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NUCLEAR EXPLOSION INTERACTION STUDIES

Volume III

The OUTPUT Code

J. R. Triplett et al.

Gulf General Atomic Incorporated
San Diego, California 92112
Contract No. F29601-67-C-0014

TECHNICAL REPORT NO. AFWL-TR-67-131, Vol III

April 1968
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FOREWORD

This report was prepared by Gulf General Atomic Incorporated, San Diego, California, under Contract F29601-67-C-0014. The research was funded by DASA under Project 5710, Subtask 07.017, Program Element 6.16.46.01H, and by ARPA Order 313, Program Element 6.25.03.01R.

Inclusive dates of research were 29 September 1966 to 27 October 1967. The report was submitted 13 March 1968 by the Air Force Weapons Laboratory Project Officer, Major John Bode (WLRT).

This report is published in four volumes: Volume I, Laser Phenomenology (classified CONFIDENTIAL); Volume II, Two-Dimensional Code Development; Volume III, The OUTPUT Code; and Volume IV, Material Property Codes. The first volume contains a classified report on interaction of laser radiation with solid targets and a brief description of calculations done in conjunction with experiments at the Air Force Weapons Laboratory. The remaining three volumes contain reports of code development efforts in the areas of radiative transfer, hydrodynamics, radiative absorption coefficients, and equations of state.

The projects described in this report are for the most part in an incomplete state of development. This is due in part to the nature of the existing computer programs themselves, which continue in a state of development as long as they are in use, and in part to the time scale involved in bringing new programs to a state of capability for solving real problems.


The cooperation of Dr. P. V. Avizonis, Major J. Bode, Capt C. C. David, Major G. Spillman, and Lt L. Stoessel of AFRL is gratefully acknowledged.


This technical report has been reviewed and is approved.

J. R. Triplett

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CLAUDE K. STAMBAUGH
Colonel, USAF
Chief, Research Division
ABSTRACT

(Distribution Limitation Statement No. 2)

The OUTPUT code is designed for the analysis of early-time nuclear explosions. The equations for radiative transfer (characteristic method) and conservation of total (fluid and radiation) momentum and energy are solved in one-dimensional (plane or spherical) geometry. The radiation equations include first-order Compton scattering, and the hydrodynamic equations are treated in explicit Lagrangian form. The code is undergoing continuing development; the formulation, flow charts, glossary, and listings presented represent its status as of 27 October 1967.
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SECTION I
INTRODUCTION

PURPOSE OF CODE

The OUTPUT code, a version of the SPUTTER code, is a one-dimensional, Lagrangian, radiative hydrodynamics program written in the FORTRAN IV language. The purpose of this code is to calculate and to predict the radiative spectrum and the early-time evolution of nuclear devices. The code has been applied to several devices; the results of these applications are reported in reference 1.

PHYSICAL MODEL

The solution to the radiation transport equation is similar to that in the standard SPUTTER program (Ref. 2) in that the characteristic ray approach is employed for grey or multifrequency problems with mixed diffusion and transport regimes. However, OUTPUT includes as options Thomson scattering (conservative scattering) and first-order Compton scattering (nonconservative). The physics of the hydrodynamics routine have also been improved by including the radiation pressure as a tensor. This improvement has the effect of adding a term to the momentum equation and a corresponding work term to the energy equation. The energy equation also now uses the radiation pressure as calculated in the radiation routines rather than the usual equilibrium diffusion assumption $P_r = (1/3) a \theta^4$, where $\theta = kT$ is the local material temperature multiplied by the Boltzmann constant, in electron volts, and $a = 137 \text{ ergs/cm}^3 / \text{eV}^4$ is the Stefan radiation constant in appropriate units.
LOGIC OF CONSTRUCTION

The logic followed in constructing OUTPUT has been that the SPUTTER format and bookkeeping subroutines should be employed as fully as possible so as to achieve the following benefits: (1) opacities and material properties developed for SPUTTER would be immediately usable; (2) problems run with SPUTTER could be picked up (i.e., off a tape) and continued with OUTPUT or vice versa, using at most a simple transcription program; (3) the job of redeveloping the auxiliary subroutines could be avoided; and (4) personnel familiar with SPUTTER and its derivatives could easily learn the differences between the codes. The differences between the current SPUTTER program and the OUTPUT code are described in Appendix I. It should be noted that the OUTPUT code was designed for a particular class of problems; consequently, the capability of the SPUTTER program to handle certain problems, e.g., conduction, boiling, etc., has been eliminated. This has increased the efficiency of the code and released needed core storage and common variables. The user is cautioned that variables used solely in deleted sections of the SPUTTER code are now employed differently.

This report is not a complete documentation of the OUTPUT code but rather a documentation of those portions of the SPUTTER code that have been substantially altered. The SPUTTER code has been documented in reference 3. As an aid to gaining experience in using the OUTPUT code, a list of the cards used in making the successful comparison between the Gulf General Atomic and Air Force Weapons Laboratory versions of the OUTPUT code is included in Appendix II.
SECTION II
THEORETICAL MODEL

The equations presented in this section represent the theoretical model employed in the OUTPUT code. In general, the constraints stated with the equations limit the applicability of the model to low-temperature devices, with one-dimensional (plane or spherical) symmetry.

RADIATION EQUATIONS

The derivation of the transport equation for the intensity \(I(v, \Omega, r, t)\), i.e., the radiant energy per unit frequency \(v\), per unit solid angle \(d\Omega\), about the direction (unit vector) \(\Omega\) per unit time \(dt\), per unit area, including the effects of Compton scattering, is presented in reference 4. A summary of this derivation is presented below.

The contributions to the rate of change of the intensity, or to \(1/c (\partial I/\partial t) + \Omega \cdot \nabla I\), are assumed to be absorption, emission, and the scattering of photons by free electrons. The absorption coefficient includes all significant processes by which photons are absorbed subject to the local-thermodynamic-equilibrium (LTE) assumption. The scattering coefficient and the differential cross section for scattering are determined from the Klein-Nishina formula (Ref. 5). A series of further approximations are required to complete the hypothesis. They are:

1. The electron states before and after scattering are nondegenerate.
2. Doppler effects can be ignored.
3. Polarization is unimportant.
4. \(h \nu / m_o c^2 \leq 0.2\), \(m_o c^2 = \) rest energy of an electron.
5. Stimulated scattering is negligible.
6. Retardation effects are omitted.

7. The spectrum is a smooth function of $\nu$.

With these assumptions, the following transport equation in plane geometry is obtained for the quantity $I(\mu) = \frac{2\pi}{c} \text{I(\|)}$:

$$\mu \frac{dI}{dx} = \mu'_a (B_\nu - I) + S$$

(1)

where $\mu = \frac{\mathbf{n} \cdot \mathbf{v}}{c}$ is the normal direction cosine;

$$\mu'_a = \mu'_a (1 - e^{-\hbar \nu / \theta_0})$$

(2)

is the linear absorption coefficient corrected for induced emission;

$$B_\nu = \frac{4\pi \hbar \nu}{c^3} \left( e^{\hbar \nu / \theta_0} - 1 \right)^{-1}$$

(3)

is the Planck distribution function for radiation from a blackbody at temperature $\theta$ eV, multiplied by $\frac{2\pi}{c}$.

$$S = -\mu_s \left[ I - \frac{3}{16} \int_1^4 d\mu' I(\mu') \left[ 3 - \mu'^2 + (3\mu'^2 - 1)(\mu')^2 \right] 
+ \gamma \left\{ -2I + \frac{3}{16} \int_1^4 d\mu' \left( I(\mu') - \gamma \frac{dI(\mu')}{d\nu} \right) \left[ 3 - \mu'^2 + \mu' (3\mu'^2 - 5) 
+ (3\mu'^2 - 1)(\mu')^2 + \mu (3 - 5\mu'^2)(\mu')^3 \right] \right\} + O(\gamma^2)$$

(4)

where

$$\mu_s = \frac{8}{3} \pi r_c^2 N_e$$

(5)

is the Thomson scattering coefficient valid for $\hbar \nu \ll m_0 c^2$, in which

$r_c = e^2 / (m_0 c^2)$ is the classical electron radius and $N_e$ is the number of electrons per cm$^3$; and

$$\gamma = \frac{\hbar \nu}{m_0 c^2}$$

(6)
Examination of the form of the scattering source term reveals that the low-energy limit, i.e., \( \gamma \rightarrow 0 \), gives the Thomson scattering law. To determine the material heating due to Compton scattering, it is necessary to integrate over all solid angles and over the entire frequency range. The result, with sign reversed, is

\[
-\nabla \cdot \vec{F} = -c \int_0^\infty \mu_a (2B_\nu - E_\nu) \, d\nu + \mu_s \frac{hc}{m_c^2} \int_0^\infty \nu E_\nu \, d\nu
\]

where

\[
\vec{F} = \int_0^\infty d\nu \int_{4\pi} I(\nu, \hat{n}) \hat{n} \, d\Omega
\]

is the total flux and

\[
E_\nu = \frac{1}{c} \int_{4\pi} I(\nu, \hat{n}) \, d\Omega
\]

is the radiant energy density spectrum. The first term on the right is the usual heating term due to absorption or emission, whereas the second term is the heating rate due to Compton scattering.

**NUMERICAL INTEGRATION OF THE TRANSPORT EQUATION**

The development of a scheme to integrate the transport equation along a ray requires that several numerical approximations be made. Rewrite the source term \( S \) by rearranging and by employing the notation

\[
I_n = \int_{-1}^1 I(\mu)\mu^n \, d\mu
\]

Thus, \( I_0, I_1, \) and \( I_2 \) are proportional to the radiation energy density, the flux, and the radiation pressure, respectively. Now,
It is convenient to expand the last term as follows:

\[ \gamma^2 \frac{\partial f(\gamma)}{\partial \gamma} = \frac{\partial [\gamma^2 f(\gamma)]}{\partial \gamma} - 2\gamma f(\gamma) \]

Using this substitution, one finds

\[ S = -\mu \left\{ I(1 - 2\gamma) - \frac{3}{16} \left[ (3 - \mu^2) I_o + (3\mu^2 - 1) I_2 \right] 
+ \frac{9}{16} \gamma \left[ (3 - \mu^2) I_o + \mu(3\mu^2 - 5) I_4 + (3\mu^2 - 1) I_2 + \mu(3 - 5\mu^2) I_3 \right] 
- \frac{3}{16} \frac{\partial \gamma^2}{\partial \gamma} \left[ (3 - \mu^2) I_o + \mu(3\mu^2 - 5) I_4 + (3\mu^2 - 1) I_2 + \mu(3 - 5\mu^2) I_3 \right] \} \]  

In Eq. (1), the intensity \( I(\mu, \nu) \) is the monochromatic intensity. In most computer codes employing radiation transport, the quantity being calculated is the integral of the intensity over some frequency band \( (\nu_j, \nu_{j+1}) \), for which one finds

\[ \mu \frac{dI_j}{dx} = \mu_{a_j} (B_j - I_j) + S_j \]

where

\[ S_j = \int_{\nu_j}^{\nu_{j+1}} S(\nu) \, d\nu \]

\[ I_j = \int_{\nu_j}^{\nu_{j+1}} I(\nu) \, d\nu \]
The notation \( I_j \) must not be confused with \( I_n \) as previously defined. However, the distinction between symbols is fairly clear. Note that single subscripts denote intensities and double subscripts represent moment quantities. Continuing, one finds

\[
S_j = -\mu_s \left\{ I_j (1 - 2\gamma_j) - \frac{3}{46} \left[ (3 - \mu^2) I_{o_j} + (3\mu^2 - 4) I_{2_j} \right] \right.
\]

\[
+ \frac{9}{46} \gamma_j \left[ (3 - \mu^2) I_{o_j} + \mu(3\mu^2 - 5) I_{1_j} + (3\mu^2 - 4) I_{2_j} + \mu(3 - 5\mu^2) I_{3_j} \right]
\]

\[
- \frac{3}{46} \frac{h}{m_o c^2} \nu_{j+1}^2 \left[ (3 - \mu^2) I_{o} (\nu_{j+1}) + \mu(3\mu^2 - 5) I_{1} (\nu_{j+1}) + (3\mu^2 - 4) I_{2} (\nu_{j+1}) + \mu(3 - 5\mu^2) I_{3} (\nu_{j+1}) \right]
\]

\[
+ \frac{3}{46} \frac{h}{m_o c^2} \nu_j^2 \left[ (3 - \mu^2) I_{o} (\nu_j) + \mu(3\mu^2 - 5) I_{1} (\nu_j) + (3\mu^2 - 4) I_{2} (\nu_j) + \mu(3 - 5\mu^2) I_{3} (\nu_j) \right]
\]

\[
+ (3\mu^2 - 4) I_{2} (\nu_j) \right\}
\]

where the assumption has been made that \( \overline{\gamma}_j = (1/2)(\gamma_j + \gamma_{j+1}) \) and

\[
I_{x, j} = \int_{\nu_j}^{\nu_{j+1}} I_{X} (\nu) \, d\nu
\]

Moreover,

\[
\overline{\gamma}_j I_{x, j} = \int_{\nu_j}^{\nu_{j+1}} \gamma I_{X} (\nu) \, d\nu
\]

One should note that in Eq. (15) the quantities \( I_{o} (\nu_j) , I_{1} (\nu_j) , \text{ etc.} \) are the moment quantities evaluated at the boundaries of the frequency group of interest, i.e., \( \nu_j \) and \( \nu_{j+1} \). At this point, an approximation must be made relating the boundary quantities and the frequency average quantities. If
the spectrum is flat over the various frequency groups, then

$$I_{x_j} \approx I_x (\nu_j) (\nu_{j+1} - \nu_j)$$  \hspace{1cm} (18)

Thus,

$$S_j = -\mu_s \left\{ I_j (1 - 2 \gamma_j) - \frac{3}{16} [(3 - \mu^2) I_{o_j} + (3\mu^2 - 1) I_{2_j}] \right. \right.$$

$$+ \frac{9}{16} \gamma_j [(3 - \mu^2) I_{o_j} + \mu(3\mu^2 - 5) I_{4_j} + (3\mu^2 - 1) I_{2_j} + \mu(3 - 5\mu^2) I_{3_j}]

$$- \frac{3}{16} \frac{h}{m_o c^2} \frac{\nu_{j+1}^2}{(\nu_{j+2} - \nu_{j+1})} [(3 - \mu^2) I_{o_{j+1}} + \mu(3\mu^2 - 5) I_{4_{j+1}} + (3\mu^2 - 1) I_{2_{j+1}}

$$+ \mu(3 - 5\mu^2) I_{3_{j+1}}

$$+ \frac{3}{16} \frac{h}{m_o c^2} \frac{\nu_j^2}{\nu_{j+1} - \nu_j} [(3 - \mu^2) I_{o_j} + \mu(3\mu^2 - 5) I_{4_j} + (3\mu^2 - 1) I_{2_j}

$$+ \mu(3 - 5\mu^2) I_{3_j} \left\} \right.$$  \hspace{1cm} (19)

Reorganizing the source term with $\Delta \nu_j = \nu_{j+1} - \nu_j$, one obtains

$$S_j = -\mu_s \left\{ I_j (1 - 2 \gamma_j) - \frac{3}{16} A_2 [(3 - \mu^2) I_{o_j} + (3\mu^2 - 1) I_{2_j}] \right. \right.$$

$$+ \frac{3}{16} A_4 \mu [(3\mu^2 - 5) I_{4_j} + (3 - 5\mu^2) I_{3_j}]$$

$$- \frac{3}{16} A_3 [(3 - \mu^2) I_{o_{j+1}} + (3\mu^2 - 1) I_{2_{j+1}}

$$- \frac{3}{16} A_3 \mu [(3\mu^2 - 5) I_{4_{j+1}} + (3 - 5\mu^2) I_{3_{j+1}} \left\} \right.$$  \hspace{1cm} (20)
where

\[ A_1 = 3 \gamma_j + \frac{h}{m_0 c^2} \frac{v_j^2}{\Delta v_j} \] (21)

\[ A_2 = 1 - A_1' \] (22)

\[ A_3 = \frac{h}{m_0 c^2} \frac{v_{j+1}^2}{\Delta v_{j+1}} \] (23)

At this point, the integration of the transport equation can be performed as follows:

\[ \frac{dI_j}{dx} = \mu_a j (B_j - I_j) - \mu_s I_j (1 - 2 \gamma_j) + \mu_s \{ \} \] (24)

where

\[ \{ \} = \frac{S_i}{\mu_s} + I_j (1 - 2 \gamma_j) \] (25)

Using

\[ \left( \frac{x}{\mu} \right) \left[ \mu_a j + \mu_s (1 - 2 \gamma_j) \right] \] (26)

as an integrating factor, one finds

\[ I_j (x_2) = I_j (x_1) e^{-\alpha_j (x_2 - x_1)} + \frac{\mu_a j}{\mu} \int_{x_1}^{x_2} B_j (x') e^{-\alpha_j (x_2 - x')} dx' \]

\[ + \frac{\mu_s}{\mu} \int_{x_1}^{x_2} \{ \} e^{-\alpha_j (x_2 - x')} dx' \] (27)

where

\[ \alpha_j = \frac{1}{\mu} \left[ \mu_a j + \mu_s (1 - 2 \gamma_j) \right] \] (28)
The first two terms on the right-hand side have already been evaluated in radiation-transport routines currently employed by SPUTTER (Ref. 2). Here, the integral in the third term will be evaluated. The term in braces [Eq. (25)] contains such terms as \( I_{0j} \), \( I_{0j+1} \), etc. At this point, some approximation must be made concerning the spatial dependence of these quantities. Following the assumption used in developing the Thomson scattering code, assume that all functions vary linearly in geometric space.

Performing the required integration, one then finds

\[
\frac{\mu_s}{\mu} \int_{x_1}^{x_2} \{ \} e^{-\alpha_j (x_2 - x')} dx'
\]

\[
= \frac{\mu_s}{\mu} \frac{3}{16} A_2 \left[ \left( \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j} \right) A + B \left( \frac{x_2 - x_1 e^{-\alpha_j \Delta}}{\alpha_j} - \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j^2} \right) \right]
\]

\[
- \frac{\mu_s}{\mu} \frac{3}{16} \mu \mu \frac{3}{16} A_1 \left[ \left( \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j} \right) C + D \left( \frac{x_2 - x_1 e^{-\alpha_j \Delta}}{\alpha_j} - \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j^2} \right) \right]
\]

\[
+ \frac{\mu_s}{\mu} \frac{3}{16} A_3 \left[ \left( \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j} \right) E + F \left( \frac{x_2 - x_1 e^{-\alpha_j \Delta}}{\alpha_j} - \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j^2} \right) \right]
\]

\[
+ \frac{\mu_s}{\mu} \frac{3}{16} \mu \mu \frac{3}{16} A_2 \left[ \left( \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j} \right) G + H \left( \frac{x_2 - x_1 e^{-\alpha_j \Delta}}{\alpha_j} - \frac{1 - e^{-\alpha_j \Delta}}{\alpha_j^2} \right) \right]
\]

where \( \Delta = \Delta x = x_2 - x_1 \)

\[
A = \frac{1}{\Delta x} \left\{ x_2 (F10(J, x_1) + \mu^2 F12(J, x_1)) - x_1 (F10(J, x_2) + \mu^2 F12(J, x_2)) \right\}
\]

\[
B = \frac{1}{\Delta x} \left\{ F10(J, x_2) - F10(J, x_1) + \mu^2 (F12(J, x_2) - F12(J, x_1)) \right\}
\]
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\[ C = \frac{1}{\Delta x} \left\{ x_2 F I 3(J, x_1) - x_4 F I 3(J, x_2) + \mu^2 (x_2 F I 1(J, x_1) - x_4 F I 1(J, x_2)) \right\} \]

\[ D = \frac{1}{\Delta x} \left\{ F I 3(J, x_2) - F I 3(J, x_1) + \mu^2 (F I 1(J, x_2) - F I 1(J, x_1)) \right\} \]

\[ E = \frac{1}{\Delta x} \left\{ x_2 (F I 0(J + 1, x_1) + \mu^2 F I 2(J + 1, x_1)) - x_4 (F I 0(J + 1, x_2) + \mu^2 F I 2(J + 1, x_2)) \right\} \]

\[ F = \frac{1}{\Delta x} \left\{ F I 0(J + 1, x_2) - F I 0(J + 1, x_1) + \mu^2 (F I 2(J + 1, x_2) - F I 2(J + 1, x_1)) \right\} \]

\[ G = \frac{1}{\Delta x} \left\{ x_2 F I 3(J + 1, x_1) - x_4 F I 3(J + 1, x_2) + \mu^2 (x_2 F I 1(J + 1, x_1) - x_4 F I 1(J + 1, x_2)) \right\} \]

\[ H = \frac{1}{\Delta x} \left\{ F I 3(J + 1, x_2) - F I 3(J + 1, x_1) + \mu^2 (F I 1(J + 1, x_2) - F I 1(J + 1, x_1)) \right\} \]

with \( F I 0(J, x_1) = 3 I_{o,j} (x_1) - I_{o,j} (x_1) \)

\( F I 2(J, x_1) = 3 I_{2,j} (x_1) - I_{o,j} (x_1) \)

\( F I 3(J, x_1) = 3 I_{3,j} (x_1) - 5 I_{1,j} (x_1) \)

\( F I 1(J, x_1) = 3 I_{1,j} (x_1) - 5 I_{3,j} (x_1) \)

\[ A_1 = 3 \bar{\nu}_j + \frac{h}{m_o^2 c^2} \Delta \nu_j^2 \]

\[ A_2 = 1 - A_1 \]

\[ A_3 = \frac{h}{m_o^2 c^2} \frac{\nu_j^{j+1}}{\Delta \nu_{j+1}} \]
Computationally, the solution to the integral will be inaccurate due to figure loss when \( \alpha_j \Delta < 0.01 \). Under this condition, one finds the following solution:

\[
\frac{\mu_s}{\mu} \int_{x_1}^{x_2} \{ -\alpha_j(x_2-x') \} e^{-\alpha_j x'} \, dx'
\]

\[
= \frac{\mu_s}{\mu} \frac{3}{16} A_2 \Delta \left[ (F10(J, x_1) + \mu^2 F12(J, x_1))(1 - \frac{1}{2} \alpha_j \Delta) + \frac{\Delta B}{2} \right] 
- \frac{\mu_s}{\mu} \Delta \frac{3}{16} \mu A_4 \left[ \left( 1 - \frac{1}{2} \alpha_j \Delta \right) (F13(J, x_1) + \mu^2 F14(J, x_1)) + \frac{\Delta D}{2} \right] 
+ \frac{\mu_s}{\mu} \Delta \frac{3}{16} \mu A_3 \left[ \left( 1 - \frac{1}{2} \alpha_j \Delta \right) (F10(J + 1, x_1) + \mu^2 F12(J + 1, x_1)) + \frac{\Delta F}{2} \right] 
+ \frac{\mu_s}{\mu} \Delta \frac{3}{16} \mu A_3 \left[ \left( 1 - \frac{1}{2} \alpha_j \Delta \right) (F13(J + 1, x_1) + \mu^2 F14(J + 1, x_1)) + \frac{\Delta H}{2} \right] 
\]

(32)

**DISCUSSION OF THE NUMERICAL APPROXIMATIONS**

In Eq. (7), the material heating rate for Compton scattering was derived. It will now be shown that numerically the heating rate calculated will be consistent with the analytical result. Integrating Eq. (24) over \( \mu \), one finds

\[
\frac{1}{c} \nabla \cdot F_j = \mu_a a_j (2B_j - E_j) - \mu_s E_j (1 - 2\overline{\nu}_j) + \mu_s E_j \left( 1 - 3\overline{\nu}_j - \frac{h}{m_c^2 \Delta \nu_j} \nu_j^2 \right) 
+ \mu_s \frac{h}{m_c^2 \Delta \nu_{j+1}} \nu_{j+1}^2 E_{j+1} 
\]

(33)
Summing on \( j \), one obtains

\[
\frac{1}{c} \nabla \cdot F = \sum_{j=1}^{N} \mu_{a_j} (2B_j - E_j) - \mu_s \sum_{j=1}^{N} \gamma_j E_j - \mu_s \frac{h}{m_o c^2} \sum_{j=1}^{N} \left( \frac{\nu_j^2}{E_j \Delta \nu_j} - \frac{\nu_{j+1}^2}{E_{j+1} \Delta \nu_{j+1}} \right)
\]

(34)

For the third term on the right-hand side, note that

\[
\sum \left( \frac{\nu_j^2}{E_j \Delta \nu_j} - \frac{\nu_{j+1}^2}{E_{j+1} \Delta \nu_{j+1}} \right) = \frac{\nu_{N+1}^2}{E_{N+1} \Delta \nu_{N+1}}
\]

(35)

Now, by definition, \( \nu_1 = 0 \) and \( E_{N+1} = 0 \). Thus, the calculated heating rate due to Compton scattering is

\[
\mu_s c \sum_{j=1}^{N} \gamma_j E_j
\]

(36)

Comparing this with the analytical result, i.e.,

\[
\mu_s \frac{hc}{m_o c^2} \int_0^\infty \nu E_{\nu} d\nu
\]

(37)

one notes that the above sum is consistent with assuming a flat spectrum over the various groups. Moreover, in the limit as the number of groups increases, the sum approaches the analytical result.

A computational difficulty encountered with the present formulation is that the intensity \( I_j(\mu) \) depends on the solid-angle moments of the intensity \( I_{0j}, I_{1j}, \) etc. However, if these quantities were known, the evaluation of the intensity would be superfluous, since one is usually concerned with finding heating rates. The solution to the transport equation gives the intensity \( I_j^{n+1}(\mu) \) at time \( n + 1 \) in terms of the moment quantities at time \( n \). The assumption made is that the moment quantities at time \( n \) are close to those at time \( n + 1 \). Computationally, there are two equivalent ways to achieve this result. The first way, and the least desirable, is to run the
calculation with very small time increments. Now, even if the moment quantities are rapidly changing, the errors involved are proportional to the time step. After a number of cycles in which the solution is constant, the moment quantities will also converge. A second way to guarantee that the moment quantities are consistent with the transport equation is to iterate on these quantities. A test can be made to see if the values at time \( n + 1 \) are different from those at time \( n \). If differences are detected, the complete transport calculations can be rerun using the new, updated, moment quantities. This scheme is presently employed in the Thomson and Compton scattering subroutines. To calculate the number of iterations required for convergence, consider the following model for the case of Thomson scattering, i.e., \( \gamma \to 0 \). For this case, the transport equation becomes

\[
\mu \frac{dI_n^{n+1}}{dx} = \mu_a (B - I_n^{n+1}) - \mu_s I_n^{n+1} + \frac{3}{16} \mu_s [(1 - \mu^2) I_o^n + (3\mu^2 - 1) I_2^n] \tag{38}
\]

If spatial homogeneity is assumed, then

\[
I_n^{n+1} = \frac{\mu_a}{\mu} B + \frac{3}{16} \frac{\mu_s}{\mu} [(3 - \mu^2) I_o^n + (3\mu^2 - 1) I_2^n] \tag{39}
\]

where \( \mu = \mu_a + \mu_s \).

To find \( I_o^{n+1} \) and \( I_2^{n+1} \), integrate over \( d\mu \). Thus,

\[
I_o^{n+1} = \frac{2\mu_a}{\mu} B + \frac{\mu_s}{\mu} I_o^n \tag{40}
\]

and

\[
I_2^{n+1} = \frac{2}{3} \frac{\mu_a}{\mu} B + \frac{3}{10} \frac{\mu_s}{\mu} I_o^n + \frac{4}{10} \frac{\mu_s}{\mu} I_2^n \tag{41}
\]

Solving these recursion equations, one finds that

\[
I_o^n = 2B \left[ 1 - \left( \frac{\mu_s}{\mu} \right)^n \right] \tag{42}
\]
From the form of these equations, one can conclude that the convergence rate is controlled by the ratio $\mu_s/\mu_a + \mu_s$. Thus, in the interesting situation in which scattering is the dominant mechanism, the number of iterations required to achieve a given accuracy increases as the amount of absorption decreases. This fact should be kept in mind when employing the scattering code in regions with dominant scattering.

For the case of Compton scattering, consider the homogeneous transport equation, or

$$0 = \mu_a (B - I^{n+1}) - \mu_s I^{n+1} (1 - 2\gamma) + \frac{3}{16} \mu_s [(3 - \mu^2) I_o^{n+1} + (3\mu^2 - 4) I_2^{n+1}]$$

$$- \frac{9}{16} \mu_s \gamma [(3 - \mu^2) I_o^n + \mu(3\mu^2 - 5) I_1^n + (3\mu^2 - 4) I_2^n + \mu(3 - 5\mu^2) I_3^n]$$

$$+ \frac{3}{16} \mu_s \frac{a}{\partial \gamma} \gamma^2 [(3 - \mu^2) I_o^n + \mu(3\mu^2 - 5) I_1^n + (3\mu^2 - 4) I_2^n + \mu(3 - 5\mu^2) I_3^n]$$

If one now integrates over $\int_1^1 d\mu$, one finds

$$\bar{\mu} I_0^{n+1} = 2\mu_a B + \mu_s (1 - 3\gamma) I_o^{n+1} + \mu_s \frac{\partial (\gamma I_o^n)}{\partial \gamma}$$

(43)

where $\bar{\mu} = \mu_a + \mu_s (1 - 2\gamma)$.

Integrating over frequency, one finds

$$I_0^{n+1} = \frac{2\mu_a}{\bar{\mu}_j} B_j + \frac{\mu_s}{\bar{\mu}_j} (1 - 3\gamma_j) I_0^{n+1} + \frac{\mu_s}{\bar{\mu}_j} \frac{h}{m c^2} \left[ \frac{\nu_j^{j+1}}{\Delta \nu_{j+1}} I_0^{n+1} - \frac{\nu_j^{j+1}}{\Delta \nu_j} I_0^{n+1} \right]$$

(46)

where

$$\bar{\mu}_j = \mu_a + \mu_s (1 - 2\gamma_j)$$

(47)
Since $I_{o_{k+1}} = 0$, the solution to the last equation is

$$I_{o_k}^n = B_k \frac{2\mu_{a_k}}{\left(\mu_{a_k} + \mu_s \left(\gamma_k + \frac{h}{m_c^2 \Delta \gamma_k}\right)\right)} \left(1 - \left[\frac{\mu_s}{\mu_k} \left(1 - 3\gamma_k - \frac{h}{m_c^2 \Delta \gamma_k}\right)\right]^n\right)$$

The remaining equations can be solved in reverse order. What is significant is that the coefficient that determines the convergence is frequency-dependent, and, moreover, the higher the frequency, the faster the convergence, provided $\gamma < 0.2$.

Computationally, the iteration scheme has been developed for both the Thomson and Compton scattering subroutines. Presently, the options available for the iteration scheme are the number of iterations, an accuracy criterion to determine convergence, and an extrapolation switch allows one to use the rate of convergence to extrapolate to find the desired function.

SPHERICAL GEOMETRY

In spherical geometry, the transport equation (1) is replaced by

$$\frac{dI}{dx} = \mu_a (B - I) + S$$

where $x$ is now a coordinate measuring distance along a characteristic ray. In the treatment of the scattering terms, the only difference is that the $\mu$'s appearing in $S(x)$ must be replaced by an average $\bar{\mu} = (\mu_1 + \mu_2)/2$, where $\mu_1$ and $\mu_2$ are the cosines of the angles of the characteristic ray with respect to the normal at $x_1$ and $x_2$, respectively.

EQUATIONS OF MOTION

The same independent coordinates are used in the OUTPUT code and SPUTTER code: mass $m$ and time $t$. The Lagrangian mass
coordinate \( m \) is defined by

\[
m = \int_0^r \alpha r^{\alpha-1} \rho(r', t) \, dr'
\]

where \( \rho \) is the density and

\[
\alpha = \begin{cases} 
1 \text{ plane} \\
3 \text{ sphere}
\end{cases}
\]

This coordinate has the physical units of mass/cm\(^2\) in plane geometry; whereas in spherical geometry, it is the mass interior to radius \( r \) divided by \( 4\pi/3 \). If \( \tau \) is the specific volume, the equation of continuity is given by

\[
\frac{d\tau}{dt} = \tau r - (\alpha - 1) \frac{\partial}{\partial r} \left( r^{\alpha-1} \frac{d\tau}{dt} \right)
\]

(54)

The momentum equation can be written as

\[
\rho \frac{du}{dt} = \frac{\partial \sigma_{ij}}{\partial r} + \frac{(\alpha - 1)}{2r} (2\sigma_{11} - \sigma_{22} - \sigma_{33})
\]

where \( u = \frac{dr}{dt} \) and \( \sigma_{ij} \) is the symmetric stress tensor. The stress tensor is given by

\[
\sigma_{ij} = -P_m \delta_{ij} - P_{ij}
\]

(53)

where \( P_m \) is the material pressure including artificial viscosity, and \( P_{ij} \) is the radiation pressure tensor. In the case of plane geometry, \( \alpha = 1 \) and the equation becomes

\[
\rho \frac{du}{dt} = -\frac{\partial(P_m + P_r)}{\partial r}
\]

(54)

where \( P_r \) is the radiation pressure in the \( r \) direction.

This is the standard equation solved in the SPUTTER code. However, the SPUTTER program assumes that \( P_r = (1/3) a \theta^4 \), the equilibrium diffusion value. This assumption is not made in the OUTPUT program; instead, the
radiation pressure for a zone is computed as the second angular moment of the intensity. Since the intensity and its moments are computed on zone boundaries \( r_i \), the radiation pressure in the zone \( r_i \leq r < r_{i+1} \) is defined as the arithmetic mean

\[
P_{\frac{r_i + r_{i+1}}{2}} = \frac{1}{2} (P_{r_i} + P_{r_{i+1}})
\]  

(55)

In the case of spherical symmetry, one must evaluate \( \sigma_{22} \) and \( \sigma_{33} \). The radiation pressure tensor is given by

\[
(P) = \frac{1}{2\pi} \int_{4\pi} d\Omega \tilde{\Omega} \tilde{Q} I(\mu)
\]  

(56)

where \( \mu \) is the cosine of the angle \( \theta \) between the ray \( \vec{r} \) and the radius vector \( \vec{r} \). If \( \vec{r} \) is taken as a polar axis, an azimuthal angle \( \phi \) can be introduced and \( \tilde{\Omega} \) can be expressed as a column vector

\[
\tilde{\Omega} = 
\begin{bmatrix}
\Omega_1 \\
\Omega_2 \\
\Omega_3
\end{bmatrix} = 
\begin{bmatrix}
\mu \\
\sqrt{1 - \mu^2} \cos \phi \\
\sqrt{1 - \mu^2} \sin \phi
\end{bmatrix}
\]  

(57)

The dyadic \( \tilde{\Omega} \tilde{Q} \tilde{I} \) is then obtained by postmultiplying this column by its transpose, which gives

\[
(P) = \frac{1}{2\pi} \int_{-1}^{1} d\mu \int_{0}^{2\pi} d\phi I(\mu) 
\begin{bmatrix}
\mu^2 & \mu \sqrt{1 - \mu^2} \cos \phi & \mu \sqrt{1 - \mu^2} \sin \phi \\
\mu \sqrt{1 - \mu^2} \cos \phi & \mu \sqrt{1 - \mu^2} \cos \phi (1 - \mu^2) \cos^2 \phi & (1 - \mu^2) \cos \phi \sin \phi \\
\mu \sqrt{1 - \mu^2} \sin \phi & \mu \sqrt{1 - \mu^2} \sin \phi (1 - \mu^2) \sin \phi & \mu \sqrt{1 - \mu^2} \sin \phi \cos \phi (1 - \mu^2) \sin^2 \phi
\end{bmatrix}
\]  

(58)

On evaluating the integrals, one obtains

\[
P_{ij} = 0 \quad i \neq j
\]  

(59)
\[ P_{11} = \int_{-1}^{1} \mu^2 I(\mu) \, d\mu \]  
\[ P_{22} = P_{33} = \frac{1}{2} \int_{-1}^{1} (1 - \mu^2) I(\mu) \, d\mu \]  

Since \( P_{22} = P_{33} = 1/2 \left[ E_r - P_{11} \right] \), where \( E_r \) is the radiation energy density,

\[ E_r = \int_{-1}^{1} I \, d\mu \]  

Eq. (52) for the spherical case is

\[ \rho \frac{du}{dt} = - \frac{\partial (P_m + P_r)}{\partial r} - \frac{1}{r} \left( 3P_r - E_r \right) \]  

where \( P_r = P_{11} \). The last term in Eq. (63) is in some cases a source of numerical noise, particularly at small radius \( r \). Hence, a parameter \( S_4 \) is employed in the code such that in the "diffusion" case, \( P_r/E_r < S_4 \), a difference representation of Eq. (63) is used. However, in the "streaming" case, \( P_r/E_r > S_4 \), one can rewrite Eq. (63) as

\[ \rho \frac{du}{dt} = - \frac{\partial P_m}{\partial r} - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 P_r \right) - \frac{1}{r} \left( P_r - E_r \right) \]  

and a difference equation based on this form is used.

\( S_4 \) should be assigned on the basis of the characteristics of the problem being solved; a typical value is \( 1/2 \).

ENERGY EQUATION

The equation for conservation of energy is given by

\[ \frac{dE_m}{dt} + \tau \left[ \frac{\partial E_r}{\partial t} + \nabla \cdot \mathbf{F} \right] = -(P_m + P_r) \tau + \left( \frac{\alpha - 1}{2r} \right) \left( 3P_r - E_r \right) u + \dot{c}_s \]  

where \( E_m \) is the material specific internal energy, \( \mathbf{F} \) is the flux of radiation,
and \( \dot{e}_s \) is an external energy source rate per unit mass. The standard SPUTTER code takes the radiative heating rate per unit mass, \( \dot{e}_r \), as

\[
\dot{e}_r = - (\nabla \cdot \vec{F}) \tau .
\]  

(66)

However, the OUTPUT code includes the radiative streaming contribution, and \( \dot{e}_r \) is calculated in the spherical case as

\[
\dot{e}_r = \left[ -\nabla \cdot \vec{F} + \frac{1}{r} (3P_r - E_r) u \right] \tau .
\]  

(67)

The form \( \tau (\partial E_r / \partial t) \) is combined with \( dE_m/dt \), so that Eq. (65) is treated as an equation for the rate of change of total specific energy. The difference equations are then developed as in the standard SPUTTER code.

The energy source rates \( \dot{e}_s \) are determined by the source routines QUE8, QUE9, QUE10. These source routines are described briefly in Appendix I.

After determining the change in total specific energy during a time interval due to \( \dot{e}_r \), \( \dot{e}_s \), and the radiative and material work terms as shown in Eq. (65), the material temperature is advanced by inverting the equation of state to find the temperature corresponding to the new values of \( E_m \) and \( \tau \).

In order to avoid excessive restriction of the time step during the early portion of a calculation, it has been found advisable to utilize the equilibrium diffusion approximation

\[
E_r = a \theta^4
\]

for the energy density of the radiation field in the energy conservation equation. This approximation is valid in the core, where most of the energy exchange between the field and the material occurs.
LOGIC OF THE CODE

The two basic independent variables in the OUTPUT code are mass and time. Each of these continuous variables is divided into discrete elements, the mass into an array of zones of variable size $M_i$ and the time into a series of time steps $\Delta t^n$. The properties of the system are calculated for all zones at time $t^{n+1}$, from given values at time $t^n$. There are essentially two kinds of properties which define the system: kinematic properties, such as the positions $R_i$ and velocities $\dot{R}_i$ of the zone boundaries; and thermodynamic properties, such as the specific volumes $V_i$, the temperatures $\theta_i$, the specific internal energies $E_i$, the material pressures $P_i$, and the specific heats at constant volume $C_v$. A specification of $M_i$, $R_i$, $V_i$, and $\theta_i$ completely defines the system at any given time, assuming that local thermodynamic equilibrium prevails throughout the system. All kinematic quantities are functions of the first two variables, and all thermodynamic properties are functions of the last two. The heating rates in each zone—specifically, heating from an external source ($\dot{e}_{s_i}$) and heating by radiation transport within the system ($\dot{e}_{r_i}$)—will generally depend on all four variables and the time.

The program is divided into two segments (MP1 and MP2) to reduce computer storage requirements. Segment MP1 sets up the initial conditions. The temperatures $\theta_i$, masses $M_i$, velocities $\dot{R}_i$, and interface positions $R_i$ for each zone are provided by card input. All other quantities necessary for complete problem specification are calculated within the MP1 segment. This segment is also utilized to redefine new zones during the course of a
calculation if, for reasons of economy or greater definition, a rezoning of
the system is desired. MP1 performs the required peripheral operations,
such as reading or changing dump tapes on restarts of a calculation.

Segment MP2 consists of two loops (see Fig. 1). The main loop
represents one cycle, representing an advance in time, and the secondary
loop represents a radiation subcycle. The division of the main loop into
separate boxes corresponds to the way various phases of the calculation
are parceled out to subroutines, called in turn by the main program (MP2).
The labels next to the boxes are in some cases the names of the subroutines;
in two cases, EOS and TEMPIT, they are names of "sub-subroutines" called
by the subroutines. Unlabeled boxes signify computations (or logical
decisions) made in the main program itself.

The cycle begins with a computation of the time increment $\Delta t^{n+1/2}$.
The time increment is set equal to the minimum of (1) DTMAX1, DTMAX2,
and DTMAX3, which are external time controls, (2) Courant stability con-
ditions $\Delta t_c^{n+1/2}$, and (3) nine-tenths of the radiation time step $\Delta t_R^{n-1/2}$. The
nine-tenths used for the radiation time calculation is introduced to prevent
radiation subcycling due to small changes in $\Delta t_R$ from one cycle to the next.
DTR, the time increment used by those subroutines involved in the sub-
cycling loop, is set to $\Delta t_R^{n+1/2}$. The hydrodynamics portion of the cycle is
entered next, where the kinematic quantities are updated. The artificial
viscosity term $P_{2i}$, which enters the calculation as a pressure to be added
to the material pressure, acts as a shock-smoothing term, spreading the
shock structure over three zones. The next two phases of the calculation
compute the heat term $\Delta Q$ in the equation expressing the first law of thermo-
dynamics, $\Delta Q = \Delta E + P\Delta V$. The heating rate due to an external source
($c_{si}^{n+1/2}$) is calculated in one of several choices of sub-subroutines, depend-
ing on the nature of the source.

The heating rate due to radiation ($c_{ri}^{n+1/2}$) is calculated in the RADTN
section. The main subroutine of this section calls an auxiliary subroutine,
Figure 1. The OUTPUT code: Summary Flow Logic
KAPPA, for absorption coefficients. The calculation may be either frequency-independent (grey) or frequency-dependent. In the latter case, the absorption coefficients and radiation fluxes are computed in each of a desired set of frequency groups, and the fluxes are summed up to obtain the total flux at each boundary. Absorption coefficients in the grey case are Rosseland-averaged opacities and are computed from analytic fits to the numerical calculations of Stewart and Pyatt (Ref. 6). For multifrequency calculations, the program uses a table of group-averaged opacities, which are read from a data tape prepared by an auxiliary program.

ENCALC, where the next calculations are performed, gathers the heating rates, computes the work done by each zone from the pressure and the rate of volume change, and finds the increment of internal energy from the first-law equation. One then obtains the corresponding increment of temperature in each zone. After the temperature and specific volume are updated, the equation of state is used to find the internal energy $E$, the material pressure $P$, and the two partial derivatives of $E$, $\partial E/\partial T$ and $\partial E/\partial T$. The difference between the first-law internal energy $E_1$ and the equation-of-state energy $E_2$ is compared to the change in internal energy $\Delta E_1$ as a check on the accuracy of the calculation. If the comparison shows they do not agree to within a certain specified value $S19$, the subroutine TEMPIT uses the regula-falsi and interval halving iteration methods to find a temperature for which the equation of state will return an acceptable $E$. At the conclusion of the energy checks in ENCALC, all quantities describing the system have been incremented from time $t^n$ to $t^{n+1}$. The main program (MP2) checks the time step $DTR$ to determine if the radiation subroutines have induced subcycling. If radiation stability requires subcycling, $DTR$ will have been set such that $\Delta t^{n+1/2}$ is an even multiple of $DTR$. Subcycling continues until the sum of all subcycling time steps is equal to $\Delta t^{n+1/2}$.

The last act in the cycle is the updating of the time $t^n$ to $t^{n+1}$ and increasing of the cycle number by unity. After some decisions have been
made regarding whether to print the information for this cycle and/or to
dump common on a binary dump tape for future restarts, control is passed
to the beginning of the loop and a new cycle begins.

In table I, a brief definition of terms used in figure 1 is given and
the variables used for storage in the code itself are specified.

FLOW CHART OF RADIATION

A flow chart of the radiation hierarchy is presented in figure 2
through 6. The figures that show the various subroutines are as follows:

RAD ............. Figure 2
STRANS .......... Figure 3
PTRANS .......... Figure 4
SCAT ............. Figure 5
STEP ............. Figure 6

OUTPUT CODE GLOSSARY

This section contains a complete list of the FORTRAN variables
appearing in several subroutines of the OUTPUT code. For each variable,
a brief definition or description is given. A number in parentheses super-
script to a variable indicates that the variable has been defined in the list
of another subroutine, as follows:

(1) SCAT
(2) STEP
(3) STRANS
(4) PTRANS
(5) RAD

Variables appear in alphabetical order within a subroutine list. The storage
allocation—Blank Common, a name common, or private storage, which
is used only within the subroutine—is also given.
<table>
<thead>
<tr>
<th>Term</th>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta t_{c}^{n+1/2} )</td>
<td>DTH2</td>
<td>time increment for cycle loop</td>
</tr>
<tr>
<td>( \Delta t_{R}^{n+1/2} )</td>
<td>RDIA</td>
<td>maximum time step for Courant stability</td>
</tr>
<tr>
<td>( \Delta t_{R}^{n+1/2} )</td>
<td>DTRMIN</td>
<td>maximum time step for radiation stability</td>
</tr>
<tr>
<td>( R_{i}^{n+1} )</td>
<td></td>
<td>zone boundary positions at time TH</td>
</tr>
<tr>
<td>( \Delta R_{i}^{n+1} )</td>
<td>DELTAR(i)</td>
<td>( R(i + 1) - R(i) )</td>
</tr>
<tr>
<td>( A_{i}^{n+1} )</td>
<td>A(i)</td>
<td>area of zone ( \sigma r_{i}^{-1} )</td>
</tr>
<tr>
<td>( V_{r}^{n+1/2} )</td>
<td>VD(i)</td>
<td>rate of change of zone volumes</td>
</tr>
<tr>
<td>( P_{2i}^{n+1} )</td>
<td>P2(i)</td>
<td>artificial viscosity pressure</td>
</tr>
<tr>
<td>( e_{s}^{n+1/2} )</td>
<td></td>
<td>rate of energy deposition by source</td>
</tr>
<tr>
<td>( e_{r}^{n+1/2} )</td>
<td>ER(i)</td>
<td>rate of energy deposition by radiation</td>
</tr>
<tr>
<td>( v_{i}^{n+1} )</td>
<td>SV(i)</td>
<td>specific volume of zone</td>
</tr>
<tr>
<td>( \Delta E_{f}^{n+1} )</td>
<td>PB(i)</td>
<td>first-law increment of energy change</td>
</tr>
<tr>
<td>( \theta_{i}^{n+1} )</td>
<td>THETA(i)</td>
<td>temperature of zone</td>
</tr>
<tr>
<td>( E_{i}^{n+1} )</td>
<td>E(i)</td>
<td>first-law internal energy</td>
</tr>
<tr>
<td>( E_{i}^{n+1} )</td>
<td>E(i)</td>
<td>equation-of-state internal energy</td>
</tr>
<tr>
<td>( P_{1i}^{n+1} )</td>
<td>P1(i)</td>
<td>equation-of-state material pressure</td>
</tr>
<tr>
<td>( (\delta E/\theta_{i})^{n+1} )</td>
<td>CV(i)</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>( t )</td>
<td>TH</td>
<td>time</td>
</tr>
</tbody>
</table>
Called from RADTN

Initialize

Call KAPPA for grey opacities.

Calculate $\Delta t_{n+1/2}$.

Set up Y-lines.

Initialize frequency loop (IHNU=1).

If multifrequency, call KAPPA for opacities and define sources.

Set boundary source conditions, retrieve moment quantities from disk, and set Compton terms.

Define source gradients and diffusion regions.

Do diffusion or transport by defined regions. Diffusion is done directly, transport by calls to PTRANS or STRANS.

Multifrequency print if called for.

Update scattering moments. If iteration is required, loop; otherwise, proceed.

Update moments and advance frequency. (IHNU=IHNU+1) If last frequency, proceed; otherwise, loop.

Calculate ER and RETURN.

Figure 2. RAD
Figure 3 (continued). STRANS(N, M)
Figure 3 (continued). STRANS(N, M)
Figure 3 (continued).  STRANS(N, M)
Figure 3 (continued). STRANS(N,M)
Figure 3 (continued). STRANS(N, M)
Figure 3 (continued). STRANS(N, M)
Figure 3 (continued). STRANS(N, M)
Figure 3 (continued). STRANS(N,M)
Figure 3 (continued). STRANS(N, M)
Figure 3 (concluded). STRANS(N, M)
Figure 4. PTRANS
Figure 4 (continued). PTRANS
Figure 4 (continued). PTRANS
Figure 4 (continued). PTRANS
Figure 4 (continued).  PTRANS
Figure 4 (concluded). PTRANS
Figure 5. SCAT
FS = 0.375*FMUS*DX*(((FI0(I1)) + SQMU*FI2(I1))* (1.-HD) + 0.5*BB)

RETURN

AA = R2*(FI0(I1) + SQMU*FI2(I1)) - R1*(FI0(I2) + SQMU*FI2(I2))

FS = 0.375*FMUS/(HD+HD)*(AA*EST+BB* (DX+R1*EST)) - BB*DX*EST/(HD+HD)

Figure 5 (continued). SCAT
Figure 5 (continued). SCAT
Figure 5 (concluded). SCAT
FIGURE 6. STEP
The following table describes the meaning and usage of certain variables in the context of the SCAT code.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AA</strong></td>
<td>Following the notation of Eq. (30). AA is $A \cdot \Delta x$ as defined there. Private.</td>
</tr>
<tr>
<td><strong>A1</strong></td>
<td>This is defined in Eq. (31). It is evaluated in RAD. JIM Common.</td>
</tr>
<tr>
<td><strong>A3</strong></td>
<td>This is defined in Eq. (31). It is evaluated in RAD. JIM Common.</td>
</tr>
<tr>
<td><strong>BB</strong></td>
<td>This is $B \cdot \Delta x$ as defined in Eq. (30). Private.</td>
</tr>
<tr>
<td><strong>CC</strong></td>
<td>This is $C \cdot \Delta x$ as defined in Eq. (30). Private.</td>
</tr>
<tr>
<td><strong>CVB</strong></td>
<td>This input quantity, the negative intensity abort flag, is described on p. 87 of this report. Blank Common.</td>
</tr>
<tr>
<td><strong>DD</strong></td>
<td>This is $D \cdot \Delta x$ as defined in Eq. (30). Private.</td>
</tr>
<tr>
<td><strong>DELTAR</strong></td>
<td>Used in definition of DX in the plane case only. This variable in $r_{i+1}^{n+1} - r_i^{n+1}$, evaluated in HYDRO. Since $r^n$, the SPUTTER variable $C$, is used elsewhere in the radiation routines to define coordinates, it is recommended that the statement two lines below statement 100 be deleted and the GO TO 110 three lines below that be replaced by GO TO 106. This would give a more consistent, cheaper definition of DX. Blank Common.</td>
</tr>
<tr>
<td><strong>DX</strong></td>
<td>The distance from the initial to the final point of the current step along the characteristic ray, given in Eqs. (29) and (30), both as $\Delta$ and $\Delta x$. Private.</td>
</tr>
<tr>
<td><strong>EE</strong></td>
<td>This is $E \cdot \Delta x$ as defined in Eq. (30). Private.</td>
</tr>
<tr>
<td><strong>EST</strong></td>
<td>This is $1 - e^{-\Delta \tau}$, where $\Delta \tau$ is the optical depth of the current step. Evaluated in STEP and kept to avoid redundant calculation. JIM Common.</td>
</tr>
<tr>
<td><strong>FF</strong></td>
<td>This is $F \cdot \Delta x$ as defined in Eq. (30). Private.</td>
</tr>
</tbody>
</table>
| **FIO**  | This is the mixture of moment quantities $3I_0 - I_2$, as defined in Eq. (31). The last character of the variable is a zero. Although doubly indexed (frequency, zone) in Eq. (31), it is singly indexed (zone) in the OUTPUT code. Since the code calculates downward in frequency, the variables for the next upper (previous)
F10 (continued) frequency are available in FOQ, etc. The full set of F10, etc., over frequency is stored on drum or disc, since there is no longer room in core storage for the doubly indexed array. PALMER Common.

F11 This is the mixture of moment quantities $3I_4 - 5I_3$, as defined in Eq. (31). Used only in Compton scattering. PALMER Common.

F12 This is the mixture of moment quantities $5I_2 - 6I_1$, as defined in Eq. (31). PALMER Common.

F13 This is the mixture of moment quantities $3I_3 - 5I_1$, as defined in Eq. (31). F10, F11, F12, and F13 are all evaluated in RAD. PALMER Common.

FMS This zone array is $1/2 \mu_s$, defined in Eq. (5), evaluated in RAD, using SOLID(37), an input quantity, for $\kappa_s$. Equivalenced to SMLA in Blank Common.

FMU This is a linkage variable that provides information about the characteristic line. In plane geometry, it is simply $|\mu|$, the absolute value of the cosine of the angle with respect to the normal. In spherical geometry, however, it is $y^2$, the square of the impact parameter, evaluated once each $y$-line in STRANS. An approximate average value for $\mu$ is calculated in SCAT in this case. In both plane and spherical geometry, the derived parameter is the absolute value of the cosine. JIM Common.

FMUS This variable, set to FMS(K) early in SCAT, is multiplied by GMP before being used in STEP to calculate an approximate $\mu_a/(\mu_a + \mu_g)$. The multiplication by GMP is done only in the case of Compton scattering. It is not clear that this variable is needed as linkage. A slight speed gain is achieved at the cost of clarity. JIM Common.

FMUX This is the angular variable $\mu$ defined beneath Eq. (1). It is unfortunate that the angular variable $\mu$ can be so easily confused with the photon absorption and scattering coefficients $\mu_a$ and $\mu_g$, but it appears to be the common notation and is reflected in this computer program. Private.
FQ0
This is the mixture of moment quantities $3I_{01} - 1_2$ and corresponds to the FI0 $(J + 1, x)$ used in Eq. (30). That is, it is the FI0 zone array of the next higher (previously treated) frequency group. For the first group, these quantities are set zero in RAD and are not used in SCAT. It should be noted that the zero assumption is a particularly poor one as discussed on p. 82 of this report. The last character of FQ0 is a zero, and the array is in PALMER Common.

FQ1
The array corresponding to FI1. PALMER Common.

FQ2
The array corresponding to FI2. PALMER Common.

FQ3
The array corresponding to FI3. PALMER Common.

FS
This is the scattering intensity, the "result" of executing the SCAT routine. It is the right-hand side of Eq. (29). JIM Common.

FS1
The right-hand side of Eq. (29), which gives the detailed formulation of the scattering intensity, has four lines. Except for the factor $\mu_0/\mu \cdot 3/16$, FS1 is an intermediate term representing the first line. Private.

FS2
Represents the second line of the same equation. Private.

FS3
Represents the third line of the same equation. Private.

FS4
Represents the fourth line of the same equation. Private.

GG
This is $G \cdot \Delta x$ as defined in Eq. (30). Private

GMP
This is $1 - 2y$, mentioned on p. 82 of this report. It appears in Eqs. (19) ff. JIM Common.

HD
This is $1/2 \kappa \rho \Delta x$, where $\Delta x$ is the geometrical length of the current step along the characteristic ray, $\rho$ is the density, and $\kappa$ is the photon coefficient $\kappa_g + \kappa_\phi$. Thus, HD, evaluated in PTRANS or STRANS and used in STEP and SCAT, is one-half the optical depth of the step. JIM Common.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>This is $H \cdot \Delta x$ as defined in Eq. (30). Private.</td>
</tr>
<tr>
<td>HVB</td>
<td>This input quantity, the negative scattering intensity debug print flag, is described on p. 87 of this report. Blank Common.</td>
</tr>
<tr>
<td>IALPHA</td>
<td>The geometry flag ($1 = $plane, $3 = $sphere). Used in SCAT to obtain FMUX and SQMU, given FMU. Blank Common.</td>
</tr>
<tr>
<td>IHNU</td>
<td>The frequency group index, used in Compton scattering to branch to simpler coding for the first frequency group. LINDLY Common.</td>
</tr>
<tr>
<td>IMHAD</td>
<td>A divide check abort flag. Private.</td>
</tr>
<tr>
<td>I1</td>
<td>The index of the left-hand boundary of the current step. Evaluated in PTRANS or STRANS, used in STEP and SCAT. JIM Common.</td>
</tr>
<tr>
<td>I2</td>
<td>The corresponding index of the right-hand boundary. JIM Common.</td>
</tr>
<tr>
<td>K</td>
<td>The zone index of the current step, naturally the lesser of I1 and I2 provided that the characteristic ray at closest approach always is tangent to a zone boundary, as it is for the current standard SPUTTER and the OUTPUT code. Replacing K with IZN (see STEP) would remove this as a necessary condition, rendering SCAT safely generalizable. Private.</td>
</tr>
<tr>
<td>NN</td>
<td>This is a thick-thin flag set in STEP to allow SCAT to execute either Eq. (29) or Eq. (32), whichever is appropriate. JIM Common.</td>
</tr>
<tr>
<td>R1</td>
<td>The x-position of the left side of the step, not the radial position in spherical geometry. In STRANS, R1 and R2 are evaluated directly. In PTRANS, R1 and R2 are set to the slab coordinates and then adjusted for slant angle in SCAT. JIM Common.</td>
</tr>
<tr>
<td>R2</td>
<td>The x-position of the right side of the step. JIM Common.</td>
</tr>
<tr>
<td>SQMU</td>
<td>$\mu^2$, where $\mu$ is given by the variable FMUX. Private.</td>
</tr>
</tbody>
</table>
The SPUTTER error flag. See pp. 88-91 of this report. Blank Common.

This is an intermediate quantity

\[ \frac{x_2 - x_1 e^{-\alpha_1 \Delta}}{\alpha_1} - \frac{1 - e^{-\alpha_1 \Delta}}{\alpha_1^2} \]

appearing four times in Eq. (29). Private

This quantity, equivalenced to ACO3T4 in Blank Common, is the transport debug print flag. Zero for normal operation and no print, non-zero for transport debug print. It is changed in SCAT to trigger a print and then turned back in PTRANS or STRANS to forestall more prints, if HVB is set and a negative intensity is encountered.

This is R1**2, used to find a \( \mu \) in the spherical case. Private.

This is R2**2, used to find a \( \mu \) in the spherical case. Private.

This is the square of the impact parameter, used to find \( \mu \) in the spherical case. Private.

This is the intensity on the right side of the step, the "result" of executing the STEP routine. It appears as \( I_j(x_2) \) on the left side of Eq. (27). JIM Common.

This is a zone array of \( \kappa \rho = \mu^i + \mu^s \), equivalenced to BIGB, evaluated in RAD, and used in the formation of \( Q \). Blank Common.
This is a zone array of $e^{-1/2\Delta r}$, where HD (see SCAT) is $1/2\Delta r$. It is formed in STEP if one is proceeding inward in spherical geometry or forward in slab geometry and is available in the reverse case. It is equivalenced to SMLH in Blank Common.

This is the index of the zone being traversed in the current step along the characteristic ray. Evaluated in PTRANS or STRANS, used in STEP. JIM Common.

In subsection 2.1.5 of reference 2, the three initial boundary conditions for $I_{j-1}$ (initial value of $F_2$) are described. For the first two of these, $F_2$ is defined in PTRANS or STRANS, and for the general case, $F_2$ is left over from the previous step. However, for the third case, diffusion, the initial intensity is given by Eq. (2.19) of reference 2, and this is executed in STEP. If LDF is 1, the diffusion boundary condition is applied. If LDF is 2, $F_2$ is assumed to be properly initialized. LDF, in JIM Common, is evaluated in PTRANS or STRANS.

This is the left-right index, used to decide whether to evaluate $H_4$. It is set in STRANS or PTRANS. JIM Common.

This is $NN + 1$, set to allow a computed GO TO on the thick-thin flag. A code reform would increase $NN$ by 1 in the several places in PTRANS and STRANS where it is set, and would eliminate the need for a second variable. Private.

This is, or should be, $\mu_a/(\mu_a + \mu_s)$. In the current, rather crude calculation of this quantity, $Q$ is set zero in case the value should go negative. Private.

This is the slant source gradient along the characteristic ray, evaluated at the left side of the current step, $R_1$. This quantity appears as $\mu_{1-1}(\partial B/\partial h)|_{1-1}$ in Eq. (2.19) of reference 2. JIM Common.

This is the corresponding quantity evaluated at the right side, $R_2$. JIM Common.
This is a zone array of the source $B_i$ in Eq. (24), evaluated at the zone center. Calculated in RAD. Blank Common.

This is a zone array of the source evaluated at the zone boundary according to criteria discussed in subsections 3.2 and 5.1.3 of reference 2 (see also the model in subsection 2.1.1 of reference 2). Equivalenced to X5 in Blank Common, evaluated in RAD, and adjusted in STRANS if TG (see STRANS) is zero.

**STRANS**

Variables defined in the STEP or SCAT lists are given first.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELTAR$^{(1)}$</td>
<td>I2$^{(1)}$</td>
</tr>
<tr>
<td>EST$^{(1)}$</td>
<td>LDF$^{(2)}$</td>
</tr>
<tr>
<td>FMU$^{(1)}$</td>
<td>LRI$^{(2)}$</td>
</tr>
<tr>
<td>FMUS$^{(1)}$</td>
<td>NN$^{(1)}$</td>
</tr>
<tr>
<td>FS$^{(1)}$</td>
<td>R1$^{(1)}$</td>
</tr>
<tr>
<td>F2$^{(2)}$</td>
<td>R2$^{(1)}$</td>
</tr>
<tr>
<td>H$^{(2)}$</td>
<td>S1$^{(1)}$</td>
</tr>
<tr>
<td>HD$^{(1)}$</td>
<td>TG1$^{(2)}$</td>
</tr>
<tr>
<td>H4$^{(2)}$</td>
<td>TG2$^{(2)}$</td>
</tr>
<tr>
<td>IALPHA$^{(1)}$</td>
<td>TRDBG$^{(1)}$</td>
</tr>
<tr>
<td>IHNU$^{(1)}$</td>
<td>X6$^{(2)}$</td>
</tr>
<tr>
<td>IZN$^{(2)}$</td>
<td>Y2$^{(2)}$</td>
</tr>
<tr>
<td>I$^{(1)}$</td>
<td></td>
</tr>
<tr>
<td>ALPHA</td>
<td>The geometry indicator in real form (see IALPHA$^{(1)}$). SPUTTER has both real and integer forms for this variable. Blank Common.</td>
</tr>
<tr>
<td>C</td>
<td>The &quot;old&quot; space variable, $r_i^n$, used throughout the radiation codes. Blank Common.</td>
</tr>
<tr>
<td>CNT1</td>
<td>The updated cycle number, evaluated only if a transport debug print is called. Private.</td>
</tr>
<tr>
<td>Name</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
<td>CSQD</td>
<td>Evaluated in RAD and used there and in STRANS. This is a zone array of the square of the coordinate, C^2, and is provided to save multiplications within frequently executed loops. It probably saves relatively little time and can be considered one of the more expendable arrays. Equivalenced to CRTZ in Blank Common.</td>
</tr>
<tr>
<td>C1</td>
<td>For IAX &gt; 1, there is an interior solid or diffusion region. Even if this consists of many zones, one requires, nevertheless, only a few y-lines. As stated in subsection 2.2.2 of reference 2, &quot;If a diffusion region having an outer boundary r_D exists inside the transport region, y-lines are placed as near as possible to 0.5 r_D, 0.75 r_D, and r_D penetrating the diffusion region.&quot; C1 is set first to 0.5 r_D and then to 0.75 r_D to provide this placement. Private.</td>
</tr>
<tr>
<td>DHNU</td>
<td>The width, in eV, of the current frequency group. Set in STRANS and it is probably not used. LINDLY Common.</td>
</tr>
<tr>
<td>EPSI</td>
<td>Radius of OUTPUT sample tube. Blank Common.</td>
</tr>
<tr>
<td>FL</td>
<td>A separate array of the first moment of the intensity, ( \int_{-1}^{1} \psi \phi ), normalized differently from X2 and used in the evaluation of the scattering moment quantities. Equivalenced to SMLB in Blank Common.</td>
</tr>
<tr>
<td>FLX</td>
<td>An intermediate quantity, ( I_+ + I_- ), along the previous y-line, used in calculating the intensity moments. Private.</td>
</tr>
<tr>
<td>FM</td>
<td>The array of ( I_+ ) along the current y-line. Four intensities are required to calculate the moment quantities for a particular zone boundary and y-band. One can be used immediately after being calculated; the others have to be available in zone array storage. Equivalenced to ER in Blank Common.</td>
</tr>
<tr>
<td>FM1</td>
<td>The intermediate quantity ( (I_+ + I_-) ) along the current y-line minus ( (I_+ + I_-) ) along the previous y-line. Private.</td>
</tr>
<tr>
<td>FNL</td>
<td>The intermediate quantity ( (I_+ - I_-) ) along the previous y-line. Private.</td>
</tr>
<tr>
<td>FNU</td>
<td>The intermediate quantity ( (I_+ - I_-) ) along the current y-line. Private.</td>
</tr>
</tbody>
</table>
Evaluated and used in the "top slice" section, this intermediate quantity is \( I_\text{T} = I_\text{T} + I_\text{T} + I_\text{T} \), where \( I_\text{T} \) is an interpolated value at the top of the arc. See subsections 2.3.2 and 5.2.5 of reference 2. Private.

This is \( 2(I_\text{T} + I_\text{T}) + I_\text{T} \). See discussion of FP. Private.

The zone array of scattering intensity on the inward sweep. Stored for display in the debug print. Equivalenced to SMLD in Blank Common.

The corresponding array of scattering intensity for the outward sweep. Equivalenced to SMLE in Blank Common.

\( I_\text{T} \). See discussion of FP. Private.

On the zone boundary, the x of the previous y-line minus the x on the current y-line. Private.

On the zone boundary, the x of the previous y-line plus the x on the current y-line. Private.

The cube of FXP, used twice in evaluating the third moment of the intensity. Private.

If positive, there is a blackbody exterior to the radiation region. The initial intensity is set accordingly. Blank Common.

The frequency (eV) of the lower limit of the current frequency group. Set to \( 10^{-3} \) for a grey problem. LINDLY Common.

The frequency (eV) at the upper limit of the current frequency group. LINDLY Common.

Depending on SOLID(10), an input quantity. H2 is a zone array of one-half either the Planck or Rosseland optical depth. Planck if SOLID(10) is zero; otherwise, Rosseland. Used to define optical depth in the transport routines. Evaluated in RAD. Equivalenced to EC in Blank Common.

Generally used in STRANS to denote the zone or zone-boundary index. Private.

The left (interior) zone limit of the current transport subregion, inside of which may lie a diffusion region, an interior blackbody, or the center. Private, but linked to RAD by the first argument of the subroutine.
IAXP

An index denoting the lower limit of zone boundaries for which a top-slice calculation is to be made. Private.

IAXP1

The lower limit of zone boundaries to be treated in the transport debug print for a given y-line; this is the maximum of IAX, IAXP-1. Private.

IBX

The right (exterior) zone limit of the current transport subregion. Private, but linked to RAD by the second argument.

IBXP1

This is IBX+1 and is the upper zone-boundary limit of the current transport subregion. Private.

IM

The right (exterior) zone limit of the whole radiation region, set in RAD. LINDLY Common.

IN

The left (interior) zone limit of the whole radiation region, set in RAD. LINDLY Common.

IT

A running index used in setting up y-lines; its function is probably obsolete. Private.

ITUBE

Index of zone boundary at which OUTPUT sample tube is affixed. Set to LMDA(26), the input quantity, near the beginning of STRANS. Used only because subscripted subscripts are illegal in FORTRAN. Private.

J

Used as the running index of the top-slice DO loop, where I is a limit of the loop. Private.

JJ

The y-line index. Private.

JJJ

Set to JJ throughout, its function as distinct from JJ is to be found in the coding given in reference 2 (since deleted). It was involved in a y-line skipping procedure, now prohibited by scattering considerations. A code cleanup would remove JJJ. Private.

K

This is the index for the variable X. Private.

KK

A counter updated from the X array which gives a "master index" for a given y-line. Private.

KKK

An index for X used in the transport debug print. Private.
A dummy argument used in CALL DVCHK, analogous to IMHAD(1). When codes were processed from FORTRAN II to FORTRAN IV by SIFT, the dummy variable KOOOFX was invented, where the middle characters were zeros. By historical accident, the characters in this variable are letter O’s. A code cleanup would change this to KX. Private.

KX
A dummy argument used in CALL DVCHK. Private.

LMDA(26)
The input quantity that defines ITUBE. Blank Common.

NY
The number of y-lines. LINDLY Common.

OX
A zone array storing X for the previous y-line. Since all the X are available, OX could be dispensed with if an index analogous to K but for the previous y-line were defined. Equivalenced to W in Blank Common.

PR
A zone array for the second moment of intensity, $\int_1^\infty \mu^2 \, d\mu$. Used in calculating the scattering moment quantities. Equivalenced to X7 in Blank Common.

Q
Same meaning as $Q^{(2)}$, but different storage. Private.

RHO
A zone array for the zeroth moment of intensity, $\int_1^\infty I \, d\mu$. Used in calculating the scattering moment quantities and, in RAD and elsewhere, as radiation energy density. Blank Common.

SOLID(18)
The current cycle number. Blank Common.

SUMX3
A zone array for $I_-$ along the previous y-line. See discussion of FM. Equivalenced to CHIC in Blank Common.

SUMX4
A zone array for $I_+$ along the previous y-line. See discussion of FM. Equivalenced to BC in Blank Common.

TEMP(1)
In SPUTTER, the TEMP array is used by many subroutines for scratch storage. Liberal use of it was made in STRANS, especially for "top-slice" coding. (See subsections 2.1.2, 2.3.2, and 5.2.5 of reference 2.) All the TEMP variables are in Blank Common. TEMP(1) is used to give half the optical depth of a top slice, or the full optical depth of one side of it.

TEMP(2)
This is $\Delta_1^{1/2}$ (defined in subsection 2.1.2 of reference 2).
This is $r_{1BXP1}^2 - y_j^2$, where $j$ is the index of the last $y$-line used. Intermediate quantity in calculating intensity moments in a top-slice calculation.

$X^2$ of a point on the previous $y$-line.

The corresponding $X^2$ for the current $y$-line. Also used for an entirely different purpose, e.g., TEMP(2) in a top-slice calculation.

Used as the $y$-value of the last $y$-line treated, which need not be $Y(JJ-1)$, since lines may be skipped.

This is $r_i^2 - y_j^2$, used in calculating $FU$ in a top slice.

A special top-slice source gradient, $F$, defined in Eq. (2.13), subsection 2.1.2, reference 2.

$G$ defined in Eq. (2.13), subsection 2.1.2, reference 2.

In the top-slice thin approximation, this is the last term of Eq. (2.15), subsection 2.1.2, reference 2.

The source gradient, defined in RAD. It may be set zero depending on conditions discussed in subsection 5.1.3 of reference 2. See also table II in this report. This is a zone array equivalenced to $V$ in Blank Common.

This is the "OUTPUT-output," in gross outward flux down a tube of specified radius at specified zone boundary. LINDLY Common.

A zone array for the third moment of intensity, $\int_1^\infty \mu^3 d\mu$. Used in calculating the scattering moment quantities. Equivalenced to SMLC in Blank Common.

This is an array of all the values of $\sqrt{r_i^2 - y_j^2}$ for all $y$-lines set up early in RAD. The $X$ array is evaluated in RAD, outside the frequency loop, in order to save taking thousands of square roots within the frequency loop, say, in STRANS. For each $y$-line, in addition to the set of $x$ values, there are stored in the $X$ array the number of $x$ values and the negative of $y^2$. DAVIS Common.

In the final top-slice calculation, this variable is $r_{1BXP1+j}^2 - r_{1BXP1}^2$. Used to calculate an initial diffusion intensity. Private.

For a given cell boundary, this is $X^2$ for the previous $y$-line. Used as an intermediate quantity in evaluating moments. Private.

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XTUBE

This is $\sqrt{\frac{x_{\text{tube}}}{\text{EPSI}}}^2$, the x-value of the intersection of the OUTPUT sample tube with the zone boundary at which the tube is affixed. Private.

X2

This is the first moment of the intensity $\int_{-1}^{1} \mu d\mu$, normalized before exiting STRANS to serve as the radiation flux for the calculation of heating rates. Blank Common.

X8

This is a zone array of the slant source gradient, $\text{TG} \cdot x/r$. Equivalenced to X4 in Blank Common.

Y

This is the array of values of $y$, the impact parameter, for y-lines. Equivalenced to BIGA in Blank Common.

YSQDP

This is convenient storage for $y^2$ of the current y-line. Private.

YSQD1

This is convenient storage for $y^2$ of the previous y-line. Private.

PTRANS

Variables defined in the STEP, SCAT, or STRANS lists are given first.

<table>
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</table>
The left-hand (interior) zone index of the SPUTTER vapor region. Since this may not correspond to the left limit of the radiation region, IN (defined in RAD) should be used and not redefined (perhaps erroneously) in PTRANS. Code revision is needed here. Blank Common.

The right-hand zone limit of region with source. Set to IM in RAD, it is retained solely to procrastinate on cleanup in PTRANS. DAVIS Common.

The left-hand zone limit of region with source. See ICX. DAVIS Common.

IM+1. See IM\(^{(3)}\). Private.

IN-1. See IN\(^{(3)}\). Private.

The index of a characteristic line, analogous to the y-line index JJ\(^{(3)}\). Maximum legitimate value is 5. Private.

JJ+1, used in the transport debug print. Private.

The input quantity specifying the number of characteristic lines to be used (a maximum of 6). Blank Common.

The index of Gauss weights. See RR. Private.

The index of angles. See RR. Private.

The upper limit to JJ, set to LMDA(37)-1. LINDLY Common.

The plane transport calculation is a scheme of evaluating intensities along characteristic lines and integrating these (forming the moments) by the "double Gaussian" method, where special weighting quantities corresponding to the angular intervals are stored. A table of average angles and corresponding Gauss weights for 2, 3, 4, 5, and 6 angular intervals is stored by DATA statement in RR and used in the finite sum formulation in the code. There are 40 entries. Private.

This is \(1/2 \rho \kappa \Delta x\), one-half the slant optical depth, calculated in the case of no source, an option now eliminated. See ICX, ICY, and p. 79 of this report. A code cleanup would delete reference to TEMP(1). Blank Common.
**RAD**

Variables defined in SCAT, STEP, STRANS, or PTRANS are given first.

<table>
<thead>
<tr>
<th>Variable</th>
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<td>FMS&lt;sup&gt;(1)&lt;/sup&gt;</td>
<td>IAX&lt;sup&gt;(3)&lt;/sup&gt;</td>
</tr>
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</table>

**A**

The SPUTTER area term, $\alpha r^{-1}$, a zone array, used in calculating the radiation flux at the external boundary for either diffusion or no vapor. Blank Common.

**A1P**

This is the intermediate quantity $\frac{\hbar^2}{m_e c^2 \Delta \nu_j}$ in Eq. (34). Used in forming A1. Private.

**BETA**

The quantity $\frac{\hbar \nu}{\theta}$, often represented by the variable u. In this case, $\hbar \nu$ is the minimum photon energy for the current frequency group. Private.

**BETAP**

The quantity $\frac{\hbar \nu}{\theta}$, where $\hbar \nu$ is the maximum photon energy for the current frequency group. Private.

**BLANK3**

Used in radiation supercycling. If the time step calculation in RAD calls for supercycling, BLANK3 is set to the current time plus whatever is allowed for the supercycle. Blank Common.
CAPAC

Planck, evaluated in KAPPA or one of its subroutines and used in RAD. CAPAC(150-152) is reserved for special input quantities. See p. 86 of this report. Blank Common.

CAPAR

Rosseland, evaluated in KAPPA or one of its subroutines and used in RAD. CAPAR(121-150) used for up to 15 frequencies of "OUTPUT-output" flux rates and fluxes accumulated over time. This rather clumsy storage mechanism sets an upper limit of 120 zones and 15 frequencies for problems run on the OUTPUT code.

CNTMAX

This is the SPUTTER cycle limit and is tested so that, assuming one wants a multifrequency print with the regular SPUTTER print, a print is obtained for the last cycle. Blank Common.

CPA

This is the Rosseland κ with scattering, adjusted for Compton scattering if appropriate. Used in forming the optical depth arrays. Private.

CPB

This is the Planck κ with scattering, used in forming the H(2) and H2(3) arrays if SOLID(IO) is zero. Private.

CPC

This is the Rosseland κ with scattering, not adjusted for Compton scattering. Private.

CV

This is the array of C_v, specific heat, calculated elsewhere in SPUTTER and used in RAD to formulate the stability time step. Blank Common.

DFB

The portion of the area of the Planck function, normalized to 1, occupied by the current frequency group at a given temperature. Private.

DHNUP

The width of the previous (next higher) frequency group, used in calculating A^3(1). Private.

DTH2

The current SPUTTER time step, set in the TDELT routine. Blank Common.

DTR

The time step that governs RAD. This is usually set to DTH2 in TDELT, but is set smaller in RAD if subcycling is called for, such that the SPUTTER master time step is an integral multiple of DTR. Blank Common.
The actual minimum radiation time step calculated in RAD. This is then used to determine whether subcycling or supercycling is necessary. Blank Common.

A "running minimum" time step, set so that at the end of the loop, the smallest and the next-to-smallest time step can be saved. Private.

A "running second-smallest" time step, corresponding to DTR1 above. Private.

The SPUTTER zone array for internal energy per unit mass, used in the time step energy accuracy calculation. Blank Common.

This is the heating rate due to conduction, zeroed out at the end of RAD to avoid later trouble. Blank Common.

The multifrequency edit flag, equivalenced to S12 in Blank Common. It should be noted that one sometimes wishes to get "multifrequency" prints when running grey problems. Some of the intermediate quantities printed out are of general interest and value in troubleshooting.

This array is used to accumulate radiation energy density over frequency, serving the same purpose as the old variable SUMRHO (see Ref. 2), now deleted. RHO(3) is set to EK after exit from the frequency loop. Blank Common.

This is a special array of internal energy in a "lambda region" (see Ref. 3). It is used in RAD to form quickly the total internal energy in the radiation region for use in the time-step calculation. See WSB, WSBB. Blank Common.

A special storage for the index of the zone which has the smallest radiation time step. Used in PRINT. Blank Common.

The heating rate due to radiation transfer. While the radiation energy density and radiation pressure are used elsewhere in the code, and the flux is edited and viewed with interest, it is safe to say that calculating ER is the basic purpose of RAD and its subroutines. Blank Common.
Temporary storage for FI0 while testing whether to iterate. Unfortunately, the third character is a letter "O," not a zero. Private.

Temporary storage for FI1 while testing whether to iterate. Private.

Temporary storage for FI3 while testing whether to iterate. Private.

This SPUTTER array of mass per volume factor is used in converting energy per unit mass to energy preparatory to calculating the energy accuracy time step. Blank Common.

This is the intermediate quantity $\bar{\gamma}$ defined beneath Eq. (15). Private.

This is the right (exterior) boundary flag, specifying blackbody exterior if $GL = 1/2$, vacuum otherwise. STRANS and PTRANS initialize a blackbody intensity for any positive value of $GL$ (see $GL^{(3)}$). There are historical reasons for the inconsistency, but it is not justified. Blank Common.

Maximum allowed number of radiation supercycles. Blank Common.

See p. 87 of this report. Blank Common.

This is $2\kappa_s \bar{\gamma}$, used to adjust the absorption coefficient for Compton scattering. See Eq. (33). Private.

The fourth power of the upper limit of photon energy for the current frequency group. Perhaps it is not used in the current version of RAD. Private.

Set to the minimum of $HNUP^{(3)}$ and $10^5$ to avoid execution of Compton scattering at invalid frequencies. Used in forming $A3^{(1)}$. Private.

The fourth power of the lower limit of photon energy for the current frequency group. This, like $HNUP4$, may be expendable. Private.

This is $1/2 \rho \kappa_R \Delta r$, where $\kappa_R$ is the local Rosseland mean absorption coefficient adjusted for scattering. Used only in RAD. The array $H2^{(3)}$ is set to $H3$ if SOLID(10) is nonzero. Equivalenced to BR in Blank Common.
IA

The left-hand (interior) zone limit to the vapor region. IN(3) is set to it unconditionally in this version of RAD. IN can take on other values, if the programmer wishes to confine the radiation region to only part of the vapor (e.g., the part in local thermodynamic equilibrium), but this would require a small programming change. Blank Common.

IB

The right-hand (exterior) zone boundary limit to the vapor region. For generality, this should be replaced by IMP1(4). Blank Common.

IBM1

This is IB-1, the right-hand (exterior) zone limit to the vapor region. IM(3) set unconditionally to IBM1 in this version of RAD. Blank Common.

ICXM1

This is ICX(4)-1, evaluated as a separate variable solely for use as the upper limit of a DO statement. Private.

IDMX

The dimension limit for the X array, currently 4000. When testing an index against a dimension limit in the body of a computer program, it is good practice to have this a variable set at the beginning of the code to facilitate later changes in the size of the array. Private.

IJZILC

Purpose unknown. Private.

IMN1

This is the index of the zone with the smallest radiation time step. EO is set to IMN1. Private.

IMN2

This is the index of the zone with the second-smallest radiation time step. Private.

INP1

This is IN+1 defined solely to serve as the lower limit of a DO loop. Private.

IQEM

The index of the zone boundary suffering the worst change in FIO. Used in monitoring the scattering iteration. Private.

IR

The index of the rightmost (exterior) zone for which one wishes to calculate radiation. In reference 2, IR is set and used, but not discussed. In the fireball configuration, it was desired not to do radiation for zones colder than, say, 0.05 eV. IR was set to the index of the outermost zone warmer than that. The code is now in the rather hazardous situation that IR is set to IM if one does scattering or has a blackbody exterior. Otherwise, it is not evaluated at all. This should be repaired. Private.
A y-line index while the \( X^{(3)} \) array is being formed. Also used as a zone-boundary index while enlarging the transport subregions by five mean free paths on each side. Private.

**J**

**JDRUM**

The logical unit, either 25 or 26, of the drum being read for moment quantities. PALMER Common.

**JDRUMI**

The logical unit, either 26 or 25, of the drum on which moment quantities are being written. Private.

**JK**

The y-line index advanced while y-lines are being formed. Private.

**K**

A counter, used as other than zero only if more than 4000 entries in the X array are required to place a y-line every zone. The code will successively try to place a y-line every \((K+1)^{st}\) zone until \( K \) reaches 10, at which point the code aborts. Private.

**KK**

A zone-boundary counter advanced as the X array is formed for a given y-line. Private.

**KKK**

A special index to insert the number of X entries for the current y-line into the X array. Private.

**KMAX**

The multifrequency flag. Zero, grey; nonzero, multifrequency. This is a standard SPUTTER input quantity. Blank Common.

**K000FX**

This divide check flag has zeros, contrasting with \( K00OFFX^{(3)} \). A genuine holdover from the days of SIFT. Private.

**K1**

A counter to skip zones so that a y-line is drawn every \((K+1)^{st}\) zone. See K above. Private.

**LMDA(37)**

Same as \( \text{LMDA}(37)^{(4)} \), but used in RAD for definition of external input intensities, a feature deleted from the OUTPUT code. (See page 81 of this report.) Reference to \( \text{LMDA}(37) \) should be removed from RAD. Blank Common.

**MAXLM**

The number of lambda regions. Used in calculating WSB from ELM. Blank Common.

**NHNU**

The number of frequency groups, specified in the OUTPUT code by \( \text{LMDA}(36) \). See p. 86 of this report. LINDLY Common.

**NMU**

The number of characteristic rays in plane geometry. Set to \( \text{LMDA}(37) \). The FORTRAN statement that does this should be removed. Private.
The number of radiation subcycles, set at the end of the time-step calculation if appropriate. Abort if NRAD > 50. Blank Common.

Set to 1 at the beginning of RAD, it appears never to be used or reset. The statement should be removed. Blank Common.

The number of y-lines if spherical geometry. LINDLY Common.

A dummy variable, it is $\Delta \tau$ (optical depth) in defining source gradients for forced diffusion and serves as the normalized "OUTPUT-output" flux. Private.

The denominator in the relative difference expression for FIO, the iteration test. Private.

The relative difference of FIO, tested against CAPAC(150). See p. 86 of this report. Private.

The largest of the QE's. Used along with IQEM to monitor the iteration. Private.

The numerator in the relative difference expression for FIO, the iteration test. Private.

This is $\rho \Delta \tau$, the formulation of which is geometry-dependent. Used in forming optical depths. Private.

A zone array of $\theta^4$, used to avoid calculating $\theta^4$ within the frequency loop. Very dispensable if one is tight for storage. Equivalenced to PB in Blank Common.

Integrated optical depth, a zone array used only in the section expanding transport subregions by five mean free paths on each side. See subsection 3.2 of reference 2 and p. 80 of this report. Equivalenced to GOFR in Blank Common.

A running integrated optical depth. Quite dispensable. Private.

Zone array for log $\theta$, used in linear-in-log interpolation for multifrequency absorption coefficients in DIANA. Equivalenced to CAR in Blank Common.

Zone array for $-\log \rho$, used similarly. Equivalenced to CHIR in Blank Common.

The SPUTTER array of velocity, used in computing the radiation work term portion of the heating rate. Blank Common.
RDD

Used to store radiation energy density calculated on the previous cycle. Blank Common.

SLUG

The fraction of the internal energy of a zone allowed to leak out (or be added; the latter is causing difficulties) of a zone in the time step calculated in the energy accuracy criterion. An input quantity, usually set to 0.1. Blank Common.

SMLR

Radiation pressure, summed over frequency, formed in RAD, used in HYDRO. Blank Common.

SOLID(10)


SOLID(37)

See p. 86 of this report. Blank Common.

SUMX2

The sum over frequency groups of $X^2$. $X^2$ is set to SUMX2 at the end of RAD. Equivalenced to CRTR in Blank Common.

SUMX3

Same as SUMX3$^3$. Zeroed in RAD for no good reason.

SUMX4

Same as SUMX4$^3$. Zeroed in RAD for no good reason.

SV

The specific volume, $1/p$, a regular SPUTTER zone array. Used in forming optical depths. Blank Common.

TAUX

See the discussion of SOLID(37) on p. 86 of this report. TAUX is added to the absorption coefficient, so it is zero if SOLID(37) is negative; otherwise it assumes the value of SOLID(37). Private.

TAX

This is $2\kappa_g/m_e c^2$. See HHTAX. Private.

TD

A variable set in MP2 as a print flag. Used in RAD to make the multifrequency print coincide with the regular SPUTTER print. Blank Common.

TELM(25)

An input quantity to adjust the time step by a constant factor. Blank Common.

TELM(26)

Set to DTR1. Blank Common.

TELM(27)

Set to IMN1. Blank Common.

TELM(28)

Set to DTR2. Blank Common.

TELM(29)

Set to IMN2. Blank Common.
TEL(M30)  The advanced cycle number, making CNT4\(^{(3)}\) redundant. Blank Common.

TEMP(1,2,3)  Used as intermediate quantities in the time-step calculation. Blank Common.


THETA  The zone array of temperature in eV. Blank Common.

THETAK(103)T4  See p. 86 of this report. Blank Common.

THTAMX  The highest temperature in the problem, formerly used in evaluating IR. Its sole purpose now is to bypass the radiation calculation for problems colder than 0.05 eV throughout. Private.

TSi  Used in calculating X values to indicate the point of closest approach if negative, or x if positive. Private.


WSB  The total internal energy in the problem, used in the energy accuracy time-step criterion to ignore zones whose internal energy is less than 0.001 WSB. Private.

WSBB  The internal energy of a zone in the energy accuracy time-step calculation. Private.

X3  An array of flags to define the transport and diffusion subregions. If for a zone X3 is -1, the zone is in a diffusion region. If X3 = 0, the zone is in a transport region. Other values of X3 are nonsense. Blank Common.

X4  Used to denote y\(^{2}\) for each y-line formed. Later zeroed and used by equivalence in PTRANS and STRANS. Blank Common.

ZP1(18)  Another place for the SPUTTER time step, used in calculating NRAD and DTR if there is to be radiation subcycling. Blank Common.

ZZ  A flag in the scattering moment iteration test. If any zone requires iteration, the flag is turned on. Off: ZZ = 0. On: ZZ = 1. Private.
APPENDIX I

THE OUTPUT CODE AS A MODIFICATION OF SPUTTER

INTRODUCTION AND SUMMARY

The OUTPUT code is a version of SPUTTER (Ref. 3) that is adapted to a special class of problems. Radiation transfer is especially important in determining the behavior of such problems, and the OUTPUT code differs from SPUTTER primarily in its more sophisticated treatment of radiation transfer and in the deletion of space-consuming portions of SPUTTER that deal with matter in the solid state. It should be borne in mind that the OUTPUT code is under development and is continually being changed. Any description of it will therefore rapidly lose accuracy of detail, and major changes made in the near future may go unreported for some time. However, the present configuration is a convenient one to describe thoroughly. Since a large fraction of the content of the OUTPUT code has been described in earlier reports (Refs. 2, 3), much of the material presented in this appendix pertains to changes made in the older programs. Three areas of the OUTPUT code are discussed: (1) changes in SPUTTER subroutines, other than radiation subroutines, (2) changes in the radiation routines, and (3) progress in treating Compton scattering. Changes now in progress include input quantities whose use differs from their use in standard SPUTTER and the abort indicators.
CHANGES IN SPUTTER SUBROUTINES OTHER THAN RADIATION SUBROUTINES

Altered Routines

MP2

This routine has been changed to eliminate references to CNDCTN and the boil codes, which have been deleted. Another change eliminates division by BLANK1 when it is zero. This change has also been incorporated in the standard SPUTTER code.

HYDRO

A small section that does "dummy hydro" (sets C and DELTAR to their appropriate values, but makes no changes in radii or velocities) has been added. This section is executed if S4 is negative. The option is very useful when one wishes to observe the effects of radiation transfer in a static configuration. Changes to delete references to solid material regions and permit the use of more accurate radiation pressures available from the OUTPUT radiation routines are now under development.

EOS

This routine has been changed to incorporate the actual radiation pressure, rather than an equilibrium diffusion approximation. Reference to ZPART, an array of 760 words not used by the OUTPUT code, has been deleted.

ECALC

Changes to eliminate treatment of solids and improve the precision of the source treatment are under development.

RTAPE

This routine, which picks up the desired configuration from the SPUTTER dump tape, has been modified to pick up additional data (moment quantities and, in the future, intensities) and set up the drum storage for them.
WTAPE

This routine does the write operations corresponding to RTAPE. Also, on initial starts, where there are no data to pick up, the drum storage is set up and appropriate starting values are written.

KAPPA

A change has been made to catch division by zero before exit. This may become standard.

KAP6

This routine, which is valuable in test problems, uses the THETAK array to define the frequency table and a corresponding set of absorption coefficients (independent of temperature and density) without using a DIANE tape. It is not available in the standard SPUTTER code.

KAP12

This routine gives an analytic approximation for the grey absorption coefficient of uranium as a function of temperature and density. It is not available in the standard SPUTTER code.

QUE8

This source routine deposits energy (evaluates SMLQ) into a predetermined set of contiguous zones, the energy being distributed uniformly over "mass space." The rate varies stepwise with time. The RDK array is used as input for the zone limits, the time cuts, and the energy deposition rates.

QUE9

This source routine is similar to QUE8, except that it allows for several (up to seven) regions or sets of contiguous zones, each of which has its own set of time cuts and energy deposition rates. RDK storage limits require that no region have more than six distinct time sections.
This source routine is like QUE8 in that it allows only one region. But instead of the energy source being distributed uniformly, it is distributed as some power of the space variable, the exponent being determined by values of the energy deposition rate specified at the limits of the region.

DIVCHK

This is a machine-language routine which makes a report each time a division by zero is made. It is handy to have if one still wishes to abort the run when such a division occurs; in addition, there is a report precisely indicating where the division occurred. This routine, which was written by the Gulf General Atomic systems group, is not generally available or widely advertised, because it carries with it the hazard of uncontrolled computer behavior should the routine be overlaid by other coding subsequent to a call to it. The OUTPUT code precludes this possibility.

The MAP

All large programs on the UNIVAC-1108 make use of the MAP (Memory Allocation Processor) to fit their coding into the available storage with appropriate overlays. The SPUTTER and OUTPUT codes are no exception. The OUTPUT MAP differs from the SPUTTER MAP as follows:

1. DIVCHK is explicitly represented at the independent level with no overlay possible. (SPUTTER does not have DIVCHK at all.)
2. RADTN is segmented along with MP2, allowing an overlay with MP1, rather than being independent.
3. NAMEC (a name common block used only in MP1 and its subroutines) is segmented with MP1, so as to allow overlay with MP2, etc.
4. The overlay of the KAP routines has been deleted because, for reasons that are not now clear, it does not function properly. The storage penalty this causes has been minimized.
5. Several USE cards are provided for those routines bearing the same name as standard, undeleted SPUTTER routines.

Deleted Routines

The standard SPUTTER subroutines CNDCTN, BOIL, and CBOIL have been deleted, as have references to them in MP2. NONEQ and some other subroutines may soon also be deleted.

Dummy Routines

It has been convenient in several instances to write dummy subroutines with the same name as standard SPUTTER subroutines, since outright deletion of the latter would require extensive changes elsewhere in the code. The standard subroutines and the reasons why they are undesirable are as follows:

1. DRAD, ERAD, QUE4. These refer to a 642-word array not needed by the OUTPUT code.
2. QUE16, NONEQ. These refer to a 760-word array not used by the OUTPUT code. NONEQ, furthermore, has nearly 300 words of private data storage.
3. CMOL, KAP5. Both have considerable private data storage.

CHANGES IN RADIATION ROUTINES

Two substantial changes in program organization have been made in the past few months. (1) The codes PRAD and SRAD, executed for plane and spherical geometry, respectively, have been replaced by one code, RAD. The two codes were over 90 percent the same, line for line, and considerable duplication had taken place in maintaining them. The necessary allowances for geometry were incorporated, and the codes were unified. This change has been made both in the standard SPUTTER and the OUTPUT codes. (2) In PTRANS and STRANS, the coding to solve the transport equation was repeated many times and in several forms to allow for different
configurations. (See the STRANS program listing in reference 2.) Considerable duplication of effort has resulted because scattering led to considerable changes in these codes, other minor changes are made from time to time, and more sophisticated solutions of the transport equation are being coded. The advantages of solving the transport equation in only one place in the code became more apparent. The new subroutine, STEP, solves the transport equation in general form for one increment along a characteristic line. Calls to STEP have been incorporated in STRANS and PTRANS. The data block COMMON/JIM/, originally provided for linkage between the transport routines and SCAT, has been expanded by six words to accommodate STEP. The call hierarchy of the radiation routines was formerly as follows:

The new hierarchy, which results in considerable reduction of program storage and much more understandable codes, is as follows:
STEP has been added only in the OUTPUT code.

Another change made in both the standard SPUTTER and the OUTPUT versions of RAD is the elimination of several input parameters. The deleted parameters are described briefly in table II and in detail in reference 2. Some of the parameters listed in reference 2 were assigned different SPUTTER variable names after the reference was issued. The more recent names are also given in table II. In some cases, the option involving the parameter has been removed altogether, and for these no "built-in" value is given. Instances where built-in values differ from the suggestions in reference 2 reflect experience in running problems.

Two rather substantial deletions have been made in the OUTPUT version of RAD. (1) ICX and ICY have been eliminated as special indices indicating sourceless subregions of the radiation region. This eliminates quite a bit of complicated branching, and the time saved by doing simpler calculations in sourceless regions, although substantial in some configurations run on standard SPUTTER, would be insignificant in most applications of the OUTPUT code. (2) The multifrequency merge procedure
TABLE II
DELETED VARIABLES

<table>
<thead>
<tr>
<th>Name Used in reference 2</th>
<th>Recent name(s)</th>
<th>Value suggested in reference 2</th>
<th>Built-in value</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB*</td>
<td>DELPRT</td>
<td>0</td>
<td>---</td>
<td>Brightness print (option deleted)</td>
</tr>
<tr>
<td>GA</td>
<td>GA</td>
<td>0.33</td>
<td>0.333</td>
<td>Source, optical depth gradient criterion (TG)</td>
</tr>
<tr>
<td>GL</td>
<td>GA,GR,(0.333)</td>
<td>0.3</td>
<td>---</td>
<td>Source, MFP gradient criterion (y-lines, option deleted)</td>
</tr>
<tr>
<td>CMIN</td>
<td>AC</td>
<td>0.3</td>
<td>--(∞)</td>
<td>Minimum depth for TG criterion (option deleted)</td>
</tr>
<tr>
<td>ACO3T4</td>
<td>TA</td>
<td>0.1</td>
<td>0.01</td>
<td>Half-optical depth for thick-thin transition</td>
</tr>
<tr>
<td>S15</td>
<td>S15</td>
<td>1</td>
<td>---</td>
<td>Restart on grey calculation, not needed with current DIANA</td>
</tr>
<tr>
<td>TELM(37)</td>
<td>TELM(37)</td>
<td>0.005</td>
<td>0.001</td>
<td>Fraction of total energy in zone for time-step criteria</td>
</tr>
<tr>
<td>CVB</td>
<td>CVB</td>
<td>0</td>
<td>0.5</td>
<td>Select y-lines</td>
</tr>
<tr>
<td>HVB</td>
<td>HVB</td>
<td>5</td>
<td>5</td>
<td>Buffer of transport region in number of mean free paths</td>
</tr>
<tr>
<td>HCB</td>
<td>HCB</td>
<td>0.1</td>
<td>0.1</td>
<td>Diffusion criterion</td>
</tr>
<tr>
<td>Not mentioned</td>
<td>BOILB = NTIMES</td>
<td>--</td>
<td>50</td>
<td>Subcycle limit</td>
</tr>
</tbody>
</table>

*CB is currently used in standard SPUTTER as a multifrequency merge criterion, replacing the constant 10.0 in reference 2. The OUTPUT code has no frequency merging.
has been taken out. Although it may save time in some applications, no provision for merging has been made in the scattering coding. The variable CB, of course, is now not used in the OUTPUT code. Another, less significant feature deleted in the OUTPUT code is the provision for external nonthermal radiation intensities as an energy source in plane geometry.

A major addition to RAD is the calculation of the moment quantities, used as scattering source terms in the SCAT routine, first reported in reference 7. An iteration procedure to obtain better values of the moment quantities at cycle \((n + 1)\) (the scattering equation is implicit in this respect), described in section 1 has been incorporated. The FQ0, FQ1, FQ2, and FQ3 arrays should not be updated inside the iteration loop, since they are the moment quantities for the previous frequency. This error would cause trouble only in Compton scattering. The updating has been moved outside the iteration loop, and the scheme now works well for both Thomson and Compton scattering. The choice of three words at the end of the CAPAC array as input parameters to control the iteration has been retained. These are described in "Input Quantities" of this appendix. However, the size of the array is the standard SPUTTER value of 152.

The X array, the set of all values \(x_{ij} = \sqrt{r_i^2 - y_j^2}\), is calculated early in RAD to avoid taking many square roots within the frequency loop. In reference 2, this array was given 2400 words of essentially private data storage. (This would allow 66 zones if a y-line were drawn at each boundary.) In the standard SPUTTER version of RAD, the X array is stored in 1064 words of SPUTTER Blank Common, allowing for 41 zones with y-line every zone. Since OUTPUT calculations will have many zones as a result of complex configurations, and since accuracy in the scattering calculation requires a y-line at every boundary, the private storage has been reinstated in the OUTPUT code; this storage is 4000 words, allowing for 86 zones. The
scratch area in SPUTTER Blank Common made available by this change has been taken up by intermediate moment quantities and scattering parameters.

Three variables used in the Compton scattering calculation are dependent only on frequency and not on position:

1. \( GMP = 1 - 2\gamma \), where \( \gamma = \frac{1/2(h\nu_+ + h\nu_+ + h\nu_-)}{m_e c^2} \)

2. \( A_1 = \frac{h\nu_-^2}{m_e c^2(h\nu_+ - h\nu_-)} + 3\gamma \)

3. \( A_3 = \frac{h\nu_+^2}{m_e c^2(h\nu_+ + h\nu_-)} \)

Although evaluated in SCAT, they should be calculated only once per frequency in order to save time. This is done in RAD.

PROGRESS IN TREATING COMPTON SCATTERING

Satisfactory results have been obtained for a test problem with spherical geometry analogous to the test problem with plane geometry described in appendix IV. However, some interesting difficulties arose, which have not yet been resolved. At high frequency, for sufficiently large negative \( \frac{dl}{dv} \), where \( I \) may be intensity or some moment quantity (these usually vary similarly with \( \nu \)), the code will calculate a negative scattering intensity. At present, the code sets the negative intensity to zero and goes on. (There are input parameters to trigger a debug print or abort, or both.) The large negative \( \frac{dl}{dv} \), with the concomitant negative intensities, has appeared in two different situations.

First, in the calculation of the highest-frequency group, the starting value for \( I_x(\nu_j) \) is zero. This results in the appearance of negative intensities, but at least some sort of radiation field description is achieved. Two schemes
have been proposed for improving the formulation of $I_{x_j}(v_j)$. The better one will soon be chosen and incorporated in the code. However, this case of the first frequency is a rather artificial way of arriving at large negative $dl/dv$.

The other situation arose when an attempt was made to refine the frequency interval. The result was, for one frequency group, a bistable oscillation in the iteration procedure for obtaining updated moment quantities. It is believed that it arose in the following manner. First, given zero starting values for the moment quantities, the code had a positive $dl/dv$ and calculated scattering intensities, generating moment quantities appropriate for that frequency. Then the code noted that an iteration was necessary and set the "old" moment quantities to the "new" ones. (See "Input Quantities" in this appendix. CAPAC(151) was 0.) It proceeded to do Compton scattering, found large negative $dl/dv$, calculated negative intensities, set them to zero, generated zero moment quantities as a result, iterated again, and at the third pass was right where it started. While this may not be an exact description of what happened, it is probably close enough to indicate the trouble unambiguously. The "extrapolation switch" described in section II of this volume (CAPAC(151) in the code) could easily be used as an interpolation switch by assigning to it a value between 0 and -1. Interpolation is suggested by the fact that the first "crack" at a solution, resulting in large negative $dl/dv$, may have been an overshoot. This idea was tried out with CAPAC(151) = -0.5 and proved successful. There was no convergence difficulty in areas of negative $dl/dv$. However, at lower frequencies the convergence, while monotonic, was discouragingly slow. This suggests a possible frequency-dependent extrapolation switch.

In summary, Compton scattering in the OUTPUT code is at present a laboratory curiosity. It appears to work well, if not reliably, in an interesting test problem. (It takes about three times as long to calculate as Thomson scattering.) However, this tentative state should change shortly as experience is acquired in using Compton scattering in production
problems. It should be emphasized that Thomson scattering has been used for over six months to calculate complex problems, and a number of shortcomings, which appear only in unforeseeable configurations, have been found and overcome.

CHANGES IN PROGRESS

Several areas of current code development relevant to the OUTPUT code are not discussed in detail in this report. These include improvements in the nonequilibrium radiation diffusion code DRAD, incorporation of Compton scattering in DRAD, changes in the energy bookkeeping (routines ECHK and PRINT), and modifications to provide free drum input-output. Three other changes in the program are as follows:

1. Improved source interpolation in spherical geometry. The current code follows the original STRANS (Ref. 2) logic of step-or-linear interpolation of the source, with a linear-quadratic fit across the "top slice." This is being replaced with a linear-in-$r^2$ interpolation scheme developed by J. R. Triplett (Ref. 3). The option of a piecewise constant source, as described in reference 2, is retained. Each "step" along a y-line is divided into two intervals equal in $r^2$ to conform with the new interpolation scheme. Although this change has been essentially completely checked out, it is not included in this report.

2. The Sampson approximation of the average absorption coefficient, where $\bar{\kappa} = (b + \kappa_R)/(b + \kappa_p) \cdot \kappa_p$ and $b$ varies as the integrated optical depth, appears to be a desirable first step toward a more sophisticated transmission function treatment than the one now being used.

3. Saving intensities. Future code development in several areas, e.g., retardation, more accurate calculation of moment quantities, and improved Compton scattering recipes, will require the storage
of the entire set of intensities calculated at cell-ray intersections. This change also was completed too late for inclusion in the present report.

For each frequency, given that the \( x \) storage \((x_{ij} = \sqrt{\frac{x_i^2}{2} - y_j^2})\) for all values on one side of the plane of symmetry is 4000 words, the intensity store will require 8000 words of random access storage, plus 8000 times an upper limit to the number of frequencies for drum storage. This requirement can be met, but not economically on the UNIVAC-1108. (The UNIVAC-1108 has the odd feature of a 3/8-\( \mu \)sec fetch from opposite bank versus a 3/4-\( \mu \)sec fetch from same bank, so that it pays to put coding and data in opposite halves of the "almost-random access" fast storage.) The speed penalty can probably be minimized so as to be negligible on long, expensive problems by an adroit choice of what data are to be kept in the wrong half of the store. Other problems are definition of the appropriate set of intensities for a cell boundary treated by diffusion and definition of intensities for \( y \)-lines where a specific calculation is not made. At present, the OUTPUT code sets up a \( y \)-line at every zone boundary and calculates intensities along all of these. However, one might wish to run a problem with more than 86 zones, the maximum possible for less than 8000 intensities, or one might wish to skip \( y \)-lines to save time.

**FORTRAN LISTINGS**

Appendix III contains FORTRAN listings for the principal subroutines of the OUTPUT code: RAD, PTRANS, STRANS, STEP, SCAT, QUE8, QUE9, QUE10, HYDRO, RTAPE, WTAPE, and the MAP. For brevity, SPUTTER blank common, which is used in all the subroutines, is listed only in RAD. The equivalence table used in RAD, PTRANS, STRANS, and STEP is also listed only in RAD.
INPUT QUANTITIES

The input quantities listed below are (1) parameters used in the Compton and Thomson scattering treatment, (2) special parameters for the "OUTPUT-output," a special edit, and (3) various quantities used by the nonstandard source and opacity routines.

**THETAK(103)**  
Required input. Sets value of HNUP. Compton scattering does not allow large values of HNUP. There is no safety coding, such as "if zero, set to $10^6$." It must be entered.

**SOLID(36)**  
Compton switch. If zero, Compton scattering is calculated; if nonzero, Thomson scattering.

**SOLID(37)**  
Scattering coefficient. If negative, it is not added to the absorption coefficient from the DIANE tape, but the absolute value is used as the scattering coefficient. If positive, $\kappa(DIANE)$ is assumed to be $\sigma_a$ only, and $\kappa_s = SOLID(37)$ is added.

$SOLID(37) = 0$ would provide a very inefficient "no scattering" calculation.

**LMDA(36)**  
Required input for NHNU. The subroutines WTAPE and RTAPE, which set up the drum storage for the scattering source terms and process these on a nonstandard SPUTTER dump tape, must know the value of NHNU before the DIANE tape is read. Using the dimension limit of 20 would avoid this, but would lead to inefficiencies. (MUST BE LOADED IN FIRST SET OF CARDS.)

**CAPAC(150)**  
Scattering iteration convergence coefficient, usually 0.05.

**CAPAC(151)**  
Scattering iteration extrapolation parameter, usually 0.5.

**CAPAC(152)**  

**EPSI**  
Radius of OUTPUT sample tube.

**LMDA(26)**  
Index of zone boundary at which OUTPUT sample tube is affixed.
CVB
If CVB is zero, negative scattering intensities are set to zero and the problem continues. If CVB is nonzero and a negative scattering intensity arises, the code calls UNCLE.

HVB
If HVB is zero, negative scattering intensities go unreported. If HVB is nonzero and a negative scattering intensity arises, a brief data report is given, and a y-line print is triggered.

HCB
The normal SPUTTER RAD meaning applies here. Negative forces diffusion; positive forces transport; zero lets the code decide.

RDK
This array is used in standard SPUTTER to provide input for the various source routines, and the OUTPUT code does the same.

QUE8: Energy is supplied uniformly in mass space within a defined region (i.e., all zones within the region receive a constant ergs/sec/gm) at a rate that varies stepwise with time. RDK(1) is the index of the first zone in the region; RDK(2) is the index of the last zone in the region. Up to 50 time cuts can be specified in RDK(3) - RDK(52). The 49 rates for the corresponding periods (in ergs/sec for the whole region) are given in RDK(54) - RDK(102). Energy source rate for t > RDK(52) is zero.

QUE9: This routine is similar to QUE8, except that the fine time definition is sacrificed for some spatial definition. Up to six different contiguous regions can be defined (seven bounding zone indices provided). Each has a separate set of up to six source rates and time cuts. Detailed input specification appears in the FORTRAN listing.

QUE10: Energy is supplied as \( g^{-x} \) in mass space, where \( g \) is a mass space coordinate and \( x \) is an exponent determined by the average energy rate and by the rate specified at the right boundary. The two rates are specified for each given period. Ten periods are allowed. Time cuts are given by RDK(3) - RDK(13), the average rates by RDK(54) - RDK(63), and the boundary rates by RDK(44) - RDK(53).

THETAK
This array is used as input to KAP6/JP. If one wishes to run a multifrequency problem without using a DIANE tape, one must specify the number of frequency groups and the boundaries, in eV, of the desired...
frequency groups. KAP6 specifies these and also a constant absorption coefficient, independent of density and temperature, for each frequency group. Up to 20 frequency groups are allowed. THETAK(61) - THETAK(80) gives the lower boundary frequencies. THETAK(81) - THETAK(100) gives the absorption coefficients. THETAK(101) gives the number of frequency groups; THETAK(102) gives the grey absorption coefficient; and THETAK(103) gives the upper boundary frequency of the top group, as indicated on p. 86 of this report.

S1 FLAGS IN THE RADIATION ROUTINES

The SPUTTER code follows the practice of setting the variable S1 to some value and calling UNCLE in case of serious trouble. The integer portion of the S1 flag indicates the subroutine, and the four digits after the decimal point indicate the FORTRAN statement number. The S1 flags for the radiation routines in the OUTPUT code are listed and commented on below.

<table>
<thead>
<tr>
<th>S1 Flag</th>
<th>Immediate Cause</th>
<th>Probable Remote Causes and Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.0102</td>
<td>Zero or negative CAPAC or CAPAR (grey)</td>
<td>Zero or negative CAPAC or CAPAR. This usually happens with untested KAP routines.</td>
</tr>
<tr>
<td>13.0112</td>
<td>Small Δt causes more than 50 radiation subcycles</td>
<td>Can be caused on first cycle by improperly set TELM(25) or SLUG, or any time by actual pathologies in CV, THETA, SV, or opacity, or simply by failure to start the problem with a small enough time step.</td>
</tr>
<tr>
<td>13.0150</td>
<td>Divide check</td>
<td>Division by zero. See the preced ing DIVCHK diagnostic print for locations of divide instruction and divisor.</td>
</tr>
<tr>
<td>13.0119</td>
<td>K &gt; '0'</td>
<td>At least 10 attempts have been made to space y-lines so that x-storage is not overtaxed, and these have failed. A code change is necessary, or the problem must be rerun with fewer zones.</td>
</tr>
</tbody>
</table>
### SF Flag

<table>
<thead>
<tr>
<th>Immediate Cause</th>
<th>Probable Remote Causes and Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>Negative INM1</td>
<td>Absurd. Indicates a radiation zone with zero or negative index.</td>
</tr>
<tr>
<td>Zero or negative CAPAC or CAPAR (multifrequency)</td>
<td>See 13.0102.</td>
</tr>
<tr>
<td>Zero or negative optical depth</td>
<td>Something has gone wrong with the definition of H3 in the preceding lines of code.</td>
</tr>
<tr>
<td>GL &gt; 0.9, HCB &lt; 0</td>
<td>An attempt is being made to force diffusion and provide external source intensities, two incompatible problem specifications.</td>
</tr>
<tr>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>Positive X3</td>
<td>Absurd. Code sets X3 either zero or negative as a diffusion flag.</td>
</tr>
<tr>
<td>IALPHA = 2</td>
<td>A transport calculation in cylindrical geometry is being attempted.</td>
</tr>
<tr>
<td>IR &lt; IM</td>
<td>Absurd in present code. IR set to IM earlier.</td>
</tr>
<tr>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
</tbody>
</table>

### PTRANS

<table>
<thead>
<tr>
<th>Immediate Cause</th>
<th>Probable Remote Causes and Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>IALPHA &gt; 1</td>
<td>Somehow PTRANS called for nonplane geometry. Absurd.</td>
</tr>
<tr>
<td>INM1 negative</td>
<td>See 13.0460.</td>
</tr>
<tr>
<td>GL &gt; 0, GL ≠ 0.5</td>
<td>The external source intensity option has been deleted. GL therefore has a meaningless value.</td>
</tr>
<tr>
<td>IBX &gt; IM</td>
<td>Absurd. A portion of the transport region seems to be outside the radiation region.</td>
</tr>
</tbody>
</table>
## STRANS

<table>
<thead>
<tr>
<th>S1 Flag</th>
<th>Immediate Cause</th>
<th>Probable Remote Causes and Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.0001</td>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>14.0005</td>
<td>IALPHA ≠ 3</td>
<td>Somehow STRANS called for nonspherical geometry. Absurd.</td>
</tr>
<tr>
<td>14.0087</td>
<td>y-line index out of range</td>
<td>The most likely cause for this stop is scrambled radii.</td>
</tr>
<tr>
<td>14.0152</td>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>14.0207</td>
<td>X(K) = 0.0</td>
<td>This is an absurd stop. Any occurrence of it in the OUTPUT code would have to be carefully examined for the cause.</td>
</tr>
</tbody>
</table>

## STEP

No S1 interrupts appear in the program.

## SCAT

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>75.001</td>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>75.008</td>
<td>FS &lt; 0, CVB ≠ 0</td>
<td>Negative scattering intensity with abort flag set. If HVB also ≠ 0, a diagnostic print is given.</td>
</tr>
<tr>
<td>75.0009</td>
<td>Divide check in SCAT</td>
<td>Same as 13.0150.</td>
</tr>
</tbody>
</table>

## KAPPA

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0500</td>
<td>Bad QLM (J + 17)</td>
<td>Improper optical property specification resulting in an attempt to call a non-existent KAP routine.</td>
</tr>
<tr>
<td>15.0810</td>
<td>Divide check</td>
<td>Same as 13.0150.</td>
</tr>
<tr>
<td>SI Flag</td>
<td>Immediate Cause</td>
<td>Probable Remote Causes and Comments</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------</td>
<td>-------------------------------------</td>
</tr>
<tr>
<td></td>
<td>No SI interrupts appear in this program.</td>
<td></td>
</tr>
</tbody>
</table>

**KAP6**

**KAP12**

No SI interrupts appear in this program.
APPENDIX II

DESCRIPTION OF INPUT FOR A SAMPLE PROBLEM

The debug test problem described in this appendix was run on the UNIVAC-1108 about September 15, 1967, and on the CDC 6600 at the Air Force Weapons Laboratory, Kirtland AFB, New Mexico, on September 22. The two runs matched within expected roundoff differences. The calculation was a multifrequency transport calculation with Compton scattering, using dummy hydrodynamics and the KAP6 opacity routine.

The problem consisted of a sphere with a radius of 10 cm, which contained 10 zones, with a temperature of 8 keV, and was surrounded by a shell with a thickness of 10 cm, which contained 20 zones, at a temperature of 600 eV. The density was taken as one in all zones, and the Rosseland opacity as 0.2 cm$^2$/g in all groups.

The input deck consists of SPUTTER Common input cards only, except for an initial header card which contains arbitrary identifying information in Columns 1-72. These Common input cards have the following format:

Col. 1: 1 denotes the last card of the deck and must appear on that card. 2 specifies that the data on the card will be converted to fixed point and appear as INTEGER variables in Common.

Blank (or 1) specifies that the data on the card will appear as REAL variables (floating-decimal form) in Common.

Cols. 2-6: Location relative to start of Blank Common in which the first data word on the card is to be stored.

Col. 7: Number of data fields on the card (maximum 7).
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Cols. 8-70: Up to seven data fields, of nine columns each, contain numerical data to be read under the format OP7E9.4. Blank fields generate zeros (unless they are excluded by the field number in Col. 7).

Cols. 71-80: Card identification information.

The test problem input deck is shown in table III. Each card, identified by its initial location number, is discussed below. Data fields containing 0 can, of course, be omitted, but are included in several cases for expository reasons.

68: FREQ gives the number of cycles between prints. CNTMAX, in 69, gives the cycle number at which the problem is to be terminated.

1: Zone-boundary indices for the "lambda" regions (which are usually regions of different materials). For one lambda region of 30 zones, the two limits, 1 and 31, are given.

19: LMDA(19) is a flag which, if >0, will trigger a complete print of Blank Common as if there were an error exit, even if the problem terminates normally. Since the extra information is often useful, setting LMDA(19) to 1 has become standard practice.

36: LMDA(36) is a special quantity specifying the number of frequencies. This is discussed further on p. 86 of this report. In the present case, there were 20 frequencies.

40: Several numbers are given on this card: the SPUTTER variables IA, IB, ICA, ICB, and KMAX, respectively. The nonzero value of KMAX dictates multifrequency. The other numbers, index limits, are required by the special XCARDS as a substitute for normal SPUTTER problem generation.

53: IG is another quantity needed by XCARDS. This is the extreme upper-limit index to the problem. When one remembers that SPUTTER has indices for upper limit to vapor, upper limit to solid, upper limit to radiation regions, upper limit to non-LTE region, etc., the significance of the term "extreme upper limit" becomes clear; and IG does serve a purpose.

65: TMAX is the problem time at which the problem is terminated. An absurdly large value is set here, since one wishes to limit the run by cycle count instead.
<table>
<thead>
<tr>
<th>Column</th>
<th>COMPTON TEST</th>
<th>20 FREQ WITH CAPACITEC(151) = -0.2</th>
<th>SEPTEMBER 18, 1967</th>
</tr>
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<tr>
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<td>2-6</td>
<td>7-8-16</td>
<td>17-25</td>
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<td>6835.0</td>
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<td></td>
</tr>
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<td>31</td>
<td></td>
</tr>
<tr>
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<td>10</td>
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</tr>
<tr>
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<td></td>
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<tr>
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<td>31</td>
<td></td>
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<tr>
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<td></td>
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<td></td>
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(Table continued on next page)
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<th>WITH CAPAC C(151) = -0.2</th>
<th>SEPTEMBER 18, 1967</th>
</tr>
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<td>17-25</td>
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<td>8000.</td>
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</tbody>
</table>

TABLE III (continued)

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77: CVB, described on p. 87 of this volume, is set zero in this case because one knows that negative scattering intensities will arise in the first frequency group, and one wishes to ignore them.

78: This quantity, SLUG, is described on p. 71 of this report. Its normal value, 0.1, appears here.

79: ALPHA (see p. 17 of this report). Set 3. for spherical geometry.

81: HVB (see p. 87 of this report). Set zero to bypass debug print.

83: HCB (see p. 87 of this report). Set zero to let code decide.

90: GL (see pp. 58 and 67 of this report). Set zero to provide exterior vacuum.

93: RHOR. Special flag for air equation of state. Set big by habit. Not needed for this problem.

100: RPIA. If zero, radiation terms are added in the equation-of-state calculation. Normally set zero.

107: TC is the time before which DTMAX1 applies, and after which DTMAX2 applies.

109: TE is the time before which DTMAX2 applies, and after which DTMAX3 applies.

115: The time step is usually controlled by physical considerations (e.g., hydro or radiation time-step controls). However, if the user feels that these might be too generous in some cases, he may specify an upper limit to the time step, one for each of three specified time intervals, namely, DTMAX1, DTMAX2, DTMAX3. Some number must be specified for these. In this problem, it was desired that a quasi-steady-state radiation field be established in a time short compared with cooling times. Since retardation is not included here, an arbitrarily small time step is indicated, and one was chosen.

127: TRDBG, otherwise known as AC03T4, described on p. 54 of this report. Set zero to avoid debug print.

137: S2 is the radiation flag, tested in MP2 and in the switching routine RADTN. A value of 2 means that RAD is to be called. Zero bypasses radiation altogether.

138: S3. Set nonzero to bypass conduction. Since the appropriate subroutine has been deleted from the OUTPUT code, this input quantity is unnecessary.
S4. Its original meaning, "don't call BOIL," has, with the deletion of BOIL, been changed to allow for "dummy hydro" or no motion if set negative. This was desired for this test problem.

S5. Used to indicate the logical unit of the SPUTTER dump tape. The value 10 is a typical one.

EDITMF, otherwise known as S12. Described on p. 66 of this report. Set to 1 to deliver the multifrequency edit.

S16. This is a flag that tells MP1 (the generator-setup-rezone section of SPUTTER) whether the problem is an "initial" or cold start or a "restart." Zero, the former; one, the latter. This problem is an "initial" start, where all parameters must be supplied by the user and a zero-cycle dump prepared. Hence the value zero.

R (radii) for the 31 zone boundaries.

SV (inverse densities) for the 30 zones.

THETA (temperatures in eV) for the 30 zones.

These are the three parameters controlling iteration on the moment quantities. They are described on p. 86 of this volume and are also discussed on p. 83. It is stated on p. 83 that CAPAC(151) = 0. did not work, whereas CAPAC(151) = -0.5 worked well. CAPAC(151) = -0.2, which was used in this problem, proved even better.

OKLM(1). This equation-of-state flag results in a call to EIONM5 for hydrogen. A suitable dummy material.

The value 206. results in a call to KAP6, a special OUTPUT opacity subroutine described in Appendix A.

MAXLM is the number of "lambda" regions in this configuration.

THETAK(61-80) is used by KAP6 to define the frequency groups.

THETAK(81-100) is used by KAP6 to define the opacities.

THETAK(101-103) is used for KAP6 to define NHNU, the grey opacity, and the top frequency limit.

TELM(25) (defined on p. 71 of this report). Set to 0.5 as a holdover from another problem. Not needed as long as the time step is held down by DTMAX1, 2, 3.

SOLID(IO) (defined on p. 71 of this report). Set nonzero for Rosseland opacities. Superfluous when KAP6 is used, as in this case.
A second test calculation has been carried out to test the sensitivity of the results to the parameter S4. The problem was as follows:

Region 1

\[0 < r < 15 \text{ cm}\]
\[\rho = 1.\]
\[\nu = 8.433 r \text{ keV}\]
\[\Delta r = 1. \text{ cm}\]

Region 2

\[15 < r < 150 \text{ cm}\]
\[\rho = 31/r^3\]
\[\theta = 600 \text{ eV}\]
\[g = 10 \text{ (i.e., total mass of each zone is } 4/3 \pi \times 10 \text{ g)}\]

The Rosseland optical depth of Region 2 was 0.7. Three values of S4 were tried: 0.33, 0.5, and 1. The first of these effectively forces use of motion equation differencing based on Eq. (64) even in the diffusion limit. The singularity at \(r = 0\), represented by the last term in Eq. (64), causes the velocity to diverge near the center, as shown in figure 7 for time \(2.36 \times 10^{-9} \text{ sec}\). The use of an S4 value higher than 1/3 allows the use of Eq. (63) in the central region, and more reasonable results are obtained. No significant difference between the results for \(S4 = 0.5\) and 1.0 at low radii or between any of the results for \(r > 4 \text{ cm}\) was noted. A calculation with a very small core radius would presumably indicate that the value 0.5 is superior, but additional work is required before this point can be established.
Figure 7. Dependence of Radial Velocity on Parameter $S_4$
APPENDIX III

LISTING OF OUTPUT CODE Routines

10 IF (RDK=1) RDK(2) = .GT. 0.1 10 TO 20
   20 IL=RDk(1)
      CALL UNCLE.
      DO 30 I=1,IL
         30 IL=1
30 SUMG=SUMG+S(I)
40 DO 50 J=1,152
50 SMLQ(I)=0.
T2= TH + DTH2
IF(T2 .LT. RUK(J)) GO TO 1000
DO 60 J=4,52
IF(RDK(J) .GT. T2) GO TO 70
IF(RDK(J) .EQ. 0.) GO TO 1000
60 CONTINUE
70 J= J-1
SOLIU(I)=RUK(J+50)
IF(RDK(J) .LE. TH) GO TO 40
TEMP(I)=(RUK(J)-TH)+RUK(J+49)*RDK(J+50)*(T2-RDK(J))
SOLIU(I)= TEMP(I)/DTH2
80 CNVRT= CNVRT + SOLID(I)*DTH2
   TEMP(I) = SOLID(I) * .23873241 / SUMG
   DO 90 I=1,1K
90 SOLQ(I) = TEMP(I) * G(I)
1000 FIRST = 1.
RETURN
END
**FOR U9E9/JP1, U9E9/JP1**

**SUBROUTINE U9E9**

```c
** COMMON LMA(J37), NR, USMLR, IA, IB, ICA, ICB
1 KMAX, BLANK, BLANK2, BLANK3, IAP1, IBP1, ICAP1, ICNP1,
2 IT, IH, IKAU, ULANK4, IAM1, IEN1, ICM1, ICDN1,
3 IIP1, IOM1, IALPHA, ULANK5, TH, TMAX, BLANK6, DELPRT,
4 FHEQ, CNTMAX, AK, ASMLR, PUSHA, PUSHB, HOILA, BIOLB,
5 CVA, CVB, SLUG, ALPHA, HVA, HVB, HCA, HCB,
6 EMINA, EMIB, CA, CB, GA, GB, GL, GR,
7 RHIOL, RHOR, EPSL, ERP, RIA, RIB, ROIA, ROIAB,
8 RPIA, HPIU, HPDIA, HPDIB, TPRINT, TA, TB, TC
** COMMON T1, TE, ZTM2, ZTM2, EMKLM, EMKLM, EMKLM, EMKLM,
1 DTMAX1, DTMAX2, DTMAX3, JTR, SWITCH, CO, CMIN, DELTA,
2 GAMA, HCRT, SIGMA, AC, ACOST4, CVNTR, SUMRA, SUMRB,
3 ROIA, ROIA, ROIA, ROIA, ROIA, ROIA, ROIA, ROIA,
4 S4, S5, S6, S7, S8, S9, S10, S11,
5 S12, S13, S14, S15, S16, S17, S18, S19,
6 S20, E0, F0, TAU, ZERO, L (152), DELTAR(152),
7 ASG (152), KO (152), VT (152), RBD (152), SMLR (152),
8 DETA (37), P (152), P1 (152), PB (152), PBI (152),
** COMMON P2 (152), SV (152), RMO (152), Theta (152),
1 W (132), E (152), E1 (152), E2 (152), E3 (152),
2 V (152), G (152), D (152), C (152), X2 (152),
3 X3 (152), X4 (152), X5 (152), X6 (152), X7 (152),
4 SMLA (152), SMLH (152), SMLC (152), SMLD (152), SMLF (152),
5 ECL (152), EFR (152), SMLG (152), SMLH (152), B10 (152),
6 SIGU (152), CV (152), BC (152), BR (152), CHIC (152),
7 CHIN (152), CAPAC (152), CAPAC (152), CAPAC (152), CAPAC (152),
8 CRTPC (152), G0HI (152), FEW (152), CAR (152), OKLM (37),
** COMMON TELM (37), EKLM (37), EKLM (37), EKLM (37),
1 FPRM (37), GPRM (37), GPRM (37), GPRM (37),
2 ZP1 (37), ZP2 (37), SOLID (37), ECHCK (37), RK (104),
3 HL (37), HOK (104), ROK (104), TMETAK(104), TEMP (16),
4 HEAD (12), MAXL, MAXL
```

** DATA FIRST/0,/**

```c
** HNROK(101),**
1 IF(FIRST(101),5) 00 TO 25
DO 20 K=1,N
IL=ROK(K)
RMROK(K)=1.0
ROK(14*K)=0.
20 DO 10 I=IL,IR
10 R0K(14*K)=ROK(14*K)+G(I)
20 CONTINUE
25 DO 30 I=1,152
30 SMLG(I)=0.
```

102
DO 99 K=1,N
   T2=TH*U*TH2
   IF(T2.LT.1E-20) GO TO 100
   M=10+(K-1)*12
   IF(T2.LT.HUK(M)) GO TO 99
   M1=M+1
   MB=M+5
   DO 40 J=M1,MB
   L=J
   IF(HUK(J),GT.,12) GO TO 50
   40 CONTINUE
   GO TO 99
50 SOLU(J)=RDK(L+5)
   IF(RUK(L-1).LT.1.) GO TO 60
   Q=(RDK(L-1)-1.)*HUK(L+4)
   IF(L.EQ.0) Q=0.
   TEMP(J)=((T2-HUK(L-1))*RDK(L+5)+Q)
   SOLU(J)=TEMP(J)/U*TH2
60 CNVRT=CNVRT+SOLU(J)*U*TH2
   TEMP(J)=236.732415*SOLU(J)*HUK(194+K)
   IL=RUK(K)
   IL=IL+1.
   DO 70 I=IL+1,N
   70 SMLO(I)=TEMP(I)*G(I)
99 CONTINUE
100 FIRST=1.
   RPLB=CNVRT
   RETURN

C
C     RDK(1)-RUK(7) CONTAIN HURT OF SOURCE REGIONS
C    RUK(10)-HUK(15) CONTAIN TIME CUTS FOR FIRST GROUP
C    HUK(16)-RDK(21) CONTAIN RATES FOR FIRST GROUP
C    HUK(22)-RUK(27) TIME 2ND GROUP
C    RUK(28)-RUK(33) RATES
C    RUK(34)-RUK(39) TIME 3RD GROUP
C    RUK(40-45) RATES
C    HUK(46-51) TIMES 4TH GROUP
C    RUK(52-57) RATES
C    RUK(58-63) TIMES 5TH GROUP
C    HUK(64-69) RATES
C    RUK(70-75) TIMES 6TH GROUP
C    RUK(76-81) RATES
C    RUK(95-106) CONTAIN MASS OF EACH GROUP
C    RUK(101) NO. OF MATERIAL GROUPS
C
C K IS SOURCE REGION INDEX
C J, L ARE TIME CUT INDICES
C M(K) IS THE RDK TABLE INDICATOR
C
C VEAT. IF TIME EXCEEDS LAST CUT, SOURCE SET ZERO.
C
20 IL=RU(J)
   IR=RU(K)
   SUMG=0.
   DO 30 I=IL,IR
30   SUMG=SUMG+G(I)
   SUMG = SUMG+.5*G(I)
   DO 50 I=1,152
50   SMLQ(I)=0.
   T2*TH + DT=2
   IF(T2 .LT. RUK(J)) GO TO 1000
   DO 60 J=4,52
   IF(RUK(J) .LT. T2) GO TO 70
   IF(RUK(J) .GE. 0.) GO TO 1000
60   CONTINUE
70   J= J+1
   TEMP(2) = RUK(J)+.50
   SOLID(8)=RUK(J)+.50
   IF(RUK(J) .LE. TH) GO TO 80
   TEMP(1)=(RUK(J)-TH)*RUK(J+39)*RUK(J+40)*(T2-RUK(J))
   TEMP(2) = TEMP(2)/TH
   TEMP(2) = TEMP(2)/TH
   SOLID(8) = TEMP(1)/TH
80   BETA = 4.1879*SUMG*TEMP(2)/SOLID(8)-1.
   SUMQ = 0.
   SUMS = G(I-1)
   DO 90 I=IL,IR
   SUMS = SUMS+.5*(G(I-1)+G(I))
   TEMP(1) = TEMP(2)*SUMS/SUMG**BETA
   SMLQ(I) = TEMP(1)+G(I)
90   SUMQ = SUMQ+SMLQ(I)*4.1879
   RPIB = SUMQ-SOLID(8)
   DO 95 I=IL,IR
95   SMLQ(I) = SMLQ(I)*(SOLID(8)/SUMQ)
1000 FIRST= 1.
RETURN
END
SPUTTER COMMON

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COMMON (10-34) RESERVED FOR DESIGNATION OF DIANE TAPE UNIT.

AMASNO (10-34) RESERVED FOR DESIGNATION OF DIANE TAPE UNIT.
C
H=0
SWNL=0.
C
M = IM
IL = N
100 GO 200 J=1; HAAL.
IF(IL.LT.LMDA(J)) GO TO 300
200 CONTINUE
S1 = 15.0200
CALL UNCLE
300 IN=INU(LMDA(J)-1,X)
J= J - 1
IF(OLM(J+17))400,550,400
350 QLM(J+17)=AdS(OLM(J))
400 CONTINUE
SOLU(20)= IL
SOLU(21)=JR
SOLU(22)=J
IF(AHASH(J+17).GT.0.) GO TO 600
L = 2
MATERL=QLM(J+17) + .5
IF(MATERL.GT.200) L=MATERL - 200
IF(MATERL.EQ. 1) L=1
IF(MATERL.EQ. 13) L=5
IF(MATERL.EQ. 101 .OR. MATERL.EQ. 103) L=1
IF(MATERL.EQ. 102)L=8
IF(L.LT. 13) CO TO 500
S1= 15.0500
CALL UNCLE
500 GO TO (1=1.2,3,4,5,6,7,8,9,10,11,12,.L)
1 CALL KAP1
GO TO 700
2 CALL KAP2
GO TO 700
3 CALL KAP3
GO TO 700
4 CALL KAP4
GO TO 700
5 CALL KAP5
GO TO 700
6 CALL KAP6
GO TO 700
7 CALL KAP7
GO TO 700
8 CALL KAP8
GO TO 700
9 CALL KAP9
GO TO 700
10 CALL KAP10
GO TO 700
11 CALL KAP11
GO TO 700
12 CALL KAP12
GO TO 700
600 CALL UXHAI(IJ)
700 UJ HAO 1=IL+1R
CAPAR(I) = AKIN1(CAPAR(I) + 2.620EG/THETA(I))
600 CONTINUE
CALL UVCHK(KX)
GO TO (121, 16, 1K)
010 S1 = 15.0810
CALL UNCLE
620 IL=1R + 1
IF (IL.LT. M) HLTUNH
GO TO 100
END
AFWL-TR-67-131, Vol III

SUBROUTINE KAP6

COMPILED NOVEMBER 11, 1966

C
SPECIAL KAP ROUTINE FOR CONSTANT INPUT MULTIFREQUENCY OPACITIES
C
THETAK(61-80) SPECIFIES FREQUENCY
C
THETAK(81-100) SPECIFIES OPACITY

COMMON /LINOLT/ HNU, NNU, HNU+1, INT, IN, IN+1, HNU+1, THICK+1, HYY
IF (HNU.EQ.0) NHHU = THETAK(101)
HNU = THETAK(HNU+60)
IF (NNU.EQ.0) IL = THETAK(121)
IF (INT.EQ.0) IN = THETAK(100)
DO 10 I = 1, IL
10 IF (INT.EQ.0) GO TO 6

C
GREG KAPPA

CAPAH(11) = THETAK(102)
GO TO 10

C
MULTIFREQUENCY KAPPA
6 CAPAH(I) = THETAK(HNU+60)
10 CAPAH(I) = CAPAH(I)
RETURN
END
COMMON    LMDA(37), ITL, USMLR, IA, IB, ICA, ICB
1 KMAX, BLANK, BLANK, BLANK3, IAP1, IBP1, ICAP1, ICBP1
2 IL, IG, HRAD, BLANK, IAM1, IBM1, ICAM1, ICBM1
3 LIPI, ION1, IALPHA, BLANKS, TH, TMAX, BLANK6, DELPHI
4 FNU, CNTMAX, AH, ASHLR, PUSHA, PUSHB, BOILA, BOILB
5 CVA, CVB, SLUG, ALPHA, HVA, HVB, HCA, HCB
6 EMINA, EMINH, CA, CB, GA, GB, GL, GR
7 RHOL, RHOH, EPIL, EPSI, MIA, MIB, ROIA, ROIB
8 RPIA, RPIL, HPIA, HPILB, TPRINT, TA, TB, TC

COMMON    TO, TE, UTH2, DTH2P, DTH1, DTRWM, DTMAX
1 DTMAX1, DTMAX2, DTMAX3, DTR, SWITCH, CO, CMIN, DELTA
2 GAMM, SIGMA, SIGMA, AC, AC0374, CNVRT, SUMRA, SUMRB
3 ROL1, ROIA1, KOIN1, KOUT1, GMS, S1, S2, S3
4 S4, S5, S6, S7, S8, S9, S10, S11
5 SI2, S13, S14, S15, S16, SI7, S18, S19
6 S20, EO, FO, TAU, ZERO, R (152), DELTAR (152)
7 ASG (152), KU (152), VO (152), ROD (152), SMLR (152)
8 DELT (37), P (152), PI (152), PB (152), PBI (152)

COMMON    P2 (152), SV (152), RHO (152), THETA (152)
1 W (152), E (152), EI (152), EK (152), A (152)
2 V (152), G (152), U (152), C (152), X2 (152)
3 X3 (152), X4 (152), X5 (152), X6 (152), X7 (152)
4 SMLA (152), SMLO (152), SMLC (152), SMLO (152), SMLE (152)
5 LC (152), EH (152), SMLG (152), SMLH (152), SIGA (152)
6 SIGB (152), CV (152), BC (152), BR (152), CHIC (152)
7 CMIA (152), CMAC (152), CAPAR (152), CRIC (152), CRTE (152)
8 CRTPC (152), GUSF (152), FWH (152), CAR (152), ODLM (152)

COMMON    TELM (37), ELKL (37), ELM (37), ECLM (37), FCLM (37)
1 FRLM (37), VLM (37), GLM (37), AMASNO (37), CHROF (37)
2 ZFNL (37), ZPNL (37), SOLID (37), ECHOK (37), RK (104)
3 NL (37), RHOK (104), RD (104), THETAK (104), TEMP (104)
4 MEAD (12), MAXL, MAXLM

IL=SOLU(21)

COMMON    USMLR(120)
1 KNSOLU (22)
2 CHRM=92.0
3 AMASNO=0.3176+9
4 ZPSN=0.48E-03
5 ZPBS=0.81E+04

DO 1090 I=IK, IL
109
AFWL-TR-67-131, Vol III

CAPAH(1) = 1.2
CAPAC(1) = 1
GO TO 1000
1 10$1
TMP(1) = TTHLTA(1)**2
TMP(2) = TTHLTA(2)**2
TEMP(1) = TTHLTA(1)**1.5
C
IF(SV(1) .GE. 10.0) GO TO 200
IF(SV(1) .LE. 1.0) GO TO 10
IXS = 2
SVI = SV(1)
SVI(3:300)
DYDT = ALOGLU(SV(1)/2.3025851)
GO TO 200
5 SV(1) = 1.
TEMP(1) = CAPAH(1)
C
10 IF(TTHLTA(1) = 1004.0/12.12.14)
12 CAPAH(1) = 5.449E6/(TTHLTA(1)**(1.0+1.0)***1.0*E=0.5%SV(1)*TEMP(3))**1**1.0
C
1 SUHSTIV(SV(1))
CAPAH(1) = AMAX1(ULKLM(J+17) * CAPAH(1), -2)
GO TO 20
14 CAPAH(1) = 1.0*1.75E12/TLP(P(4))**1.27892.78421/(TEMP(3)**(7.6E13)
C
1 SUHSTIV(SV(1))
C
CAPAH(1) = AMAX1(ULKLM(J+17) * CAPAH(1), -2)
GO TO 20
20 GO TO (300+JU)+1XT
C
200 CONTINUE
IF (TTHLTA(1) = 2.0) 201,207,20Z
201 EFF = E2
GO TO 300
202 IF (TTHLTA(1) = 2.0) 203,203,204
203 EFF = E2/(TTHLTA(1)**(1.0)***2**2.5E-5*THETA(1)**5
GO TO 300
204 IF (TTHLTA(1) = 1004.0/12.12.14)
205 EFF = 0.5E5/(TTHLTA(1)**(3.0+0.5)***5*THETA(1)**3
GO TO 300
206 EFF = 0.944/THETA(1)**2
300 CONTINUE
dS = S(UH(TTHLTA(1))
WSA = P(EWi(1)+2.0/SV(1))
IF(FLH(1) .LT. 1.0) GO TO 330
IF(WSA .LT. WSA(1))
GO TO 330
330 WSA = AMAX1(WSA, 1.0)**/ACTH(1)***E+EXP(-8.5/THEMA(1)))
335 CAPAH(1) = AMAX1(0.01**/ACTH(1)***1**0.0/WSA**WSA)
C
C
SV(1) = SV(1)
C
350 IF(FLH(1) .LT. 0.0) GO TO 370
360 VSC = E7**/ACTH(TLMP(1)+HALOG(CAPAH(1)+ALOG(TEMP(6)**))
GO TO 380
370 VSC = E7
100 CONTINUE
RETURN
END

110
Q1 FOR RTAPE/ORIG: RTAPE/ORIG: RTAPE/AI
SUBROUTINE RTAPE(TAPE/TAPE/CYCLE)

RTAP0010

SPUTTER COMMON

COMMON LMLD(17), HR, YMLMLR, IA, IB, ICA, ICB, RTAP0070
1 KMAX, BLANK(1), BLANK2, BLANK3, IAP1, IBP1, ICA1, ICBP1, RTAP0080
2 IX, IO, NNAD, ULANK4, IAN1, IAN2, ICAMI, ICRM1, RTAP0090
3 IPI1, IOM1, IALPHM, ULANK5, TM, TMAX, BLANK6, DELPRY, RTAP0100
4 FHEQ, CNT-MAX, AH, ASMLR, PUSHA, PUSHB, NOILA, BOILB, RTAP0110
5 CVA, CVB, SLUG, ALPHM, HVA, HVB, MCA, MCB, RTAP0120
6 EMINA, EMINB, CA, CB, GA, GB, GL, GR, RTAP0130
7 RHUL, RHRM, EPL10, EPS1, HIA, HIB, RDA, ROIB, RTAP0140
8 RP1A, RPI1, RP1A1, RP1B1, TPRIUT, TA, TB, TC, RTAP0150
COMMON TD, J1PU, TD2, TDT2, DTHI, DTMRNM, DTMAX, RTAP0160
1 DTMAX, DTMAX2, DTMAX3, DTR, SWITCH, CO, CMIN, DELTA, RTAP0170
2 GAMA, GCRIT, SIGMA, SIGMA, AC, ACQ3, CNVP, SUNRA, SUNRO, RTAP0180
3 ROIA, ROIAM1, NOIB, ROIBP1, GHS, S1, S2, S3, RTAP0190
4 SS1, SS2, S3, S4, S5, S6, S7, S8, S9, S10, S11, RTAP0200
5 S12, S13, S14, S15, S16, S17, S18, S19, RTAP0210
6 S20, EO, FO, HAU, TAU, ZERO, R (152), DELTAR (152), RTAP0220
7 ASQ (152), HU (152), VD (152), ROD (152), SMLR (152), RTAP0230
8 DELR (152), P (152), PB (152), PB1 (152), RTAP0240

COMMON IP2 (152), SV (152), RMO (152), THETA (152), RTAP0250
1 W (152), E (152), EI (152), KE (152), A (152), RTAP0260
2 V (152), U (152), D (152), C (152), X2 (152), RTAP0270
3 X3 (152), X4 (152), X5 (152), X6 (152), X7 (152), RTAP0280
4 SMLA (152), SMLA (152), SMLB (152), SMLB (152), SMLC (152), RTAP0290
5 EC (152), ER (152), ERL (152), SMLQ (152), SMLQ (152), BNGA (152), RTAP0300
6 BNGA (152), CV (152), BC (152), BK (152), CHIC (152), RTAP0310
7 CMIN (152), CAPAC (152), CAPAR (152), CNCT (152), CRT2 (152), RTAP0320
8 CRT2C (152), UVFX (152), FEM (152), CAR (152), OKLM (152), RTAP0330
COMMON TELM (152), EEKM (152), ELM (152), ECLM (152), RTAP0340
1 FELM (152), WLM (152), GLM (152), AMASNO (152), RTAP0350
2 ELM (152), ZP2 (152), SOLID (152), ECHCK (152), RTAP0360
3 RL (152), RIOK (152), ROAM (152), THEMA (152), RTAP0370
4 MEAD (12), MAXL, RTAP0380
5 MEAD (12), MAXL, RTAP0390

RTAP0040

COMMON /LINDLY, MLHL, Sage, IMH, IMH, IMH, NTMIND, TMIND, THICK, THICK, THICK
COMMON /PALMER, F10152, F11152, F12152, F13152, F14152, F15152,
2 FG1(152), FG2(152), FG3152, JRUN, RTAP0380

RTAPE defines JOHNN on RESTARTS. RTAPE defines it on INITIAL STARTS.

NRNU = LMD(36)

READ (NTAPE) NTAPE

10 READ (NTAPE) COUNT

IF (COUNT.LT.0.OR.COUNT.GT.CYCLE) GO TO 30

IF (ABS(F0) .LT. 1.E-20) READ (NTAPE)
IF (ABS(COUNT - CYCLE) *LT* 1.E-20) GO TO 20
WRITE (6,50) COUNT
50 FORMAT (IH0 5X, 6HCYLE F6.0, 8H SKIPPED)
READ (NTAPE)
UU 15 I = 1, NNNU
15 READ (NTAPE)
GO TO 10
20 READ (NTAPE), JHUNA(11), I=1, 9405
JHUNA = 25
JHUNA(36) = NNNU
UU 25 I = 1, NNNU
READ (NTAPE) F10, F11, F12, F13
25 WRITE (JHUNA) F10, F11, F12, F13
RETURN
30 SI = 40.
WRITE (6, 50) SOLID(34)
40 FORMAT (IH0 5X, 6HCYLE F6.0, 17H NOT ON DUMP TAPE )
CALL EXIT
END
10 IF (EXKL(191) .GT. 501) GO TO 100
 501 CONTINUE
 CALL AUTOZ
 520 IF (GTK(5) .GT. 530) GO TO 530
 530 CALL DEZONE
 201 CONTINUE
 40 CALL TUELT
 T0=TU+1.
 IF (BLANK2 .LT. 1.E-15) GO TO 170
 IF (T1 .GT. FREQ) GO TO 171
 IF (TPRINT .GT. TH .AND. DTH2) GO TO 100
 TPRINT= TPRINT + BLANK2
 IF (TH .GT. TPRINT) BLANK2 = 0.
 GO TO 171
 170 IF (ANODIT(SOLID(18)+1 .EQ. FREQ).AND. (GT. 0.5)) GO TO 100
 171 TU=0.
 100 CONTINUE

C*************************************************************************
C INTEGRAL SOUR
C*************************************************************************
C*************************************************************************

IF (PUSHA) 2000+2010+2001
 2000 IU=IUM1
 GO TO 2010
 2010 IU=IA
 210 CONTINUE
 204 CALL HYDRO
 202 CALL PTWO

C SELECT SOURCE
 IF (AUS(S6) .LT. 1.E-20) GO TO 80
 CALL GUE

C*************************************************************************
 80 CONTINUE
 DTH2=DTH2
 2P1(18)=$DTH2
 2P1(19)=0.0
 84 CONTINUE
 IF (AUS(S6) .LT. 1.E-20) GO TO 88
 CHECK TO SEE IF MAUATION NEEDS TO BE DONE ON CURRENT SUBCYCLE
 85 IF (I1+L .GT. NP .AND. SOL1(11)+1.1.GT.CHTMAX) GO TO 86
 IF (BLANK3 .EQ. 0) 86=88
 CALL NADTN
 86 CONTINUE
 CALL ECALC
 CALL GCALC
 NAR=NRAD-1
 IF (NAR+1.E-9 .GT. 12P1(18)/2P1(18)) GO TO 110
 105 CALL SWITCH (6+ K00DFX)
 GO TO (106+107), K00DFX
 106 S1= 1.0106
 GO TO 660
 107 CONTINUE
 IF (WARN(1) .GT. 1) 10d= 605+.605
 108 2P1(18)=2P1(18)+S1
 2P1(19)=2P1(19)+1.0
C  FORCE SV TO BE CONSISTENT WITH R AND G
   DO 111 I=IA+IU41
   SV(I)=DELTAR(I)/G(I)
   IF(IALPHA.EQ.0) GO TO 111
   TEMP(I)=R(I+1)+K(I)
   IF(IALPHA.EQ.3) TEMP(I)=TEMP(I)*R(I+1)+R(I)**2
   SV(I)=SV(I)*TEMP(I)
111 CONTINUE

309 SOLIU(10)=SOLIU(10)+1.0
   COUNT=SOLIU(10)
   TEMP=DTH*I
   SOLIU(20)=S(I00)
   BLANK4=BLANK4+SOLIU(20)*DTH2

180 IF (COUNT = CNTMAX) 190, 301, 301
190 IF (TH .GT. THAX) GO TO 301
   IF (ABS(BLAN4) .LT. 0.1) GO TO 195
   IF (AKOD(SOLID(10),BLANK4,LT,0.5) CALL WTAP
195 IF (TU .GT. U.B) GO TO 210
   CALL PRINT
   IF(BLAN4,LT,0.5) CALL WTAP

210 CALL SWITCH(Q+KUOFA)
   GO TO(250,3)*KOOFA
250 SI = 1.0250
   GO TO 610
301 SI = 1.0301
   GO TO 610
600 SI = 1.0600
   GO TO 610
605 SI = 1.0650
610 CALL UNCLE
END
IF (I54) 10 = 20
10 IF I1 = 1
GO TO 15
I1 = I1 + 1

15 DELTAN(I1) = K(I1+1) - K(I1)

C (I1) = R(I1)
GO TO 70

20 P(I1) = 0.
Q(I1) = 0.
RHO(I1) = 0.
SV(I1) = 0.

IF (I1 .NE. 1) GO TO 23

R(I1) = R(I1) - P(I1) / G(I1) + A(I1) * DTH1
GO TO 30

23 R(I1) = R(I1) + (2. * (P(I1-1) - P(I1)) / (G(I1) + G(I1-1)) + A(I1)) * 2 * DTH1
GO TO 30

25 IF (I1 .NE. 1) GO TO 27

R(1) = 0.
GO TO 30

27 CONTINUE

CALL UVCHK(KP)
IF (KP .LT. 2) X = SQRT(KP)

30 CONTINUE

R(I1) = R(I1) + HU(I1) * DTH2
GO TO 30

S1 = 7.1
CALL UNCLE

43 A(I) = 3.*R(I)+R(I)
CALL UVCHK(KP)
IF(KP.LE.2) X=3
IF(ABS(R(I+1)*C(I+1))*ABS(R(I)*C(I))/R(I)+R(I)).LT.1.E-6 GOTO 10
10 45
VU(I) = (TEMP(I)+R(I+1)*R(I)+R(I)*R(I)+DELTA(I)+C(I+1)
* C(I+1)+C(I)+C(I+1))/(DTH2)
GO TO 50
45 VU(I) = (R(I+1)*R(I)+R(I+1)*R(I)+R(I)+R(I)).LT.1.E-6 GOTO 70
50 DELTA(I) = TEMP(I)
A(I) = ALPHA**H(I)***(ALPHA-1.)** CALL UVCHK(KP)
IF(KP .GT. 1) GO TO 70
51=7.129
X=1.
X=SQRT(X)
CALL UNCLE
70 CONTINUE
RETURN
END
<table>
<thead>
<tr>
<th>COMMON</th>
<th>LMUA(37)</th>
<th>NR</th>
<th>SMLR</th>
<th>IA</th>
<th>IB</th>
<th>ICA</th>
<th>ICB</th>
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<td>KMAA</td>
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**EQUIVALENCES (LMUA(37), IHH)**

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**SpuTTeR COMMON**

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120
DIMENSION EKL(I)
EQUIVALENCE (FRLM, EKL)

IF (PUSHMA) 10, 5, 3
1 IC = 1B
FRLM(1B) = FRLM(1B) + ITH*2(IH)
IF (ITHMA = .LT. EKL(IH)) 2, 2, 10
2 IF (IUC - 10) 3, 3, 10
3 ICM1 = IUC
IUC = ICBP1
ICBP1 = IUC + 1
CALL LOS (ICBM1)
GO TO 10
5 IC = IA
FRLM(IA) = FRLM(IA) + TH*2(TH)
IF (ITHIA = .LT. EKL(ICA)) 6, 6, 10
6 IF (ICA > EQL) 7, 7, 10
ICA = ICA + 1
CALL LOS (ICA)
10 CONTINUE
DO 160 I = 1, MAXL
ELM(I) = 0
IL = LMDA(IJ)
NVAP = 0
IF (IL + GE. IA, AND. IL. LE. IB) NVAP = 1
IF (J .EQ. MAXL) GO TO 115
IR = LMDA(J+1) - 1
DO 110 I = IL, IR
ELM(I) = ELM(I) + G(I)*K(I)
IF (IFIX(52) .LE. 0) GO TO 105
TEMP(J1) = DELTA(I)
IF (SUM1 .LE. TMB .OR. 1) GO TO 105
102 TEMP(1) = TEMP(1) + (3. * P(I) * (K(I) + TEMP(1)) + TEMP(1)**2)
GO TO 104
103 TEMP(1) = TEMP(1) + (2. * K(I) + TEMP(1))
104 ERLM(J1) = ERLM(J1) + K(1) + TEMP(1)
IF (1 .LT. 52) AND. 1) GO TO 105
ELM(J1) = ELM(J1) + K(1) + TEMP(1)
105 IF (IFIX(52) .LE. 0) GO TO 115
QLM(I1) = QLM(I1) + SMLQ(I1)*DIR
110 CONTINUE
115 IM = IL
IM = IM - 1
IF (NVAP .NE. 1) GO TO 170
FRLM(J1) = FRLM(J1) + DTR*X2(IM)
IF (IM = EQU 1) GO TO 180
GL = GL(I)
IF (IFIX(U1LA) .EQ. 0 .OR. IM = MAX 1) GO TO 160
GL = SOLID(24)*G(18B)
152 IF (PUSHMA) 160, 160, 155
155 \( \text{LM}(J) = \text{LM}(J) + (P(IA) \times U(IA) + P(IA) \times G(IA) \times DTR/(GX + 1 \times G(IA))) \times A(IA) \)

GO TO 160

160 \( \text{LM}(J) = \text{LM}(J) + (P(IM) \times U(IM) + P(IM) \times G(IM) \times DTR/(GX + 1 \times G(IM))) \times A(IM) \)

GO TO 180

170 IF (S0) 180, 175, 180

175 IF (IC * IE, IA * ANU, IC * IE, IU) \( \text{FCLM}(J) = \text{FCLM}(J) + \text{GTR} \times X7(IM) - 1 \times \text{DELT} \times (X6(IM) - X6(IM)) \)

180 CONTINUE

IF (IPX(SUMM), UIU, UIU) GOTO 200

DO 190 I = IA, IIM1

190 \( \text{RDO}(I) = \text{RDK}(I) \times S(I) + E(I) \times (RDO(I) \times RDO(I)) + 1 \times RDK(I) \times RDK(I) \times DTR \)

200 \( \text{FRLM}(14) = \text{FRLM}(14) + DTR \times X3(IH) \)

\( \text{P}(152) = \text{AMAX}1(\text{P}(152), \text{SOLID}(20)) \)

RETURN

END

ECALL110

ECALL1120
AFWL-TR-67-131 Vcl III

W T FOR EOS/OUT: EOS/OUT: EOS/OUT
SUBROUTINE EOS (EOS)
C OUTPUT VERSION OF EOS — CHANGES TO USE THE RADIATION PRESSURE
CALCULATED IN THE SCATTERING TRANSPORT CODE
C COMPILED SEPTEMBER 1, 1967 /KL
C
C COMPILED ON MARCH 31, 1966 ADDING COMMON FOR NRAR (GE)

S P U T T E R E R C O M M O N

COMMON

LMAI1 (17), NR, NMLR, IA, IB, ICA, ICB,
1 KMAX, ULAJK1, BLAIR2, BLAIR3, IAP1, IUP1, ICAP1, ICBP1,
2 II, IMAX, HMAK, IAMA, IAM1, IBM1, ICAM1, ICBM1,
3 IIP1, IGM1, IALPHA, UALK5, IMAX, ITMAX, BLANK5, DELPR,
4 FRLQ, CNTMAX, AR, NMLSR, PUSHA, PUSMH, ROIL1, BOIL1,
5 CVA, CVB, SLUG, ALPHA, UVA, HVU, NCA, HCB,
6 EMINA, EMAB, CA, CB, 6A, 6B, GL, GR,
7 RHOL, RHOM, EPI0, EPSI, RIA, RIR, KRIA, ROIB,
8 RHIA, HPIH, HPIF1, HPIF1B, TPRINT, TA, TB, TC

COMMON

TUM, TLM, TH1, UTH1, UTH2, UTM2, OTRM1, OTMAX
1 OTMA1, UTHMAX, DTMAP, UTR, SWITCH, CO, CMIN, DELTA
2 GAMMA, WCKIT, SIGMAU, AC, AC03, CNVRT, SUNRA, SUNRU,
3 ROIL, ROIL1, KM1, KOL1, RO1B, GMS, S1, S2, S3,
4 SN, SPM, SPM, S7, S8, S9, S10, S11
5 S12, S13, S14, S15, S16, S17, S18, S19,
6 S20, EO, FO, TAU, ZERO, R (152), DELTAR (152),
7 ASG (152), RU (152), UD (152), RDO (152), SPLR (152),
8 DEIK (152), P2, P1 (152), P1 (152), PH (152), PB1 (152),

COMMON

P2 (152), SV (152), RMO (152), THETA (152),
1 E (152), E1 (152), EN (152), A (152),
2 G (152), D (152), C (152), X2 (152),
3 X3 (152), XN (152), XN (152), X (152), X7 (152),
4 SMLA (152), SMLH (152), SMLRC (152), SMUR (152),
5 CE (152), ER (152), SME (152), SMUR (152), SDLG (152),
6 BIGU (152), CV (152), DC (152), GR (152),
7 CMHA (152), CAPAC (152), CR1 (152), CR2 (152),
8 CTRPC (152), GOFR (152), E FW (152), CAH (152),
9 COMMON

FILE (37), EKLM (37), ELH (37), FCLM (37),
10 FRLM (37), KLM (37), OLM (37), OHASH (37),
11 ZP1 (37), ZP2 (37), SMID (37), ECCK (37),
12 RL (37), RHOK (104), ROK (104), THERM (104),
13 HEAD (12), MAXL, MAXLM
COMMON/LMS/ E100(20)
EQUIVALENCE(E100,1.2,3'14,15,16,17,18,19,20,21)
COMMON/LM5-MATN, IlLMUM, SHAFUJ.12,13,14,15,16,17,18,19,20,21
DATA MATNM, IL/LMUM((1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21)
EQUIVALENCE (L/MATNM, L/MATNM)
EQUIVALENCE (SHAFUJ,PATH)
COMMON/LM5/ H1-1
DIMENSION Z1111, PART(11)
EQUIVALENCE(M(11), DOLMUM, (M(2),2(2)), (M(3), PART(3))
COMMON/LM5/ TLMD(30)
EQUIVALENCE(ITL=5(15), DOLTAU)
EQUIVALENCE(ITL=5(15), DOLTAU)

COMMON/CNTNL/SCYCLE*JMULT
COMMON/6L/NUMAR
REAL NUMAR
NUMAR= 1.
IF(SOLID(10), EOS, SCYCLE) = 10 = 0
1 = W
IX = NR - 1
NX = 1
IF(PSHA) = 303, 2, 302
302 IX = NR
NX = 1
IF (1 = 1) (11, 2)
303 IX = NR
NX = 1
IF (1 = 1) 21, 1
1 E(I) = CVA + THLTA(I) + 0.25
I=CV(I)
PB(11) = 0.0
I = PL(I) = 0.0
GO TO 500
2 CONTINUE
DO 1 I = 1, MAXLM
1 E(I) = CV(I)
EOS 1120
EOS 1130
EOS 1140
EOS 1150
2 GO TO 500
EOS 1170
EOS 1180
EOS 1190
EOS 1200
EOS 1210
EOS 1220

C IF (OKLM(J), EQ. 0.) GO TO 4
C MATERIAL NUMBER OF ZERO LEADS TO AN ERROR EXIT
C IF I(NUMOKLM(J)) = 0 GO TO 1
C MATERIAL NUMBER LESS THAN ZERO AND GREATER THAN +1 OR GREATER
C THAN ZERO AND LESS THAN ONE WILL BYPASS ALL EQUATIONS OF STATE.
C IF (OKLM(J)) 3,4,57
3 L= 3-OKLM(J)
ERROR = -2.
GO TO 58
4 CONTINUE
C MATERIAL MELTIFH OF ZERG LEADS TO AN ERROR EXIT
S1 = 4.0004
CALL UNCLE
C OKLMJ1 NOT PROPERLY ENTERED IN INPUT
C
57 LE = 
ERROR = 0,
IF ((1-1)x143/.XGT.0 ..AND. ZY1(126.1+X.E.0.) ) ERROR = -3.0
58 CONTINUE
IF (L.LE.200 OR.L.GE.301 AND.L.LE.400) GO TO 7
IF (L .LE. 100 .AND. S'LX(1) .GT. RHOR) GO TO 10
C TO AVOID 'EIONX', SET RHOR .EQ. 1.036.
C
GO TO 7
C 10 CONTINUE
C THIS PATH IF OKLMJ1 .LE. 200, FORCES 'EIONX' IF AND ONLY IF
C TEMPERATURE OF ZUIE AT START OF CYCLE .GT. RHOR.
C L = 102
GO TO 7
C
23 CONTINUE
C THIS PATH IF OKLMJ1 .LE. 102, FORCES 'E58' IF AND ONLY IF
C TEMPERATURE OF ZUIE AT START OF CYCLE .LE. RHOR.
C L = 208
GO TO 8
C
7 CONTINUE
IF (L .EQ. 102 .AND. SYA(1) .LE. RHOR) GO TO 23
C TO AVOID 'E58' SET RHOR .EQ. -1.380.
C
IF(L.EQ.101.OR.L.GT.102.OR.L.GE.6.OR.L.LE.306) EION(14)=RIA
IF(EHOR.EQ.(-3.0)) ZOAH = FLW(1)
IF (OKLMJ1 .GT. 106.) GO TO 60
CALL EIONX(THETA(I),SY(1)+EROR)
GO TO 61
60 CALL EIONX (THETA(I),SY(1)+EROR)
61 CONTINUE
C IF (L.GE.201 .AND. L .LE. 300 ) GO TO 9
NBARE = EION(17)
IF (S11) 54+52,51
52 IF (EROR) 53+54+55
53 S11 = EROR + 100. * FLOAT(1)
GO TO 51
51 IF (EKHR) 53+54+55
53 S11 = EROR+FLOAT(1)*100.
C 54 CONTINUE
C
C IF (EION(14) .EQ. 0. ) GO TO 901
S12 = EION(14)
CALL UNCLE
C EION(14) IS SET EQUAL TO ZERO BY ANY VALID EXIT
C
901 CONTINUE
C
C
P1(1) = EION(7)
E(1) = EION(6)
IF ((E(1) > KX.GT.0 .AND. (E1(26).GT.0.) ) GO TO 203
LVT(I) = EION(9)
AS1(I) = EION(11)
FE=1(1) = EION(13)
PBU(I) = EION(19)
IF (KIB.EQ.0.) GO TO 199
IF (SV(1).GE.0. .AND. (SOLIDU(17)/SV(1)-1).GT.0 )
ASW(I) = SGRTASBU(I)**2*2. E12*SOLIDU(17)*(SOLIDU(17)/SV(1)-1))
GO TO 199
8 CONTINUE
C 9 CONTINUE
L = L-200
GO TO (11,12,13,14,15,16,17,18,19,20,21,22) .L
11 CALL ES1
GO TO 200
12 CALL ES2
GO TO 200
13 CALL ES3
GO TO 200
14 CALL ES4
GO TO 200
15 CALL ES5
GO TO 200
16 CALL ES6
GO TO 200
17 CALL ES7
GO TO 200
18 CALL ES8
GO TO 200
19 CALL ES9
GO TO 200
20 CALL ES10
GO TO 200
21 CALL ES11
GO TO 200
22 CALL ES12
GO TO 200
6 CONTINUE
100 CONTINUE
C ASO IS SPEED OF 500Hw
200 ASO(I) = SQRT(.29*GAMA*P1(I)*SV(1))
199 IF (IFIX(SZ).EQ. 5) GO TO 404
GO TO 150
404 P1(I) = RHO(I) * 0.33333333 + P1(I)
GO TO 400
150 IF(HPIA) < 0.201 .400
201 TEMP(I) = THETA(I)*44
P1(I) = PI(I) + 5*(SHLK(I)*SHLK(I)*PI(I))
PBU(I) = 137.0*TEMP(I)*P91(I)
E1(I) = P1(I) + 137.0*TEMP(I)*SV(I)
CV(I) = CV(I) + 548.0*TEMP(I)/THETA(I)*SV(I)
EOS 1520
400 IF (SV(1).LT.60.) GO TO 401
IF (ECHCKJ(I+17).EQ.0.) ECHCKJ(I+17) = HVA * HCA - 1.5 * AMASHOT(I)
1 *(HCA/CVA) + 0.25) *(1./HVAR + FE1(I))
401 E(I) = E(I) + ECHCKJ(I+17)
500 RETURN
END
SOLID(37) IS THE SCATTERING COEFFICIENT
P
SOLID(36) IS THE COMPTON SWITCH. ZERO=COMPTON NONZERO=THOMSON.
P
F10, F11, F12, F13 FREQUENCY-DEPENDENT SCATTERING SOURCES.
P
READ IN FROM DRUM, ALTERNATELY FROM LOGICAL UNITS 25 AND 26.
P
RECIPROCAL ELECTRON REST ENERGY IN EV**-1 IS 1.95692E-6
P
IUMX = 4000
P
TN = 1.
P
TAX = ABS(SOLID(37)) * 1.95692E-6
P
IF (ABS(SOLID(36)) .GT. 1.E-20) TAX = 0.
P
JDRUM = 51 - JUHUM
P
REWIND JDRUM
P
C

NO VAPOUR ZONES

C

30 X2(IMI) = 1.043512 * A(INP1) * (THETA(IMI)**4 - THETA(IMI)**4)
Q1(IMI) = X2(IMI)
GO TO 130

40 THAMX = 0.025
DO 100 IN1 = IM1
C

SET UP FOR KAPPA INTERPOLATION

C

G1(IMI) = THETA(IMI)**4
Q37(IMI) = ALOG(THETA(IMI))
Q38(IMI) = ALOG(5V(IMI))
FMS(1) = ABS(SOLIU(57)) * 0.5 / SV(IMI)
CSGD(1) = C(IMI)**2
IF (THETA(IMI) .LT. THAMX . AND. THAMX = THETA(IMI))
P

100 CONTINUE

IF (THETA(IMI) .GT. THETA(IMI))
THAMX = THETA(IMI)
IF (IM1 .GT. 0 . AND. THAMX .LT. THETA(IMI))
THAMX = THETA(IMI)
P

2 GO TO 130

CSGD(IMI) = C(IMI)**2
IF (AD(SOLIU(57)) .GT. 1.0E-20) IN = IM

100 CONTINUE

CALL KAPPA(IN1)

C

MINIMUM RADIATION TIME STEP

C

I5 = 0
DO 161 IM1 = 1, MAXLM

161 I5 = SS + ELM(I)
DTR1 = 6.110
DTR2 = 5.10
DO 107 IM1 = IM1

IF (AMIN(CAPAC(1), CAPAR(1)) .GT. 0.1) GO TO 102
S1 = 13.0102

CALL UNCLE

102 IF (AD(SOLIU(101)) .LT. 1.2E-20) GO TO 103

TEMP(1) = CAPAR(1)
TEMP(2) = CAPAR(1)
GO TO 104

103 TEMP(1) = CAPAR(1)
TEMP(2) = CAPAR(1)

104 IF (THETA(1) .LT. 0.01) GO TO 107
H(1) = 0.5 * TEMP(1) / SV(IMI) * DELTAR(1)

129
C IF (A116(.LT.1.E-20) .OR. WSHU .LT. 1.E-08 . OR . WSHU) GO TO 105 P
C ACCURACY CRITERION P
C TEMP(1)=SLUG#*WSHU/ABS(EPH(1)) P
C STABILITY CRITERION P
105 TEMP(2) = TELM(25) - (A111(TEMP(1)) - (.5 + 3.5 * ETA(1)*2) * CV(1)) P
   IF (TEMP(2) .LT. 1.E-20) GO TO 107 P
   IF (TELMP(2) .LT. UTH1) GO TO 106 P
   DTH2=UTH1 P
   IWM2=IWM1 P
   UTH1=TELMP(2) P
   IWM1=I P
   GO TO 107 P
106 IF (TELMP(2) .LT. UTH2) GO TO 107 P
   DTH1=TELMP(2) P
   IWM2=I P
107 CONTINUE P
   DTM=IWM2*UTH1 P
   EG=IWM1 P
   IF (IGM1 .GT. TELM(26)) GO TO 108 P
   TELM(26)=UWH1 P
   TELM(27)=IWM1 P
   TELM(28)=DTH2 P
   TELM(29)=IWM2 P
   TELM(30)=SOLU(110)*1 P
108 IF (UTKMIN+UTH) 111+112+109 P
109 ULANG=UTH+AMIN1*UTKMIN+GAM+DTH2 P
   GO TO 112 P
111 NMO=ZP1(118)/UTKMIN + 1 P
   UTH2=ZP1(118)/FLUAT(NHAD) P
   IF (NMO .LT. 50) GO TO 112 P
   SL=1.0112 P
   CALL UCHL P
112 CALL UCHK(KA) P
   GO TO (150+160)+ KX P
150 S1 = 1.0100 P
   CALL UCHL P
160 IF (S1 .LT. 0.1) IR = IM P
   GO TO (1120+113+1131+1131+1131) P
C C ANACRISTICS IN NONPLANE GEOMETRY -- SET UP X AND Y OUTSIDE FREQ LOOP P
C 113 K = 0 P
C 114 KSK= P
C Y(1) = 0.0 P
C Y(1) = 0.0 P
C 115 JK = 1 P
C C******************** SET UP Y LINES P
C 1470 P
C 1500 P
C 1530 P
C 1540 P
C 1550 P
C 1560 P
C 1570 P
C 1580 P
C 1600 P
C 1610 P
C NO Y GREATER THAN C(I+1)
C IF (IN .LE. 1) GO TO 115
JK = 2
Y(2) = C(IN)
X4(2) = CSQU(I+1)
115 DO 116 I = IN+1, IN
IF (AUS(J37(I+1) - G37(I)) .LE. 0.5) 117, 117, 116
116 JK = JK+1
Y(JK) = C(I+1)
X4(JK) = CSQU(I+1)
K1 = K
GO TO 118
117 IF (K1 .LE. 0 .OR. I .EQ. IN) GO TO 116
K1 = 1
118 CONTINUE
ITY = JK
GO TO 120
119 K2 = 1
IF (K .LE. 10) GO TO 114
S1 = INTK1
CALL UNCLE

C***************************************************************
C FIND COMPLETE SET OF X VALUES  (INTERSECTIONS OF RAUII WITH Y LINES)
C***************************************************************
C**************************************************************************
C FORMAT = -X4, -NUMBER OF INTERSECTIONS+x+S.
C**************************************************************************
C
120 K2 = 1
UO = 125 J = 2
J = 1 + 1
X(K2) = -X4(J)
K2 = K2 + 2
K1 = 1
121 T51 = CSQU(I - X4(J))
IF (T51) 124, 124, 122
122 X(K2) = SQRT(T51)
K2 = K2 + 1
IF (K2 = 10) 123, 123, 119
123 I = 1
K = KK + 1
GO TO 121
124 KK = K2
X(KKK) = (KK = 1)
125 CONTINUE
C FINISH X-BLOCK WITH A NEGATIVE NUMBER
C**************************************************************************
C BEGIN FREQUENCY LOOP
C**************************************************************************
SET UP MAX FREQUENCY BOUNDARY

COMPTON SCATTERING FORBIDS VERY HIGH FREQUENCIES. THETA(I+1) CHOSEN TO BE COMPATIBLE WITH KAPPA/JP.

120 HNUP = THETA(103)
    INNU = 1
    DO 129 I=IN1,INP1
    X4(I) = 0.
    FQ1(I) = 0.
    FQ2(I) = 0.
    FQ3(I) = 0.
    RUU(I) = LK(I)
    EU(I) = 0.
    SMRI(I) = 0.
    129 SUMX2(I)=0,0
    IF (KMAX .NE. 0) GO TO 310

MONOFREQUENCY CALCULATION

210 HNUP = 1
    DO 220 I=IN1,IM
220 Xo(I)=G(I)
    DO 240 I=IN1,IM
240 DF = 1.0
    HNU = .001
    DNH = THETA(103)
    ICX = IM
    ICT = IN
    GO TO 460

TYPICAL GROUP CALCULATION OF SOURCES

310 CALL KAPPA(IN1,IM)
    ZZ = 0.
    DNHUP = DNH
    UHNUP = HNUP + HNU
    350 ICX = IM
    ICT = IN
    IF (GL .LT. 1.E-20) GO TO 370
    DO 360 I=IN1,INP
360 X6(I)=DFB*G(I)
    DO 450 I=IN1,IM
370 DO 450 I=IN1,IM
    BETA = DNH/THETA(I)
    IF (BETA .GT. 19.) GO TO 430
    BETAP = HNUP / THETA(I)
    IF (BETAP .GT. 0.01) GO TO 440
    430 X6(I)=0.0
    GO TO 450

FORM SOURCE X6

132
C 440 IF (DFB .EQ. PLNKUT(BETA + BETA1)) CALL UNCLE
450 CONTINUE
CALL UCSH(KK)
GO TO (452, 460), KK
452 S1 = 13.4452
CALL UNCLE
460 IF (INM1) 470, 490, 480
C SET BLACKBODY CONDITION FOR IA GREATER THAN 1
C 470 S1 = 13.0470
CALL UNCLE
480 IF (DFB .EQ. PLNKUT(HNU/THETA(INM1), HNU/THETA(INM1)))
Xb(INM1) = DFB * THETA(INM1)**4
C SET BLACKBODY CONDITION IF DESIRED FOR IMP1
C 490 IF (ABS(SOLU(J7)) .LT. 1.E-5) GO TO 510
IF (ABS(SOLU(IMP1)) .LT. 1.E-20) GO TO 500
FNS(INM1) = 0.
DFB = PLNKUT(HNU / THETA(IMP1), HNU / THETA(IMP1))
Xb(IMP1) = DFB * THETA(IMP1)**4
FNS(IMP1) = 0.
GO TO 510
500 Xb(IMP1) = 0.
510 IF (ABS(SOLU(J7)) .LT. 1.E-20) GO TO 515
C SCATTERING: SET RADIATION REGION ACTIVE THROUGHOUT: RING IN DATA
C FROM DRUM, AND SET UP COPTON SCATTERING FREQUENCY PARAMETERS
ICY = IN
ICX = IN
READ (1DRUM) FI0, FI1, FI2, FI3
HNUX = AMIN(HNU, 1.E5)
GAMMA = AMIN(0.2, 0.9749E-6 * (HNU + HNUX))
AIP = HNUX**2 / HNU * 1.95692E-6
IF (HNU .EQ. 1) GO TO 512
A3 = HNUX**2 / HNUUP * 1.95692E-6
512 A1 = AIP + 3. * GAMMA
GMP = 1. - 2. * GAMMA
515 Q31 = 0.0
C FORM ROSSELLAND AND PLANCK OPTICAL DEPTHS
C 520 DO 570 I = IN*IN
IF (CAPAR(I)) 530, 550, 520
520 IF (CAPAC(I)) 530, 550, 520
530 S1 = 13.0530
CALL UNCLE
540 TAU = AMAX1(SOLU(37), 0.)
C SPECIAL CODING TO FORCE KAPPA'S TO BE AT LEAST 0.2
CPC = AMAX1(CAPAR(I) + TAU, 0.2)
C HHTAX MUST BE LIMITED TO 2. * GAMMA * KAPPA(S) = .08
HHTAX = AMIN(TAX * (HNU + HNUX), .08)
GO TO (542, 344, 544), IALPHA
542 Q91 = 0.0
GO TO 546
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544 \[ u_{ij} = \frac{1}{S(1)} \cdot (C(1) - C(i)) \]
546 CPA = CPC - M/TAX
547 H3(1) = CPA * UV

C000E ALL HOSSELED IF SOLID(1) IS NONZERO

IF (ABS(SOLID(1))) LT 1.E-20 GO TO 556
H2(1) = H3(1)
H(1) = CPA / SV(1)
GO TO 560

550 CPA = AMAX1(CAPAC(1) + T/AUX, 0.2)
H2(1) = CPA - UV
H(1) = CPA / SV(1)

560 U31=U1+H3(1)
Q3(1)=U31
H1(1) = 0.6 * H(1)
H2(1)=0.6eH2(1)
H3(1)=0.6eH3(1)
X2(1)=0.0
X3(1)=0.0
X4(1)=0.0
Y2(1)=0.0
T0(1)=0.0
RHO(1)=0.0
PR(1)=0.0
FL(1)=0.0

570 T1(1)=0.0
X2(IMP1)=0.0
X3(IMP1)=0.0
X4(IMP1)=0.0
T6(IMP1)=0.0
PH(IMP1)=0.0
FL(IMP1)=0.0
TR(IMP1)=0.0
Y2(IMP0)=0.0
T6(IMP)=0.0

FOHM Y2 AND T6 SET X3=-1 IF A DIFFUSION CRITERION MET USING HCB

600 ICXH1=ICX1-1
IF (ICXH1.GT.ICX1) GO TO 650
DO 640 ICXH1=ICX1+1
IF (AMAX1(X6(I)+X6(I+1)),LT.1.E-30, .LT. 0.333) GO TO 610
IF (ABS((X6(I)-X6(I+1)) / TEMP(I))), .LT. 0.333) GO TO 610
IF (ABS((X6(I)+X6(I+1))/ (X6(I)+X6(I+1))), .LT. 0.333) GO TO 610
610 T6(I)=0.0
GO TO 640

620 T6(I)=X6(I+1)-X6(I)/TEMP(I)

FORCE TRANSPORT FOR RAPIDLY VARYING SOURCE OR POSITIVE HCB

IF (ABS(T6(I))), .GT. 0.1 * Y2(I+1), .OR. HCB, .GT. 0.) GO TO 640
630 X3(I)=1.0
CONTINUE

FORCE DIFFUSION FOR NEGATIVE HCB
650 IF (HCB) 651, 655, 655

134
IF (GL .GT. 0.9) GO TO 665
X3(I) = -1.
GO TO 665
X3(I) = -1.
GO TO 665
S1 = 13.0684
CALL UNCLE

LAST ZONE MUST BE TRANSPORT IF EXTERNAL INPUT INTENSITIES PROVIDE:
IF (GL .GT. 0.9) X3(IN) = 0.0
IF (ABS(GL - 0.5) .LT. 1.E-3) GO TO 690
Y2(IMPI) = 6(ICK)
TG(IMPI) = 0.0
GO TO 655

EXTEND TRANSPORT REGION BOUNDARIES TO PROVIDE 5 MEAN FREE PATHS:
I = IN + 1
CALL DVCHK(KX)
GO TO (692, 700, 720)
S1 = 13.0692
CALL UNCLE
IF (X3(I)) GO TO 830
J = 1
IF (Q3(J) - Q3(I-1) - 5.1) 830
X3(J) = 0.0
J = J + 1
IF (J = IN) 760
IF (I = IMPI) 810
GO TO 710
I = IN + 1
TEST TO FORM TRANSPORT REGIONS
IF (X3(IN)) 890
IAX = IN
840 IF (X3(I)) 860
850 CALL UNCLE

135
REMOVE ONE ZONE DIFFUSION REGION

850 I=I+1
  IF (I-ICX=1) 840,950,956
860 I=I+1
  IF (I-ICX=1) 870,950,950
870 IF (X(I)) 890, 875, 723
875 X(I-1) = 0.
    GO TO 840
880 IF (X(I)) 900 TO 973
    GO TO 906
890 IF (IN.GT.1) GO TO 910
C
ASSUME C(I) = 0.
    GO TO 920
900 X2(I) = 0.0
    FL(I) = 0.
    GO TO 920
910 X2(IN) = 1.0235E12 * A(I) * (X6(IN-1) - X6(IN))
    FL(IN) = 0.5 * (X6(IN-1) - X6(IN))
920 PR(IN) = Y2(IN) * 0.666667
    RH0(IN) = Y2(IN) + Y2(IN)
    FL(IN) = -0.666667 * T6(IN)
    TR(IN) = 0.6 * FL(IN)
925 IF (X(I)) 930, 940, 720
C
FORM X2 FOR DIFFUSION ZONE'S IN ORDER
930 X2(I) = -1.37812 * T6(I) * A(I)
    PH(I) = Y2(I) + Y2(I)
    RH0(I) = Y2(I) + Y2(I)
    FL(I) = -0.666667 * T6(I)
    TR(I) = 0.6 * FL(I)
720
C
IF (I-ICX-1) 925, 980, 980
C
DO TRANSPORT TO IN REGION OF NO SOURCE
720
940 IAX=I
    GO TO 850
950 IBX=IBX+1
960 IF (IBX.GT. ICX) GO TO 965
    GO TO (961, 962, 963), 1ALPHA
961 CALL PTRANS(IAX, IBX)
    GO TO 965
962 SI = 13.0962
    CALL UNCLE
963 CALL STRAUS(IAX, IBX)
965 IF (IBX.GT. ICX) 970 TO 1030, 1030
970 I=IBX+2
    GO TO 930
980 IF (1.GT.I) GO TO 981
720
981 IF (1.GT.I) GO TO 990
    CALL UNCLE

136
C "RIGHT-Hand BOUNDARY CONDITION FOR DIFFUSION ZONES"
C, VLAT. MAY NEED HNUP.. PL. TH HERE.
990 IF (GL) 1000, 1010, 1020
1000 X2(IIMP1) = 0.
GO TO 1030
1010 X2(IIMP1) = 1.0283E12 * X6(I4) * A(IIMP1)
GO TO 1025
1020 X2(IIMP1) = 1.0283E12 * (X6(I1) - X6(I1MP1)) * A(IIMP1)
1025 HN0(IIMP1) = Y2(IIMP1) + Y2(IIMP1)
PK(IIMP1) = 0.3333333 * R(I0(IIMP1))
FL(IIMP1) = X2(IIMP1) / A(IIMP1) * 4.8624E-13
TH(IIMP1) = FL(IIMP1) * 0.6
C "OPTIONAL EDIT OF X2 ETC."
C
1030 IF (ABS (EDITNF) .LT. 1.E-20) GO TO 1040
CNT1 = SOLID(16) + 1.
IF (T6 .GT. 0.) GO TO 1040
WRITE (6,4) CNT1, TH, HNU, HNUP, IN, ICX, ICY
WRITE (6,4) G0 1035 I = IN(IIMP1)
WRITE (6,6) I, C(I1), X6(I1), H2(I1), H3(I1), TG(I1), Y2(I1), X3(I1),
1X2(I1), RAO(I1), PH(I1), TR(I1)
1035 CONTINUE
3 FORMAT (9H1 CYCLE = F7.0, 9H TIME = E13.6, 12H HNU FROM FA(2),
2 4H FA TO FA(2), 10X, H1K14, 10X, H1C14, 10X, H1C14, 10X)
4 FORMAT (3XH11, 1UX11H, 10XH2H6, 10XH2H4, 10XH2H3, 10XH2HT)
, 2 10XH2Y2, 3XH2A3, 10XH2X2, 9XH3H2H, 10XH2PH, 10XH2TR)
6 FORMAT (9H1, IE(11,4), IE(12,5), OPFS, 1, PNF2, 1.3)
1040 IF (T6 .LT. CAPIC(152)) GO TO 1050
ZZ = 0.
QEM = 0.
DO 1053 I = IN(IIMP1)
F1050 = 2. * PM(I) - RH0(I)
F105 = PM(I) - RH0(I)
F105 = 2. * PM(I) - RH0(I)
QON = F1050 * F105
QU = F1050 + F1050
IF (ABS (QON) .LT. 0.1) GO TO 1051
G0 1053
1051 G0 = ABS (QON) / Q0)
IF (GE .LT. CAPIC(150)) GO TO 1052
ZZ = 1.
IF (GE .LT. QEM) GO TO 1052
1QEM = 1
QEM = 0.
DO 1052 T6 = T6 + 1.
IF (T6 .LT. 9, OK, ZZ .LT. 0.1) GO TO 1055
CNT1 = SOLID(18) + 1.0
WHITE (6,8) CNT1, IHNU, T4, DEN, IUEM
8 FORMAT (29H SCATTERING IONIZATION CYCLE F5.0, 9H IHNU = I2, 7H, 276 = F3.0, 32H LARGEST RELATIVE DIFFERENCE IS F5.3, 9H IN ZONE 13)
GO TO 515
1055 DO 1057 I = IN + IM1
   F00(I) = F10(I)
   F01(I) = F11(I)
   F02(I) = F12(I)
   F03(I) = F13(I)
   SUMX2(I) = SUMX2(I) + X2(I)
   EX(I) = -H01(I) + U(B,5) + CK(I)
   SMLR(I) = SMLR(I)+PH(I)*U(0,5)
   F10(I) = 3, * H00(I) - PH(I)
   F12(I) = 3, * PH(I) - H00(I)
   F13(I) = 3, * FL(I) - 5, * TR(I)
1057 F13(I) = 3, * TH(I) - 5, * FL(I)
   Q = THICK * 1.02612
   CAPAR(IHNU+130) = CAPAR(IHNU+135) + 0.5 * DTR * (CAPAR(IHNU+120) - 2Q)
   CAPAR(IHNU+120) = Q
   IF (A05(SOL(157), 6T, 1.0-20) WRITE(JURUM) F10, F11, F12, F13)
   ADVANCE FREQ, STROKE EMERGENT FLUX, TEST FOR COMPLETION OF GROUPS
   IF (IHFU(IHNU)) 1060, 1090, 1090
   IF (IHNU-HNU) 1060, 1060, 1060
   1060 CALL JVCNM (KQOFX)
   GO TO (1070+110) KQOFX
C*******************************************************************************
C                  S. FREQUENCY LOOP
C*******************************************************************************
1070 S1 = 13.5, 1070
   CALL UNCLE
   1080 SUMX2(1HNU) = 0.0
   JURUM = JORDM
   IHNU = JORDM
   DO 1090 I = IN, IM1
      EC(I) = 0.
      X2(I) = SUMX2(I)
      RH0(I) = EX(I)
      ER(I) = SUMX2(I) - SUMX2(I+1) + 0.5 * (A(I+1) + 3.0 * SMLR(I+1) - 2*EX(I+1) - A(I) + 3.0 * SMLR(I) - EX(I))
   1090 CONTINUE
   X2(1IM1) = SUMX2(1IM1)
   RH0(1IM1) = EX(1IM1)
   EDIT OF OUTPUT OUTPUT
   IF (10.0, 0.5, A05, CNT1, LT, CNTMAX) GO TO 1320
   WRITE (6,5) CNT1, TH
   WRITE (6,7) (CAPAR(IHNU+120), I = 1, IHNU)
   WRITE (6,7) (CAPAR(IHNU+135), I = 1, IHNU)
5 FORMAT (29H OUTPUT OUTPUT FOR CYCLE F5.0, 10X7THIME = FPE13.6)
7 FORMAT (1P10E12.5)
1320 RETURN
C*******************************************************************************
C
   END
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WIT FOR PTRANS/P, PTANS/P, PTRANS/P1
SUBROUTINE PTRANS(N,NM)
COMPILED OCTOBER 9, 1967 WUL
C PLANE CHARACTERISTIC THACE *1TH DOUBLE GAUSSIAN INTEGRATION
C USES THE RADIATION TRANSPORT *STEP* SUBROUTINE
C
C*********************************************
PTRA0050
C
C
COMMON - LMADA(37), NR, HISL, IA, IB, ICA, ICB
1 KMAX, BLANK1, BLANK2, BLANK3, IAPI, IDPI, ICAPI, ICUP1, PTRO0090
2 II, IG, IRRD, BLANK4, IAM1, IBM1, ICAM1, ICBM1, PTRO0100
3 IPI, IGML1, IALPHA, BLANK5, TM, TMAX, BLANK6, DELPRT, PTRO0110
4 FREQ, CNTMAX, AR, ASMLR, PUSHA, PUSHD, UOILA, HOILD, PTRO0120
5 CVA, CVU, SLUG, ALPHA, HVA, HVB, MCA, MCE, PTRO0130
6 EMINA, EMINB, CA, CB, GA, GU, GL, GR, PTRO0140
7 RHOA, RHON, EPAU, LPSI, MAX, RIB, KIA, RIB1, PTRO0150
8 RPIA, RPOL1, RPOID1, RPOID2, RPINT, T1, TB, TC, PTRO0160
COMMON TD, TE, OT2, DTH2, DTH1, DTRMM, DTMAX, PTRO0170
1 DTMAX1, UMAX2, DTMAX3, UTR, SWITCH, CO, CMIN, DELTA, PTRO0180
2 GAMA, MCN11, SIGNAL, AC, ACOST4, CHVR, SUMRA, SUMRB, PTRO0190
3 RIA, ROIAM1, ROIB1, ROIBP1, GSM, S1, S2, S3, PTRO0200
4 S4, S5, S6, S7, S8, S9, S10, S11, PTRO0210
5 S12, SI3, SI4, SI5, SI6, SI7, SI8, SI9, SI10, PTRO0220
6 SI2, SI2, SI2, SI2, SI2, SI2, SI2, SI2, SI2, SI2, SI2
7 ASQ (152), M1 (152), MG (152), VD (152), RRD (152), SMLR (152), PTRO0230
8 DELL (37), P (152), PI (152), PH (152), P01 (152), PTRO0250
9 COMMON W2 (152), E (152), EI (152), EK (152), A (152), PTRO0260
10 V (152), C (152), X2 (152), X2 (152), PTRO0270
11 X3 (152), X4 (152), X5 (152), X6 (152), X7 (152), PTRO0290
12 SMLA (152), SMLB (152), SMLC (152), SMLD (152), SMLE (152), PTRO0300
13 EC (152), ER (152), ERL (152), ELM (152), PTRO0310
14 U16 (152), CV (152), BR (152), CHIC (152), PTRO0320
15 CHIR (152), CAPAC (152), CAPAR (152), CRTC (152), CRTR (152), PTRO0330
16 CRTPC (152), GOPFR (152), FEW (152), CAR (152), UMLK (37), PTRO0340
17 COMMON TELM (37), EKLM (152), ELM (37), ELM (37), PTRO0360
18 FR (37), WM (37), WLM (37), QLML (37), QLML (37), PTRO0360
19 ZP1 (37), ZP2 (37), ZP3 (37), ZP4 (37), ZP5 (37), ZP6 (37), PTRO0370
20 R (37), ROK (104), T (104), T (104), PTRO0380
21 HEAD (12), MAXL, MAXL
C
C *********************************************
PTRA0400
C
C
COMMON /CTRL/ SCYLE, MHUL
COMMON /DAVIS/ X(4000), ICK, ICT
COMMON /JVM/ NN, FMU, RL, RQ, HD, EST, I1, I2, GMF, A1, A3, FMU
2 FS, LDF, LRI, I12, T61, T62, F2
C
C DIMENSION MK(40)
C
C***************************************
PTRO0540
C
C
DATA RR/2.113248E-01,7.886752E-01,1.056624E+01,3.943376E+01
1 1.127017E+01,5.000000E+01,8.872983E-01,3.130600E-02
2 2.222222E+01,2.447718E-01,6.943100E-02,3.300095E-01
3 6.999999E+01,3.056828E+01,1.207610E+02,1.076071E+01
4 2.184655E+01,1.618513E+01,4.619101E-02,2.307653E-01
C
C
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5 5.000000E-01, 7.629247E-01, 9.530894E-01, 5.557100E-03, PTRA0610
6 5.525460E-01, 1.122222E-01, 1.840889E-01, 1.129063E-01, PTRA0620
7 3.876392E-02, 1.939385E-01, 3.806903E-01, 3.810696E-01, PTRA0630
8 8.96962E-01, 1.144891E-01, 8.276980E-02, PTRA0640
9 8.906074E-01, 1.422222E-01, 1.840889E-01, 1.129063E-01, PTRA0650

C

DIMENSION CSWD (1), FM (1), H (1), M (1), H2 (1),
1 H3 (1), J4 (1), FM (1), JO (1), TG (1), G3 (1), P
2 G37 (1), J36 (1), SUMX2 (1), SUMX3 (1), SUMX4 (1), X8 (1), P
3 Y (1), Y2 (1), OX (1), FL (1), TR (1), FSM (1), P
4 FSP (1), P

C

EQUIVALENCE

(AC03T4, TRUBG), (BC, SUMX4), (BIGA, Y), P
1 (BIGB, H), (DR, J3), (CAR, G37), (CHIC, SUMX3), P
2 (CHIR, G38), (CHR, SUMX2), (X7, PR), (GOFR, G3), P
3 (PB, 01), (Si2, EDITMF), (EC, H2), (W, OX), P
4 (SMLB, FSP), (SMLB, FL), (SMLB, TR), (SMLH, H4), P
5 (ER, FM), (V, FL), (SM), (FSM, FSM), (CSQ0), P
6 (X5, X2), (X4, X8), (SMLB, FSP), P

C

OX CONTAINS X FROM THE PREVIOUS Y LINE

CSQ0 SAME AS CRTC
EDITMF SAME AS SI2
01 SAME AS PB
FM SAME AS EX
H SAME AS BIGB
M3 SAME AS EC
H3 SAME AS BN
H4 SAME AS SMLH
PR SAME AS X7
FMS SAME AS SMLA
FL SAME AS SMLB
TR SAME AS SMLC
FSM SAME AS SMLD
FSP SAME AS SMLC
Y2 SAME AS X5
OX SAME AS W
TG SAME AS V
G3 SAME AS GOFR
G37 SAME AS CAR
G3B SAME AS CHIR
SUMX2 SAME AS CRTR
SUMX3 SAME AS CHIC
SUMX4 SAME AS BC
TRUBG SAME AS AC03T4
Y SAME AS BIGA
X8 SAME AS X4

C

PLACES ONLY

140

TRAN 930
PTRA0910
PTRA0920
PTRA0930
C***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/***/**
$$A(i) = T(u(i) * H(u(i + 1)))$$

$$I1 = 1 - 1$$

$$I2 = 1$$

$$R1 = C(I-1)$$

$$R2 = C(1)$$

$$I2(I) = 1 - 1$$

$$TG1 = X8(I-1)$$

$$TG2 = X8(1)$$

$$H0 = H2(I-1)/H1(I)$$

CALL STLP

260 SUMX3(I) = F2

IF (AUS(TG(I)) .LT. 1.E-20) Y2(I) = X6(I)

I1 = I1 + 1

IF (I .GT. 10X) GO TO 320

IF (1 .LE. ICX + 1) GO TO 220

C NO SOURCE IN ZONE GREATER THAN ICX

CAVEAT. SCATTERING IN SOURCELESS REGION NOT HANDLED PROPERLY HERE

IF (F2 .EQ. 0.0) GO TO 260

TEMP(I) = H2(I-1)/H1(I)

H4(I-1) = FREXP(-T1MP(I)-T1MP(I))

F2 = F2 + H4(I-1)

GO TO 260

300 IF (F2 .EQ. 0.0) GO TO 310

TEMP(I) = H2(I-1)/H1(I)

H4(I-1) = FREXP(-T1MP(I)-T1MP(I))

F2 = F2 + H4(I-1)

310 SUMX3(I) = F2

FSM(I) = F5

LDF = 2

I1 = I1 + 1

IF (I .LT. 10X) GO TO 300 + 300 + 200

C DO NEGATIVE ANGLES SECOND

320 I1 = IBX(I)

FS = 0.0

LAI = 2

IF (IBX(I) .LT. ISJ) GO TO 330 + 330 + 200

330 IF (GL) .GT. 500 + 200 + 300

GL = 0 MEANS BLACKBODY CONDITION SET AT IMPI

C POSITIVE INTEGER INPUT OPTION DELETED

C GL = 0 MEANS VACUUM AT IAP1

C NEGATIVE MEANS REFLECTIVE CONDITION AT IMPI

340 IF (GL .LT. 0.5) GO TO 350

F2 = X8(IMPI)

GO TO 480

350 S1 = 14.0350

CALL UNCLE

C ERROR IF INDEX EXCEEDS NORMAL RANGE

360 S1 = 14.0350

CALL UNCLE

C
C 370 LDF = 1
GO TO 400

399 IF (Abs(IR(T(-1) + 1)) .LT. 1.E-20) Y2(I+1) = X6(I)

C

C REGULAR INTEGRATION STEP FOR F2, NEGATIVE MU
C

400 IF (Abs(IR(T(I))) .LT. 1.E-20) Y2(I) = X6(I)
I1 = I + 1
I2 = I
H1 = -C(I+1)
R2 = -C(I)
IZN = I
TG1 = -X8(I+1)
TG2 = -X6(I)
MU = H2(I)/HK(NMU)
CALL STIP

440 SUMX4(I)=F2

C FORM CONTRIBUTION TO X2
C

X2(I)=X2(I)-(F2-SUMX3(I))*RR(NGS)
RHO(I) = RHO(I) + (F2 + SUMX3(I)) * RR(NGS) / RR(NMU)
PR(I) = PR(I) + (F2 + SUMX3(I)) * RR(NGS) * RR(NMU)
FL(I) = X2(I)
TR(I) = TR(I) - (F2 - SUMX3(I)) * RR(NGS) * RR(NMU) * RR(NMU)
IF (Abs(IR(T(I))) .LT. 1.E-20) Y2(I) = X6(I-1)
I1=I-1

IF (I-IAX) 530,450,450

450 IF (I-ICY) 500,400,400

C NO SOURCE IN ZONE LESS THAN ICY
C

470 IF (F2 .LT. 0.0) GO TO 480
TEMP(I)=H2(I)/RR(NMU)
H2(I)=FRExP(-TEMP(I)-TEMP(I))
F2=F2*H2(I)

480 SUMXN(I)=F2

490 X2(I)=X2(I)-(F2-SUMX3(I))*RR(NGS)
RHO(I) = RHO(I) + (F2 + SUMX3(I)) * RR(NGS) / RR(NMU)
PR(I) = PR(I) + (F2 + SUMX3(I)) * RR(NGS) * RR(NMU)
FL(I) = X2(I)
TR(I) = TR(I) - (F2 - SUMX3(I)) * RR(NGS) * RR(NMU) * RR(NMU)
LDF = 2
FSP(I) = F5
I1=I-1
IF (I-1-ICX) 399,470,470

C NO SOURCE IN ZONE LESS THAN ICY
C

500 IF (F2 .LT. 0.0) GO TO 510
TEMP(I)=H2(I)/RR(NMU)
H4(I)=FRExP(-TEMP(I)-TEMP(I))
F2=F2*H4(I)

C

C

143
AFWL-TR-67-131, Vol III

510 SUMX4(I)=F2
X2(I)=X2(I)-(F2-SUMX3(I)) * RR(NGS)
RHO(I) = RHO(I) + (F2 + SUMX3(I)) * RR(NGS) / RR(NMU)
PR(I) = PR(I) + (F2 + SUMX3(I)) * RR(NGS) / RR(NMU)
FL(I) = X2(I)
TH(I) = TH(I) - (F2 - SUMX3(I)) * RR(NGS) / RR(NMU) * RR(NMU)
I=I+1
IF (I=IAX) 530,500,550
520 F2=0.0
GO TO 480
530 CONTINUE
IF (ABS(TROBG) .LT. 1.E-20) GO TO 539
C DEBUG PRINT OF INTENSITIES
JJJ = JJ + 1
IF (JJJ .EQ. 1) WRITE (6,8)
WRITE (6,9)
DO 65 I = IAX, IXPL1
55 WRITE (6,5) I, SUMX3(I), SUMX4(I)
5 FORMAT (1X, 1PE11.4, E14.7)
8 FORMAT (28H1PLANE TRANSPORT DEBUG PRINT/)
9 FORMAT (7X1HI, 7X7HI RIGHT, 6X7HI LEFT)
10 FORMAT (27H GAUSSIAN QUADRATURE ANGLE I2, 17TH WHOSE COSINE IS F11, RED1660)
2,8/
IF (THDBG .GT. 69.) THDBG = THDBG - 69.
539 CONTINUE
540 JJ = JJ + 1
NMU = NMU + 1
NGS = NGS + 1
IF (JJ=NY) 140,140,550
550 UO 560 =IAX, IXPL1
560 X2(I) = X2(I)+ 2.052E12
RETURN
END
C subroutine TRANS(S)  TRANS(S)  TRANS(S)
COMPILED OCTOBER 9, 1967
MOUIFIED FOR COMPUTER SCATTERING AND IMPROVED LOGIC
ANGULAR INTEGRATIONS ON LINEAR FORM
CONNECTED LINEAR-WQUADRATIC INTERPOLATION AT X=0.
EPSI IS Y LIMIT OF EDIT TUBE
LMOD(20) IS INTERFACE INDEX OF EDIT TUBE APERTURE

C SPUTTER COMMON
C
COMMON LMOD(37), NR, NSMLR, IA, IB, ICA, ICB
1 KMAX, BLANK, BLANK2, BLANK3, IAPI, IUP1, ICAP1, ICHP1
2 I, IG, AHAD, BLANK4, IAMA, IBMI, ICAMI, ICBM1
3 IPI, IG1, ALPHAP, BLANK5, TH, TMAX, BLANK6, DELPR
4 FREU, CNT, MAX, AR, ASMLR, PUSH, PUSHB, BOLLA, BOILU
5 CVA, CVO, SLUG, ALPHA, HVA, HVB, HCA, HCB
6 EMNIA, EMIH, IA, CH, GA, GB, GL, GR
7 RHOL, HRHOM, EPI0, EPSI, NIA, RIB, RIA, RUIB
8 RPIA, RPIO, APUIA, RPO1B, TPRINT, TA, TB, TC
COMMON TO, TE, UTH2, DTHBP, DTH1, DTAMIN, DTMAX
1 DTMAX1, UTHK2, DTHAX3, UTHR, SWITCH, CO, CMIN, DELTA
2 GAMA, TCH, SIGMA, AC, AC034, CVNRT, SUMRA, SUMRB
3 ROIA, ROU1M1, ROU1, ROIBP1, GMS, S1, S2, S3
4 S4, S5, S6, S7, S8, S9, S10, S11
5 S12, SS, TRESP, TMAH, TBSH, S16, S17, S18, S19
6 S20, SU, FO, TAU, ZENO, R (152), DELTAR (152)
7 ASG (152), RU (152), RU (152), RDU (152), SMLR (152)
8 DELR (37), P (152), P1 (152), P2 (152), PB1 (152)
COMMON S15 (152), V2 (152), SV (152), SMLR (152), THETA (152)
1 E (152), EL (152), EK (152), A (152)
2 V (152), G (152), D (152), C (152), X2 (152)
3 X3 (152), X4 (152), X5 (152), X6 (152), X7 (152)
4 SMLR (152), SMLR (152), SMLR (152), SMLR (152), SMLR (152)
5 EC (152), EH (152), SMLQ (152), SMLQ (152), SMLQ (152)
6 BIGB (152), CV (152), BC (152), BR (152), CHIC (152)
7 CHIK (152), CAPAC (152), CAPAR (152), CRIC (152), CTX (152)
8 CRTPC (152), GOFR (152), FEW (152), OCR (152), ORML (37)
COMMON EKLM (37), EKLX (37), EKLX (37), EKLX (37), EKLX (37)
1 FMLM (37), FLM (37), ELM (37), AMASNO (37), CHRM0 (37)
2 ZP1 (37), ZP2 (37), SOLLO (37), ECMD (37), RK (104)
3 RL (37), RHOK (104), HUK (104), THEMAT (104), TEMP (161)
4 HEAD (12), MAXL, MAXLM

C
C COMMON /LINOLY/ MNU, SGNL, IHNU, NHNU, NHGNU, THICK, NT
C COMMON /DAVIS/ (X(4000), ICX, ICT)

145
COMMON /JIM/ N1, FMUS, N1, H2, H3, H4, EST, 11, I2, GMP, A1, A3, FMUS,
2 FS, LLF, LKH, T01, T02, F2
DIMENSION CSOU (1), PH (1), FM (1), H (1), H2 (1),
1 H3 (1), H4 (1), FM (1), G1 (1), TG (1), Q3 (1),
2 037 (1), W36 (1), SUMX2 (1), SUMX3 (1), SUMX4 (1), X8 (1), P
3 T (1), Y (1), OX (1), FL (1), TR (1), FSM (1), P
4 FSP (1)

EQUIVALENCE (AC03T4, TNDHG), (BC, SUMX4), (BISA, Y), P
1 (BIG5, H), (ER, H3), (CAR, Q37), (CHIC, SUMX3), P
2 (CHIR, J36), (CRTH, SUMX2), (X7, PR), (GOFT, X3), P
3 (PB, W1), (S12, EDITMF), (IEC, H2), (W, OX), P
4 (SMLA, FM5), (SMLH + FL), (SMLC, TR), (SMLH + H4), P
5 (ER, FM), (V, TG), (SMLD, FSP), (CRIC, CSO2), P
6 (X5, T2), (X4, X8), (SMLE, FSP), P

OX CONTAINS X FROM THE PREVIOUS Y LINE

CSOU SAME AS CRTC
EDITMF SAME AS SI2
G1 SAME AS PB
FM SAME AS EM
H SAME AS BIG5
H2 SAME AS EC
H3 SAME AS BR
H4 SAME AS SMLH
PR SAME AS X7
FM5 SAME AS SMLA
FL SAME AS SMLD
TR SAME AS SMLC
FSM SAME AS SMLD
FSP SAME AS SMLE
Y2 SAME AS X5
OX SAME AS W
TG SAME AS V
Q3 SAME AS GOFR
Q37 SAME AS CAR
Q38 SAME AS CHIR
SUMX2 SAME AS CTIR
SUMX3 SAME AS CHIC
SUMX4 SAME AS BC
TROBG SAME AS AC03T4
T SAME AS BISA
X8 SAME AS X4

IAX3H
IBX3H
THICK = 0.0
ITUBE = LMDA(20)
IF (C(TUBE) < LT. EPSI) GO TO 1
XTUBE = SQRT(CSOU(1)ITUBE) - EPSI**2
CALL DVCHRM(KK)
GO TO (1,3) + KK
1 S1=14,0001
CALL UNCLE
3 IBX1=IDX+1
IALPHA=ALPHA
C C ERROR IF NOT SPHERE
C GO TO (5+5,7) + IALPHA
5 S1=14,0005
CALL UNCLE
C C SPHERE ONLY
C 7 I=IBX+1
FS = 0.0
FMU = 0.0
LXI = 1
IF (IXL-IM) J7, 6, 11
8 F2 = 0.0
IF (GL .LT. 0.1) F2 = X6(IM+1)
GO TO 23
C C ERROR IF INDEX EXCEEDS NORMAL RANGE
C 11 S1=14,0001
CALL UNCLE
C******************************************************************************
C CALCULATE Y = 0 RAY
C******************************************************************************
C LHS OF RAY FIRST; STORE F2 IN SUMX3.
C 13 II = I + 1
II = I
RI = -C(I+1)
R2 = -C(I)
HO = H2(I)
IZN = I
T61 = -T6(I+1)
T62 = -T6(I)
CALL STEP
C C SAVE LHS INTENSITIES IN SUMX3
C 23 SUMX3(I)=F2
FSM(I) = FS
IF (ABS(T6(I)) .LT. 1.E-20) Y2(I) = X6(I-1)
I2=1
LDF = 2
IF (ABS(T6(I)) .LT. 1.E-20) Y2(I) = X6(I)
IF (I=IAK) 47,13+13
DIFFUSION BOUNDARY CONDITION AT IBXP1

37 IF (ABS(T6(I)) .LT. 1.E-20) Y2(I) = X6(I)
LUF = 1
GO TO 13

RHS OF RAY

47 I=IA
IAXP=IA
LRI = 2
IF (IAX = IN) 11, 48, 51
48 IF (IN - 1) 11, 49, 50
49 IAXP=IN+1
RHO(I)=SUMXJ(I)*2.
PH(I)=SUMXJ(I)*6067
GO TO 60

DIFFUSION BOUNDARY CONDITION AT IA

50 IAXP = IN
F2 = X6(IN-1)
GO TO 69

REGULAR INTEGRATION STEP (Y=0, RHS)

59 I1 = I - 1
I2 = I
R1 = C(I-1)
R2 = C(I)
MU = H2(I-1)
IN = I - 1
T61 = T6(I-1)
T62 = T6(I)
CALL STEP

SAVE RHS INTENSITIES IN SUMX4

69 SUMX4(I) = F2
FSP(I) = FS
QK(I)=C(I)
IF (ABS(T6(I)) .LT. 1.E-20) Y2(I) = X6(I)
LDF = 2
II=1+1
IF (ABS(T6(I)) .LT. 1.E-20) Y2(I) = X6(I-1)

COMPLETE Y=0 RAY

************************************************************TRAN3470
*TRAN3480
*TRAN3490
*TRAN3500
*TRAN3510

148
SEARCH FOR Y-LINE NEAR HALF OF C(IAX)

C

JJ=1
JJJ=1
KK=1

C

SEARCH FOR Y-LINE NEAR THREE QUARTERS OF C(IAX)

C

103 C1=C1+0.25*C(IAX)
C

COMPLETE X2 INTEGRATION WHEN LAST Y-LINE USED

C

127 DO 139 IX=IAAP,IBXP1

C

ANGULAR INTEGRATION OF X2 USING LINEAR INTERPOLATION

C

FNL = SUMX(1)/2 - SUMX3(1)
X50 = 0X(I)+X2
X2(I) = X2(I) + FNL * (X50 + X50)

C

129 IF (IBX-LI) 131,135,137
131 IF (1-IAAP) 171,133,137
133 X5=SUMCSAD(IBXP1+1)-CSAD(IBXP1))
FM(1UXP1) = 12(1UXP1) + XS / C(1UXP1+1) + Tg(1UXP1+1) * 
2 FHAP(-XS + H(1UXP1)) 
GO TO 137

135 FM(1UXP1)=0.0
137 TEMP(11)=CSOQ(1UXP1)-TS,1
TEMP(11)=CSOQ(11)-TS,1
FUX(TEMP(11))*(FX(1UXP1)+FX(1UXP1))*(CSQD(1UXP1)-
1)
CSQD(1UXP1)*(CSQD(1UXP1)+CSQD(1UXP1))/TEMP(11)
FLX = SUMX3(I) + SUMX4(I)
FP = FLX + FU
FPL = FP + FLX
RHO(I)=RHO(I)+OX(I)+FP
PK(I) = PK(I)+ASU * (FPL + FLX) * OX(I)
TR(I) = TR(I)+XS2+2 * FNL * 4,

139 CONTINUE

C%%%%%%%%%%%%%%%%%%%%%%%%%%=%TRAN4350
C COMPLETION OF X2 INTEGRATION AT END OF TRANS REGION %TRAN4520
C

GO 151 I=1UXP1
IF (I.EQ.1) GO TO 151
PH(I)=PH(I)+0.0334/(CSD(I)+C(I))
RHO(I)=RHO(I)+0.5U/C(I)
FL(I) = 1.666667 * X2(I) / CSQD(I)
IF (I.GT.1) GO TO 147
TR(I) = 0,
GO TO 151
147 TR(I) = TR(I)+0.5 / CSQD(I)+2
151 X2(I)=X2(I)+1.020212
CALL UVCHK(KK)
GO TO (152, 153, 153)
152 S1 = 14.0152
CALL UNEC

153 RETURN

C%%%%%%%%%%%%%%%%%%%%%%%%%%=%TRAN4520
C TYPICAL Y-LINE INTEGRATION
C%%%%%%%%%%%%%%%%%%%%%%%%%%=%TRAN4520

C LHS CALCULATION: FIRST STOKE F2 IN FM

167 I=1UXP1
FS = 0.0
LRI = 1
IF (IHX = IM) 173, 168, 171
168 FS = 0.0
IF (6L = GT 0.) F2 = xbl(M+1)
GO TO 205

C ERROR IF INDEX EXCEEDS NORMAL RANGE

171 S1=14.0171
CALL UNCLE

DIFFUSION BOUNDARY CONDITION AT IEXP1

173 IF (ABS(TG(I)) .LT. 1.E-20) Y2(I) = X6(I)
X6(I+1) = X(K-1) / C(I+1) * TG(I+1)
LDF = 1

SAVE X6 FOR RHS=|DIFF INTENSITY|

181 X6(I) = X(K)/C(I) * T(I)
HD = (X(K-1) - X(K)) * H(I)

REGULAR INTEGRATION STEP(LHS)

II = I + 1
I2 = I
R1 = -X(K-1)
R2 = -X(K)
IZN = 1
T61 = -XU(I)
T62 = -XU(I)
CALL STEP

SAVE F3 OF LHS IN FH FOR INTEGRATION.

191 FH(I) = F2
FSM(I) = FS
IF (ABS(TG(I)) .LT. 1.E-20) Y2(I) = X6(I-1)
I = I-1
LDF = 2
KX = 1
IF (ABS(TG(I)) .LT. 1.E-20) Y2(I) = X6(I)
IF (I = IAX) 299, 193, 193
193 IF (X(K)) 195, 207, 181

Y-LINE HAS MADE CLOSEST APPROACH

195 TEMP(2) = SQRT(DELTAR(I) + 4.0*DELTA(I) + DELTAR(1)) * H(I)
IF (ABS(TEMP(2)) .LT. 1.E-20) GO TO 223
HU = X(K-1) * H(I)
TEMP(1) = HU
I = I + 1
I2 = I
R1 = -X(K-1)
R2 = 0.
IF (TEMP(1) .LT. 0.2) 197, 197, 201

THIN LHS

197 TEMP(16) = (Y2(I+1) + X6(I)) * 0.5 * TEMP(1) + (0.667 * Y2(I) - 0.5 * X6(I+1))
1 = 0.167 * X6(I) + TEMP(2)
NN = 0
CALL SCAT
Q = AMAX1(0., 1. - F1 / H(I))
F2 = FS + F2 + (1. - TEMP(1)) + Q * TEMP(16)
GO TO 223

201 H4(I) = FREQP(-TEMP(1))

NORMAL LHS

XX = 0.

TRAN5380  TRAN5390  TRAN5410
TRAN5730  TRAN5740  TRAN5750  TRAN5760
TRAN5780  TRAN5790  TRAN5800
TRAN6110  TRAN6120  TRAN6130  TRAN6140
TRAN6150  TRAN6160  TRAN6170
TRAN6190  TRAN6200
TRAN6210
TRAN6310  TRAN6290  TRAN6300
TRAN6380  TRAN6390  TRAN6430
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\[ U = 1 \]
\[ \text{TEMP}(1) = \text{FHEXP}(\text{-TEMP}(2)) \]
\[ \text{TEMP}(13) = (\text{X6}(1) - \text{Y2}(1)) / (\text{TEMP}(2) + 2 + 2.0) \]
\[ \text{TEMP}(15) = (\text{X6}(1) - \text{Y2}(1))/((\text{TEMP}(1) - \text{TEMP}(2))) \]

C_\text{VEAT}, H is the sum of its normal value.

\[ \text{EST} = 1. - H_4(1) \]
\[ \text{CALL SCAT} \]
\[ Q = \text{XMAX}(0.), 1. - \text{FMAX}/H(1)) \]
\[ F2 = \text{FS} + F2 - H4(1) + \left( Y2(1) + \text{TEMP}(13) + \text{TEMP}(7) \right) \cdot (\text{-TEMP}) \]
\[ Z(15) = \text{TEMP}(13) \cdot \left( \text{TEMP}(2) + 1.0 \right) + H4(1) \cdot \left( \text{TEMP}(15) - Y2(1 + 1) \right) \]

GO TO 223

C

FIRST TRANSPORT ZONE ON LHS

\[ 205 \text{X8}(1) = \text{X}(1) / C(1) \cdot G(1) \]

GO TO 191

C

\[ X = \text{ZERO ERROR} \]

\[ \text{GO TO 221} \]

\[ \text{IF } F2 \leq 0.0 \text{ GO TO 221} \]

\[ \text{FM}(1) = \text{FS} \]

\[ \text{FSM}(1) = \text{FS} \]

C

\[ \text{SUM TOP SLICE CONTRIBUTIONS TO X2} \]

DO 231 \[ J = \text{IAXP}, 1 \]
\[ \text{TEMP}(11) = \text{CSQU}(J) - \text{YSQ}1 \]
\[ \text{FNL} = \text{SUMX4}(J) - \text{SUMX3}(J) \]
\[ \text{XSQ} = \text{OX}(J) \cdot \text{XSQ} \]
\[ \text{X2}(J) = \text{X2}(J) + \text{FNL} \cdot \left( \text{XSQ} + \text{XSG} \right) \]
\[ \text{FU} = \left( \text{TEMP}(11) + (\text{FM}(1) + \text{F2}) + (\text{YSQG} - \text{CSQU}(J)) + \text{SUMX4} \right) / \text{TEMP}(5) \]
\[ \text{FLX} = \left( \text{SUMX3}(J) + \text{SUMX4}(J) \right) \]
\[ \text{FP} = \text{FLX} + \text{FU} \]
\[ \text{FNL} = \text{FP} + \text{FLX} \]
\[ \text{RHO}(J) = \text{RHO}(J) + \text{OX}(J) + \text{FP} \]
\[ \text{PR}(J) = \text{PR}(J) + \text{XSQ} \cdot (\text{FPL} + \text{FLX}) \cdot \text{OX}(J) \]
\[ \text{TR}(J) = \text{TR}(J) + \text{XSQ} \cdot \text{FNL} \cdot \text{Q} \]

231 CONTINUE

\[ \text{LNI} = 2 \]
\[ \text{LOF} = 2 \]
\[ \text{IAXP} = 1 + 1 \]
\[ \text{SUMX3}(1) = \text{F2} \]
\[ \text{SUMX4}(1) = \text{F2} \]
\[ \text{I} = \text{IAXP} \]
\[ \text{K} = 1 \]

152
C IF(ABS(TTMI(LTN.LT.1.E-20)) GO TO 257 DELETED 3/16/67, FCT
11 = 1 - 1
12 = 1
R1 = 0.
H2 = X(IK)
IF (TEMP(I) - .02) 233.233.237
C SMALL OPTICAL DEPTH EXPANSION (X=0,RHS)
C
233 N4 = 0
CALL SCAT
U = AMAX1(0., 1. - FMUS / H(I-1))
F2 = +5 + F2 + (1. - TEMP(I) + G * TEMP(16))
GO TO 257
C NORMAL RHS (X=0)
237 N4 = 1
C VELAT. H4 IS THE SQUARE OF ITS NORMAL VALUE.
EST = 1., - H4(I-1)
CALL SCAT
U = AMAX1(0., 1. - FMUS / H(I-1))
F2 = F5 + F2 + H4(I-1) + G * (T2(I) + TEMP(15) + FREXP(-TEMP(1)) +
2 TEMP(2)) + (TEMP(1) + TEMP(15) * (1. - TEMP(2))) + H4(I-1) *
3 (T2(I-1) + TEMP(15)))
257 IF (F2 + LT. U.) F2 = 0.
GO TO 259
C CALCULATE RHS Y-LINE INTEGRATION
C ARRIVE HERE IF Y-LINE INTERSECTS INNER RADIALS
C
239 IAXPIAX
I=IAX
K=K-1
LKI = 2
LF = 1.
IF (IAX **NE. IN .GR. IN .EQ. 1) GO TO 238
C BLACKOUT
F2 = X6(IN-1)
GO TO 259
238 IF (X(K1)) 240, 207, 241
C DIFFUSION BOUNDARY CONDITION WHEN X=O.
240 XBI(I-1) = 0.
HD = X(K) * H(I-1)
R1 = 0.
GO TO 242
241 XBI(I-1) = X(K+1)/C(I-1)*TO(I-1)
HD = (X(K) - X(K+1)) * H(I-1)
R1 = X(K+1)
242 H4(I-1) = FREXP(-H0)
C DIFFUSION BOUNDARY CONDITION AT IAX
C
GO TO 250
249 HU = X(K) - X(K+1) + H(I-1)
R1 = X(K+1)
250 I1 = 1 - 1
I2 = 1
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C ANGULAR INTEGRATION OF X2 USING LINEAR INTERPOLATION

259 FNU = F2 - FH(1)
FLX = SUMX4(I) = SUMX3(I)
X2(I) = X2(I) + ((X(K) + X(I) + OX(I)) + FNU + (OX(I) + OX(I) + 2 X(K)) * FNL) + FXM:
IF (1.*FH. ITUH) GO TO 263
IF (EPSL < LT. TEMP(V))) GO TO 261
IF (THICK = THICK + ((XHUE + XHIUE + OX(I)) + F2 + (OX(I) + OX(I) + 2 XHUE) * SUMX4(I) * (OX(I) - XHUE))
GO TO 263
261 IF (THICK + ((X(K) + X(I) + OX(I)) + F2 + (OX(I) + OX(I) + 2 X(K)) * SUMX4(I) + (OX(I) - X(K)))
263 IF (XHUE = FH(I) + PLX + SUMX4(I)
TEMP(6) = OX(I) + OX(I)
TEMP(7) = X(K) + X(I)
FPR = (OX(I) + X(K))
FM3 = FM3 * 3
FM1 = FLX - FH(I)
HNO(1) = (I + FH(I) + FM3 + FLX)
PK(I) = FH(I) + FPR + (TEMP(6) + TEMP(7)) * FM1 + 4. *
2 (FLX + OX(I) + TEMP(V) - FU + X(K) = TEMP(7))
TR(I) = TH(I) = FH(I) = FM3 + (FM3 + OX(I) + TEMP(6) - 2 TEMP(7)) * FNU + (FM3 + OX(I) + (3. * TEMP(7) = TEMP(6))
C C C SAVE F2 AND FM FOR NEXT Y-LINE

SUMX4(I) = 2
SUMX3(I) = FH(I)
FSF(I) = F3
OX(I) = X(K)
IF (ABS(TG(I)) + LT. 1.E-20) Y2(I) = X6(I)
IMI = 1
MK = 1
LD = 2
IF (ABS(TG(I)) + LT. 1.E-20) Y2(I) = X6(I-1)
IF (I-BXP3(1) = 249.249.263
C C C DEBUG PRINT

1.3 CONTINUAL

MHz+HNU+HNU
IF (AUS(TROB(1)) + LT. 1.E-20) GO TO 301
CNT1 = SOLU(T1) + 1.0
WRITE (6*307) MHz, HNU, IMI, CNH, CNF1
WRITE (6*309) YIUJ

154
WRITE (6,311) JJ, IAX, IGX
WRITE (6,315)
IAXP1 = MAX(0, IAX, IAXP-1)
DO 299 I = IAXP1, IBXP1
IF (JJ .GE. 1) GO TO 297

PRINT Y=0 INTEGRATION

295 WRITE (6,313) I, X(I), X(I+1), XH(I)+XH(I+1), FSM(I),
2FSPI(I), SUMX3(I), SUMX4(I), X2(I)
GO TO 299

PRINT REGULAR Y-LINE INTEGRATION

297 KKK = KK + 1 + J*3
WRITE (6,313) I, X(KK), X6(I), X(I)+XH(I), FSM(I),
2FSPI(I), SUMX3(I), SUMX4(I), X2(I)
299 CONTINUE
IF (THUG .GE. 68.) THUG = THUG - 69.
301 IF (JJ .EQ. 1) GO TO 303

C******************************************************************************************************
C EDIT AND Y-LINE ADVANCE
C******************************************************************************************************

KK = KK - IFIX(X(KK + 1)) + 2
303 YSQ01 = YSQUP
TEMP(J) = Y(J)
JJ = JJ + 1
GO TO 67

307 FORMAT (13HLINE PRINT FOR FREQUENCY BAND FROM F9.3 TO F9.3)
309 FORMAT (5H Y = 1PE10.5)
311 FORMAT (6H JJ = I3, 10X, CYCLE F5.0)
313 FORMAT (14H IFIX(X) = I3)
315 FORMAT (3X1HI, 1X1IM1, 1X2I1XH1, 12X1H8X, 212X1H4X, 11X3HFSM, 11X3HFSPI, 9X5HSUMX3,
39XSHSUMX4, 12X2H5X2)
END
IF DIFFUSION ZONE, DEFINE STARTING INTENSITY

20 F2 = Y2(I1) - TG1
21 IF (HO .GT. 0.01) GO TO 30

C THIN ZONE

30 NN = 1
31 GO TO (35, 40) LRI

C NORMAL ZONE -- DEFINE EXP(-DELTATAU)

35 H4(I2N) = FREXP(-HO)
36 EST = 1. - H4(I2N)**2
37 CALL SCAT
38 Q = AMAX1(0., 1. - FMUS / H(I2N))
39 NNI = NN + 1
40 GO TO (60, 70) HNI
41 F2 = FS + F2 * (1. - HD - HD) + Q * (((Y2(I1) + Y2(I2)) + 0.5 +
42 X6(I2N)) * HD)
43 GO TO 100

70 F2 = FS + F2 * H4(I2N)**2 + Q * (Y2(I1) - TG1 + (TG1 - Y2(I1)) * T
44 2 H4(I2N) + HG2 - TG1 + H4(I2N))

100 IF (F2 .LT. 0.) F2 = 0.
101 RETURN

END
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Q.T FOR SCAT/PP/SCAT/PP/SCAT/JP
SUBROUTINE SCAT

COMMON LMDA(37), IA, IB, ICA, ICO
1 KMAX, ULAMK1, ULAMK2, ULAMK3, IAM1, IAM2, IAMH, ICAM2, ICAMH
2 II, IG, HEAU, ULAMK4, IAM3, IDM, ICHAM, ICHMH
3 III1, IG1, TLM, ULAMK5, TM1, BLANK6, DELPHT
4 FCHE, CNTMAX, AK, ASMLR, PUSH, PUSHB, BOLLA, BOLLB
5 CVA, CUF, SLUG, ALPH, HVA, HVB, HCA, HCB
6 EM2A, EM2H, CA, CH, GL, GH
7 RHOL, RHOR, EPISI, RHA, RIB, RDI, ROIB
8 FP1A, RPUB, RPUIC, FPDIB, TPHNT, TA, TP, TC
COMMON IV, TD, TE, DTHP, DTHI, DTHMIN, DTMAX
1 DTMAX2, DTMX2, JTR, SWAT, CO, CMIN, DELTA
2 GANA, CH, AC, AC03TN, CNVR, SUMA, SUMR
3 ROIA, ROIAN, ROID, ROlUP, GMS, SI, S2, S3
4 S4, S5, S6, S7, S8, S9, SI0, S11
5 SL, S13, S14, S15, S16, S17, S18, S19
6 S20, E0, F0, TA, ZERO, R (152), DELTAK(152)
7 ASQ (152), RU (152), VD (152), R0D (152), SMLR (152)
8 UELA (37), P (152), P1 (152), PB (152), PHI (152)
COMMON P2 (152), SV (152), RHO (152), TMETA (152)
1 W (152), E (152), E1 (152), EK (152), A (152)
2 V (152), G (152), D (152), C (152), X2 (152)
3 X3 (152), X4 (152), X5 (152), X6 (152), X7 (152)
4 SMLA (152), SMLB (152), SMLC (152), SMLD (152), SMSLE (152)
5 EC (152), ER (152), SMLQ (152), SMLR (152), SME (152)
6 BI2O (152), CV (152), UC (152), BR (152), CMIC (152)
7 BHIH (152), CAPAC (152), CAPAR (152), CRTC (152), CRTD (152)
8 CRTC (152), GOFN (152), FEN (152), CAR (152), OMKL (37)
COMMON TELM (37), EKLM (37), ELM (37), FCLM (37)
1 FRLH (37), MLM (37), GLM (37), AMASNO (37), CHR10 (37)
2 ZP1 (37), ZPI (37), SOLID (37), ECHCK (37), R (104)
3 RL (37), ROK (104), ROK (104), TMETA (104), TEMP (16)
4 HEAU (12), MAXL, MAXLM
COMMON /LINULT/ HU, S0G1, HMHU, HUHU, HUPH, NT, IM, IN, DHMU, THICK, NT
COMMON CNTKL, SCYCLE, JMUJ
COMMON /PALMER/ FN (152), F1 (152), F11 (152), F12 (152), F13 (152), F00 (152)
2 F01 (152), F02 (152), F03 (152), JURUM
COMMON /IM/ HNU, FMU, RI, R2, RD, EST, ET, 12, GMP, A1, A3, FMUS
2 FS: LUF, LNH, T2, TEL, T27, F2
DIMENSION CGSU (1), PR (1), FM (1), H (1), H2 (1), T
1 H3 (1), H4 (1), H5 (1), T6 (1), T7 (1), T8 (1), T9 (1)
2 GST (1), GSB (1), SUNX2 (1), SUNX3 (1), SNUXX (1), X8 (1), T
3 T (1), Y2 (1), OX (1), PL (1), TR (1), FSK (1)
CALL UVCHK(IHAU) 60 TO (100, 102, 103, 104) IALPHA

CALL UNCLE

2 60 TO (100, 102, 103, 104) IALPHA

100 S0MU = FMU * FMU
FMUX = FMU
R1 = R1 / FMU
R2 = R2 / FMU
GO TO 106

102 TSQ = FMU
XX = R2 * R2
XP = R1 * R1
IF (YSO .GT. 0.) GO TO 104
SMUX = 1.
FMUX = 1.
GO TO 106
104 SMUX = 0.5 * (XP / (AP + T5U) + XX / (XX + YSO))
FMUX = SQRT(SMUX)
106 UX = H2 = H1
110 FMUS = FMS(I2H)
BB = F1U(I2) - F1U(I1) + SMUX * (F12(I2) - F12(I1))
IF (AUS(SOLU(I3)) .LT. 1.E-20) GO TO 115
C THOMSON SHORT CUT
IF (INU .NE. 0) GO TO 112
FS = .375 * FMUS * UX + ((F10(I1) + SMUX * F12(I1)) * (1. - HD) + 2.5 * BB)
GO TO 26
112 AA = H2 * (F10(I1) + SMUX*F12(I1)) - R1 * (F10(I2) + SMUX*F12(I2))
FS = .375 * FMUS / (HD + HD) * (AA * EST + BB * (DX + R1 * EST) - 2 BB * DX * EST / (HD + HD))
GO TO 26
C NORMAL COMP:ON PATH
115 DD = F13(I2) - F13(I1) + SMUX*F12(I1)
IF (INU .EQ. 1) GO TO 5
FF = F02(I2) - F02(I1) + SMUX * (F02(I2) - F02(I1))
HM = F03(I2) - F03(I1) + SMUX * (F01(I2) - F01(I1))
5 IF (INU .GT. 0) GO TO 10
C THIN RECIPE
FS1 = (1. - A1) * (1. - HD) * (F1U(I1) + SMUX * F12(I1)) + .5*BB
FS2 = -A1 * FMUX * (1. - HD) * (F13(I1) + SMUX * F11(I1)) + .5*DD
IF (INU .EQ. 1) GO TO 9
FS3=FS2*(1.-(HD)*FQ0(I1)+SMUX*FQ2(I1))*+.5*FF)
FS4 = A2 + FMUX * ((1. - HD) * (FQ3(I1) + SMUX * FQ1(I1)) + .5*HH)
FS = .375 * FMUS / (FS1 + FS2 + FS3 + FS4) * DX
GO TO 25
9 FS = .375 * FMUS / (FS1 + FS2) * DX
GO TO 25
10 AA = R2 * (F10(I1) + SMUX*F12(I1)) - R1 * (F10(I2) + SMUX*F12(I2))
CC = R2 * (F13(I1) + SMUX*F11(I1)) - R1 * (F13(I2) + SMUX*F11(I2))
IF (INU .EQ. 1) GO TO 15
EE=2*(F02(I1)+SMUX*F02(I1))-R1*(F02(I2)+SMUX*F02(I2))
EE=2*(F03(I1)+SMUX*F03(I1))-R1*(F03(I2)+SMUX*F03(I2))
15 TERM2 = DX + EST = (R1 - DX / (HD + HD))
FS1 = (1. - A1) * (AA * EST + BB + TERM2)
FS2 = A1 + FMUX * (EST + CC + DD + TERM2)
IF (INU .EQ. 1) GO TO 20
FS3=FS2*(EE*EST+FF*TERM2)
FS4 = A3 + FMUX * (EST = GG + HM + TERM2)
FS5=FS1+FS2+FS3+FS4+FS5)
GO TO 25
20 FS=FS5*(FMUX/(HD+HD))-(FS1+FS2)
25 FMUS = FMUS = GMP
26 IF (FS .GT. (-1.E-20)) GO TO 29
29 IF (AUS(INVB) .LT. 1.E-20) GO TO 128
WHITE (6b. 27) FS, 1Z3, IHN, FMUX
27 FORMAT (6H FS = IPE13.6) ET661Z2N = 13, 7X7IHNUN = 12, 7X6FH = 1
2PE13.6
WHITE (6b. 28) H1, H2, HD, EST, F51, F52, F53, F54, AA, BB, CC, DD,
2 EL: FF, 66+ HH = F1U(I1), F1O(I2), F1I(I1), F11(I2), F12(I1), F12(I2)
28 F12(I1), F13(I1), F13(I2)
28 F12(I1), F13(I1), F13(I2)
28 F12(I1), F13(I1), F13(I2)
28 F12(I1), F13(I1), F13(I2)
28 F12(I1), F13(I1), F13(I2)
29 IF (THDUBG .LT. 68) THDUG = THDUBG + 69.
128 FS=0.
IF (AUS(CVB) .LT. 1.E-20) uO TO 29
S1=75,0004
CALL UNCLE
29 CALL UVCHE(INHAU)
GO TO (30+40), INHAU
30 S1=75,0009
CALL UNCLE
40 RETURN
END
APPENDIX IV

APPLICATIONS OF THE PLANE GEOMETRY CODE

The plane geometry code was applied in conjunction with the standard SPUTTER code to several one-dimensional problems. The problem definition can be summarized as follows:

\[
\begin{align*}
T_o &= 600 \text{ eV} \\
\rho_o &= 1 \text{ g/cm}^3 \\
20 \text{ zones} & \quad T_{\text{boundary}} = 8 \text{ keV}
\end{align*}
\]

The only physical process considered was radiation transport, i.e., no hydrodynamics. The equation of state for the material was that of CH\textsubscript{2}. The opacity used was that of a hypothetical material in which \(\kappa_R = 0.2\) at all frequency groups.

The results of the calculations are shown in figures 8 through 13. Figure 8 shows the flux as a function of \(u = hv/kT\) for the Thomson scattering, i.e., \(1/m_o c^2 = 0\). The difference between the curves at 0 and 2 mfp represents the energy deposited into the material. However, in the case of Thomson scattering, no material heating should occur. One should look at this discrepancy as a convergence problem. The results of figure 8 are plotted for cycle 20. Extending the calculation further would improve the accuracy at the cost of further computer time.

Figure 9 is the same calculation with Compton scattering. One should note the spectrum changes at the high-frequency end. This reduction in the spectrum for the Compton case is qualitatively in agreement with theory.
Figure 8. Spectrum at Various Optical Depths Employing Thomson Scattering
Figure 9. Spectrum at Various Optical Depths Employing Compton Scattering
Figure 10. Exit Spectrum for Compton Scattering
Figure 11. Spectrum using Compton Scattering
Figure 12. Comparison of Theoretical and Computed Heating Rates

\[ \mu S C_2 \sum_{j=1}^{10} \frac{\dot{\gamma} \rho H_0}{j} \]
The conclusion one can draw is that the Comp Scat code converges faster than the Thom Scat. (~16% faster $\frac{12-10}{12} = 1/6$)

Figure 13. Comparison of Convergence Rates for Thomson and Compton Scattering
In the first two problems, illustrated in figures 8 and 9, the frequency groups were (0 to 8 keV), (8 to 16 keV), (16 to 24 keV), (24 to 32 keV), (32 to 40 keV), (40 to 48 keV), (48 to 56 keV), (56 to 64 keV), (64 to 72 keV), and (72 to 100 keV). The question arises as to how dependent is the solution on the number and definition of these groups. In figures 9 and 10, two different groupings were considered. In figure 10, the frequency groups were chosen to include 10 percent of the energy in an 8-keV blackbody. It is apparent from figure 10 that the spectrum has been distorted by unequal frequency groups. This conclusion was also supported by the fourth problem, i.e., the case in which the number of frequency groups was doubled. Initially, the first nine frequency groups were halved, whereas the (72 to 100 keV) group remained constant. This problem was unsuccessful. The unequal frequency grouping caused discontinuities to be propagated through the spectrum. This difficulty was eliminated by changing the (72 to 100 keV) group to (72 to 76 keV), where all the groups were then 4 keV wide. The results of this problem are shown in figure 11. Notice that figures 9 and 11 have converged to approximately the same solution. With the previous remarks in mind, one should be hesitant about solving problems in which the frequency groupings are unequal.

In figure 9, the difference between the curve at 2 and 0 mfp is the amount of energy used to heat the material. To determine if this is a reasonable estimate, a comparison of the material heating ratio (SMLE) and the theoretical heating rates

$$\left(\mu_{sc} \sum_{j=1}^{N} \gamma_j E_j\right)$$

as a function of zone is presented in figure 12. The difference between these two curves can be used as a criterion for convergence.

A further point of interest is how fast the solution converges to its steady-state value at the edge of the window away from the source. Figure 13 graphically displays this convergence.
For comparative purposes, a plot of the flux from an 8-keV blackbody as a function of frequency is presented in figure 14.

Figure 14. Spectrum from 8-keV Blackbody
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APPENDIX V

COMPTON AND INVERSE COMPTON SCATTERING

INTRODUCTION

Scattering of photons by free electrons in the Compton scattering process results in a modification of the radiative intensity in angular distribution and spectrum by virtue of the resulting energy and momentum exchange. This rate of change of intensity is described by the equation of radiative transfer, a Boltzmann equation for the photons. The objective here is to derive an approximation to the radiative-transfer equation which is valid when the scattering takes place from electrons in a Maxwellian distribution having temperature $T_e$. For this temperature to be maintained, it is necessary for electrons to undergo numerous energy-exchanging collisions between each photon scattering event. These collisions are then sufficient to maintain a Maxwellian distribution at the same temperature. Compton collisions with moving electrons may result in gain of photon energy (inverse Compton collisions) as well as energy loss. Both of these processes in the general case are necessary to describe the events which take place in a Compton scattering medium. In particular, the realization of a state of equilibrium between the photons and the electrons requires the inclusion of inverse Compton collisions and the direct Compton collisions to a consistent degree of approximation.

SCATTERING EQUATIONS

The equation for the transfer of photons in a scattering medium has been given in general form by Sampson (Ref. 8) and by Bond, Watson, and
Welch (Ref. 9). This equation has been applied by Freeman (Ref. 4) to
the Compton scattering of photons by electrons at rest. In that work the
equations are derived for photons having energies which are small compared
with $mc^2$. An expansion is performed of the Klein-Nishina cross section
through second order in the quantity $\gamma = \hbar \nu / mc^2$. A further Taylor series
expansion in frequency of the scattered intensity about the frequency of the
photon beam in question is performed to obtain an "agelike" approximation.
A similar treatment of the equation of transfer for scattering has been given
by Fraser (Ref. 5). He takes into account the Compton scattering to first
order in $\gamma$ and, in addition, includes terms to first order in the quantity
$\alpha = \theta / mc^2$, thereby including the inverse Compton effect in first order.
The objective in this appendix is to reduce the results of Fraser to a form
more suitable for calculation and to test his results in certain known limiting cases. The equation of scattering transfer will be derived first in a
form suitable for application to a general geometric situation. The equations
are then simplified by specialization to the plane and spherically
symmetric geometries, and finally the diffusion approximation is derived.

The equations for the Compton scattering are contained in Fraser's
expressions for $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ as given in his Eqs. (31), (32), and (33).
When the drift velocity $u_e$ is zero, these equations become

$$\Gamma_1 = -N\Phi_0 \left[\frac{3c^2}{16\hbar \nu^3} I(\nu, \Omega) \gamma \left(1 - \nu \frac{\partial}{\partial \nu}\right) \int d\Omega' I(\nu, \Omega')(1 - \mu + \mu^2 - \mu^3) \right]$$

$$\Gamma_2 = -N\Phi_0 \frac{3c^2}{16\hbar \nu^3} \left\{\gamma \left(1 - \nu \frac{\partial}{\partial \nu}\right) \int d\Omega' I(\nu, \Omega')(1 + \mu^2)(1 - \mu) \right\}$$

$$\Gamma_3 = -N\Phi_0 \frac{3}{16\pi} \left\{\gamma \left(1 - \nu \frac{\partial}{\partial \nu}\right) \int d\Omega' I(\nu, \Omega')(1 + \mu) \right\}$$

$$\left[1 + \alpha \left(2 - 2\nu \frac{\partial}{\partial \nu} + \nu^2 \frac{\partial^2}{\partial \nu^2}\right) \right] \int d\Omega' I(\nu, \Omega') + \alpha \left(4 - 2\nu \frac{\partial}{\partial \nu} + \nu^2 \frac{\partial^2}{\partial \nu^2}\right) \int d\Omega' I(\nu, \Omega') $$

$$\left[1 - \alpha \left(6 + 2\nu \frac{\partial}{\partial \nu} - \nu^2 \frac{\partial^2}{\partial \nu^2}\right) \right] \int d\Omega' I(\nu, \Omega') \mu^2 - \alpha \left(4 + 2\nu \frac{\partial}{\partial \nu} - \nu^2 \frac{\partial^2}{\partial \nu^2}\right) \int d\Omega' I(\nu, \Omega') \mu^3$$

(68)
where $N$ is the total number density of electrons and $\phi_0 = \frac{8\pi}{3} \left( \frac{z^2}{r_0^2} \right) = 6.65 \times 10^{-25} \text{ cm}^{-2}$ is the Thomson scattering. In Eq. (68), the values of $M_0$, $M_1$, $M_2$, and $M_3$ have been substituted for as follows:

$$M_0 = \frac{3}{16\pi} \int d\Omega' I(\nu, \Omega')$$

$$\Omega \cdot M_1 = \frac{3}{16\pi} \int d\Omega' I(\nu, \Omega')\mu$$

$$\Omega \cdot M_2 = \frac{3}{16\pi} \int d\Omega' I(\nu, \Omega')\mu^2$$

$$\Omega \cdot M_3 = \frac{3}{16\pi} \int d\Omega' I(\nu, \Omega')\mu^3$$

(69)

$$M_0 - \Omega \cdot (M_1 - M_2 + M_3) = \frac{3}{16\pi} \int d\Omega' I(\nu, \Omega')(1 + \mu^2)(1 - \mu)$$

where the cosine of the angle of scattering is $\Omega \cdot \Omega' = \mu$. These terms represent the scattering contributions to the equation of radiative transfer for the intensity $I(\nu, \Omega)$. The first term represents the scattering out of the beam proportional to the total Klein-Nishina cross section expanded to first order in $\gamma$. The second term represents the contribution of stimulated scattering to both of the processes of scattering from the beam and scattering into the beam. The quantity $\Gamma_3$ describes the scattering of photons into the beam by all of the electrons contained in the Maxwell distribution.

A comparison shows that the $\alpha$-independent terms are the same as Freeman's first order in $\gamma$ terms. These terms can be regrouped to display the order of the terms more effectively. Their sum, comprising the scattering contributions to the radiative transfer equation, is
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\[
\begin{align*}
&- \mu_s \left[ (1 - 2\gamma) I(\nu, \Omega) - \frac{3}{16\pi} \right] \int \text{d}\Omega' \ I(\nu, \Omega')(1 + \mu^2) \\
&- \gamma \left( 1 - \nu \frac{\partial}{\partial \nu} \right) \int \text{d}\Omega' \ I(\nu, \Omega')(1 + \mu^2)(1 - \mu) \\
&+ \alpha \left[ 2 \int \text{d}\Omega' \ I(\nu, \Omega')(1 - 2\mu - 3\mu^2 + 2\mu^3) \\
&+ \left( \nu^2 \frac{\partial^2}{\partial \nu^2} - 2\nu \frac{\partial}{\partial \nu}, \int \text{d}\Omega' \ I(\nu, \Omega')(1 + \mu^2)(1 - \mu) \right] \\
&- \frac{c^2}{h\nu^3} I(\nu, \Omega) \gamma \left( 1 - \nu \frac{\partial}{\partial \nu} \right) \int \text{d}\Omega' \ I(\nu, \Omega')(1 + \mu^2)(1 - \mu) \right] \\
&\text{(70)}
\end{align*}
\]

where \( \mu_s = \phi_0 N \).

In order to place the equation in conservative form in anticipation of formation of difference equations for numerical work, it is desirable to reformulate the frequency derivative terms. This reformulation corresponds to isolating those terms which contribute to exchange of energy within the photon distribution from the terms which contribute to exchange of energy between the electrons and photons. The desired substitutions are as follows:

\[
\begin{align*}
\gamma \left( 1 - \nu \frac{\partial}{\partial \nu} \right) I &= \frac{h}{mc^2} \left[ 3\nu I - \frac{\partial (\nu^2 I)}{\partial \nu} \right] \\
\nu^2 \frac{\partial^2 I}{\partial \nu^2} - 2\nu \frac{\partial I}{\partial \nu} &= \frac{\partial^2 (\nu^2 I)}{\partial \nu^2} - 6 \frac{\partial (\nu I)}{\partial \nu} + 4I \\
&\text{(71)}
\end{align*}
\]

For the case of spherical or plane geometry, symmetry of the radiative intensity about the polar direction can be assumed. This symmetry permits the integration of the azimuthal component of the solid angle in the photon scattering terms. If the results of Freeman (Ref. 4, p. 10) are used, the scattering terms become

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\[ -\mu_s \left[ (1 - 2\gamma) I(\nu, \Omega) - \frac{3}{16} \left( \int d\mu_3 I_3 \left[ 3 - \mu_1^2 + (3\mu_1^2 - 1) \mu_3^2 \right] \right) \right] \]

\[ -\gamma \left( 1 + \frac{c^2}{3h\nu} I(\nu, \Omega) \right) \left( 1 - \nu \frac{\partial}{\partial \nu} \right) \int d\mu_3 I_3 f(\mu_1, \mu_3) \]

\[ + \alpha \left( 2 \int d\mu_3 I_3 \left[ (3\mu_1^2 - 1)(3\mu_3^2 - 1) + 2\mu_1 (3\mu_1^2 - 1) \mu_3^2 + 2\mu_1 (5\mu_1^3 - 3) \mu_3^3 \right] \right) \]

\[ + \left( \nu^2 \frac{\partial^2}{\partial \nu^2} - 2\nu \frac{\partial}{\partial \nu} \right) \int d\mu_3 I_3 f(\mu_1, \mu_3) \right) \]  

\[ \text{where} \]

\[ f(\mu_1, \mu_3) = 3\mu_1^2 - 1 + \mu_1 (3\mu_1^2 - 5) \mu_3 + (3\mu_1^2 - 1) \mu_3^2 + \mu_1 (3 - 5\mu_1^2) \mu_3^3 \]

In Eq. (72), the inclusion of the additional terms for induced emission and inverse Compton scattering is not appreciably more difficult than for the Compton scattering term already considered. The same angular integrals enter as in Freeman's Eq. (18) (Ref. 4). The main difference arises in the presence of the second derivative with respect to frequency in addition to the first derivative occurring in the Compton formulation.

The diffusion approximation can also be carried out in parallel fashion to the derivation of Compton scattering for cold electrons. In this case two equations result, which are obtained from the zeroth and first moments of the equation of transfer. The Compton terms entering in the zeroth-moment equation are

\[ -\mu_s \left[ -\gamma \left( I_o + \nu \frac{\partial I_o}{\partial \nu} \right) + \frac{c^2}{h\nu} \gamma I_o \left( \frac{2}{15} I_1 - \nu I_o \frac{\partial I_o}{\partial \nu} + \frac{2}{15} \nu I_1 \frac{\partial I_1}{\partial \nu} \right) \right] \]

\[ - \alpha \left( \nu^2 \frac{\partial^2 I_o}{\partial \nu^2} - 2\nu \frac{\partial I_o}{\partial \nu} \right) \]  

\[ \text{where} \]

\[ f(\mu_1, \mu_3) = 3\mu_1^2 - 1 + \mu_1 (3\mu_1^2 - 5) \mu_3 + (3\mu_1^2 - 1) \mu_3^2 + \mu_1 (3 - 5\mu_1^2) \mu_3^3 \]
and those for the first-moment equation are

\[
- \mu_s \left[ I_1 \left( 1 - \frac{12}{5} \gamma \right) + \frac{2}{5} \gamma v \frac{\partial I_1}{\partial v} + \frac{c^2}{hv^3} \gamma \left( \frac{3}{5} I_0 I_1 + \frac{2}{5} v I_0 \frac{\partial I_1}{\partial v} \right) - \nu I_1 \frac{\partial I_0}{\partial v} \right] + \frac{2}{5} \alpha \left( I_1 + \nu^2 \frac{\partial^2 I_1}{\partial v^2} - 2 \nu \frac{\partial I_1}{\partial v} \right) \right]
\]

TESTS OF THE EQUATIONS

The equations derived above can be subjected to a number of tests to determine whether known results are obtained. Consider a scattering medium containing a weak radiation field for which the stimulated emission terms can be neglected. Integrations can then be carried over the solid angle and the entire frequency spectrum. The resultant integrated radiative-transfer equation becomes an equation for the rate of change of radiation energy within a particular volume element. The Compton contributions to the rate of change represent the rate of loss or gain of energy from the radiation field. They also correspond to the gain or loss of energy by the electrons. These terms are

\[
\int \mathrm{d} \nu \int \mathrm{d} \Omega (I_1 + I_3) = \mu_s c \left[ \int \gamma E_\nu \mathrm{d} \nu + \int \gamma^2 \frac{\partial E_\nu}{\partial \gamma} \mathrm{d} \nu + \alpha \int \left( \nu^2 \frac{\partial^2 E_\nu}{\partial \nu^2} - 2 \nu \frac{\partial E_\nu}{\partial \nu} \right) \mathrm{d} \nu \right]
\]

\[
= \mu_s \left( 4 \theta_e \int E_\nu \mathrm{d} \nu - \int h E_\nu \mathrm{d} \nu \right)
\]

where \( E_\nu = 1/c \int \mathrm{d} \Omega \nu (\nu, \Omega) \) is the spectral radiation energy. This is precisely the result given by Grasberger (Ref. 10).

The terms taking account of the stimulated scattering can also be included provided the radiation is isotropic. The contribution to the heating rate is
If, in addition, the radiation energy is given by a Planck distribution having a temperature $\theta_r$, as given by
\[ I = B = \frac{2h}{c} \frac{\nu^3}{e^{\frac{h
u}{\theta_r}} - 1} \]  
the frequency integrations can be performed. The total resulting heating rate of the electrons is
\[ \frac{dE_e}{dt} = \frac{1}{m^2c^2} \frac{\sigma \nu^4}{\theta_r} \left( \theta_r - \theta_e \right) \]  
where $\sigma$ is the Stefan-Boltzmann constant. This rate is applicable to a blackbody enclosure in which the radiation intensity is somehow maintained in a Planck distribution corresponding to a radiation temperature which is different from the temperature of the electrons within the enclosure. The derived electron heating rate is a well-known result (Ref. 10) which displays the characteristic that the radiation and electron temperatures will approach each other by virtue of this scattering interaction, yielding an equilibrium state in which the temperatures are equal.

These results test all of the Compton terms, including the induced scattering terms. Consequently, they constitute a strong confirmation of the correctness of these equations.

**APPROXIMATIONS**

Several approximations have been made in deriving these equations which limit their applicability. First, the neglect of high-order terms in $\gamma$ and $\alpha$ restricts validity of the transfer equation to $\nu$ and $\theta_e$ of the order of 100 kV. The inclusion of the terms corresponding to $\gamma^2$ given in reference 4 would allow application of the equations to somewhat higher photon energies.
The second approximation, resulting from the expansion of the frequency dependence of the intensity and omission of quadratic terms in the frequency difference, implies that $I_\mu$ must be a smooth function of frequency. Near a photoelectric edge or in the neighborhood of a line profile, these conditions may not be satisfied. An investigation of the validity of this approximation has been carried out by Chandrasekhar (Ref. 11), who evaluated the transmission of line radiation through a cold scattering atmosphere. His result shows that even in the case of a $\delta$-function source of radiation, a relatively small error, as measured by the fraction of energy erroneously scattered to higher frequencies (see Fig. 33, Ref. 11, p. 334), results (approximately 15 percent for an atmosphere containing two-thirds of a mean free path).

The neglect of the specific effects of polarization may also be of some consequence. Chandrasekhar (Ref. 12) has compared the diffuse reflection resulting from radiation incident on a semi-infinite scattering medium when the correlation of photon polarization after scattering is followed or neglected. Differences of the order of 5 percent in the scattered intensity are obtained (see Figs. 24 and 25, Ref. 12, pp. 262 and 263).

Scattering in the treatment of this report results from electrons which are free. Effects of binding of the electrons and the localization of the electrons within the atom give rise to modifications of the incoherent scattering Klein-Nishina formula for free-electron scattering and produce additional scattering which is coherent with the incident radiation. These effects are small when the energy of the photon is much greater than the binding energy of the atom and when the temperature of the material is sufficiently high that the probability of bound electrons being present is small.
EQUILIBRIUM SPECTRA

In this subsection, the solutions to the above equations corresponding to scattering of radiation in a homogeneous enclosure are examined. The equation for the spectral energy density is

\[ \frac{1}{c} \frac{\partial I}{\partial t} = \mu_n (B - I_o) - \mu_s \left[ -2\gamma I_o + \left( 1 + \frac{c^2}{3h^2} \right) \left( I_o - \nu \frac{\partial I_o}{\partial \nu} \right) \right. \\
\left. - a \left( \nu^2 \frac{\partial^2 I_o}{\partial \nu^2} - 2\nu \frac{\partial I_o}{\partial \nu} \right) \right] \] (79)

The steady-state solution of these equations should admit a Planck function for the radiation intensity corresponding to the temperature of the electrons. Substitution of the Planck function into Eq. (79) shows that such is the case. For the Planck function to satisfy this equation, however, it is necessary that the correct number density of photons be present. This condition will not in general be satisfied, however, if only the scattering interaction is present. More generally, the equilibrium will be achieved with too few or too many photons for the thermodynamic equilibrium solution to apply.

If too few photons are present, the induced scattering terms quadratic in \( I_o \) will be negligible, giving the steady-state equation

\[ \nu^2 \frac{\partial^2 I_o}{\partial \nu^2} + \left( \frac{\gamma}{\alpha} - 2 \right) \nu \frac{\partial I_o}{\partial \nu} + \frac{\gamma}{\alpha} I_o = 0 \] (80)

This equation is satisfied by the function \( I_o = Av^3 e^{-hv/\theta} \), the Wien approximation to the Planck function, but it has an arbitrary normalization, depending on the number of photons present.

If, on the other hand, there are too many photons for the thermodynamic equilibrium solution, only the quadratic terms need be retained,
In this case, the solution is \( I_0 = B \nu \), where \( B \) depends on the photon number density. However, this solution is valid only for frequencies such that \( \nu^2 < \left( \frac{c^2}{\hbar} \right) B \).

**CRITERIA FOR COMPTON HEATING**

The inclusion of the Compton and inverse Compton scattering contributions to the heating rate is required only when the heating by pure absorption is negligible by comparison. In order to estimate the conditions where the Compton terms must be included, an estimate of the absorption contribution to the heating is derived and is then compared with the corresponding Compton scattering formula. As shown above, if the radiation intensity can be characterized by a Planck distribution with \( \theta \), the electron heating rate from Compton scattering is

\[
\frac{dE_e}{dt} = 2.1 \times 10^{-17} N_e \theta^4 (\theta - \theta_e) \quad \text{(ergs/cm}^3\text{sec)}
\]

when temperatures are measured in eV and \( N_e \) in cm\(^{-3}\).

At high temperatures, the pure absorption mechanism giving the largest contribution will be that from the free-free absorption. By using the absorption coefficient for the free-free process (Ref. 13), it is possible to derive a formula for the rate at which electrons exchange energy with the photons:

\[
\frac{dE_e}{dt} = 1.5 \times 10^{-25} Z^2 N_e N_i \theta_e^{-1/2} (\theta - \theta_e) \quad \text{(ergs/cm}^3\text{sec)}
\]

where \( Z^2 \) is the effective square of the ionic charge and \( N_i \) is the ionic number density.
The ratio $R$ of the absorption heating rate to the scattering heating rate is given by

$$R = \frac{7.2 \times 10^{-9} Z^2 N_i}{\theta_e^{1/2} \theta_r^4}$$

(84)

A large value of $R$ implies that scattering can be neglected in the electron heating rate. As an illustration of the application of this formula, one can obtain the temperature and density conditions at which heating due to scattering and absorption are equal. Figure 15 displays the temperature-density dependence of this condition, $R = 1$, for several typical materials, assuming that $\theta_e = \theta_r$. In applying this result, it should be kept in mind that deviations of the radiation spectrum from Planckian will strongly affect the answer.

The above criterion for the electron heating rate does not directly apply to the relative importance of absorption and scattering on the spectrum itself, since the frequency dependence of the absorption and scattering terms is quite different. If $R \geq 1$, the low-frequency portion of the spectrum will be dominated completely by absorption. The high-frequency portion of the spectrum will be modified predominately by direct Compton scattering if $\theta_r \gg \theta_e$ and by inverse Compton scattering when $\theta_r \ll \theta_e$. 

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Figure 15. Temperature and Density at which Heating Rates Due to Absorption and Scattering are Equal
In references 14 and 15, quantities termed "transmission functions" are proposed for use in the analysis of radiative transfer by the method of characteristics. These functions are mean values of the monochromatic attenuation factor \( \exp(-\rho_{\nu} \Delta x) \) for a range of frequencies \( \Delta \nu \), where \( \Delta x \) is an interval along a ray within which the density \( \rho \) and opacity \( \kappa_{\nu} \) are assumed to be uniform. The frequency average of this exponential is taken with two different weighting functions \( B_{\nu} \) and \( dB_{\nu}/d\theta \). In an idealized problem with uniform temperature only the first of these averages, denoted by \( S(\Delta x) \), is needed. This function, defined by

\[
S_{ij}(\Delta x) = \frac{\int_{v_j}^{v_{j+1}} B(\nu, \theta) \exp[-\rho_{ij}(\nu, \rho, \theta) \Delta x_i] d\nu}{\int_{v_j}^{v_{j+1}} B(\nu, \theta) d\nu}
\]  

(85)

has a limiting form for very small \( \Delta x \) given by

\[
S_{ij}(\Delta x_i) \sim \exp(-\kappa_{ij} P \Delta x_i)
\]  

(86)

where \( \kappa_{ij} P \) is the usual Planck mean. For larger \( \Delta x \) (in most cases still rather small) the function flattens out rapidly; i.e., \(-(1/\rho \Delta x) \log S\) is a decreasing function of \( \Delta x \), eventually limiting to the minimum value of \( \kappa_{\nu} \) in the frequency group at sufficiently large \( \Delta x \), plus terms of order \( \Delta x^{-1} \).
In a medium with uniform composition, temperature, and density, the transport equation for the intensity in group $j$ at a point $x_n$ along a ray $x_o < x < x_n$ due to radiation moving in the positive $x$ direction is given in references 14 and 15 as

$$I_j(x_n) = B_j(x_n) + [I(x_o) - B(x_o)] S_j(x_n - x_o) \quad (87)$$

If the interval $(x_o, x_n)$ is now subdivided into $n$ zones of width $\Delta x = (x_n - x_o)/n$, the intensity at each of the subdivision points is

$$I_j(x_i) = B_j(x_i) + [I(x_i-1) - B(x_i-1)] S_j(\Delta x)$$

$$= B_j(x_i) + [I(x_i-2) - B(x_i-2)] S_j^2(\Delta x), \text{ etc.} \quad (88)$$

By repeated application of Eq. (88), one readily obtains

$$I_j(x_n) = B_j(x_n) + [I(x_o) - B(x_o)] S_j^n(\Delta x) \quad (89)$$

which, with Eq. (87), implies that

$$S_j(n\Delta x) = S_j^n(\Delta x) \quad (90)$$

a condition which is satisfied if $S$ is an exponential function, but is unfortunately violated by functions of the type described in references 14 and 15. With sufficiently fine subdivision of the medium in this example, the attenuation would in fact be that given by the Planck opacity, an incorrect result apart from the limiting case in which $\rho \alpha P(x_n - x_o) < 1$.

This example exhibits the limitations resulting from one of the assumptions made in references 14 and 15, and in Eq. (87), namely that the frequency dependence of $I(\nu)$ within each group is proportional to that of $B(\nu)$ at every point along the ray. Actually, the frequency dependence of $I(\nu)$ depends inherently upon conditions along the entire ray, and no method of frequency averaging based only upon locally evaluated temperature and density can be expected to be valid.
A second and unrelated difficulty occurs in spherical geometry, namely that consistent and simple models for the spatial dependence of the source function do not reduce to the correct diffusion limit, and other models which limit correctly are somewhat inconsistent and have a tendency to yield negative values for the intensity (Ref. 2, Vol III).

In this appendix a formulation is proposed which employs (1) nonlocal "transmission functions" of exponential form and (2) an apparently consistent model for the source function which has the correct diffusion limit. The method utilizes only the currently available Planck and Rosseland group mean opacities, but is capable of generalization to a formulation which, by describing the frequency dependence of opacity within each group in more detail, may allow use of fewer groups to attain the needed accuracy.

For monochromatic radiation, the intensity along a ray at optical depth $\tau_n$ is given by

$$I(\tau_n) = I(\tau_0) e^{-(\tau_n - \tau_0)} + \int_{\tau_0}^{\tau_n} B e^{-((\tau_n - \tau_o) d\tau}$$

with integration by parts,

$$I(\tau_n) = B(\tau_n^+) + [I(\tau_o^-) - B(\tau_o^-)] e^{-(\tau_n - \tau_o)} - \int_{\tau_0}^{\tau_n} \frac{dB}{d\tau} e^{-(\tau_n - \tau)} d\tau$$

The last term, representing source gradient contributions to the intensity, may be represented as a sum of individual zone contributions:

$$\int_{\tau_0}^{\tau_n} \frac{dB}{d\tau} e^{-(\tau_n - \tau)} d\tau = \sum_{k=1}^{n} Q_{kn}$$
where

\[ Q_{kn} = \int_{s_{k-1}}^{s_k} \frac{dB}{d\tau} \left( \frac{1}{\tau} - \frac{1}{\tau_s} \right) d\tau + B_k^+ - B_k^- \]

\[ = e^{-\left(\frac{1}{\tau} - \frac{1}{\tau_s}\right)} \left[ \int_{s_{k-1}}^{s_k} \frac{dB}{ds} e^{-\sigma_k(s - s_s)} ds + B_k^+ - B_k^- \right] \quad (94) \]

Here \( s \) is the geometric distance along the ray, \( \sigma_k = \rho_k \omega_k \) is assumed constant in each interval \( s_{k-1} < s < s_k \), and the possibility of discontinuous \( B(s) \) at interval boundaries is allowed for.

It is now necessary to represent the source gradient \( dB/ds \) in an explicit manner. Let the direction cosines of a ray with respect to Cartesian axes \( x, y, \) and \( z \) be denoted respectively by \( \xi, \eta, \) and \( \zeta \); then

\[ ds = \frac{1}{\xi} dx = \frac{1}{\eta} dy = \frac{1}{\zeta} dz \quad (95) \]

The quadratic form

\[ B(x, y, z) = B(x_k, y_k, z_k) + \alpha_k (z - z_k) + \beta_k \left( x^2 + y^2 - x_k^2 - y_k^2 \right) \]

\[ + \gamma_k \left( x^2 + y^2 + z^2 - x_k^2 - y_k^2 \right) \]

\[ x_{k-1} < x < x_k \]

\[ y_{k-1} < y < y_k \]

\[ z_{k-1} < z < z_k \quad (96) \]

has plane symmetry if \( \beta_k = y_k = 0 \), cylindrical symmetry if \( \alpha_k = y_k = 0 \), spherical symmetry if \( \alpha_k = \beta_k = 0 \), and two-dimensional (finite) cylindrical symmetry if just \( y_k = 0 \). This form may be simplified somewhat if the Cartesian axes can be chosen independently for each ray, so that, for example, the \( x \)-axis is parallel to the direction of the ray in spherical
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gameometry, or to the projection of the ray on the plane z = 0 in cylindrical geometry.

From Eqs. (95) and (96) the source gradient along the ray is

\[
\frac{dB}{ds} = \alpha_k \xi + 2\beta_k (\xi x + \eta y) + 2\gamma_k (\xi x + \eta y + \xi z)
\]

so that

\[
Q_{kn} = \left[ \frac{\alpha_k}{\sigma_k} \xi + 2 \frac{\beta_k}{\sigma_k} (\xi \sigma_k x_k + \eta \sigma_k y_k - \xi^2 - \eta^2) + 2 \frac{\gamma_k}{\sigma_k} (\xi \sigma_k x_k + \eta \sigma_k y_k + \xi z_k - 1) \right] e^{-(\tau_n - \tau_k)}
\]

\[
- \left[ \frac{\alpha_k}{\sigma_k} \xi + 2 \frac{\beta_k}{\sigma_k} (\xi \sigma_k x_{k-1} + \eta \sigma_k y_{k-1} - \xi^2 - \eta^2) + 2 \frac{\gamma_k}{\sigma_k} (\xi \sigma_k x_{k-1} + \eta \sigma_k y_{k-1} + \xi z_{k-1} - 1) \right] e^{-(\tau_n - \tau_{k-1})}
\]

On comparing Eqs. (97) and (98), it is apparent that when \( \tau_n - \tau_{n-1} \gg 1 \) the intensity limits to the value

\[
I(\tau_n) = B(\tau_n) - \left. \frac{dB}{d\tau} \right|_{\tau = \tau_n} = I(\tau_n)
\]

where for plane geometry \( \tau_n = \tau_n \) and otherwise \( \tau_n = \tau_n - 1 \). This is essentially consistent with the diffusion approximation.

So far, the analysis has been restricted to the single-frequency case. Equation (92) contains several types of terms, each with a different frequency dependence, corresponding to a variety of different materials, temperatures, and densities. Since frequency averaging destroys the separability of these contributions (as discussed above), approximation methods of a nonlocal character are needed.

The transmission functions of reference 15 can easily be generalized to nonlocal form. For example, the Planck transmission function defined
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for a single zone by Eq. (85) is to be replaced by the kernel

\[ S_j(s_i, s_n) = \int_{\nu_j}^{\nu_{j+1}} \sigma_i B(\theta_i, \nu) \exp \left[ -\int_{\nu_i}^{\nu_{i+1}} \sigma_k B(\theta_k, \nu) \, d\nu \right] \sum_{k=1}^{n-1} [-\sigma_k(\nu, \rho_k, \theta_k) \Delta s_k] \, d\nu / \int_{\nu_i}^{\nu_{j+1}} \sigma_i B(\theta_i, \nu) \, d\nu \]  

(100)

which describes the attenuation of radiation emitted at \( s_i \) between \( s_i \) and a field point \( s_n \). Of course, the direct evaluation of the terms in Eq. (100) is scarcely feasible in practice. Instead, a representation of the entire function is required which can be readily evaluated and which preserves some of the characteristics of Eq. (100), in particular, the thin limit

\[ \lim_{s_n \to s_i} S_j(s_i, s_n) = 1 - (s_n - s_i) \sigma_j^P(s_i) \]  

(101)

where \( \sigma_j^P(s_i) \) is the group Planck mean. A necessary condition for the diffusion limit is that for opacity independent of position

\[ \lim_{s_i \to -\infty} \sigma_j^R(s_i) \int_{s_i}^{s_n} S_j(s, s_n) \, ds = 1 \]  

(102)

where \( \sigma_j^R(s_i) \) is the group Rosseland mean. In general, \( S_j \) as a function of \( s_n \) should be uniformly positive, while the first derivative should be uniformly negative; also, the function should be short-range in the sense that for

\[ s_i \to -\infty, \quad S_j(s_i, s_n) \sim (s_n - s_i)^{-m}, \quad m > 1 \]  

(103)

where if \( m \) is not infinite, as for an exponential, it is at least large enough to restrict the significant contributions to the integral in Eq. (102) to source positions within a very few Rosseland mean free paths of the field point in all except pathologically nongrey cases.

The following construction is a nonlocal "picket-fence" (Refs. 16 and 17) transmission function of this type. It is assumed that the group
Planck and Rosseland means are known at each point, and that a parameter $b_{kj}$ characteristic of the opacity distribution within group $j$ at point $s_k$ can be chosen. (For brevity, the group index $j$ will be suppressed in the following formulas.) The first step is to construct the following composite opacities for each point:

$$\sigma_A = \frac{\sigma^P + b\sigma^R - \text{sgn}(1 - b^2) \left[ \left( \sigma^P - \sigma^R \right) \left( \sigma^R - b^2 \sigma^P \right) \right]^{1/2}}{1 + b} \quad (104)$$

$$\sigma_B = \frac{\sigma^P - b\sigma^R + \text{sgn}(1 - b^2) \left[ \left( \sigma^P - \sigma^R \right) \left( \sigma^R - b^2 \sigma^P \right) \right]^{1/2}}{1 - b} \quad (105)$$

where $b$ is to be chosen so that $\sigma_A$ and $\sigma_B$ are real, and

$$0 < \sigma_A \leq \sigma_B \quad (106)$$

These conditions are satisfied if $\sigma^P = \sigma^R$, or if $\text{sgn}(1 - b^2) = \text{sgn}(\sigma^P - \sigma^R)$, i.e.,

$$b^2 < 1 < \frac{\sigma^P}{\sigma^R} \quad (107)$$

or

$$b^2 > 1 > \frac{\sigma^P}{\sigma^R} \quad (108)$$

Equation (108) only applies to cases which are nearly grey, or which have substantial scattering contributions in the Rosseland mean. Values of $\sigma_A$ and $\sigma_B$ for two ratios $\sigma^P/\sigma^R$ are shown in figure 16.

The second step is to define the optical distances

$$\tau^A(s_i, s_n) = \int_{s_i}^{s_n} \frac{\sigma^A(s)}{\sigma^A} ds \quad (109)$$

$$\tau^B(s_i, s_n) = \int_{s_i}^{s_n} \frac{\sigma^B(s)}{\sigma^B} ds \quad (110)$$
Figure 16. Dependence of Component Opacities on the Parameter b
Next, the transmission function is represented by

\[ S(s_i, s_n) = \frac{1}{2} (1 + b_n) e^{-\tau(s_i, s_n)} + \frac{1}{2} (1 - b_n) e^{-\tau(s_i, s_n)} \]  

(111)

and, finally, the 'nonlocal opacity' by

\[ \sigma(s_i, s_n) = -\frac{\sigma}{ds_n} \log S(s_i, s_n) \]

\[ = \frac{1}{S(s_i, s_n)} \left[ \frac{1}{2} (1 + b_n) e^{-\tau(s_i, s_n)} + \frac{1}{2} (1 - b_n) e^{-\tau(s_i, s_n)} \right] \]

(112)

The significance of the parameter \( b \) is now clearer. Assuming \( \sigma^P \) to be considerably larger than \( \sigma^R \) along the ray, two limiting cases can be distinguished. The nongrey character may be due to the presence within the group of a few strong line components, with the absorption minima, or windows, occupying most of the group width. The appropriate values of \( b \) will then be those near 1, so that according to Eq. (105) the 'line' opacity \( \sigma^B \) is large compared with \( \sigma^P \). The transmission, as shown in figure 17, attenuates at the Planck rate for a very short distance, with most of the contributions attenuated at a rate \( \sigma^A \) which is slightly less than the Rosseland mean. At the other extreme, there is the less probable situation in which the absorption maxima are broad, with the nongrey character due to the presence of isolated narrow but deep windows. For \( b \to -1 \), \( \sigma^A \to 0 \), and \( \sigma^B \to \sigma^P \). The transmission attenuates at nearly the Planck mean rate for perhaps several Planck mean free paths, but then levels out in such a way that small contributions are transmitted with little attenuation from remote sources. For intermediate cases, values of \( b \) near zero may be used. In particular, for \( b = 0 \),

\[ \begin{align*}
\sigma^A_B &= \sigma^P \pm \left[ \sigma^P (\sigma^P - \sigma^R) \right]^{1/2} \\
\end{align*} \]

(113)
Figure 17. Transmission Function \( S(\Delta x) \)
and just half of the emitted radiation attenuates by each of the rates $\sigma_A$ and $\sigma_B$. For $\sigma_P < \sigma_R$, in order that the $S$ function defined by Eq. (111) be positive at large distances, $b$ must be restricted to positive values $b > 1$. The transmission functions also have a more appropriate shape near the source for $b > 1$, as shown in figure 17.

The evaluation of the parameter $b$ will be discussed in a subsequent report, in terms of a distribution function for mean free paths at each point (Ref. 17). In the absence of such information, a "defined" value such as 0 may be used. It should be noted that the Milne-Eddington model is not explicitly assumed; however, $b$ is not treated as a function of position in the definitions of the derivative and integral of the transmission function, Eqs. (112) and (102).

The quantities of the form $e^{-(\tau_B - \tau_k)}$ and $\sigma_k$ appearing in Eqs. (92) and (98) may then be evaluated by the expressions given in Eqs. (111) and (112), respectively, to obtain a computationally feasible frequency-averaged form.

The source gradient coefficients $\alpha_k$, $\beta_k$, $\gamma_k$ in Eqs. (96), (97), and (98) are readily defined by Rosseland optical depth interpolation. The index $k$ may be regarded as ordering the intersections of the ray with successive surfaces, which are alternately boundary surfaces and mid-surfaces for each cell; furthermore, $k$ and $s$ are assumed to increase in the direction of the ray. Thus, if $k$ indexes a boundary point and $k-1$ a zone midpoint, for one-dimensional cases

$$\alpha_k = \frac{B_k - B_{k-1}}{z_k - z_{k-1}}, \quad \beta_k = \frac{B_k - B_{k-1}}{r_k^2 - r_{k-1}^2}, \quad \gamma_k = \frac{B_k - B_{k-1}}{R_k^2 - R_{k-1}^2} \tag{114}$$

where $r^2$ and $R^2$ are, respectively, $x^2 + y^2$ and $x^2 + y^2 + z^2$, and

$$B_k = B_{k-1} + \epsilon_k \left[ \frac{(B_{k+1} - B_k) \sigma_k}{\sigma_k (z_k - z_{k-1}) + \sigma_{k+1} (z_{k+1} - z_k)} \right] \tag{115}$$
for plane geometry, and

$$B_k = B_{k-1} + \epsilon_k \frac{(B_{k+1} - B_{k-1}) (\sigma_R^2 R_k^2 - R_{k-1}^2)}{(\sigma_k^2 (R_k^2 - R_{k-1}^2) + (\sigma_{k+1}^2 R_{k+1}^2 - R_k^2)}$$

for spherical geometry. One-dimensional cylindrical geometry may employ an interpolation formula like Eq. (116) with $R$ replaced by $r$. The coefficient $\epsilon_k$ is 1 unless the step model is to be used, in which case $\epsilon_k = 0$ and consequently $\alpha_k = \beta_k = \gamma_k = 0$. In two-dimensional cylindrical geometry, the corresponding formulae for bilinear Rosseland optical depth interpolation in the variables $z$ and $r^2$ are left as an exercise for the reader. If $k$ indexes a midpoint, similar formulae apply, with

$$\alpha_k = \frac{B_k - B_{k-1}}{z_k - z_{k-1}}, \text{ etc.}$$

A final remark is that for the plane case, the integration over ray orientation should be performed analytically. The exponentials in Eq. (111) are then to be replaced by $E$-functions. The detailed evaluation of this approach is left for a subsequent report.
REFERENCES


12. Ibid., p. 254.


The OUTPUT code is designed for the analysis of early-time nuclear explosions. The equations for radiative transfer (characteristic method) and conservation of total (fluid and radiation) momentum and energy are solved in one-dimensional (plane or spherical) geometry. The radiation equations include first-order Compton scattering, and the hydrodynamic equations are treated in explicit Lagrangian form. The code is undergoing continuing development; the formulation, flow charts, glossary, and listings presented represent its status as of 27 October 1967.
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