrix**
**Reduced Upper Triangular Stiffness Matrix**

**Row 1**

<table>
<thead>
<tr>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.84379E+00</td>
<td>-0.19372E+00</td>
<td>0.41384E+00</td>
<td>-0.11143E+00</td>
<td>0.18915E+00</td>
<td>-0.19496E+00</td>
<td>0.49571E+00</td>
<td>0.18548E+00</td>
<td>-0.19382E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.74471E+00</td>
<td>-0.16308E+00</td>
<td>0.19353E+00</td>
<td>-0.33616E+00</td>
<td>0.64852E+00</td>
<td>-0.32658E+00</td>
<td>0.17295E+00</td>
<td>0.21454E+00</td>
<td>-0.32658E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.37921E+00</td>
<td>-0.12756E+00</td>
<td>0.36536E+00</td>
<td>-0.62593E+00</td>
<td>0.27459E+00</td>
<td>-0.31971E+00</td>
<td>0.30642E+00</td>
<td>0.58225E+00</td>
<td>-0.32691E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.13159E+00</td>
<td>-0.21893E+00</td>
<td>0.58094E+00</td>
<td>-0.87589E+00</td>
<td>0.11367E+00</td>
<td>-0.16149E+00</td>
<td>0.41384E+00</td>
<td>0.13917E+00</td>
<td>-0.32691E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.12292E+00</td>
<td>-0.13187E+00</td>
<td>0.23394E+00</td>
<td>-0.43929E+00</td>
<td>0.28905E+00</td>
<td>0.11367E+00</td>
<td>-0.28526E+00</td>
<td>0.38681E+00</td>
<td>-0.87589E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.11898E+00</td>
<td>-0.13598E+00</td>
<td>0.58944E+00</td>
<td>-0.87589E+00</td>
<td>0.11367E+00</td>
<td>-0.28526E+00</td>
<td>0.83681E+00</td>
<td>0.58694E+00</td>
<td>-0.16149E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.11262E+00</td>
<td>-0.13598E+00</td>
<td>0.58944E+00</td>
<td>-0.87589E+00</td>
<td>0.11367E+00</td>
<td>-0.28526E+00</td>
<td>0.83681E+00</td>
<td>0.58694E+00</td>
<td>-0.16149E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15917E+00</td>
<td>-0.28905E+00</td>
<td>0.26822E+00</td>
<td>-0.21412E+00</td>
<td>0.41384E+00</td>
<td>0.13917E+00</td>
<td>-0.16149E+00</td>
<td>0.83681E+00</td>
<td>-0.16149E+00</td>
<td>0.13917E+00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Reduced Upper Triangular Weight Matrix**

<table>
<thead>
<tr>
<th>Row</th>
<th>Weight Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.50000F-01</td>
</tr>
<tr>
<td>2</td>
<td>0.11500E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.11500E+00</td>
</tr>
<tr>
<td>4</td>
<td>0.15900E+00</td>
</tr>
<tr>
<td>5</td>
<td>0.15900E+00</td>
</tr>
<tr>
<td>6</td>
<td>0.35500E+00</td>
</tr>
<tr>
<td>7</td>
<td>0.35500E+00</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Note: The table continues with rows 8 and 9 not shown in the image.
### Eigenvalues and Eigenvectors

#### Eigenvalue Number 1
**Corresponding to 1.9E+12.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.967E+16</td>
<td>2.73E+21</td>
</tr>
<tr>
<td>4.96E+16</td>
<td>7.11E+21</td>
</tr>
<tr>
<td>3.12E+21</td>
<td>1.72E+21</td>
</tr>
<tr>
<td>2.73E+21</td>
<td>8.79E+21</td>
</tr>
<tr>
<td>2.73E+21</td>
<td>1.45E+21</td>
</tr>
<tr>
<td>1.99E+21</td>
<td>2.94E+21</td>
</tr>
<tr>
<td>1.19E+21</td>
<td>9.35E+21</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 2
**Corresponding to 6.6E+16.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.87E+16</td>
<td>-4.76E+16</td>
</tr>
<tr>
<td>-9.13E+16</td>
<td>-1.21E+16</td>
</tr>
<tr>
<td>-9.13E+16</td>
<td>-4.76E+16</td>
</tr>
<tr>
<td>-9.13E+16</td>
<td>-1.21E+16</td>
</tr>
<tr>
<td>-9.13E+16</td>
<td>-4.76E+16</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 3
**Corresponding to 8.5E+16.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.18E+21</td>
<td>2.70E+21</td>
</tr>
<tr>
<td>1.76E+21</td>
<td>1.23E+21</td>
</tr>
<tr>
<td>6.90E+21</td>
<td>2.88E+21</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 4
**Corresponding to 2.9E+16.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.33E+21</td>
<td>3.99E+21</td>
</tr>
<tr>
<td>3.33E+21</td>
<td>3.99E+21</td>
</tr>
<tr>
<td>3.33E+21</td>
<td>3.99E+21</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 5
**Corresponding to 1.3E+16.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E+00</td>
<td>4.20E+00</td>
</tr>
<tr>
<td>5.40E+00</td>
<td>-6.21E+00</td>
</tr>
<tr>
<td>1.44E+01</td>
<td>-1.44E+01</td>
</tr>
<tr>
<td>2.90E+00</td>
<td>1.22E+01</td>
</tr>
<tr>
<td>5.10E+00</td>
<td>-4.90E+00</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 6
**Corresponding to 2.9E+07.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E+00</td>
<td>4.80E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>-1.00E+00</td>
</tr>
<tr>
<td>-2.84E+07</td>
<td>-2.84E+07</td>
</tr>
<tr>
<td>-3.13E+07</td>
<td>-3.13E+07</td>
</tr>
<tr>
<td>-2.72E+12</td>
<td>-2.72E+12</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 7
**Corresponding to 5.1E+06.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 8
**Corresponding to 6.0E+03.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 9
**Corresponding to 1.0E+00.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
</tbody>
</table>

#### Eigenvalue Number 10
**Corresponding to 1.0E+00.**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>1.00E+00</td>
<td>1.00E+00</td>
</tr>
</tbody>
</table>
Here are the natural frequencies:

<table>
<thead>
<tr>
<th>Natural Frequency Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>44a</td>
<td>44a</td>
<td>41a</td>
<td>41a</td>
<td>41a</td>
<td>41a</td>
<td>41a</td>
<td>41a</td>
<td>41a</td>
</tr>
</tbody>
</table>

|                             | 89.045 CPS | 129.898 CPS | 486.842 CPS | 474.760 CPS | 918.938 CPS | 496.859 CPS | 1887.344 CPS | 2238.934 CPS | 1379.942 CPS |
APPENDIX B

Program FLUENC Listing
RI-AIN INPUT OF DATA

READ(IN,10) (TITLE(I),I=1,24)
IF(IN,N.E.0) CALL EXIT
REWIND MDISC
REWIND NDISC
REWIND IDISC
REWIND KDISC
WRITE(OUT,9) (TITLE(I),I=1,24)
READ(IN,10) NJTS,NR,NBE,NPE,NMODE,MKEY,NLUMP
NJTS=NO. OF JOINTS, NR=NO. OF JOINTS WITH RESTRAINTS
NHF=NO. OF BEAM ELEMENTS, NPE=NO. OF TRIANGULAR PLATE ELEMENTS
NMODE=NO. OF EIGENVALUES AND EIGENVECTORS DESIRED
MKEY = 1 DO NOT COMPUTE ELEMENTAL CONSISTENT MASS TERMS
MKEY = 2 COMPUTE ELEMENTAL CONSISTENT MASS TERMS
NLUMP = NO. OF LUMPED MASSES INPUT
WRITE(OUT,9) NJTS,NR,NBE,NPE,NMODE,MKEY,NLUMP

INPUT MATERIAL PROPERTIES
READ(IN,10) NMAT
DO 10 I=1,NMAT
READ(IN,10) YM(I),PR(I),GE(I),DENS(I)
10 YM=YOUNG'S MOD./1.0**K, PR=POISSON RATIO, GE=MOD. OF RIGIDITY
DENS=DENSITY
YF=GE(I)*EO.0.0.) GE(I)=YM(I)/(2.*(1.+PR(I)))
YM(I)=YM(I)*F6
IF(GE(I).GT.1.0) GO TO 10
WRITE(OUT,9) (1,YM(I),PR(I),GE(I),DENS(I),I=1,NMAT)
DO 20 I=1,NMAT
20 DENS(I)=DENS(I)/(.5*.1412.)

INPUT JOINT COORDINATES
READ(IN,10) (X(M),Y(M),M=1,NJTS)
WRITE(OUT,9) (M,X(M),Y(M),M=1,NJTS)
WRITE(OUT,9) (M,X(M),Y(M),M=1,NJTS)

INPUT JOINT RESTRAINT CODE
N=FREE
N=CLAMPED
DO 12 I=1,NJTS
NR(I)=0
NPR(I)=0
NR3(I)=0
N1(I)=0
N2(I)=0
12 NR(I)=0
IF(NR.EQ.0) GO TO 12
WRITE(OUT,9) (M,X(M),Y(M),M=1,NJTS)
DO 11 I=1,NR
READ(IN,10) JT,M1,M2,M3
Nh1(JT) = M1
Nh2(JT) = M2
Nh3(JT) = M3
WRITE(OUT, 'UH') JT, M1, M2, M3
1 CONTINUE
50 CONTINUE
C GENERATE COORDINATE NUMBERS FOR EACH DEGREE OF FREEDOM. IF
C CLAMPED, NORMAL DISPLACEMENTS ARE NUMBERED FIRST
C N1, N2, N3 CONTAIN COORD. NUMBERS FOR EACH JOINT
C NDF = NO. OF NORMAL DISPLACEMENTS
C NDF = NO. OF DEGREES OF FREEDOM INCLUDING ROTATIONS
CALL COORD(NHR, NR, NR1, M1, M2, M3, NJ, (NREU,NDF))
WRITE(OUT, 'U2')
DO 50 I = 1, NJTS
IF(NR1(I) .EQ. 1) GO TO 50
WRITE(OUT, 'U2') I, NR1(I)
50 CONTINUE
C INPUT LUMPED MASSES
IF(NLUMP .EQ. 0) GO TO 260
WRITE(OUT, 'U2') ((JMASS(I), RMSS(I)), I = 1, NLUMP)
WRITE(OUT, 'U2')
DO 50 I = 1, NLUMP
WRITE(OUT, 'U2') JMASS(I), RMSS(I)
RMSS(I) = RMSS(I) / (j1 * 4 * d.)
50 CONTINUE
C CONTINUE
250 CONTINUE
NSSTF = NDF * (NDF + 1) / 2
DO 13 I = 1, NSSTF
STF(I) = 0.
IF(NRE .EQ. 2) GO TO 204
PRINT TO GENERATE BEAM STIFFNESS IFNS
WRITE(OUT, 'U4')
DO 14 NN = 1, NRE
WRITE(OUT, 'U4')
14 CONTINUE
C INPUT BEAM ELEMENT PROPERTIES
READ(NIN, 14) AR, XI, YJ, MAT, JTNR, JTFR
C AR = AREA OF BEAM CROSS SECTION, XI = AREA MOMENT OF INERTIA,
C YJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA, MAT = MATERIAL CODE
C JTNR, JTFR = JOINT NUMBERS AT ENDS
WRITE(OUT, 'U2') NN, AR, XI, YJ, MAT, JTNR, JTFR
C SET UP COORD NUMBERS
NOSC(1) = NI(JTNR)
NOSC(2) = N2(JTNR)
NOSC(3) = N3(JTNR)
NOSC(4) = NI(JTFR)
NOSC(5) = N2(JTFR)
NOSC(6) = N3(JTFR)
IF(MKEY .EQ. 1) GO TO 250
C STORE INFO. FOR LATER USE
WRITE(OUT, 'U2')
CONTINUE
DI(SCR) AR, XI, YJ, MAT, JTNR, JTFR, (NOSC(I), I = 1, 6)
29 CONTINUE
X1 = XI(JTNR)
X2 = XI(JTFR)
Y1 = Y(JTNR)
Y = Y(JTR)

FLNTH = SQRT(((X2 - X1) * 2 + (Y2 - Y1) * 2)

CALL TRANS(X1, X2, Y1, Y2, FLNTH, DCS)

E = YM(MAT)

G = GE(MAT)

CALL HFAMK(FLNTH, E, G, X1, YJ, STM, DCS)

DO 15 K = 1, N

IF(NOSC(K) .EQ. 0) GO TO 15

I = NOSC(K)

DO 16 N = 1, I

IF(NOSC(N) .EQ. 0) GO TO 16

J = NOSC(N)

IF(J .LT. I) GO TO 16

MM = (-1) * (1 - NDF - I) / 2

SSTF(MM) = SSTF(MM) + STM(K, N)

1A CONTINUE

1B CONTINUE

1C CONTINUE

230 CONTINUE

IF(NP .EQ. 0) GO TO 230

C BEGIN TO GENERATE TRIANGULAR PLATE STIFFNESS TERMS

WRITE(OUT, 10) N

NO = 1, NPE = 1

C INPUT TRIANGULAR PLATE ELEMENT PROPERTIES

READ(IN, 10) PTH, MAT, JT1, JT2, JT3, NUX

C PTH = PLATE THICKNESS, MAT = MATERIAL CODE.

C JT1, JT2, JT3 = JOINT NUMBERS AT CORNERS, ANGLE AT JT1 MUST NOT BE

C 90 DEGREES

C DX, DY, D1, DXY, HETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL

C PRINCIPAL AXES WRT TRIANGLE LOCAL AXES

IF(NDX .EQ. 0) READ(IN, 10) DX, DY, D1, DXY, HETA

IF(NDX .EQ. 0) GO TO 30

HETA = 0.

DY = YH(MAT) * PTH / (12. * (1. - PR(MAT) / 2))

DN = PR(MAT) * DX

DXY = (1. - PR(MAT)) / 2. * DX

30 HETA = HETA / 180.

WRITE(OUT, 111) NO, PTH, MAT, JT1, JT2, JT3, DX, DY, D1, DXY, HETA

C SET UP COEF NUMBERS

NOSCL(1) = N1(JT1)

NOSCL(2) = N2(JT1)

NOSCL(3) = N3(JT1)

NOSCL(4) = N4(JT2)

NOSCL(5) = N5(JT2)

NOSCL(6) = N6(JT2)

NOSCL(7) = N7(JT3)

NOSCL(8) = N8(JT3)

NOSCL(9) = N9(JT3)

IF(MKEY .EQ. 0) GO TO 230

C STORE INFO. FOR LATER USE

WRITE(1DISC) PTH, MAT, JT1, JT2, JT3, (NOSC(1), I = 1, 9)
254 CONTINUE
RX1=X(JT1)
RX2=X(JT2)
RY1=Y(JT1)
RY2=Y(JT2)
Y2=SQR((HX-RX1)**2+(RY2-RY1)**2)
CALL TRANS((HX,X1,RY1,RY2,DCS)
X1=DCS(2)*(X(JT3)-RX1)-DCS(1)*(Y(JT3)-RY1)
Y1=DCS(1)*(X(JT3)-RX1)+DCS(2)*(Y(JT3)-RY1)
DO 19 K=1,N
IF (NOSC(K).EQ.0) GO TO 19
I=NOSC(K)
DO 20 N=1,N
IF (NOSC(N).EQ.0) GO TO 20
J=NOSC(N)
IF (J.LT.J) GO TO 20
MM=(?+1+1)*(2*NDF-1)/2
SSTF(MM)=SSTF(MM)+PLTK(K,N)
19 CONTINUE
17 CONTINUE
15 CONTINUE
C STORE FOR REDUCTION
DO 1 1=1,NDF
NS=(?+1+1)*(2*NDF-1)/2
NE=(?+1+1)*(2*NDF-1)/2
WHERE(MDISC(SSTF(J),J=NS,NE)
REWIND IDISC
DO 29 I=1,NSSTT
SM(I)=0.
IF (MKPL.EQ.1) GO TO 25
IF (MKPL.EQ.1) GO TO 201
C GENERATE H+AM MASS MATRICES
DO 29 I=3 WM=1,NRE
REAH(MDISC(AK,XI,JTNR,JTRF,(NOSC(1),I=1,6)
X1=X(JTNR)
X2=X(JTFR)
Y1=Y(JTNR)
Y2=Y(JTFR)
F/LN=H=SQR((X2-X1)**2+(Y2-Y1)**2)
CALL TRANS(X1,X2,Y1,Y2,FLNTH,DCS)
RH0=DENS(MA1)
CALL HEAMM(F/1NTH,RH0,AR,XI,YJ,SM,DCS)
DO 29 K=1,N
1+(NOSC(K),.EQ.1) GO TO 24
I=NOSC(K)
DO 29 N=1,N
IF (NOSC(N).EQ.0) GO TO 25
J=NOSC(N)
IF (J.LT.J) GO TO 25
MM=(?+1+1)*(2*NDF-1)/2
SM(MM)=SM(MM)+SM(K,N)
25 CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
IF (NPE.EQ.0) GO TO 301
C
GENERATE TRIANGULAR PLATE MASS MATRICES
DO 6 NM=1,NPE
READ (DIDISC) PTH,MAT,JI,JT,JT3,(NOSC(I),I=1,9)
RX1=X(JT1)
RX2=X(JT2)
RY1=Y(JT1)
RY2=Y(JT2)
Y=SQRT((RX1-RX2)**2+(RY1-RY2)**2)
CALL TRANS(RX1,RX2,RY1,RY2,DCS)
X=DCS(2)*(X(JT3)-RX1)+DCS(1)*(Y(JT3)-RY1)
Y=DCS(1)*(X(JT3)-RX1)+DCS(2)*(Y(JT3)-RY1)
PRHO=DENS(MAT)
CALL PLATEM(Y*,X*,Y*,PRHO,P2H,DCS,PLTH)
DO 7 K=1,9
IF(NOSC(K).EQ.0) GO TO 27
I=NOSC(K)
DO 8 N=1,9
IF(NOSC(N).EQ.0) GO TO 28
J=NOSC(N)
IF(J.LT.I) GO TO 28
MM=(2*K-1)*(2*DF-1)/
SM(MM)=SM(MM)+PLTH(K,N)
7 CONTINUE
6 CONTINUE
CONTINUE
C
STORE FOR REDUCTION
CONTINUE
DO 9 I=1,MLUMP
NN=JMASS(I)
IF(N1(NN).EQ.0) GO TO 298
NN=N1(NN)
NS=(2*NNN+(NNN-1)*(2*DF-NNN))/2
SM(NS)=SM(NS)+SMASS(NNN)
9 CONTINUE
8 CONTINUE
DO 9 I=1,NIF
NS=(2*I-1)*(2*DF-1)/
NF=(2*DF-1-1)*(2*DF-1)/2
WK1F(NDISC)*(SM(J),J=NS,NF)
NUMASS=DF-NREDU
CALL FITGEN(A,VALU,TEMU,B,CHU,F,1DFM,1DISC,JDISC,KDISC,DISC,FIDISC,NDISC,NUMASS)
DO TO 100
100 CONTINUE
FORTRAN DECK

COORDN ASSIGNS A COORD. NO. TO EACH DEGREE OF FREEDOM AT EACH CONTACT

NR1, NR2, NR3 = ARRAYS CONTAINING RESTRAINT INFORMATION FOR EACH DEGREE

OF FREEDOM AT EACH JOINT (FREE=0, CLAMPED=1)

NL, NR, NJ = COORD. NO. FOR EACH DEGREE OF FREEDOM (NORMAL DISPLACEMENTS ARE NUMBERED FIRST)

NJTS = NO. OF JOINTS

NRFDU = NO. OF NORMAL DISPLACEMENTS

NDF = TOTAL NO. OF DEGREES OF FREEDOM (INCLUDING ROTATIONS)

SUBROUTINE COORDN(NR1, NR2, NR3, NL, NR, NJTS, NRFDU, NDF)

DIMENSION NR1(50), NR2(50), NR3(50), NL(50), NR7(50), N0(50)

N0=1

N0 = NO. OF NORMAL DISPLACEMENTS

IF(NM(NU), EQ, 0) GO TO 10

N1(I) = NO

NO = NO + 1

CONTINUE

NRFDU = NO + 1

GO TO 10

CONTINUE

NDF = NO - 1

RETURN

END
FORTRAN DECK

TRANSFORMATION DIRECTION COSINES

C X1,Y1 = COORDS. OF POINT 1
C X2,Y2 = COORDS. OF POINT 2
C FL = DISTANCE BETWEEN POINTS 1 AND 2
C DCS = DIRECTION COSINES OF VECTOR FROM POINT 1 TO POINT 2
SUBROUTINE TRANS(X1,X2,Y1,Y2,FL,DCS)
DIMENSION DCS(?)
DCS(1)=(X2-X1)*FL
DCS(?)=(Y2-Y1)/FL
RETURN
END
5 FORTRAN DECK
CHFAMK
PLANE GRID BEAM ELEMENT STIFFNESS MATRIX IN SYSTEM COORD.
C FL = BEAM LENGTH
C E = YOUNG'S MODULUS
C G = MODULUS OF RIGIDITY
C XI = AREA MOMENT OF INERTIA
C YJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA
C SIM = STIFFNESS MATRIX
C DCS = DIRECTION COSINES
SURROUN E HFMK(FL,E,G,FI,FI,STIM,DCS)
DIMENSION STIM('A'), DCS(0)
Z1=1.0*XI/FL
Z2=G*YJ/FL
STIM(1,1)=1.0*Z1/(FL*FL)
STIM(2,1)=6.0*Z1*DCS(0)/FL
STIM(3,1)=4.0*Z1*DCS(0)*DCS(1)+Z2*DCS(1)*DCS(1)
STIM(4,1)=-4.0*Z1*DCS(1)/FL
STIM(5,1)=(-4.0*Z1*Z2+DCS(1)*DCS(2)
STIM(6,1)=4.0*Z1*DCS(1)+Z2*DCS(2)*DCS(2)
STIM(4,2)=-STIM(1,2)
STIM(4,3)=-STIM('4,1)
STIM(4,4)=STIM(1,4)
STIM(4,5)=STIM(2,1)
STIM(5,2)=2.0*Z1*DCS(0)+Z2*DCS(1)*DCS(1)
STIM(5,3)=-(2.0*Z1*Z2+DCS(0)*DCS(2)
STIM(5,4)=-STIM(2,1)
STIM(5,5)=STIM(2,2)
STIM(6,2)=STIM(3,1)
STIM(6,3)=STIM(4,3)
STIM(6,4)=-STIM(3,1)
STIM(6,5)=STIM(3,2)
STIM(6,6)=STIM(3,3)
NO 10 1=2,J,N
N=1-1
NO 10 J=1,N
STIM(J,1)=STIM(1,J)
RETURN
ENDE
FORTRAN Deck

CHFAMM   PLANE GRID BEAM ELEMENT MASS MATRIX IN SYSTEM COORDS.

C  FL = BEAM LENGTH
C  RHO = DENSITY
C  A = CROSS SECTIONAL AREA
C  XI = AREA MOMENT OF INERTIA
C  YJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA
C  SMH = MASS MATRIX
C  DCS = DIRECTION COSINES

SUBROUTINE CHFAMM(FL,RHO,A,XI,YJ,SMH,DCS)

DIMENSION SMH(4,6),DCS(2)

ZI=RHO*A*FL
77=FL**7
74=XI*A

MM=1*(13.*Z3*(2.*Z2))
GG=1*(11.*FL**11.*Z3*(10.*FL))
AA=Z1*(Z2**10.*Z3**9.*FL**10.)
TT=Z1*YJ/A

RR=Z1*(9.*YJ**9.*Z2)/(5.*Z2)
OO=Z1*(13.*YJ**13.*Z3/(10.*FL))

SS=-Z1*(Z2**10.*Z3**9.*FL)

PP=Z1*YJ**9.*A

SMH(1,1)=DD

SMH(2,1)=CC*DCS(1)

SMH(2,2)=AA*DCS(1)*DCS(2)+TT*DCS(1)*DCS(1)

SMH(3,3)=AA+TT+DCS(1)*DCS(z)

SMH(4,4)=DCS(1)*DCS(1)*DCS(1)*DCS(1)*DCS(1)

SMH(4,1)=RR

SMH(4,2)=OO*DCS(2)

SMH(4,3)=-OO*DCS(1)

SMH(4,4)=SMH(1,1)

SMH(5,1)=-SMH(4,2)

SMH(5,2)=SS*DCS(1)*DCS(1)*PP*DCS(1)*DCS(1)

SMH(5,3)=-SS*PP*DCS(1)*DCS(1)

SMH(5,4)=-SMH(5,1)

SMH(5,5)=SMH(2,2)

SMH(6,1)=-SMH(4,4)

SMH(6,2)=SMH(5,1)

SMH(6,3)=SS*DCS(1)*DCS(1)*PP*DCS(1)*DCS(1)

SMH(6,4)=-SMH(6,1)

SMH(6,5)=SMH(3,2)

SMH(6,6)=SMH(3,3)

DO I=1,2
N=I-1
DO J=1,N
SMH(J,1)=SMH(1,1)
RETURN
FND
C FORTRAN DECK
C PLATEK
C THIS SUBROUTINE DETERMINES THE STIFFNESS MATRIX OF A
C TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.
C Y2,X2,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C DX,DY,D1,D2,D3,R=E = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL
C PRINCIPAL AXES W/O TRIANGLE LOCAL AXES
C DCS = DIRECTION COSINES
C PLTK = STIFFNESS MATRIX
SUBROUTINE PLATEK(Y2,X2,Y3,DX,DX,Y1,DXY,RETA,DCS,PLTK)
DIMENSION PLTK(9,9),C(9,9),CINV(9,9),P(9,9),R(9,9)
DIMENSION T(9,9),STIFF(9,9),DCS(9)
EQUVALENC(P(1,1),STIFF(1,1)), (R(1,1)), (1,1))
CALL CMAT(Y2,X2,Y3,C)
CALL MINV(C,CINV)
CALL DINVAT(Y2,X2,Y3,DX,DX,Y1,DXY,RETA,P)
CALL MATMPY(P,CINV,R,9)
DO 10 J=1,N
10 N=1-J
DO 10 J=1,N
ZZ=CINV(I,J)
ZZ=CINV(J,I)
CINV(I,J)=ZZ
CINV(J,I)=ZZ
CONTINUE
CALL MATMPY(CINV,R,STIFF,9)
NO =00 J=1,N
NO =00 J=1,N
400 T(1,1)=H.
T(1,1)=I.
T(4,4)=1.
T(3,3)=1.
T(2,2)=DCS(I)
T(3,3)=DCS(I)
T(4,4)=DCS(I)
T(5,5)=DCS(I)
T(6,6)=DCS(I)
T(7,7)=DCS(I)
T(8,8)=DCS(I)
T(9,9)=DCS(I)
T(1,2)=DCS(I)
T(2,2)=DCS(I)
T(3,3)=DCS(I)
T(4,4)=DCS(I)
T(5,5)=DCS(I)
T(6,6)=DCS(I)
CALL MATMPY(STIFF,T,C,9)
T(2,2)=DCS(I)
T(3,3)=DCS(I)
T(4,4)=DCS(I)
T(5,5)=DCS(I)
T(6,6)=DCS(I)
T(7,7)=DCS(I)
T(8,8)=DCS(I)
T(9,9)=DCS(I)
RETURN
92
THIS SUBROUTINE FORMS THE C MATRIX RELATING THE CORNER
DISPLACEMENTS TO THE POLYNOMIAL DEFLECTION COEFFICIENTS
FOR THE TRIANGULAR PLATE ELEMENT
Y1, X3, Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
SUBROUTINE CMAT(Y1,X3,Y3,C)
DIMENSION C(9,9)
DO 10 I=1,9
DO 10 J=1,9

10 C(I,J)=0.
C(1,1)=1.
C(1,2)=Y1
C(2,1)=1.
C(2,2)=X3
C(3,3)=1.
C(4,4)=Y2
C(4,5)=Y2**2
C(4,6)=Y2**3
C(5,5)=J.
C(5,6)=Y2
C(6,6)=Y2**2
C(6,7)=Y2**3
C(7,7)=1.
C(7,8)=X3
C(7,9)=Y3
C(8,8)=X3**2
C(8,9)=X3**3
C(9,9)=X3**4
C(1,3)=X3
C(1,4)=X6
C(2,6)=X3
C(2,7)=X3**2
C(3,5)=X6
C(3,6)=X3**2
C(4,7)=X3**3
C(5,8)=X3**2
C(5,9)=X3**3
RETURN
END
FORTRAN DECK
MATRIX INVERSION SUBROUTINE

A = MATRIX TO BE INVERTED
U = INVERTED MATRIX
NM = ORDER OF MATRIX (.LE. 9)

SUBROUTINE MINV(A, U, NM)
DIMENSION A(NM, NM), U(NM, NM)

DO 4001 I = 1, NM
DO 4001 J = 1, NM
U(I, J) = 0
IF (I .EQ. J) U(I, J) = 1

9001 CONTINUE
EPS = 0.0000001
DO 9015 I = 1, NM

K = 1
IF (I .EQ. NM) GO TO 9021
9021 IF (A(I, I) .LT. EPS) GO TO 9006, 9007
9005 IF (-A(I, I) .LT. EPS) GO TO 9006, 9007
9006 K = K + 1
DO 9023 J = 1, NM

U(I, J) = U(I, J) + U(K, J)
9023 A(I, J) = A(I, J) + A(K, J)
GO TO 9021

9007 DIV = A(I, I)
DO 9009 J = 1, NM

U(I, J) = U(I, J) / DIV
9009 A(I, J) = A(I, J) / DIV
DO 9015 MM = 1, NM

DELTA = A(MM, I)
IF (ABS(DELTA) .LT. EPS) GO TO 9012, 9013
9016 IF (MM .NE. I) GO TO 9010
9010 DO 9011 J = 1, NM

U(MM, J) = U(MM, J) - U(I, J) * DELTA
9011 A(MM, J) = A(MM, J) - A(I, J) * DELTA
9015 CONTINUE
DO 9033 I = 1, NM
DO 9033 J = 1, NM

A(I, J) = U(I, J)
RETURN
END
C FORTRAN DECK
C
C THIS SUBROUTINE DETERMINES THE DOUBLE INTEGRAL MATRIX FOR
C THE K EQUATION FOR THE TRIANGULAR PLATE ELEMENT
C
C Y2.X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C DX,DY,D1,DXY,BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL
C PRINCIPAL AXES W/O TRIANGLE LOCAL AXES
C
C P = DOUBLE INTEGRAL MATRIX
C
C SUBROUTINE DINTMAT(Y2,X3,Y3,DX,DY,D1,DXY,BETA,P)
C DIMENSION P(9,9),D(3,3)
C
C DO JN=1,N
C DO JN=1,N
C P(1,J)=0.
C CALL DMAT(DX,DY,D1,DXY,BETA,D)
A1=UBLINT(Y2.X3,Y3,.0,.0)
A2=UBLINT(Y2.X3,Y3,.1,.0)
A3=UBLINT(Y2.X3,Y3,.2,.0)
A4=UBLINT(Y2.X3,Y3,.0,.1)
A5=UBLINT(Y2.X3,Y3,.6,.2)
A6=UBLINT(Y2.X3,Y3,.1,.1)
P(4,4)=4.*D(1,1)*A1
P(4,5)=4.*D(1,3)*A1
P(4,6)=4.*D(1,2)*A1
P(4,7)=12.*D(1,1)*A2
P(4,8)=4.*D(1,1)*A4+D(1,2)*A2+2.*D(1,3)*(A2+A4)
P(4,9)=12.*D(1,2)*A4
P(5,5)=4.*D(1,3)*A1
P(5,6)=4.*D(3,2)*A1
P(5,7)=12.*D(3,1)*A2
P(5,8)=4.*D(3,1)*A4+D(3,2)*A2+2.*D(3,3)*(A2+A4)
P(5,9)=12.*D(3,2)*A4
P(6,6)=4.*D(2,2)*A1
P(6,7)=12.*D(2,1)*A2
P(6,8)=4.*D(2,1)*A4+D(2,2)*A2+2.*D(2,3)*(A2+A4)
P(6,9)=12.*D(2,2)*A4
P(7,7)=36.*D(1,1)*A3
P(7,8)=12.*D(1,1)*A6+D(1,2)*A3+2.*D(1,3)*(A3+A6)
P(7,9)=36.*D(1,2)*A6
P(8,8)=4.*D(1,1)*A5+D(1,2)*A3+2.*D(1,3)*(A3+A6)
P(8,9)=12.*D(1,2)*A5+D(2,2)*A3+2.*D(2,3)*(A3+A6)
P(9,9)=36.*D(2,2)*A5
C DO JN=1,N
N=J+1
C RETURN
C END

95
FORTRAN DECK

SUBROUTINE MATHPY(A, B, C, N)

DIMENSION A(N,N), B(N,N), C(N,N)

DO 10 I = 1, N
    DO 20 J = 1, N
        C(I,J) = 0.
    20 CONTINUE

10 C(I,J) = C(I,J) + A(I,K) * B(K,J)

RETURN

END
SUBROUTINE DETERMINES THE FLEXURAL RIGIDITY MATRIX IN

C
C THIS SUBROUTINE DETERMINES THE FLEXURAL RIGIDITY MATRIX IN
C TRIANGLE LOCAL COORDINATES
C
C D = FLEXURAL RIGIDITY MATRIX IN TRIANGLE LOCAL COORDINATES.
C
C DIMENSION D (3, 3)
C
T11 = COS (BETA) + x2
T12 = SIN (BETA) + COS (BETA)
T13
T21
T22
T23
T31
T32
T33
Z11 = DXX * T11 + DYY * T12 + DZZ * T13
Z12 = DXX * T21 + DYY * T22 + DZZ * T23
Z13 = DXX * T31 + DYY * T32 + DZZ * T33
Z21
Z22
Z23
Z31
Z32
Z33
RETURN
END
FORTRAN DECK

C CORLINT

C THIS SUBROUTINE EVALUATES THE DOUBLE INTEGRALS APPPEARING IN THE
C EQUATIONS FOR K AND M FOR THE TRIANGULAR PLATE ELEMENT
C Y3, X3, Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C M, N = POWER OF X AND Y RESPECTIVELY, PRZEMIENIECKI. PAGE 305

FUNCTION CORLINT(Y3, X3, Y3, M, N)

DIMENSION A1(2), B1(7), P1(7), P2(7), P3(7)

EQUIVALENCE(A1(1), P3(1))

IF(M-1) 40, 41, 42

40 P1(1)=1.0
N1=6
GO TO 43

41 P1(1)=-1.0
P1(7)=1.0
N1=1
GO TO 43

42 CONTINUE
A1(1)=-1.0
A1(7)=1.0
B1(1)=-1.0
B1(7)=1.0
M=1

MN=M-1
DO 10 J=1, MN
CALL PLYMP(A1,1,B1,M1,P1,N1)

10 CONTINUE

43 CONTINUE

IF(N-1) 50, 51, 52

50 P2(1)=1.0
N2=0
GO TO 53

51 P2(1)=-Y3+Y2
P2(7)=Y3
N2=1
GO TO 53

52 CONTINUE
A1(1)=-Y3+Y2
A1(7)=Y3
B1(1)=-Y3+Y2
B1(7)=Y3
M=1

MN=N-1
DO 70 J=1, MN
CALL PLYMP(A1,1,B1,M1,P2,N2)

70 CONTINUE

53 CONTINUE

IF(M=1) 40, 41, 42
M1=N2
20 CONTINUE
22 CONTINUE
CALL PLYMP(P1,N1,P2,N2,P3,N3)
MN=MN+1
SOL=0.
DO 30 I=1,NN
SOL=SOL+(X**((M+1))*Y2*(1./FLOAT(M+N+2))*P3(I)*(1./FLOAT(NJ+2-1)))
30 CONTINUE
/ai INT=SOL
RETURN
END
FORTRAN DECK
C POLYNOMIAL MULTIPLY
C SUBROUTINE PLUMP(A,L,B,M,C,N)
C
DIMENSION A(L),B(M),C(N)

I I = N + 1
D0 I = 1,N 1
C(I) = 0
L2 = L + 1
M2 = M + 1
D0 J = 1,M2
K = 1 + J
C(K-1) = C(K-1) + A(I)*B(J)
IF(C(K) = 0) CONTINUE
K = K
I1 = K
IF(I1 = N2) CONTINUE
N = L1 - 1
N2 = N + 1
D0 J = 1,N2
N1 = J + I1 - 1
C(J) = C(N1)
RETURN
END
FORTRAN DICK

CPFATEM

THIS SUBROUTINE DETERMINES THE MASS MATRIX OF A
TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.
Y1, X3, Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
PRHO = DENSITY
PIN = PLATE THICKNESS
DCS = DIRECTION COSINES
PLTM = MASS MATRIX

SUBROUTINE PLATEM(Y2, X3, Y3, PRHO, PIN, DCS, PLTM)
DIMENSION PLTM(9, 9), PIN(9, 9), DCS(2)
DIMENSION T(9, 9), FMASS(9, 9), DCS(2)
EQUIVALENCE (P(1, 1), FMASS(1, 1)), (R(1, 1), T(1, 1))
CALL CMAT(Y2, X3, Y3, C)
CALL MINV(C, CINV, C)
CALL UINMTM(Y2, X3, Y3, PRHO, PIN, P)
CALL MAMTPY(P, CINV, R, C)
DO 10 J = 2, 9
DO 10 J = 1, N
771 = CINV(1, J)
772 = CINV(J, 1)
CINV(1, J) = 771
CINV(J, 1) = 772
CONTINUE
DO 40 J = 1, N
DO 40 J = 1, N
CALL MAMTPY(CINV, R, FMASS, C)
T(1, J) = 0.
T(1, 1) = 1.
T(4, 4) = 1.
T(7, 7) = 1.
T(8, 8) = DCS(2)
T(4, 3) = DCS(2)
T(6, 5) = DCS(2)
T(8, 6) = DCS(2)
T(9, 9) = DCS(2)
T(7, 3) = DCS(1)
T(6, 6) = DCS(1)
T(9, 9) = DCS(1)
T(7, 2) = DCS(1)
T(6, 5) = DCS(1)
T(9, 9) = DCS(1)
CALL MAMTPY(FMASS, T, C, C)
T(7, 4) = DCS(1)
T(6, 6) = DCS(1)
T(9, 9) = DCS(1)
T(3, 7) = DCS(1)
T(6, 6) = DCS(1)
T(9, 9) = DCS(1)
CALL MAMTPY(T, C, PLTM, C)
RETURN
END
FORTRAN DECK

DIMENSION P(9,9)
P(1,1)=DBLINT(Y2,X3,Y3,0,0)
P(2,1)=DBLINT(Y2,X3,Y3,1,0)
P(3,1)=DBLINT(Y2,X3,Y3,2,0)
P(3,1)=DBLINT(Y2,X3,Y3,0,1)
P(4,2)=DBLINT(Y2,X3,Y3,1,1)
P(4,3)=DBLINT(Y2,X3,Y3,0,2)
P(4,4)=P(2,2)
P(4,5)=DBLINT(Y2,X3,Y3,3,0)
P(4,6)=DBLINT(Y2,X3,Y3,2,1)
P(4,7)=P(4,7)
P(5,1)=P(3,2)
P(5,2)=P(4,3)
P(5,3)=DBLINT(Y2,X3,Y3,1,2)
P(5,4)=DBLINT(Y2,X3,Y3,3,1)
P(5,5)=DBLINT(Y2,X3,Y3,2,2)
P(6,1)=P(3,3)
P(6,2)=P(5,3)
P(6,3)=DBLINT(Y2,X3,Y3,0,3)
P(6,4)=P(5,5)
P(6,5)=DBLINT(Y2,X3,Y3,1,3)
P(6,6)=DBLINT(Y2,X3,Y3,0,4)
P(7,1)=P(4,7)
P(7,2)=P(4,4)
P(7,3)=P(5,4)
P(7,4)=DBLINT(Y2,X3,Y3,5,0)
P(7,5)=DBLINT(Y2,X3,Y3,4,1)
P(7,6)=DBLINT(Y2,X3,Y3,3,2)
P(7,7)=DBLINT(Y2,X3,Y3,0,4)
P(8,1)=P(5,3)+P(4,3)
P(8,2)=P(6,4)+P(6,4)
P(8,3)=P(5,5)+P(9,5)
P(8,4)=P(7,6)+P(7,5)
P(8,5)=DBLINT(Y2,X3,Y3,2,3)+P(7,6)
P(8,6)=DBLINT(Y2,X3,Y3,1,4)+DBLINT(Y2,X3,Y3,5,1)
P(8,7)=DBLINT(Y2,X3,Y3,4,2)+DBLINT(Y2,X3,Y3,3,2)
P(8,8)=DBLINT(Y2,X3,Y3,2,4)+DBLINT(Y2,X3,Y3,4,1)
P(9,1)=P(7,7)
P(9,2)=P(8,4)
P(9,3)=P(9,4)
P(9,4)=DBLINT(Y2,X3,Y3,2,1)
P(9,5)=DBLINT(Y2,X3,Y3,1,4)

102
\begin{verbatim}
\textbf{P(0,6)} = \textbf{DBL.INT} (Y2, X3, Y3, U, J) \\
\textbf{P(0,7)} = \textbf{DBL.INT} (Y2, X3, Y3, J, J) \\
\textbf{P(0,8)} = \textbf{DBL.INT} (Y2, X3, Y3, 1, 2) \times \textbf{DBL.INT} (Y2, X3, Y3, J, J) \\
\textbf{P(0,9)} = \textbf{DBL.INT} (Y2, X3, Y3, U, J) \\
\textbf{DO} \textbf{JO} \textbf{ I}=1,9 \\
\textbf{DO} \textbf{JO} \textbf{ J}=1,1 \\
\textbf{10} \textbf{ P(1,J)} = \textbf{P(1,J)} \times \textbf{P(NU*PTH}} \\
\textbf{DO} \textbf{20} \textbf{ I}=2,9 \\
\textbf{N}=1-1 \\
\textbf{DO} \textbf{20} \textbf{ J}=1,N \\
\textbf{P(J,I)} = \textbf{P(1,J)} \\
\textbf{CONTINUE} \\
\textbf{RETURN} \\
\textbf{END} \\
\end{verbatim}
S FORTRAN DECK
CEIGFN REDUCES STIFFNESS MATRIX AND INVERTS IT, REDUCES MASS MATRIX
C DETERMINES EIGENVALUES AND EIGENVECTORS
C THE ARGUMENTS ARE =
C A - VECTOR OF LENGTH NRDF*(NRDF+1)/2
C VALU - VECTOR OF LENGTH NRDF OR NMASS (SMALLER)
C E - MATRIX OF DIMENSION (NRDF, 3)
C IDUM4 - VECTOR OF LENGTH NRDF OR NMASS (SMALLER)
C ITAPE, JTAPE, MTAPE, NTAPE - THESE ARE VARIOUS TAPES
C NRDF - NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
C NEIG - NUMBER OF EIGENVALUES DESIRED
C NVFC - NUMBER OF EIGENVECTORS DESIRED
C NMASS = NO. OF NORMAL DISPLACEMENTS
C NOMASS= NO. OF ROTATIONAL DEGREES OF FREEDOM
C STIFF IS ON MTAPE IN COMPACT FORM
C MASS IS ON NTAPE IN COMPACT FORM
C SUBROUTINE EIOFN(A, VALU, TEMP, B, C, DUM3, E, IDUM4, ITAPE, JTAPE, KTAPE, 1 MTAPE, JTAPE, MTAPE, NTAPE, KTAPE)
1 DIMENSION DUM3(NRDF), IDUM4(1), A(1), VALU(1), B(1), C(1), E(NRDF, 3), 1 TEMP(1)
1 DIMENSION [LOW(50), HIGH(50)]
1 INTEGER OUT = 6
1 REWIND MTAPE
1 REWIND NTAPE
1 NTREM = NMASS
1 CALL DIVID(NMASS, NOMASS, MTAPE, JTAPE, KTAPE, A, R)
1 CALL ZROMAK(A, B, C, DUM3, NMASS, NOMASS, ITAPE, JTAPE, MTAPE, KTAPE)
1 CALL DIVID(NMASS, NOMASS, MTAPE, JTAPE, KTAPE, A, B)
1 CALL ZROMAK(A, B, C, DUM3, NMASS, NOMASS, ITAPE, JTAPE, MTAPE, KTAPE)
345 CONTINUE
1 REWIND MTAPE
1 REWIND NTAPE
1 NRFDU = NMASS
1 NRME = NRDU*(NRDU+1)/2
1 READ(MTAPE) (A(I), I = 1, NRME)
1 WRITE(OUT, 5500)
1 FORMAT(15BHR REDUCED UPPFR TRIANGULAR
1 STIFFNESS MATRIX)
1 1D 5501 I = 1, NRDU
1 NS = (2*I+1-1)*(2*NREDU-1)/2
1 NE = (2*NREDU+1-1)*(2*NREDU-1)/2
1 WRITE(OUT, 5502) 1, (A(J), J = NS, NE)
5502 FORMAT(20I4, 15H(/9E14.5))
1 1D 5503 CONTINUE
1 CALL SYMINV(A, NREDU)
1 WRITE(OUT, 5503)
5503 FORMAT(15BHR REDUCED UPPFR TRIANGULAR
1 FLEXIBILITY MATRIX)
1 PUNCH 5602, ((LOW(K), HIGH(K)), K = 1, NREDU)
5602 FORMAT (18l4)
1 D 5504 I = 1, NREDU
1 NS = (2*I+1-1)*(2*NREDU-1)/2
1 NE = (2*NREDU+1-1)*(2*NREDU-1)/2
1 WRITE(OUT, 5505) 1, (A(J), J = NS, NE)
5505 FORMAT (18I4)
PUNCH 6011. Q1. Q2. Q3

6011 FORMAT ('A6')
DO 5507 I=1, NREDU
II=I-1
IF (II.EQ.0) GO TO 5508
DO 5509 J=1, II
NU=(2*I+J-J)/2+(J-1)*(NREDU-I)
5509 R(J)=A(NU)
5508 CONTINUE
NS=(?I+I-1)*(2*NREDU-I))/2
NE=(?NREDU+(I-1)*(2*NREDU-I))/2
J=1
DO 5510 JJ=NS, NE
R(J)=A(JJ)
5510 J=J+1
PUNCH 6010, (R(J), J=1, NREDU)
6010 FORMAT (1P6E17.5)
5507 CONTINUE
C OPTION TO EXPAND REDUCED FLEXIBILITY MATRIX TO FULL MATRIX BY
C INSERTING 1 OR 2 ZERO ROWS AND COLUMNS REPRESENTING ATTACH POINTS.
C CONFD . NCOD = 1 OPTION EXECUTED , NCOD = 0 OPTION NOT EXECUTED
READ (5, 560) NCOD
560 FORMAT (16)
IF (NCOD) 580, 580, 570
570 CALL FULLFL (A, NDFD)
580 READ (NTAPE) (A(I), I=1, NRMX)
DO 6017 I=1, NRMX
6017 A(I)=A(I)+32.17417
WRITE (OUT, 5515)
5515 FORMAT ('///79REDUCED UPPER TRIANGULAR
1FigHT MAtrIX')
DO 5406 I=1, NDFD
NS=(?I+I-1)*(2*NREDU-I))/2
NE=(?NREDU+(I-1)*(2*NREDU-I))/2
5406 WRITE (OUT, 5502) I, (A(J), J=NS, NE)
PUNCH 6011, Q4. Q5. Q6
DO 5511 I=1, NDFD
II=I
IF (II.EQ.0) GO TO 5512
DO 5513 J=1, II
NU=(?I+J-J)/2+(J-1)*(NREDU-I)
5513 R(J)=A(NU)
5512 CONTINUE
NS=(?I+I-1)*(2*NREDU-I))/2
NE=(?NREDU+(I-1)*(2*NREDU-I))/2
J=1
DO 5514 JJ=NS, NE
B(J)=A(JJ)
5514 J=J+1
PUNCH 6010, (B(J), J=1, NREDU)
5511 CONTINUE
IF (NEIG.EQ.0) RETURN
NMAX=NTEMP*(NTFMP+1)*2
30 CONTINUE
C READ IN THE MASS MATRIX
REWIND NTAPF
READ (NTAPE) (A(I), I=1, NRMX)
REWIND NTAPF
355 CONTINUE
CALL EIGMAT (NTEMP, A, VALU, TEMP, R, DUH3, E, IDUM4, NTAPF, JTAPF,
1 NTAPF, NEIG, NVEG)
DO 60 I=1,MEIG
   DUM3(I)=SORT(VALU(I))/6.2831853
60 CONTINUE
WRITE(OUT,9009)
WRITE(OUT,9005) (I,DUM3(I),I=1,MEIG)
9009 FORMAT(1H1.43X.33HHERE ARE THE NATURAL FREQUENCIES //)
9005 FORMAT(35X*29HTHE NATURAL FREQUENCY NUMBER 13,2X,2HIS F12.5,2X,
13HCPS)
9008 FORMAT(1H1.3AX.43HHERE ARE THE EIGENVALUES AND EIGENVECTORS //)
   RETURN
END
$ FORTRAN

**DECK CFULFL**
EXPANDS REDUCED FLEXIBILITY MATRIX BY INSERTING 1 OR 2 ZERO ROWS AND COLUMNS REPRESENTING ATTACH POINTS.

THE ARGUMENTS ARE
- 
- **B(1)** = REDUCED FLEXIBILITY MATRIX IN COMPACT FORM
- **NXC** = ORDER OF REDUCED FLEX. MATRIX
- INPUT DATA REQUIRED
- **NR** = NO. OF ATTACH POINTS (1 OR 2)
- **NNF, NWO** = MASS NUMBERS OF ATTACH POINTS 1 AND 2 RESP.

**SURROUINTE FULFL(R, NXC)**

**DIMENSION B(1), D(1275), C(50)**

**DATA 07/6HEXPA, Q8/6HED FLE , Q9/6HXIBILI, Q10/6HTY MAT , Q11/6HR I**

**1X / REA D(5,1)NR, NNE, NWO**

**1 FORMAT (918)**

**MS=NXC+NR**

**MM=MS*(MS+1)/2**

**DO 50 I=1, MS**

**50 N(I)=0.0**

**JJJ=0**

**KK=0**

**JJ=0**

**DO 100 J=1, MS**

**IF (J.EQ.NNE, OR, J.EQ.NWO) GO TO 99**

**I=JJ+1**

**JJ=I+NXC-J+1**

**KKK=J-1**

**DO 98 JK=1, JJ**

**98 KKK=KKK+1**

**IF (KKK.EQ.NNF, OR, KKK.EQ.NWO) GO TO 96**

**GO TO 97**

**97 KK=KK+1**

**98 D(KK)=R(JK)**

**99 CONTINUE**

**GO TO 100**

**99 KK=KK+MS-J+1**

**JJJ=JJJ+1**

**100 CONTINUE**

**WRITE (6, 2)**

**2 FORMAT (/86HUPPER TRIANGLE - EXPANDED FLEXIBILITY MATRIX)**

**DO 10 I=1, MS**

**NS=(2*I*(I-1)*(2*MS-1))/2**

**NE=(2*MS*(I-1)*(2*MS-1))/2**

**WRITE (6, 3) I, (D(J), J=NS, NE)**

**3 FORMAT (/3HRW, 14/(9F14.5))**

**10 CONTINUE**

**PUNCH 4, 07, 09, 010, 011**

**4 FORMAT (5A6)**

**DO 20 I=1, MS**

**11=I-1**

**IF (11.EQ.0) GO TO 18**

**DO 19 J=1, I**

**19 C(J)=D(NU)**

**20 CONTINUE**

**NS=(2*I*(I-1)*(2*MS-1))/2**

**NE=(2*MS*(I-1)*(2*MS-1))/2**

**J=1**
DO 16 JJ=NS,NE
C(J)=D(JJ)
16 J=J+1
PUNCH 5,(C(J),J=1,NS)
5 FORMAT(1P6E12.5)
20 CONTINUE
RETURN
END
FORTRAN DFCK

/*
 */

C N=NO. OF NORMAL DISPLACEMENTS
C M=NO. OF ROTATIONAL D.O.F.
C NTPE=CONTAINS STIFFNESS (OR MASS) MATRIX
C MTPF-K1; (M1?) STORED
C ITPF-K1; (M1?) STORED
C A= DUMMY STORAGE VECTOR, LARGER OF (N*(N+1)/2 OR M*(M+1)/2)
SUBROUTINE DIVID (N,M,NTPE,MTPF,ITPF,A,B)
DIMENSION A(*),B(*)

RFWIND ITPF
RFWIND NTPE
RFWIND MTPF
M=NTPE=M
M=N*M
ICNT=0
DO 10 I=1,N
11=NM-I+1
READ(NTPE) (R(J),J=1,II)
UU=II-M
DO 10 J=1,II
ICNT=ICNT+1
A(ICNT)=B(J)
10 U=U+1
ICNT=0
DO 10 J=1,II
ICNT=ICNT+1
R(J,ICNT)=B(J)
10 WRITE(ITPF) (A(J),J=1,M)
CONTINUE
WRITE(ITPF) (A(J),J=1,NMAX)
RFWIND MTPF
RFWIND ITPF
II=0
ICNT=II
DO 10 I=1,M
II=II+ICNT
READ(NTPE) (R(J),J=1,II)
ICNT=ICNT+1
DO 10 J=1,II
II=II+1
A(IU)=R(J)
10 CONTINUE
RETURN
END

109
CZROMAK

C D is a dummy vector with storage N or M (LARGER)
C A is a dummy vector with storage N*(N+1)/2 OR M*(M+1)/2 (LARGER)
C R is a dummy vector with storage N or M (LARGER)
C C is a dummy vector with storage N or M (LARGER)
C N=NU. of normal displacements
C M=NU. of rotational DOF.
C NTPF contains K1 matrix
C NTPF contains K2 matrix
C ITPE contains K2 matrix
C KTPE stores K1'*K2*(-1)
C A initially contains K1 inverse
C *** reduced stiffness matrix is stored on ITPE
C ONROUTINE ZROMAK(A,B,C,D,N,M,NTPF,HTPE,ITPE,KTPE)
C DIMENSION A(1),B(1),C(1),D(1)
C DOUBLE PRECISION SUM,DP1,DP2
C CALL SYMINV(A,M)
C rewind ntpf
C rewind itpe
C rewind itpe
C rewind itpe
C NMAX=N*(N+1)/2
C MMAX=M*(M+1)/2
C I=1
READ(NTPF) (R(I),I=1,M)
ICNT=0
DO 10 I=1,N
   10 K=I
   DO 20 J=1,M
      20 C(J)=A(I)
      ICNT=ICNT+1
      JA=M
      ID=I
      DO 30 J=1,J
         IF(JJ.EQ.ID) GO TO 30
         C(J)=A(ID)
         JA=JA-1
         ID=ID+JA
      30 CONTINUE
      SUM=0.000
      DO 40 J=1,M
         DP1=R(J)
         DP2=C(J)
         SUM=SUM+DP1*DP2
         N(JK)=SUM
      40 CONTINUE
      WRITE (ITPE) (D(J),J=1,M)
      WRITE (KTPE) (D(J),J=1,M)
      CONTINUE
      Rewind itpe
REWIND MTPE
REWIND NTPF
REWIND KTPF
READ (NTPE) (A(J), J=1, NMAX)
ICNT=0
DO 60 KK=1, N
READ (ITPE) (D(J), J=1, M)
KI=KK
DO 10 KJ=1, N
READ(MTPE)(C(J), J=1, M)
KP=KJ
IF(KP.LT.KI) GO TO 60
SUM=0, ADB
DO 30 KR=1, M
DP1=D(KR)
DP2=C(KR)
SUM=SUM + DP1 * DP2
ICNT=ICNT + 1
SM=SUM
A(ICNT)=A(ICNT)-SM
/ A CONTINUE
REWIND MTPE
/ A CONTINUE
REWIND NTPF
REWIND KTPF
REWIND ITPF
WRITE(ITPE) (A(I), I=1, NMAX)
REWIND ITPE
RETURN
END
$ \text{FORTRAN BACK} \\
\text{CZROMAM} \\
\text{C N=NO. OF NORMAL DISPLACEMENTS} \\
\text{C M=NO. OF ROTATIONAL D.O.F.} \\
\text{C NTPE CONTAINS M1 MATRIX} \\
\text{C MTP CONTAINS M2 MATRIX} \\
\text{C ITPE SCRATCH TAPE} \\
\text{C KTPE CONTAINS K1?K2?**(-1)} \\
\text{C** REDUCED MASS MATRIX IS STORED ON ITPE} \\
\text{SURROUNTE ZROMAM(A,B,C,D,N,M,NTPE,MTP,ITPE,KTPF)} \\
\text{DIMENSION A(1),B(1),C(1),D(1)} \\
\text{DOUBLE PRECISION SUM1,SUM2,DP1,DP2,DP3} \\
\text{NMASS=N} \\
\text{REWIN NTPE} \\
\text{REWIN ITPE} \\
\text{REWIN MTP} \\
\text{NMAX=N*(N+1)/2} \\
\text{DO } 10 \text{ KK}=1,N \\
\text{READ(KTPE) (A(I),I=1,M)} \\
\text{ICNT=0} \\
\text{DO 1000 } \text{JK}=1,M \\
\text{JK=IK} \\
\text{JK=IK} \\
\text{DO 20 } \text{J=JJ,M} \\
\text{ICNT=ICNT+1} \\
20 \text{C(J)}=A(ICNT) \\
\text{JJ=JJ-1} \\
\text{JA=M} \\
\text{1D=IK} \\
\text{DO } 50 \text{ J=1,JJ} \\
\text{IF(JJ.EQ.0) GO TO 30} \\
\text{C(J)}=A(1D) \\
\text{JA=JA-1} \\
\text{10=1D+JA} \\
\text{GO TO 10} \\
30 \text{CONTINUE} \\
\text{SUM1}=0.0 \\
\text{DO } 50 \text{ J=1,M} \\
\text{DP1=B(J)} \\
\text{DP2=C(J)} \\
50 \text{SUM1}=SUM1+DP1*DP2 \\
\text{SUM1} \\
\text{GO TO 10} \\
\text{CONTINUE} \\
\text{WRITE(ITPE) (O(J),J=1,M)} \\
\text{CONTINUE} \\
\text{REWIN ITPE} \\
\text{REWIN NTPE} \\
\text{REWIN MTP} \\
\text{READ(MTPE) (A(J),J=1,NMAX)} \\
\text{DO } 60 \text{ KK}=1,N \\
\text{READ(MTPE) (R(J),J=1,M)} \\
\text{END}
READ (ITPE) (O(J), J=1,M)
DO 10 KJ=1,N
READ (KTE) (C(J), J=1,M)
SUM = 0.00
SUM = 0.00
DO 10 KR=1,M
DP1 = D(KR)
DP2 = D(KR)
DP1 = C(KR)
SUM = SUM + DP1 * DP1
SUM = SUM + DP2 * DP2
SM1 = SUM;
SM2 = SUM;
IF (KJ.GE.KK) MM = (*KJ*(KK-1)*(.NMASS-KK))/
IF (KJ.GE.KK) A(MM) = A(MM) - SM1 + SM2
IF (KJ.LE.KK) MM = (*KK*(KJ-1)*(.NMASS-KJ))/
IF (KJ.LE.KK) A(MM) = A(MM) - SM1
CONTINUE
REWIND KTF
CONTINUE
REWIND KTF
REWIND KTF
REWIND KTF
REWIND KTF
REWIND KTF
DO 10 1=1,NMAX
REWIND KTF
REWIND KTF
REWIND KTF
REWIND KTF
REWIND KTF
C SYMINV
C A IS THE UPPER TRIANGLE OF THE SYMMETRIC MATRIX TO BE INVERTED.
C ELEMENTS ARE STORED ROWWISE.
C N = ORDER OF MATRIX
C PROGRAM INVERTS IN PLACE.
SUBROUTINE SYMINV(A,N)
DIMENSION A(l)
MMAX=N*(N+1)/2
A(l)=SQRT(A(l))
DO 100 I=1,N
 100 A(I)=A(I)/A(1)
IM1=1
IJ=N
DO 190 I=1,N
 190 A(I)=A(I)/A(I)
 200 DO 150 J=1,N
      L=IJ-A(1)
      150 A(IJ)=A(IJ)-A(I)*A(J)
   L=L-1
   IJ=IJ-A(I)
   210 DO 130 L=1,N-1
      A(IJ)=A(IJ)-A(IJ)*A(L)
      L=L-1
   230 DO 170 J=1,N
      IF(J-IJ).GE.0. AN0 170
   300 IF(J-IJ).LE.0. ANO 170
      400 A(IJ)=A(IJ)+A(JL)*A(L)
 470 A(IJ)=A(IJ)/A(I)
 500 JJ=J+1
 590 IF(I-JJ).LT.0. ANO 590
   600 IF(I-JJ).GT.0. ANO 590
      IF(1-N).GT.0. ANO 690
   700 IF(1-N).LT.0. ANO 690
      800 IF(I-JJ).GT.0. ANO 890
      900 IF(I-JJ).LT.0. ANO 890
     100 IM1=1
     110 I=1

DO Z000 I=1,W 
J=I 
I=I 
DO 1400 J=1,W 
A(IJ)=A(IJ)*A(JJ) 
JP1=J+1 
IF(JP1-1) LT 0,1180,1400 
1180 IL=IJ 
JL=JJ 
DO 1240 L=JP1,W 
IL=IL+1 
JL=JL+1 
1240 A(IJ)=A(IJ)+A(IL)*A(JL) 
JJ=JL+1 
1400 IJ=IJ+1 
2000 II=IJ 
RETURN 
END
SUBROUTINE EIGMAT
C THIS SUBROUTINE FINDS THE EIGENVALUES AND EIGENVECTORS FOR
C SYMMETRIC MASS AND STIFFNESS MATRICES.
C THE ARGUMENTS ARE--
C N- ORDER OF MATRICES.
C A- DUMMY VECTOR WITH DIMENSION IN MAIN PROGRAM OF N*(N+1)/2
C VALU- STORAGE FOR EIGENVALUES. MUST BE DIMENSIONED IN THE MAIN
C PROGRAM AS A VECTOR OF LENGTH NEIG.
C TEMP,B,C,D,E- DUMMY VECTORS WITH DIMENSION OF N IN MAIN PROGRAM.
C F- DUMMY ARRAY WITH DIMENSIONS OF (N,3) IN MAIN PROGRAM.
C IDUM- DUMMY INTEGER VECTOR WITH DIMENSION OF N IN MAIN PROGRAM.
C NTAPE- TAPE WHERE STIFFNESS MATRIX IS STORED IN COMPACT FORM.
C NTAPE- TAPE WHERE MASS MATRIX IS STORED IN COMPACT FORM.
C JTAPE,JTAPE- SCRATCH TAPES.
C NEIG- NUMBER OF EIGENVALUES DESIRED.
C NVEC- NUMBER OF EIGENVECTORS DESIRED. MUST BE EQUAL TO OR LESS
C THAN NEIG.
C THE MASS AND STIFFNESS MATRICES ARE STORED IN COMPACT FORM AS
C VECTORS. ONLY THE UPPER TRIANGLE OF THESE MATRICES (BY ROWS) IS
C STORED.
C SUBROUTINE EIGMAT(N,A,VALU,TEMP,B,C,D,E,NTAPE,NTAPE,JTAPE,
C IDUM,NEIG,NVEC)
C DIMENSION A(L),TEMP(L),VALU(L),B(L),C(L),D(L),F(N,N),IDUM(N)
C DOUBLE PRECISION SUM,SUMJ
INTEGER OUT
OUT=0
REWIND NTAPE
REWIND JTAPE
REWIND NTAPE
REWIND NTAPE
N=$((N+1)/2

SIFP 1
C READ IN M BY ROWS IN COMPACTED FORM
C REPLACE M BY (1)TRANSPOSE, WHERE M=L*(L)TRANSPOSE
C CALCULATE FIRST ROW
READ (NTAPE) (A(I),I=1,NMAX)
REWIND NTAPE
C CONTINUE
A(1)=SORT(A(1))
DO 10 I=2,N
10 A(I)=A(I)/A(1)
C CALCULATE ALL THE OTHER ROWS
IND=N
DO 40 I=2,N
40 IND=IND+1
SUM=0.0
K=I-1
DO 50 JJ=1,K
50 M=(N-JJ)*(JJ-1)/2+I

116
SUM = SUM + A(MJ) * A(MJ)
A(IND) = SQRT(A(IND) + SUM)
IF (IND.EQ.NMAX) GO TO 100
SUM = A(IND)
K1 = I + 1
DO 99 J = K1, N
IND = IND + 1
SUM = 0, D0
I = I - 1
DO 99 JJ = I, I
K = (M - JJ) *(JJ - 1)/2
K1 = K + 1
JJ = K + J
99 SUM = SUM + A(KI) * A(KJ)
A(IND) = (A(IND) + SUM)/SUM
CONTINUE
100 CONTINUE
1 CONTINUE
C CHECK FOR SINGULAR MASS MATRIX
DO 90 J = 1, N
KI = (M - J) *(J - 1)/2 + 1
I = (A(KI) < 0.99) GO TO 109
1 CONTINUE
C THIS COMPLETES STEP 1
C STEP 2
C WRITE (L) TRANSPOSE ON TAPE BY COLUMNS
C (L) TRANSPOSE INTO TEMPORARY STORAGE (TEMP = A VECTOR)
C AND THEN WRITE TEMP ON TAPE
KTAPE = NTAPE
IND = 0
DO 144 J = 1, N
IND = IND + 1
M11 = (M - 1) *(J - 1)/2 + J
TEMP(IND) = A(M11)
144 CONTINUE
WRITE (KTAPE) (TEMP(JJ), JJ = 1, IND)
IND = 0
1 CONTINUE
C THIS COMPLETES STEP 2
C STEP 3
C (L) TRANSPOSE-1 INVERSE REPLACES (L) TRANSPOSE IN CORF
C REPLACEMENT IS DONE BY LAST COLUMN FIRST - WORKING UP THE COLUMN
DO 444 I = 1, N
IND = (I * (M + 1))/2 - N
444 A(IND) = 1/A(IND)
DO 499 J = 2, N
JJ = (N + J) - J
DO 499 1 = 2, JJ
IND = (N + J - 1) *(JJ - 1)/2
SUM = 0, D0
K = JJ - 1 + 2
DO 450 K=K, JJ
IND=IND+K
MK=(M-K)*(K-1)/2+JJ
450 SUM=SUM+A(IDK)*A(MK)
IND=IND+JJ
IND=IND-1
IND=IND-1!
450 CONTINUE
C END OF STEP 1
C
C STEP 2
C U=(L)TRANSPOSE INVERSE
C WRITE U ON TAPE BY ROWS
WRITE(JTAPF) (A(I), I=1, NMAX)
C FINISHED WITH STEP 2
C
C STEP 3
C WRITE U ON TAPE BY COLUMNS STARTING WITH THE LAST COLUMN FIRST
C PUT U (LAST COLUMN FIRST) INTO TEMP AND THEN WRITE ON TAPE
IND=0
DO 555 K=1, N
J=K-N+1
DO 555 J=1, J
IND=IND+1
MJ=(M-J)*(J-1)/2+J
TEMP(IND)=A(MJ)
555 CONTINUE
WRITE(JTAPF) (TEMP(JJ), JJ=1, IND)
IND=0
555 CONTINUE
C END OF STEP 3
C
C STEP 6
C FORM KU
C READ K INTO CORF
C READ U INTO CORF A COLUMN AT A TIME IN REVERSE ORDER
C REPLACE K BY KU COLUMN BY COLUMN STARTING WITH THE LAST COLUMN
C AND WORKING UP THE COLUMN
READ(MTAPE) (A(I), I=1, NMAX)
REWIND JTAPF
DO 655 JJ=1, N
I=K+1-JJ
READ(JTAPF) (IFMP(I), I=1, JJ)
DO 655 I=1, JJ
SUM=SUM+DM
DO 655 K=I, I
MK=(M-K)*(K-1)/2+1
655 SUM=SUM+A(MK)*TEMP(K)
IND=(M-I)*(I-1)/2+J
IF (I.EQ.J) GO TO 655
118
K1 = (N-1) * (I-1) / 2
I = 1 + 1
DO 660 K = 1, J
K1K = K1 + K
660 SUM = SUM + A(K1K) * TEMP(K)
CONTINUE
A(IND) = SUM
CONTINUE
END OF STEP 6

STEP 7
FORM((L) INVERSE) * KU
KU IS IN CORF
READ IN COLUMN BY COLUMN AND CALCULATE ((L) INVERSE) * KU
ROW BY ROW
CALCULATE THE FIRST ROW
REWIND TAPE
READ (NTAPE) TEMP(1)
DO 710 I = 1, N
A(1) = A(1) / TEMP(1)
710 NOW CALCULATE THE REST OF THE ROWS
IND = N
DO 799 I = 2, N
READ (NTAPE) (TEMP(JJ), JJ=1, I)
DO 799 J = 1, N
IND = IND + 1
JJ = I - 1
SUM = 0.0
DO 751 K = 1, JJ
MK2 = (M-K) * (K-1) / 2 + J
751 SUM = SUM + TEMP(K) * A(MK2)
799 A(IND) = (A(IND) - SUM) / TEMP(1)
STEP 7 IS COMPLETE

STEP 8
DIFFRMINING EIGENVALUES AND EIGENVECTORS OF THE NEW MATRIX
CHANGE THE SIGN OF A IN ORDER TO OBTAIN THE SMALLEST EIGENVALUE FIRST
DO 890 I = 1, NMAX
890 A(I) = -A(I)
CALL HGMAT(A, VALU, TEMP, B, C, D, E, IDUM, N, NEIG, NVEC, MTPE)
CHANGE EIGENVECTORS
DO 850 I = 1, N
850 VALU(I) = -VALU(I)
STEP 8 IS COMPLETE

STEP 9
CHANGE EIGENVECTORS BACK
READ U INTO CORF BY ROWS
READ UNCHANGED EIGENVECTORS INTO CORE ONE AT A TIME
CHANGE AND PRINT EIGENVECTORS
IF (NVEC.EQ.0) GO TO 2000
WRITE(OUT, 1001)
REWIND ITAPE
READ(TAPE) (A(I),I=1,NMAX)
REWIND ITAPE
DO 909 JJ=1,NVEC
READ(TAPE) (TFMP(I),I=1,N)
IND=0
DO 411 I=1,N
SUM=0.D0
DO 909 J=1,N
IND=IND+1
909 SUM=SUM+A(IND)*TFMP(J)
910 TFMP(I)=SUM
C NORMALIZE THE EIGENVECTORS
SUM=TFMP(I)
DO 934 II=1,N
IF(ABS(SUM)-ABS(TFMP(II))) 935 935,935
935 SUM=TFMP(II)
936 CONTINUE
TFMP(II)=SUM
937 CONTINUE
947 CONTINUE
990 WRITE(OUT,95) JJ,VALU(JJ),(TFMP(I),I=1,N)
C STEP 9 IS COMPLETE
GO TO 200
400 FORMAT (1H9,'19H EIGENVECTOR NUMBER I5/12X,17H CORRESPONDING TO
11PF,5.7/(1H 1PAE19.7,))
410 FORMAT(1H15X,1H WHERE ARE THE EIGENVALUES AND EIGENVECTORS */,*
420 FORMAT(1H15X,2H THE MASS MATRIX IS SINGULAR */,*
430 WRITE(OUT,-2)
200 RETURN
END


120
FORTRAN DECK

SUBROUTINE HIGMAT(A,VALU,VALL,UPPERD,DIAG,V,T,INTER,N,NH,NEIG,NVFL,MTAPE)

DIMENSION A(I),VALU(1),VALL(I),UPPERD(1),DIAG(1),V(1),T(NN,3)

INTER(1)

REWIND MTAPE

N=0

IF (N.LT.2) GO TO 49

NP1=N+1

IF (N.LT.1) N1=N-1

IF (N.GT.1) N1=N1+1

IF (N.EQ.1) N1=N1+1

DO 10 I=1,NM2

SIGMA=0.

IP1=I+1

DO 10 J=IP1,N

IJJ=I+J

1 SIGMA2=SIGMA2+A(IJ)**2

SIGMA=SIGMA+SIGMA2

II=IX+I

IA(I)=A(I)

UPPERD(I)=-SIGN(SIGMA,A(IIP1))

T(I,2)=SIGMA2

10 (AHSGAMMA,GI,ABS(A(IIP1))) GO TO 49

UPPERD(I)=A(IIP1)

A(IIP1)=0.

GO TO 10

2 (IIP1)=SORT(I+.AHSGAMMA/ABS(A(IIP1)))

SIGMAH=-SIGN(SIGMA*A(IIP1),UPPERD(I))

IP2=IP1

DO 20 J=IP2,N

IJJ=IX+J

2 A(IJ)=A(IJ)/SIGMAH

JK1=I+(2*N-I-1)/2

JX=JK1

IX=JK1

DO 20 J=IP1,N

VALL(J)=0.

JK=JK1+J

DO 20 K=IP1,J

IK=IX+K

VALL(J)=VALL(J)+A(JK)*A(IK)

4 JK=JK+K

IF (J.EQ.N) GO TO 6

CALL LOOP(I+2,NP1,VALL(J),A(JX),A(IX))

6 IX=IX+N-J

6 SIGMAH=0.

DO 20 J=IP1,N

121
IJ=1X+J

7 IEGAN=DELGAN+A(IJ)*VALL(J)
DG00=.5*DELGAN
DO 6 J=1P1,N
IJ=1X+J

8 T(J,J-1)=VALL(J)-DG00*A(IJ)
DO 4 II=1P1,N
II=II+II
CALL LOOP2(A(IIX),A(IIX),T(NZ,1),T(I1,1),A(I11),I11+1,NP1)

10 IIX=1XI+X+1
16 IIX=1XI+N-I
M=N*(N+1)/B
UPPERD(NM1)=A(N-1)
T(NM1)=UPPERD(NM1)**?
DIAG(NM1)=A(N-2)
DIAG(N)=A(N)
ENORM=AMAX1(ABS(DIAG)+ABS(UPPER),ABS(DIAG(N))+ABS(UPPERD(NM1)))
DO 11 I=2,NM1
ENRMP=ABS(DIAG(I))+ABS(UPPERD(I))+ABS(UPPERD(I-1))

11 IF(ENRMP,N.T..ENORM)ENORM=ENRMP
DO 12 I=1,NEIG
VALL(I)=ENORM
19 VALL(I)=ENORM
DO 12 I=1,NEIG
13 ROOT=+*(VALL(I)+VALL(I))
IF(ROOT.EQ.VALL(I).OR.ROOT.EQ.VALL(I))GO TO 4
NAGREE=0
PM2=0.
PM1=1.
DO 15 J=1,N
IF(PM2.NE.+*)GO TO 15
14 PM1=SIGN(1.,PM1)
GO TO 17
15 IF(PM1.NE.-*)GO TO 17
16 PM2=SIGN(1.,PM2)
IF(J(J-1,2))=16,19,18
17 PM=DIAG(J-J-1)*PM2/PM1
PM2=-1.
18 IF(PM.EQ.1,14,20
19 PM2=PM1
IF(PM2.EQ.1,0,20
20 NAGREE=NAGREE+1
21 PM1=P
DO 23 J=1,NEIG
IF(J.LE.NAGREE)GO TO 22
IF(VALL(J).LE.ROOT)GO TO 13
VALL(J)=ROOT
GO TO 13
22 CONTINUE
23 CONTINUE
122
CONTINUE
IF(NVEC.EQ.0)GO TO 49
FPSLON=ENORM*1.E-9
COMPL=COMPL(I)
DO 30 J=1,N
V(J)=1.
TI(J)=DIAG(J)-VALU(I)
IF(J.EQ.N)GO TO 31
TU(J)=UPPFRD(J)
31 T(J+1,1)=UPPFRD(J)
30 DO 39 J=1,N
IF(ABS(T(J,J)).LT.1.E-17)T(J,J)=EPSLON
INTER(J)=J
JP1=J+1
IF(ABS(T(JP1,J)).LE.ABS(T(J,1)))GO TO 28
INTER(J)=1
DO 35 K=J+1,N
TEMP=T(J,K)
T(J,K)=T(JP1,K)
35 T(JP1,K)=TEMP
28 T(JP1,J)/T(J,1)
VAL(J)=OR(INTER(J),AND(TMULP,COMPL1))
T(JP1,J)/T(JP1,J)-TMULP*T(J,J)
29 T(JP1,J)=T(JP1,J)-TMULP*T(J,J)
INTER=1
DO 40 J=1,N
L=N+1-J
20 V(I)=(V(L)-1(L,2)\*V(L+1)-1(L,3)\*V(L+2))/T(L,1)
V(NORM)=0.
DO 33 L=1,N
33 V(NORM)=V(NORM)+V(L)**2
V(NORM)=SORT(V(NORM))
DO 40 J=1,N
40 V(J)=V(J)/V(NORM)
IF(INTER.EQ.2)GO TO 36
INTER=1
DO 45 L=2,N
45 L=M1-1
TRY=VALL(LM1)
IF(AND(TRY.1.E-9)GO TO 35
VTEM=V(LM1)
V(LM1)=V(L)
V(L)=VTEM
15 V(I)=V(L)-VALL(LM1)\*V(LM1)
GO TO 31
36 IF(V(NORM.EQ.0.)V(I)=1.
11X=(N*N-N-N/2)
DU .7 KK=1,NM2
IIPL=N-KK
UTV=0.
CALL LOOPS(UTV,A(IIX),V(NZ),IIPL+1,MP1)
CALL LOOP4(A(IIX),V(NZ),MP1,IIPL+1,UTV)
IIX=IIX+IIPL-N-2
WRITE(MTAPF) (V(ICH),ICH=1,N)
CONTINUE
RETURN
END
$FORTRAN\ DECK$

CLOUP1
SURROUITE LUMP(JP2, NP1, SGAMPJ, AJX, AIX)
DIMENSION \( AIX(1), AIX(1) \)
DO 1 \( L=JP2, NP1 \)
1 \( SGAMPJ=SGAMPJ, AJX(L)*AIX(L) \)
RETURN
END

$FORTRAN\ DECK$

CLOUP2
SURROUITE LUMP2(AII, AIX, SI, AII, IP1, NP1)
DIMENSION \( AIIX(1), AIX(1), S(1) \)
DO \( JJ=IP1, NP1 \)
1 \( AIX(JJ)=AIIX(JJ)-AIIX(S(JJ)-SI*AIX(JJ) \)
RETURN
END

$FORTRAN\ DECK$

CLOUP3
SURROUITE LUMP3(UTV, AII, V, IP2, NP1)
DIMENSION \( AIIX(1), V(1) \)
DO \( J=IP2, NP1 \)
1 \( UTV=UTV*AI\(X(J)*V(J) \)
RETURN
END

$FORTRAN\ DECK$

CLOUP4
SURROUITE LUMP4(AII, V, NP1, IP2, UTV)
DIMENSION \( AIIX(1), V(1) \)
DO \( K=IP2, NP1 \)
1 \( V(K)=V(K)-AIIX(K)*UTV \)
RETURN
END

125
MAIN PROGRAM FLUENC-FOR GENERATING STIFFNESS, FLEXIBILITY AND MASS
MATRICES FROM PLANE GRID BEAM AND TRIANG. PLATE ELEMENTS

DIMENSIONED VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>24</td>
<td>TM</td>
<td>10</td>
<td>PR</td>
<td>10</td>
<td>SE</td>
<td>10</td>
<td>DEN8</td>
<td>10</td>
</tr>
<tr>
<td>X</td>
<td>50</td>
<td>Y</td>
<td>50</td>
<td>NR1</td>
<td>50</td>
<td>NR2</td>
<td>50</td>
<td>NR3</td>
<td>50</td>
</tr>
<tr>
<td>MI</td>
<td>50</td>
<td>NE</td>
<td>50</td>
<td>N3</td>
<td>50</td>
<td>NOEC</td>
<td>9</td>
<td>CCA</td>
<td>2</td>
</tr>
<tr>
<td>SIM</td>
<td>6.6</td>
<td>SMM</td>
<td>6.6</td>
<td>PLTK</td>
<td>9.9</td>
<td>PLTH</td>
<td>9.9</td>
<td>SSIF</td>
<td>11328</td>
</tr>
<tr>
<td>SM</td>
<td>11328</td>
<td>DSMAS</td>
<td>50,1E25 (28), VA</td>
<td>LUF</td>
<td>TEMP</td>
<td>50</td>
<td>0</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>CUN3</td>
<td>150</td>
<td>F</td>
<td>150,3</td>
<td>IDUN4</td>
<td>50</td>
<td>JNASS</td>
<td>50</td>
</tr>
</tbody>
</table>

127
PLANE GRID BEAM ELEMENT STIFFNESS MACROS

FL = BEAM LENGTH
E = YOUNG'S MODULUS
G = MODULUS OF RIGIDITY
I = AREA MOMENT OF INERTIA
J = EFFECTIVE TORSIONAL MOMENT OF INERTIA
SM = STIFFNESS MATRIX
DCS = DIRECTION COSINES

DICTIONED VARIABLES

SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE

STH 4,6 DCS 2

SUBROUTINE BEAM(F,E,I,E,STH,DCS)

START

STH(1,1)=12.*E/FL
STH(2,1)=6.*E/FL
STH(2,2)=6.*E/FL

STH(3,1)=6.*I/FL
STH(3,2)=6.*I/FL
STH(3,3)=6.*I/FL

STH(4,1)=6.*J/FL
STH(4,2)=6.*J/FL
STH(4,3)=6.*J/FL
STH(4,4)=6.*J/FL

STH(5,1)=6.*I/FL
STH(5,2)=6.*I/FL
STH(5,3)=6.*I/FL
STH(5,4)=6.*I/FL

STH(6,1)=6.*J/FL
STH(6,2)=6.*J/FL
STH(6,3)=6.*J/FL
STH(6,4)=6.*J/FL

STH(7,1)=6.*J/FL
STH(7,2)=6.*J/FL
STH(7,3)=6.*J/FL
STH(7,4)=6.*J/FL

REPEAT TO 18 FOR
i=8,9,...,n

STH(1,1)=STH(1,1)
STH(2,1)=STH(2,1)
STH(3,1)=STH(3,1)
STH(4,1)=STH(4,1)
STH(5,1)=STH(5,1)
STH(6,1)=STH(6,1)
STH(7,1)=STH(7,1)

REPEAT TO 18 FOR
j=1,2,...,m

STH(1,1)=STH(1,1)
STH(2,1)=STH(2,1)
STH(3,1)=STH(3,1)
STH(4,1)=STH(4,1)
STH(5,1)=STH(5,1)
STH(6,1)=STH(6,1)
STH(7,1)=STH(7,1)

RETURN
This subroutine determines the double integral matrix for the triangular plate matrix: PZEMMIEK, PAGE 304

$r, s, t$ = Coords. of plate corners in local coordinates

$\rho$ = Density

$p$ = Plate thickness

$P$ = Double integral matrix

**DIMENSIONED VARIABLES**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>5</td>
</tr>
<tr>
<td>$s$</td>
<td>5</td>
</tr>
<tr>
<td>$t$</td>
<td>5</td>
</tr>
</tbody>
</table>
MATRIX INVERSION SUBROUTINE

A = MATRIX TO BE INVERTED
U = INVERTED MATRIX
MN = ORDER OF MATRIX (i.e., n)

DIMENSIONS VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9,9</td>
<td>U</td>
<td>9,9</td>
</tr>
</tbody>
</table>
SUBROUTINE MINY(A, U, NH)

9010 A(NH, J) = A(NH, J) + K(J) + DELT
9012 CONTINUE
9013 REPEAT TO 9033 FOR I = 1, 1 + 1, ..., NH
9030 A(IN, J) = U(IN, J)
9032 EXIT

REPEAT TO 9033 FOR J = 1, 1 + 1, ..., NH

RETURN
EIGEN REDUCES STIFFNESS MATRIX AND INVERTS IT, REDUCES MASS MATRIX
DETERMINES EIGENVALUES AND EIGENVECTORS
THE ARGUMENTS ARE:
A - VECTOR OF LENGTH NRDP*(NRDP+1)/2
VALU - VECTOR OF LENGTH MEIS
TEMP,B,C,DUM3 - VECTORS OF LENGTH NRDP OR NHASS (SMALLER)
E - MATRIX OF DIMENSION (NRDP,3)
IDUM4 - VECTOR OF LENGTH NRDP OR NHASS (SMALLER)
ITAPE,NTAPE, HTAPE, NTAPE. - THESE ARE VARIOUS TAPES
NRDF - NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
MEIS - NUMBER OF EIGENVALUES DESIRED
HEVE - NUMBER OF EIGENVECTORS DESIRED
NHASS - NO. OF NORMAL DISPLACEMENTS
NRHASS - NO. OF ROTATIONAL DEGREES OF FREEDOM
STIFF IS ON HTAPE IN COMPACT FORM
MASS IS ON HTAPE IN COMPACT FORM

DIMENSIONED VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUM3</td>
<td>NRDF</td>
<td>IDUM4</td>
<td>S</td>
<td>A</td>
<td>S</td>
<td>VALU</td>
<td>S</td>
<td>B</td>
<td>S</td>
</tr>
<tr>
<td>C</td>
<td>E</td>
<td>NRDF,S</td>
<td>TEN</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

141
SUBROUTINE EIGEN(A,VALU,TEMP,B,C,DUM3,E,DUM4,TAPE,ITAPE,XTAPE,

5511  5511  CONTINUE  NO  RETURN  NHNZ=NHNZ+1  READ(MEIG,CR,6)  THEN

10  CONTINUE  NO  RETURN  NHNZ=NHNZ+1  READ(MEIG,CR,6)  THEN

70  CONTINUE  NO  RETURN  NHNZ=NHNZ+1  READ(MEIG,CR,6)  THEN

100  CONTINUE  NO  RETURN  NHNZ=NHNZ+1  READ(MEIG,CR,6)  THEN

40  CONTINUE  NO  RETURN  NHNZ=NHNZ+1  READ(MEIG,CR,6)  THEN

WRITE(OUT,9009)  I,DUM3(I),I+1,MEIG)
COORDINates assigns a coord. no. to each degree of freedom at each joint

M1, M2, M3 = arrays containing restraint info. for each degree of freedom at each joint (free=-1, clamped=4)

N1, N2, N3 = coord. no. for each degree of freedom (normal displacements are numbered first)

NITS = no. of joints

NREDU = no. of normal displacements

NDF = total no. of degrees of freedom (including rotations)

DIMENSIONS VARIABLES

Symbol Storages Symbol Storages Symbol Storages Symbol Storages Symbol Storages Symbol Storages
M1 M2 M3 M4 M5 M6 M7 M8 M9 M10

SUBROUTINE COORDIN(M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, NREDU, NDF)

Page 1
SUBROUTINE

THIS SUBROUTINE EVALUATES THE DOUBLE INTEGRALS APPEARING IN THE
EQUATIONS FOR \( a \) AND \( b \) FOR THE TRIANGULAR PLATE ELEMENT

\( x, y, z \) = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

\( n, m \) = POWER OF \( x \) AND \( y \) RESPECTIVELY, PRZEMIECHOCKI, PAGE 383

**DIMENSIONED VARIABLES**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2</td>
<td>B1</td>
<td>7</td>
<td>P1</td>
<td>7</td>
<td>P2</td>
<td>7</td>
</tr>
</tbody>
</table>
FUNCTION DELIMITRE(XS,YH,N,H)

CONTINUE

RETURN

PAGE 2
DIMENSIONED VARIABLES

symbol       storages  symbol  storages  symbol  storages  symbol  storages  symbol  storages
D: 0,1,3

SUBROUTINE DHAT(DX,DY,DI,D2,Y,BETA,B)

START

T10= (COS(BETA)) * K
T11= (COS(BETA)) * K
T12= T11
T13= 2.0 * SIN(BETA) * COS(BETA)
T14= T12

T20= T11
T21= T13
T22= T13
T23= T13

T30= T11
T31= T13
T32= T13
T33= T13

Z1= DT1 + DT2 + D1 + D2
Z2= DT1 + DT2 + D1 + D2
Z3= DT1 + DT2 + D1 + D2
Z4= DT1 + DT2 + D1 + D2

RETURN

PAGE 1

149
TRANS  TRANSFORMATION DIRECTION COSINES
X1,Y1 = COORDS. OF POINT 1
X2,Y2 = COORDS. OF POINT 2
FL = DISTANCE BETWEEN POINTS 1 AND 2
DCS = DIRECTION COSINES OF VECTOR FROM POINT 1 TO POINT 2

DIMENSIONED VARIABLES
SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE SYMBOL STORAGE
DCS 2

SUBROUTINE TRANS(X1, X2, Y1, Y2, FL, DCS)

START
DCS(1) = (X2 - X1) / FL
DCS(2) = (Y2 - Y1) / FL
RETURN

150
DIMMAT

This subroutine determines the double integral matrix for
the $k$ equation for the triangular plate element.

$T_x, T_y, T_z = \text{COORDS. OF PLATE CORNERS IN LOCAL COORDINATES}$

$dx, dy, dz, dV, \beta = \text{FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL}$

PRINCIPAL AXES WRT TRIANGLE LOCAL AXES

$P = \text{DOUBLE INTEGRAL MATRIX}$

DIMENSIONED VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>0,0</td>
<td>$d$</td>
<td>1,1</td>
<td>$d$</td>
<td>1,1</td>
<td>$d$</td>
<td>1,1</td>
</tr>
</tbody>
</table>
CHAT

**THIS SUBROUTINE FORMS THE C MATRIX RELATING THE CORNER DISPLACEMENTS TO THE POLYNOMIAL REFLECTION COEFFICIENTS FOR THE TRIANGULAR PLATE ELEMENT**

\[ y_2, x_3, y_3 = \text{COORDS. OF PLATE CORNERS IN LOCAL COORDINATES} \]

\[ C = C \text{ MATRIX} \]

**DIMENSIONED VARIABLES**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>9,9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

153
PLATE

THIS SUBROUTINE DETERMINES THE STIFFNESS MATRIX OF A
TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.

$X_a, X_b, X_c$ = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

$X, Y, Z = X_{EXT}, 

YET_A = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL

PRINCIPAL AXES W/O TRIANGLE LOCAL AXES

DCS = DIRECTION COSINES

PLTK = STIFFNESS MATRIX

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLTK</td>
<td>9,9</td>
<td>C</td>
<td>9,9</td>
<td>CINV</td>
<td>9,9</td>
<td>P</td>
<td>9,9</td>
</tr>
<tr>
<td>T</td>
<td>9,9</td>
<td>STIFF</td>
<td>9,9</td>
<td>DCS</td>
<td>9,9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PLATE

THIS SUBROUTINE DETERMINES THE MASS MATRIX OF A
TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.

Y2, X3, Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

PHO = DENSITY

PIN = PLATE THICKNESS

DCS = DIRECTION COSINES

PLTM = MASS MATRIX

DIMENSIONS VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLTM</td>
<td>9,9</td>
<td>C</td>
<td>9,9</td>
<td>CINV</td>
<td>9,9</td>
<td>F</td>
<td>9,9</td>
<td>R</td>
<td>9,9</td>
</tr>
<tr>
<td>T</td>
<td>9,9</td>
<td>PMASS</td>
<td>9,9</td>
<td>DCS</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

157
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1MM</td>
<td>0,0</td>
<td>DCS</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**BEAM**
- Plane grid beam element mass matrix in system coords.
- PL = beam length
- RHO = density
- A = cross sectional area
- XI = area moments of inertia
- XJ = effective torsional moment of inertia
- 1MM = mass matrix
- DCS = direction cosines
DIVID

N*NO. OF NORMAL DISPLACEMENTS
N*NO. OF ROTATIONAL D.O.F.
MTE-CONTAINS STIFFNESS (OR MASS) MATRIX
MTE-N12 (N12) STORED
MTE-N11 (N11) STORED

A - CUMM. STORAGE VECTOR, LARGER OF (M*(M+1)/2 OR N*(N+1)/2)

DIMENSIONED VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>B</td>
<td>2</td>
<td>C</td>
<td>1</td>
<td>D</td>
<td>2</td>
</tr>
</tbody>
</table>
I am a dummy vector with storage \( n \) or \( n (\text{larger}) \)

A is a dummy vector with storage \( n(n+1)/2 \) or \( n^2(n+1)/2 \) (larger)

B is a dummy vector with storage \( n \) or \( n(n+1)/2 \) (larger)

C is a dummy vector with storage \( n \) or \( n (\text{larger}) \)

\( m \) = no. of normal displacements

\( N \) = no. of rotational d.o.f.

MPE contains \( K_{11} \) matrix

MPE contains \( K_{12} \) matrix

MPE scratch tape

MPE stores \( K_{12} \times K_{12}^{(+)}(-1) \)

A initially contains \( K_{22}^{-1} \) inverse

*** Reduced stiffness matrix is stored on MPE

---

**Dimensioned Variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Storage</th>
<th>Symbol</th>
<th>Storage</th>
<th>Symbol</th>
<th>Storage</th>
<th>Symbol</th>
<th>Storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>B</td>
<td>1</td>
<td>C</td>
<td>1</td>
<td>D</td>
<td>1</td>
</tr>
</tbody>
</table>
SUBROUTINE ZROMAE(A,B,C,D,H,N,ITYPE,ITYPE,ITYPE,ITYPE)

GO TO 10
IF (KLT.K1) GO TO 12

SUN=0.000

REPEAT TO 88
FOR K=1,1+1,...,N

DPI=0(DR)

OPE=C(KR)

SUM=SUM+DP1+DP2

88

IF (ICH=ICH+1) THEN

SH=SUM

A(ICH)=A(ICH)-SH

CONTINUE

WRITE(ITYPE)(A(I),I=1,NMAX)

RETURN
**ZEORAN**

- M=NO. OF NORMAL DISPLACEMENTS
- M=NO. OF ROTATIONAL D.O.F.
- NIFE CONTAINS M15 MATRIX
- NIFE CONTAINS M12 MATRIX
- NIFE SCRATCH TAPE
- NIFE CONTAINS K1R*K12**(-1)

### REDUCED MASS MATRIX IS STORED ON NIFE

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
</tr>
</tbody>
</table>

166
MAINFPT
MULTIPLIES MATRICES A AND B TO GET C, ALL OF ORDER MxN

DIMENSIONED VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9,9</td>
<td>B</td>
<td>9,9</td>
<td>C</td>
<td>9,9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SUBROUTINE MAINFPT(A,B,C,M)

START

REPEAT TO 10 FOR I=1,1+4,...,M

REPEAT TO 10 FOR J=1,1+4,...,M

C(I,J)=A(I,J)+B(I,J)

RETURN

REPEAT TO 10 FOR K=1,1+4,...,M

C(I,J)=C(I,J)+A(I,K)*B(K,J)

RETURN
DIMENSIONS VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
<th>SYMBOL</th>
<th>STORAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>AiX</td>
<td>1</td>
<td>AiX</td>
<td>1</td>
<td>Bi</td>
<td>5</td>
<td>Ci</td>
<td>1</td>
<td>Di</td>
<td>1</td>
</tr>
</tbody>
</table>

SUBROUTINE LOOP2(AiX,AiX,Bi,Ci,Di,Ip1,Ip1)

START

REPEAT TO Z FOR

AiX(I)+Di(I)+AiX(I)+Bi(I)+AiX(I)+Ip1(I)

RETURN
DIMENSIONED VARIABLES

SYMBOL      STORAGE    SYMBOL      STORAGE    SYMBOL      STORAGE    SYMBOL      STORAGE    SYMBOL      STORAGE
AIIX        I           V           I

SUBROUTINE LOOPS(U1V, AIIX, V, IIp2, MP1)
SUBROUTINE LOOP4(A1I,X,Y,MP1,MP2,UTV)

START

REPEAT TO 4

FOR

K = 1, MP2, ..., MP1

V(K) = V(K) + AIIX(K) * UTV

RETURN

173
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>SYMBOL</th>
<th>SYMBOL</th>
<th>SYMBOL</th>
<th>SYMBOL</th>
<th>SYMBOL</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I</td>
<td>VALU</td>
<td>I</td>
<td>VALL</td>
<td>I</td>
<td>UPPERS</td>
</tr>
<tr>
<td>V</td>
<td>S</td>
<td>T</td>
<td>NH, S</td>
<td>INIER</td>
<td>I</td>
<td>DIA</td>
</tr>
</tbody>
</table>
SUBROUTINE BIGNATH, VALU, VALL, UPPERC, DIAG, Y, T, INTER, MT, N, MEIG, MVEC.

DELGAM = DELGAM + A(J) * VALL(J) -> DGOT = DGELAM

REPEAT TO 8 FOR J = IP1, IP1 + 1, ..., M -> J = J + 2

T(J, J) = VALL(J) * DGOT + A(J, J)

REPEAT TO 9 FOR I = IP1, IP1 + 1, ..., M -> I = I + 2

TRANSFER TO SUBROUTINE LOOP

A(I, J), A(I, I), T(IZ, I), T(I, J), A(J, J), I = I + 1, M

END OF LOOP

DIAG(M) = A(M)

CHORN = MAX(ABS(DIAG) + ABS(UPPERC), ABS(DIAG) + ABS(UPPERC(M)))

REPEAT TO 11 FOR I = 2, 2 + 1, ..., M

CHRKP = ABS(DIAG(I)) + ABS(UPPERC(I)) + ABS(UPPERC(I - 1))

CONTINUE -> CHRKP > CHORN YES

CHORN = CHRKP

REPEAT TO 12 FOR I = 1, 1 + 1, ..., NEIG

VALU(I) = CHORN

REPEAT TO 14 FOR I = 1, 1 + 1, ..., NEIG

GO TO 14

I15

ROOT1 = VALUE(I) + VALL(I)

ROOT 0 = VALU(I) OR ROOT 0 = VALUE(I)

AGREE = 0

PHI(0)

PM01

REPEAT TO 21 FOR J = 1, 1 + 1, ..., N

176
SYNTHV

A is the upper triangle of the symmetric matrix to be inverted.

Elements are stored rowwise.

N = order of matrix

Program inverts in place.

DIMENSIONED VARIABLES

SYMBOL   STORAGES   SYMBOL   STORAGES   SYMBOL   STORAGES   SYMBOL   STORAGES   SYMBOL   STORAGES

A  I
EIGEN

THIS SUBROUTINE FINDS THE EIGENVALUES AND EIGENVECTORS FOR
SYMMETRIC MASS AND STIFFNESS MATRICES.

THE ARGUMENTS ARE--
N- ORDER OF MATRICES.
A- DUMMY VECTOR WITH DIMENSION IN MAIN PROGRAM OF N*(N+1)/2
VALU- STORAGE FOR EIGENVALUES MUST BE DIMENSIONED IN THE MAIN
PROGRAM AS A VECTOR OF LENGTH N*EIG.
TEMP,A,C,E,- DUMMY VECTORS WITH DIMENSION OF N IN MAIN PROGRAM.
E- DUMMY ARRAY WITH DIMENSIONS OF (N,E) IN MAIN PROGRAM.
IDUM- DUMMY INTEGER VECTOR WITH DIMENSION OF N IN MAIN PROGRAM.
MTAPE- TAPE WHERE STIFFNESS MATRIX IS STORED IN COMPACT FORM.
NTAPE- TAPE WHERE MASS MATRIX IS STORED IN COMPACT FORM.
TAPE,ITAPE- SCRATCH TAPES.
EIG- NUMBER OF EIGENVALUES DESIRED.
NEIG- NUMBER OF EIGENVECTORS DESIRED. MUST BE EQUAL TO OR LESS
THAN EIG.

THE MASS AND STIFFNESS MATRICES ARE STORED IN COMPACT FORM AS
VECTORS. ONLY THE UPPER TRIANGLE OF THESE MATRICES (BY ROWS) IS
STORED.

DIMENSIONED VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
<th>SYMBOL</th>
<th>STORAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>TEMP</td>
<td>1</td>
<td>VALU</td>
<td>1</td>
<td>B</td>
<td>1</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>E</td>
<td>n,3</td>
<td>IDUM</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SUBROUTINE EIGENAV, A, VALU, TEMPH, B, C, D, E, IDUM, HTAPE, HFILE, HTAPE, JTAPE,

1. EQ. 1
   NO
   K1=K-1,1=I+1
   A(IND)=SUM
   CONTINUE

   REPEAT TO 699
   FOR
   K1=1,2,3,...,NMAX

   A(IND)=A(IND)-SUM/TEMPh
   REPEAT TO 699
   FOR
   K1=1,2,3,...,NMAX

   TRANSFER TO SUBROUTINE
   SIGMAT
   A, VALU, TEMPH, B, C, D, E, IDUM, HFILE, HTAPE, HFILE

   REPEAT TO 855
   FOR
   I=1,2,3,...,NHEG
   VALU(I)=VALU(I)
   HFILE, ER. D
   NO

   WRITE
   WRITE(OUT,400)
   REPEAT TO 855
   FOR
   I=1,2,3,...,NHEG
   HFILE

186
APPENDIX D

Symbol List
Symbol List

Listed below by their FORTRAN names are some of the input quantities to the program and their equivalent names in Section 3.0.

<table>
<thead>
<tr>
<th>Input Quantity</th>
<th>Symbol in Section 3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>YM</td>
<td>E</td>
</tr>
<tr>
<td>PR</td>
<td>v</td>
</tr>
<tr>
<td>GE</td>
<td>G</td>
</tr>
<tr>
<td>DENS</td>
<td>( \rho )</td>
</tr>
<tr>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>RSMASS</td>
<td>( M_1 )</td>
</tr>
<tr>
<td>AR</td>
<td>A</td>
</tr>
<tr>
<td>XI</td>
<td>I</td>
</tr>
<tr>
<td>YJ</td>
<td>J</td>
</tr>
<tr>
<td>PTH</td>
<td>t</td>
</tr>
<tr>
<td>DX</td>
<td>( D_x )</td>
</tr>
<tr>
<td>DY</td>
<td>( D_y )</td>
</tr>
<tr>
<td>DI</td>
<td>( D_1 )</td>
</tr>
<tr>
<td>DXY</td>
<td>( D_{xy} )</td>
</tr>
<tr>
<td>BETA</td>
<td>( \beta )</td>
</tr>
</tbody>
</table>
THIS STUDY COVERS THE DEVELOPMENT OF A SET OF COMPUTER PROGRAM TO PERFORM FLUTTER ANALYSIS BY THE COLLOCATION METHOD. WHILE THIS METHOD HAS BEEN KNOWN FOR SOME TIME, ONLY RECENTLY HAVE ADVANCES IN COMPUTER TECHNOLOGY MADE THE METHOD TECHNICALLY AND FINANCIALLY FEASIBLE. THE INGREDIENTS OF A COLLOCATION FLUTTER ANALYSIS ARE 1) A FLEXIBILITY MATRIX, 2) AERODYNAMIC INFLUENCE COEFFICIENT MATRIX, AND 3) AN EIGENVALUE SOLUTION. THIS STUDY IS PRESENTED IN FOUR VOLUMES. VOLUME I CONTAINS A GENERAL PROGRAM DISCUSSION. VOLUME II CONTAINS THE PROGRAM FLUENC WHICH CALCULATES THE FLEXIBILITY MATRIX. VOLUME III CONTAINS A SET OF THREE PROGRAMS TO CALCULATE AERODYNAMIC INFLUENCE COEFFICIENTS FOR SUBSONIC, TRANSONIC, AND SUPERSONIC FLIGHT REGIMES. VOLUME IV CONTAINS THE PROGRAM COPA WHICH SETS UP AND SOLVES THE FLUTTER EIGENVALUE MATRIX.