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AUTHORITY
USNWC ltr 24 Mar 1972
ABSTRACT. A study is made of the use of the horizontal dipole as a VLF transmitting antenna. Two types of multiconductor-loaded dipoles are considered: (1) an off-center-fed dipole one wavelength long, with wave velocity compensation; and (2) a center-fed dipole about 0.2 wavelength long, end-loaded with unterminated radials. Theoretical equations are derived to determine the efficiency, beamwidth, bandwidth, maximum power radiating capability, and maximum antenna voltages, over a frequency range of 10-20 kHz and an earth conductivity range of $10^{-3}$ to $10^{-4}$ mho/meter. Measurements made on a 6-km experimental antenna constructed in Hawaii confirm the equations for radiation efficiency and bandwidth.
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FOREWORD

The work described in this report was conducted in the Electronics Division, Research Department, from November 1967 through February 1968, in connection with the development of a survivable submarine-communication system. Funding was provided by the Naval Electronics Systems Command (PME 117-22) under P. O. 8-0553.

Released by
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May 1968

Under authority of
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INTRODUCTION

The very low frequency (VLF) research program at the Naval Weapons Center Corona Laboratories has shown that there are certain advantages in the design and construction of horizontal-dipole transmitting antennas, from the standpoint of both economy and efficiency (Ref. 1-4). The purpose of this report is to present theoretical and experimental data that will provide the basis for selecting the optimum horizontal-dipole transmitting antenna to operate in a practical environment.

The performances of two types of horizontal dipoles are considered: (1) an off-center-fed dipole one wavelength long, with wave velocity compensation, and (2) a center-fed dipole about 0.2 wavelength long, end-loaded with unterminated radials. Both types are self-resonant and need no tuning networks; both types are unterminated and require no ground plane.

Theoretical equations are derived to determine the radiation efficiency, maximum antenna voltage, power-radiating capability, bandwidth, and beamwidth of the antennas, over a frequency range of 10-20 kHz and an earth conductivity range of $10^{-3}$ to $10^{-4}$ mho/meter. Experimental data are then presented of measurements made on a short, end-loaded horizontal dipole constructed in Hawaii, to confirm the theoretical performance in terms of radiation efficiency and bandwidth and to establish the relatively low cost of this type of antenna.

THEORETICAL PERFORMANCE OF A HORIZONTAL DIPOLE ONE WAVELENGTH LONG

Consideration is first given to a representative dipole one wavelength long, elevated 10 meters above the earth, and made up of six No. 12 wires in a cage. The dipole is loaded at every kilometer with a series capacitor to bring the wave velocity up to the velocity of light, where the wave antenna has been found to have the greatest efficiency.

By assuming different effective diameters of the cage of No. 12 wires, two values of characteristic impedance can be considered.
A $Z_0$ of 200 $\Omega$ results from using a cage effective diameter of 1.482 meters. A smaller cage yields a characteristic impedance of 400 $\Omega$. The amount of series capacitance used depends upon the resonant frequency. For $Z_0 = 200 \Omega$, 0.45 $\mu$F-km is needed at 10 Hz, 0.215 $\mu$F-km is needed at 15 Hz, and 0.13 $\mu$F-km is needed at 20 Hz to bring the wave velocity up to that of free space.

By using the physical dimensions of the antenna and the conductivity of the earth under the antenna, the parameters required to evaluate the dipole performance—such as $Z_0$, the attenuation constant $\alpha \lambda$, and $c/v$, the ratio of free-space wave velocity to that along the dipole—can be computed from the following equations (Ref. 3):

$$Z_0 = \frac{2\pi \frac{c}{v} - j\alpha \lambda}{2\pi C \frac{c}{p}}$$  \hspace{1cm} (1)

$$\alpha \lambda = \frac{c}{f} \sqrt{\omega r C_p} \left[1 + \frac{\omega^2}{Q^2} \cos \left(\frac{\omega}{2} \arctan \left(\frac{1}{Q} \right)\right)\right]$$  \hspace{1cm} (2)

$$\frac{c}{v} = \frac{c}{\omega} \sqrt{\omega r C_p} \left[1 + \frac{\omega^2}{Q^2} \sin \left(\frac{\omega}{2} \arctan \left(\frac{1}{Q} \right)\right)\right]$$  \hspace{1cm} (3)

where $Q = \omega L/r$ and the angle $\arctan (-1/Q)$ is in the second quadrant. Here the antenna series inductance and resistance per unit length are computed from equations derived by Carson (Ref. 5):

$$L = 2 \times 10^{-7} \ln \frac{710}{a \sqrt{a}} \text{ (H/m)}$$  \hspace{1cm} (4)

$$r = \pi f \times 10^{-7} \left(1 + \frac{R_m}{R_s}\right) + R_w \text{ (}\Omega/\text{m})$$  \hspace{1cm} (5)

where $R_m/R_s$ is taken from the curve in Fig. 1 for a certain ratio of dipole separation to skin depth.

The capacitance to earth is obtained from the solution of LaPlace's equation and is
FIG. 1. Ratio of Mutual Impedance to Self Impedance for Two Parallel Conductors Near Earth.
where $h \gg a$.

**EFFICIENCY**

An expression for the efficiency of a single unterminated dipole has been derived in Eq. 57 of Ref. 3. When the dipole is resonant and fed one-fourth the length from one end, the efficiency is

$$\eta = \frac{16\pi^3 f C \cos^2 \theta}{3 \sigma \left( 2\pi \frac{\sigma}{\nu} - j \alpha \lambda \right) \left( \tanh \alpha l_{1\lambda} + \tanh \alpha l_{2\lambda} \right)}$$

where $l_1 = 3/4$ and $l_2 = 1/4$ of the dipole length. This efficiency is derived by equating the ground-wave radiated field strength of a perfect, short, vertical antenna with that of the horizontal dipole (Ref. 3, p. 14).

The radiation efficiency of a single dipole, as described above, is plotted as a function of earth conductivity in Fig. 2 for several VLF frequencies and two values of characteristic impedance ($Z_0$). The radiation efficiency is not affected greatly by the characteristic impedance for the values shown. However, as $Z_0$ becomes small and more complex, it will affect the efficiency more. It is fortunate that this has little effect on the maximum antenna voltage since low characteristic impedance results in low antenna voltage and high power radiating capability. The dipole efficiency rises very rapidly for low earth conductivity; this is to be expected since $\sigma$ occurs in the denominator of Eq. 7.
FIG. 2. Radiation Efficiency of a Long Horizontal Dipole.
and, in addition, most of the antenna parameters are to some extent dependent upon \( \sigma \). The efficiency rises quite rapidly with frequency in view of the fact that the antenna is twice as long, physically, at 10 kHz as it is at 20 kHz.

The efficiency plotted in Fig. 2 is for one dipole, with consideration given for the mutual resistance induced into it by parallel dipoles on either side at a distance of 1 km. Therefore, for an array of "n" dipoles spaced 1 km apart, the effective efficiency for comparison with a perfect vertical antenna is "n" times the efficiency shown in Fig. 2.

MAXIMUM ANTENNA VOLTAGE

The maximum voltage on the resonant wave antenna one wavelength long occurs at a distance of \( \lambda/4 \) from the feed point and is easily computed from transmission line theory (Ref. 6) as

\[
V_{\text{max}} = \frac{I_{\text{in}} Z_0}{\cosh \frac{a\lambda}{4}}
\]

The input current may be written in terms of the radiated power, characteristic impedance, efficiency, and attenuation constant along the antenna as

\[
I_{\text{in}} = \sqrt{\frac{P_{\text{in}}}{R_{\text{in}}}}
\]

\[
P_{\text{in}} = \frac{P_r}{\eta}
\]

and the input resistance at resonance as

\[
R_{\text{in}} = Z_0 \left( \tanh \frac{a\lambda}{4} + \tanh \frac{3}{4} a\lambda \right)
\]

Therefore

\[
I_{\text{in}} = \sqrt{\frac{P_r}{\eta Z_0 \left( \tanh \frac{a\lambda}{4} + \tanh \frac{3}{4} a\lambda \right)}}
\]
By combining Eq. 8 and 11, the maximum voltage on a wave antenna fed \( \lambda/4 \) from one end is found to be

\[
V_{\text{max}} = \sqrt{\frac{P_r Z_0}{\eta \left( \tanh \frac{\alpha \lambda}{4} + \tanh \frac{3}{4} \alpha \lambda \right)}} \cosh \frac{\alpha \lambda}{4}
\]

(12)

The maximum voltage on the dipole antenna for 1 MW of effective radiated power from 60 dipoles is plotted in Fig. 3 as a function of earth conductivity for various VLF frequencies and two values of characteristic impedance. The power radiated from each dipole is only 278 W. The efficiencies from Fig. 2 and the attenuation constants computed with Eq. 2 are used in Eq. 12 to compute the maximum antenna voltage. The physical configuration of the antenna is the same as the representative dipole described earlier. The curves in Fig. 3 indicate that the characteristic impedance should be as small as possible without deteriorating the efficiency. The lowest characteristic impedance considered was 200 \( \Omega \). In cases where it is feasible to lay the antenna directly on the ground, the antenna voltage can be reduced considerably by using a low characteristic impedance. The use of higher frequencies and lower earth conductivities results in low antenna voltage because of the increase in dipole efficiency. Even the higher antenna voltages in Fig. 3 are well below the corona onset voltage.

POWER RADIATING CAPABILITY

The maximum effective power radiated by a group of dipoles one wavelength long can be computed from an equation obtained by rearranging Eq. 12:

\[
P_{r,\text{max}} = \left( N \eta V_c \cosh \frac{\alpha \lambda}{4} \right) \frac{2 \eta \left( \tanh \frac{\alpha \lambda}{4} + \tanh \frac{3}{4} \alpha \lambda \right) Z_0}{2 \eta \left( \tanh \frac{\alpha \lambda}{4} + \tanh \frac{3}{4} \alpha \lambda \right) Z_0}
\]

(13)

where \( N \) = the number of dipoles and \( V_c \) = the maximum antenna voltage before the onset of corona. This voltage was computed from an equation derived by Smith and Gustafson (Ref. 7). At an altitude of 5,000 ft above sea level, corona onset voltage is approximately 35,000 V. As shown
FIG. 3. Voltage on 60 Long Horizontal Dipoles Radiating 1 MW of Effective Power.
in Fig. 4, 60 dipoles have the capability to radiate several hundred megawatts of effective power. Nearly one billion watts of effective power can be radiated at 20 kHz with an earth conductivity of $10^{-4}$ mho/meter (see Fig. 4). Of course, since each dipole is only 5% efficient, this would require 5 MV of input power to each dipole. Only one value of characteristic impedance (200 - j25) is considered in Fig. 4.

**BANDWIDTH**

The bandwidth of a dipole one wavelength long is derived from the input impedance. The dipole input impedance is the sum of the input impedance of each portion of the antenna on either side of the feed point treated as an open-terminated transmission line. The equation is written:

$$Z_{in} = Z_0 \left[ \text{coth} \left( \omega \lambda l_1 + j\frac{2\pi}{v} f_1 \lambda \right) + \text{coth} \left( \omega \lambda l_2 + j\frac{2\pi}{v} f_2 \lambda \right) \right]$$

where $Z_0$ and $\omega \lambda$ are computed from Eq. 1, 2, and 3, and $l_1$ and $l_2$ are the dipole lengths on either side of the feed point. The input impedance of the representative dipole is plotted on Smith charts in Fig. 5, 6, and 7. In all cases, $\sigma = 5 \times 10^{-4}$ $\mu$m. The dipole length is changed and the series loading capacitance value is changed to resonate the dipole at 10, 15, and 20 kHz. The input impedance is first computed over a ±20% frequency range about each of the resonant frequencies. It is then normalized to the resonant resistance and plotted on the Smith chart to obtain the half-power bandwidth with a generator whose internal impedance is equal to the resonant resistance. For the characteristic impedance chosen (200 - j25 at resonance), the half-power bandwidth is much greater than 40%. The 1.3 dB power bandwidth is about 40% for all three dipoles.

The input impedance for the three dipoles of different lengths, resonating at 10, 15, and 20 kHz, are plotted as a function of frequency in Fig. 8, 9, and 10. In addition, $c/v$, $\omega \lambda$, and $Z_0$ are plotted in the same figures. These parameters are nearly constant over the VLF band if no series capacitance is used in the antenna. In this case, however, where series capacitance is used to increase the wave velocity and thereby increase the efficiency, the parameters vary over the frequency band, as shown in Fig. 8, 9, and 10.
FIG. 4. Power-Radiating Capability of 60 Long Horizontal Dipoles Before Corona Onset.

ALTITUDE = 5000 FT ABOVE SEA LEVEL
V_max = 35 KV
Z_0 = 200 - j25 Ω
DIPOLE SEPARATION = 1 KM
FIG. 5. Input Impedance of 30-km Horizontal Dipole.
FIG. 6. Input Impedance of 20-km Horizontal Dipole.
FIG. 7. Input Impedance of 15-km Horizontal Dipole.
FIG. 8. Antenna Parameters for 30-km Horizontal Dipole.
FIG. 9. Antenna Parameters for 20-km Horizontal Dipole.
FIG. 10. Antenna Parameters for 15-km Horizontal Dipole.
RADIATION PATTERNS

The power radiation pattern is computed with Eq. 7, where $\theta$ is the angle in the horizontal plane off the long end of the dipole. Increasing the wave velocity along the antenna broadens the beamwidth, as may be seen in Fig. 11. An antenna one wavelength long with $c/v = 1.0$ yields an 80° half-power beamwidth. The beam may be flattened on top and broadened by using higher antenna wave velocities (lower values of $c/v$); see Fig. 11 where $c/v = 0.9$. When many parallel dipoles are used in an array, the dip near $\theta = 0^\circ$ can be used to compensate for the array beam factor, which is maximum at $\theta = 0^\circ$. This compensating effect could give a flat main beam 40° wide for an array of dipoles.

The patterns in Fig. 11 are for all three of the resonant frequencies—10, 15, and 20 kHz—and the efficiency is normalized for this purpose. Off resonance the patterns would change, but the change would be slow because $c/v$ varies quite slowly with frequency (see Fig. 8, 9, and 10).

COST

The cost of an antenna depends in part upon the terrain over which the antenna is constructed. One total cost estimate, including construction, materials, and right-of-way, is $11,800 per mile in flat farming areas, $16,000 per mile in rolling-hill-type terrain, and $24,800 per mile in the mountains. These figures, obtained from Edwin Devaney of the Naval Electronics Laboratory Center, provide for three No. 000 copper cables on a cross arm supported by poles 40 ft high and spaced 300 ft apart. However, such large conductors are not needed for the horizontal dipole, since the antenna current is only 40 A and the skin depth in copper at VLF frequencies is so small that most of the copper in a No. 000 wire would be wasted. For the VLF range, No. 12 wire gives optimum copper usage. The dipole considered uses six No. 12 wires, which is 25 lb per mile. (Three No. 000 wires weigh 8,040 lb per mile.) The difference between the cost of the No. 000 wire and the No. 12 wire, assuming copper wire costs $1.00 per lb, is $7,400 per mile. The total antenna cost, based on six No. 12 copper-weld wires, is then $7,400 per mile in flat farm land, $8,600 per mile in rolling-hill-type terrain, and $17,400 per mile in the mountains; these prices include the right-of-way. The estimated cost of an array of dipoles under various conditions is shown in Table 1.
FIG. 11. Radiation Patterns of a Long Horizontal Dipole, in the Horizontal Plane.
TABLE 1. Estimated Cost of Horizontal Dipole Antenna
One Wavelength Long.

<table>
<thead>
<tr>
<th>Unit cost</th>
<th>Flat farmland</th>
<th>Rolling hills</th>
<th>Mountains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per mile</td>
<td>$ 4,400</td>
<td>$ 9,600</td>
<td>$ 17,400</td>
</tr>
<tr>
<td>Per 10 kHz dipole</td>
<td>82,000</td>
<td>161,000</td>
<td>325,000</td>
</tr>
<tr>
<td>Per 15 kHz dipole</td>
<td>55,000</td>
<td>107,000</td>
<td>216,000</td>
</tr>
<tr>
<td>Per 20 kHz dipole</td>
<td>40,000</td>
<td>78,000</td>
<td>157,000</td>
</tr>
<tr>
<td>10 kHz Dipole array, 50% efficient</td>
<td>$14 million</td>
<td>$14.5 million</td>
<td>$7.1 million</td>
</tr>
<tr>
<td>$ = 10^-3</td>
<td>170 dipoles</td>
<td>55 dipoles</td>
<td>22 dipoles</td>
</tr>
<tr>
<td>$ = 5 \times 10^{-4}</td>
<td></td>
<td></td>
<td>10^-4</td>
</tr>
<tr>
<td>15 kHz Dipole array, 50% efficient</td>
<td>$6.5 million</td>
<td>$6 million</td>
<td>$3 million</td>
</tr>
<tr>
<td>$ = 10^-3</td>
<td>119 dipoles</td>
<td>55 dipoles</td>
<td>14 dipoles</td>
</tr>
<tr>
<td>$ = 5 \times 10^{-4}</td>
<td></td>
<td></td>
<td>10^-4</td>
</tr>
<tr>
<td>20 kHz Dipole array, 50% efficient</td>
<td>$3.3 million</td>
<td>$2 million</td>
<td>$1.57 million</td>
</tr>
<tr>
<td>$ = 10^-3</td>
<td>83 dipoles</td>
<td>25 dipoles</td>
<td>10 dipoles</td>
</tr>
<tr>
<td>$ = 5 \times 10^{-4}</td>
<td></td>
<td></td>
<td>10^-4</td>
</tr>
</tbody>
</table>
The efficiency of the horizontal dipole one wavelength long can be improved about 30% by replacing the \( \lambda/4 \) portion (\( \ell_2 \) in Eq. 7) with tuning radials. When \( \theta = 0^\circ \) (off the end of the dipole), this short portion of the antenna contributes a very small amount to the efficiency (see the second term in the brackets of Eq. 7), and its presence adds 33% to the input resistance, which reduces the efficiency by that much. Five parallel tuning radials would have much less input resistance and would also have a shorter total physical length, thus reducing construction costs. The performance of this type of dipole may be analyzed at a later date.

**THEORETICAL PERFORMANCE OF A SHORT HORIZONTAL DIPOLE LOADED WITH END RADIALS**

A shorter, end-loaded, center-fed dipole is now considered as a means of obtaining greater efficiency per unit length, with less bandwidth. The same physical configuration as the long dipole is used except that the short dipole is only 6 km long and is loaded on the ends with 5 radials (see Fig. 12). The length of the radials determines the resonant frequency. The equations for the antenna parameters, Eq. 1-6, apply equally well for this type of dipole.

**EFFICIENCY**

The efficiency of this short antenna is enhanced by a nearly uniform current distribution since the length is much less than a wavelength (approximately 0.2 \( \lambda \)). The efficiency of an incremental length dipole near the earth, derived in Eq. 52 of Ref. 3, is

\[
\eta = \frac{160\pi^2 \omega_0 \cos^2 \theta}{\sigma_R \sin \frac{1}{\lambda} \left( \frac{I_{d}}{I_{in}} \right)^2}
\]  

(15)

The current moment of the short dipole, \( I_{d}/I_{in} \), is actually a damped cosine function which, when integrated over the length of the dipole and substituted into Eq. 15, gives the following equation for the dipole efficiency:
FIG. 12. Radiation Efficiency of a Short Horizontal Dipole.
\[ \eta = \frac{3.508 \times 10^{-7} l_{a\lambda}}{\sigma_{\text{oin}}} \left( \frac{\sin 2\pi \frac{C}{v_{a}} l_{a\lambda}}{2\pi \frac{C}{v_{a}} l_{a\lambda}} \right)^2 \exp \left( -\frac{\alpha_{a}}{\frac{C}{v_{a}} l_{a\lambda}} \right) \cos^2 \theta \] (16)

where \( l_{a\lambda} \) is the half-length of the dipole.

The input impedance must be computed at resonance to obtain the efficiency. The equation for the input impedance of an end-loaded dipole, from transmission line theory, is

\[ Z_{\text{in}} = \frac{2Z_{oA} \left[ Z_{r} + Z_{oA} \tanh \left( a_{r} \lambda l_{r\lambda} + j2\pi \frac{C}{v_{r}} l_{r\lambda} \right) \right]}{Z_{oA} + Z_{r} \tanh \left( a_{r} \lambda l_{r\lambda} + j2\pi \frac{C}{v_{r}} l_{r\lambda} \right)} \] (17)

where \( Z_{r} \), the input impedance of the radials at the ends of the dipole, is written

\[ Z_{r} = \frac{Z_{oR}}{n} \coth \left( a_{r} \lambda l_{r\lambda} + j2\pi \frac{C}{v_{r}} l_{r\lambda} \right) \] (18)

The resonant input resistance is determined by a computer reiteration process in which various radial lengths are assumed.

The radiation efficiency is plotted as a function of earth conductivity in Fig. 12 for the above-described 6-km dipole. Purely resistive characteristic impedances were used to compute the curves. A few points using the more realistic complex \( Z_{o} \) are also plotted at the lower value of \( Z_{o} \) to show that \( Z_{o} \) should be as real as possible to obtain the highest efficiency. The reactance part of \( Z_{o} \) for a conductor near the earth tends to remain constant, so higher \( Z_{o} \) values give greater efficiency because the reactance part of \( Z_{o} \) is a smaller fraction of the resistive part. More work is needed to determine the optimum characteristic impedance with regard to efficiency. The efficiency of a group of parallel dipoles 1 km apart can be found from the curves in Fig. 12 by multiplying the efficiency of one dipole by the number of dipoles in the group, since the mutual impedance was programmed into the equations to compute those efficiency curves.
MAXIMUM ANTENNA VOLTAGE

The maximum antenna voltage occurs at the end of the antenna radiating portion and the junction of the loading radials. Found by multiplying the input impedance of the radials by the antenna current at that point, it is

\[ V_{\text{max}} = X_{\text{ar}} \sqrt{\frac{P_r}{\eta \rho_{\text{oin}}} \exp \left( -\frac{a \lambda l_{\text{a}}}{{\lambda}^2} \right) \cos \left( 180^\circ \frac{c}{v_a} l_{\text{a}} \right)} \]  

(19)

where \( l_{\text{a}} \) is the total length of the dipole, in wavelengths.

The maximum voltage on a single dipole in a group of 60 dipoles that radiate 1 MW of effective power is shown in Fig. 13. The lower conductivities give lower maximum voltage because of the higher efficiencies, and the higher values of characteristic impedance give larger maximum voltages. A small amount of reactance in \( Z_0 \) has very little effect upon \( V_{\text{max}} \) since the efficiency times the resonance resistance occurring in the equation for \( V_{\text{max}} \) is nearly constant as a function of the amount of reactance in \( Z_0 \). The antenna voltages appear to be below corona onset even for the lowest frequencies and highest conductivities. Comparison with Fig. 3 shows that the voltages on the short dipole are higher than on the long dipole.

POWER RADIATING CAPABILITY

The maximum effective power radiated by a group of short dipoles can be computed from an equation obtained by rearranging Eq. 19.

\[ P_{\text{r max}} = \left[ \frac{V_{\text{c}} N}{X_{\text{ar}} \exp \left( -\frac{a \lambda l_{\text{a}}}{{\lambda}^2} \right) \cos \left( 180^\circ \frac{c}{v_a} l_{\text{a}} \right)} \right]^2 \eta \rho_{\text{oin}} \]  

(20)

where \( V_{\text{c}} \), the maximum antenna voltage before corona onset, replaces \( V_{\text{max}} \). This voltage, computed earlier in the report, is 35 kV. The maximum effective radiated power is plotted as a function of earth conductivity for several frequencies in Fig. 14. Several hundred megawatts of effective power could be radiated by an array of 60 dipoles over low conductivity earth. Only one value of characteristic impedance is considered.
FIG. 13. Voltage on 60 Short Horizontal Dipoles Radiating 1 MW of Effective Power.
FIG. 14. Power-Radiating Capability of 60 Short Horizontal Dipoles Before Corona Onset.
BANDWIDTH

The bandwidth of the short dipole is obtained by plotting the input impedance on a Smith Chart (see Fig. 15, 16, and 17). The radial lengths of the 6-km dipole are varied to obtain resonance at each of three frequencies: 10, 15, and 20 kHz. The amount of series capacitance is then varied at each of these resonant frequencies to obtain an antenna wave velocity equal to that of free space \( c/v = 1 \). The input impedance is computed over a frequency range of \pm 20\% about each resonant frequency for each dipole configuration. The Smith chart plots indicate an average half-power bandwidth of about 32\% when the dipole is driven with a transmitter whose internal impedance is equal to the dipole resonant resistance. The characteristic impedance at resonance of these dipoles is 200 - j25 \( \Omega \). Lower bandwidths would result for higher characteristic impedances. In Fig. 18, 19, and 20, the input resistance and reactance, as well as antenna wave velocity, attenuation, and characteristic impedance, are plotted as a function of the frequencies around resonance for the three dipole configurations.

RADIATION PATTERN

The half-power beamwidth of the short dipole is nearly 90\(^\circ\). The radiation pattern is approximately a cosine pattern about the dipole axis in the horizontal plane.

COST

The cost of the short dipole depends upon the terrain over which the antenna is constructed and on the conductivity of the earth. See Table 2. It appears that the total costs of an array of long dipoles and an array of short dipoles, using the same type of line and the same terrain, are about equal, both having an effective efficiency of 50\%.
FIG. 15. Input Impedance of Short Horizontal Dipole with Five 825-m Radials.
FIG. 16. Input Impedance of Short Horizontal Dipole with Five 328-m Radials.
FIG. 17. Input Impedance of Short Horizontal Dipole with Five 115-m Radials.
FIG. 18. Antenna Parameters for Short Horizontal Dipole End-Loaded with Five 825-m Radials.
FIG. 19. Antenna Parameters for Short Horizontal Dipole End-Loaded with Five 328-m Radials.

\[ f_0 = 15 \text{ KHz} \]
\[ C_s = 0.215 \text{ UF-KM} \]
\[ \sigma = 5 \times 10^{-4} \text{ U/M} \]
\[ Z_{or} = 600 \Omega \]
\[ a_{sl} = 0.3 \]
\[ c/v_T = \frac{1}{4} \]
FIG. 20. Antenna Parameters for Short Horizontal Dipole End-Loaded with Five 115-m Radials.
TABLE 2. Estimated Cost of Short Horizontal Dipole
\( (Z_0 = 400 \Omega) \)

<table>
<thead>
<tr>
<th>Unit cost</th>
<th>Type of terrain</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flat farmland</td>
<td>Rolling hills</td>
<td>Mountains</td>
</tr>
<tr>
<td>Per mile</td>
<td>$ 4,400</td>
<td>$ 8,600</td>
<td>$17,400</td>
</tr>
<tr>
<td>Per 10 kHz dipole</td>
<td>22,700</td>
<td>44,000</td>
<td>90,000</td>
</tr>
<tr>
<td>Per 15 kHz dipole</td>
<td>19,000</td>
<td>37,000</td>
<td>74,000</td>
</tr>
<tr>
<td>Per 20 kHz dipole</td>
<td>17,000</td>
<td>33,000</td>
<td>68,000</td>
</tr>
<tr>
<td>10 kHz Dipole array, 50% efficient</td>
<td>( \sigma = 10^{-3} )</td>
<td>( \sigma = 5 \times 10^{-4} )</td>
<td>( \sigma = 10^{-4} )</td>
</tr>
<tr>
<td></td>
<td>620 dipoles</td>
<td>360 dipoles</td>
<td>82 dipoles</td>
</tr>
<tr>
<td></td>
<td>$14.1 million</td>
<td>$15.8 million</td>
<td>$7.4 million</td>
</tr>
<tr>
<td>15 kHz Dipole array, 50% efficient</td>
<td>( \sigma = 10^{-3} )</td>
<td>( \sigma = 5 \times 10^{-4} )</td>
<td>( \sigma = 10^{-4} )</td>
</tr>
<tr>
<td></td>
<td>330 dipoles</td>
<td>167 dipoles</td>
<td>38 dipoles</td>
</tr>
<tr>
<td></td>
<td>$6.3 million</td>
<td>$6.1 million</td>
<td>$2.8 million</td>
</tr>
<tr>
<td>20 kHz Dipole array, 50% efficient</td>
<td>( \sigma = 10^{-3} )</td>
<td>( \sigma = 5 \times 10^{-4} )</td>
<td>( \sigma = 10^{-4} )</td>
</tr>
<tr>
<td></td>
<td>220 dipoles</td>
<td>116 dipoles</td>
<td>26 dipoles</td>
</tr>
<tr>
<td></td>
<td>$3.7 million</td>
<td>$3.8 million</td>
<td>$1.75 million</td>
</tr>
</tbody>
</table>
EXPERIMENTAL PERFORMANCE OF A SHORT, END-LOADED HORIZONTAL DIPOLE

The antenna shown in Fig. 21 has been constructed on the lava beds of Hawaii. Measurements were made on this dipole to confirm the theoretical performance, in terms of efficiency and bandwidth, shown in Fig. 12, 15, 16, 17, 18, 19, and 20. The radiating portion of the dipole is a cable made up of ten No. 19 conductors, each 6-km long, elevated 4 ft above the lava on 1½-in. diameter poles. The dipole is center-fed and is loaded on each end with 9 equally spaced radials made of No. 12 conductors 900 ft long, and each radial is loaded with 8 equally spaced 225 μH inductors.

The antenna cost $3800 for labor and materials. This low cost—about 10% of the amount estimated—was possible because the dipole was constructed on the Pohakuloa Army Base, and therefore no rights-of-way had to be purchased. In addition, the antenna was elevated only 4 ft above the ground on small poles. (A 20-kHz antenna that would be 50% efficient, with a 4-kHz bandwidth, would cost $200,000 to construct at this same location.)

Conductivity measurements were made over the area beneath the antenna to a depth of about 330 ft by the Right Angle Array Method (Ref. 8). The average conductivity between 10 and 20 kHz was about 0.3 millimho/meter.
The radiation efficiency was found to be very close to that predicted theoretically (see Fig. 12). To determine the efficiency of the single dipole, the ground wave field strength is measured with a calibrated loop (Ref. 9) at a distance beyond the near fields of the antenna. This field strength is used to compute the radiated power from (Ref. 10)

\[ P_r = \frac{(E_f R)^2}{90} \]  

(21)

where \( E_f \) = field strength, in volts/meter, and \( R \) = range from center of antenna, in meters. The radiated power is then compared with the input power to obtain the dipole efficiency as compared with that of a perfect vertical monopole.

The dipole radiation efficiency was measured both with and without end-loading (see Fig. 22 and 23). The dipole was resonant at 17.66 kHz without radials and at 10.6 kHz with radials. Measurements were made at several different distances off the end of the antenna to minimize the two sources of error for this type of measurement. These sources of errors are contamination from sky waves and contamination from near field components. Measurements were made at several ranges to make certain that the radiated field strength was decreasing inversely proportional to the distance from the center of the dipole. An average of the curves in Fig. 22 and 23 represents the most accurate values for the efficiency of the dipole. The efficiency increased near the resonant frequency of each dipole configuration even though the reactance was tuned out for the efficiency measurements. This increase in efficiency is due to the more uniform current distribution on the dipole. The end radials nearly double the efficiency of the antenna near 10 kHz, and the efficiency continues to increase with frequency. However, the dipole with no radials is most efficient near its resonant frequency (17.66 kHz). The dipole with radials is approaching full wave resonance at the highest frequency used (20 kHz). Full wave resonance reduces the radiation efficiency off the end of the dipole but enhances the sky wave radiation. Series capacitance loading of 0.5 μF each 1250 ft was used to increase the wave velocity along the antenna (see Fig. 22). This loading increased the efficiency over the whole frequency range but was most effective at the high end. This resulted from the antenna being electrically shorter and further from full wave resonance and the current distribution more uniform.
$f_0 = 10.6$ KHz

Measurements made at three different distances from the dipole center off the end of the dipole

Series capacitance
0.5 UF-1250. FT
R = 26.5 KM

R = 7.7 KM
R = 14.3 KM
R = 26.5 KM

No series capacitance

NO SERIES CAPACITANCE LOADING

\( f_0 = 17.66 \text{ KHz} \)

MEASUREMENTS MADE AT TWO DIFFERENT DISTANCES FROM THE DIPOLE CENTER OFF THE END OF THE DIPOLE.

FIG. 23. Efficiency of Hawaiian Dipole Without End Loading.
Impedance measurements were made on the different dipole configurations to determine bandwidth and other antenna parameters such as wave velocity, attenuation factor, and characteristic impedance. The bandwidth of the dipole loaded with end radials is nearly equal to the bandwidth of the dipole with no end loading (see Fig. 24 and 25). The half-power bandwidth is 25%, which is near that predicted theoretically (see Fig. 15). This means the antenna will accept half of the output power, or more, from a transmitter whose output impedance is equal to the resonant resistance of the dipole over a 25% bandwidth about the resonant frequency. In Table 3, the half-power bandwidth and also the 1-dB bandwidths, in both cycles and percent of the resonant frequencies, are tabulated for the different dipole configurations. If the dipole or the end-loading radials are on the ground the bandwidth is greater, but this dipole configuration is not recommended because of the observed changes in resonant frequency and resonant resistance during rainstorms. Raising the dipole a few feet above the ground considerably reduces this effect. The change in phase of the radiated electric field due to weather must be evaluated. Plots of the dipole input resistance and reactance versus frequency are shown in Fig. 26 and 27.

TABLE 3. Comparison of Bandwidth for Several Dipole Configurations.

<table>
<thead>
<tr>
<th>Type of antenna</th>
<th>Resonant frequency, kHz</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single conductor on lava; no radials</td>
<td>14.2</td>
<td>18% = 2.5 kHz, 35% = 5 kHz</td>
</tr>
<tr>
<td>Single conductor, elevated; no radials</td>
<td>17.7</td>
<td>14% = 2.4 kHz, 24% = 4.2 kHz</td>
</tr>
<tr>
<td>Single conductor on lava; 5 radials on lava</td>
<td>10.2</td>
<td>18% = 1.8 kHz, 35% = 3.6 kHz</td>
</tr>
<tr>
<td>Single conductor, elevated; 5 radials on lava</td>
<td>10.8</td>
<td>17% = 1.8 kHz, 35% = 3.8 kHz</td>
</tr>
<tr>
<td>Single conductor, elevated; 9 radials, elevated</td>
<td>10.6</td>
<td>13% = 1.4 kHz, 25% = 2.7 kHz</td>
</tr>
</tbody>
</table>
FIG. 24. Input Impedance of End-Loaded Hawaiian Dipole.
FIG. 25. Input Impedance of Hawaiian Dipole Without Radials.
FIG. 27. Input Resistance and Reactance of Hawaiian Dipole Without End Radials.
The efficiency-bandwidth product is a measure of information-transmitting capability and also an indicator of the phase and amplitude stability of an antenna. Table 4 is a comparison of an Hawaiian antenna made up of 10 parallel horizontal dipoles and of the largest Navy transmitting antenna at Cutler, Maine. At 10.6 kHz the Hawaiian antenna would have an efficiency-bandwidth product 18 times greater than the Cutler antenna, and at 17.7 kHz the efficiency-bandwidth product would be 4 times greater.

**TABLE 4. Comparison of Efficiency-Bandwidth Product of 10-Dipole Horizontal Antenna, Hawaii, and Navy Antenna, Cutler, Maine.**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>At 10.6 kHz</th>
<th>At 17.7 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hawaiian antenna w/ 9 radials</td>
<td>Navy, Cutler, Maine</td>
</tr>
<tr>
<td>Efficiency, %</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>Bandwidth, Hz</td>
<td>2700</td>
<td>6</td>
</tr>
<tr>
<td>Efficiency-bandwidth product, Hz</td>
<td>54</td>
<td>3</td>
</tr>
</tbody>
</table>
NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>Antenna conductor radius</td>
</tr>
<tr>
<td>c</td>
<td>Free-space wave velocity ((3 \times 10^8))</td>
</tr>
<tr>
<td>C_P</td>
<td>Antenna distributed capacitance to ground ((\text{farads/meter}))</td>
</tr>
<tr>
<td>C_s</td>
<td>Lumped series capacitance inserted in antenna</td>
</tr>
<tr>
<td>f</td>
<td>Frequency</td>
</tr>
<tr>
<td>h</td>
<td>Antenna height ((\text{meters}))</td>
</tr>
<tr>
<td>I_{in}</td>
<td>Antenna input current</td>
</tr>
<tr>
<td>I_m</td>
<td>Imaginary term</td>
</tr>
<tr>
<td>l_1</td>
<td>Length of longest part of dipole from feed point to the end</td>
</tr>
<tr>
<td>l_2</td>
<td>Length of shortest part of dipole from feed point to the end</td>
</tr>
<tr>
<td>L</td>
<td>Antenna series inductance per unit length</td>
</tr>
<tr>
<td>n</td>
<td>Number of loading radials</td>
</tr>
<tr>
<td>N</td>
<td>Number of dipoles</td>
</tr>
<tr>
<td>P_{in}</td>
<td>Antenna input power</td>
</tr>
<tr>
<td>P_r</td>
<td>Radiated power</td>
</tr>
<tr>
<td>Q</td>
<td>(\frac{\omega L}{r})</td>
</tr>
<tr>
<td>r</td>
<td>Antenna series resistance per unit length</td>
</tr>
<tr>
<td>R</td>
<td>Range from center of antenna in meters</td>
</tr>
</tbody>
</table>
$R_{in}$  Antenna input resistance

$R_m$  Mutual resistance

$R_{oin}$  Antenna resonant input impedance

$R_s$  Antenna self resistance

$R_w$  Conductor ac resistance

$Re$  Real term

$s$  Distance between conductors

$v$  Wave velocity

$v_a$  Wave velocity along antenna

$v_r$  Wave velocity along radials

$V_c$  Antenna voltage before corona onset

$V_{max}$  Maximum antenna voltage

$x$  Distance from antenna feed point

$X_{ar}$  Input reactance to radials

$X_m$  Antenna mutual reactance

$X_s$  Antenna self reactance

$Z_{in}$  Antenna input impedance

$Z_o$  Characteristic impedance

$Z_{oA}$  Characteristic impedance of antenna

$Z_{or}$  Characteristic impedance of radials

$Z_r$  Input impedance of radials at ends of dipole

45
Termination impedance
Attenuation constant
Antenna attenuation constant
Radial attenuation constant
$\beta = \frac{2\pi}{\lambda}$; free-space propagation phase constant
$\beta_1 = \frac{2\pi (\varepsilon)}{\lambda \sigma}$; propagation phase constant along antenna
Skin depth (meters)
Free-space dielectric constant ($3.85 \times 10^{-12}$)
Antenna radiation efficiency compared with that of a perfect monopole
Free-space wavelength
$2\pi f$
Earth conductivity (mhos/meter)
Angle in azimuthal plane measured from antenna axis
REFERENCES


A study is made of the use of the horizontal dipole as a VLF transmitting antenna. Two types of multiconductor-loaded dipoles are considered: (1) an off-center-fed dipole one wavelength long, with wave velocity compensation; and (2) a center-fed dipole about 0.2 wavelength long, end-loaded with unterminated radials. Theoretical equations are derived to determine the efficiency, beamwidth, bandwidth, maximum power radiating capability, and maximum antenna voltages, over a frequency range of 10-20 kHz and an earth conductivity range of $10^{-3}$ to $10^{-4}$ mho/meter. Measurements made on a 6-km experimental antenna constructed in Hawaii confirm the equations for radiation efficiency and bandwidth.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th></th>
<th>LINK B</th>
<th></th>
<th>LINK C</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>VLF transmitting antennas</td>
<td></td>
<td></td>
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<tr>
<td>VLF communications</td>
<td></td>
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<tr>
<td>Horizontal dipole antennas</td>
<td></td>
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</tbody>
</table>