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AN EMPIRICAL EQUATION FOR THE DETERMINATION OF THE MAXIMUM SIDE-ASPECT TARGET STRENGTH OF AN INDIVIDUAL FISH

JANUARY 1969

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ABSTRACT

Experiments are described in which the target strengths of a number of individual fish were measured at various frequencies. The results of these experiments are combined with results from six other sources and an empirical equation approximating the maximum side-aspect target strength of an individual fish is found to be:

\[ T = 24.1 \log L - 4.1 \log \lambda - 33.2, \]

for \( 1 \leq L/\lambda \leq 100 \), where \( T \) is the target strength at one yard in dB, \( L \) is the fish length in feet, and \( \lambda \) is the acoustic wavelength in feet.

This result is combined with a theoretical estimate of the resonance of a fish containing a swimbladder, to produce a curve approximating the maximum side-aspect target strength of an individual bladder-fish for \( L/\lambda \leq 100 \).

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APPROVED FOR RELEASE: ____________
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DATE: 28 January 1969
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INTRODUCTION

Knowledge of the sound reflection characteristics or target strength of underwater targets is important for the effective utilization of active sonar systems. The size of an individual target or group and its reflecting properties are important factors in considering any object as a potential target. Individual or schools of marine organisms represent specific targets for fish-finding sonars in particular, and potential targets for any active sonar system in general, such as an echo-sounder, or military sonar. The target strength of a school of fish depends on the average size, number, distribution and aspect of the individuals in the school. It is therefore of interest to determine the average size and number of individuals in a fish school which are required to produce a given target strength. The minimum size and number of individuals required to produce a given target strength will occur at the aspect for which the target strength of an individual fish is a maximum. This aspect will be near the side-aspect of the fish. Thus, the initial step in the determination of the maximum target strength of a school of fish is the determination of the maximum side-aspect target strength of an individual fish.

Cushing, et al. (1) have stated that the variation of target strength, or acoustic cross-section, of a fish depends on the ratio of fish length (L) to acoustic wavelength (\(\lambda\)). They state that for \(L/\lambda \leq 8\), the acoustic cross-section (\(\sigma\)) of a fish follows Rayleigh's law; that is, \(\sigma\) is proportional to \(\lambda^{-4}\), and that for \(L/\lambda \geq 100\) the reflection will be geometric; that is, \(\sigma\) is proportional to \(\lambda^{-2}\). For the range \(8 \leq L/\lambda \leq 100\) they find that the acoustic cross-sections of different fish of the same size and species can vary by as much as a factor of ten. Haslett's(2) experiments indicate that this mid-range extends down to at least \(L/\lambda = 4\).

Cushing, et al. suggest that this mid-range be avoided in all quantitative measurements. This range, however, is the range of greatest interest for most sonar applications. Therefore, a means to approximate the target strength of an individual fish in this mid-range is of interest.

Several researchers have measured parameters which relate to the target strength of fish, but little has been done to correlate their results. This report discusses experiments conducted by the U. S. Naval Oceanographic Office to determine the maximum side-aspect target strengths of small fish as a function of species, size, and acoustic frequency. The data obtained are combined with all presently available applicable data into a single non-dimensional curve, which can be used to determine the maximum side-aspect target strength of an individual fish.
EXPERIMENTAL METHOD

The experiments were conducted in October 1967 and March 1968 at the Naval Ordnance Laboratory Acoustic Facility (NOLAF), Brighton, Maryland. This facility is fully described in reference 3. The primary purpose of these tests was to determine the target strengths of individual live fish at various frequencies.

A live fish was anesthetized and mounted with monofilament line on a rotatable frame whose axis of rotation was one yard from the transmitting and receiving transducers. The transducers were either a pair of QBG or EDO 327 transducers.

After the fish was mounted, the apparatus was lowered into the water until the fish was slightly below the surface. At this point all bubbles were removed from the fish by directing a stream of water over its body and into its mouth and gills. The apparatus was then lowered to the test depth of 20 feet and an electric shocker was activated to clear the area of unwanted fish. A diagram of the experimental set-up is shown in Figure 1.

The mounted fish was slowly rotated about its dorsiventral axis and insonified with a 0.4 msec long acoustic pulse. A signal gate was used to eliminate all received signals except the echo from the test fish. This echo was recorded on a polar plotter which was synchronized to rotate with the frame on which the fish was mounted. Two typical polar plots are shown in Figure 2. Interference caused by the frame invalidated the data near the head and tail aspects of the fish. The polar plots obtained have been used only to determine the maximum side-aspect target strengths of the fish.

The fish tested ranged in length from 2 to 8.1 inches, and were insonified with as many as eight different acoustic frequencies which ranged from 20 to 130 kHz. The species tested were Fundulus diaphanus (banded killifish), Notemigonus crysoleucas crysoleucas (eastern golden shiner), Pomoxis nigromaculatus (black crappie), Pomoxis annularis (white crappie), and Perca flavescens (yellow perch).

Because the available test time was not sufficient to calibrate the entire experimental acoustic system, the target strength determinations were made utilizing an indirect calibration procedure incorporating reference targets. Calibration of the experimental system was accomplished by substituting two hollow reference spheres for the fish and comparing the level of the target fish to the level of the reference target. The spheres were 1.5 and 2.25 inches in diameter. The measured relative amplitude values obtained from the two spheres were averaged for each frequency. Setting these average values equal to the theoretical target strength, referenced to one yard, of a sphere whose diameter
FIGURE 2. TARGET STRENGTH OF A POMOXIS NIGROMACULATUS (BLACK CRAPPIE) VS. ASPECT
is equal to the geometric mean of the diameters of the two reference spheres resulted in the determination of scale values for the mean sphere at each frequency utilized, and enabled absolute target strengths for the fish to be calculated.

The speed of sound in water was assumed to be 5000 ft/sec, although the experiments were conducted in fresh water. The error involved is insignificant in the final result.

PRESENT RESULTS

The results of previous target strength experiments have been presented in various ways, using both dimensional and non-dimensional parameters. In order to compare data the non-dimensional presentation is obviously better. Of all earlier researchers only Haslett (2) has presented non-dimensionalized data.

Figure 3 shows the present data plotted using Haslett's non-dimensional parameters of $\sigma/L^2$ and $L/\lambda$, where $L$ is the fish length, $\lambda$ the acoustic wavelength, and $\sigma$ the acoustic cross-section of the fish. It can be seen that there are no obvious conclusions that can be drawn from this presentation. Examination of Figures 3 and 4 of reference 2 also shows the same wide scatter in previous data plotted in this fashion.

A small modification of Haslett's parameters, replacing $\sigma/L^2$ by $\sigma/\lambda^2$, results in a significant improvement in the presentation of the data. Figure 4 shows that $\sigma/\lambda^2$ definitely increases with increasing $L/\lambda$. Using the method of least squares, the regression line through the data in Figure 4 is

$$\frac{\sigma}{\lambda^2} = 0.12 \left(\frac{L}{\lambda}\right)^{2.01},$$

with a correlation coefficient of 0.92 for the 112 data points.

Using the data of Figure 4, the regression lines and standard errors of estimate were determined for each of the four families in the three orders (Cyprinodontiformes, Perciformes, and Cypriniformes) of fish which were tested. It was found that the variation in the data for one family was of the same order of magnitude as the variations among the families.

Figure 5 shows $\sigma/\lambda^2$ vs. $L/\lambda$ curves for representative fish of each of the five species tested in the present experiments. It is seen that the curve for each fish follows, in general, the regression line of Figure 4. This is true for all fish tested. Hence, this regression line is an actual approximation of the acoustic cross-sections of specimens representing three different orders of fish and is not just an average of a number of widely divergent curves.
FIGURE 3. $\sigma/L^2$ VS. $L/\lambda$ FOR THE PRESENT DATA
\[
\sigma / \lambda^2 = 0.12 (L / \lambda)^{2.01}
\]

**Figure 4.** \(\sigma / \lambda^2\) vs. \(L / \lambda\) for the present data

- **Fundulus Diaphanus** (Banded Killifish)
- **Pomoxis Annularis** (White Crappie)
- **Pomoxis Nigromaculatus** (Black Crappie)
- **Perca Flavescens** (Yellow Perch)
- **Notemigonus Crysoleucas** (Eastern Golden Shiner)
FIGURE 5. $\frac{\sigma}{\lambda^2}$ vs. $L/\lambda$ for fish representative of the five species tested
PREVIOUS RESULTS

A study has been made of all available data on target strengths of fish. All known target strength data for complete individual fish have been examined and all data for which fish length, maximum side-aspect target strength or acoustic cross-section, and insonifying frequency could be determined have been converted into the parameters of $\frac{\sigma}{\lambda^2}$ and $L/\lambda$. Any data which were referenced to one meter were corrected to one yard. Pertinent data were obtained from six sources: Volberg (4), Hashimoto and Maniwa (5), Yudanov, Gan’kov, and Shatoba (6), Haslett (2, 7), Smith (8), and Jones and Pearce (9).

Volberg (4) measured target strength as a function of aspect for skipjack and yellowfin tunas, yellowtails, crappie, and large mouth bass at frequencies of 20, 40, 50 and 280 kHz. The fish were 13 to 32 inches long. Using the method of least squares, the regression line through Volberg's 70 data points is

$$\frac{\sigma}{\lambda^2} = 0.22 \frac{(L/\lambda)^2}{L/\lambda}$$

with a correlation coefficient of 0.96.

Hashimoto and Maniwa (5) measured the reflection loss of mackerel, beryx, and plaice at frequencies of 28, 100, 200, 300 and 400 kHz. The fish were 8 to 15 inches long. The regression line through their 31 data points is

$$\frac{\sigma}{\lambda^2} = 0.0088 \frac{L/\lambda^2}{L/\lambda}$$

with a correlation coefficient of 0.97.

Yudanov, et al. (6) measured the acoustic cross-sections of flounder, 6 to 12 inches long, at 30 kHz. The regression line through their 44 data points is

$$\frac{\sigma}{\lambda^2} = 0.032 (L/\lambda)^2$$

with a correlation coefficient of 0.81.

Haslett (2, 7) measured the acoustic cross-sections of sticklebacks, guppies, and minnows at frequencies of 360, 625, and 1480 kHz. The fish were 0.4 to 2.5 inches long. The correlation coefficient for 31 data points from reference 7 is 0.93, whereas for 67 data points from reference 2 the correlation coefficient is 0.65. The regression line through all 98 data points is
\[
\sigma/\lambda^2 = 0.0033 (L/\lambda)^3.19,
\]
with a correlation coefficient of 0.88.

Smith (8) measured the target strength of a sea bass whose length was calculated for the present study to be approximately 8 inches. Fifteen data points were obtained at frequencies ranging from 13 to 27 kHz, and the regression line for these data is

\[
\sigma/\lambda^2 = 0.047 (L/\lambda)^2.69,
\]
with a correlation coefficient of 0.91.

Jones and Pearce (9) measured the target strengths of perch, 7.5 to 9 inches long, as a function of aspect at a frequency of 30 kHz. Only 5 data points were obtained, which is an insufficient number to calculate a regression line of any significance.

DISCUSSION

Combining all data and utilizing the method of least squares, the equation for the regression line through all 375 data points is

\[
\sigma/\lambda^2 = 0.054 (L/\lambda)^2.41,
\]
with a correlation coefficient of 0.93. This result is shown in Figure 6.

Figure 6 shows that the results of all seven experimenters are in good agreement. Haslett's data show some deviation from the other data near \(L/\lambda = 10\), but this is not too surprising inasmuch as the data from reference 2 has the lowest correlation coefficient of all the data.

Figure 6 is based on results obtained using fish from eleven families in six different orders; Perciformes, Cypriniformes, Cyprinodontiformes, Pleuronectiformes, Beryciformes, and Gasterosteiformes. The fish ranged in length from under one inch to almost one yard and were insonified with frequencies ranging from 13 to 1480 kHz.

Some of the fish tested had swimbladders, while others did not. Volberg (4) found that for tuna of the same size, the presence or absence of a swimbladder made little difference in target strength. Jones and Pearce (8) estimated that the swimbladder accounts for about half the target strength of perch. Obviously a change of 50 per cent in the magnitude of any single data point in Figure 6 would not be noticeable. Therefore, although the presence of a swimbladder is
\[ \frac{\sigma}{\lambda^2} = 0.054 (L/\lambda)^{2.41} \]

**FIGURE 6.** \( \frac{\sigma}{\lambda^2} \) VS. \( \frac{L}{\lambda} \) FOR ALL PERTINENT DATA
very important in studies connected with the fish of the deep scattering layer, it is of little significance in the detection of specific biological targets. This is because the resonating swimbladders of small fish account for most of the sound scattering caused by the deep scattering layer, whereas the $L/\lambda$ ratios of interest in many sonar applications are much greater than those required for resonance.

Equation 7 may be converted into target strength using the equation

$$
T = 10 \log \left( \frac{\sigma}{4\pi} \right),
$$

where $\sigma$ is in square yards and $T$ is the target strength at one yard. The target strength of an individual fish is, therefore, from equations 7 and 8,

$$
T = 24.1 \log L - 4.1 \log \lambda - 33.2,
$$

where $L$ and $\lambda$ are in feet. It is seen from the data of Figure 6 that this equation is valid in the range of $1 \leq L/\lambda \leq 100$.

The fish utilized in the determination of equations 7 and 9 were what might be called "regularly-shaped" fish; that is, fish with body depth ($d$) to length ratios of about one to three or one to four. Hence, the target strengths of elongated fish, such as eels, or truncated fish, such as the ocean sunfish, would not be accurately approximated by equation 9. Replacing the parameter $L/\lambda$ with one which reflects the area of a fish, such as $\sqrt{d \times L}/\lambda$, would enable the target strengths of all fish to be approximated more accurately. Obviously, the parameter $L/\lambda$ is much more convenient than $\sqrt{d \times L}/\lambda$, and for the majority of fish this convenience far outweighs any inaccuracies in the results.

It is possible to estimate the variation of $\sigma/\lambda^2$ with $L/\lambda$ for a fish with a swimbladder for values of $L/\lambda < 1$. A variation of Andreeva and Chindova's (10) equation for the resonant frequency ($f_0$) of a fish swimbladder is

$$
f_0 = 1.8 \left( \frac{D + 100}{R} \right)^{1/2},
$$

where $R$ is the equivalent spherical radius of the swimbladder in feet, $D$ is the depth in feet, and $f_0$ is in Hertz. From Haslett (11) the relation of $R$ to $L$ can be given as

$$
R \approx 5 \times 10^{-2} L.
$$
Combining equations 10 and 11 and assuming the speed of sound in water to be 5000 ft/sec,

\[(L/\lambda_o) \approx \left(\frac{D + 100}{140}\right)^{1/2}\]  

(12)

Assuming \(D = 20\) ft, which is the depth at which the present experiments were conducted, \(L/\lambda_o = 0.08\).

Andreeva and Chindova also give an equation for the acoustic cross-section of a resonating swimbladder, which can be modified using equation (11) to give

\[
\frac{\sigma_o}{\lambda_o^2} = \pi \times 10^{-2} Q^2 \left(\frac{L}{\lambda_o}\right)^2,
\]

(13)

where \(Q\) is the quality factor of resonance, having a value of about 5 at \(D = 20\) ft. Therefore, for \(L/\lambda_o = 0.08\), \(\sigma_o/\lambda_o^2 = 5 \times 10^{-3}\). Assuming that the \(\sigma/\lambda^2\) vs. \(L/\lambda\) curve is symmetrical near the swimbladder resonance peak, the one-half power points can also be determined.

For frequencies somewhat below and above the resonance peak, the lower limit of a fish's maximum acoustic cross-section can be approximated by a spherical bubble equal in volume to the fish's swimbladder. For frequencies less than the resonant frequency, a bubble follows Rayleigh's law of scattering. For frequencies greater than the resonant frequency, the acoustic cross-section of a bubble is equal to its geometric cross-section.

By utilizing the resonance peak, the bubble curves, and equation 7, it is possible to plot a curve which approximates the acoustic cross-section of a bladder-fish for \(L/\lambda \leq 100\). This curve is shown in Figure 7 for the values of \(Q\) and \(R/L\) which were assumed. The dotted portions of this curve are fairied and the segment between \(10^{-1} \leq L/\lambda \leq 5 \times 10^{-1}\) could easily be drawn differently. Figure 7 enables one to estimate the target strength of an individual fish for all values of \(L/\lambda \leq 100\). It seems probable that the region of Rayleigh scattering for bladder-fish begins somewhere between \(10^{-2} \leq L/\lambda \leq 10^{-1}\), and not for values of \(L/\lambda > 1\), as stated by Cushing, et al. (1).

Figure 7 shows that for a given wavelength, a large, non-resonant fish can have a greater acoustic cross-section than a smaller, resonant fish. By replotting these curves in the manner shown in Figure 8, it can be seen that the acoustic cross-section of a specific size fish is probably greater at the resonant frequency than at any other frequency for which \(L/\lambda \leq 100\).
$\frac{\sigma}{\lambda}^2 = 0.054 (\frac{L}{\lambda})^{2.41}$

Fish with resonant swimbladder

Bubble; geometric scattering

Bubble; Rayleigh scattering

**Figure 7.** $\frac{\sigma}{\lambda}^2$ vs. $\frac{L}{\lambda}$ for a bladder-fish
Figure 8. $\frac{\sigma}{L^2}$ vs. $L/\lambda$ for a bladder-fish
CONCLUSIONS

The target strength of an individual fish is quite variable in the $L/\lambda$ range of interest for most sonar applications; however, an empirical equation has been developed which approximates the maximum side-aspect target strength of an individual fish within this range. This equation was developed using the non-dimensionalized results of experiments conducted by the U. S. Naval Oceanographic Office in conjunction with those of six other sources.

An estimate of the acoustic cross-section of an individual bladder-fish at $L/\lambda$ values below the range of experimental data has been combined with the regression line through the data to produce an estimate of the maximum side-aspect target strength of an individual bladder-fish for all values of $L/\lambda \leq 100$. 

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REFERENCES


Experiments are described in which the target strengths of a number of individual fish were measured at various frequencies. The results of these experiments are combined with results from six other sources and an empirical equation approximating the maximum side-aspect target strength of an individual fish is found to be:

$$T = 24.1 \log L - 4.1 \log \lambda + 33.2,$$

for $1 \leq L/\lambda \leq 100$, where $T$ is the target strength at one yard in dB, $L$ is the fish length in feet, and $\lambda$ is the acoustic wavelength in feet.

This result is combined with a theoretical estimate of the resonance of a fish containing a swimbladder, to produce a curve approximating the maximum side-aspect target strength of an individual bladder-fish for $L/\lambda \leq 100$. 
AN EMPIRICAL EQUATION FOR THE DETERMINATION OF THE MAXIMUM SIDE-ASPECT TARGET STRENGTH OF AN INDIVIDUAL FISH