AD NUMBER

AD829895

NEW LIMITATION CHANGE

TO
Approved for public release, distribution unlimited

FROM
Distribution authorized to U.S. Gov’t. agencies and their contractors; Administrative/Operational Use; Feb 1968. Other requests shall be referred to Director, Air Force Aero Propulsion Laboratory, Wright-Patterson AFB, OH, 45433.

AUTHORITY

AFAPL ltr, 21 Sep 1971
AFAPL-TR-65-45
Part VII

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY
Part VII: The Three Lobe Bearing and Floating Ring Bearing

J. Lund
Mechanical Technology Incorporated

TECHNICAL REPORT AFAPL-TR-65-45, PART VII
February 1968

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Air Force Aero Propulsion Laboratory (APFL), Wright-Patterson Air Force Base, Ohio 45433.

Air Force Aero Propulsion Laboratory
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio
AFAPL-TR-65-45
Part VII

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY
Part VII: The Three Lobe Bearing and Floating Ring Bearing

J. Lund
Mechanical Technology Incorporated

TECHNICAL REPORT AFAPL-TR-65-45, PART VII
February 1968

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Air Force Aero Propulsion Laboratory (APFL), Wright-Patterson Air Force Base, Ohio 45433.

Air Force Aero Propulsion Laboratory
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio
FOREWORD

This report was prepared by Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110 under USAF Contract No. AF33(615)-3238. The contract was initiated under Project No. 3048, Task No. 304806. The work was administered under the direction of the Air Force Aero Propulsion Laboratory, with Mr. Michael R. Chasman (AFPL) acting as project engineer.

This report covers work conducted from 1 August 1966 to 1 August 1967.

This report was submitted by the authors for review on 31 August 1967. Prior to assignment of an AFAPL document number, this report was identified by the contractor's designation HTI 67T147. This report is Part VII of final documentation issued in multiple parts.

This technical report has been reviewed and is approved.

ARTHUR V. CHURCHILL, Chief
Fuels, Lubrication and Hazards Branch
Support Technology Division
Air Force Aero Propulsion Laboratory
ABSTRACT

This volume treats three special bearing types selected for study because of their favorable stability characteristics and, hence, their potential for use in high speed rotating machinery applications. The three bearing types are:

a. The Three Lobe Journal Bearing
b. The Floating Sleeve Bearing with an Incompressible Lubricant
c. The Floating Sleeve Bearing with a Compressible Lubricant.

In the floating sleeve bearings, the ring is prevented from rotating but is otherwise free to move. The ring is floated by pressurizing the outer film of the bearing. In the case of a compressible lubricant, the inner film is pressurized as well.

The volume gives extensive design data in form of charts and tables from which the bearing dimensions can be obtained for a given application. Data are given for bearing flow, friction power loss and the speed at which hydrodynamic instability sets in. In addition, two computer programs accompany the volume, and instructions and listings of the programs are included. The programs may be used to obtain data for cases not covered by the presented design data.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>THE THREE LOBE JOURNAL BEARING</td>
<td>5</td>
</tr>
<tr>
<td>THE HYDRODYNAMIC-HYDROSTATIC RING BEARING</td>
<td>15</td>
</tr>
<tr>
<td>THE HYBRID-HYDROSTATIC RING BEARING</td>
<td>25</td>
</tr>
<tr>
<td>APPENDIX I: The Static and Dynamic Performance of a Partial Arc Bearing with Turbulent Film</td>
<td>95</td>
</tr>
<tr>
<td>APPENDIX II: The Static and Dynamic Performance of the Three Lobe Bearing with Turbulent Film</td>
<td>119</td>
</tr>
<tr>
<td>APPENDIX III: The Stiffness and the Damping of a Hydrostatic Bearing with an Incompressible Lubricant</td>
<td>127</td>
</tr>
<tr>
<td>APPENDIX IV: The Stability of the Hydrodynamic - Hydrostatic Ring Bearing with an Incompressible Lubricant</td>
<td>147</td>
</tr>
<tr>
<td>APPENDIX V: The Stiffness and the Damping of a Hybrid Journal Bearing with a Compressible Lubricant</td>
<td>155</td>
</tr>
<tr>
<td>APPENDIX VI: The Stability of the Hybrid-Hydrostatic Ring Bearing with a Compressible Lubricant</td>
<td>177</td>
</tr>
<tr>
<td>APPENDIX VII: Computer Program - The Static and Dynamic Performance of a Lobed Bearing with Turbulent Film</td>
<td>179</td>
</tr>
<tr>
<td>APPENDIX VIII: Computer Program - The Performance and Stability of a Hybrid Journal Bearing with Flexible, Damped Support</td>
<td>231</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>267</td>
</tr>
<tr>
<td>Figure</td>
<td>Illustration Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>The 3 Lobe Bearing, Schematic</td>
</tr>
<tr>
<td>2</td>
<td>The Hydrodynamic-Hydrostatic Ring Bearing, Schematic</td>
</tr>
<tr>
<td>3</td>
<td>The Hybrid-Hydrostatic Ring Bearing, Schematic</td>
</tr>
<tr>
<td>4</td>
<td>3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Stability Map</td>
</tr>
<tr>
<td>5</td>
<td>3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Stability Map</td>
</tr>
<tr>
<td>6</td>
<td>3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Friction</td>
</tr>
<tr>
<td>7</td>
<td>3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Friction</td>
</tr>
<tr>
<td>8</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Stability Map</td>
</tr>
<tr>
<td>9</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Stability Map</td>
</tr>
<tr>
<td>10</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Friction</td>
</tr>
<tr>
<td>11</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Friction</td>
</tr>
<tr>
<td>12</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Minimum Film Thickness</td>
</tr>
<tr>
<td>13</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Minimum Film Thickness</td>
</tr>
<tr>
<td>14</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = \frac{1}{2}$, Flow</td>
</tr>
<tr>
<td>15</td>
<td>3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Flow</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>16</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing, ( C_X ), ( R_e = 0, \frac{c_o}{W} = 0.7 ), Stability Map</td>
</tr>
<tr>
<td>17</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing, ( C_X ), ( R_e = 0, \frac{c_o}{W} = 0.4 ), Stability Map</td>
</tr>
<tr>
<td>18</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing, ( C_X ), ( R_e = 0, \frac{c_o}{W} = 0.1 ), Stability Map</td>
</tr>
<tr>
<td>19</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing, ( C_X ), ( R_e = 30,000, \frac{c_o}{W} = 0.7 ), Stability Map</td>
</tr>
<tr>
<td>20</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing, ( C_X ), ( R_e = 30,000, \frac{c_o}{W} = 0.4 ), Stability Map</td>
</tr>
<tr>
<td>21</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing, ( C_X ), ( R_e = 30,000, \frac{c_o}{W} = 0.1 ), Stability Map</td>
</tr>
<tr>
<td>22</td>
<td>Hydrostatic Bearing, Incompressible, Single Plane Admission, Stiffness</td>
</tr>
<tr>
<td>23</td>
<td>Hydrostatic Bearing, Incompressible, Double Plane Admission, Stiffness</td>
</tr>
<tr>
<td>24</td>
<td>Hydrostatic Bearing, Incompressible, Single Plane Admission, Damping</td>
</tr>
<tr>
<td>25</td>
<td>Hydrostatic Bearing, Incompressible, Double Plane Admission, Damping</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>26</td>
<td>Hydrostatic Bearing, Incompressible, Flow</td>
</tr>
<tr>
<td>27</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 2, \frac{(P_s - P_a)}{L D} = 0.24 ), Stability Map</td>
</tr>
<tr>
<td>28</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 2, \frac{(P_s - P_a)}{L D} = 0.24 ), Instability Frequency</td>
</tr>
<tr>
<td>29</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 2, \frac{(P_s - P_a)}{L D} = 0.18 ), Stability Map</td>
</tr>
<tr>
<td>30</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 2, \frac{(P_s - P_a)}{L D} = 0.18 ), Instability Frequency</td>
</tr>
<tr>
<td>31</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 2, \frac{(P_s - P_a)}{L D} = 0.09 ), Stability Map</td>
</tr>
<tr>
<td>32</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 2, \frac{(P_s - P_a)}{L D} = 0.09 ), Instability Frequency</td>
</tr>
<tr>
<td>33</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 5, \frac{(P_s - P_a)}{L D} = 0.24 ), Stability Map</td>
</tr>
<tr>
<td>34</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 5, \frac{(P_s - P_a)}{L D} = 0.24 ), Instability Frequency</td>
</tr>
<tr>
<td>35</td>
<td>Hybrid-Hydrostatic Ring Bearing, ( \frac{P_C K}{P} = 5, \frac{(P_s - P_a)}{L D} = 0.18 ), Stability Map</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>36</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 5$, $\frac{C^0}{P_a} = 0.18$, Instability Frequency</td>
</tr>
<tr>
<td>37</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 5$, $\frac{C^0}{P_a} = 0.09$, Stability Map</td>
</tr>
<tr>
<td>38</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.24$, Instability Frequency</td>
</tr>
<tr>
<td>39</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.24$, Instability Frequency</td>
</tr>
<tr>
<td>40</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.18$, Stability Map</td>
</tr>
<tr>
<td>41</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.18$, Stability Map</td>
</tr>
<tr>
<td>42</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.09$, Instability Frequency</td>
</tr>
<tr>
<td>43</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.09$, Instability Frequency</td>
</tr>
<tr>
<td>44</td>
<td>Hybrid-Hydrostatic Ring Bearing, $P_s = 10$, $\frac{C^0}{P_a} = 0.09$, Instability Frequency</td>
</tr>
<tr>
<td>45</td>
<td>Hydrostatic Bearing, Compressible, $\frac{P_s}{P_a} = 1.25$, Stiffness</td>
</tr>
</tbody>
</table>
### Illustrations (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 2 ), Stiffness</td>
<td>87</td>
</tr>
<tr>
<td>47</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 5 ), Stiffness</td>
<td>88</td>
</tr>
<tr>
<td>48</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 10 ), Stiffness</td>
<td>89</td>
</tr>
<tr>
<td>49</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 1.25 ), Damping</td>
<td>90</td>
</tr>
<tr>
<td>50</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 2 ), Damping</td>
<td>91</td>
</tr>
<tr>
<td>51</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 5 ), Damping</td>
<td>92</td>
</tr>
<tr>
<td>52</td>
<td>Hydrostatic Bearing, Compressible, ( \frac{P}{P_a} = 10 ), Damping</td>
<td>93</td>
</tr>
<tr>
<td>53</td>
<td>Hydrostatic Bearing, Compressible, Flow</td>
<td>94</td>
</tr>
<tr>
<td>54</td>
<td>Geometry of Partial Arc Bearing</td>
<td>97</td>
</tr>
<tr>
<td>55</td>
<td>Finite Difference Grid</td>
<td>103</td>
</tr>
<tr>
<td>56</td>
<td>Finite Difference Grid Lines</td>
<td>104</td>
</tr>
<tr>
<td>57</td>
<td>Three Lobe Bearing</td>
<td>119</td>
</tr>
<tr>
<td>58</td>
<td>Geometry of Single Lobe</td>
<td>119</td>
</tr>
<tr>
<td>59</td>
<td>Axial Strip</td>
<td>131</td>
</tr>
<tr>
<td>60</td>
<td>Axial Strip</td>
<td>134</td>
</tr>
<tr>
<td>61</td>
<td>Hydrodynamic-Hydrostatic Ring Bearing</td>
<td>147</td>
</tr>
<tr>
<td>Figure</td>
<td>Illustration</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------</td>
<td>------</td>
</tr>
<tr>
<td>62</td>
<td>Axial Strip</td>
<td>164</td>
</tr>
<tr>
<td>63</td>
<td>Feeding Hole</td>
<td>205</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Table 1</td>
<td>3 Lobe Bearing, Vertical Rotor</td>
<td>37</td>
</tr>
<tr>
<td>Table 2</td>
<td>3 Lobe Bearing, Preload:6 = 0.5</td>
<td>39</td>
</tr>
<tr>
<td>Table 3</td>
<td>Plain Cylindrical Bearing</td>
<td>41</td>
</tr>
</tbody>
</table>
SYMBOLS

\( a \)
Orifice radius (or radius of laminar restrictor tube), inch

\( B \)
Damping coefficient, lbs/sec/inch

\( B_0 \)
Damping coefficient for outer film, lbs/sec/inch

\( B_{xx}, B_{xy}, B_{yx}, B_{xx} \)
Damping coefficients for bearing, lbs/sec/inch

\( B_{rr}, B_{rt}, B_{tr}, B_{tt} \)
Damping coefficients for bearing, lbs/sec/inch

\( \bar{B}_B \)
Dimensionless effective bearing film damping

\( C \)
Radial clearance, inch

\( C_0 \)
Radial clearance for outer film, inch

\( C_D \)
Discharge coefficient or vena contracts coefficient

\( C_f \)
Coefficient of friction

\( D \)
Journal diameter, inches

\( D_0 \)
Diameter of outer bearing, inch

\( d \)
Feeder hole diameter, inch

\( e \)
Journal center eccentricity, inch

\( e_B \)
Eccentricity of journal center with respect to bearing center, inch

\( e_0 \)
Journal center eccentricity, steady-state, inch

\( F_f \)
Friction force, lbs.

\( F_r, F_t \)
Radial and tangential components of bearing force, lbs.

\( F_x, F_y \)
Vertical and horizontal components of bearing force, lbs.

\( F_{x0}, F_{y0} \)
Static components of \( F_x \) and \( F_y \), lbs.

\( f_r \)
\( f_t \)

\( G_x, G_z \)
Turbulent flow coefficients, see Ref. 1
\[ G = G_x / G_x \]

\[ G_0, G_1, G_2 \]

Defined by Eq. (A-19), Appendix I

\[ G_0, G_1, G_2 \]

Defined through Eqs. (E-17) to (E-19), Appendix V

\[ H = h^{3/2} G_x^{1/2} \]

\[ H_i, H_j, H_k \]

Defined by Eq. (A-18), Appendix I

\[ H_1, H_2 \]

Defined by Eq. (C-56), Appendix III

\[ \overline{h} \]

Film thickness, inch

\[ h = \overline{h}/C, \text{ dimensionless film thickness} \]

\[ i = \sqrt{-1} \]

\[ i, j \]

Finite difference indices, Appendix I

\[ j = \sqrt{-1}, \text{ Appendix V} \]

\[ K \]

Spring coefficient, lbs/inch

\[ K_o \]

Spring coefficient for outer film, lbs/inch

\[ K_{xx}, K_{xy}, K_{yx}, K_{yy} \]

Spring coefficients for bearing, lbs/inch

\[ K_{rr}, K_{rt}, K_{tr}, K_{tt} \]

Spring coefficients for bearing, lbs/inch

\[ \overline{K}_B \]

Dimensionless effective bearing film stiffness

\[ K_R \]

Stiffness of shaft, lbs/inch

\[ K_{eff} \]

Effective bearing film stiffness, lbs/inch

\[ L \]

Bearing length, inch

\[ L_o \]

Length of outer bearing, inch

\[ L_1 \]

Distance between admission planes, inch

\[ L_2 = L - L_1, \text{ inch} \]

\[ L_z \]

Length of feeder hole, inch

\[ M \]

Journal mass (half rotor mass for rigid rotor), lbs/sec^2/inch

\[ M_B \]

Mass flow into bearing from one feeder hole, lbs/sec/inch

\[ M_c \]

Mass flow through feeder hole orifice, lbs/sec/inch

xvi
Total bearing mass flow, lbs/sec/inch

\( M \)

Mass of rotor disc, lbs/sec\(^2\)/inch

\( M_R \)

Dimensionless mass flow (Eq. (C-13), App. III or Eq. (E-31), App. V)

\( m \)

Number of finite difference increments

\( N \)

Rotor speed, rps

\( \bar{N} \)

Rotor speed, rpm

\( n \)

Number of feeder holes

\( n \)

Number of finite difference increments on half bearing length

Film pressure, psi

\( \bar{F} \)

Dimensionless film pressure

\( P \)

Supply pressure, psi

\( P_s \)

Ambient pressure, psi

\( P_a \)

Film pressure at rim of feeder hole, psi

\( P'_c \)

Bearing flow, inch\(^3\)/sec

\( Q \)

Flow in circumferential direction, inch\(^3\)/sec.

\( Q_x \)

Side flow, inch\(^3\)/sec.

\( Q_z \)

Dimensionless flow (Eq. (C-47), App. III or Eq. (E-40), App. V)

\( q \)

Journal radius, inch

\( R \)

Radius of outer bearing, inch

\( R_o \)

Reynolds number for bearing

\( R_e = \frac{\rho R_o C}{\mu} \)

Local Reynolds number for film

\( R_h = \frac{\rho R_h C}{\mu} = h R_e \)

\( R \)

Gas constant, inch\(^2\)/sec\(^2\)*\( \Omega \)

\( r \)

Distance of lobe centers from bearing center, 3 lobe bearing, inch

\( xvii \)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_p$</td>
<td>Distance of lobe center from bearing center, inch</td>
</tr>
<tr>
<td>$s$</td>
<td>$\mu ND \left( \frac{E}{C} \right)$, Sommerfeld number</td>
</tr>
<tr>
<td>$T$</td>
<td>Total temperature of film, °F.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, seconds</td>
</tr>
<tr>
<td>$W$</td>
<td>Static load on bearing, lbs</td>
</tr>
<tr>
<td>$V$</td>
<td>$\frac{P}{P_a}$, supply pressure ratio</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Feeder hole volume</td>
</tr>
<tr>
<td>$x, y$</td>
<td>Journal center amplitudes, inch</td>
</tr>
<tr>
<td>$z$</td>
<td>Axial coordinate, inch</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\frac{v}{\omega}$, frequency ratio</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$a^2/dC$, inherent compensation factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$r/C$, preload</td>
</tr>
<tr>
<td>$\delta_p$</td>
<td>$r_{p}/C$, preload for lobe $p$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$e/C$, eccentricity ratio</td>
</tr>
<tr>
<td>$\epsilon_B$</td>
<td>$e_{B}/C$, eccentricity ratio for bearing</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Eccentricity ratio for lobe $p$</td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>Eccentricity ratio, steady-state</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>$z/R$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular coordinate, radians</td>
</tr>
<tr>
<td>$\theta_1, \theta_2$</td>
<td>Angular coordinates for leading and trailing edges of lobe, radians</td>
</tr>
<tr>
<td>$\theta_{in}$</td>
<td>Angle from load vector to leading edge of lobe, radians</td>
</tr>
<tr>
<td>$\theta_{out}$</td>
<td>Angle from load vector to trailing edge of lobe, radians</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$6\mu \omega \left( \frac{E}{C} \right)^2 /P_a$, compressibility number</td>
</tr>
</tbody>
</table>
\( \Lambda \)  
Restrictor coefficient for incompressible flow, defined by Eq. (C-12), App. III. For compressible flow, defined by Eq. (E-33), App. V.

\( \lambda \)
Spacing factor (pressure correction factor), Eqs. (E-49) and (E-50), App. V.

\( \mu \)
Lubricant viscosity, lbs.sec/inch\(^2\)

\( \nu \)
Frequency, radians/sec

\( \xi_1 \)
\( = \frac{L_1}{D} \)

\( \xi_2 \)
\( = \frac{L_2}{D} \)

\( \rho \)
Mass density of lubricant, lbs.sec\(^{-2}\)/inch\(^4\)

\( \sigma \)
Squeeze number (Eq. (C-8), App. III or Eq. (E-10), App. V)

\( \tau \)
Dimensionless time

\( \varphi \)
Attitude angle, radians

\( \varphi_B \)
Attitude angle for bearing, radians

\( \varphi_0 \)
Attitude angle, steady-state, radians

\( \psi_p \)
Angle between load vector and preload direction for lobe \( p \), radians

\( \psi \)
\( = \varphi \), Appendix I

\( \psi \)
Defined by Eq. (C-39), Appendix III

\( \psi_1, \psi_2, \psi_3, \psi_4 \)
Perturbations of \( \psi = \varphi \), Appendix I

\( \psi_0', \psi_1' \)
Defined by Eqs. (E-41) and (E-42), Appendix V

\( \psi_0, \psi_1 \)
Defined by Eqs. (E-60) and (E-61), Appendix V

\( \omega \)
Angular speed of rotor, radians/sec.
Subscripts

0  Steady-state or static condition
o  Outer film or outer bearing
p  Lobe number
c  Conditions in bearing film at rim of feeder hole
x  In x-direction
y  In y-direction
i,j  Finite difference coordinates
The current trend towards high-speed rotating machinery in many applications has focused attention on the bearings supporting the rotor. Most conventional bearings are limited to relatively low speeds for a variety of reasons such as operating life, high friction power loss and stability, and at the present time there is no universal bearing which will eliminate all the problem areas.

The present volume analyzes three special journal bearing types which are capable of insuring stable rotor operation to rather high speeds. As conceived, the bearings assume low viscosity lubricants such as a gas or a low viscosity fluid (first a liquid metal) in order to hold the friction power loss to an acceptable level. The three bearings are intended to provide the designer of high speed machinery with a greater variety of bearing types to choose from, a choice which at present is mostly limited to the tilting pad journal bearing.

The volume gives extensive design data in form of charts and tables from which the dimensions and performance data of the bearings can be obtained. Several numerical examples are included to illustrate the actual use of the data in making a design. Furthermore, two computer programs accompany the volume, and instructions for using the programs and listings of the programs are given.

The bearing considered first is the three lobe bearing which is shown schematically in fig. 1. This bearing has been used for many years because of its known ability to suppress instability, but up to this time, no information has been available on its actual stability limit. Also, data has been lacking on the friction power loss and flow of the bearing. The influence on the bearing performance of turbulence in the bearing film is included in the investigation to cover lubricants such as liquid metals which are frequently used in space power plants or in machinery for nuclear reactors.

The major advantages of the three lobe bearing are that it is relatively simple in construction, it has no moving parts like a tilting pad bearing or a floating
sleeve bearing, it can be designed to operate stable at rather high speeds and it is also able to run with a vertical rotor or a rotor in a zero-g field which is not possible with conventional cylindrical bearings. The limitations of the bearing are its inability to accommodate any appreciable shaft misalignment unless specially mounted, and its relatively high friction power loss.

The second bearing type is of the floating sleeve variety and is shown in Figure 2. It is given the name of the hydrodynamic-hydrostatic ring bearing. The ring inserted between the journal and the outer bearing is prevented from rotating but is otherwise unrestrained. The ring is floated by pressurizing the outer film through feeder holes in the outer wall. By proper selection of bearing dimensions and operating parameters, this bearing can be designed to have a very high stability limit (theoretically, the bearing can be inherently stable if the mass of the sleeve is ignored). Furthermore, because of its construction the bearing can accommodate more shaft misalignment than the three lobe bearing. It is found, on the other hand, that to achieve the improved stability it is necessary to have a relatively big clearance in the outer film which means that the flow requirements of the bearing are quite high compared to non-pressurized bearings. Also, the friction power loss of the bearing becomes prohibitive at speeds considerably lower than the speed at which the bearing would otherwise become unstable such that the bearings high speed potential cannot be utilized fully. Even so, the bearing is a more truly high-speed bearing than the three lobe bearing.

The third bearing type is also a floating sleeve bearing and quite similar to the one considered above except that both the inner film and the outer film are pressurized and the lubricant is a gas instead of a liquid. The bearing is called the hybrid-hydrostatic ring bearing and is shown in fig. 3. It has the same desirable stability characteristics as in the previous case and because of the gaseous lubricant, the friction power loss is not nearly as serious. Hence, the hybrid-hydrostatic bearing can operate at very high speeds, say 100,000 rpm or more, depending on the application. The bearing, however, requires somewhat tighter clearances than the liquid lubricated ring bearing which means that its ability to accommodate misalignment is more restricted.
In the following, each of the three bearing types are treated in separate sections. The sections are devoted to a detailed discussion of the design procedure for the bearings and explain the use of the design charts contained in figs. 4 to 53. In addition, complete numerical examples have been worked out for each bearing type. The theoretical analyses from which the data have been obtained, are given for reference in 6 appendices in back of the volume. Two computer programs have been written, one for the three lobe bearing which also applies to the inner film of the hydrodynamic-hydrostatic ring bearing, and one for the hybrid-hydrostatic ring bearing. Manuals for the programs are given in Appendices VII and VIII.
THE THREE LOBE JOURNAL BEARING

The geometry of the 3 lobe bearing is shown schematically in figure 1. The bearing is composed of 3 circular arcs whose centers of curvature are removed from the center of the bearing by the distance r. Thus, even when the journal is centered in the bearing, the pads are loaded. In this way, the stability threshold of the bearing is raised and even a vertical rotor, or a rotor operating in a "zero-g field," can run stably which is not possible with a conventional full circular bearing. However, the improved stability threshold is paid for by an increase in friction power loss and a smaller operating minimum film thickness which makes the bearing more sensitive to impurities in the lubricant.

The journal diameter is D, its radius is R and the bearing length is L. The lobes are separated by axial grooves with an arc width of 20 degrees. The machined radial clearance of the lobes, common to all three lobes, is C such that the radius of curvature of the lobes is R + C. The preload is defined as:

Preload: \( \delta = \frac{F}{C} \) (1)

Thus, when \( \delta = 0 \), the bearing becomes cylindrical and when \( \delta = 1 \), the journal touches all three lobes.

The preload is an important design parameter for the 3 lobe bearing. It strongly influences the stability threshold and the friction power loss of the bearing.

The other design parameters are the length-to-diameter ratio \( \frac{L}{D} \), the Sommerfeld number \( S \) and the Reynolds number \( R_e \):

Sommerfeld Number: \( S = \frac{u \pi D L}{W} \left( \frac{R}{C} \right)^2 \) (2)

Reynolds Number: \( R_e = \frac{2 \pi \rho R N C}{\mu} \) (3)
where:

\[ W = \text{bearing load, lbs} \]
\[ \Omega = \text{rotor speed, rps} \]
\[ \mu = \text{lubricant viscosity, lbs} \cdot \text{sec/inch}^2 \]
\[ \varrho = \text{lubricant mass density, lbs} \cdot \text{sec}^2 /\text{inch}^4 \]

For a given bearing geometry (i.e. for known values of \( \frac{L}{D} \) and \( \delta \)) and for a given operating condition (i.e. for known values of \( S \) and \( R_e \)), the bearing performance can be calculated as shown in the analyses in Appendices I and II. The actual calculations are carried out on a computer by means of the program described in Appendix VII where a listing and the instructions for using the program are given.

For known values of the four parameters, the bearing performance is defined by a set of dimensionless quantities:

the bearing eccentricity ratio: \[ \varepsilon_B = \frac{e_B}{C} \] (4)

the bearing attitude angle: \[ \phi_B \] (5)

flow parameter: \[ \frac{Q_f}{MDL} \] (6)

the friction factor: \[ \frac{F_f}{C} \] (7)

the stability mass parameter: \[ \frac{OBD}{W} \] (8)

the instability frequency ratio: \[ \gamma = \frac{v}{\omega} \] (9)

the dimensionless spring coefficients: \[ \frac{CK_{xx}, CK_{xy}, CK_{yx}, CK_{yy}}{W} \] (10)

the dimensionless damping coefficients: \[ \frac{ODB_{xx}, ODB_{xy}, ODB_{yx}, ODB_{yy}}{W} \] (11)

where:

\[ e_B = \text{the distance between the bearing center and the journal center, inch} \]
\[ Q_f = \text{the total hydrodynamic leakage flow, inch}^3/\text{sec} \]
\[ F_f = \text{the total friction force, lbs} \]
\[ M = \text{the mass of the journal, lbs} \cdot \text{sec}^2 /\text{inch} \]
\[ \omega = 2 \pi N, \text{the angular speed of rotation, radians/sec} \]
\[ v = \text{the whirl frequency at onset of instability, radians/sec} \]
\[ K_{xx}, K_{xy}, K_{yx}, K_{yy} = \text{spring coefficients, lbs/inch} \]
\[ B_{xx}, B_{xy}, B_{yx}, B_{yy} = \text{damping coefficients, lbs.sec/inch} \]

The \( x \)-\( y \)-coordinate system has its origin in the steady-state position of the journal center with the \( x \)-axis in the direction of the applied static load:

![Diagram of bearing center and journal center]

Under dynamic load, the journal center motion is described by the amplitudes \( x \) and \( y \). Then the dynamic forces acting on the journal are:

\[
\begin{align*}
F_x &= -K_{xx} x - B_{xx} \frac{dx}{dt} - K_{xy} y - B_{xy} \frac{dy}{dt} \\
F_y &= -K_{yx} x - B_{yx} \frac{dx}{dt} - K_{yy} y - B_{yy} \frac{dy}{dt}
\end{align*}
\]

(12)

The 8 dynamic coefficients are used to represent the bearing in a rotor response calculation as described in Volume 5 (see also reference 3).

The hydrodynamic leakage flow, \( Q_g \), is the sum of the end leakage flows from the lobes and also includes any net excess flow into the grooves (see Appendix VII). To get the flow in gallons per minute (gpm), multiply \( Q_g \) by 60 and divide by 231.

Usually, the lubricant is supplied to the bearing under some pressure. The flow induced in this way (the so called "zero speed flow") must be added to the hydrodynamic leakage flow to obtain the total bearing flow.
The friction power loss can be calculated from the friction force, $F_f$, as:

$$\text{Friction power loss} = \frac{H \times F_f}{6600} \text{ HP}$$  (13)

If it is assumed that all of the heat generated by the friction power loss goes into the lubricant film, the corresponding temperature rise of the lubricant can be calculated based on the obtained flow. With this temperature the operating viscosity of the lubricant can be determined.

The eccentricity ratio $e_B$ and the attitude angle $\phi_B$ defines the steady-state position of the journal center relative to the bearing center and the static load line (the x-axis). From this the minimum film thickness in the bearing can be determined (see Appendix II). Under almost all conditions, the minimum film thickness occurs at the bottom lobe (see fig. 1) in which case:

$$\text{Minimum Film Thickness, inch} = \min \left\{ \frac{C \left[ 1 - \sqrt{e_B^2 + \delta^2 + 2\delta e_B \cos \phi_B} \right]}{C \left[ 1 + \cos (2\phi - \tan^{-1} \left( \frac{e_0 \sin \phi_B}{e_0 \cos \phi_B} \right)) \right] e^2 + \delta^2 + 2\delta e_B \cos \phi_B \right\}$$

The minimum film thickness gives a relative measure of how heavily the bearing is loaded. The acceptable lower value for the minimum film thickness depends on the condition of the lubricant.

The threshold of instability is defined through the stability mass parameter in eq. (8). It applies to a rigid rotor with total mass $2M$ supported in two similar bearings, and the parameter defines the mass the rotor must have in order for the rotor-bearing system to be on the threshold of instability for the specified operating condition. If the actual rotor mass is bigger, the bearing is unstable. At the threshold of instability the steady-state equilibrium position of the journal is neutrally stable and the journal center whirs in an infinitesimal small closed orbit with a frequency $\nu$, given through the whirl frequency ratio $\gamma = \frac{\nu}{\omega}$. The stability mass parameter can also be used to determine the threshold of instability for a flexible rotor. Let the rotor consist of a shaft with stiffness
K on which is mounted a central disc with mass 2M. K and M are defined such that the natural frequency of the motor simply supported at the bearing centerlines is equal to \( \sqrt{K/M} \). Then the threshold of stability is defined by the parameter:

\[
\text{Stability Mass Parameter for Flexible Rotor: } \frac{CM_\omega^2}{W} = \frac{CK_\omega^2}{W} + \frac{CM_\omega^2}{W}
\]

where \( CM_\omega^2 \) and \( \gamma \) have the values obtained for the rigid motor. It is seen that the flexibility of the motor lowers the threshold of instability.

If the rotor is not symmetric, the computer program described in Volume 5 can be used to calculate the speed at onset of instability. In this calculation, the bearing is represented by the 8 dynamic coefficients as obtained above.

The performance of the 3 lobe bearing with a preload of \( \delta = 0.5 \), and for two values of the length-to-diameter ratio: \( L/D = 0.5 \) and 1, is given in Table. The columns of the table give the values of the parameters defined above. For those conditions where no value is given for the instability frequency ratio or the stability mass parameter, the bearing is stable.

The table contains two additional quantities, namely \( Q_x/NDLC \) which is the dimensionless circumferential flow (i.e. the sum of the flows entering the three lobes from the grooves), and \( CMW/\mu DL \left( \frac{K}{C} \right) \) which is another dimensionless form of the stability mass parameter (it is equal to \( \frac{CM_\omega^2}{W} / 4 \pi^2 S \)).

The most significant results are plotted in Figs. 8 to 15. Figs. 8 and 9 give the stability mass parameter. Figs. 10 and 11 show the friction factor, Figs. 12 and 13 give the minimum film thickness normalized with respect to C, and Figs 14 and 15 give the flow parameter. The abcissa is in all cases the Sommerfeld number \( S \), and each graph contains four curves corresponding to Reynolds numbers of 0, 2000, 10000 and 30,000. The curves for \( R_e = 0 \) apply to laminar flow (i.e. when \( R_e < 2,000 \)).
The use of the graphs is best illustrated by an example. Let a rigid, symmetric rotor of 200 lbs. be supported in two similar 3 lobe bearings with a length-to-diameter ratio of 1/2. The lubricant is liquid potassium at 500°F. The data are:

- Bearing diameter: \( D = 3 \text{ inch} \)
- Bearing length: \( L = 1.5 \text{ inch} \)
- Radial clearance: \( C = 0.002 \text{ inch} \)
- Preload: \( \delta = 0.5 \)
- Bearing load: \( W = 100 \text{ lbs.} \)
- Journal mass: \( M = 100/386 = 0.259 \text{ lbf.sec}^2/\text{inch} \)
- Lubricant viscosity: \( \mu = 3.85 \times 10^{-8} \text{ lbf.sec/inch}^2 \)
- Lubricant mass density: \( \varrho = 7.3 \times 10^{-5} \text{ lbf.sec}^2/\text{inch}^4 \)

Denote the rotor speed in rpm as \( \bar{N} \). Then,

\[
\bar{R}_e = \frac{2 \pi \cdot 7.3 \times 10^{-5} \cdot 1.5 \cdot 0.002}{3.85 \times 10^{-8}} \cdot \frac{\bar{N}}{60} = 0.693 \bar{N} \]

\[
\bar{s} = \frac{3.85 \times 10^{-8} \cdot 3 \cdot 1.5 \cdot (1.5 / 0.002)^2}{100} \cdot \frac{\bar{N}}{60} = 1.625 \times 10^{-5} \bar{N} \]

\[
\ddot{c} = \frac{W_0 - 0.259}{100} \left( \frac{2 \pi \bar{N}}{60} \right)^2 = 5.68 \times 10^{-8} \bar{N}^2 \]

\[
Q_x = 3.8 \times 10^{-5} \left( \frac{Q_x}{NDL} \right) \bar{N} \text{ gpm} \]

Friction power loss = \( 3.17 \times 10^{-6} \left( \frac{RF_f}{SW} \right) \bar{N} \text{ HP} \)

With these relationships the bearing performance can be obtained from figs. 10, 12 and 14:

<table>
<thead>
<tr>
<th>Rotor speed ( \bar{N}, \text{rpm} )</th>
<th>( \bar{R}_e )</th>
<th>( \bar{s} )</th>
<th>( \ddot{c} )</th>
<th>( Q_x )</th>
<th>Power loss</th>
<th>Flow</th>
<th>Min. Film Thickness, inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000</td>
<td>2,980</td>
<td>0.0812</td>
<td>5.8</td>
<td>0.2</td>
<td>1.5b</td>
<td>0.092</td>
<td>0.32</td>
</tr>
<tr>
<td>10,000</td>
<td>5,960</td>
<td>0.1625</td>
<td>14</td>
<td>0.335</td>
<td>0.84</td>
<td>0.44</td>
<td>0.33</td>
</tr>
<tr>
<td>15,000</td>
<td>8,940</td>
<td>0.2438</td>
<td>31</td>
<td>0.4</td>
<td>0.59</td>
<td>1.48</td>
<td>0.34</td>
</tr>
<tr>
<td>18,000</td>
<td>10,710</td>
<td>0.2925</td>
<td>47</td>
<td>0.428</td>
<td>0.51</td>
<td>2.68</td>
<td>0.36</td>
</tr>
</tbody>
</table>
To determine the threshold of instability, plot the stability limit for the rotor in fig. 8:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( N, \text{ rpm} )</th>
<th>( S )</th>
<th>( \frac{C}{W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>3,300</td>
<td>0.0546</td>
<td>( \frac{C}{W} ) (stable)</td>
</tr>
<tr>
<td>10,000</td>
<td>16,800</td>
<td>0.273</td>
<td>17.9</td>
</tr>
<tr>
<td>30,000</td>
<td>50,400</td>
<td>0.819</td>
<td>81.5</td>
</tr>
</tbody>
</table>

The operating line of the rotor is given by: \( \frac{C}{W} = 215.82 \) which is a straight line with a slope of 2 in fig. 8 because of the logarithmic scales. The operating line intersects the stability limit curve at \( \frac{C}{W} = 19 \) from which the speed at onset of instability is found to be 18,300 rpm. For comparison, the instability speed of a plain cylindrical bearing can be determined from Table 3 as:

- Plain Cylindrical Bearing: 10,700 rpm for \( C = 0.002 \)
- 14,260 rpm for \( C = 0.001 \)

This gives an indication of the stability improvement obtainable with a three lobe bearing.

If the rotor instead of being rigid is flexible and consists of a shaft on which is mounted a central disc, the threshold speed will be lowered. Let the disc weigh 200 lbs and let the shaft have a stiffness of \( 10^6 \) lbs/inch. The natural frequency of the rotor simply supported is:

\[
\text{Natural frequency} = \sqrt{\frac{1,000,000}{200/386} = \sqrt{\frac{500,000}{100/386}} = 1,390 \text{ radians/sec} = 13,300 \text{ rpm}}
\]

The shaft stiffness to be used in eq. (15) is: \( k = 500,000 \) lbs/inch and the half disc mass is: \( M = 100/386 = 0.259 \text{ lbs.sec}^2/\text{inch} \). Hence:

\[
\frac{Ck}{W} = 0.002 \cdot 500,000 = 10
\]

\[
\frac{C\omega^2}{W} = 215.82
\]
The flexible rotor has the same operating line as the rigid rotor since the mass is unchanged. The stability limit, however, is lowered. From eq. (15)

\[
\frac{CM \omega^2}{W} = \frac{10 \cdot \frac{Cm^3}{W}}{10 + (0.5)^2 \frac{Cm^3}{W}}
\]

where the instability frequency ratio has been set equal to 0.5 (for a more accurate value, use Table 2). Hence, by using fig. 8:

<table>
<thead>
<tr>
<th>R, m</th>
<th>N, rpm</th>
<th>S</th>
<th>( \frac{Cm^2}{W} ) (from fig. 8)</th>
<th>( \frac{CM \omega^2}{W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>3,360</td>
<td>0.0546</td>
<td>(stable)</td>
<td>(stable)</td>
</tr>
<tr>
<td>10,000</td>
<td>16,800</td>
<td>0.273</td>
<td>17.9</td>
<td>12.35</td>
</tr>
<tr>
<td>30,000</td>
<td>50,400</td>
<td>0.819</td>
<td>81.5</td>
<td>26.35</td>
</tr>
</tbody>
</table>

The operating line intersects the stability curve at \( \frac{CM \omega^2}{W} = 12 \) to which corresponds a threshold speed of 14,500 rpm.

When the rotor is vertical or operates in a "zero g field", the bearing is unloaded, i.e. \( W = 0 \). Hence, the eccentricity ratio \( e_B \) is zero, and the Sommerfeld number becomes infinite and can no longer be employed as a parameter. In this case the performance parameters of eqs. (7), (8), (10) and (11) are redefined.

**Vertical Rotor**

Friction factor:

\[
\frac{F_f}{C} = \frac{R C}{2 \mu D L (\frac{R}{C})^2} = \frac{1}{S} \frac{R C}{2 \mu D L (\frac{R}{C})^2}
\]

(16)

Stability mass parameter:

\[
\frac{CMN}{D L (\frac{R}{C})^2} = \frac{1}{4 \pi^2 S} \frac{Cm^2}{W}
\]

(17)

Dimensionless spring coefficients:

\[
\frac{CK_{xx}}{\mu D L (\frac{R}{C})^2} = \frac{1}{S} \frac{CK_{xx}}{W}
\]

(18)

Dimensionless damping coefficients:

\[
\frac{CDB_{xx}}{\mu D L (\frac{R}{C})^2} = \frac{1}{S} \frac{CDB_{xx}}{W}
\]

(19)

and similarly for \( K_{xy}, K_{yx}, K_{yy}, B_{xy}, B_{yx} \) and \( B_{yy} \). It should be noted that for a vertical rotor:

\[ K_{yy} = K_{xx} \]
Calculated performance data are given in Table 1 for a 3 lobe bearing with \( \frac{L}{D} = \frac{1}{2} \) and 1, and for 4 values of the Reynolds number. The preloads are considered. The stability mass parameter and the friction factor are plotted in Figs. 4 to 7 as a function of the preload. Hence, for a given application the preload required to ensure stable operation is readily determined. To illustrate, consider the same rigid rotor as in the previous example. From the given rotor and bearing data:

\[
R_o = 0.596 \cdot \frac{N}{2} \cdot \frac{CMN}{\mu DL(\xi)^2} = \frac{0.259}{2 \cdot 3.35 \cdot 10^{-3} \cdot 1.5 \cdot (0.5)^3} \cdot \frac{N}{60} = 0.886 \cdot 10^{-4} \cdot N
\]

Friction power loss = \( 5.15 \cdot 10^{-4} \left( \frac{R}{C} \cdot \frac{F}{\mu D L(\xi)^2} \right) \cdot N^2 \) HP

Thus, by using Figs. 4 and 6:

<table>
<thead>
<tr>
<th>( R_o )</th>
<th>( N, ) rpm</th>
<th>( CMN )</th>
<th>From Fig. 4</th>
<th>( \frac{R}{C} \cdot \frac{F}{\mu D L(\xi)^2} )</th>
<th>Power Loss, HP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>3,360</td>
<td>0.297</td>
<td>0.399</td>
<td>54</td>
<td>0.03</td>
</tr>
<tr>
<td>10,000</td>
<td>16,800</td>
<td>1.49</td>
<td>0.52</td>
<td>156</td>
<td>2.27</td>
</tr>
<tr>
<td>30,000</td>
<td>50,400</td>
<td>4.40</td>
<td>0.575</td>
<td>360</td>
<td>47.1</td>
</tr>
</tbody>
</table>

Hence, the rotor can operate stably at 50,000 rpm if the preload is approximately 0.6 but the friction power loss is then close to 50 HP.

It is seen that the threshold speed for the vertical rotor with a bearing preload of 0.5 is approximately 16,000 rpm which is a slight reduction of the speed determined for a horizontal rotor where the bearing is loaded.
THE HYDRODYNAMIC-HYDROSTATIC RING BEARING

The bearing is shown schematically in figure 2. It actually consists of two bearings (two lubrication films) separated by a floating ring. The ring is restrained from rotating by a pin but is otherwise free to move. In the inner film, pressures are developed by the hydrodynamic action from the rotating journal. The outer film, which is a hydrostatic bearing, is supplied with pressurized lubricant through restricted feeder holes. In this way the bearing can be designed such that the damping from the outer film stabilizes the inner bearing thereby raising the stability limit of the bearing significantly. Furthermore, by this construction the bearing's ability to accommodate misalignment is appreciably improved. On the other hand, the flow requirements of the bearing are large as discussed in the following.

The detailed analysis of the bearing is given in Appendices III and IV. The computer program described in Appendix V is used to calculate the performance characteristics of the inner film. Some typical data are given in Table 3.

The stability characteristics of the bearing are given by figures 16 to 21. The length-to-diameter ratio is equal to 1, based on the journal diameter.

The charts are based on a given outer film stiffness, \( K_0 \), and the curves give the stability mass parameter \( C K_0 W^2 / W \) as a function of the damping, \( B_0 \), of the outer film. The curves are plotted for several values of the Sommerfeld number \( S \) of the inner film, and two Reynolds numbers are considered: 0 and 30,000.

The outer film stiffness and damping are used in the dimensionless forms:

\[
\text{for outer film stiffness: } \frac{C K_0}{W} \tag{20}
\]
\[
\text{for outer film damping: } \frac{C u B_0}{W} \tag{21}
\]

where:

\( C \) = radial clearance of inner film, inch
The stability mass parameter has been defined and discussed in the previous section on the three lobe bearing (see eq. (8)). It applies to a rigid, symmetric rotor with mass 2M.

The charts, figs. 16 to 21, show that for a certain range of the outer film damping, the bearing is inherently stable. This "corridor" of inherent stability is seen to have a slope of 2 in the charts. Since the charts employ logarithmic scales, this means that for the bearing to operate in the corridor the ratio \( \frac{\cos_0^2}{W} \) is constant. This ratio is independent of speed.

In designing the bearing, the objective is to select the outer film stiffness and damping such that the bearing operates in the stable corridor. The outer film coefficients are given by the charts in figs. 22 to 25. In these charts the coefficients have the dimensionless form:

\[
\text{Dimensionless stiffness: } \frac{1+\frac{c}{2}^2}{1+\frac{2}{3} \frac{K}{B}} \frac{C}{K} \frac{L}{B} \frac{D}{D}
\]

\[
\text{Dimensionless damping: } \frac{C}{K} \frac{L}{B} \frac{D}{D}
\]

(note: subscript "o" is left out on the charts)

where:

- \( D_o \) = bearing diameter, inch
- \( L_o \) = bearing length, inch
- \( R_o \) = bearing radius, inch
- \( C_o \) = radial clearance of outer film, inch
- \( K_o \) = spring coefficient, lbs/inch
- \( B_o \) = damping coefficient, lbs/sec/inch
- \( P_s \) = lubricant supply pressure, psig
- \( \mu \) = lubricant viscosity, "lbs/sec/inch"
The feeder holes of the bearing may either be in the centerplane of the bearing, called "single plane admission," or arranged in two planes symmetric with respect to the centerplane, called "double plane admission." If the distance between the admission planes is $L_1$, define:

$$L_2 = L_0 - L_1$$

$$\xi_1 = \frac{L_1}{D_0}$$

$$\xi_2 = \frac{L_2}{D_0}$$

Hence, for single plane admission, $\xi_1 = 0$ and $\xi_2 = L_0 / D_0$, whereas for double plane admission the above definitions apply.

The flow restriction may be provided by orifices or by the feeder holes themselves.

If there is no orifice and the feeder hole acts as a narrow tube, the bearing is said to be laminar restricted. When the predominant flow restriction is in the orifice, the bearing is orifice restricted. Finally, the flow may be restricted in the "curtain" area formed between the rim of the feeder hole and the surface of the ring. This is called "inherent compensation." The inherent compensation factor:

$$\delta = \frac{a^2}{dC_0}$$

(24)

gives the ratio between the orifice area and the curtain area. When $\delta = 0$, the feeder hole is purely orifice restricted, and when $\delta = 0$, the flow restriction takes place in the curtain area only. For intermediate values of $\delta$, both restrictors are acting. For the laminar restrictor, set $\delta = 0$. 

-17-
The spacing factor $\lambda$ accounts for the effect of the spacing of the feeder holes. Let there be $n$ feeder holes in total (i.e. for double plane admission there are $\frac{n}{2}$ holes per plane). Then $\lambda$ is given by:

$$\lambda = 1 + \frac{2}{n_p} \log_e \left( \frac{D}{n_p d} \right)$$

(25)

where $n_p$ is the number of holes per admission plane ($n_p = n$ or $n_p = \frac{1}{2} n$). For more accurate expressions, see eqs. (C-28), (C-29), (C-33) and (C-34), Appendix III. A typical value is: $\lambda = 1.5$.

In the charts, figs. 16 to 21, the abcissa is the restrictor coefficient $A_s$ which gives the ratio between the flow resistance of the bearing film and the feeder holes. It is defined as:

$$A_s = \lambda \xi_2$$

$$= \frac{3 \nu C_D n^2}{C_0^3 \sqrt{P_s} \sqrt{1 + \frac{a}{l}^4}}$$

where:

- $C_D$ = discharge coefficient ($C_D \approx 0.6$)
- $a$ = radius of orifice or feeder hole, inch
- $P_s$ = mass density of lubricant, lbs sec$^{-2}$/inch$^4$
- $l$ = length of feeder hole, inch

and the other symbols have been defined above. It is seen that the bearing is optimized with respect to stiffness when $A_s = 1$.

The charts assume that the bearing is operating with no eccentricity. This assumption is valid in most cases. Furthermore, the load displacement characteristic is close to being linear such that:

$$W = C_0 \epsilon_o K_o$$

(27)

where:

- $W$ = bearing load, lbs.
- $\epsilon_o$ = bearing eccentricity ratio
This relationship is valid for eccentricities up to: $e_o = 0.4$ to 0.5.

The bearing flow can be determined from fig. 26 where the ordinate is the dimensionless flow:

$$\frac{3uO}{P_{\text{C}}^{\frac{3}{2}}} \cdot \lambda \xi_2$$

Here:

$Q = \text{flow, inch}^3/\text{sec.}$

The chart is valid for both single plane admission and double plane admission, and for any length-to-diameter ratio.

The use of the design charts is best illustrated by an example. First, it is seen from eq. (27) that:

$$\frac{C_{o_o}}{W} = \frac{1}{e_o}$$

or:

$$\frac{C}{C_o} = e_o \frac{C_k}{W}$$  \hspace{1cm} (28)

$e_o$ should not exceed 0.4 in order for the bearing to have some load margin. Hence, the dimensionless stiffness $C_k$ should be chosen as large as possible in order for $C_o$ not to become too large which would lead to unreasonable flow requirements. On the other hand, the stability corridor narrows down with increasing outer film stiffness and eventually disappears. Thus, a compromise is necessary. In the present case, choose the largest dimensionless stiffness value given in the charts, namely $C_k/W = 0.7$, which means that figs. 16 and 19 apply. To operate in the stability corridor, set $C_{o_o}/W = 0.6$ for $C_{o_o}/W = 1$ whereby the rotor will operate along a straight line with a slope of 2 in the chart. Hence:

$$B_o \sqrt{\frac{C}{W}} = \frac{C_{o_o}/W}{(C_{o_o}/W)^2} \frac{1}{\sqrt{4}} = 0.6 = 0.6$$  \hspace{1cm} (29)

Let the rotor be rigid and symmetric with a weight of 200 lbs. The data for the rotor and its two bearings are:
bearing diameter: $D = 3$ inch
bearing length: $L = 3$ inch
outer diameter of ring: $D_o = 3.5$ inch
lubricant viscosity: $\mu = 3.85 \cdot 10^{-8}$ lbs·sec/inch
lubricant mass density: $\rho = 7.3 \cdot 10^{-5}$ lbs·sec$^2$/inch
bearing load: $W = 100$ lbs.
journal mass: $M = 100/386 = 0.259$ lbs·sec$^2$/inch

The lubricant is liquid potassium at 500°F.

The outer bearing is chosen as a hydrostatic bearing with single plane admission and laminar restricted feeder holes. Hence, fig. 24 applies where the dimensionless damping is:

$$\frac{C_o B_o R^2}{\mu L D_o (\frac{R}{C_o})^3} = \frac{B_o}{2 \mu L D_o (\frac{R}{C_o})^3} = \frac{1}{2 \mu L D_o (\frac{R}{C_o})^3} \sqrt{\frac{W}{C}} \cdot \frac{0.6}{\sqrt{I}}$$

or:

$$\left(\frac{R}{C_o}\right)^3 = \frac{1}{2 \mu L D_o (\frac{R}{C_o})^3} \sqrt{\frac{W}{C}} \cdot \frac{0.6}{\sqrt{I}} \left/ \frac{C_o P_o}{\mu L D_o (\frac{R}{C_o})^2}\right.$$

Now, from eq.(28) it is seen that $C/C_o < 0.4 \cdot 0.7 = 0.28$, i.e. even if it is desired to have a small value of $C_o$ to keep the flow down, $C_o$ cannot be too small if $C$ shall have a practical value. Set $C = 0.0007$ inch and try with a dimensionless damping value of 4 (corresponds to $\Lambda_s = 0.25$). Hence, from eq. (31):

$$\left(\frac{R}{C_o}\right)^3 = \frac{0.6}{2 \cdot 3.85 \cdot 10^{-8} \cdot 3} \cdot \sqrt{\frac{0.259 \cdot 100}{0.0007}} / 4.0 = 1.25 \cdot 10^8$$

or:

$$\frac{R}{C_o} = 5 \cdot 10^2$$

-20-
\[ C_0 = \frac{1.75}{500} = 0.0035 \text{ inch} \]

Since \( C/C_0 = 0.0007/0.0035 = 0.2 < 0.28 \), this value of \( C_0 \) can be accepted. The chosen dimensionless damping corresponds to a restrictor coefficient of:

\[ \Lambda_b = 0.25. \]

Thus, from fig. 22:

\[ \frac{C_0 K}{F L D_o} \xi_2 = 0.28 \]

Now:

\[ \frac{C_0 K}{F L D_o} \xi_2 = \frac{C_0 W}{F L D_o} \frac{\xi_2 C K}{W} = 0.28 \]

or:

\[ P_s = \frac{C_0 W}{G} \frac{L o}{D_o} \xi_2 \frac{C K}{W} \frac{1}{0.28} \quad (32) \]

Here: \( \xi_2 = L_o/D_o \) and \( \lambda \) shall be set equal to 1.5. Thus:

\[ P_s = \frac{0.0035}{0.0007} \frac{100}{3.3} \cdot 1.5 \cdot \frac{3}{3.5} \cdot 0.7 \cdot \frac{1}{0.28} = 150 \text{ psig.} \]

which establishes the required supply pressure.

From the definition of the restrictor coefficient:

\[ \Lambda_b = 0.25 = \frac{3}{8} \frac{n a^4}{C_0 I} \xi_2 \]

or:

\[ \frac{n a^4}{I} = \frac{8}{3} \frac{(0.0035)^3}{1.5 \cdot 3/3.5} = 8.89 \cdot 10^{-8} \]

Let the feeder hole have a diameter: \( d = 2a = 0.020 \text{ inch} \) and let the number of holes be: \( n = 10 \) whereby:

\[ I = 1.125 \text{ inch} \]

Check the spacing factor:

\[ \lambda = 1 + \frac{2}{n \xi_2} \log_e \left( \frac{D}{nd} \right) = 1 + \frac{2}{10 \cdot 3/3.5} \cdot \log_e \left( \frac{3.5}{10 \cdot 0.02} \right) = 1.67 \]
which is close enough to the estimated value of 1.5. It should be noted that if the obtained feeder hole dimensions are not acceptable for various reasons, the bearing can be made orifice restricted instead or a different $A_2$-value can be chosen which would have little influence on the selection of $C$ and $C_0$. Going to a value of $A_2 = 0.5$ would reduce the required feeder hole length to $l = 0.56$ inch, the supply pressure would become $P_s = 105$ psig whereas the outer film clearance only would change from 0.0035 inch to 0.0034 inch. In other words, it is very important to select the proper value of the outer film clearance but the bearing is not too sensitive to even quite larger changes in the other parameters. This is quite readily deduced from eq. (31) where, for a given rotor weight, lubricant viscosity and overall bearing dimensions, the clearance $C_0$ is pretty well defined once it is required that the bearing must operate in the stable corridor. Thus, eq. (31) can be considered to be the governing design equation with eq. (28) as a necessary condition.

From fig. 26 the dimensionless flow is found to be 0.2. Hence, the flow becomes:

$$Q = \frac{\pi P C^3}{3 \mu \lambda e_2} \cdot 0.2 = \frac{\pi \cdot 150 \cdot (0.0035)^3}{3 \cdot 3.85 \cdot 10^{-8} \cdot 1.5 \cdot 3/3.5} \cdot 0.2 = 27.2 \text{ inch}^3 / \text{sec} = 7.1 \text{ gpm}$$

This is seen to be 20 times the flow required for the three lobe bearing for the same application. However, whereas the three lobe bearing becomes unstable at 18,000 rpm, the hydrodynamic-hydrostatic bearing is stable to much higher speeds.

The actual stiffness of the hydrostatic bearing is calculated to be:

$$K_0 = \frac{150 \cdot 3 \cdot 3.5}{0.0035} \cdot 0.28 = 126,000 \text{ lbs/inch}$$

The damping coefficient becomes:

$$B_0 = \frac{3.85 \cdot 10^{-8} \cdot 3 \cdot 3.5}{0.0035} \cdot \left(\frac{1.75}{0.0035}\right)^2 \cdot 4.0 = 116 \text{ lb.sec/inch}$$

Assuming a linear load-displacement relationship, the eccentricity ratio becomes:
which is considerably less than 0.4. Hence, the bearing should be able to withstand dynamic loads of at least the same magnitude as the static load.

From Table 3, the stability mass parameter of the inner film is found to be approximately equal to 6 with a whirl frequency ratio of 0.5. Now, the onset of instability can be likened to a resonance where:

\[ \nu^2 = \frac{K_{eff}}{M} \]

where \( K_{eff} \) represents the effective stiffness of the inner film. Hence:

\[ K_{eff} = \nu^2 M = \left( \frac{\nu}{\omega} \right)^2 \frac{C_M d^2}{W} \cdot \frac{W}{C} = (0.5)^2 \cdot 6 \cdot \frac{100}{0.0007} = 214,000 \text{ lbs/inch} \]

The ring has an inner diameter of 3 inches and an outer diameter of 3.5 inches. Hence, its mass is:

\[ \frac{\pi}{4} (3.5^2 - 3^2) \cdot 3 \cdot 0.283 = 2.17 \text{ lbs} = 0.0056 \text{ lbs\cdotsec}^2/\text{inch} \]

The natural frequency of the ring is:

\[ \sqrt{\frac{214,000 + 126,000}{0.0056}} = 7,800 \text{ radians/sec} = 75,000 \text{ rpm} \]

With an estimated instability frequency ratio of 0.5, the ring may become unstable at 150,000 rpm. This, then should be considered the top speed of the rotor to be on the safe side.

Making use of Table 3, the friction power loss of the bearing can be computed:

<table>
<thead>
<tr>
<th>Rotor Speed (rpm)</th>
<th>Friction Power Loss (HP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000</td>
<td>9.7</td>
</tr>
<tr>
<td>40,000</td>
<td>64.4</td>
</tr>
<tr>
<td>60,000</td>
<td>195</td>
</tr>
<tr>
<td>80,000</td>
<td>432</td>
</tr>
</tbody>
</table>
Thus, even if the bearing is stable up to at least 150,000 rpm, the power loss becomes prohibitive at half that speed because of turbulence in the film.

In summary, the calculated bearing dimensions and the bearing performance data are:

- **Journal diameter:** \( D = 3 \text{ inch} \)
- **Outer diameter of ring:** \( D_o = 3.5 \text{ inch} \)
- **Bearing length:** \( L = 3 \text{ inch} \)
- **Radial clearance of inner film:** \( C = 0.0007 \text{ inch} \)
- **Radial clearance of outer film:** \( C_o = 0.0035 \text{ inch} \)
- **Number of feeder holes for hydrostatic bearing:** \( n = 10 \)
- **Feeder hole diameter:** \( d = 0.020 \text{ inch} \)
- **Feeder hole length:** \( l = 1.25 \text{ inch} \)
- **Supply pressure for hydrostatic bearing:** \( P_s = 150 \text{ psig} \)
- **Bearing flow:** \( Q = 7.1 \text{ gpm} \)
- **Maximum stable speed (conservative estimate):** 150,000 rpm
- **Friction power loss:**
  - at 70,000 rpm: 9.7 HP
  - at 40,000 rpm: 64.4 HP
  - at 60,000 rpm: 195 HP
  - at 80,000 rpm: 432 HP
THE HYBRID-HYDROSTATIC RING BEARING

The bearing is shown schematically in figure 3. It differs from the previously considered hydrodynamic-hydrostatic ring bearing by employing a gas as a lubricant instead of a liquid. Furthermore, both the inner and the outer film are pressurized thereby enhancing the load carrying capacity which would otherwise be rather limited with gas as a lubricant. The ring separating the inner and the outer film, is restrained from rotating but is otherwise free to follow the motions of the journal. As shown in fig. 3, the inner film is supplied with pressurized gas through feeder holes in the centerplane of the ring. The load carrying capacity of the inner film is then produced by both hydrostatic and hydrodynamic action which is known as a hybrid bearing. The outer film is purely hydrostatic and, as shown in fig. 3, consists of two bearings supplied with pressurized gas through feeder holes in a central plane. The ends of the bearings are vented to atmosphere. Figure 3 is only intended to show one possible arrangement and other designs would be possible.

The major objective of this type of bearing design is to improve the stability limit of the bearing. Furthermore, the bearing offers the advantage of being able to accommodate a larger amount of misalignment than more conventional bearing types.

The analysis of the bearing is given in Appendices V and VI. Furthermore, a computer program for calculating the bearing has been written, and the instructions for using the program and a listing of the program are given in Appendix VIII.

The stability characteristics of the bearing are given by the charts in figs. 27 to 44. Three values of the supply pressure for the inner film have been considered:

\[
\frac{P_s}{P_a} = 2, 5 \text{ and } 10.
\]
where:

\( P_s \) = supply pressure for inner film, psia
\( P_a \) = ambient pressure, psia

Furthermore, three values of the stiffness of the outer film are considered. In dimensionless form:

\[
\frac{\frac{C K_o}{(P_s - P_a)LD}}{L} = 0.24, 0.18 \text{ and } 0.09
\] (33)

where:

\( D \) = journal diameter, inch
\( L \) = bearing length, inch
\( C \) = radial clearance of inner film, inch
\( K_o \) = stiffness of outer film, lbs/inch

Curves are given for 7 values of the compressibility number \( \Lambda \):

\[
\Lambda \equiv \frac{6 \rho \omega}{P_a} \left( \frac{K}{C} \right)^2 = 0.3, 1, 2, 5, 10, 30 \text{ and } 100
\] (34)

where:

\( \mu \) = gas viscosity, lbs·sec/inch\(^2\)
\( \omega \) = angular speed of journal, radians/sec

The charts, figs. 27, 29, 31 --- 43, give the value of the stability mass parameter:

\[
\text{Stability Mass Parameter: } \frac{C m o^2}{(P_s - P_a)LD}
\] (35)

as a function of the dimensionless damping of the outer film:

\[
\frac{C m B_o}{P_a L D}
\] (36)

where:

\( M \) = journal mass (half the rotor mass), lbs·sec\(^2\)/inch
\( B_o \) = damping coefficient of outer film, lbs·sec/inch

As in the case of the hydrodynamic-hydrostatic ring bearing, it is seen that for a certain range of the outer film damping the bearing is inherently stable. The
objective when designing the bearing is to select the bearing dimensions such that the bearing operates in the "stability corridor" established by the charts.

The charts, figs. 28, 30, 37, --44, give the whirl frequency ratio:

Whirl frequency ratio: \( \frac{V}{\omega} \) \hspace{1cm} (37)

as a function of the stability mass parameter. Here:

\( V \) = whirl frequency at onset of instability, radians/sec

These charts are used in calculating the stiffness and damping of the outer film as discussed later.

All the charts, figs. 27 to 44, are based on the following data:

- length-to-diameter ratio: \( \frac{L}{D} = 1 \)
- restrictor coefficient: \( \Lambda_8 = 0.7 \)
- Inherent compensation factor: \( \Lambda = 1000 \)
- spacing factor: \( \Lambda = 1.5 \)
- eccentricity ratio: \( \epsilon = 0.02 \)

For \( \Lambda = 0.7 \), the bearing is optimized with respect to the hydrostatic stiffness. The bearing is inherently compensated (i.e. there are no orifices) to eliminate the possibility of pneumatic hammer instability. The journal is assumed to operate essentially in its concentric position but since the load is reasonably linear with displacement, the charts should be valid up to an eccentricity ratio of approximately 0.4 which covers the acceptable design range.

Figures 45 to 52 give the dimensionless stiffness and dimensionless damping of the hydrostatic outer film in the form:

Dimensionless Stiffness: \( \frac{C}{C_0} \left( \frac{P_s - P_0}{P} \right) \frac{L}{D_0} \) \hspace{1cm} (38)

Dimensionless Damping: \( \frac{B_0}{\mu L} \frac{P_0}{P_0} \) \hspace{1cm} (39)
(Note: in the charts, subscript "o" has been left out).

The abscissa in the charts is the Squeeze number \( \sigma \):

\[
\text{Squeeze Number: } \sigma = \frac{12\mu \nu}{P_o} \left( \frac{R_o}{C_o} \right)^2
\]

The symbols are:

- \( D_o \) = outer diameter of ring, inch
- \( R_o \) = outer radius of ring, inch
- \( L_o \) = length of outer bearing, inch
- \( C_o \) = radial clearance of outer film, inch
- \( (P_o)_o \) = supply pressure for outer film, psia
- \( (P_o)_a \) = ambient pressure for outer film, psia
- \( \mu \) = gas viscosity, lbs·sec/inch²
- \( \nu \) = vibratory frequency, radians/sec
- \( K_o \) = spring coefficient of outer film, lbs/inch
- \( B_o \) = damping coefficient of outer film, lbs·sec/inch

The charts are valid for a single admission plane of feeder holes in the center-plane of the bearing. The feeder holes are inherently compensated (\( \delta = 1000 \)) to avoid pneumatic hammer and the spacing factor is \( \lambda = 1.5 \). The length-to-diameter ratio is 0.4 and four supply pressure ratios have been considered: \( \frac{P_o}{P_a} = 1.25, 2, 5 \) and 10. A wide range of the restrictor coefficient is covered:

\( \Lambda_1 \) = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5 and 1. The restrictor coefficient is defined as:

\[
\Lambda_1 = \frac{6\pi n d \sqrt{RT}}{P_o C_o^2} \quad \text{(inherent compensation)}
\]

where:

- \( n \) = number of feeder holes
- \( d \) = feeder hole diameter, inch
- \( R \) = gas constant, inch²/sec² °R
- \( T \) = total temperature, °R

The gas flow of the outer film is given in dimensionless form in the chart, fig. 53:

\[
\frac{6\pi \Lambda_1 G}{2P_o C_o^3 D_o} \quad \text{(42)}
\]
where:
\[ \bar{M} = \text{mass flow, lbs-sec/inch} \]

The chart is actually valid for all length-to-diameter ratios, for both single and double plane admission, and for both orifice restriction and inherent compensation. Hence, \( \frac{L_0}{D_0} \) is replaced by \( \frac{L_2}{D} \) and the restrictor coefficient \( D_0 \) is given in its general form (see Appendix V).

The use of the charts is best illustrated by an example. The or is assumed to be rigid and symmetric and it weighs 90 lbs. The given data are:

- Bearing load: \( W = 45 \) lbs.
- Journal mass: \( M = \frac{45}{386} = 0.117 \) lbs-sec \( ^2 / \) inch
- Journal diameter: \( D = 3 \) inch
- Bearing length: \( L = 3 \) inch
- Radial clearance, inner film: \( C = 0.0015 \) inch
- Outer diameter of ring: \( D_o = 3.75 \) inch
- Length of one outer bearing: \( L_0 = 1.25 \) inch
- Ambient pressure: \( \bar{P}_a = 14.7 \) psia
- Gas viscosity (air at 120°F): \( \mu = 2.8 \times 10^{-9} \) lbs-sec/inch \( ^2 \)
- Gas constant (air): \( R = 2.472 \times 10^5 \) inch \( ^2 / \) sec \( ^2 \)^0R
- Total temperature: \( 120 + 460 = 580^\circ R \)

Hence:
\[ \sqrt{RT} = 1.434 \times 10^8 \text{ inch}^2/\text{sec}^2 \]

or:
\[ \frac{RT}{\bar{M}} = 1.196 \times 10^4 \text{ inch/sec} \]

The load on the bearing is 45 lbs and the bearing area is: \( L \times D = 3 \times 3 = 9 \) inch \( ^2 \).

The load per square inch is then 5 psi which requires that the available pressure across the bearing be at least 20 psi and preferably more, i.e.:

\[ \frac{P_s}{P_a} > 20 \]

or \[ \frac{P_s}{P_a} > 2.4 \]
The stability charts have the dimensionless support stiffness in the form:

\( \frac{C}{K} (P_s - P_a) LD \). The eccentricity ratio of the outer film is determined from the relationship:

\[ C_o \varepsilon_o K_o = \frac{W}{K_o} \]

from which:

\[ \varepsilon_o = \frac{W}{C_o \varepsilon_o} = \frac{W}{(P_s - P_a) LD} \frac{C_o}{C_0} \frac{P_s - P_a}{LD} \]

\( \varepsilon_o \) should be small and should not exceed 0.4. Thus, in order to keep the clearance ratio around 1, the dimensionless outer film stiffness should be chosen as large as possible and the supply pressure should also be kept high. Choose the largest value of the dimensionless support stiffness in the charts, namely:

\[ \frac{CK_o}{(P_s - P_a) LD} = 0.24 \]

Estimate that \( C/C_0 \approx 1 \) and set \( \varepsilon_o = 0.2 \) to get:

\[ (P_s - P_a) = \frac{1}{\varepsilon_o} \frac{W}{C_0 \varepsilon_o \frac{1}{0.24}} = 100 \text{ psi} \]

or:

\[ P_s = 120 \text{ psia} \]

which means:

\[ \frac{P_s}{P_a} = \frac{120}{14.7} \approx 8 \]

The stability charts to use for this case are then fig. 33 and fig. 39. To operate in the stability corridor it is found from the charts that the dimensionless outer film damping should be:

\[ \frac{C_{dB}}{P_s LD} = 2.5 \text{ at } \frac{C_{dB}^2}{(P_s - P_a) LD} = 1 \]

For constant support damping, the rotor will operate along a straight line with a slope of 2 in the charts as the speed increases. The corresponding damping can be found from the relationship:
From figs. 50 to 53 it is seen that the dimensionless outer film damping will be around 1. Set:

\[ \frac{B_0}{\left(\frac{R_0}{C_0}\right)^3} = 1 \]

Since there are two outer film bearings, the damping per bearing is half the required damping given in eq. (46). The damping per bearing can be expressed by means of eq. (46) as:

\[ \frac{B_0}{\mu L_0 \left(\frac{R_0}{C_0}\right)^3} = 1 = \frac{1}{2} \frac{P_a}{\mu L_0 \left(\frac{R_0}{C_0}\right)^3} \cdot \sqrt{\frac{MLD}{C(P_s - P_a)}} \cdot \frac{2}{1} \]

or:

\[ \left(\frac{R_0}{C_0}\right)^3 = \frac{1}{2} \frac{P_a}{\mu L_0} \cdot \frac{MLD}{C(P_s - P_a)} \cdot \frac{2}{1} \cdot \frac{1}{1} = \frac{1}{2} \cdot 14.7 \cdot \frac{0.117 \cdot 3.3}{2.8 \cdot 10^{-9} \cdot 1.25} \cdot \frac{0.117 \cdot 3.3}{2 \cdot 1} \]

Set \( C = 0.0015 \) inch whereby:

\[ \left(\frac{R_0}{C_0}\right)^3 = 10.84 \cdot 10^9 \]

or:

\[ \frac{R_0}{C_0} = 2.21 \cdot 10^3 \]

i.e.

\[ C_0 = \frac{1.875 \cdot 10^{-3}}{2.21} = 0.00085 \text{ inch} \]

Eq. (47) (together with eqs. (44 or (45) ) must be considered the key design equation. For a given rotor-bearing system, the clearance of the outer film can only vary within a very limited range if the rotor is to operate within the stability corridor. Hence, \( C_0 \) is the critical design dimension whereas the other design parameters can vary appreciably without having serious effects.
To check the estimated outer film damping, the following relationship is readily deduced:

\[
\frac{C_{\text{ef}}^2}{(P_s - P_\alpha) LD} = \frac{\Delta^2}{72} \frac{\mu F^2}{L(P_s - P_\alpha)(R/G)^5}
\]

(48)

Here:

\[
\frac{\mu F^2}{L(P_s - P_\alpha)(R/G)^5} = \frac{0.117(14.7)^2}{(2.8 \times 10^{-5})^2 \cdot 3 \cdot (120 - 14.7)} \frac{1.5}{0.0015} = 10.21
\]

whereby:

\[
\frac{C_{\text{ef}}^2}{(P_s - P_\alpha) LD} = 0.142 \cdot \Delta^2
\]

From the definition of \( \Lambda \) and \( \sigma \) (eqs. (34) and (40)) it is seen that:

\[
\sigma = 2 \cdot \frac{\nu}{\sigma} \cdot \Lambda
\]

(49)

Making use of these relationships, figs. 34 and 40 can be employed to set up the following table:

<table>
<thead>
<tr>
<th>( \Delta )</th>
<th>( \frac{C_{\text{ef}}^2}{(P_s - P_\alpha) LD} )</th>
<th>( \frac{\nu}{\sigma} )</th>
<th>( \sigma )</th>
<th>( \frac{\nu}{\sigma} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.567</td>
<td>0.3</td>
<td>2.0</td>
<td>0.3</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>0.202</td>
<td>2.02</td>
<td>0.31</td>
<td>3.1</td>
</tr>
<tr>
<td>10</td>
<td>14.2</td>
<td>0.105</td>
<td>2.10</td>
<td>0.185</td>
<td>3.7</td>
</tr>
<tr>
<td>30</td>
<td>128</td>
<td>0.036</td>
<td>2.16</td>
<td>0.075</td>
<td>4.5</td>
</tr>
<tr>
<td>100</td>
<td>1420</td>
<td>0.0112</td>
<td>2.24</td>
<td>0.0245</td>
<td>4.9</td>
</tr>
</tbody>
</table>

In this table, \( \frac{\nu}{\sigma} \) is the instability frequency ratio obtained by interpolation between figs. 34 and 40. As seen from figs. 33 and 39, the stability corridor passes between two regions of instability, and the instability frequency is different for the two regions as shown by figs. 34 and 40. The left hand region is identified in figs. 34 and 40 as the region where \( B_0 \rightarrow 0 \), and the right hand region is identified as the region where \( B_0 \rightarrow \infty \).

At the onset of instability, the journal whirls in closed orbit with frequency \( \nu \). Hence, \( \nu \) becomes the vibratory frequency as seen by the outer film and the
stiffness and damping of the outer film must, therefore, be evaluated at the corresponding values of the Squeeze number as given in the table. Since the outer film decreases with increasing squeeze number it is only necessary to consider the Squeeze number corresponding to the left hand instability region ($B_o = 0$).

The charts, figs. 33 and 39, are based on a dimensionless support stiffness of 0.24. Hence:

$$K_o = \frac{(120-14.7)\cdot 3.3}{0.0015} = 151,600 \text{ lbs/inch}$$

Since there are two bearings, the required stiffness per bearing is:

$$K_o \text{ per bearing} = 75,800 \text{ lbs./inch}$$

Assuming a restrictor coefficient value such that:

$$\Lambda = 0.05$$

the dimensionless stiffness per bearing is found by interpolation between figs. 47 and 48 as ($\sigma = 2.2$):

$$\frac{C_o E_o}{(P_s - P_o) L_o D_o} = 0.15$$

or:

$$\frac{(P_s - P_o)}{L_o D_o} = \frac{0.00085 \cdot 75,800}{1.25 \cdot 3.75 \cdot 0.15} = 91.6$$

Therefore:

$$P_{s0} = 106.3 \text{ psia} \cong 120 \text{ psia}$$

whereby:

$$\frac{P_s}{P_o} = 8$$

Thus, both films have the same supply pressure.

At this calculated pressure ratio, with $\Lambda = 0.05$ and $\sigma = 2.2$ the dimensionless damping per bearing is found from figs. 51 and 52 to be equal to the earlier assumed value of 1. Hence, the assumed value of $\Lambda = 0.05$ is correct.

Now:

$$\xi_2 = \frac{L_o}{D_o} = \frac{1.25}{3.75} = \frac{1}{3}$$

-33-
Note: even if the charts, figs. 45 to 52, are based on \(\xi_2 = 0.4\), the dimensionless stiffness and damping are not too sensitive to variations in \(\xi_2\) and the charts may, therefore, also be used for \(\xi_2 = \frac{1}{3}\). Hence:\n\[A_B = 0.05^\prime 3 = 0.15\]

From the definition of \(A_B\) in eq. (41):
\[
\frac{n_0 d_0}{\rho \omega n_0 g^4} = \frac{120 \cdot (0.00085)^2 \cdot 0.15}{6 \cdot 2.8 \cdot 10^{-2} \cdot 1.198 \cdot 10^4} = 0.0646 \text{ inch}
\]

set:
\[n_0 = 10\]

whereby:
\[d_0 = 0.0065 \text{ inch}\]

The spacing factor becomes:
\[
\lambda = 1 + \frac{2}{n_0 \xi_2} \log \left( \frac{D_0}{n_0 d_0} \right) = 3.4
\]

which is higher than the value of 1.5 for which the charts are valid. This, however, has only very minor influence on the damping but the stiffness is somewhat reduced. On the other hand, the supply pressure has been set 14 psi larger than actually required which should offset the reduction due to the spacing factor.

Even so, the calculated feeder hole diameter is small and should preferably be larger (0.01 to 0.02 inches) although the obtained value can be used as is.

The mass of the ring is:
\[
\frac{M}{4} \left(3.75^2 - 3.0^2\right)^{\frac{1}{2}} \cdot 3.083 = 3.38 \text{ lbs} = 0.00875 > \text{ lbs-sec}^2/\text{inch}
\]

If the ring was rigidly supported, figs. 35 and 39 give:
\[B_0 = \infty : \frac{C^2}{(P_s - P_a)LD} = 1.4\]

The corresponding frequency ratio is: \(\frac{\nu}{\omega} = 0.5\). Define an effective stiffness of the inner film by:
\[\nu^2 = K_{\text{eff}} \frac{M}{M}
\]
or:
\[
\frac{C K_{\text{eff}}}{(P_s - P_a)LD} \left(\frac{\nu}{\omega}\right)^2 = \frac{C^2}{(P_s - P_a)LD} = 0.25 \cdot 1.4 = 0.35
\]

(50)
whereby:

$$K_{eff} = \frac{(120 - 14.7) \cdot 3}{0.0015} \cdot 0.35 = 221,000 \text{ lbs/inch}$$

With a total outer film stiffness of: $K_o = 151,600 \text{ lbs/inch}$, the natural frequency of the ring is:

$$\text{Natural frequency of ring} \sqrt{\frac{221,000 + 151,600}{0.00875}} = 6.530 \text{ radians/sec} = 62,400 \text{ rpm}$$

Assuming an instability frequency ratio of 0.5, the ring may become unstable by itself at 125,000 rpm. This is a conservative estimate but to be safe, this speed should be considered the maximum speed for the bearing.

With $\Lambda_g \xi_2 = 0.05$, the dimensionless flow for the outer film is found from fig. 53 to be 0.034. Hence, the flow per bearing becomes:

$$\frac{\bar{M}}{N} = \frac{\bar{M}}{N} \cdot (0.00085)^3 \cdot 0.034 = 1.172 \cdot 10^{-6} \frac{\text{lbs} \cdot \text{sec}}{\text{inch}} = 4.52 \cdot 10^{-4} \frac{\text{lbs}}{\text{sec}}$$

$$\bar{M} = 0.33 \text{ scfm}$$

For the two outer bearings, the flow is 0.0009 lbs/sec = 0.66 scfm.

For the inner film, $\Lambda_g \xi_2 = 0.7$. Since $\xi_2 = 1$, $\Lambda_g = 0.7$ or:

$$n_d = \frac{120 \cdot (0.0015)^2}{6 \cdot 2.8 \cdot 10^{-9} \cdot 1.434 \cdot 10^8} \cdot 0.7 = 0.938$$

Set $n = 12$ whereby:

$$d = 0.076 \text{ inch}$$

The corresponding spacing factor is: $\lambda = 1.22$ which is close to the value of $\lambda = 1.5$ on which the charts are based.

The dimensionless flow for the inner film is determined from fig. 53 to be 0.39. Hence, the flow becomes:

$$\frac{\bar{M}}{N} = \frac{\bar{M}}{N} \cdot (0.00085)^3 \cdot 0.39 = 2.475 \cdot 10^{-5} \frac{\text{lbs} \cdot \text{sec}}{\text{inch}} = 9.55 \cdot 10^{-3} \frac{\text{lbs}}{\text{sec}} = 7.1 \text{ scfm}$$

The calculated bearing dimensions and performance data are summarized below:

- total rotor weight: $2W = 90 \text{ lbs}$.
- journal diameter: $D = 3 \text{ inch}$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter of ring:</td>
<td>$D_o = 3.75$ inch</td>
</tr>
<tr>
<td>Length of ring:</td>
<td>$L = 3$ inch</td>
</tr>
<tr>
<td>Length of one outer bearing:</td>
<td>$L_o = 1.25$ inch</td>
</tr>
<tr>
<td>Radial clearance of inner film:</td>
<td>$C = 0.0015$ inch</td>
</tr>
<tr>
<td>Radial clearance of outer film:</td>
<td>$C_o = 0.00085$ inch</td>
</tr>
<tr>
<td>Number of feeder holes for inner film:</td>
<td>$n = 12$</td>
</tr>
<tr>
<td>Diameter of feeder holes for inner film:</td>
<td>$d = 0.076$ inch</td>
</tr>
<tr>
<td>Number of feeder holes for one outer bearing:</td>
<td>$n_o = 10$</td>
</tr>
<tr>
<td>Diameter of feeder holes for outer bearing:</td>
<td>$d_o = 0.0065$ inch</td>
</tr>
<tr>
<td>Supply pressure for both films:</td>
<td>$P_s = 120$ psia</td>
</tr>
<tr>
<td>Flow for inner film:</td>
<td>$7.1$ scfm</td>
</tr>
<tr>
<td>Flow for two outer bearings:</td>
<td>$0.66$ scfm</td>
</tr>
<tr>
<td>Maximum stable speed (conservative estimate):</td>
<td>$125,000$ rpm</td>
</tr>
</tbody>
</table>
### 3 LOBE BEARING, VERTICAL ROTOR

<table>
<thead>
<tr>
<th>Preload</th>
<th>( R )</th>
<th>( R_y^* )</th>
<th>( Q_y^* )</th>
<th>( Q_y^* )</th>
<th>( v/a )</th>
<th>( CKN )</th>
<th>( CK_x^* )</th>
<th>( CK_y^* )</th>
<th>( CaB_x^* )</th>
<th>( CaB_y^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R )</td>
<td>( R_y^* )</td>
<td>( Q_y^* )</td>
<td>( Q_y^* )</td>
<td>( v/a )</td>
<td>( CKN )</td>
<td>( CK_x^* )</td>
<td>( CK_y^* )</td>
<td>( CaB_x^* )</td>
<td>( CaB_y^* )</td>
</tr>
<tr>
<td>0</td>
<td>18.04</td>
<td>0.0812</td>
<td>4.345</td>
<td>0.4978</td>
<td>0.01503</td>
<td>1.499</td>
<td>1.585</td>
<td>3.184</td>
<td>-0.00577</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>44.55</td>
<td>0.0892</td>
<td>4.351</td>
<td>0.4983</td>
<td>0.02122</td>
<td>2.150</td>
<td>2.536</td>
<td>5.090</td>
<td>-0.0139</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>133.8</td>
<td>0.0946</td>
<td>4.355</td>
<td>0.4986</td>
<td>0.04833</td>
<td>0.4893</td>
<td>6.179</td>
<td>12.39</td>
<td>-0.0300</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>306.3</td>
<td>0.0964</td>
<td>4.357</td>
<td>0.4987</td>
<td>0.1011</td>
<td>1.027</td>
<td>13.24</td>
<td>26.56</td>
<td>-0.0678</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>21.11</td>
<td>0.1994</td>
<td>3.787</td>
<td>0.4935</td>
<td>0.07009</td>
<td>0.6783</td>
<td>2.542</td>
<td>5.152</td>
<td>-0.00912</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>47.48</td>
<td>0.2185</td>
<td>3.803</td>
<td>0.4952</td>
<td>0.09369</td>
<td>0.9174</td>
<td>3.815</td>
<td>7.703</td>
<td>-0.0207</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>139.6</td>
<td>0.2331</td>
<td>3.814</td>
<td>0.4962</td>
<td>0.2039</td>
<td>2.004</td>
<td>8.983</td>
<td>18.10</td>
<td>-0.0432</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>317.7</td>
<td>0.2381</td>
<td>3.818</td>
<td>0.4965</td>
<td>0.4206</td>
<td>4.143</td>
<td>18.96</td>
<td>38.18</td>
<td>-0.0992</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>24.66</td>
<td>0.2933</td>
<td>3.313</td>
<td>0.4886</td>
<td>0.1935</td>
<td>1.832</td>
<td>3.986</td>
<td>8.159</td>
<td>-0.0171</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>50.77</td>
<td>0.3207</td>
<td>3.337</td>
<td>0.4918</td>
<td>0.2464</td>
<td>2.367</td>
<td>5.732</td>
<td>11.66</td>
<td>-0.0307</td>
<td></td>
</tr>
<tr>
<td>0.375</td>
<td>145.8</td>
<td>0.3445</td>
<td>3.357</td>
<td>0.4938</td>
<td>0.5111</td>
<td>4.952</td>
<td>17.98</td>
<td>26.28</td>
<td>-0.0635</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>329.7</td>
<td>0.3528</td>
<td>3.363</td>
<td>0.4944</td>
<td>1.038</td>
<td>10.08</td>
<td>27.11</td>
<td>54.84</td>
<td>-0.1413</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>29.74</td>
<td>0.3810</td>
<td>2.828</td>
<td>0.4817</td>
<td>0.5339</td>
<td>4.905</td>
<td>6.912</td>
<td>14.35</td>
<td>-0.0301</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>55.38</td>
<td>0.4154</td>
<td>2.860</td>
<td>0.4869</td>
<td>0.6456</td>
<td>6.067</td>
<td>9.391</td>
<td>19.29</td>
<td>-0.0497</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>154.0</td>
<td>0.4502</td>
<td>2.890</td>
<td>0.4909</td>
<td>1.258</td>
<td>12.02</td>
<td>20.22</td>
<td>41.19</td>
<td>-1.015</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>345.2</td>
<td>0.4628</td>
<td>2.900</td>
<td>0.4921</td>
<td>2.501</td>
<td>24.01</td>
<td>41.66</td>
<td>84.65</td>
<td>-2.183</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>41.48</td>
<td>0.4817</td>
<td>2.156</td>
<td>0.4673</td>
<td>2.521</td>
<td>21.78</td>
<td>17.61</td>
<td>37.69</td>
<td>-1.025</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>66.00</td>
<td>0.5223</td>
<td>2.198</td>
<td>0.4762</td>
<td>2.836</td>
<td>25.44</td>
<td>22.29</td>
<td>46.80</td>
<td>-1.032</td>
<td></td>
</tr>
<tr>
<td>0.6667</td>
<td>170.72</td>
<td>0.5769</td>
<td>2.243</td>
<td>0.4861</td>
<td>4.876</td>
<td>45.61</td>
<td>44.57</td>
<td>91.68</td>
<td>-1.235</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>376.2</td>
<td>0.5984</td>
<td>2.261</td>
<td>0.4891</td>
<td>9.264</td>
<td>87.73</td>
<td>88.79</td>
<td>182.5</td>
<td>-1.484</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>52.27</td>
<td>0.5198</td>
<td>1.807</td>
<td>0.4560</td>
<td>6.646</td>
<td>54.63</td>
<td>32.89</td>
<td>72.13</td>
<td>-1.817</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>75.92</td>
<td>0.5617</td>
<td>1.849</td>
<td>0.4667</td>
<td>7.198</td>
<td>62.01</td>
<td>40.07</td>
<td>85.86</td>
<td>-2.182</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>189.6</td>
<td>0.6301</td>
<td>1.904</td>
<td>0.4832</td>
<td>11.27</td>
<td>104.1</td>
<td>76.51</td>
<td>158.3</td>
<td>-4.248</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>401.0</td>
<td>0.6585</td>
<td>1.926</td>
<td>0.4883</td>
<td>20.58</td>
<td>194.2</td>
<td>150.1</td>
<td>307.5</td>
<td>-8.336</td>
<td></td>
</tr>
<tr>
<td>Preload</td>
<td>R</td>
<td>F_x</td>
<td>Q_x</td>
<td>Q_y</td>
<td>u</td>
<td>CMN</td>
<td>CK_xx</td>
<td>CK_xy</td>
<td>CoB_xx</td>
<td>CoB_xy</td>
</tr>
<tr>
<td>---------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-------</td>
<td>-------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td></td>
<td>δ</td>
<td>R_e</td>
<td></td>
<td></td>
<td></td>
<td>μDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SW</td>
<td>SW</td>
<td></td>
<td>H^2</td>
<td>SW</td>
<td>SW</td>
<td>SW</td>
<td>SW</td>
</tr>
<tr>
<td>0</td>
<td>18.04</td>
<td>0.0434</td>
<td>4.318</td>
<td>0.4968</td>
<td>0.02665</td>
<td>0.2584</td>
<td>3.343</td>
<td>6.729</td>
<td>0.00256</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>44.55</td>
<td>0.0500</td>
<td>4.323</td>
<td>0.4976</td>
<td>0.03991</td>
<td>0.3912</td>
<td>5.788</td>
<td>11.63</td>
<td>-0.00230</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>133.8</td>
<td>0.0549</td>
<td>4.326</td>
<td>0.4979</td>
<td>0.06622</td>
<td>0.9486</td>
<td>14.89</td>
<td>29.90</td>
<td>-0.0141</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>306.3</td>
<td>0.0567</td>
<td>4.328</td>
<td>0.4980</td>
<td>0.2059</td>
<td>2.035</td>
<td>32.47</td>
<td>63.21</td>
<td>-0.0389</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>21.12</td>
<td>0.1054</td>
<td>3.718</td>
<td>0.4908</td>
<td>0.1230</td>
<td>1.171</td>
<td>5.194</td>
<td>10.58</td>
<td>-0.00222</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>47.50</td>
<td>0.1209</td>
<td>3.730</td>
<td>0.4930</td>
<td>0.1732</td>
<td>1.667</td>
<td>8.496</td>
<td>17.23</td>
<td>-0.00881</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>139.7</td>
<td>0.1338</td>
<td>3.740</td>
<td>0.4942</td>
<td>0.4003</td>
<td>3.875</td>
<td>21.20</td>
<td>42.90</td>
<td>-0.03069</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>317.8</td>
<td>0.1386</td>
<td>3.744</td>
<td>0.4945</td>
<td>0.8447</td>
<td>8.186</td>
<td>45.79</td>
<td>92.59</td>
<td>-0.0619</td>
<td></td>
</tr>
<tr>
<td>0.375</td>
<td>24.67</td>
<td>0.1533</td>
<td>3.207</td>
<td>0.4840</td>
<td>0.3358</td>
<td>3.110</td>
<td>7.985</td>
<td>16.50</td>
<td>-0.0115</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>50.84</td>
<td>0.1747</td>
<td>3.226</td>
<td>0.4882</td>
<td>0.4469</td>
<td>4.214</td>
<td>12.35</td>
<td>25.29</td>
<td>-0.0200</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>146.0</td>
<td>0.1953</td>
<td>3.242</td>
<td>0.4906</td>
<td>0.9847</td>
<td>9.381</td>
<td>29.92</td>
<td>60.99</td>
<td>-0.0477</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>330.1</td>
<td>0.2031</td>
<td>3.248</td>
<td>0.4913</td>
<td>2.054</td>
<td>19.63</td>
<td>40.01</td>
<td>130.3</td>
<td>-0.1217</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>29.85</td>
<td>0.1960</td>
<td>2.682</td>
<td>0.4747</td>
<td>0.9105</td>
<td>8.112</td>
<td>13.22</td>
<td>27.85</td>
<td>-0.02706</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>55.57</td>
<td>0.2219</td>
<td>2.708</td>
<td>0.4816</td>
<td>1.140</td>
<td>10.45</td>
<td>19.37</td>
<td>40.23</td>
<td>-0.0374</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>346.2</td>
<td>0.2628</td>
<td>2.740</td>
<td>0.4880</td>
<td>4.833</td>
<td>45.54</td>
<td>95.89</td>
<td>196.5</td>
<td>-1.957</td>
<td></td>
</tr>
<tr>
<td>0.6667</td>
<td>41.89</td>
<td>0.2405</td>
<td>1.954</td>
<td>0.4560</td>
<td>4.141</td>
<td>34.01</td>
<td>31.05</td>
<td>68.09</td>
<td>-0.0670</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>379.5</td>
<td>0.3308</td>
<td>2.031</td>
<td>0.4864</td>
<td>16.82</td>
<td>156.1</td>
<td>193.7</td>
<td>399.9</td>
<td>-4.056</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>53.07</td>
<td>0.2545</td>
<td>1.576</td>
<td>0.4414</td>
<td>10.58</td>
<td>81.38</td>
<td>54.72</td>
<td>124.0</td>
<td>-0.03635</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>77.07</td>
<td>0.2824</td>
<td>1.605</td>
<td>0.4554</td>
<td>11.57</td>
<td>94.80</td>
<td>71.23</td>
<td>156.4</td>
<td>-1.660</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>187.4</td>
<td>0.3336</td>
<td>1.638</td>
<td>0.4776</td>
<td>18.75</td>
<td>169.0</td>
<td>152.7</td>
<td>319.8</td>
<td>-3.023</td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td>407.2</td>
<td>0.3573</td>
<td>1.653</td>
<td>0.4848</td>
<td>35.18</td>
<td>326.7</td>
<td>315.0</td>
<td>649.7</td>
<td>-6.616</td>
<td></td>
</tr>
</tbody>
</table>
I

aN

-

,

°...

.

. .

.

. .

. .

S I-l

o.i

,.N

~
P"S

~ ~
Pug"

.."

•+
....
•

-

S
Sn=zi

228"

:32

oq*

o .0°

34

Z

,-

a"]: "1=1

.. . . .

. .n. .

. ....
.

~**oo0000cca ~

~

1..2,IZ*

..

.. .. .

@.
66666ti
66666*

.

4

-P

-1"

.o

5f

o

. .. . .

.

iJl

al

.

q..9.9

i

f4~lf
i

bar~a

i.i

X!1

m

-xieH

Z.

.

.

.

AM.
Al.4
.9~f

..

.

~

0a.1O~~9

a
___

__ __

__

__

__

__

__

g
i

___-39-_

. ......
- ...

2"-!.....
HUI-

zgg

131

.


<table>
<thead>
<tr>
<th>$F_{\text{min}}$</th>
<th>Film</th>
<th>Thickness</th>
<th>$\gamma$</th>
<th>$h$</th>
<th>$g$</th>
<th>$b$</th>
<th>$A$</th>
<th>$E$</th>
<th>$\tan \delta$</th>
<th>$Q$</th>
<th>$Q_{\text{inc}}$</th>
<th>$Q_{\text{disc}}$</th>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$\phi$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.043</td>
<td>63.32</td>
<td>478.5</td>
<td>1.003</td>
<td>55.84</td>
<td>2.242</td>
<td>2.705</td>
<td>4813</td>
<td>46.60</td>
<td>1.172</td>
<td>12.76</td>
<td>19.84</td>
<td>-19.34</td>
<td>9.503</td>
<td>17.90</td>
<td>-89.03</td>
</tr>
<tr>
<td>0.1</td>
<td>0.124</td>
<td>62.50</td>
<td>370.0</td>
<td>1.611</td>
<td>9.120</td>
<td>0.506</td>
<td>2.658</td>
<td>6720</td>
<td>12.99</td>
<td>2.063</td>
<td>4.747</td>
<td>4.948</td>
<td>-2.388</td>
<td>2.095</td>
<td>1.85</td>
<td>5.000</td>
</tr>
<tr>
<td>0.15</td>
<td>0.287</td>
<td>59.86</td>
<td>300.6</td>
<td>0.9339</td>
<td>5.720</td>
<td>0.788</td>
<td>1.674</td>
<td>6450</td>
<td>12.26</td>
<td>3.317</td>
<td>4.869</td>
<td>4.265</td>
<td>-1.349</td>
<td>1.783</td>
<td>9.822</td>
<td>1.003</td>
</tr>
<tr>
<td>0.2</td>
<td>0.364</td>
<td>56.73</td>
<td>236.5</td>
<td>0.5935</td>
<td>3.870</td>
<td>0.973</td>
<td>2.567</td>
<td>3759</td>
<td>15.10</td>
<td>6.646</td>
<td>5.564</td>
<td>4.128</td>
<td>-1.151</td>
<td>1.124</td>
<td>10.17</td>
<td>1.345</td>
</tr>
<tr>
<td>0.25</td>
<td>0.415</td>
<td>53.06</td>
<td>1.179</td>
<td>0.03882</td>
<td>2.729</td>
<td>3.140</td>
<td>2.723</td>
<td>1237</td>
<td>128.3</td>
<td>90.22</td>
<td>4.632</td>
<td>4.290</td>
<td>-0.706</td>
<td>1.686</td>
<td>9.045</td>
<td>1.523</td>
</tr>
<tr>
<td>0.3</td>
<td>0.468</td>
<td>47.62</td>
<td>1.341</td>
<td>0.02678</td>
<td>1.991</td>
<td>1.347</td>
<td>2.542</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>8.817</td>
<td>4.328</td>
<td>-1.506</td>
<td>1.239</td>
<td>9.662</td>
<td>1.451</td>
</tr>
<tr>
<td>0.02</td>
<td>0.048</td>
<td>63.16</td>
<td>478.2</td>
<td>4083</td>
<td>63.12</td>
<td>2.353</td>
<td>1.79</td>
<td>4868</td>
<td>39.07</td>
<td>2.624</td>
<td>10.60</td>
<td>19.05</td>
<td>-18.28</td>
<td>8.166</td>
<td>41.44</td>
<td>1.90</td>
</tr>
<tr>
<td>0.1</td>
<td>0.127</td>
<td>63.59</td>
<td>364.2</td>
<td>0.0466</td>
<td>10.60</td>
<td>6.207</td>
<td>2.681</td>
<td>4865</td>
<td>10.52</td>
<td>3.996</td>
<td>4.062</td>
<td>4.921</td>
<td>-5.067</td>
<td>1.853</td>
<td>11.11</td>
<td>5.233</td>
</tr>
<tr>
<td>0.15</td>
<td>0.312</td>
<td>61.30</td>
<td>284.6</td>
<td>0.3913</td>
<td>0.410</td>
<td>0.866</td>
<td>2.634</td>
<td>4766</td>
<td>0.977</td>
<td>5.878</td>
<td>4.076</td>
<td>4.376</td>
<td>-1.840</td>
<td>1.601</td>
<td>9.317</td>
<td>9.606</td>
</tr>
<tr>
<td>0.2</td>
<td>0.379</td>
<td>58.17</td>
<td>219.4</td>
<td>0.02504</td>
<td>4.264</td>
<td>0.133</td>
<td>3.556</td>
<td>4576</td>
<td>0.574</td>
<td>8.672</td>
<td>4.472</td>
<td>4.205</td>
<td>-1.141</td>
<td>1.659</td>
<td>8.509</td>
<td>1.275</td>
</tr>
<tr>
<td>0.25</td>
<td>0.430</td>
<td>55.31</td>
<td>1.721</td>
<td>0.03641</td>
<td>2.964</td>
<td>2.17</td>
<td>2.355</td>
<td>3951</td>
<td>10.17</td>
<td>15.52</td>
<td>5.261</td>
<td>4.414</td>
<td>-6.649</td>
<td>1.408</td>
<td>0.358</td>
<td>1.473</td>
</tr>
<tr>
<td>0.3</td>
<td>0.488</td>
<td>49.59</td>
<td>1.259</td>
<td>0.01335</td>
<td>2.124</td>
<td>2.426</td>
<td>1.84</td>
<td>2938</td>
<td>11.59</td>
<td>25.54</td>
<td>6.631</td>
<td>4.699</td>
<td>-0.265</td>
<td>1.345</td>
<td>8.556</td>
<td>1.216</td>
</tr>
<tr>
<td>0.02</td>
<td>0.048</td>
<td>63.66</td>
<td>428.1</td>
<td>1900</td>
<td>0.684</td>
<td>2.661</td>
<td>2.738</td>
<td>4848</td>
<td>37.17</td>
<td>4.956</td>
<td>10.13</td>
<td>18.02</td>
<td>-17.99</td>
<td>7.824</td>
<td>40.76</td>
<td>1.926</td>
</tr>
<tr>
<td>0.1</td>
<td>0.230</td>
<td>64.36</td>
<td>364.9</td>
<td>0.03123</td>
<td>11.08</td>
<td>0.320</td>
<td>2.691</td>
<td>4929</td>
<td>9.904</td>
<td>8.033</td>
<td>3.866</td>
<td>4.885</td>
<td>-3.051</td>
<td>1.797</td>
<td>10.93</td>
<td>5.127</td>
</tr>
<tr>
<td>0.15</td>
<td>0.319</td>
<td>63.64</td>
<td>292.2</td>
<td>0.01289</td>
<td>6.666</td>
<td>0.619</td>
<td>2.645</td>
<td>6899</td>
<td>0.232</td>
<td>1.160</td>
<td>3.024</td>
<td>4.260</td>
<td>-1.798</td>
<td>1.547</td>
<td>8.970</td>
<td>9.394</td>
</tr>
<tr>
<td>0.2</td>
<td>0.384</td>
<td>58.65</td>
<td>2.268</td>
<td>0.01178</td>
<td>4.436</td>
<td>0.256</td>
<td>2.596</td>
<td>4813</td>
<td>7.285</td>
<td>15.66</td>
<td>4.098</td>
<td>4.208</td>
<td>-1.132</td>
<td>1.654</td>
<td>8.315</td>
<td>1.263</td>
</tr>
<tr>
<td>0.25</td>
<td>0.435</td>
<td>55.00</td>
<td>1.496</td>
<td>0.00783</td>
<td>3.064</td>
<td>3.245</td>
<td>2.543</td>
<td>4556</td>
<td>0.015</td>
<td>22.70</td>
<td>4.185</td>
<td>4.645</td>
<td>-1.051</td>
<td>1.187</td>
<td>8.165</td>
<td>1.435</td>
</tr>
<tr>
<td>0.5</td>
<td>0.568</td>
<td>50.16</td>
<td>1.929</td>
<td>0.00333</td>
<td>2.176</td>
<td>2.460</td>
<td>2.567</td>
<td>4764</td>
<td>3.968</td>
<td>18.76</td>
<td>5.645</td>
<td>4.750</td>
<td>-0.970</td>
<td>1.104</td>
<td>8.294</td>
<td>1.274</td>
</tr>
</tbody>
</table>
### TABLE 3

**PLAIN CYLINDRICAL BEARING**

<table>
<thead>
<tr>
<th>$L/D$</th>
<th>$R$</th>
<th>$e$</th>
<th>$S$</th>
<th>$\frac{R}{C}$</th>
<th>$\frac{F}{W}$</th>
<th>$\frac{C+R}{W}$</th>
<th>$\frac{\chi \omega^2}{W}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0907</td>
<td>4.778</td>
<td>94.74</td>
<td>7.202</td>
<td>0.9964</td>
<td>1.353</td>
<td>26.88</td>
<td>6.418</td>
</tr>
<tr>
<td>0.1578</td>
<td>2.654</td>
<td>53.14</td>
<td>7.039</td>
<td>0.1672</td>
<td>0.7594</td>
<td>15.29</td>
<td>6.381</td>
</tr>
<tr>
<td>0.2902</td>
<td>1.278</td>
<td>26.50</td>
<td>6.719</td>
<td>0.3068</td>
<td>0.3757</td>
<td>7.937</td>
<td>6.300</td>
</tr>
<tr>
<td>0.4149</td>
<td>0.7377</td>
<td>16.19</td>
<td>6.483</td>
<td>0.4366</td>
<td>0.2267</td>
<td>5.168</td>
<td>6.368</td>
</tr>
<tr>
<td>0.5138</td>
<td>0.4791</td>
<td>11.23</td>
<td>6.701</td>
<td>0.5379</td>
<td>0.1549</td>
<td>3.850</td>
<td>6.767</td>
</tr>
<tr>
<td>0.5995</td>
<td>0.3198</td>
<td>8.111</td>
<td>7.592</td>
<td>0.6246</td>
<td>0.1095</td>
<td>3.009</td>
<td>7.698</td>
</tr>
<tr>
<td>0.6777</td>
<td>0.2107</td>
<td>5.884</td>
<td>10.75</td>
<td>0.7017</td>
<td>0.07730</td>
<td>2.388</td>
<td>10.41</td>
</tr>
<tr>
<td>0.8182</td>
<td>0.07786</td>
<td>2.884</td>
<td>-</td>
<td>-</td>
<td>0.8365</td>
<td>0.03433</td>
<td>1.466</td>
</tr>
<tr>
<td>0.0969</td>
<td>2.878</td>
<td>148.7</td>
<td>6.318</td>
<td>0.1019</td>
<td>0.7948</td>
<td>41.13</td>
<td>5.716</td>
</tr>
<tr>
<td>0.1680</td>
<td>1.609</td>
<td>83.47</td>
<td>6.190</td>
<td>0.1766</td>
<td>0.4458</td>
<td>23.21</td>
<td>5.639</td>
</tr>
<tr>
<td>0.3075</td>
<td>0.7835</td>
<td>41.24</td>
<td>5.968</td>
<td>0.3230</td>
<td>0.2202</td>
<td>11.72</td>
<td>5.439</td>
</tr>
<tr>
<td>0.4366</td>
<td>0.4610</td>
<td>24.84</td>
<td>5.846</td>
<td>0.4578</td>
<td>0.1327</td>
<td>7.331</td>
<td>5.305</td>
</tr>
<tr>
<td>0.5365</td>
<td>0.3061</td>
<td>16.97</td>
<td>6.058</td>
<td>0.5617</td>
<td>0.09076</td>
<td>5.238</td>
<td>5.473</td>
</tr>
<tr>
<td>0.6213</td>
<td>0.2091</td>
<td>12.00</td>
<td>7.222</td>
<td>0.6495</td>
<td>0.06441</td>
<td>3.921</td>
<td>6.158</td>
</tr>
<tr>
<td>0.6977</td>
<td>0.1415</td>
<td>8.493</td>
<td>9.920</td>
<td>0.7266</td>
<td>0.04583</td>
<td>2.982</td>
<td>7.780</td>
</tr>
<tr>
<td>0.8314</td>
<td>0.05618</td>
<td>3.899</td>
<td>-</td>
<td>-</td>
<td>0.8562</td>
<td>0.02139</td>
<td>1.699</td>
</tr>
<tr>
<td>0.1013</td>
<td>1.149</td>
<td>180.3</td>
<td>5.785</td>
<td>0.1058</td>
<td>0.3133</td>
<td>49.20</td>
<td>5.288</td>
</tr>
<tr>
<td>0.1756</td>
<td>0.6440</td>
<td>101.3</td>
<td>5.667</td>
<td>0.1833</td>
<td>0.1759</td>
<td>27.75</td>
<td>5.197</td>
</tr>
<tr>
<td>0.3209</td>
<td>0.3165</td>
<td>50.26</td>
<td>5.469</td>
<td>0.3346</td>
<td>0.08720</td>
<td>13.98</td>
<td>4.952</td>
</tr>
<tr>
<td>0.4541</td>
<td>0.1891</td>
<td>30.48</td>
<td>5.381</td>
<td>0.4727</td>
<td>0.05279</td>
<td>8.684</td>
<td>4.714</td>
</tr>
<tr>
<td>0.5560</td>
<td>0.1277</td>
<td>20.95</td>
<td>5.549</td>
<td>0.5780</td>
<td>0.03624</td>
<td>6.146</td>
<td>4.532</td>
</tr>
<tr>
<td>0.6414</td>
<td>0.08900</td>
<td>14.91</td>
<td>6.464</td>
<td>0.6662</td>
<td>0.02579</td>
<td>4.535</td>
<td>4.676</td>
</tr>
<tr>
<td>0.7170</td>
<td>0.06171</td>
<td>10.61</td>
<td>8.053</td>
<td>0.7430</td>
<td>0.01836</td>
<td>3.377</td>
<td>5.031</td>
</tr>
<tr>
<td>0.8470</td>
<td>0.02295</td>
<td>4.830</td>
<td>-</td>
<td>-</td>
<td>0.8714</td>
<td>0.00851</td>
<td>1.791</td>
</tr>
<tr>
<td>0.1028</td>
<td>0.5313</td>
<td>191.5</td>
<td>5.664</td>
<td>0.1072</td>
<td>0.1442</td>
<td>51.98</td>
<td>5.144</td>
</tr>
<tr>
<td>0.1783</td>
<td>0.2979</td>
<td>107.6</td>
<td>5.493</td>
<td>0.1857</td>
<td>0.08102</td>
<td>29.33</td>
<td>5.065</td>
</tr>
<tr>
<td>0.3257</td>
<td>0.1469</td>
<td>53.46</td>
<td>5.302</td>
<td>0.3388</td>
<td>0.04021</td>
<td>14.77</td>
<td>4.799</td>
</tr>
<tr>
<td>0.4604</td>
<td>0.08818</td>
<td>32.52</td>
<td>5.213</td>
<td>0.4781</td>
<td>0.02441</td>
<td>9.174</td>
<td>4.543</td>
</tr>
<tr>
<td>0.5632</td>
<td>0.05992</td>
<td>22.43</td>
<td>5.357</td>
<td>0.5840</td>
<td>0.01679</td>
<td>6.486</td>
<td>4.369</td>
</tr>
<tr>
<td>0.6492</td>
<td>0.04204</td>
<td>16.02</td>
<td>6.183</td>
<td>0.6723</td>
<td>0.01197</td>
<td>4.777</td>
<td>4.314</td>
</tr>
<tr>
<td>0.7243</td>
<td>0.02941</td>
<td>11.45</td>
<td>7.458</td>
<td>0.7488</td>
<td>0.008535</td>
<td>3.544</td>
<td>4.271</td>
</tr>
<tr>
<td>0.8532</td>
<td>0.01269</td>
<td>5.270</td>
<td>-</td>
<td>-</td>
<td>0.8754</td>
<td>0.003933</td>
<td>1.839</td>
</tr>
</tbody>
</table>
Figure 1. The 3 Lobe Bearing, Schematic
Figure 2. The Hydrodynamic-Hydrostatic Ring Bearing, Schematic
Figure 3. The Hybrid-Hydrostatic Ring Bearing, Schematic
Figure 4. 3 Lobe Bearing, Vertical Rotor, $L = \frac{1}{2}$, Stability Map

STABILITY MASS PARAMETER $\frac{\text{CM}}{D L \left(\frac{L}{D}\right)^2}$

PRELOAD $\delta$

REYNOLDS NO. = 50,000

10,000

2,000

0
Figure 5. 3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = 1$, Stability Map
Figure 6. 3 Lobe Bearing, Vertical Rotor, $\frac{L}{D} = \frac{1}{2}$, Friction
Figure 7. 3 Lobe Bearing, Vertical Rotor, \( \frac{L}{D} = 1 \), Friction
Figure 9. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Stability Map
Figure 10. 3 Lobe Bearing, Preload = 0.5, \( \frac{L}{D} = \frac{1}{2} \), Friction
Figure 11. 3 Lobe Bearing, Preload = 0.5, $\frac{L}{D} = 1$, Friction
Figure 14. 3 Lobe Bearing, Preload = 0.5, S = 1, Flow = 10

SOMMERFELD NO.: $S = \frac{r_1}{r_2}$

DIMENSIONLESS FLOW: $\frac{\text{Net}}{Q}$
Figure 16. Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 0$, $\frac{C_k}{W} = 0.7$, Stability Map
Figure 17. Hydrodynamic-Hydrostatic Ring Bearing, $Re = 0$, $\frac{CK}{W} = 0.4$, Stability Map
Figure 18. Hydrodynamic-Hydrostatic Ring Bearing, \( R_e = 0 \), \( \frac{C_k}{W} = 0.1 \), Stability Map.
Figure 21. Hydrodynamic-Hydrostatic Ring Bearing, $R_e = 30,000$, $\frac{C_k}{W} = 0.1$, Stability Map
Figure 22. Hydrostatic Bearing, Incompressible, Single Plane Admission, Stiffness
Figure 24. Hydrostatic Bearing, Incompressible, Single Plane Admission, Damping
Figure 27. Hybrid-Hydrostatic Ring Bearing, \( \frac{P_s}{P_a} = 2, \frac{CK_0}{(P_s - P_a)LD} = 0.24 \), Stability Map
Figure 28. Hybrid-Hydrostatic Ring Bearing, $\frac{P}{P_a} = 2$, $\frac{C \eta_0}{(P_s - P_a)L_D} = 0.24$, Instability Frequency
Figure 29. Hybrid-Hydrostatic Ring Bearing, $\frac{P_a}{P_o} = 2$, $\frac{Ck_o}{(P_o-P_a)LD} = 0.18$, Stability Map
Figure 30. Hybrid-Hydrostatic Ring Bearing, $\frac{P_s}{P_a} = 2$, $\frac{C_k}{(P_s - P_a)LD} = 0.18$, Instability Frequency
Figure 31. Hybrid-hydrostatic Ring Bearing, \( \frac{p_s}{p_a} = 2 \), \( (p_s - p_a) \text{L/D} = 0.09 \), Stability Map

\[
\frac{(p_s - p_a) \text{L/D}}{C_m m^2}
\]
Figure 32. Hybrid-Hydrostatic Ring Bearing, \( \frac{P_b}{P_a} = 2 \), \( \frac{CK_b}{(P_a - P_b)LD} = 0.09 \), Instability Frequency
Figure 33. Hybrid-Hydrostatic Ring Bearing, $\frac{P_a}{P_a} = 5, \frac{C_k}{(P_a - P_a)LD} = 0.24$, Stability Map.
Figure 34. Hybrid-Hydrostatic Ring Bearing, $\frac{P_a}{P_s} = 5$, $\frac{CK_0}{(P_s - P_a)LD} = 0.24$, Instability Frequency
Figure 35. Hybrid-Hydrostatic Ring Bearing, \( \frac{P_t}{P_a} = 5 \), \( \frac{CK_o}{(P_t - P_a)LD} = 0.18 \), Stability Map
Figure 36. Hybrid-Hydrostatic Ring Bearing, \( P_a = 5 \), \( \frac{C_{F_0}}{(P_0 - P_d)LD} = 0.18 \), Instability Frequency
Figure 37. Hybrid-Hydrostatic Ring Bearing, \( \frac{P_s}{P_a} = 5, \frac{C_K}{(P_s - P_a)LD} = 0.09 \), Stability Map
Figure 30. Hybrid-Hydraulic Ring Bearing, $p_i = 5$, $(p_i - p_o)LD = 0.09$, Instability Frequency
Figure 40. Hybrid-Hydrostatic Ring Bearing, $P_e = 10$, $(P_e - P_a)LD = 0.24$, Instability Frequency
Figure 41. Hybrid-Hydrostatic Ring Bearing, $\frac{P_a}{F_a} = 10$, $\frac{C_k}{(P_b - P_a)LD} = 0.18$, Stability Map
Figure 43. Hybrid-Hydrostatic Ring Bearing, $\frac{p_s}{p_a} = 10$, $\frac{CK_o}{(P_s - P_a)LD} = 0.09$, Stability Map
Figure 44. Hybrid-hydrostatic Ring Bearing, \( P_a = 10 \), \( (P_s - P_a) / LD = 0.09 \), Instability Frequency

\[ \frac{CMu^2}{(P_s - P_a) / LD} \times \frac{1}{2} \]
Figure 45. Hydrostatic Bearing, Compressible, $\frac{p}{p_a} = 1.25$, Stiffness

\[ \sigma = \frac{12\mu v}{P_a} \left( \frac{R}{C} \right)^2 \]
Figure 46. Hydrostatic Bearing, Compressible, $P = 2$, Stiffness

\[
\sigma = \frac{12 \pi R \delta}{P_d} \left( \frac{P - P_0}{P_d} \right)
\]
Figure 51. Hydrostatic Bearing, Compressible, $\frac{P}{P_a} = 5$, Damping
Figure 52. Hydrostatic Bearing, Compressible, \( \frac{p_n}{p_a} = 10 \), Damping
APPENDIX I: The Static and Dynamic Performance of a Partial Arc Bearing with Turbulent Film

To calculate the load carrying capacity, the friction, the flow and the dynamic coefficients of a turbulent lubricant film it is necessary to solve Reynolds equation. This is done numerically as described in this appendix.

In a turbulent lubricant film Reynolds equation is given by (see reference 1):

\[
\frac{1}{R}\frac{d}{d\theta}\left[G_x \frac{h^3}{12\mu} \frac{d\bar{P}}{d\theta}\right] + \frac{1}{2} \frac{d}{dz}\left[G_x \frac{h^3}{12\mu} \frac{d\bar{P}}{dz}\right] = \frac{1}{2} R \omega \frac{d\bar{h}}{d\theta} + \frac{d\bar{h}}{dt}
\]

(A-1)

where \(\bar{P}\) is the film pressure, \(\mu\) is the lubricant viscosity, \(R\) is the journal radius, \(\omega\) is the angular speed of the journal, \(\Theta\) is the angular coordinate, (see fig. 54), \(z\) is the axial coordinate, \(t\) is time, \(G_x\) and \(G_z\) are turbulent flow coefficients and \(\bar{h}\) is the local film thickness:

\[
\bar{h} = C + e \cos \theta
\]

(A-2)

Here, \(C\) is the radial clearance (the difference between the radius of curvature of the bearing surface, and the radius of the journal, see fig. 54) and \(e\) is the eccentricity (the distance between the journal center and the center of curvature of the bearing).

Reynolds equation, eq. (A-1), is made dimensionless by setting:

\[
\xi = \frac{z}{R}
\]

(A-3)

\[
\tau = \omega t
\]

(A-4)

\[
h = \frac{\bar{h}}{C} = \frac{1}{\lambda} \epsilon \cos \theta
\]

(A-5)

\[
\epsilon = \frac{e}{C}
\]

(A-6)

\[
P = \frac{\bar{P}}{\mu \omega (\xi)^2} = \frac{1}{2\pi S} \frac{\bar{P}}{W/LD}
\]

(A-7)
where \( W \) is the load carried by the bearing, \( L \) is the bearing length, \( D \) is the journal diameter and:

\[
\text{Sommerfeld Number: } S = \frac{\mu N D L}{W} \left( \frac{R}{C} \right)^2
\]  

(A-8)

\( N \) is the journal speed in rps, i.e. \( \omega = 2\pi N \)

Substituting eqs. (A-3) to (A-7) into eq. (A-1) yields Reynolds equation in dimensionless form:

\[
\frac{d}{dh} \left[ G_x h^3 \frac{dP}{dh} \right] + \frac{d}{dh} \left[ G_z h^3 \frac{dP}{dh} \right] = 6 \frac{dh}{dh} + 12 \frac{dh}{dh}
\]  

(A-9)

The dimensionless turbulent flow coefficients \( G_x \) and \( G_z \) are functions of the local Reynolds number \( R_h \):

\[
R_h = h R_e
\]  

(A-10)

where:

\[
\text{Reynolds Number: } R_e = \frac{\varrho R_0 \omega C}{\mu}
\]  

(A-11)

\( \varrho \) is the mass density of the lubricant.

The actual functional relationships between \( G_x \), \( G_z \) and \( R_h \) are given in reference 1. When comparing with ref. 1, however, it should be noted that \( G_x \) and \( G_z \) as used in the present analysis are larger than the values given in ref. 1 by a factor of 12 such that for laminar flow \( (R_h = R_e = 0) \), \( G_x \) and \( G_z \) are equal to unity rather than 1/12 as in ref. 1.

From eq. (A-10) it is seen, that for a given Reynolds number \( R_e \), \( G_x \) and \( G_z \) are functions of \( \Theta \) only (since they are functions of \( h \)) and do not depend on the axial coordinate \( f \).
Before solving eq. (A-9) it is convenient to give it another form. Introduce the new variables:

\[ G = \frac{G_x}{G} \quad (A-12) \]

\[ H = h^{\frac{1}{2}} G_x \quad (A-13) \]

\[ \psi = PH = Ph^{\frac{3}{2}} G_x \quad (A-14) \]

whereby eq. (A-9) can be written:

\[ \frac{\partial^2 \psi}{\partial \theta^2} + G \frac{\partial^2 \psi}{\partial \phi^2} - \frac{1}{H} \frac{\partial^2 \psi}{\partial \theta^2} \psi = \frac{1}{H} \left[ b \frac{\partial h}{\partial \theta} + 12 \frac{\partial h}{\partial \phi} \right] \quad (A-15) \]

Assume that the journal operates under static conditions with an eccentricity ratio \( \varepsilon \) and an attitude angle \( \phi \), where, in the present analysis, the attitude angle \( \phi \) is the angle from a fixed x-axis to the line connecting the center of the bearing with the center of the journal:

![Figure 54: Geometry of Partial Arc Bearing](image-url)
The static equilibrium position \((E_0, \varphi_0)\) is perturbed by giving the journal center a small amplitude motion with amplitudes \((\xi_1, \varphi_1)\) and corresponding velocities \((\dot{\xi}_1, \dot{\varphi}_1)\) where "dot" refers to \(\frac{d}{dt}\). Hence, the instantaneous position of the journal center is defined by:

\[
E = E_0 + \xi_1 \\
\varphi = \varphi_0 + \varphi_1
\]

where \(\xi_1 \ll E_0\) and \(\varphi_1 \ll \varphi_0\). This motion causes similar perturbations in the film pressure and the film thickness such that:

\[
\psi = \psi_0 + \xi_1 \psi_1 + \xi_2 \psi_2 + \xi_3 \psi_3 + \xi_4 \psi_4 \\
H = H_0 + \xi_1 H_1 + \xi_2 H_2 \\
G = G_0 + \xi_1 G_1 + \xi_2 G_2 \\
h = 1 + \xi_1 \cos \theta + \xi_2 \cos \theta + \xi_3 \sin \theta = h_0 + \xi_1 \cos \theta + \xi_2 \theta \sin \theta
\]

Substitution of eqs. (A-17) to (A-20) into eq. (A-15) yields 5 equations to determine the static pressure and its four perturbation components:

\[
\frac{d^2 \psi_0}{d\theta^2} + G_0 \frac{d^2 \psi_1}{d\theta^2} - \frac{1}{H_0} \frac{d^2 H_0}{d\theta^2} \psi_0 = \frac{6}{H_0} \frac{dh_0}{d\theta} = - \frac{6 \sin \theta}{H_0} E_0
\]

\[
\frac{d^2 \psi_1}{d\theta^2} + G_0 \frac{d^2 \psi_1}{d\theta^2} - \frac{1}{H_0} \frac{d^2 H_0}{d\theta^2} \psi_1 = \frac{1}{H_0} \left( \frac{d^2 H_0}{d\theta^2} - \frac{H_0}{H_0} \frac{d^2 H_0}{d\theta^2} \right) \psi_0 - G_1 \frac{d^2 \psi_0}{d\theta^2} - \frac{6}{H_0} (\sin \theta + \frac{H_0}{H_0} \frac{dh_0}{d\theta})
\]

\[
\frac{d^2 \psi_2}{d\theta^2} + G_0 \frac{d^2 \psi_2}{d\theta^2} - \frac{1}{H_0} \frac{d^2 H_0}{d\theta^2} \psi_2 = \frac{1}{H_0} \left( \frac{d^2 H_0}{d\theta^2} - \frac{H_0}{H_0} \frac{d^2 H_0}{d\theta^2} \right) \psi_0 - G_2 \frac{d^2 \psi_0}{d\theta^2} - \frac{6}{H_0} (-\cos \theta + \frac{H_0}{H_0} \frac{dh_0}{d\theta})
\]

\[
\frac{d^2 \psi_3}{d\theta^2} + G_0 \frac{d^2 \psi_3}{d\theta^2} - \frac{1}{H_0} \frac{d^2 H_0}{d\theta^2} \psi_3 = \frac{12 \cos \theta}{H_0}
\]
Comparing eqs. \((A-21)\) and \((A-25)\), it is seen that:

\[
\psi_0 = -\frac{1}{2} \varepsilon \psi_4 \tag{A-26}
\]

The boundary conditions are that the pressure is ambient (i.e. zero) along the periphery of the film. Hence, at the sides of the bearing, \(P = 0\) or:

\[
\xi = \pm \frac{1}{2} \quad (z = \pm \frac{1}{2}); \quad \psi = 0
\]

Since there is symmetry with respect to \(\xi = 0\), this condition can also be written:

\[
\xi = 0; \quad \psi = 0 \quad \therefore \quad \psi_0 = \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \tag{A-27}
\]

\[
\xi = 0; \quad \frac{d\psi}{dz} = 0 \quad \therefore \quad \frac{d\psi_0}{dz} = \frac{d\psi_1}{dz} = \frac{d\psi_2}{dz} = \frac{d\psi_3}{dz} = \frac{d\psi_4}{dz} = 0 \tag{A-28}
\]

If the film is complete over the entire bearing surface, the two remaining boundary conditions are that the pressure is zero at the leading edge and at the trailing edge of the bearing arc:

\[
\begin{align*}
\theta &= \Theta; \quad \psi = 0 \quad \therefore \quad \psi_0 = \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \tag{A-29}
\end{align*}
\]

\text{No film rupture}
On the other hand, the film is frequently not complete. If any portion of the bearing arc is in the diverging part of the film (i.e. $180^\circ < \Theta < 360^\circ$, see fig. A-1), Reynolds equation predicts subambient film pressures in this region. These subambient pressures cannot, in general, exist unless the bearing is submerged completely in its own lubricant. Instead, the film contracts such that it does not take up the full length of the bearing, and the pressure in the contracted film is zero. This is called film rupture. On the boundary between the complete film and the contracted film, the pressure is zero (i.e. eq. (A-29) is valid when $\Theta_1$ and $\Theta_2$ are adjusted to give the angular location of the boundary).

In addition, a second condition is:

$$\frac{\partial \bar{p}}{\partial \Theta} = \frac{\partial \bar{p}}{\partial \xi} = 0$$

or since $\psi = 0$ on the boundary:

$$\frac{\partial \psi}{\partial \Theta} = \frac{\partial \psi}{\partial \xi} = 0$$

Let a point on the boundary have the coordinates $(\Theta, \xi)$, i.e.:

on free boundary: $\psi(\Theta, \xi) = 0 \quad \frac{\partial \psi}{\partial \Theta} = \frac{\partial \psi}{\partial \xi}$ (A-30)

Under static conditions the boundary has the coordinates $(\Theta_o, \xi_o)$. For small scale motion of the journal the boundary is perturbed such that:

$$\Theta = \Theta_o + \delta \Theta$$
$$\xi = \xi_o + \delta \xi$$ (A-31)

where $\delta \Theta$ and $\delta \xi$ are small quantities. This results in a similar perturbation of the pressure variable $\psi$:

$$\psi = \psi_o + \delta \psi$$ (A-32)
Expand \( \psi \) in a Taylor series from a point on the boundary:

\[
\psi(\xi, \zeta) = \psi(\xi_0, \zeta_0) + \frac{\partial \psi}{\partial \xi} \delta \xi + \frac{\partial \psi}{\partial \zeta} \delta \zeta = 0
\]

where terms of order \( \delta^2 \) have been ignored. Introduce eq. (A-32) to get:

\[
\psi(\xi, \zeta) = \psi_0(\xi, \zeta) + \delta \psi(\xi, \zeta) + \frac{\partial \psi}{\partial \xi} \delta \xi + \frac{\partial \psi}{\partial \zeta} \delta \zeta = 0 \tag{A-33}
\]

With \( \psi_0(\xi, \zeta) = \frac{\partial \psi}{\partial \xi} \bigg|_{0} = \frac{\partial \psi}{\partial \zeta} \bigg|_{0} = 0 \) as the given boundary condition, this equation results in:

\[
\delta \psi(\xi_0, \zeta_0) = 0 \tag{A-34}
\]

Comparing eq. (A-32) with eq. (A-17) it is seen that:

\[
\delta \psi(\xi_0, \zeta_0) = \left[ \varepsilon_i \psi_i + \varepsilon_i \phi_i \psi_i + \varepsilon_i \phi_i \psi_i \right]_{\xi_0, \zeta_0}
\]

Since \( \varepsilon_i, \phi_i, \xi, \) and \( \phi_i \) are independent variables, each of the four pressure perturbations must vanish independently on the same free boundary as obtained under static conditions. Hence, the complete boundary conditions at the free boundary becomes:

\[
\begin{align*}
\text{on "zero order" } & \quad \psi_0 = \frac{\partial \psi}{\partial \xi} = \frac{\partial \psi}{\partial \zeta} = 0 \tag{A-35} \\
\text{free boundary } & \quad \psi_1 = \psi_2 = \psi_3 = \psi_4 = 0 \tag{A-36}
\end{align*}
\]

Hence, the location of the free boundary is determined from the solution of \( \psi_0 \) (i.e. the steady-state solution). Once the boundary has been established, the perturbations \( \psi_1, \psi_2, \psi_3 \) and \( \psi_4 \) are to be solved over the same domain such that they vanish along the periphery of the domain.

To determine the coordinates of the free boundary, substitution of eq. (A-35) into eq. (A-21) yields:

-191-
Let the free boundary have the coordinates \((\theta, \xi)\) where \(\theta\) is considered a function of \(\xi\):

\[
\frac{\partial^2 \psi}{\partial \xi^2} + C_x \frac{\partial^3 \psi}{\partial \xi^3} = \frac{6}{H_0} \frac{\partial h}{\partial \theta} \tag{A-37}
\]

The inward directed normal to the boundary is \(n\). Since all derivatives along the boundary are zero, \(\psi\) can be expanded in a Taylor series from a point on the boundary as:

\[
\psi = \frac{1}{2} \frac{\partial^2 \psi}{\partial \xi^2} (\Delta n)^2 \tag{A-38}
\]

From the above figure:

\[
(\Delta n)^2 = \theta^2 \left[1 + \left(\frac{\partial \theta}{\partial \xi}\right)^2\right] = \theta^2 (1 + \tan^2 \beta) \tag{A-39}
\]

By a coordinate transformation, \(\frac{\partial^2 \psi}{\partial \xi^2}\) can be found from eq. \((A-37)\) as:

\[
\frac{\partial^2 \psi}{\partial \xi^2} \left[\cos^2 \beta + C_x \sin^2 \beta\right] = \frac{\partial^3 \psi}{\partial \theta^2} + C_x \frac{\partial^3 \psi}{\partial \xi^3} = \frac{6}{H_0} \frac{\partial h}{\partial \theta} \tag{A-40}
\]

Combining eqs. \((A-38)\) to \((A-40)\) yields:

\[
\frac{3}{H_0} \frac{\partial h}{\partial \theta} \frac{1 + \tan^2 \theta}{\cos^2 \beta + C_x \sin^2 \beta} \theta^2 - \psi = 0 \tag{A-41}
\]
or since \( \frac{1}{\cos^2 \theta} = 1 + \tan^2 \phi = 1 + \left( \frac{d\phi}{d\phi} \right)^2 \):

\[
\frac{3}{H_0} \frac{\partial \phi}{\partial \theta} \left[ 1 + \left( \frac{d\phi}{d\phi} \right)^2 \right] e^2 - \gamma = 0 \tag{A-42}
\]

This equation must be solved together with eq. (A-21) as described later.

Equations (A-21) to (A-25) are solved by finite difference methods. A finite difference grid is introduced with i-axis along the \( f \)-direction and \( j \)-axis along the \( \theta \)-direction:

There are \( n \) increments in the \( f \)-direction such that the length of an increment becomes:

\[
\Delta f = \frac{1}{n} \frac{L}{D} \tag{A-43}
\]

In the \( \theta \)-direction, the arc length of the bearing pad is subdivided into equal increments of length \( \Delta \theta \).

In order to write eqs. (A-21) to (A-25) as finite difference equations, consider the grid point \((i,j)\):
Here, $a_i^{(+)}$, $a_i^{(-)}$ and $\beta_j$ are introduced to take into account the location of the free boundary. Hence, for most points $a_i^{(+)} = a_i^{(-)} = \beta_j = 1$, and only for the $j$-grid lines closest to the boundaries do $a_i^{(+)}$, $a_i^{(-)}$ or $\beta_j$ differ from 1.

Eqs. (A-21) to (A-25) can be written in the general form:

$$\frac{\partial^4 \psi}{\partial \theta^4} + C_0 \frac{\partial^4 \psi}{\partial \delta^4} + \ell_1 \psi = \ell_2 + \ell_3 \psi_0 + \ell_4 \frac{\partial \psi_0}{\partial \delta}$$  \hspace{1cm} (A-44)

where:

$$\ell_1 = - \frac{1}{H_0} \frac{\partial H}{\partial \delta^2}$$  \hspace{1cm} (A-45)

and where $\ell_2$, $\ell_3$ and $\ell_4$ are only functions of $\Theta$ which can be determined by comparing eq. (A-44) with any one of eqs. (A-21) to (A-25). Thus, $\ell_2 = \ell_3 = 0$ for $\psi = \psi_0$, $\psi_3$ and $\psi_4$.

Referring to fig. 56, eq. (A-44) can be written in finite difference form as:

$$\frac{2G_{ij}}{\beta_j (a_i^{(+)} \Delta \delta)^2} \psi_{i,j} + \left[ \ell_{ij} - \frac{2}{a_i^{(+)} \psi_0 \Delta \delta^2} - \frac{2G_{ij}}{(1+\beta_j) \Delta \delta^2} \right] \psi_{ij} + \frac{2G_{ij}}{(1+\beta_j) \Delta \delta^2} \psi_{i+1,j}$$

$$+ \frac{2}{a_i^{(+)}(a_i^{(+)} + a_i^{(-)}) \Delta \delta^2} \psi_{i,j-1} + \frac{2}{a_i^{(+)}(a_i^{(+)} + a_i^{(-)}) \Delta \delta^2} \psi_{i,j+1} = \ell_{ij} \psi_{0,i} + \ell_{ij} \psi_{3,i} + \ell_{ij} \psi_{4,i}$$  \hspace{1cm} (A-46)
Introduce the n-dimensional vector \( \phi_j \):

\[
\phi_j = \begin{bmatrix} \psi_{ij} \\ \psi_{ij} \\ \vdots \\ \psi_{ij} \end{bmatrix}
\]

whereby eq. (A-46) can be written as:

\[
A_j \phi_j + B_j \phi_{j+1} + C_j \phi_{j+1} = F_j
\]  

Here, \( F_j \) is an n-dimensional vector whose elements are given by the right hand side of eq. (A-46), \( A_j \) is an \( nxn \) - matrix, and \( B_j \) and \( C_j \) are \( nxn \) - diagonal matrices. Noting that \( \psi_{ij} = 0 \) and \( \psi_{n+i,j} = \psi_{n-i,j} \) from the given boundary conditions, the elements of the matrix \( A_j \) become (first index gives row number, second index gives column number):

\[
(A_j)_{ii} = \frac{\ell_j}{\beta_j \Delta \theta^2} - \frac{2 \psi_{ij}}{\beta_j \Delta \theta^2} - \frac{2 \psi_{ij}}{\beta_j \Delta \theta^2} \quad 1 \leq i \leq n
\]

\[
(A_j)_{i,i+1} = \frac{2 \psi_{ij}}{\beta_j \Delta \theta^2} \quad 2 \leq i \leq n
\]

\[
(A_j)_{i,i-1} = \frac{2 \psi_{ij}}{\beta_j \Delta \theta^2} \quad i = n
\]

\[
(A_j)_{i,i+1} = \frac{2 \psi_{ij}}{(i+\beta_j \Delta \theta^2) \Delta \theta^2} \quad 1 \leq i \leq n-1
\]

All other elements are zero such that \( A_j \) is a tri-diagonal matrix. The elements of the \( B_j \) -matrix and \( C_j \) -matrix are:

\[
(B_j)_{ii} = \frac{2}{d_i^{(n)}(d_i^{(n)} + d_i^{(-n)}) \Delta \theta^2} \quad 1 \leq i \leq n
\]

\[
(C_j)_{ii} = \frac{2}{d_i^{(n)}(d_i^{(n)} + d_i^{(-n)}) \Delta \theta^2} \quad 1 \leq i \leq n
\]

All other elements are zero.

To solve eq. (A-48), define an \( nxn \)-matrix, \( D_j \), and an n-dimensional vector \( E_j \).
such that:

$$\phi_{j+1} = D_j \phi_j + E_{j+1}.$$  \hspace{1cm} (A-51)

Substitution of this expression into eq. (A-48), yields the following recurrence relationships:

$$D_j = -(A_j + B_j D_{j-1}) C_j = -G_j C_j$$ \hspace{1cm} (A-52)

$$E_j = (A_j + B_j D_j)^{-1} (F_j - B_j E_{j-1}) = G_j (F_j - B_j E_{j-1})$$  \hspace{1cm} (A-53)

In figure 55, the points on the free boundary should be at j=0 and j=m. Since $\psi = 0$ on the boundary, the boundary condition becomes:

$$\phi_0 = \phi_m = 0.$$ \hspace{1cm} (A-54)

Hence, from eq. (A-51):  

$$D_0 = E_0 = 0$$

after which eqs. (A-52) and (A-53) can be used to calculate $D_j$ and $E_j$, step by step, starting with $j = 1$, until $j = m-1$. Then eq. (A-51) is employed, letting $j$ go from $j = m$, where $\phi_m = 0$, to $j = 2$.

In this way the solution for $\psi$ has been obtained. It should be noted, that since $A_j$, $B_j$ and $C_j$ are the same for all the $\psi$-equations (i.e. eq. (A-21) to (A-25)), then the $D_j$-matrices and the $G_j$-matrices are also the same. Thus, $D_j$ and $G_j$ can be stored in the first $\psi$-calculation and used unchanged in all the remaining $\psi$-calculations.

In order to avoid a singularity when $\epsilon_0 = 0$, it is most convenient to solve for $\psi_4$ first from $\psi$. (A-25). With $\psi_0 = -\frac{1}{2} \epsilon_0 \psi_4$ from eq. (A-26), the equation to determine the free boundary, eq. (A-42), becomes:

-106-
\[(6\sin \theta) \left[ \frac{[1+(\frac{d\phi_j}{dz})^2]}{1+C_\phi(\frac{d\phi_j}{dz})^2} \right] \theta^2 - (\psi_j)_{ij} = 0 \]

\[\text{To solve this equation it is necessary to have a first estimate of the } \psi_j \text{ distribution available. For this purposes eqs. (A-52) and (A-53) are used directly such that } j=0 \text{ corresponds to the leading edge of the bearing arc and } j=m-1 \text{ corresponds to the trailing edge of the arc. In the "back-sweep", employing eq. (A-51), any negative value in a } \phi_j \text{-vector is set equal to zero before computing the next } \phi_j \text{-vector. Based on the } \psi_4 \text{-distribution determined in this way, eq. (A-55) is solved step by step, letting } i \text{ go from } i=n \text{ to } i=1. \text{ At } i=n, \frac{d\phi}{dz} = 0 \text{ such that eq. (A-55) is readily solved for the corresponding } \theta \text{-value. At subsequent } i \text{-values, } \frac{d\phi}{dz} \text{ can be expressed in terms of } \theta \text{ and the previous } \theta \text{-values, allowing the complete boundary to be established.}

\text{Once the free boundary is known, } d_i^{(0)}, d_i^{(-1)} \text{ and } \beta_i \text{ can be found for use in the finite difference equations (see fig. .53), a new } \psi_4 \text{-distribution can be computed and the corresponding new boundary can be found. This process is repeated until the relative difference between two subsequent boundaries is smaller than some preassigned error limit. When the final boundary has been established, the solutions for } \psi_1, \psi_2 \text{ and } \psi_3 \text{ are obtained simply by matrix multiplications.}

\text{The film forces have two components acting in the journal center: a radial component } F_r \text{ directed towards the bearing center and a tangential component positive in the direction of rotation, (see fig. .54). These forces are obtained by integration of the film pressures } F:\]

\[F_r \begin{cases} \frac{1}{2} \int_0^{\theta_2} \bar{P} \left\{ -\cos \theta \right\} R d\theta dz \\ \int_0^{\theta_1} \bar{P} \left\{ \sin \theta \right\} R d\theta dz \end{cases}\]

\[(A-56)\]
or by substitution from eqs. (A-3), (A-7) and (A-8):

\[
\xi_f = \frac{F_r}{SW} = \frac{F_r}{\mu NDL(\delta)^2} \left\{ \frac{1}{b} \int_{\theta_0}^{\theta_2} \int_{\phi_0}^{\phi_2} \left\{ \frac{-\cos \theta}{\sin \theta} \right\} \, d\phi \, d\theta \right\} = \frac{\pi}{b} \left\{ \frac{1}{\theta_2 - \theta_0} \right\} \left\{ \frac{1}{\phi_2 - \phi_0} \right\}
\]

(A-57)

Combining eqs. (A-14), (A-17) and (A-18):

\[
P = \frac{\Psi}{H_0} = \frac{\Psi_i}{H_0} + \left[ \frac{\Psi_i}{H_0} - \frac{H_0}{H_0} \right] \varepsilon_i + \left[ \frac{\Psi_i}{H_0} - \frac{H_0}{H_0} \right] \varepsilon_i \phi_i + \frac{\Psi_i}{H_0} \varepsilon_i + \frac{\Psi_i}{H_0} \varepsilon_i \phi_i
\]

(A-58)

At the same time, \( f_r \) and \( f_t \) can be expanded as:

\[
f_r = f_{r_0} + \frac{\partial f_r}{\partial \varepsilon} \varepsilon_i + \frac{\partial f_r}{\varepsilon_\phi} \varepsilon_i \phi_i + \frac{\partial f_r}{\partial \phi} \varepsilon_i + \frac{\partial f_r}{\varepsilon_\phi} \varepsilon_i \phi_i
\]

(A-59)

and similarly for \( f_t \). Thus, substituting eq. (A-58) into eq. (A-57) and comparing with eq. (A-59) results in:

\[
\frac{\partial f_r}{\partial \varepsilon} \left\{ \frac{1}{\theta_2 - \theta_0} \right\} \left\{ \frac{1}{\phi_2 - \phi_0} \right\} \left\{ \frac{-\cos \theta}{\sin \theta} \right\} \, d\phi \, d\theta = \frac{\pi}{b}
\]

(A-60)

\[
\frac{\partial f_t}{\varepsilon_\phi} \left\{ \frac{1}{\theta_2 - \theta_0} \right\} \left\{ \frac{1}{\phi_2 - \phi_0} \right\} \left\{ \frac{-\cos \theta}{\sin \theta} \right\} \, d\phi \, d\theta
\]

(A-61)
\[
\frac{\partial Fr}{\partial \xi} \left\{ \begin{array}{c}
\frac{1}{H_0} \\
0
\end{array} \right\} = \frac{\pi}{L} \int_0^\alpha \frac{1}{H_0} \left\{ \begin{array}{c}
-cos\theta \\
sin\theta
\end{array} \right\} d\theta d\xi
\]

(A-62)

\[
\frac{\partial Fr}{\partial \Phi} \left\{ \begin{array}{c}
\frac{1}{H_0} \\
0
\end{array} \right\} = \frac{\pi}{L} \int_0^\alpha \frac{1}{H_0} \left\{ \begin{array}{c}
-cos\theta \\
sin\theta
\end{array} \right\} d\theta d\xi
\]

(A-63)

\[
f_{r0} \left\{ \begin{array}{c}
\frac{\partial Fr}{\partial \xi} \\
\frac{\partial Fr}{\partial \Phi}
\end{array} \right\} = -\frac{1}{2} \varepsilon_0 \left\{ \begin{array}{c}
\frac{\partial Fr}{\partial \xi} \\
\frac{\partial Fr}{\partial \Phi}
\end{array} \right\}
\]

(A-64)

For convenience in combining several bearing lobes and also to eliminate the changing polar coordinate system (i.e., \(\phi_0\) changes with applied static load), a fixed cartesian in x-y-coordinate system is introduced as shown in fig. 54. The following coordinate transformations are readily deduced:

\[
-F_x = F_r \cos \phi_0 + F_t \sin \phi_0
\]

(A-65)

\[
-F_y = F_r \sin \phi_0 - F_t \cos \phi_0
\]

(A-66)

\[
\frac{\Delta}{\varepsilon} = \varepsilon \cos \phi
\]

(A-67)
\[ y = \frac{1}{c} \sin \phi \] (A-68)

from which:

\[ \frac{d\phi}{dx} = \frac{1}{c} \cos \phi \quad \frac{d\phi}{dy} = \frac{1}{c} \sin \phi \] (A-69)

Thus, from eqs. (A-65) and (A-66):

\[ \frac{C_{Kxx}}{SW} = - \frac{C}{d} \frac{d}{dx} \left( \frac{F_x}{SW} \right) = \left( \frac{dF_x}{dx} \cos \phi_0 + \frac{dF_x}{d\phi} \sin \phi_0 \right) \cos \phi_0 - \left( \frac{dF_x}{d\phi} \cos \phi_0 + \frac{dF_x}{dx} \sin \phi_0 \right) \sin \phi_0 \] (A-70)

\[ \frac{C_{Kxy}}{SW} = - \frac{C}{d} \frac{d}{dy} \left( \frac{F_x}{SW} \right) = \left( \frac{dF_x}{dy} \cos \phi_0 + \frac{dF_x}{d\phi} \sin \phi_0 \right) \sin \phi_0 + \left( \frac{dF_x}{d\phi} \cos \phi_0 + \frac{dF_x}{dy} \sin \phi_0 \right) \cos \phi_0 \] (A-71)

\[ \frac{C_{Kyx}}{SW} = \frac{C}{d} \frac{d}{dx} \left( \frac{F_y}{SW} \right) = \left( \frac{dF_y}{dx} \sin \phi_0 - \frac{dF_y}{d\phi} \cos \phi_0 \right) \cos \phi_0 - \left( \frac{dF_y}{d\phi} \sin \phi_0 - \frac{dF_y}{dx} \cos \phi_0 \right) \sin \phi_0 \] (A-72)

\[ \frac{C_{Kyy}}{SW} = \frac{C}{d} \frac{d}{dy} \left( \frac{F_y}{SW} \right) = \left( \frac{dF_y}{dy} \sin \phi_0 - \frac{dF_y}{d\phi} \cos \phi_0 \right) \sin \phi_0 + \left( \frac{dF_y}{d\phi} \sin \phi_0 - \frac{dF_y}{dy} \cos \phi_0 \right) \cos \phi_0 \] (A-73)

\[ \frac{C_{\omega Bxy}}{SW} = - \frac{C}{\omega d} \frac{d}{dx} \left( \frac{F_x}{SW} \right) = \left( \frac{dF_x}{dx} \cos \phi_0 + \frac{dF_x}{d\phi} \sin \phi_0 \right) \cos \phi_0 - \left( \frac{dF_x}{d\phi} \cos \phi_0 + \frac{dF_x}{dx} \sin \phi_0 \right) \sin \phi_0 \] (A-74)

\[ \frac{C_{\omega Bxy}}{SW} = - \frac{C}{\omega d} \frac{d}{dy} \left( \frac{F_x}{SW} \right) = \left( \frac{dF_x}{dy} \cos \phi_0 + \frac{dF_x}{d\phi} \sin \phi_0 \right) \sin \phi_0 + \left( \frac{dF_x}{d\phi} \cos \phi_0 + \frac{dF_x}{dy} \sin \phi_0 \right) \cos \phi_0 \] (A-75)
The forces acting on the journal can, therefore be written:

\[ F_x = F_{x_0} - K_{xx} \frac{dx}{dt} - B_{xx} \frac{dx}{dt} - K_{xy} y - B_{xy} \frac{dy}{dt} \]  

\[ F_y = F_{y_0} - K_{yx} x - B_{yx} \frac{dx}{dt} - K_{yy} y - B_{yy} \frac{dy}{dt} \]  

The static load on the bearing pad is \( W \) where:

\[ W = \sqrt{F_{r_0}^2 + F_{ro}^2} \]
or, with the definition of \( f_r \) and \( f_t \) from eq. (A-57):

\[
S = \frac{1}{\sqrt{f_{ro}^2 + f_{to}^2}} \tag{A-83}
\]

The friction force \( F_f \) acting on the journal is determined by integrating the shear stress over the wetted area of the journal. The shear stress is:

\[
\tau = \frac{1}{2} R_n C_f \frac{\mu R \omega}{h} + \frac{1}{2} h \frac{\partial P}{R \partial \theta} \tag{A-84}
\]

where \( C_f \) is the turbulent friction coefficient. In laminar flow with \( R_e = 0 \), \( \frac{1}{2} R_n C_f = 1 \). For details, see reference 1.

The total friction force then becomes:

\[
F_f = 2 \int_0^{\pi} \tau R d\theta dz
\]

Introducing \( \tau \) from eq. (A-84) and making the equation dimensionless by means of eqs. (A-3), (A-5), (A-7) and (A-8) results in:

\[
\frac{R}{C} \frac{F_f}{5W} = \pi \int_0^{\pi} \bar{R} C_f \frac{1}{h} d\theta + \frac{1}{2} \frac{\pi}{R \theta} \int_0^{\pi} h \frac{\partial P}{\partial \theta} d\theta d\phi \tag{A-85}
\]

where \( R \cdot F_f \) is the total friction torque. The last integral in this equation can be reduced as follows:

\[
\int_0^{\pi} h \frac{\partial P}{\partial \theta} d\theta = \int_0^{\pi} h dP = \int_0^{\pi} \frac{\partial P}{\partial \theta} d\theta = \int_0^{\pi} P \sin \theta d\theta
\]
whereby eq. (A-85) becomes:

\[
\frac{R}{c} \frac{F_x}{5W} = \pi \left[ R \frac{h}{h} \frac{C_f}{C_f} \right] d\theta + \frac{c^2 f_o}{2} f_o \tag{A-86}
\]

Turning next to the flow, the volume flow per inch in the circumferential direction is:

\[
q_x = \frac{1}{2} R \omega h - G_x h^3 \frac{\partial \bar{P}}{\partial \theta} \tag{A-87}
\]

The volume flow per inch in the axial direction is:

\[
q_z = -G_x \frac{h^3}{12\mu} \frac{\partial \bar{P}}{\partial z} \tag{A-88}
\]

The total flow is found by integration which, in dimensionless form becomes:

\[
\frac{Q_x}{NDLC}_{\text{lead-edge}} = \left[ \frac{\pi}{2} h - \frac{\pi}{12} G_x h^3 \left( \frac{b}{b} \right) \frac{\partial \bar{P}}{\partial \theta} \right]_{0}^{\theta} \tag{A-89}
\]

\[
\frac{Q_x}{NDLC}_{\text{trail-edge}} = \left[ \frac{\pi}{2} h - \frac{\pi}{12} G_x h^3 \left( \frac{b}{b} \right) \frac{\partial \bar{P}}{\partial \theta} \right]_{0}^{\theta} \tag{A-90}
\]

\[
\frac{Q_z}{NDLC}_{\text{sides}} = -\frac{\pi}{12} \frac{1}{b} \left[ G_x h^3 \left( \frac{b^2}{b^2} \right) \frac{\partial \bar{P}}{\partial \theta} \right]_{0}^{\theta} \tag{A-91}
\]

where \(Q_x\) is the total side flow (i.e. for both sides of the bearing). For flow continuity:

\[
(Q_x)_{\text{lead-edge}} = (Q_x)_{\text{trail-edge}} + (Q_x)_{\text{sides}} \tag{A-92}
\]
If there is film rupture at either the leading edge, at the trailing edge or at both edges, the integrals in eqs. (A-87) and/or (A-88) are zero, and the remaining term, \( \frac{\Pi}{2} h \) must be integrated along the boundary.

Having described the calculation of the dynamic coefficients, the load carrying capacity, the friction force and the flow, it remains to determine the various functions in the basic eqs. (A-21) to (A-25). With:

\[
h_0 = 1 + \varepsilon_1 \cos \theta
\]

(A-91)

the derivatives of \( H_0 \) become:

\[
\frac{1}{H_0} \frac{dH_0}{d\theta} = \frac{1}{H_0} \frac{dH_0}{d\theta} \frac{dh_0}{d\theta}
\]

(A-92)

\[
\frac{1}{H_0} \frac{d^2H_0}{d\theta^2} = \frac{1}{H_0} \frac{dH_0}{d\theta} \frac{d^2h_0}{d\theta^2} + \frac{1}{H_0} \frac{dH_0}{d\theta} \left( \frac{dh_0}{d\theta} \right)^2
\]

(A-93)

where (see eq. (A-13)):

\[
H_0 = h_0^{\frac{3}{2}} \sigma_n \]

(A-94)

Expansion of \( H \) yields:

\[
H = H_0 + \frac{dH_0}{dh_0} (h - h_0) = H_0 + \varepsilon_1 \frac{dH_0}{dh_0} \cos \theta + \varepsilon_2 \frac{dH_0}{dh_0} \sin \theta
\]

(A-95)

or, by comparison with eq. (A-18):

\[
\frac{1}{H_0} H_1 = \frac{1}{H_0} \frac{dH_0}{dh_0} \cos \theta
\]

(A-96)

\[
\frac{1}{H_0} H_2 = \frac{1}{H_0} \frac{dH_0}{dh_0} \sin \theta
\]

(A-97)
from which:

$$\frac{1}{H_0} \frac{d^3 H_0}{d \theta^3} = - \cos \theta \frac{1}{H_0} \frac{d H_0}{d h_0} + \left[ \frac{d^2 H_0}{d \theta^2} \cos \theta - 2 \frac{d^2 H_0}{d \theta^2} \sin \theta \right] \frac{1}{H_0} \frac{d^2 H_0}{d h_0^2} + \left( \frac{d h_0}{d \theta} \right)^2 \cos \theta \frac{1}{H_0} \frac{d^3 H_0}{d h_0^3}$$

(A-98)

$$\frac{1}{H_0} \frac{d^3 H_0}{d \theta^3} = - \sin \theta \frac{1}{H_0} \frac{d H_0}{d h_0} + \left[ \frac{d^2 H_0}{d \theta^2} \sin \theta + 2 \frac{d^2 H_0}{d \theta^2} \cos \theta \right] \frac{1}{H_0} \frac{d^2 H_0}{d h_0^2} + \left( \frac{d h_0}{d \theta} \right)^2 \sin \theta \frac{1}{H_0} \frac{d^3 H_0}{d h_0^3}$$

(A-99)

These equations contain the first three derivatives of $H_0$ with respect to $h_0$.

The derivatives are found from Eq. (A-94):

$$\frac{1}{H_0} \frac{d H_0}{d h_0} = \frac{3}{2} \frac{1}{h_0} + \frac{1}{2} \frac{1}{G_x} \frac{d G_x}{d h_0}$$

(A-100)

$$\frac{1}{H_0} \frac{d^2 H_0}{d h_0^2} = \frac{3}{4} \frac{1}{h_0^3} + \frac{1}{2} \frac{1}{h_0} \frac{d G_x}{d h_0} - \frac{1}{4} \left( \frac{1}{G_x} \frac{d G_x}{d h_0} \right)^2 + \frac{1}{2} \frac{1}{G_x} \frac{d^2 G_x}{d h_0^2}$$

(A-101)

$$\frac{1}{H_0} \frac{d^3 H_0}{d h_0^3} = - \frac{3}{8} \frac{1}{h_0^3} + \frac{9}{8} \frac{1}{h_0^2} \frac{d G_x}{d h_0} - \frac{9}{8} \frac{1}{h_0} \left( \frac{1}{G_x} \frac{d G_x}{d h_0} \right)^2 + \frac{3}{8} \left( \frac{1}{G_x} \frac{d G_x}{d h_0} \right)^3$$

$$- \frac{3}{4} \frac{1}{G_x} \frac{d G_x}{d h_0} \frac{1}{G_x} \frac{d^2 G_x}{d h_0^2} + \frac{9}{4} \frac{1}{h_0^2} \frac{d G_x}{d h_0} \frac{1}{G_x} \frac{d^2 G_x}{d h_0^2} + \frac{1}{2} \frac{1}{G_x} \frac{d^3 G_x}{d h_0^3}$$

(A-102)
\[ \frac{1}{G_x} \frac{dG_x}{dh_o} = R_e \left( \frac{1}{G_x} \frac{dG_x}{dR_h} \right) \]

\[ \frac{1}{G_x} \frac{d^2G_x}{dh_o^2} = R_e^2 \left( \frac{1}{G_x} \frac{d^2G_x}{dR_h^2} \right) \quad (A-103) \]

\[ \frac{1}{G_x} \frac{d^3G_x}{dh_o^3} = R_e^3 \left( \frac{1}{G_x} \frac{d^3G_x}{dR_h^3} \right) \]

Since \( G_x \) is a given function of the local Reynolds number \( R_h \), the three derivatives of \( G_x \) are known. For laminar flow, these derivatives are zero.

Similarly, eq. (A-12) can be expanded:

\[ G_1 = G_0 + \frac{dG_o}{dh_o} (h - h_o) = G_0 + \varepsilon \frac{dG_o}{dh_o} \cos \theta + \varepsilon \varphi \frac{dG_o}{dh_o} \sin \theta \quad (A-104) \]

where:

\[ G_0 = G_x \quad (A-105) \]

Comparing eqs. (A-104) and (A-119):

\[ G_1 = \frac{dG_o}{dh_o} \cos \theta = R_e \frac{d}{dR_h} \left( \frac{G_x}{G_x} \right) \cos \theta \quad (A-106) \]

\[ G_2 = \frac{dG_o}{dh_o} \sin \theta = R_e \frac{d}{dR_h} \left( \frac{G_x}{G_x} \right) \sin \theta \]
In this way, all the coefficients in eqs. (A-14) to (A-25) can be evaluated and the equations can be solved as previously discussed.
APPENDIX II: The Static and Dynamic Performance of the Three Lobe Bearing with Turbulent Film

The three lobe bearing is made up of three arcs:

![Three Lobe Bearing](image)

Figure 57: Three Lobe Bearing

The centers of curvature of the arcs do not coincide with the center of the bearing. Instead, the centers lie on a small circle with radius \( r \). In this way the lobes are pre-loaded.

For the purpose of generality, assume that each lobe has its own pre-load radius \( r = r' \). Introduce a cartesian \( x-y \) - coordinate system with origin in the bearing center and the \( x \)-axial in the direction of the applied static load:

![Geometry of Single Lobe](image)

Figure 58: Geometry of Single Lobe
The journal radius is \( R \) and the lobes have the common radius of curvature: \( R + C \), where \( C \) is the radial clearance. The center of the lobe is removed from the bearing center by a distance \( r_p \), and the angle between the \( x \)-axis and the line connecting the two centers is \( \psi_p \). The distance between the journal center and the lobe center is the lobe eccentricity \( e_p \), and \( \psi_p \) is the angle between the \( x \)-axis and the line connecting the two centers. With respect to the bearing center, the journal center has the eccentricity \( e_0 \) and the attitude angle \( \psi_0 \). From fig. 58.

\[
\begin{align*}
\cos \psi_0 &= e_p \cos \psi_p + r_p \cos \psi_p \\
\sin \psi_0 &= e_p \sin \psi_p + r_p \sin \psi_p
\end{align*}
\]  
(B-1)

Introduce the eccentricity ratios:

\[
\begin{align*}
e_0 &= \frac{e_0}{C} \\
e_p &= \frac{e_p}{C}
\end{align*}
\]  
(B-2)

and the preload:

\[
d_p = \frac{r_p}{C}
\]  
(B-3)

whereby eq. (B-1) becomes:

\[
\begin{align*}
\cos \psi_p &= e_0 \cos \psi_0 - d_p \cos \psi_p \\
\sin \psi_p &= e_0 \sin \psi_0 - d_p \sin \psi_p
\end{align*}
\]  
(B-4)

Hence:

\[
\begin{align*}
e_p &= \sqrt{(e_0 \cos \psi_0 - d_p \cos \psi_p)^2 + (e_0 \sin \psi_0 - d_p \sin \psi_p)^2}
\end{align*}
\]  
(B-5)
\[ \phi_p = \tan^{-1} \left[ \frac{E_p \sin \varphi_0 - \delta_p \sin \psi_p}{E_0 \cos \varphi_0 - \delta_p \cos \psi_p} \right] \] (8-6)

Thus, for given coordinates of the journal center with respect to the bearing center, the corresponding lobe eccentricity ratio and attitude angle can be determined. The angles from the line of centers to the beginning and the end of the lobe become:

\[ \Theta_1 = \Theta_{p,in} - \phi_p \] (8-7)

\[ \Theta_2 = \Theta_{p,out} - \phi_p \]

Thereby all the data required for calculating the lobe are known. The calculation is described in Appendix I. Performing calculations for each of the \( n \) lobes making up the bearing, the properties of the composite bearing are obtained by a simple summation over the lobes:

\[ \frac{F_{00}}{\mu NDL (8^1)} = \sum_{i=1}^{n} \left( - \frac{F_{00}}{SW} \right)_i \] (8-8)

\[ \frac{F_{00}}{\mu NDL (8^1)} = \sum_{i=1}^{n} \left( \frac{F_{00}}{SW} \right)_i \] (8-9)

\[ \frac{R}{C} \frac{F_x}{\mu NDL (8^1)} = \sum_{i=1}^{n} \left( \frac{R}{C} \frac{F_x}{SW} \right)_i \] (8-10)

\[ \frac{Q_x}{NDLC} = \sum_{i=1}^{n} \left( \frac{Q_x}{NDLC} \right)_i \] (8-11)
\[
\frac{C_{K_{xx}}}{\mu NDL(\xi)^2} = \sum_{i=1}^{n} \left( \frac{C_{K_{xx}}}{SW} \right)_i
\]
(B-12)

\[
\frac{C_{\omega B_{xx}}}{\mu NDL(\xi)^2} = \sum_{i=1}^{n} \left( \frac{C_{\omega B_{xx}}}{SW} \right)_i
\]
(B-13)

and similarly for the 6 remaining dynamic coefficients.

The total hydrodynamic bearing flow is the side flow \(Q_x\). To this should be added, however, any "surplus" flow from the grooves between the lobes. To illustrate, the circumferential flow from lobe \(p\) into the groove separating lobe \(p\) and lobe \((p+1)\), is \(Q_x\)_{p, trail edge} (see Appendix I, eq. (A-88)). Similarly, the circumferential flow out of the groove is \(Q_x\)_{p+1, lead edge}. If \(Q_x\)_{p, trail edge} is greater than \(Q_x\)_{p+1, lead edge}, the difference should be added to \(Q_x\). Otherwise, there is no surplus flow.

If in eqs. (B-8) to (B-13), the x-direction is considered to be the direction of the applied load \(W\), then for static equilibrium:

\[
F_{y_0} = 0
\]
(B-14)

\[
-F_{x_0} = W
\]
(B-15)

This condition is satisfied by performing the calculations with a fixed value of \(\xi_B \cos \varphi_B\) and vary \(\xi_B \sin \varphi_B\) until eq. (B-14) is fulfilled. Then, combining eqs. (B-15) and (B-8):

\[
S = \frac{\mu NDL}{W} \left( \frac{R}{C} \right)^2 = \left( -\frac{F_{x_0}}{\mu NDL(\xi)^2} \right)^{-1}
\]
(B-16)
where $S$ is the Sommerfeld number for the bearing. Thereafter the bearing friction and the dynamic coefficients can be given in a different dimensionless form than in eqs. (B-10), (B-12) and (B-13):

$$\frac{R F_x}{C W} = S \left( \frac{R F_x}{C \mu NDL(G)} \right)$$

(B-17)

$$\frac{C K_{xx}}{W} = S \left( \frac{C K_{xx}}{\mu NDL(G)} \right)^2$$

(B-18)

$$\frac{C \omega B_{xx}}{W} = S \left( \frac{C \omega B_{xx}}{\mu NDL(G)} \right)^2$$

(B-19)

and similarly for the 6 remaining coefficients. This latter form is to be preferred except when the bearing is unloaded in which case the original expressions are employed.

In order to express the stability properties of the bearing, a symmetrical, rigid rotor with a mass of $2M$ is considered. The rotor is supported in two identical bearings. Assuming the translatory critical speed to be the lowest (corresponds approximately to requiring that the transverse radius of gyration of the rotor is less than half the bearing span which is normally the case), the motions of the two journals in their bearings will be in phase and be the same. Hence, the mass of the rotor can be divided in two equal parts, each of mass $M$, and lumped at the journals. Hence, the equations of motion for a journal become:

$$M \frac{d^2 x}{dt^2} = -K_{xx} x - B_{xx} \frac{dx}{dt} - K_{xy} y - B_{xy} \frac{dy}{dt}$$

(B-20)

$$M \frac{d^2 y}{dt^2} = -K_{yx} x - B_{yx} \frac{dx}{dt} - K_{yy} y - B_{yy} \frac{dy}{dt}$$

-123-
At the threshold of instability, \( x \) and \( y \) are pure harmonic motions with frequency \( \nu \), i.e.:

\[
x = x_c \cos(\nu t) - x_s \sin(\nu t) = (x_c + ix_s)e^{i\nu t}
\]

(B-21)

and similarly for \( y \) where:

\[
i = \sqrt{-1}
\]

(B-22)

Hence, eq. (B-20) can be written:

\[
\begin{pmatrix}
(K_{xx} - M\nu^2 + i\nu B_{xx}) & (K_{xy} + i\nu B_{xy}) \\
(K_{yx} + i\nu B_{yx}) & (K_{yy} - M\nu^2 + i\nu B_{yy})
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} = 0
\]

(B-23)

These two equations only have a non-trivial solution when the determinant is zero. Equating the real part and the imaginary part of the determinant to zero results in:

\[
M\nu^2 = \frac{K_{xx} B_{yy} + K_{yy} B_{xx} - K_{xy} B_{yx} - K_{yx} B_{xy}}{B_{xx} + B_{yy}}
\]

(B-24)

\[
y^2 = \frac{(K_{xx} - M\nu^2)(K_{yy} - M\nu^2) - K_{xy} K_{yx}}{B_{xx} B_{yy} - B_{xy} B_{yx}}
\]

(B-25)

Computing eq. (B-24) first, \( M\nu^2 \) can be substituted into eq. (B-25) whereby the instability frequency \( \nu \) is determined. Thereafter the corresponding journal mass \( M \) is readily obtained.

The results are most conveniently represented in dimensionless form. For this
purpose, eqs. (B-24) and (B-25) are written:

\[
4 \pi^2 \frac{C_M N}{\mu D L (\xi)^2} \left( \frac{\nu}{\omega} \right)^2 = \frac{C_{Kx} C_{CwBxy} + C_{Kxy} C_{CwBxy} - C_{Kxy} C_{CwBxy} - C_{Kx} C_{CwBxy}}{\frac{C_{CwBxy}}{SW} + \frac{C_{CwBxy}}{SW}}
\]

(B-26)

\[
\left( \frac{\nu}{\omega} \right)^2 = \frac{\left[ C_{Kx} \frac{C_M N}{\mu D L (\xi)^2} (\xi) \right] \left[ C_{Kxy} - \frac{C_M N}{\mu D L (\xi)^2} \right] - C_{Kxy} C_{Kxy}}{\frac{C_{CwBxy}}{SW} \frac{C_{CwBxy}}{SW} - \frac{C_{CwBxy}}{SW} \frac{C_{CwBxy}}{SW}}
\]

(B-27)

where the 8 bearing coefficients are in dimensionless form as defined by equations (B-12) and (B-13).

The ratio \( \frac{\nu}{\omega} \) between the instability frequency and the rotational speed is known as the frequency ratio. Under most conditions, \( \frac{\nu}{\omega} \approx \frac{1}{2} \). The instability mass can either be expressed in the form given by eq. (B-26) or in the form:

\[
\frac{C_M \omega^2}{W} = 4 \pi^2 S \frac{C_M N}{\mu D L (\xi)^2}
\]

(B-28)
APPENDIX III: The Stiffness and the Damping of a Hydrostatic Bearing with an Incompressible Lubricant

In the hydrodynamic-hydrostatic ring bearing where the lubricant is a liquid (incompressible), the outer bearing is a purely hydrostatic bearing. This appendix describes the analysis for calculating the flow, the stiffness and the damping of a hydrostatic bearing.

The basic equation governing the flow through the bearing film is Reynolds equation in which there is no contribution from journal rotation:

\[
\frac{\partial}{\partial \theta} \left[ \frac{h^3}{12\mu} \frac{\partial \bar{P}}{\partial \theta} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12\mu} \frac{\partial \bar{P}}{\partial z} \right] = \frac{\partial h}{\partial t}
\]  

(C-1)

Here, \( \bar{P} \) is the film pressure, \( \mu \) is the lubricant viscosity, \( t \) is time, \( R \) is journal radius (outer radius of ring), \( \theta \) is the angular coordinate, \( z \) is the axial coordinate and \( h \) is the film thickness:

\[
\bar{h} = C + e \cos \theta
\]  

(C-2)

where \( C \) is the radial clearance and \( e \) is the eccentricity between the bearing center and the journal center.

To make Reynolds equation dimensionless, set:

\[
P = \frac{\bar{P}}{R^2}
\]  

(C-3)

\[
h = \frac{\bar{h}}{C}
\]  

(C-4)

\[
I = \frac{z}{R}
\]  

(C-5)
Thereby eq. (C-1) becomes:

\[
\frac{d}{dh} (h^3 \frac{dP}{dh}) + \frac{d}{dt} (h^3 \frac{dP}{dh}) = \sigma \frac{dh}{dt}
\]

where:

\[
\sigma = \frac{12 \pi \nu}{l^2} \left( \frac{R}{h} \right)^2
\]

\[P_s\] is the supply pressure of the lubricant to the bearing and \(\nu\) is the frequency of the motion of the journal.

The flow is supplied to the bearing through \(n\) restricted feeder holes. The flow restriction may either be accomplished by an orifice or by a thin tube, the latter method denoted as the laminar restrictor. In the case of an orifice, the mass flow through the feeder hole is given by:

\[
M_B = C_D \pi a^2 \sqrt{\phi (P_s - P_c') \left( \frac{R}{h} \right)^2} \frac{\sqrt{1 + (\frac{h}{l})^2}}{1 + \left( \frac{h}{h_0} \right)^2}
\]

where \(M_B\) is the mass flow in lbs/sec/inch, \(C_D\) is a discharge coefficient, \(a\) is the radius of the orifice, \(\phi\) is the mass density of the lubricant, \(P_c'\) is the pressure downstream of the feeder hole at the inlet to the bearing film, \(h\) is the dimensionless film thickness at the feeder hole and \(\delta\) is the inherent compensation factor:

\[
\delta = \frac{a^2}{dC}
\]
d is the diameter of the feeding hole. Thus eq. (C-9) assumes a pressure drop both through the orifice and through the "curtain" area between the rim of the feeder hole and the journal surface. When $\delta \equiv 0$, the flow restriction takes place only in the orifice whereas $\delta \to \infty$ means that the curtain area dominates as the restricting mechanism.

It is also possible to restrict the flow by simply relying on the feeder hole itself to provide the restriction. The viscous drag in the hole causes a drop in pressure as the flow passes through. The relationship is given by:

\[
M_B = \frac{\pi \rho a^4}{8 \mu l} (p_3 - p_c')
\]

where $l$ is the length of the feeder hole, $a$ is the radius of the feeder hole and the other symbols are defined above.

The two restrictor equations can be written in dimensionless form as:

\[
\frac{3 \mu n M_B}{\pi g C^3 P_s} \lambda_{f_2} = \Lambda_5 m
\]

where:

\[
\Lambda_5 = \begin{cases} \frac{3 \mu C_0 n a^2}{C^3 \sqrt{P_s} \sqrt{1+\delta}} \lambda_{f_2} & \text{Orifice Restrictor} \\ \frac{2}{g} \frac{n a^4}{C^3 l} \lambda_{f_2} & \text{Laminar Restrictor} \end{cases}
\]

-129-
A. is known as the restrictor coefficient and it is the governing parameter for the performance of the hydrostatic bearing. It defines the ratio between the flow resistance of the bearing film and the flow resistance of the feeder holes.

To calculate the bearing performance, Reynolds equation, eq. (C-7), must be solved together with the feeder hole flow equation, eq. (C-11). The present solution will be based on the fact that the journal in a hydrostatic bearing normally operates close to its concentric position. This means that the journal center eccentricity \( \varepsilon \) is small compared to the radial clearance \( C \), or in terms of the eccentricity ratio \( \varepsilon \):

\[
\varepsilon = \frac{\varepsilon}{C} \quad (C-14)
\]

\( \varepsilon \) is much smaller than 1 (in practice the solution is a valid approximation for \( \varepsilon \) values as large as 0.4 to 0.5). Thus, the dimensionless pressure can be written as:

\[
P = P_0 + \varepsilon P_1 + \varepsilon^2 P_2 \quad (C-15)
\]

where:

\[
\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{1}{C} \frac{d\varepsilon}{dt} \quad (C-16)
\]
The dimensionless film thickness is found from eqs. (C-2) and (C-4) as:

\[ h = 1 + \epsilon \cos \theta \]  

(C-17)

Substitute eqs. (C-15) and (C-17) into eq. (C-7) and collect terms according to the perturbation variables:

\[ \frac{\partial P}{\partial \epsilon^2} + \frac{\partial^2 P}{\partial \xi^2} = 0 \]  

(C-18)

\[ \frac{\partial^2 P}{\partial \epsilon^2} + \frac{\partial^2 P}{\partial \xi^2} = 3 \sin \theta \frac{\partial P}{\partial \epsilon} \]  

(C-19)

\[ \frac{\partial^2 P}{\partial \epsilon^2} + \frac{\partial^2 P}{\partial \xi^2} = \sigma \cos \theta \]  

(C-20)

Consider first eq. (C-18) which gives the solution for the pressure in the film when the journal is concentric in the bearing. Under that condition each feeder hole has the same flow and in the analysis it is only necessary to consider an axial strip belonging to one feeder hole:

The strip extends over the length \( L_z \) of the bearing and its width is \( \frac{2\pi R}{n} \). Hence, the ranges of the coordinates are:
or dimensionless:

\[-\frac{L}{2} \leq z \leq \frac{L}{2}\]
\[-\frac{2R}{n} \leq R\theta \leq \frac{2R}{n}\]

where:

\[\xi = \frac{L}{D}\]

and \(D = 2R\) is the diameter of the bearing.

The boundary conditions are that the pressure is ambient, (i.e. it is zero) at the ends of the bearing and there is no flow across the sides of the strip:

\[\text{for } \xi = \pm \xi_2: \quad P_0 = 0\]

\[\text{for } \theta = \pm \frac{\pi}{n}: \quad \frac{\partial P}{\partial \theta} = 0\]

The solution of eq. (C-18) is then the solution of the potential equation for a rectangle with a source in the center. If the source strength in \(C\), the solution becomes:

\[P_0 = \frac{1}{2} C \sum_{k=-\infty}^{\infty} (-1)^k \log \left[ \frac{\cosh((\xi - 2k\xi_2)n) - 1}{\cosh((\xi + 2k\xi_2)n) - \cos(n\theta)} \right]\]
(see reference 2). To determine the source strength, the flow out of the strip which, of course, is the same as the flow $M_{z0}$ through the feeder hole, can be computed:

$$M_{z0} = 4 \int_0^\pi g \frac{C^3}{12\mu} \left(-\frac{\partial P}{\partial z}\right)_{z=\frac{h}{2}} R \, d\theta$$

or:

$$\frac{3\mu M_{z0}}{g C^3 P_s} = -\int_0^\pi \left(\frac{\partial P}{\partial z}\right)_{z=\frac{h}{2}} \, d\theta = \frac{1}{2} \bar{C} \sum_{k=-\infty}^{\infty} (-1)^k \frac{\sinh \left((\bar{F} + 2k)\lambda \right)}{\cosh \left((\bar{F} + 2k)\lambda \right) - \cos \lambda} = \frac{\pi}{2} \bar{C}$$

Hence:

$$\frac{1}{2} \bar{C} = \frac{3\mu M_{z0}}{\pi g C^3 P_s} \quad (C-26)$$

The pressure $P'_o$ at which the flow enters the film, is taken at a point on the rim of the feeder hole with the coordinates $\zeta = 0, \theta = d/D$ where $d$ is the feeder hole diameter. Thus, from eq. (C-24):

$$P'_o = \frac{1}{2} \bar{C} n \lambda F^2 = \frac{3\mu M_{z0}}{\pi g C^3 P_s} \lambda F^2$$

where:

$$\lambda = \frac{1}{\lambda F^2} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[ \frac{\cosh(nF^2(2k+1)) - 1}{\cosh(2kF^2) - \cos(nF^2)} \right] \quad (C-28)$$

**Single Plane Admission**

Since $\frac{d}{D} << 1$, eq. (C-28) can be written with good approximation as:
\[
\lambda = \frac{1}{n_{fs}} \log \left[ \frac{\cosh(n_{fs}) - 1}{1 - \cos(n_{fs})} \right] = 1 + \frac{2}{n_{fs}} \log \left( \frac{D}{n_d} \right)
\]

(C-29)

**Single Plane Admission**

If there are two admission planes, there are two feeder holes per axial strip and fig. 59 is modified to:

![Axial Strip Diagram](image)

**Figure 60: Axial Strip**

With a total of \( n \) feeder holes, there are \( 1/2n \) holes per admission plane. The length between admission planes is \( L_1 \), and the total bearing length is:

\( L = L_1 + L_2 \) such that \( L_2 \) becomes the combined length outside the admission planes.

The ranges of the dimensionless coordinates for the strip are:

\[
-(\xi_1 + \xi_2) \leq \xi \leq (\xi_1 + \xi_2)
\]

(C-30)

\[
-\frac{2\pi}{n} \leq \theta \leq \frac{2\pi}{n}
\]

where:

\[
\xi_1 = \frac{L_1}{D}
\]

(C-31)
The solution for the pressure distribution is found from the method of sources and sinks to be:

\[
P_n = \frac{c}{n} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[ \frac{\cosh \left( \frac{q}{2} \left( \frac{c}{2} + 2k \frac{c}{2} \right) \right) - 1}{\cosh \left( \frac{q}{2} \left( \frac{c}{2} + 2k \frac{c}{2} \right) \right) - \cos (\theta)} \right]
\]  

(C-32)

Defining the pressure, \( P'_{oc} \), downstream of the feeder hole to be the pressure at:

\( (x = x_1, \theta = \frac{d}{D}) \), eq. (C-27) can be employed with:

\[
\lambda = \frac{1}{h_{fz}} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[ \frac{\cosh \left( \frac{q}{2} \left( \frac{c}{2} + 2k \frac{c}{2} \right) \right) - 1}{\cosh \left( \frac{q}{2} \left( \frac{c}{2} + 2k \frac{c}{2} \right) \right) - \cos \left( \frac{ad}{2D} \right)} \right]
\]  

(C-33)

**Double Plane Admission**

In practice, \( \frac{nd}{2D} \ll 1 \) whereby an approximate expression for \( \lambda \) becomes:

\[
\lambda \approx \frac{1}{h_{fz}} \log \left[ \frac{\cosh \left( \frac{q}{2} \left( \frac{c}{2} \right) \right) - 1}{1 - \cos \left( \frac{ad}{2D} \right) \cosh \left( \frac{q}{2} \left( \frac{c}{2} \right) \right) - \cos \left( \frac{ad}{2D} \right)} \right] \approx 1 + \frac{2\pi}{h_{fz}} \log \left( \frac{2D}{nd} \right)
\]  

(C-34)

The perturbation of the film pressure defined by eq. (C-15) causes a similar perturbation in the flow. From eq. (C-13):

\[
m = m_0 + \varepsilon \left[ \left( \frac{dm}{d\theta} \right) P'_{oc} + \left( \frac{dm}{d\theta} \right) \cos \theta \right] + \varepsilon \left( \frac{dm}{d\theta} \right) P'_{oc}
\]  

(C-35)

\[
m_0 = \begin{cases} 
\sqrt{1 - P'_{oc}} & \text{Orifice Restrictor} \\
1 - P'_{oc} & \text{Laminar Restrictor}
\end{cases}
\]  

(C-36)
Define:

\[ \psi = - \frac{A_s}{\delta z} \left( \frac{dm}{dp_<} \right) \]  \hspace{1cm} (C-39)

whereby eq. (C-35) can be written:

\[ m = m_o + \varepsilon \left[ (\frac{dm}{dh_o}) \cos \theta - \frac{F_s}{A_s} \psi P'_{ic} \right] - \varepsilon \frac{F_s}{A_s} \psi P'_{ic} \]  \hspace{1cm} (C-40)

In solving eqs. (C-18) to (C-20), only for the first equation is an exact solution readily obtained as shown by eqs. (C-24) and (C-32). To solve the two remaining equations it is necessary to perturb the source strength. In the present analysis this will be done by an approximate method.

Assume that there are infinitely many feeder holes. Thereby, the feeder holes form a continuous line feed from which the flow is purely axial. The mass flow per inch of circumference becomes:
where \( z'_1 = z - \frac{1}{2} L_1 \). In dimensionless form:

\[
\left[ \frac{dP}{d\bar{Z}} - \frac{dP}{d\bar{Z}_2} \right]_{\bar{Z} = \bar{Z}_1} = \frac{\frac{1}{2} n M_0}{2\pi R}
\]  

(C-41)

\[
\left[ \frac{dP}{d\bar{Z}} - \frac{dP}{d_{\bar{Z}_2}} \right]_{\bar{Z} = \bar{Z}_1} = \frac{3\mu n M_0}{\pi \eta \rho C^2} \frac{1}{R} = \frac{A_{21}}{h} \frac{m}{\bar{R}_2}
\]  

(C-42)

where:

\[
\bar{Z} = \frac{Z}{R} = \bar{Z} - \bar{Z}_1
\]  

(C-43)

Substitute from eqs. (C-15), (C-17) and (C-40) into eq. (C-42) to get:

\[
\left[ \frac{dP}{d\bar{Z}} - \frac{dP}{d_{\bar{Z}_2}} \right]_{\bar{Z} = \bar{Z}_1} = q
\]  

(C-44)

\[
\left[ \frac{dP}{d\bar{Z}_2} - \frac{dR}{d_{\bar{Z}_2}} \right]_{\bar{Z} = \bar{Z}_1} = -3q \frac{1 + \frac{3}{2} \delta^2}{1 + \delta^2} \cos \theta - \psi \frac{1}{\lambda} \rho_{\text{ce}}
\]  

(C-45)
where:

\[ q = \frac{A_4 m_0}{\lambda_2} = \frac{3\mu h M_{B0}}{\pi \eta C^3 p} \]  

(C-47)

and \( \delta \) is defined to be zero for the laminar restrictor. Making use of eqs. (C-47) and (C-37), eq. (C-39) becomes:

\[
\psi = \begin{cases} 
\frac{A_4^2}{2q \lambda_2} & \text{Orifice Restrictor} \\
\frac{A_4}{\lambda_2} & \text{Laminar Restrictor}
\end{cases}
\]  

(C-48)

With the assumption of a line feed instead of discrete feeder holes, there is no variation circumferentially in the pressure when the journal is concentric in the bearing. Therefore:

\[ \frac{\partial P_0}{\partial \theta} = 0 \]  

(C-49)

Hence, the solution for \( P_0 \) can be found directly from eqs. (C-18) and (C-44) as:

\[
P_0 = \begin{cases} 
q (F_2 - F_1) & 0 \leq F_2 \leq F_1 \text{ (or: } F_1 \leq F_2 \leq (F_1 + F_2) ) \\
q F_2 & 0 \leq F_2 \leq F_1
\end{cases}
\]  

(C-50)

-138-
The pressure at the line feed is:

\[ P_{oc} = q \frac{F_2}{\lambda} = \frac{\Lambda_3 m_0}{\lambda} \]  

(C-51)

Comparing eqs. (C-51) and (C-27):

\[ P'_{oc} = A_3 m_o = \lambda P_{oc} \]  

(C-52)

which gives the actual pressure downstream of the feeder holes in terms of the approximate line feed pressure. Inserting eq.(Q6) into eq. (C-52), the feeder hole downstream pressure is found to be:

\[ P'_{oc} = \begin{cases} \frac{1}{2} \Lambda_3 \left[ -\Lambda_3 + \sqrt{\Lambda_3^2 + 4} \right] & \text{Orifice Restrictor} \\ \frac{\Lambda_3}{1 + \Lambda_3} & \text{Laminar Restrictor} \end{cases} \]  

(C-53)

Then:

\[ q = \frac{1}{\lambda F_2} P'_{oc} \]  

(C-54)

Thus, the exact solution for \( P_o \), eq. (C-24) or eq. (C-32) can be replaced by the approximate solution of eq. (C-50). The total flow is the same in the two cases which means that only in the immediate neighborhood of the feeder holes is there any significant difference between the two solutions. This localized effect can be ignored in computing the load carrying capacity of the bearing.
Turning to the solution of the perturbed pressures, it is again assumed that the feeder holes can be represented by a line feed but with a correction introduced for the pressure downstream of the feeder holes to insure a correct flow. Actually, this correction factor must be determined from a perturbation of the source strength in eqs. (C-24) and (C-32). However, in the present analysis it shall be assumed that the same correction factor as derived for $P_0$, namely $\lambda$, also applies to the perturbed pressure. In other words, it is assumed that:

\[
P_{1c}' = \lambda P_{1c} \tag{C-55}
\]

\[
P_{2c}' = \lambda P_{2c} \tag{C-56}
\]

The solutions are taken in the form:

\[
P_1 = H_1 \cos \theta \tag{C-57}
\]

\[
P_2 = \delta H_2 \cos \theta \tag{C-58}
\]

where $H_1$ and $H_2$ are functions of $\xi$ only. Thereby eqs. (C-19), (C-20), (C-45) and (C-46) become:

\[
\frac{d^2 H_1}{d \xi^2} - H_1 = 0 \tag{C-59}
\]

\[
\frac{d^2 H_2}{d \xi^2} - H_2 = 1 \tag{C-60}
\]

\[
\left[ \frac{dH_1}{d \xi} - \frac{dH_2}{d \xi_2} \right]_{\xi=\xi} = -3q \frac{\xi + \delta}{1 + \delta^2} - \gamma H_{1c} \tag{C-61}
\]
\[
\left[ \frac{dH_1}{d\xi} - \frac{dH_2}{d\xi_2} \right]_{\xi=\xi_1} = -\Psi H_{2c} \tag{C-60}
\]

where eqs. (C-59) and (C-60) serve as boundary conditions to eqs. (C-57) and (C-58). The other boundary conditions are:

\[
\text{at } \xi=0: \quad \frac{dH_1}{d\xi} = \frac{dH_2}{d\xi} = 0 \tag{C-61}
\]

\[
\text{at } \xi_1 = \xi_2 (\xi = \xi_1, \xi_2): \quad H_1 = H_2 = 0 \tag{C-62}
\]

The solutions are obtained directly as:

\[
H_1 = \begin{cases} 
    H_{1c} \frac{\cosh \xi}{\cosh \xi_1}, & 0 \leq \xi \leq \xi_1 \\
    H_{1c} \frac{\sinh (\xi_2 - \xi_2)}{\sinh \xi_2}, & 0 \leq \xi_2 \leq \xi_2 
\end{cases} \tag{C-63}
\]

\[
H_2 = \begin{cases} 
    (H_{2c} + 1) \frac{\cosh \xi}{\cosh \xi_1} - 1, & 0 \leq \xi \leq \xi_1 \\
    (H_{2c} + 1) \frac{\sinh (\xi_2 - \xi_2)}{\sinh \xi_2} + \frac{\sinh \xi_2}{\sinh \xi_2} - 1, & 0 \leq \xi_2 \leq \xi_2 
\end{cases} \tag{C-64}
\]
where:

\[ H_{1c} = \frac{-3 \frac{a^3}{\beta^2} \sinh \delta_2}{\cosh \delta_2 + [\beta + \tanh \delta_1] \sinh \delta_2} \]  
(C-65)

\[ H_{2c} + 1 = \frac{\psi \sinh \delta_1 + 1}{\cosh \delta_2 + [\beta + \tanh \delta_1] \sinh \delta_2} \]  
(C-66)

The force \( F \) exerted on the journal by the film is found by integrating the pressure \( P \) in the film:

\[ F = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} \bar{P} \cos \theta R d\theta \, dz \]  
(C-67)

Making use of eqs. (C-3), (C-5), (C-22), (C-31) and (C-15), eq. (C-67) becomes:

\[ F = -2 \pi R^2 P_0 \int_0^{\frac{\pi}{2}} \int_0^{2\pi} (P_0 + \varepsilon P_1 + \varepsilon P_2) \cos \theta \, d\theta \, d\zeta \]  
(C-68)

With \( P_0 \) given by eq. (C-50), it is seen that \( P_0 \), of course, gives no contribution to the force, i.e. \( F_0 = 0 \). Substituting for \( P_1 \) and \( P_2 \) from eq. (C-56) yields:

\[ F = -2 \pi R^2 P_0 \int_0^{\frac{\pi}{2}} \int_0^{2\pi} (\varepsilon H_1 + \varepsilon H_2) \, d\zeta \]  
(C-69)
Since the solution has been linearized, the force can be expressed in terms of a spring coefficient $K$ and a damping coefficient $B$:

\[ F = Ke + B \frac{de}{dt} = CKe + CVB \dot{e} \]  \hspace{1cm} (C-70)

where \( e \) is the displacement (the eccentricity) of the journal center, see eqs. (C-14) and (C-16). Comparing eqs. (C-69) and (C-70):

\[ CK = -2\pi R^2 P^2 \int_0^{\xi_1+\xi_2} \frac{H_1}{\xi} \, d\xi \]  \hspace{1cm} (C-71)

\[ CVB = -2\pi R^2 P^2 \delta \int_0^{\xi_1+\xi_2} \frac{H_2}{\xi} \, d\xi \]  \hspace{1cm} (C-72)

In dimensionless form:

\[ \frac{CK}{(L_1+L_2)DP} = - \frac{\pi/2}{\xi_1+\xi_2} \int_0^{\xi_1+\xi_2} \frac{H_1}{\xi} \, d\xi \]  \hspace{1cm} (C-73)

\[ \frac{CB}{\mu(L_1+L_2)D(\xi)^2} = - \frac{6\pi}{\xi_1+\xi_2} \int_0^{\xi_1+\xi_2} \frac{H_2}{\xi} \, d\xi \]  \hspace{1cm} (C-74)
With substitution from eqs. (C-63) and (C-65), eq. (C-73) yields:

\[
\lambda = \frac{3\pi}{2(\xi_1 + \xi_2)} \frac{q\lambda}{\cosh \xi_2 + [\psi + \tanh \xi_2] \sinh \xi_2} \left[ \cosh \xi_2 - 1 + \sinh \xi_2 \tanh \xi_2 \right]
\]

(C-75)

In this form, the dimensionless stiffness is a function of three parameters only: \(\lambda_s, \xi_1\), and \(\xi_2\). The factor \(q\lambda\) depends on \(\lambda_s\) only (see eqs. (C-53) and (C-54) and the parameter \(\psi\)), therefore, is also a function of \(\lambda_s\) only (see eq. (C-48)).

With substitution from eqs. (C-64) and (C-66), eq. (C-74) yields:

\[
\frac{CB}{\mu(L_1 + L_2)D(R^2)} = 6\pi \left[ 1 - \frac{1}{\xi_1 + \xi_2} \frac{\sinh \xi_2 + \cosh \xi_2 \tanh \xi_2 + \psi(2 \cosh \xi_2 - 2 + \sinh \xi_2 \tanh \xi_2)}{\cosh \xi_2 + (\psi + \tanh \xi_2) \sinh \xi_2} \right]
\]

(C-76)

The dimensionless damping is a function of three parameters only: \(\lambda_s, \xi_1\), and \(\xi_2\).

The total volume flow to the bearing, \(Q\) inch\(^3\)/sec, can be defined in dimensionless form from eq. (C-47) as:

\[
\frac{3\mu Q}{\pi^2 \xi_2^3} \lambda_s \xi_2 = q_0 \xi_2 = \rho_0 \]

(C-77)
which depends only on $\Lambda_2$ and is independent of $F_1$ and $F_2$.

Eqs. (C-75) to (C-77) are used to calculate the design charts for the stiffness, the damping and the flow for a hydrostatic journal bearing with an incompressible lubricant.
APPENDIX IV: The Stability of the Hydrodynamic - Hydrostatic Ring Bearing with an Incompressible Lubricant

In the hydrodynamic-hydrostatic ring bearing there is a ring between the journal and the bearing. The ring is prevented from rotating but is otherwise free to move. The inner diameter of the ring is \( D \) and the outer diameter is \( D_0 \). The radial clearance of the inner film is \( C \) and that of the outer film is \( C_0 \). The bearing length is \( L \).

![Figure 61: Hydrodynamic-Hydrostatic Ring Bearing](image)

As shown in Appendix I, the dynamic forces of the inner film can be expressed in terms of 8 coefficients: \( K_{xx}, K_{xy}, K_{yx}, K_{yy}, B_{xx}, B_{xy}, B_{yx}, \) and \( B_{yy} \). The outer film, on the other hand, requires only two coefficients, a spring coefficient \( K_0 \) and a damping coefficient \( B_0 \), as given in Appendix III.

In calculating the stability properties of this bearing, the rotor is assumed to be rigid and symmetrical with a mass of \( 2M \). As discussed in connection with eq. (B-20) in Appendix II, the rotor mass can be lumped at the journals whereby each journal is assigned a mass of \( M \).

Then the equations of motion become:

\[
M \frac{d^2 x}{dt^2} = -K_{xx}(x-x_0) - B_{xx} \frac{dx}{dt} - K_{xy}(y-y_0) - B_{xy} \frac{dy}{dt} = -K_0 x_0 - B_0 \frac{dx_0}{dt}
\]

\[
M \frac{d^2 y}{dt^2} = -K_{yx}(x-x_0) - B_{yx} \frac{dx}{dt} - K_{yy}(y-y_0) - B_{yy} \frac{dy}{dt} = -K_0 y_0 - B_0 \frac{dy_0}{dt} \quad \text{(D-1)}
\]
where \( x \) and \( y \) are the amplitudes of the journal center, and \( x_0 \) and \( y_0 \) are the amplitudes of the center of the ring. In these equations, the mass of the ring itself has been ignored because it is small compared to the journal mass. Eq. (D-1) is made dimensionless by setting:

\[
\bar{x} = \frac{x}{c} \quad \bar{y} = \frac{y}{c} \quad \text{(D-2)}
\]

\[
\bar{x}_0 = \frac{x_0}{c} \quad \bar{y}_0 = \frac{y_0}{c} \quad \text{(D-3)}
\]

\[
\tau = \omega t \quad \text{(D-4)}
\]

\[
\bar{K}_0 = \frac{Ck_0}{\mu NDL(\sigma)^2} = \left( \frac{c}{c_0} \right) \frac{P s L D_0}{SW} \frac{c_k k_0}{P_s L D_0} \quad \text{(D-5)}
\]

\[
\bar{B}_0 = \frac{C_s B_0}{\mu NDL(\sigma)^2} = 2\pi \left( \frac{c}{c_0} \right)^3 \frac{D_s^3}{D^4} \frac{c_s B_0}{\mu L D_s (\sigma_0)^2} \quad \text{(D-6)}
\]

\[
\bar{M} = \frac{C_m^2 M}{SW} = 4\pi^2 \frac{C N M}{\mu D L (\sigma)^2} \quad \text{(D-7)}
\]

\[
\bar{K}_{xx} = \frac{C_{km}}{SW} = \frac{C_{km}}{\mu D L (\sigma)^2} \quad \text{(D-8)}
\]
\[ \overline{B}_{xy} = \frac{CwB_{xx}}{SW} = \frac{CwB_{xx}}{\mu NDL(\xi)} \]  

(D-9)

and similarly for \( \overline{B}_{xy}, \overline{B}_{yx}, \overline{B}_{yy}, \overline{B}_{yx} \) and \( \overline{B}_{yy} \). Here, \( \omega \) is the angular speed of the journal and \( S \) is the Sommerfeld number. Thereby eq. (D-1) becomes:

\[ \overline{M}\ddot{x} = - \overline{K}_{xx}(\dot{x} - \dot{x}_0) - \overline{B}_{xx}(\ddot{x} - \ddot{x}_0) - \overline{K}_{yy}(\dot{y} - \dot{y}_0) - \overline{B}_{yy}(\ddot{y} - \ddot{y}_0) = -\overline{K}_0 \dot{x}_0 - \overline{B}_0 \ddot{x}_0 \]  

(D-10)

\[ \overline{M}\ddot{y} = - \overline{K}_{yx}(\dot{x} - \dot{x}_0) - \overline{B}_{yx}(\ddot{x} - \ddot{x}_0) - \overline{K}_{yy}(\dot{y} - \dot{y}_0) - \overline{B}_{yy}(\ddot{y} - \ddot{y}_0) = -\overline{K}_0 \dot{y}_0 - \overline{B}_0 \ddot{y}_0 \]

where "dot" refers to \( \frac{d}{dt} \).

At the threshold of instability, the motion is purely harmonic with frequency \( \nu \) such that:

\[ \overline{x} = \overline{x}_0 \cos(\nu t) - \overline{x}_0 \sin(\nu t) = \mathcal{O}\left\{ (\overline{x}_0 + i\overline{x}_0) e^{i\nu t} \right\} \]  

(D-11)

and similarly for \( \overline{y}, \overline{y}_0 \) and \( \overline{y}_0 \), where:

\[ \gamma = \frac{\nu}{\omega} \]  

(D-12)

Then eq. (D-10) can be written as:

\[ \overline{M}\gamma^2 \overline{x} = (\overline{K}_{xx} + i\gamma \overline{B}_{xx})(\overline{x} - \overline{x}_0) + (\overline{K}_{yy} + i\gamma \overline{B}_{yy})(\overline{y} - \overline{y}_0) = (\overline{K}_0 + i\gamma \overline{B}_0)\overline{x}_0 \]  

(D-13)

\[ \overline{M}\gamma^2 \overline{y} = (\overline{K}_{yx} + i\gamma \overline{B}_{yx})(\overline{x} - \overline{x}_0) + (\overline{K}_{yy} + i\gamma \overline{B}_{yy})(\overline{y} - \overline{y}_0) = (\overline{K}_0 + i\gamma \overline{B}_0)\overline{y}_0 \]
Solve for $x_0$ and $y_0$:

$$x_0 = \frac{M_y^2}{K_0 + iyB_0} \bar{x}$$  \hspace{1cm} (D-14)$$

$$y_0 = \frac{M_y^2}{K_0 + iyB_0} \bar{y}$$

and substitute into eq. (D-13) to get:

$$\left[ \begin{array}{cc} (\bar{K}_{xx} - \frac{(K_0 + iyB_0)M_y^2}{K_0 - M_y^2 + iyB_0} + iy\bar{B}_{xx}) & (\bar{K}_{xy} + iy\bar{B}_{xy}) \\ (\bar{K}_{yx} + iy\bar{B}_{yx}) & (\bar{K}_{yy} - \frac{(K_0 + iyB_0)M_y^2}{K_0 - M_y^2 + iyB_0} + iy\bar{B}_{yy}) \end{array} \right] \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = 0$$  \hspace{1cm} (D-15)$$

At the threshold of instability, the determinant of the coefficient matrix vanishes. For convenience, introduce the abbreviations:

$$E = \frac{(K_0 + iyB_0)M_y^2}{K_0 - M_y^2 + iyB_0}$$  \hspace{1cm} (D-16)$$

$$Z_{xx} = \bar{K}_{xx} + iy\bar{B}_{xx} \hspace{1cm} \text{(similarly for } Z_{xy}, Z_{yx}, Z_{yy})$$  \hspace{1cm} (D-17)$$

With this notation, the determinant of eq. (D-15) can be equated to zero to give:

$$E^2 - (Z_{xx} + Z_{yy})E + (Z_{xx}Z_{yy} - Z_{xy}Z_{yx}) = 0$$  \hspace{1cm} (D-18)$$
with the solution:

\[ E = \frac{1}{2} \left( Z_{xx} + Z_{yy} \right) \pm \sqrt{\frac{1}{4} \left( Z_{xx} - Z_{yy} \right)^2 + Z_{xy} Z_{yx} } \]  \hspace{1cm} (D-19)

where only the root with the minus sign in front of the square root is of interest when studying stability. Set:

\[ G = \text{Re} \left\{ \frac{1}{2} \left( Z_{xx} - Z_{yy} \right)^2 + Z_{xy} Z_{yx} \right\} = \frac{1}{2} (R_{xx} - R_{yy})^2 + R_{xy} R_{yx} - \frac{1}{2} \gamma^2 (B_{xx} - B_{yy}) - \frac{1}{2} \gamma^2 B_{xy} B_{yx} \]  \hspace{1cm} (D-20)

\[ H = \frac{1}{2} \text{Im} \left\{ \frac{1}{2} \left( Z_{xx} - Z_{yy} \right)^2 + Z_{xy} Z_{yx} \right\} = \gamma \left[ \frac{1}{4} (R_{xx} - R_{yy}) (B_{xx} - B_{yy}) + \frac{1}{2} (R_{xy} B_{yx} + R_{yx} B_{xy}) \right] \]  \hspace{1cm} (D-21)

whereby eq. (D-19) becomes:

\[ E = \frac{1}{2} \left( Z_{xx} + Z_{yy} \right) - \left\{ \sqrt{\frac{1}{4} \left( G + \sqrt{G^2 + 4H^2} \right)} + i \sqrt{\frac{1}{4} \left( -G + \sqrt{G^2 + 4H^2} \right)} \right\} \]  \hspace{1cm} (D-22)

Set:

\[ E = K_\theta + i \gamma \bar{B}_\theta \]  \hspace{1cm} (D-23)

where:

\[ K_\theta = \frac{1}{2} \left( R_{xx} + R_{yy} \right) - \sqrt{\frac{1}{4} \left( G + \sqrt{G^2 + 4H^2} \right)} \]  \hspace{1cm} (D-24)

\[ \gamma \bar{B}_\theta = \frac{1}{2} \gamma \left( B_{xx} + B_{yy} \right) + \sqrt{\frac{1}{4} \left( -G + \sqrt{G^2 + 4H^2} \right)} \]  \hspace{1cm} (D-25)
Combining eqs. (D-16) and (D-23) yields:

$$\bar{K}_0 = \bar{M}_J^2 \left[ 1 + \bar{M}_J^2 \frac{\bar{K}_0 - \bar{M}_J^2}{(\bar{K}_0 - \bar{M}_J^2)^2 + (\gamma \bar{B}_0)^2} \right]$$

(D-26)

$$\gamma \bar{B}_0 = - (\bar{M}_J)^2 \frac{\gamma \bar{B}_0}{(\bar{K}_0 - \bar{M}_J^2)^2 + (\gamma \bar{B}_0)^2}$$

(D-27)

Eliminate $\bar{M}_J^2 / [(\bar{K}_0 - \bar{M}_J^2)^2 + (\gamma \bar{B}_0)^2]$ between the two equations to get:

$$\bar{M}_J^2 = \frac{\bar{K}_0 \gamma \bar{B}_0 + \bar{K}_0 \gamma \bar{B}_0}{\gamma \bar{B}_0 + \gamma \bar{B}_0}$$

(D-28)

Substitute for $\bar{M}_J^2$ into eq. (D-27) to obtain an equation which contains $\gamma$ as the only unknown:

$$\gamma \bar{B}_0 \left[ \bar{K}_0^2 + (\gamma \bar{B}_0)^2 \right] + \gamma \bar{B}_0 \left[ \bar{K}_0^2 + (\gamma \bar{B}_0)^2 \right] = 0$$

(D-29)

Since $\bar{K}_0$ and $\gamma \bar{B}_0$, as seen from eqs. (D-24) and (D-25), are rather complicated functions of $\gamma$, eq. (D-29) is a higher order polynomial in $\gamma$ which cannot be solved in closed form. Instead, the solution is found numerically. Once the roots for $\gamma$ have been obtained, the corresponding dimensionless journal mass $\bar{M}$ is found from eq. (D-28).

If $\bar{M}$ is plotted as a function of $\bar{B}_0$ for a fixed value of $\bar{K}_0$, it is found that under certain circumstances the rotor is inherently stable, i.e. $\bar{M} \to \infty$. When $\bar{M} \to \infty$, $\gamma \to 0$ such that eqs. (D-24) and (D-25) yield:

$$\bar{K}_0 \to \frac{1}{2} (\bar{K}_{xx} + \bar{K}_{yy})$$

(D-30)

$$\gamma \bar{B}_0 \to - \sqrt{\frac{1}{4} (\bar{K}_{xx} - \bar{K}_{yy})^2 + \bar{R}_{xy} K_{yy}}$$
where \( \frac{1}{2}(K_{x^2} - K_{y^2}) + K_{y^2} \) which holds true except at large eccentricity ratios. Solve eq. (D-29) for \( \frac{\gamma \bar{B}_v}{\gamma \bar{B}_o} \):

\[
\frac{\gamma \bar{B}_o}{\gamma \bar{B}_v} = - \frac{1}{2} \left( \frac{K_{x^2} + \gamma \bar{B}_o^2}{\gamma \bar{B}_v^2} \right) + \sqrt{\frac{1}{4} \left( \frac{K_{x^2} + \gamma \bar{B}_o^2}{\gamma \bar{B}_v^2} \right)^2 - \frac{K_{y^2}}{(\gamma \bar{B}_v)^2}}
\]  

(D-31)

For a given value of \( K_o \) and with \( \bar{K}_o \) and \( \gamma \bar{B}_o \) defined by eq. (D-30), this equation allows calculation \( \frac{\gamma \bar{B}_o}{\gamma \bar{B}_v} \) for \( \gamma = 0 \).

Noting that:

\[ \gamma^2 = \left( \frac{\gamma \bar{B}_o}{\gamma \bar{B}_v} \right)^2 \left( \frac{\gamma \bar{B}_o}{\bar{B}_o} \right)^2 \]

eq. (D-28) yields:

\[
\bar{M} = \frac{1}{\gamma^2} \left( \frac{K_{x^2} \gamma \bar{B}_v}{\gamma \bar{B}_o} + K_o \right) = \frac{K_o + \gamma \bar{B}_o \gamma \bar{B}_o}{(\bar{B}_o)^2 \left( \frac{1}{\gamma \bar{B}_o} \right)^2 \left( \frac{K_{x^2} \gamma \bar{B}_v}{\gamma \bar{B}_o} + K_o \right) \bar{B}_o^2}
\]

(D-32)

Corresponding to the two roots for \( \frac{\gamma \bar{B}_o}{\gamma \bar{B}_v} \) from eq. (D-31), eq. (D-32) yields two possible values for \( \bar{M} \). The rotor is stable between these two values.
BLANK PAGE
In the hybrid-hydrostatic ring bearing which is gas lubricated, the outer bearing is purely hydrostatic whereas the inner bearing, supplied with pressurized gas from the outer bearing, is a hybrid bearing. This appendix describes the analysis for calculating the load carrying capacity, the flow, the stiffness and the damping of the hybrid journal bearing but the analysis applies equally well to the hydrostatic bearing.

For a compressible lubricant under isothermal conditions, Reynolds equation becomes:

\[
\frac{\partial}{R \partial \theta} \left[ \frac{h^3}{12 \mu} \frac{\partial P}{R \partial \theta} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12 \mu} \frac{\partial P}{\partial z} \right] = \frac{1}{2} R \omega \frac{\partial (P \gamma)}{\partial \theta} + \frac{\partial (P h)}{\partial t} \tag{E-1}
\]

where the symbols are defined in connection with eq. (A-1), Appendix I. The equation is made dimensionless by setting:

\[
\frac{r}{R} = \frac{z}{R} \tag{E-2}
\]

\[
\tau = \nu t \tag{E-3}
\]

\[
h = \frac{h}{C} = 1 + \varepsilon \cos \theta \tag{E-4}
\]

\[
\varepsilon = \frac{\varepsilon}{C} \tag{E-5}
\]

\[
P = \frac{P}{P_a} \tag{E-6}
\]

whereby eq. (E-1) can be written in dimensionless form as:
\[ \frac{A}{\lambda} \left[ h \frac{d(Ph)^2}{dx} - 2 \frac{d}{dx} (Ph) + \frac{d}{dx} \left[ h \frac{d(Ph)}{dx} \right] \right] = 2A \frac{d(Ph)}{dx} + 4 \gamma \Lambda \frac{d(Ph)}{dt} \]  

(E-7)

where:

Compressibility Number: \[ \Lambda = \frac{6 \mu \omega}{P_o} \left( \frac{R}{C} \right)^2 \]  

(E-8)

Frequency Ratio: \[ \gamma = \frac{\nu}{\omega} \]  

(E-9)

In the purely hydrostatic bearing, \( \omega = 0 \) which means that \( \Lambda = 0 \). In that case:

Squeeze Number: \[ \sigma = 2\gamma \Lambda = \frac{12 \mu \nu}{P_o} \left( \frac{R}{C} \right)^2 \]  

(E-10)

Eq. (E-7) is non-linear. It shall be solved under the assumption that the eccentricity ratio \( \varepsilon \) is small whereby the pressure variable can be expanded in a perturbation:

\[ Ph = P_o + \varepsilon \]  

(E-11)

In making use of eq. (E-11) it shall be assumed that the pressure, \( P_o \), which is the film pressure when the journal is concentric in the bearing, does not depend
on $\Theta$ \((i.e., \frac{dp_0}{d\Theta} = 0)\). This assumption implies that the flow to the bearing is considered to be supplied by a line feed rather than discrete feeding holes. A correction factor will be added later to adjust the flow.

With this assumption, eqs. (E-11) and (E-4) can be substituted into eq. (E-7) whereby two equations are obtained. The first equation is simply:

$$\frac{d^2P_0^2}{d\gamma^2} = 0 \quad (E-12)$$

In deriving the second equation it should be noted that eq. (E-4) is based on $\Theta$ being measured from the line connecting the bearing center with the journal center. The angle between this line and the static load line is the attitude angle $\Phi$. Therefore:

$$\frac{\partial (P_0)}{\partial \tau} = - \frac{\partial (P_0)}{\partial \Theta} \frac{d\Phi}{d\tau} + \frac{\partial (P_0)}{\partial \tau} \left[ \Theta \text{ constant} \right] \quad (E-13)$$

Thereby the second perturbation equation becomes:

$$\frac{d^2(P_0P_1)}{d\Theta^2} + \frac{d^2(P_0P_1)}{d\gamma^2} - \frac{A}{P_0} \left( 1 - 2^{\gamma} \phi \right) \frac{d(P_0P_1)}{d\Theta} - 2^{\gamma} \frac{A}{P_0} \frac{d(P_0P_1)}{d\tau} = -\varepsilon \omega \Theta P_0^2 \quad (E-14)$$

Under static conditions, the journal center operates at an eccentricity ratio $E_o$ and an attitude angle $\Phi_0$. Letting the journal center perform a harmonic small amplitude motion $f(t) = E_1 e^{i\nu t}, \Phi(t) = \Phi_1 e^{i\nu t}$ with frequency $\nu$ around this equilibrium position, the journal center coordinates become:

$$E = E_0 + E_1 e^{i\nu t} \quad (E-15)$$

$$\Phi = \Phi_0 + \Phi_1 e^{i\nu t}$$
The corresponding changes in the pressure variable can be expressed by:

\[ \epsilon P_0 P_1 = \epsilon_0 q_0 + \epsilon_1 e^{i \gamma} q_1 + \epsilon_0 \rho_1 e^{i \gamma} q_2 \]  

(E-16)

Set:

\[ q_0 = \Re \left\{ G_0(\gamma) e^{i \theta} \right\} \]  

(E-17)

\[ q_1 = \Re \left\{ e^{i \gamma} \Re \left\{ G_1(\gamma) e^{i \theta} \right\} \right\} \]  

(E-18)

\[ q_2 = \Re \left\{ e^{i \gamma} \Re \left\{ G_2(\gamma) e^{i \theta} \right\} \right\} \]  

(E-19)

where: \( i = \sqrt{-1} \) and \( j = \sqrt{-1} \)  

With these definitions, eq. (E-14) gives rise to 3 equations:

\[ \frac{d^2 G_0}{d \gamma^2} - (1 + \frac{A}{P_0}) G_0 = -P_0^2 \]  

(E-20)

\[ \frac{d^2 G_1}{d \gamma^2} - (1 + \frac{A}{P_0} + i 2 \gamma \frac{A}{P_0}) G_1 = -P_0^2 \]  

(E-21)

\[ \frac{d^2 G_2}{d \gamma^2} - (1 + \frac{A}{P_0} + i 2 \gamma \frac{A}{P_0}) G_2 = -i \gamma 2 \gamma \frac{A}{P_0} G_0 \]  

(E-22)

The length of the bearing is \( L \). There are two admission planes separated by the distance \( L_1 \), and the combined length outside the admission planes is \( L_2 = L - L_1 \). Since there is symmetry with respect to the centerplane of the bearing, the dimensionless axial coordinate ranges from \( \gamma = 0 \) to \( \gamma = (\gamma_1 + \gamma_2) \).
where:

\[ \xi_1 = \frac{L_1}{D} \]  

\[ \xi_2 = \frac{L_2}{D} \]  

It is convenient to introduce an additional axial coordinate:

\[ \xi_2 = \xi - \xi_1 \]

so that \( 0 \leq \xi_2 \leq \xi_2 \).

At the end of the bearing the pressure is ambient and equal to \( P_e \), i.e.

\[ P = \frac{P}{P_e} = 1 \text{ at } \xi_2 = \xi_2 \]

which means:

\[ \begin{cases} P_0 = G_0 = G_1 = 1 \\ G_2 = 0 \end{cases} \]  

At the centerplane of the bearing, \( \frac{\partial P}{\partial \xi} = 0 \) whereby:

\[ \begin{align*} 
\xi = 0 : \quad \frac{\partial P}{\partial \xi} &= \frac{dG_0}{d\xi} = \frac{dG_1}{d\xi} = \frac{dG_2}{d\xi} = 0 
\end{align*} \]  

There are \( \frac{1}{2}n \) feeder holes per admission plane. The mass flow through a hole is \( M_B \). Thus, when the admission plane is represented by a line feed, the flow per inch is \( nM_B / 4 \pi R \). Hence, the boundary condition at the admission plane becomes:

\[ \begin{align*} 
\xi = \xi_1, \xi = 0 : \quad \frac{b \mu RT}{4 \pi^2 c^2} nM_B &= h \left[ \left( \frac{d(P_h)}{d\xi} \right)_{\xi = \xi_1} - \left( \frac{d(P_h)}{d\xi} \right)_{\xi = 0} \right] 
\end{align*} \]  

-159-
where $\mathcal{R}$ is the gas constant and $T$ is the total temperature. Expand the right hand side by means of eqs. (E-11), (E-16) and (E-4):

$$
\frac{\rho m^* \mathcal{R} T}{n c^2} n M_0 = (1 + \epsilon \cos \theta) \left( \frac{d}{dt} - \frac{\partial}{\partial \xi} \right) \left[ P_0^2 + 2\xi \rho \rho_p \right] = \left( \frac{d}{dt} - \frac{\partial}{\partial \xi} \right) \left[ P_0^2 + 2(\epsilon \rho_p + \epsilon \rho \cos \theta) \right]
$$

$$
= \left( \frac{d}{dt} - \frac{\partial}{\partial \xi} \right) \left[ P_0^2 + 2\epsilon \rho (q_0 + \frac{1}{2} P_0^2 \cos \theta) + 2\epsilon \rho \epsilon^T (q_1 + \frac{1}{2} P_0^2 \cos \theta) + 2\epsilon \rho \epsilon^T q_2 \right] \quad (E-27)
$$

where subscript $c$ refers to the condition at the admission plane.

Next, the left hand side of eq. (E-27) must be expanded. Let the volume of a feeder hole be represented by $V_c$, and let the flow into the feeder hole be $M_c$. The pressure in the feeder hole is $P_c$. Then a flow balance for the feeder hole volume can be set up:

$$
M_c = M_c - \frac{\partial (V_c \rho^*)}{\partial t} \quad (E-28)
$$

where $\rho^* = \frac{P_c}{\mathcal{R} T}$ is the mass density of the gas in the feeder hole. To make eq. (E-28) dimensionless, set:

$$
\frac{M_c \sqrt{\mathcal{R} T}}{\pi a^2 P_s} = \frac{m}{\sqrt{1 + \left( \frac{r}{h} \right)^2}} \quad (E-29)
$$

where $\delta$ is the inherent compensation factor:

$$
\delta = \frac{r^2}{d} \quad (E-30)
$$

Here, $P_s$ is the supply pressure, $a$ is the radius of the orifice and $d$ is the diameter of the feeding hole. The dimensionless orifice flow $m$ is given by the standard equation:

$$
-160-
$$
where $C_D$ is the vena contracta coefficient. Actually, $C_D$ is a function of the pressure ratio $P_c'/P_s$.

The dimensionless form of eq. (E-28) becomes:

$$m = \left\{ \begin{array}{l}
C_b \sqrt{2k} \left( \frac{2}{k+1} \right)^{1/2} \frac{P_c'}{P_s} \leq \left( \frac{2}{k+1} \right)^{1/2} \\
C_b \sqrt{2k} \left( \frac{P_c'}{P_s} \right)^{3/2} \sqrt{1 - \left( \frac{P_c'}{P_s} \right)^{k+1}} \left( \frac{2}{k+1} \right)^{1/2} \leq \frac{P_c'}{P_s} \leq 1
\end{array} \right. \quad (E-31)$$

where $C_D$ is the vena contracta coefficient. Actually, $C_D$ is a function of the pressure ratio $P_c'/P_s$.

The dimensionless form of eq. (E-28) becomes:

$$\frac{6\mu R T}{\pi P_s^2 C^2} n M_0 = \Lambda_s \sqrt{\frac{1+\delta^2}{1+\left(\frac{h}{R}\right)^2}} m - 4\gamma \Lambda \frac{n V_c}{\pi D L c} \left( \frac{P_c'}{P_s} \right) \frac{dP_c'}{d\tau} \quad (E-32)$$

where:

Restrictor Coefficient: $\Lambda_s = \frac{6\mu n a^2 \sqrt{R T}}{P_s C^2 \sqrt{1+\delta^2}} \quad (E-33)$

Pressure Ratio: $V = \frac{P_s}{P_r} \quad (E-34)$

$P_c' = \frac{P_c'}{P_s} \quad (E-35)$

The restrictor coefficient, $\Lambda_s$, is, together with $\Lambda$, the governing performance parameter. It gives the ratio between the flow resistance of the bearing film and the flow resistance of the feeder hole restrictor.
From eqs. (E-11) and (E-16):

\[ P = \left(\frac{P_h}{h}\right) = (1 - \varepsilon \cos \theta) \left( P_{oc} + \varepsilon P_{ic}\right) = P_{oc} + \frac{1}{P_{oc}} \left[ \left( \varepsilon P_h P_{ic}\right) - \varepsilon P_{oc}^2 \cos \theta \right] \]

\[ = P_{oc} + \frac{1}{P_{oc}} \left[ \varepsilon_0 \left( g_{oc} - P_{oc}^2 \cos \theta \right) + \varepsilon_1 e^{i\nu} (g_{ic} - P_{oc}^2 \cos \theta) + \varepsilon_2 Q_e e^{iG_{tc}} \right] \]  

(E-36)

On this basis and making use of eq. (E-13), it is found that:

\[ \frac{dP'}{d\tau} = \frac{i}{P_{oc}} \left[ \varepsilon_1 e^{i\nu} \left( g_{ic}^* - P_{oc}^2 \cos \theta \right) + \varepsilon_2 Q_e e^{i\nu} \left( g_{tc}^* - \left( \frac{\partial \theta}{\partial \xi}\right)_c - P_{oc}^2 \sin \theta \right) \right] \]  

(E-37)

Furthermore, \( m \) must also be expanded:

\[ m = m_o + \left( \frac{\partial m}{\partial \left( \frac{P_{oc}'}{P_{oc}} \right)} \right) \frac{1}{V} \left( P_{oc}' - P_{oc} \right) = m_o + \left( \frac{\partial m}{\partial \left( \frac{P_{oc}'}{P_{oc}} \right)} \right) \frac{1}{V P_{oc}^2} \left[ \varepsilon_0 \left( g_{oc}^* - P_{oc}^2 \cos \theta \right) + \varepsilon_1 e^{i\nu} g_{ic}^* - P_{oc}^2 \cos \theta + \varepsilon_2 Q_e e^{i\nu} g_{tc}^* \right] \]  

(E-38)

where:

\[ m_o = m \left( \frac{\partial}{\partial V} \right) \]  

(E-39)

Introduce the abbreviations:

\[ q = A_s V^2 m_o \]  

(E-40)
Eqs. (E-37) to (E-42) are substituted into eq. (E-32) which is then compared to eq. (E-27). Collecting terms according to \( \varepsilon_0, \varepsilon_1 \), and \( \varphi \), four equations are obtained:

\[
\psi'_0 = -\frac{A_n V}{2 P_{oc}} \left( \frac{dm}{d\psi'_0} \right) 
\]

\[
\psi'_1 = \frac{r V_L}{\pi D L C} \frac{F_1 + F_2}{P_{oc}} 
\]

\[
\left( \frac{dP'_c}{d\zeta'_c} \right) = -q + \left( \frac{dP'_0}{d\zeta'_c} \right) 
\]

\[
\left( \frac{\partial q'_0}{\partial \zeta'_c} \right) = \frac{q}{2(1+\delta)} \cos \theta + \psi'_0 (q'_0 - P_{oc} \cos \theta) + \left( \frac{\partial q'_0}{\partial \zeta'_c} \right) 
\]

\[
\left( \frac{\partial q'_1}{\partial \zeta'_c} \right) = \frac{q}{2(1+\delta)} \cos \theta + (\psi'_0 + i \lambda \psi'_1) (q'_0 - P_{oc} \cos \theta) + \left( \frac{\partial q'_1}{\partial \zeta'_c} \right) 
\]

\[
\left( \frac{\partial q'_1}{\partial \zeta'_c} \right) = (\psi'_0 + i \lambda \psi'_1) q'_0 - i \lambda \psi'_1 \left( \frac{\partial q'_1}{\partial \psi'_0} + P_{oc} \sin \theta \right) + \left( \frac{\partial q'_1}{\partial \zeta'_c} \right) 
\]

In these equations, the primed quantities refer to the actual conditions at the feeder holes whereas the unprimed quantities derive from the solution based on the line feed assumption. Thus, it is necessary to establish a relationship between the two conditions in order to apply the equations. This is done by the same method as employed in Appendix III. In other words, consider the case where the journal is concentric in the bearing \( \varepsilon = \theta \). Because of symmetry, it is then only necessary to consider an axial strip.
The solution for the gas film pressure, $P_0'$, can be obtained from the method of sources and sinks as:

$$P_0'^2 = 1 + \frac{6\mu RT}{\pi h^2 C^3} M_0 \sum_{k=-\infty}^{\infty} (-1)^k \left[ \frac{\cos\left(\frac{n_k f_3 f_2 + 2k f_3 f_3}{2}\right) - 1}{\cos\left(\frac{n_k f_3 f_3}{2}\right) - \cos(\pi \theta)} \right]$$

(E-47)

If there is only one admission plane ($f_3 = 0$) with $n$ feeder holes, the solution becomes (see reference 2):

$$P_0'^2 = 1 + \frac{6\mu RT}{\pi h^2 C^3} M_0 \sum_{k=-\infty}^{\infty} (-1)^k \left[ \frac{\cos\left(\frac{n_k f_3 f_3}{2}\right) - 1}{\cos\left(\frac{n_k f_3 f_3}{2}\right) - \cos(\pi \theta)} \right]$$

(E-48)

In these two equations, $M_B$ is the mass flow from one feeder hole (concentric journal). The pressure, $P_{oc}$, downstream of the feeder hole is defined as the pressure on the rim of the feeder hole where $f_2 = 0$ (i.e., $f = f_3$) and $\theta = d/D$ ($d =$ feeder hole diameter, $D =$ journal diameter). Since $d \ll D$ the dominant term in the series in eqs. (E-47) and (E-48) is the one where $k = 0$, which is the only term that needs to be considered. Define:
Double Plane Admission

\[
\lambda = \frac{1}{nF_2} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[ \frac{\cosh((\hat{p}_2 F_2) - 1)}{\cosh(nkF_2) - \cos(\frac{\pi}{B})} \right] \approx \frac{1}{nF_2} \log \left[ \frac{\cosh(nF_2) - 1}{\cos(\frac{\pi}{B})} \right] \approx 1 + \frac{2}{nF_2} \log \left( \frac{D}{nd} \right) \tag{E-49}
\]

Single Plane Admission

\[
\lambda = \frac{1}{nF_2} \sum_{k=-\infty}^{\infty} (-1)^k \log \left[ \frac{\cosh(nF_2 (2k+1)) - 1}{\cos(2nkF_2) - \cos(\frac{\pi}{B})} \right] \approx \frac{1}{nF_2} \log \left( \frac{\cosh(nF_2) - 1}{1 - \cos(\frac{\pi}{B})} \right) \approx 1 + \frac{2}{nF_2} \log \left( \frac{D}{nd} \right) \tag{E-50}
\]

Then the downstream feeder hole pressure becomes:

\[
P_{oc}'^2 = 1 + \frac{6\mu\sqrt{T_n m_{oc}}}{\pi P_{oc}^2 C^3} \frac{F_2}{\lambda F_2} = 1 + \lambda_s V^2 m_o S_2 \lambda = 1 + \lambda_s' \lambda \tag{E-51}
\]

Now, \( m_0 \) is a function of \( \frac{P_{oc}'}{V} \) (see Eqs. (E-31) and (E-39)). From Eq. (E-51):

\[
\frac{1}{\lambda_s' \lambda} \left[ \left( \frac{P_{oc}'}{V} \right)^2 - \frac{1}{V^2} \right] = m_0 = m \left( \frac{P_{oc}'}{V} \right) \tag{E-52}
\]

This equation can be solved graphically:

\[
m = \frac{M_0 \sqrt{RT}}{\pi a^2 P_s} \left[ \frac{1}{\lambda_s' \lambda} \left( \frac{P_{oc}'}{V} - \frac{1}{V^2} \right) \right]
\]
Knowing \( \frac{P'}{V} \), \( q \) can be calculated directly from Eq. (E-51). In addition, the slope \( \left( \frac{dP}{dP'} \right) \) of \( \frac{P'}{V} \) can be computed whereby \( \Psi' \) is given through Eq. (E-41).

The corresponding line feed solution can be determined from solving Eq. (E-12) with boundary conditions specified by Eqs. (E-24), (E-25) and (E-63):

\[
0 \leq \xi \leq \xi_c \quad P_0^2 = 1 + q \xi_c^2
\]

\[
0 \leq \xi_a \leq \xi_c \quad P_0^2 = 1 + q (\xi_a - \xi_c)
\]

from which

\[
P_{ac}^2 = 1 + q \xi_c^2
\]  

(E-54)

A comparison between Eqs. (E-54) and (E-51) yields:

\[
\frac{P_{ac}^2 - 1}{P_0^2 - 1} = \lambda
\]

(E-55)

This establishes the relationship between the actual feeder hole downstream pressure and the one obtained from the line feed solution, based on a concentric journal. When the journal is eccentric, the flow from the feeder holes is no longer confined to the axial strip shown in Fig. 62 and, furthermore, the source strength changes. However, to get an approximate solution, consider a rectangular strip as in Fig. 62 with a uniform film thickness. In that case, the ratio:

\[
\lambda = \frac{P_{ac}^2 - 1}{P_0^2 - 1}
\]

(E-56)

stays constant, independent of the film thickness. Assuming that this relationship is valid even under dynamic conditions, Eq. (E-36) can be used to expand Eq. (56):

\[
\lambda = \frac{P_{ac}^2 - 1 + 2\xi ((P_0 P)_{ac}^2 - P_{ac}^2 \cos \phi)}{P_0^2 - 1 + 2\xi ((P_0 P)_{ac}^2 - P_{ac}^2 \cos \phi)} = \frac{1}{P_{ac}^2 - 1 + 2\xi ((P_0 P)_{ac}^2 - P_{ac}^2 \cos \phi)} \left[ P_{ac}^2 - 1 + 2\xi ((P_0 P)_{ac}^2 - P_{ac}^2 \cos \phi) \right]
\]

-166-
from which
\[ \frac{P_{rc}^2 - 1}{P_{rc}^2 - 1} = \lambda \]

\[ (\varepsilon P_{rc}) - \varepsilon P_{rc}^2 \cos \theta = \lambda \left[ (\varepsilon P_{rc}) - \varepsilon P_{rc}^2 \cos \theta \right] \]

The first equation checks with Eq. (E-55). The second equation can be expanded further by means of Eqs. (E-15) and (E-17) to yield:

\[ q_{0c} - P_{rc}^2 \cos \theta = \lambda \left[ q_{0c} - P_{rc}^2 \cos \theta \right] \quad (E-57) \]

\[ q_{ic} - P_{rc}^2 \cos \theta = \lambda \left[ q_{ic} - P_{rc}^2 \cos \theta \right] \quad (E-58) \]

\[ q_{ec} = \lambda q_{2c} \quad (E-59) \]

Define the parameters:

\[ \psi_0 = \lambda \psi_0' = -\frac{\lambda A_z V}{2 P_{rc}^2} \left( \frac{\partial m}{\partial P_{rc}^2} \right) \quad (E-60) \]

\[ \psi_i = \lambda \psi_i' = \frac{n V}{\pi D_L C} \frac{A(f_i + f_s)}{P_{rc}^2} \quad (E-61) \]

Thus, Eqs. (E-57) to (E-61) can be substituted into Eqs. (E-44) to (E-46).

Noting that:

\[ P_{rc}^2 \cos \theta = R_e \{ P_{rc}^z e^{j\theta} \} \]

\[ P_{rc}^z \sin \theta = R_e \{ -j P_{rc}^z e^{j\theta} \} \]

and making use of Eqs. (E-17) to (E-19), the resulting equations become:

\[ \left( \frac{dG_o}{d\delta_c} \right) = \frac{q}{\varepsilon (\varepsilon + j)} + \psi_0 (G_{rc} - P_{rc}^z) + \left( \frac{dG_o}{d\delta_c} \right) \quad (E-62) \]

\[ \left( \frac{dG_i}{d\delta_c} \right) = \frac{q}{\varepsilon (\varepsilon + j)} + (\psi_0 + i \gamma \chi \psi_i) (G_{ic} - P_{rc}^z) + \left( \frac{dG_i}{d\delta_c} \right) \quad (E-63) \]
These equations together with Eqs. (E-24) and (E-25) are the boundary conditions for Eqs. (E-20) to (E-22).

It is seen directly that:

$$G_2 = j(G_0 - G_1)$$  \hspace{1cm} (E-65)

It remains to determine the solutions for $G_0$ and $G_1$.

$P_0$ is constant between the feeding planes, i.e. in the range $0 \leq \zeta \leq \zeta_f$, as given by the first of Eqs. (E-53). Under these circumstances, $G_0$ and $G_1$ have closed form solutions:

$$G_0 = \frac{P_{oc}^2}{1+j \frac{A}{P_{oc}}} + \left[ G_{oc} - \frac{P_{oc}^2}{1+j \frac{A}{P_{oc}}} \frac{\cosh (d_0 + j \beta_0) \zeta}{\cosh (d_0 + j \beta_0) \zeta_f} \right]$$  \hspace{1cm} (E-66)

$$G_1 = \frac{P_{oc}^2}{1+j \frac{A}{P_{oc}} + i \frac{A}{P_{oc}}} + \left[ G_{ic} - \frac{P_{oc}^2}{1+j \frac{A}{P_{oc}} + i \frac{A}{P_{oc}}} \frac{\cosh (d_0 + j \beta_0) \zeta}{\cosh (d_0 + j \beta_0) \zeta_f} \right]$$  \hspace{1cm} (E-67)

where:

$$d_0 + j \beta_0 = \sqrt{1+j \frac{A}{P_{oc}}}$$  \hspace{1cm} (E-68)

$$d + j \beta + i (d_0 + j \beta_0) = \sqrt{1+j \frac{A}{P_{oc}} + i \frac{A}{P_{oc}}}$$  \hspace{1cm} (E-69)

$$d_0 = \left[ \frac{1}{2} (1 + \sqrt{1 + \frac{A}{P_{oc}}} \right)^{1/2}$$  \hspace{1cm} (E-70)

$$\beta_0 = \left[ \frac{1}{2} (-1 + \sqrt{1 + \frac{A}{P_{oc}}} \right)^{1/2}$$  \hspace{1cm} (E-71)

$$d = \left[ \frac{1}{4} (x_0 + 1 + \sqrt{(x_0 + 1)^2 + (x_0 + \frac{A}{P_{oc}})^2} \right)^{1/2}$$  \hspace{1cm} (E-72)
\[ \beta = \left\{ \frac{1}{2} \left[ -x_1 - 1 + \sqrt{(x_1 + 1)^2 + (x_2 + \frac{A}{p_c})^2} \right] \right\}^{1/2} \]  
(E-73)

\[ \alpha = \left\{ \frac{1}{2} \left[ -x_1 - 1 + \sqrt{(x_1 - 1)^2 + (x_2 - \frac{A}{p_c})^2} \right] \right\}^{1/2} \]  
(E-74)

\[ \beta_1 = \left\{ \frac{1}{2} \left[ -x_1 + 1 + \sqrt{(x_1 - 1)^2 + (x_2 - \frac{A}{p_c})^2} \right] \right\}^{1/2} \]  
(E-75)

\[ \alpha_1 = \left\{ \frac{1}{2} \left[ 1 - \left( \frac{A}{p_c} \right)^2 + \left( \frac{A}{p_c} \right)^2 \right] + \sqrt{\left( 1 - \left( \frac{A}{p_c} \right)^2 + \left( \frac{A}{p_c} \right)^2 \right)^2 + 4 \left( \frac{A}{p_c} \right)^2} \right\} \]  
(E-76)

\[ \alpha_1 = \left\{ \frac{1}{2} \left[ 1 - \left( \frac{A}{p_c} \right)^2 + \left( \frac{A}{p_c} \right)^2 \right] + \sqrt{\left( 1 - \left( \frac{A}{p_c} \right)^2 + \left( \frac{A}{p_c} \right)^2 \right)^2 + 4 \left( \frac{A}{p_c} \right)^2} \right\}^{1/2} \]  
(E-77)

Hence:

\[ \left( \frac{dG_o}{dt_c} \right) = \left[ G_\infty - \frac{p_c}{E} \right] (a_o + j \beta_o) \tan h(a_o + j \beta_o) \]  
(E-78)

\[ \left( \frac{dG_1}{dt_c} \right) = \left[ G_\infty - \frac{p_c}{E} (1 + j \beta + i(a_o + j \beta_o)) \right] \tan h(a_o + j \beta + i(a_o + j \beta_o)) \]  
(E-79)

These expressions can be substituted into Eqs (E-62) and (E-63) to eliminate the two derivatives.

Eqs. (E-20) and (E-21) are solved by numerical integration. They have the general form:

\[ \frac{d^2 G}{d \tau^2} - E G = F \]  
(E-80)

where:

\[ E = \begin{cases} 1 + j \frac{A}{p_o} & \text{for } G = G_o \\ 1 + j \frac{A}{p_o} + i \frac{A}{p_o} & \text{for } G = G_1 \end{cases} \]  
(E-81)

\[ F = -p_o^2 = -(1 + q (f_2 - f_1)) \]  
(E-82)

-169-
The boundary conditions are:
\[
\begin{align*}
\frac{J_2}{J_2} &= G = 1 \quad (E-83) \\
\frac{dG}{dJ_2} &= a + b \, G_c \quad (E-84)
\end{align*}
\]

where:
\[
\begin{align*}
G &= \left( \frac{\Theta}{2(1+\delta_2)} \right) - \left[ \frac{\alpha + (\alpha+i\beta_0) \tanh(\alpha+i\beta_0) \delta_2}{1 + j \frac{\alpha}{\delta_2}} \right] (1 + q \delta_2) \\
a &= \left\{ \begin{array}{l}
\frac{\Theta}{2(1+\delta_2)} - \left[ \frac{\alpha (e^{\alpha+i\beta_0}) \tanh(\alpha+i\beta_0) \delta_2}{1 + j \frac{\alpha}{\delta_2}} \right] (1 + q \delta_2) \\
\frac{\Theta}{2(1+\delta_2)} - \left[ \frac{\alpha (e^{\alpha+i\beta_0}) \tanh(\alpha+i\beta_0) \delta_2}{1 + j \frac{\alpha}{\delta_2}} \right] (1 + q \delta_2)
\end{array} \right.
\end{align*}
\]

Integrate Eq. (E-80) twice to get:
\[
G_n = \int_0^{\delta_2} (r_2 - r_1) (E \mathcal{G} + F) \, dJ_2 + a J_2 + (b J_2 + 1) G_c \quad (E-87)
\]

Subdivide the length of integration into m increments of length \( \Delta J = \delta_2 / m \) and write Eq. (E-87) in finite difference form:
\[
G_{n+1} = G_n + (\Delta J)^2 \left[ \frac{1}{4} H_0 + H_1 + \cdots + H_n \right] + \Delta J (a + b G_c) \quad 0 \leq n \leq m-1 \quad (E-88)
\]

where \( H = E \mathcal{G} + F \). Setting:
\[
G = t_0 + J \mathcal{G}_0 + i(\nu_0 + J \nu_0) + [\nu_0 + J \nu_0 + i(\nu_0 + J \nu_0)] G_c \quad (E-89)
\]

eq. (E-88) can be used to calculate \( G_n \) step by step, keeping \( G_c \) as an unknown constant ( \( G_0 = G_c \)). At \( J_2 = \delta_2 \), \( G_m = 1 \) from which \( G_c \) can be computed. Thereafter, \( G \) can be determined at each point by back substitution.
The force acting on the journal is found by integration of the film pressure. The force has a radial component \( F_r \) and a tangential component \( F_t \):

\[
F_r = 2 \int_0^\pi \int_0^{R(z)} P \{ -\cos \theta \} R \, d\theta \, dz = 2 R^2 \int_0^\pi P \{ -\cos \theta \} \, d\theta \, d\zeta
\]

or, in dimensionless form, making use of Eqs. (E-16), (E-4) and (E-15):

\[
F_t = \frac{F_r}{K_{LD}} = \frac{1}{2(E_{t+\alpha})} \left( \int_0^{2\pi} P_0 + E_0 \frac{R}{P_0} + E_0 \frac{R^2}{P_0} \left[ \frac{\cos \theta}{1 + E_0 \cos \theta} \right] + E_0 \frac{R^3}{P_0} \left[ \frac{\cos \theta}{1 + E_0 \cos \theta} \right] \, d\theta \right) \tag{E-91}
\]

The following integrals can be derived:

\[
\int_0^{2\pi} \frac{d\theta}{1 + E_0 \cos \theta} = \frac{2\pi}{\eta} \tag{E-92}
\]

\[
\int_0^{2\pi} \cos \theta \frac{d\theta}{1 + E_0 \cos \theta} = -\frac{2\pi E_0}{\eta (1+\eta)} \tag{E-93}
\]

\[
\int_0^{2\pi} \sin \theta \frac{d\theta}{1 + E_0 \cos \theta} = 0 \tag{E-94}
\]

\[
\int_0^{2\pi} \cos^2 \theta \frac{d\theta}{1 + E_0 \cos \theta} = \frac{2\pi}{\eta (1+\eta)} \tag{E-95}
\]

\[
\int_0^{2\pi} \sin^2 \theta \frac{d\theta}{1 + E_0 \cos \theta} = \frac{2\pi}{(1+\eta)} \tag{E-96}
\]

\[
\int_0^{2\pi} \cos \theta \sin \theta \frac{d\theta}{1 + E_0 \cos \theta} = 0 \tag{E-97}
\]

\[
\int_0^{2\pi} \frac{d\theta}{(1 + E_0 \cos \theta)^2} = \frac{2\pi}{\eta^3} \tag{E-98}
\]

\[
\int_0^{2\pi} \frac{\cos \theta \, d\theta}{(1 + E_0 \cos \theta)^3} = -\frac{2\pi E_0}{\eta^3} \tag{E-99}
\]
\[ \int_0^{2\pi} \frac{\sin \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = 0 \]  
(E-100)

\[ \int_0^{2\pi} \frac{\cos^2 \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = \frac{2\pi}{\eta^2(1 + \eta)} \left( 1 + \eta - \eta^2 \right) \]  
(E-101)

\[ \int_0^{2\pi} \frac{\sin^2 \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = \frac{2\pi}{\eta(1 + \eta)} \]  
(E-102)

\[ \int_0^{2\pi} \frac{\cos \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = 0 \]  
(E-103)

\[ \int_0^{2\pi} \frac{\cos^2 \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = -\frac{2\pi}{\varepsilon \eta^2(1 + \eta)} \left( 1 - \eta \right)(1 + 2\eta) \]  
(E-104)

\[ \int_0^{2\pi} \frac{\cos \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = 0 \]  
(E-105)

\[ \int_0^{2\pi} \frac{\cos \theta \, d\theta}{(1 + \varepsilon \cos \theta)^2} = -\frac{2\pi(1 - \eta)}{\varepsilon \eta(1 + \eta)} \]  
(E-106)

where:
\[ \eta = \sqrt{1 - \varepsilon^2} \]  
(E-107)

From Eqs. (E-17) and (E-18):
\[ q_0 = \text{Re}_\gamma \left\{ \text{G}_0 \right\} \cos \theta - \text{Im}_\gamma \left\{ \text{G}_0 \right\} \sin \theta \]  
(E-108)

\[ q_1 = \text{Re}_\gamma \left\{ \left[ \text{Re}_\gamma \left\{ \text{G}_1 \cos \theta - \text{Im}_\gamma \left\{ \text{G}_1 \sin \theta \right\} e^{i\gamma} \right] \right\} \]  
(E-109)

Thus, the integrals in Eq. (E-91) can be computed as:
\[ f_r = \frac{2\varepsilon_0}{\eta(1 + \eta)} H_{pr} + \varepsilon e^{i\gamma} \left[ \frac{2(1 - \eta)(2\gamma)}{\eta^3(1 + \eta)} H_{pr} + \frac{2}{\eta(1 + \eta)} (H_{pr} + i J_{ir}) \right] + \varepsilon_0 \varepsilon e^{i\gamma} \frac{2}{\eta(1 + \eta)} \left[ -H_{pr} + H_{pr} + i J_{ir} \right] \]  
(E-109)
\( \xi_\varepsilon = \frac{2\varepsilon_0}{H_0} H_{0x} + \varepsilon_0 e^{i\pi} \left[ \frac{2(M_m)}{H_{0x} + i\varepsilon_0} (H_{0x} + i\varepsilon_0) \right] - \varepsilon_0 Q e^{i\pi} \left[ -H_m + H_m + i J_m \right] \)  

(E-110)

where:

\[
H_{0x} = \frac{\pi}{2(\xi_0 + \xi_2)} \int_0^{\xi_2} \left( P_0 - \frac{\xi_0}{P_0} \right) d\xi
\]

(E-111)

\[
H_{0y} = -\frac{\pi}{2(\xi_0 + \xi_2)} \int_0^{\xi_2} \frac{J_m \left( G_0 \right)}{P_0} \left( P_0 - \frac{G_0}{P_0} \right) d\xi
\]

(E-112)

\[
H_{1x} + J_{1y} + i(J_m + J_{1y}) = \frac{\pi}{2(\xi_0 + \xi_2)} \int_0^{\xi_2} \left( P_0 - \frac{G_0}{P_0} \right) d\xi
\]

(E-113)

With substitution from Eqs. (E-53), (E-66) and (E-57) these integrals become:

\[
H_{0x} + J_{1y} = \frac{\pi}{2(\xi_0 + \xi_2)} \left\{ \xi_0 \sqrt{1 + q_{12}} + \frac{2}{3} \left[ (q_2 + 2\xi_0) - 1 \right] - \xi_0 \sqrt{1 + q_{12}} \right\} - \left[ \frac{G_{0e} - \sqrt{1 + q_{12}}}{P_{0e}} \right] \frac{\theta_{e}^{a+\beta}}{d_{a+\beta}}
\]

(E-114)

\[
H_{1x} + J_{1y} + i(J_m + J_{1y}) = \frac{\pi}{2(\xi_0 + \xi_2)} \left\{ \xi_0 \sqrt{1 + q_{12}} + \frac{2}{3} \left[ (1 + q_{12}) - 1 \right] - \xi_0 \sqrt{1 + q_{12}} \right\} - \left[ \frac{G_{0e} - \sqrt{1 + q_{12}}}{H_{1y} + \sqrt{1 + q_{12}}} \right] \frac{\theta_{e}^{a+\beta}}{d_{a+\beta}}
\]

(E-115)

where \( P_{0c} \) is given by:

\[
P_{0c} = \sqrt{1 + q_{12}}
\]

(E-116)

The two integrals on the right hand side are evaluated by numerical integration from the previously determined values of \( G_0 \) and \( G_1 \).
When the static load on the bearing is \( W \), the total force components acting on the journal in the radial and tangential directions are:

Radial direction (opposite \( \varepsilon_r \)): \[ F_r = F_r - W \cos \varphi \] (E-177)

Tangential direction (opposite to \( \varepsilon_t \)): \[ F_t = F_t + W \sin \varphi \]

With \( \varphi \) given by Eq. (E-15), these components can be written in dimensionless form as:

\[ f_r^* = \frac{f_r - \frac{W}{P_L D} \cos \varphi}{f_r^*} = \frac{f_r - \frac{W}{P_L D} (\cos \varphi - \varphi, e^{i\pi} \sin \varphi)}{f_r^*} \] (E-118)

\[ f_t^* = f_t + \frac{W}{P_L D} \sin \varphi = -f_t + \frac{W}{P_L D} (\sin \varphi + \varphi, e^{i\pi} \cos \varphi) \]

Set:

\[ f_{ro} = \frac{2 f_r}{\gamma (1+\eta)} H_{or} \] (E-119)

\[ f_{to} = \frac{2 f_t}{\gamma (1+\eta)} H_{ot} \] (E-120)

\[ f_r^* = f_r - \frac{W}{P_L D} (\cos \varphi = (K_{rr} + i \theta) \varepsilon_r e^{i\pi} + (K_{rr} + i \theta) \varepsilon_r e^{i\pi}) \] (E-121)

\[ f_t^* = f_t + \frac{W}{P_L D} \sin \varphi = (K_{tt} + i \theta) \varepsilon_t e^{i\pi} + (K_{tt} + i \theta) \varepsilon_t e^{i\pi} \] (E-122)

Then:

\[ \frac{W}{P_L D} \cos \varphi = f_{ro} \] (E-123)

\[ \frac{W}{P_L D} \sin \varphi = f_{to} \] (E-124)

\[ \frac{W}{P_L D} = \sqrt{f_{ro}^2 + f_{to}^2} \] (E-125)

\[ \varphi = \tan^{-1} \left( \frac{f_{to}}{f_{ro}} \right) \] (E-126)

\[ Z_{rr} = K_{rr} + i \theta \theta_{rr} = \frac{2(1-\eta)(2\eta+1)}{\eta^2(1+\eta)} H_{or} + \frac{2}{\eta(1+\eta)} (H_{or} + i J_{rr}) \] (E-127)
\[ Z_{rt} = \bar{K}_r + i\bar{B}_r = -\frac{2(i\eta)}{\eta(1+\eta)} H_y + \frac{2}{\eta(1+\eta)} (H_x + i J_y) \]  
(E-128)

\[ Z_{tr} = \bar{K}_t + i\bar{B}_t = -\frac{2(i\eta)}{\eta(1+\eta)} H_x - \frac{2}{i+\eta} (H_y + i J_y) \]  
(E-129)

\[ Z_{tt} = \bar{K}_{tt} + i\bar{B}_{tt} = \frac{2(1-\eta)}{\eta(1+\eta)} H_{or} + \frac{2}{i+\eta} (H_y + i J_y) \]  
(E-130)

Here, \( \bar{K}_{rr}, \bar{B}_{rr}, \bar{K}_{rt}, \bar{B}_{rt} \) etc., are the dimensionless spring and damping coefficients:

\[ \bar{K}_{rr} = \frac{\partial f_x^*}{\partial \epsilon} = \frac{C K_{r}}{P_a L D} \]

\[ \bar{B}_{rr} = \frac{\partial f_x^*}{\gamma \partial \epsilon} = \frac{C \omega B_{r}}{P_a L D} \]  
(E-131)

and analogously for the four remaining coefficients. If it is desired, the coefficients can readily be expressed in an x-y-coordinate system with the x-axis in the direction of the static load \( W \) as in the preceding appendices. This is done by making use of Eqs. (A-70) to (A-77), Appendix I where \( S W \) is replaced by \( P_a L D, f_x \) by \( f_x^* \) and \( (-f_t) \) by \( f_t^* \).

For the purely hydrostatic bearing, \( \Lambda = 0 \) but such that \( \frac{1}{2} \Lambda^* \delta \) where \( \delta \) is the squeeze number (see Eq. (E-10)). In that case \( H_x = H_y = J_y = 0 \) and \( \beta = \beta_0 = 0 \) which considerably simplifies the calculations. Furthermore, for the hydrostatic bearing, the static load increases almost proportional with displacement and, therefore, \( \eta \) should be set equal to 1 in the above equations. In total, then, for a hydrostatic bearing:

**Purely Hydrostatic Bearing**

\[ \frac{W}{P_a L D} = f_{re} = \epsilon_H H_{or} \]

\[ f_{r_0} = 0 \quad \beta_0 = 0 \]  
(E-132)
Thus, for a purely hydrostatic bearing there is only one spring coefficient and one damping coefficient which shall be denoted as $K_o$ and $B_o$. It is convenient to express these coefficients and the load carrying capacity in the dimensionless form:

\[
\frac{W}{(P_o - P_a)LD} = \frac{1}{V-1} \frac{W}{P_a LD} \quad \text{(E-134)}
\]

\[
\frac{C K_o}{(P_o - P_a)LD} = \frac{1}{V-1} \bar{K}_{rr} \quad \text{(E-135)}
\]

\[
\frac{B_o}{\mu L (\xi)^n} = \frac{24}{\sigma} \gamma \bar{B}_{rr} \quad \text{(E-136)}
\]
APPENDIX VI: THE STABILITY OF THE HYBRID-HYDROSTATIC RING BEARING WITH A COMPRESSIBLE LUBRICANT

The analysis of the stability of the hybrid-hydrostatic ring bearing is almost identical to the analysis of the hydrodynamic-hydrostatic ring bearing given in Appendix IV. Therefore, the present appendix will only describe the differences between the two analyses and otherwise refer to Appendix IV.

The equations of motion are given by Eqs. (D-1) where $K_{xv}, B_{xv}, K_{yy}$ and so on are the dynamic coefficients of the inner film, and $K_o$ and $B_o$ are the coefficients of the outer film. They are determined from the analysis in Appendix V (note: instead of $K_{xv}, B_{xv}, K_{yy},$ etc can be used $K_{rr}, B_{rr}, K_{tt}$, etc. or the latter set of coefficients can be transformed to the first set of coefficients as discussed in Appendix V). Eq. (D-1) is made dimensionless by setting:

$$K_o = \frac{Ck_o}{P_a L D} = \left(\frac{C}{C_o}\right) \left(\frac{D}{D_o}\right) \left(\frac{2}{P_a - 1}\right) \frac{C_o K_o}{(P_a - P_0)L D_o}$$

$$B_o = \frac{C_o B_o}{P_a L D} = \left(\frac{C}{C_o}\right) \left(\frac{D}{D_o}\right) \frac{C_o B_o}{2 \mu L (\frac{C}{C_o})^3}$$

$$\bar{M} = \frac{C_o M}{P_a L D}$$

$$K_{xx} = \frac{C k_{xx}}{P_a L D}$$

$$B_{xx} = \frac{C_o B_{xx}}{P_a L D}$$

and similarly for $K_{yy}, K_{vy}, K_{yx}, B_{yy}, B_{yx}$ and $B_{yy}$. The symbols are defined in Appendix IV and Appendix E.

The stability analysis is performed for fixed values of the compressibility number $\Lambda$ and the static eccentricity ratio $\varepsilon_0$ (i.e. for a constant static load $W$ and a given speed $\omega$). Hence, the dynamic coefficients are only functions of the frequency ratio $\gamma = \frac{\omega}{f}$.

-177-
Appendix D can be used directly as long as it is remembered, that whereas the
dynamic coefficients in Appendix D are independent of $\gamma$ because the lubricant is
incompressible, they now depend implicitly on $\gamma$. The equations, however, are
the same in the two cases.
APPENDIX VII: Computer Program - The Static and Dynamic Performance of a Lobed Bearing with Turbulent Film

This appendix describes the computer program PN0375: "The Static and Dynamic Performance of a Lobed Bearing with Turbulent Film" and gives the detailed instructions for using the program. The program is based on the analyses contained in Appendices I and II. It calculates the Sommerfeld number, the flow, the coefficient of friction, the 8 dynamic bearing coefficients and the critical journal mass, for a journal bearing with up to 12 lobes whose lubricant film may be turbulent. Film rupture is included.

COMPUTER INPUT

An input data form is given in back of this appendix for quick reference when preparing the computer input. In the following, the more detailed instructions are given.

Card 1 (15)

This card contains one value:

NRET which gives the number of film Reynolds numbers in the table of turbulent flow coefficients (1 ≤ NRET ≤ 250)

Table of Turbulent Flow Coefficients (4E15.7)

In a bearing film with fully developed turbulence where the Couette flow is the dominating flow component, the effect of turbulence on the lubrication action can be accounted for by means of two coefficients, $G_W$ and $G_Z$. These coefficients modify the Poiseuille flow (the pressure induced flow) and they are functions of the film Reynolds number $R_f$:

$$R_f = \frac{9R_w h}{\mu} = h R_e$$
where \( h \) is the local film thickness, normalized with respect to the radial clearance \( C \), and \( R_e \) is the Reynolds number for the bearing. \( G_r \) and \( G_z \) are given in reference 1 and can also be found in the table in the input data for the sample calculation later in this appendix. It should be noted, that the values used for \( G_r \) and \( G_z \) in the present program are twelve times the values found in reference 1. By doing this, \( G_r \) and \( G_z \) are equal to 1 when \( R_e = 0 \) (i.e., for laminar flow).

For use in the calculation of the dynamic performance of the bearing it is necessary to specify the first three derivatives of \( G_r \) with respect to \( R_e \). Also, the first derivative of \( G_z \) is required. Finally, to determine the friction loss of the bearing, the coefficient of friction, \( C_f \), must be given. In the table, \( C_f \) is specified by giving the product \( \frac{1}{2} R_e C_f \) which is equal to 1 for \( R_e = 0 \). In total, then, there are 7 quantities for each film Reynolds number. They are given on two cards:

**First Card \( (4E15.7) \)**

1. \( R_e = \frac{6 \omega R_e}{\mu} = h R_e \), the film Reynolds number. The first value in the table should preferably be 0. In any case, the table must span over the range:

\[
(l-c_p) \leq R_e \leq (1+c_p) R_e
\]

where \( R_e \) is the input value of the bearing Reynolds number (see later input list) and \( c_p \) is the maximum eccentricity ratio for any lobe in the particular calculation. Since the largest possible value of \( c_p \) is 1, it is safest to let the range of \( R_e \) be: \( 0 \leq R_e \leq 2 R_e \).

To use the table, the program employs linear interpolation. The local dimensionless film thickness \( h \) is computed from which the film Reynolds number is determined as \( R_h = h R_e \) where \( R_e \) is given in the input. Then the table is scanned, starting with the first table value of \( R_h \), until \( (R_h)_{table} \geq (R_h)_{calculated} \). Assume this to be at the \( k'th \) table value (i.e. \( (R_h)_{table} = (R_h)_{k} \)).

The proper value of \( G_x \) is then computed as:

\[
G_x = \frac{(R_h)_{k} - (R_h)_{calculated}}{(R_h)_{k} - (R_h)_{k-1}} (G_x)_{k} + \frac{(R_h)_{calculated} - (R_h)_{k-1}}{(R_h)_{k} - (R_h)_{k-1}} (G_x)_{k-1}
\]
and similarly for the remaining 6 quantities. If $(R_h)_{\text{calc.}}$ is greater than the last value in the table, the program sets $(R_h)_{\text{calc.}} = (R_h)_{\text{last value}}$. If $(R_h)_{\text{calc.}}$ is smaller than the first value in the table, the program will be in error.

2. $\frac{1}{8} R_h C_f$, the dimensionless friction factor. For $R_h = 0$ (laminar flow), $\frac{1}{8} R_h C_f = 1$.

3. $G_x$, the dimensionless Poiseuille flow coefficient for the circumferential direction. For $R_h = 0$ (laminar flow), $G_x = 1$.

4. $\frac{1}{G_x} \frac{dG_x}{dR_h}$, dimensionless. This is the first derivative of $G_x$ divided by $G_x$. For $R_h = 0$ (laminar flow), $\frac{1}{G_x} \frac{dG_x}{dR_h} = 0$.

Second Card (4E15.7)

1. $\frac{1}{G_x} \frac{d^2 G_x}{dR_h^2}$, dimensionless. This is the second derivative of $G_x$ divided by $G_x$. For $R_h = 0$ (laminar flow), $\frac{1}{G_x} \frac{d^2 G_x}{dR_h^2} = 0$.

2. $\frac{1}{G_x} \frac{d^3 G_x}{dR_h^3}$, dimensionless. This is the third derivative of $G_x$ divided by $G_x$. For $R_h = 0$ (laminar flow), $\frac{1}{G_x} \frac{d^3 G_x}{dR_h^3} = 0$.

3. $\frac{G_x}{G_y}$, dimensionless. This is the ratio between the Poiseuille flow coefficient, $G_x$, for flow in the axial direction, and $G_y$. For $R_h = 0$ (laminar flow), $\frac{G_x}{G_y} = 1$.

4. $\frac{d}{dR_h} \left( \frac{G_x}{G_y} \right)$, dimensionless. This is the first derivative of $\frac{G_x}{G_y}$. For $R_h = 0$ (laminar flow), $\frac{d}{dR_h} \left( \frac{G_x}{G_y} \right) = 0$.

In total the table contains NRET entry values of $R_h$ arranged in sequence, starting with the smallest value. Thus, the table consists of 2 NRET cards. There can be a maximum of 250 entry values.
Card (72H)

This card is used as a title card, identifying the calculation. Any descriptive text can be given in columns 2 to 72. If a 1 is punched in Column 1, the output listing will start on a new page for each case.

Control Card (1213)

This card controls the input and the output. It has 12 values:

1. M. This is the number of finite difference increments in the circumferential direction per lobe, see Fig. 53, Appendix I. When the lobe arc is $(\Theta_{out} - \Theta_{in})$, degrees, the circumferential increment is:

   $$\Delta \Theta = \frac{(\Theta_{out} - \Theta_{in})}{M} \text{ degrees}$$

   It is suggested to let $\Delta \Theta$ be 10 to 15 degrees, depending on the eccentricity ratio (the larger the eccentricity ratio, the smaller should $\Delta \Theta$ be). The largest allowable value of $M$ is 36. Also, $M$ must be larger than or equal to 3.

2. N. This is the number of finite difference increments in the axial direction on half of the bearing length (see Fig. 53, Appendix I). If the increment is $\Delta Z$ and the bearing length is $L$, then:

   $$\Delta Z = \frac{L/2}{N}$$

   In dimensionless form as used by the program:

   $$\Delta Z = \frac{L/D}{N}$$

   $N$ should be chosen such that $\Delta Z$ is approximately the same as $\Delta \Theta$ when $\Delta \Theta$ is measured in radians. However, since there are pressure gradients in the circumferential direction which are considerably larger than the axial pressure gradients, $\Delta Z$ can be made somewhat larger than $\Delta \Theta$. Furthermore, it is of great importance to keep $N$ as small as possible since the calculation time is roughly proportional to $N^2$ or $N^3$. 

-182-
Usual values for $N$ are 5 to 7, depending, of course, on the $L/D$ ratio. The allowable range of $N$ is: $3 \leq N \leq 13$.

3. NLD Specifies the number of $L/D$-values in the designated input list ($L/D$ is the length-to-diameter ratio of the bearing). The allowable range is: $1 \leq \text{NLD} \leq 20$.

4. NRE Specifies the number of bearing Reynolds numbers $Re$ in the designated input list. The allowable range is: $1 \leq \text{NRE} \leq 30$.

5. NPAD Specifies the number of bearing lobes. The allowable range is: $1 \leq \text{NPAD} \leq 12$.

6. NECA Specifies the number of $E_b \cos \phi_b$ values in the designated input list. ($E_b$ is the bearing eccentricity ratio, $\phi_b$ is the bearing attitude angle) The allowable range is: $1 \leq \text{NECA} \leq 20$.

7. NAB Specifies the maximum allowable number of calculations to be performed in determining that bearing attitude angle for which the horizontal force $F_y$ is zero. A detailed discussion is given later in connection with the input list for $E_b \cos \phi_b$ and $E_b \sin \phi_b$. NAB should be greater than NAT (the number of $E_b \sin \phi_b$-values in designated input list).

8. NRP The program provides for the case of film rupture where no sub-ambient film pressures are permitted. In the regions where the pressure otherwise would have been less than ambient, the film contracts such that the pressure in the contracted film becomes ambient. The boundary separating the full film and the contracted portion is determined by iterations as discussed later in connection with the convergence limit for the rupture calculation. NRP specifies the maximum allowable number of such iterations to be performed. A usual value for NRP is 5 to 7. If it is not desired to include the effect of rupture, set NRP = 1.
9. NPW  In most cases, only the final results of the calculations for the composite bearing are of interest. Then NPW is set equal to zero. On occasions, however, it may be desired to get the results for each bearing lobe before they are combined into the composite bearing. In that case, set NPW = 1 or NPW = -1. If NPW = -1, the results will include the pressure distribution. If NPW = 1, the pressure distribution will be omitted from the output.

10. NRW  Under certain conditions the calculations of the boundary between the complete film and the contracted film does not readily converge. In that case it may be desired to explore the matter in more detail. If NRW = 1, the output lists the "error" (the relative difference between two successive calculations) for each rupture iteration of each lobe. If NRW = -1, the output gives, in addition to the "error", the coordinates of the rupture boundary for each iteration. If NRW = -2, the output gives, in addition to the output for NRW = -1, the finite difference coefficients $a_i^{(n)}$, $a_i^{(n-1)}$, and $\beta_j$ (see Fig. 56, Appendix I) for each iteration.

When the calculations experience difficulties, the "error" does not converge smoothly. It will normally be found that the reason is that the rupture boundary intersects a $j$-gridline (see Fig. 55, Appendix I). Thus, the trouble can frequently be eliminated by changing the gridline spacing (ie. change M, item 1, control card).

Only under unusual circumstances is it of interest to explore the rupture boundary calculations. By setting NRW = 0, the output will not include any results of these calculations. Hence, the normal value of NRW is zero.

11. NPUN  As discussed in connection with Item 7, the program may perform several calculations in order to determine the attitude angle for which the horizontal force is zero. If it is desired to get the results from each calculation, set NPUN = -1 or NPUN = -2. In the latter case, the
results for the composite bearing are punched on cards after each attitude angle calculation but only if there is no more than 1 lobe. No punched card output is given if NPUN = -1. If it is desired to get the final results only after the correct attitude angle has been determined, set NPUN = 0 or NPUN = 1. In the latter case, the output is also punched on cards for bearings with 1 lobe only. This is omitted when NPUN = 0.

Thus, the usual value of NPUN is zero (or 1, if punched card output is also desired).

12. INP When INP = 0, a new set of input data follows the present set of data. The new set of data starts with the text card (Card 72H). The table of turbulent flow coefficients should not be repeated and can only be given with the first set of input. If INP = 1, the present set of data is the last or only set.

Card (1P5E14.6)

This card contains two values:

1. CVLA This is the convergence limit for the bearing attitude angle $\phi _0$. After each complete bearing calculation, the total horizontal force $F_h$ and the total vertical force $F_v$ are computed by a summation over all bearing lobes, see eqs. (B-8) and (B-9), Appendix I. The correct attitude angle is the one for which $\theta _v = 0$. This condition is accepted to be satisfied by the program when:

$$\left| \frac{F_v}{F_h} \right| \leq CVLA$$

or when the total number of attitude angle calculations exceeds NAB (Item 7, control card).

The attitude angle is $\phi _0$. If the deviation between the attitude angle for which $F_v = 0$, and the calculated attitude angle is called $\Delta \phi _\alpha$, then:
\[ \Delta \theta_b = \tan^{-1} \left( \frac{F_{ps}}{F_{wa}} \right) \]

When \( \frac{F_{ps}}{F_{wa}} \) is small this is equivalent to:

\[ \Delta \theta_b \approx \frac{F_{ps}}{F_{wa}} \]

Hence, the given convergence criteria corresponds to requiring:

\[ |\Delta \theta_b|, \text{radians} < CVLA \]

or

\[ |\Delta \theta_b|, \text{degrees} < 57.3 \cdot CVLA \]

If it is desired to obtain the attitude angle within 0.01 degrees, then set \( CVLA \approx 1.7 \cdot 10^{-4} \). A typical value for \( CVLA \) is \( 10^{-4} \).

2. \( CVLR \) This is the convergence limit for the calculation of the rupture boundary for each bearing lobe. The program calculates the angular coordinates of the boundary for each \( i \) gridline (see fig. 55, Appendix 1) let these coordinates be \( \Theta_i, i=1,2,\ldots,n \). After the \( k \)th rupture iteration, the program tests the convergence of the calculations by:

\[ \frac{\sum |\Theta_i^{(k)} - \Theta_i^{(k-1)}|}{\sum |\Theta_i^{(k-1)}|} \leq CVLR \]

When this condition is satisfied or when the total number of iterations is equal to \( NRP \) (Item 8, control card), i.e. when \( k=\text{NRP} \), the program assumes that the calculations have converged. A typical value for \( CVLR \) is \( 10^{-5} \).

When \( \text{NRP} = -1 \), set \( CVLR = 100 \).

List of Bearing Length-to-Diameter Ratios (1P5E14.6)

This list gives the input values of the length-to-diameter ratio, \( L/D \), where \( L \) is the bearing length and \( D \) is the journal diameter. In total there are \( NLD \) values (Item 3, control card), maximum 20.
List of Bearing Reynolds Numbers (1P5E14.6)

This list gives the input values for the bearing Reynolds number:

\[ R_e = \frac{\rho R \omega C}{\mu} \]

where \( \rho \) = mass density of lubricant, \( \text{lbs-sec}^2/\text{inch}^4 \), \( \mu \) = viscosity of lubricant, \( \text{lbs-sec} / \text{inch}^2 \), \( \omega \) = angular speed of journal, radians/sec, \( R \) = journal radius, inch, and \( C \) = radial clearance of lobes, inch. When \( R_e = 0 \), the film is laminar. In total, there are \( NRE \) values of \( R_e \) (item 4, control card), maximum 30.

Data for Journal Center Position

Referring to fig. 58, Appendix II, the coordinates of the journal center with respect to the bearing center can be expressed by means of the eccentricity ratio \( \varepsilon_b \) and the attitude angle \( \varphi_b \). \( \varepsilon_b \) is the ratio between the journal center eccentricity, \( \varepsilon_b \), and the machined radial clearance, \( C_e \), of the lobes:

\[ \varepsilon_b = \frac{\varepsilon_c}{C_e} \]

The attitude angle, \( \varphi_b \), is the angle between the static load line (the x-axis) and the line connecting the bearing center and the journal center, measured in the direction of journal rotation. The corresponding coordinates in the x-y-coordinate system are: \( \varepsilon_b \cos \varphi_b \) and \( \varepsilon_b \sin \varphi_b \).

A given bearing calculation is performed for a fixed value of \( \varepsilon_b \cos \varphi_b \). With this value fixed, \( \varepsilon_b \sin \varphi_b \) is varied over a range in specified increments to determine when the horizontal force \( F_{y_b} \) becomes zero. For each value of \( \varepsilon_b \sin \varphi_b \), the program calculates the static horizontal force \( F_{y_b} \). When \( F_{y_b} \) changes sign, \( F_{y_b} \) is calculated at the midpoint of the last \( \varepsilon_b \sin \varphi_b \)-interval and quadratic interpolation is employed to obtain the first "guess" of the value of \( \varepsilon_b \sin \varphi_b \) for which \( F_{y_b} \) should be zero. If the corresponding value of \( F_{y_b} \) is not sufficiently small, as tested by means of the convergence criteria explained above, the latest obtained value of \( F_{y_b} \) is used together with the two closest previously obtained values to calculate a new value of \( \varepsilon_b \sin \varphi_b \) by quadratic interpolation. This procedure is repeated until either the convergence criteria is satisfied or the total number of calculations exceeds the
allowable limit NAB (Item 7, control card).

In total, there are NECA values of $E_0 \cos \phi_0$ (Item 6, control card), maximum 20. The card specifying $E_0 \cos \phi_0$, contains two items:

**Card (1PE14.6, 15)**

1. $E_0 \cos \phi_0$. It can have any value as long as the corresponding eccentricity ratios, $E_p$, for any of the bearing lobes do not equal or exceed 1, see eq. (B-5), Appendix II.

2. $NAT$ If $NAT \neq 1$, the program determines the attitude angle for which the horizontal force is zero. Then $NAT$ gives the number of values of $E_0 \sin \phi_0$ in the following input list. Item 7, control card: NAB should be greater than $NAT$ to allow for several interpolations in determining $\phi_0$ (suggested value: NAB = NAT + 3).

If $NAT \leq -1$, no interpolation is performed to determine when the horizontal force is zero. The program calculates the bearing for each specified value of $E_0 \sin \phi_0$ and gives the results of each calculation (set NPUN = -1 or -2, item 11, control card). The absolute value of NAT, $|NAT|$, specifies the number of values of $E_0 \sin \phi_0$ in the following input list. Item 7, control card: NAB should be set to $|NAT|$.

Note: NAT cannot be zero. The maximum value of NAT (or $|NAT|$) is 25.

This card is followed by:

**List of $E_0 \sin \phi_0$ Values (1PE14.6)**

This list gives the input values of $E_0 \sin \phi_0$ for the above specified $E_0 \cos \phi_0$ value. In total there are $NAT$ values, maximum 25. $E_0 \sin \phi_0$ may have any value as long as the eccentricity ratio, $E_p$, of any bearing lobe does not equal or exceed 1, see eq. (B-5), Appendix II.
If it is desired to calculate the bearing with a concentric journal, set $\varepsilon_0 \cos \varphi_0 = 0$ and give only one value of $\varepsilon_0 \sin \varphi_0$, namely $\varepsilon_0 \sin \varphi_0 = 0$. Furthermore, set NAT = 1 and NAB = 0.

In the case where $\varepsilon_0 \cos \varphi_0 = 0$ and the journal is not concentric in the bearing, no value of $\varepsilon_0 \sin \varphi_0$ should be zero.

If it is desired to calculate the attitude angle for which the horizontal force is zero (i.e. NAT is positive), the $\varepsilon_0 \sin \varphi_0$ values should be listed in sequence, starting with the lowest value (it may be zero when $\varepsilon_0 \cos \varphi_0 \neq 0$). The range of $\varepsilon_0 \sin \varphi_0$ should be large enough that it includes the point where $F_y = 0$. Otherwise, the program cannot obtain a solution.

The preceeding input, namely the card with $\varepsilon_0 \cos \varphi_0$ and the list of $\varepsilon_0 \sin \varphi_0$ values, must be repeated together NECA times (Item 6, control card).

**Bearing Geometry Data (1P5E14.6)**

The bearing is made up of NPAD lobes (Item 5, control card). The geometry of the lobe is specified by giving the angles from the static load line (the x-axis) to the leading edge and the trailing edge of the lobe, and by giving the coordinates of the center of curvature of the lobe relative to the bearing center and the static load line, see figure 56, Appendix II. The input consists of NPAD cards, maximum 12, and on each card are four values:

1. $\theta_{p,in}$, degrees. This is the angle from the load line (the negative x-axis) to the leading edge of the lobe, measured in the direction of journal rotation (see fig. 58, Appendix II). The value for $\theta_{p,in}$ should be such that: $0 \leq \theta_{p,in} \leq 360$. $\theta_{p,in}$ must be smaller than $\theta_{p,out}$.

2. $\theta_{p,out}$, degrees. This is the angle from the load line (the negative x-axis) to the trailing edge of the lobe, measured in the direction of journal rotation (see Fig. 58, Appendix II). The value for $\theta_{p,out}$ should lie in
range: $\theta_{p,in} < \theta_{p,out} \leq 360 + \theta_{p,in}$ ($\theta_{p,out} > 0$).

3. $\delta_p$. This is the preload of the lobe:

$$\delta_p = \frac{\rho}{C}$$

where $\rho$ is the distance between the bearing center and the center of curvature of the lobe. $C$ is the machined radial clearance of the lobe (i.e. the difference between the lobe radius of curvature and the journal radius. $C$ is the same for all lobes). For a cylindrical bearing, $\delta_p = 0$. For the elliptical bearing or a three-lobe bearing, $\delta_p$ is the same for all lobes and equal to the preload of the bearing in which case $\delta_p < 1$.

4. $\psi$ degrees. This is the angle from the static load line (the positive x-axis) to the line connecting the bearing center and the center of curvature of the lobe, measured in the direction of journal rotation (see Fig. 58, Appendix II). For a cylindrical bearing, set $\psi = 0$. In the elliptical bearing or the three-lobe bearing, set $\psi = \frac{1}{2}(\theta_{p,out} + \theta_{p,in})$.

**COMPUTER OUTPUT**

The output first repeats the input data for checking purposes (the table of turbulent flow coefficients is not included). Then follow the results of the calculations. Because of the several output format options, a complete sequential description of the output will not be given. Instead, the output for the normal type of calculation will be described where only the final results for the composite bearing are given. The final results are in five lines where the items are identified by the titles:

- **L/D** is the specified length-to-diameter ratio of the bearing.
- **REYN.NO.** is the specified bearing Reynolds number $R_e$.
- **ECC*COS(ATT)** is the specified fixed value of $e_0 \cos \varphi_0$.
- **ECC*SIN(ATT)** is the final value of $e_0 \sin \varphi_0$.
- **ECC.RATIO** is the bearing eccentricity ratio $e_0 = \sqrt{(e_0 \cos \varphi_0)^2 + (e_0 \sin \varphi_0)^2}$.
- **ATT.ANGLE** is the bearing attitude angle $\varphi_0 = \tan^{-1} \left( e_0 \sin \varphi_0 / e_0 \cos \varphi_0 \right)$, degrees.
- **I/S** is the inverse Sommerfeld number for the bearing: $\frac{1}{S} = \frac{W}{\mu NDL(\varphi)}$.
F-Y/S*W is the residual dimensionless horizontal force: $F_y / \mu_{N D L}(\xi)$, eq. (B-9)

$1/(C-F/W)$ is the friction factor: $\frac{F_x}{C} = \frac{F_x}{\mu_{N D L}(\xi)}$, eq. (B-10)

$(Q-Z/N D L)_{S I D E}$ is the dimensionless flow out of both bearing sides: $Q_z/N D L$, see eq. (Q/N D L)TOTAL is the total dimensionless flow consumption of the bearing: $Q/N D L$

For a lobed bearing with NPAD lobes, the total flow is:

$$Q = \sum_{P=1}^{NPAD} \left[ (Q_x)_p \left\{ (Q_x)_{p,\text{trail.edge}} - (Q_x)_{p,\text{lead.edge}} \right\} \right.$$

$0 \quad \text{(when above quantity is negative)}$

(see remarks following eq. (5-13), Appendix II)

When NPAD = 1: $Q = Q_x + (Q_x)_{\text{trail.edge}}$

$(Q-X/N D L)_{C I R C}$ is the dimensionless total flow into the bearing lobes from the grooves:

$$\sum_{P=1}^{NPAD} (Q_x)_{p,\text{lead.edge}}$$

F-X/S*W is the dimensionless vertical bearing reaction: $-F_x / \mu_{N D L}(\xi)$, eq. (B-8). When $F_y = 0$, this is equal to $1/5$, see eq. (B-16)

CMN/N D L(R/C)2 is the dimensionless critical journal mass at the threshold of instability: $\frac{CMN}{\mu_{N D L}(\xi)}$, calculated from eqs. (B-26) and (B-27).

FREQ/W is the frequency ratio: $\gamma = \frac{\omega}{\omega_0}$, at the threshold of instability.

(FREQ/W)² is the square of the instability frequency ratio: $\gamma^2 = (\frac{\omega}{\omega_0})^2$

calculated from eqs. (B-27). When $\gamma^2$ is negative, there is no instability threshold.

CKXX/SW is the dimensionless bearing spring coefficient for the x-direction:

$$C_{kxx} = \frac{C_{kxx}}{\mu_{N D L}(\xi)}$$, see eq (B-12)

CKXY/SW is the dimensionless bearing cross-coupling spring coefficient for the x-direction:

$$C_{kxy} = \frac{C_{kxy}}{\mu_{N D L}(\xi)}$$

CKYY/SW is the dimensionless bearing spring coefficient for the y-direction:

$$C_{kyy} = \frac{C_{kyy}}{\mu_{N D L}(\xi)}$$

CYYX/SW is the dimensionless bearing cross-coupling spring coefficient for the y-direction:

$$C_{xyy} = \frac{C_{xyy}}{\mu_{N D L}(\xi)}$$

CWX/SW is the dimensionless bearing damping for the x-direction:

$$C_{wxx} = \frac{C_{wxx}}{\mu_{N D L}(\xi)}$$, see eq. (B-13).
CWBXY/SW is the dimensionless bearing cross-coupling damping for the x-direction:
\[ \frac{C_{B_{XY}}}{S} = \frac{C_{w B_{XT}}}{\mu N D L (\xi)} \]

CWBXY/SW is the dimensionless bearing cross-coupling damping for the y-direction:
\[ \frac{C_{B_{XY}}}{S} = \frac{C_{w B_{YT}}}{\mu N D L (\eta)} \]

CWBYY/SW is the dimensionless bearing damping for the y-direction:
\[ \frac{C_{B_{YY}}}{S} = \frac{C_{w B_{YT}}}{\mu N D L (\eta)} \]

SOMMERFELD NO is the Sommerfeld number:
\[ S = \frac{\mu N D L}{W} \left( \frac{R}{L} \right)^2 \]

F-X/W is a second form of the dimensionless vertical bearing reaction:
\[ F_{x/w} \]
\[ ( = 1 \text{ when } F_{yo} = 0) \]

F-Y/W is a second form of the dimensionless residual horizontal force:
\[ F_{y/w} \]

\[ (F/C) \times (F/W) \] is the most usual form of the friction factor:
\[ \frac{B}{C} \frac{F}{W}, \text{ see eq. (B-17)} \]

CMBXX/W is a second form of the dimensionless critical journal mass:
\[ C_{M_{XX}} \]
\[ \text{see eq. (B-28)} \]

CKXX/W is a second form of the dimensionless bearing spring coefficient in the
x-direction:
\[ \frac{C_{K_{XX}}}{W} \text{, see eq. (B-18)} \]

CKXY/W = \[ C_{K_{XY}} \]
CKXY/W = \[ C_{K_{Xy}} \]
CKYY/W = \[ C_{K_{YY}} \]

CWBXX/W is a second form of the dimensionless bearing damping in the x-direction:
\[ \frac{C_{w B_{XX}}}{W} \text{, see eq. (B-19)} \]

CWBXY/W = \[ C_{w B_{XY}} \]
CWBXY/W = \[ C_{w B_{YX}} \]
CWBYY/W = \[ C_{w B_{YY}} \]

The nomenclature is:

B_{xx}, B_{xy}, B_{yx}, B_{yy} \hspace{1cm} \text{Damping coefficients for the bearing, lbs sec/inch}
B_{xy}, B_{yx}, B_{yy} \hspace{1cm} \text{Damping coefficients for the bearing, lbs sec/inch}
C \hspace{1cm} \text{The machined radial clearance of the lobes, inch}
D \hspace{1cm} \text{The journal diameter, inch}
\varepsilon_{B} \hspace{1cm} \text{The eccentricity between the journal center and the bearing center, inch}
F_{x0} \hspace{1cm} \text{The static bearing reaction in the x-direction, lbs.}
F_{y0} \hspace{1cm} \text{The static bearing reaction in the y-direction, lbs.}
F_{f} \hspace{1cm} \text{The friction force, lbs (the friction torque = RF_{f})}

-192-
$K_{xx}$, $K_{xy}$, $K_{yx}$, $K_{yy}$  
Spring coefficients for the bearing, lbs/inch

$L$  
The bearing length, inch

$M$  
The critical mass of the journal at the threshold of instability.

$N$  
The rotor speed, rps

$Q$  
The total hydrodynamic bearing flow, cub. inch/sec

$Q_x$  
The hydrodynamic flow in the circumferential direction, cub. inch/sec

$Q_z$  
The hydrodynamic flow out of both bearing sides, cub. inch/sec

$R$  
The journal radius, inch

$Re$  
The Reynolds number for the bearing: $Re = \frac{\nu R \omega C}{\mu}$
dimensionless

$S$  
The Sommerfeld number for the bearing: $S = \frac{\mu NDL}{W} (\frac{R}{\epsilon})^2$
dimensionless

$W$  
The static bearing load, lbs

$\gamma$  
The instability frequency ratio: $\gamma = \frac{\omega}{\omega^*}$
dimensionless

$\xi_0$  
The eccentricity ratio for the bearing, $\xi_0 = \frac{\xi_0}{C}$
dimensionless

$\mu$  
The lubricant viscosity, lbs.sec/inch$^2$

$\nu$  
The frequency at the threshold of instability, radians/sec

$\rho$  
The mass density of the lubricant, lbs.sec$^2$/inch$^4$

$\phi_0$  
The attitude angle for the bearing. It is the angle from the X-axis to the line connecting the bearing center and the journal center, measured in the direction of journal rotation, degrees.

$\omega$  
The angular speed of the journal, radians/sec.

When it is desired to obtain the data for the individual lobes in addition to the composite bearing results, $NPW + 0$ in the input (Item 9, control card). Then there will be output for each lobe for each attitude angle iteration. Each set of output is identified by the title: BEARING PAD NO., followed by the number of the particular lobe (the lobes are numbered consecutively in the
sequence as given in the input). The results are identified by titles:

**SOMMERFELD NO** is the Sommerfeld number for the lobe, \( S_p = \frac{\mu NDLC}{R^2} \)

**ECC. RATIO** is the lobe eccentricity ratio \( \epsilon_p \), see eq. (B-5)

**ATT. ANGLE** is the lobe attitude angle \( \theta_p \), see eq. (B-6), degrees.

**CALC. ATT. ANG** is the angle: \( \tan^{-1} \left( \frac{F_x}{F_y} \right) \), see Appendix I, eqs (A-64), (A-78) and (A-79)

**F-R/S*W** is the dimensionless radial static force component for the lobe: \( f_{r*} = \frac{F_{r*}}{S_p W_p} \)

**F-T/S*W** is the dimensionless tangential static force component for the lobe: \( f_{t*} = \frac{F_{t*}}{S_p W_p} \)

**1/S*(R/C*F/W)** is the dimensionless friction factor for the lobe: \( R \frac{F_{t*}}{C \mu NDLC} = \frac{R}{C} \frac{F_{t*}}{\mu NDLC \epsilon_p^2} \)

**Q-X/NDLC\_IN** is the dimensionless flow into the lobe across the leading edge: \( \frac{Q_x}{NDLC} \)\_IN

**Q-X/NDLC\_OUT** is the dimensionless flow out of the lobe across the trailing edge: \( \frac{Q_x}{NDLC} \)\_OUT

**Q-Z/NDLC\_SIDE** is the dimensionless flow out of the lobe across both sides of the lobe: \( \frac{Q_z}{NDLC} \)

**F-X/S*W** is the dimensionless x-component of the static force on the lobe: \( -\frac{F_{x*}}{S_p W_p} \), see eq (A-78)

**F-Y/S*W** is the dimensionless y-component of the static force on the lobe: \( -\frac{F_{y*}}{S_p W_p} \), see eq. (A-79)

**DFR/SDE** is the dimensionless radial stiffness: \( \frac{\delta F_r}{\delta \epsilon} \), see eqs. (A-59) and (A-60)

**DFT/SDE** is the dimensionless tangential cross-coupling stiffness: \( \frac{\delta F_t}{\delta \epsilon} \), see eq. (A-60)

**DFR/SDA** is the dimensionless radial cross-coupling stiffness: \( \frac{\delta F_r}{\delta \phi} \), see eqs. (A-59) and (A-61)

**DFT/SDA** is the dimensionless tangential stiffness: \( \frac{\delta F_t}{\delta \phi} \), see eq. (A-61)

**DFR/SDADT** is the dimensionless radial damping: \( \frac{\delta F_r}{\delta \phi} \), see eqs. (A-59) and (A-62)

**DFT/SDADT** is the dimensionless radial damping: \( \frac{\delta F_t}{\delta \phi} \), see eqs. (A-62) and (A-63)
DFT/SADDT is the dimensionless tangential damping: \[ \frac{\omega_{it}}{\omega_{p}} \], see eq. (A-63)

\[ \begin{align*}
\text{CXXX/SW} & = \frac{C_{K_{xx}}}{S_{W}} , \quad \text{eq. (A-71)} \\
\text{CKX/SW} & = \frac{C_{K_{x}}}{S_{W}} , \quad \text{eq. (A-72)} \\
\text{CKXY/SW} & = \frac{C_{K_{xy}}}{S_{W}} , \quad \text{eq. (A-73)} \\
\text{CWX/SW} & = \frac{C_{W_{x}}}{S_{W}} , \quad \text{eq. (A-74)} \\
\text{CBYX/SW} & = \frac{C_{B_{y}x}}{S_{W}} , \quad \text{eq. (A-75)} \\
\text{CWBY/SW} & = \frac{C_{W_{by}}}{S_{W}} , \quad \text{eq. (A-76)} \\
\text{CWB/SW} & = \frac{C_{W_{b}}}{S_{W}} , \quad \text{eq. (A-77)}
\end{align*} \]

\[ \text{CM/MUDL(R/C)2} \] is the dimensionless critical journal mass for the lobe: \[ \frac{C_{M_{N}}}{\mu_{DL}(R)^{2}} \]

\[ \text{CMW*/W} \] is another form of the dimensionless critical journal mass: \[ CM_{w}/W_{p} \]

\[ \text{FREQ/W} \] is the instability frequency ratio for the lobe: \[ \frac{\Omega}{\omega_{p}} \]

\[ (\text{FREQ/W})^{2} \] is the square of the instability frequency ratio for the lobe: \[ \frac{\Omega}{\omega_{p}}^{2} \]

If \[ \frac{\Omega}{\omega_{p}} \] is negative, there is no instability threshold for the lobe.

In addition to these data, the output for a lobe also includes the results from the last rupture boundary calculation. First is a line giving the number of iterations and the final error (the left hand side of the equation given in the discussion of the input data for CVLR). This is followed by a 5 column list, giving the coordinates of the rupture boundary. The five columns are identified by titles:

- \( \text{I} \) is the i-index for the gridline (see fig. 55., Appendix I).
- \( \text{BEGIN, INDEX} \) gives the j-coordinate (see fig. 55.) of the rupture boundary at the leading edge. If \( j=1 \), leading edge rupture does not occur.
- \( \text{END INDEX} \) gives the j-coordinate (see fig. 55.) of the rupture boundary at the trailing edge. If \( j=M+1 \) (item 1, control card in input), trailing edge rupture does not occur.
- \( \text{BEGIN, ANGLE} \) gives the angular coordinate in degrees of the rupture boundary at the leading edge. The angle is measured from the line connecting the bearing center and the journal center.
**END ANGLE** gives the angular coordinate in degrees of the rupture boundary at the trailing edge. The angle is measured in the same way as for the leading edge angle.

When **NPW** = -1 (item 10, control card in input), the above data are repeated for each rupture calculation.

When **NPW** = -1, the output includes the pressure distribution for each lobe. It is identified in the output by the title: **PRESSURE DISTRIBUTION**. The dimensionless pressure \( P = \frac{F}{\mu \omega (\frac{E}{L})^2} \frac{\ell_0}{D} \). (see eq. (A-7)), where \( P \) is the actual pressure in psi, is listed starting at the leading edge of the lobe (i.e. at \( j = 0 \) in figure 55). The first value in each line is at \( i = 1 \) and the last value is at the centerline \( (i = N) \), see fig. 55.

When **NPUN** = 1 or -2 (item 11, control card in input), punched card output will be given. The first card is given at the start of a calculation for a new value of the L/D-ratio or new value of the Reynolds number (it is only given if **NPAD** = 1). The card contains four values:

**Card (4E13.5)**

1. **L/D**
2. The lobe arc, degrees ( \( \Theta_{p,ext} - \Theta_{p,ini} \) )
3. The relative angular location of the center of pressure ( \( \frac{180^\circ - \Theta_{p,ini}}{(\Theta_{p,ext} - \Theta_{p,ini})} \) )
4. The Reynolds number \( \Re \)

This card is followed by the results of the composite bearing for each value of \( E_0 \cos \Psi_0 \) (and if **NPUN** = -2, also for each value of \( E_0 \sin \Psi_0 \) ), there will be two cards:

**Card (5E13.5)**

1. \( E_0 \cos \Psi_0 \)
2. \( 1/\xi = W/\mu \pi DL \left( \frac{\ell_0}{c} \right)^2 \), the inverse Sommerfeld number
3. The friction factor: \( \frac{D}{c} \left( \frac{\frac{F_x}{c}}{\frac{\frac{F_y}{c}}{\pi DL WC_0}} \right) \)
4. The dimensionless flow across the leading edge: \( \frac{\Theta_{c}}{NDLc} \)
5. The bearing eccentricity ratio \( E_0 \)
Card (5E13.5)

1. The dimensionless radial stiffness for an inertialess tilting pad:
\[
\frac{C}{SW} \left[ K_{xx} - \Re \left\{ \frac{(K_{yy} + i\omega B_{xy})(K_{yy} + i\omega B_{yy})}{K_{yy} + i\omega B_{yy}} \right\} \right]
\]

2. The dimensionless radial damping for an inertialess tilting pad:
\[
\frac{C}{SW} \left[ \omega B_{xy} - \Im \left\{ \frac{(K_{yy} + i\omega B_{xy})(K_{yy} + i\omega B_{yy})}{K_{yy} + i\omega B_{yy}} \right\} \right]
\]

3. The dimensionless stiffness for the y-direction:
\[
\frac{C_k}{SW}
\]

4. The dimensionless damping for the y-direction:
\[
\frac{C_d}{SW}
\]

5. The attitude angle \( \phi_y \), degrees
INPUT FORM FOR COMPUTER PROGRAM
PN0375: THE STATIC AND DYNAMIC PERFORMANCE OF A LOADED BEARING WITH TURBULENT FILM

Card 1 (15)

NRENT = Number of film Reynolds numbers in table for turbulent flow coefficients

Table of Turbulent Flow Coefficients (4E15.7)

For each Reynolds number, give two cards with four values per card.

First Card:
1. \( R_h = \frac{\rho \omega R_h}{\mu} \), the film Reynolds number (dimensionless)
2. \( \frac{1}{2} c_f R_h \), the friction factor (dimensionless). For \( R_h = 0 \), \( \frac{1}{2} c_f R_h = 1 \)
3. \( G_w \), the turbulent flow coefficient for the circumferential direction (dimensionless). For \( R_h = 0 \), \( G_w = 1 \)
4. \( \frac{1}{c_x} \frac{d}{dR_h} G_w \), the first derivative of \( G_w \) divided by \( c_x \) (dimensionless) For \( R_h = 0 \), \( \frac{1}{c_x} \frac{d}{dR_h} G_w = 0 \)

Second Card:
1. \( \frac{1}{c_x} \frac{d}{dR_h} G_w \), the second derivative of \( G_w \) divided by \( c_x \) (dimensionless). For \( R_h = 0 \), \( \frac{1}{c_x} \frac{d}{dR_h} G_w = 0 \)
2. \( \frac{1}{c_x} \frac{d^2}{dR_h^2} G_w \), the third derivative of \( G_w \) divided by \( c_x \) (dimensionless). For \( R_h = 0 \), \( \frac{1}{c_x} \frac{d^2}{dR_h^2} G_w = 0 \)
3. \( \frac{G_T}{c_T} \), where \( c_T \) is the turbulent flow coefficient for the axial direction (dimensionless). For \( R_h = 0 \), \( \frac{G_T}{c_T} = 1 \)
4. \( \frac{d}{dR_h} \left( \frac{G_T}{c_T} \right) \), the first derivative of \( \frac{G_T}{c_T} \) (dimensionless). It is zero for \( R_h = 0 \).

Card (72)

Text
Control Card (1215)

1. **M** = Number of finite difference increments in the circumferential direction per lobe (3 ≤ M ≤ 36)
2. **N** = Number of finite difference increments in the axial direction on half the bearing length (3 ≤ N ≤ 13)
3. **NLQ** = Number of L/D-ratios in input list (1 ≤ NLQ ≤ 20)
4. **NRE** = Number of bearing Reynolds numbers, \( R_e \), in input list (1 ≤ NRE ≤ 30)
5. **NPAD** = Number of lobes (1 ≤ NPAD ≤ 12)
6. **NECA** = Number of \( \varepsilon_0 \cos \phi_0 \) - values in input list (1 ≤ NECA ≤ 20)
7. **NAB** = Maximum allowable number of attitude angle iterations.
8. **NRP**; If NRP ≥ 1: Maximum allowable number of iterations to determine rupture boundary.

   - If NRP = 0: Film rupture tables place, but there are no iterations.
   - If NRP = -1: No film rupture, and no iterations.
9. **NPW**: If NPW = 0: Only the composite bearing results are given, not the results for the individual lobes.

   - If NPW = 1: The results for the individual pads and the composite bearing are given, but not the pressure distribution.
   - If NPW = -1: Same as for NPW = 1, but with pressure distribution.
10. **NRW**: If NRW = 0: No results are given from the rupture boundary iteration.

     - If NRW = 1: The convergence error for each rupture boundary iteration is given.
     - If NRW = -1: Same as for NRW = 1, but output also includes the coordinates of the rupture boundary for each iteration.
     - If NRW = -2: Diagnostic for rupture boundary calculation.
11. **NPUN**: If NPUN = 0: The composite bearing results are only given after the final attitude angle calculation, no cards are punched.

     - If NPUN = 1: Same as NPUN = 0, but the results are also punched on cards.
     - If NPUN = -1: The composite bearing results are given after each attitude angle calculation, no cards are punched.
If NPUN = -2: Same as NPUN = -1, but the results are also punched on cards.

12. INF; If INF = 0: More input data follows the present set of data, starting with the text card.
   If INF = 1: This is the last set of input data.

Card (1P5E14.6)

1. CVLA = Convergence limit for attitude angle calculation.
2. CVLR = Convergence limit for rupture boundary iteration.

List of L/D-Ratios (1P5E14.6)

Give NLD-values of the length-to-diameter ratio, L/D, 5 values per card.

List of Bearing Reynolds Numbers (1P5E14.6)

Give NRe-values of the bearing Reynolds number, Re, 5 values per card.

Data for Journal Center Position

The following two input items, namely the card with \( \varepsilon_0 \cos \phi_0 \) and the list of \( \varepsilon_0 \sin \phi_0 \), must be repeated together NECA times (Item 6, control card).

Card (1P5E14.6, 15)

1. \( \varepsilon_0 \cos \phi_0 \)
2. NAT; If NAT = 1: Number of \( \varepsilon_0 \sin \phi_0 \)-values in input list which follows. The program iterates on the attitude angle to make horizontal force zero (1 \( \leq \) NAT \( \leq \) 25)
   If NAT \( \leq \) -1: The absolute value of NAT gives the number of \( \varepsilon_0 \sin \phi_0 \) values in input list which follows (1 \( \leq |NAT| \leq 25 \)). No attitude angle iteration takes place. Set NAB = |NAT| , Item 7, control card.
List of $\varepsilon_0 \sin \varphi_0$ Values (LPE14.6)

Give |NAT| -values of $\varepsilon_0 \sin \varphi_0$, 5 values per card.

Bearing Geometry Data (LPE14.6)

Give NPAD cards (Item 5, control card). Each card contains 4 values:

1. $\Theta_{in}$, degrees. The angle from the load line to the leading edge of the lobe

2. $\Theta_{out}$, degrees. The angle from the load line to the trailing edge of the lobe

3. $\delta_p = \frac{r_p}{R_p}$ . The lobe preload ($0 \leq \delta_p$), dimensionless.

4. $\psi_p$ . Preload angle, degrees.
DELY3=1.0/0Z/DZ
DZ2=DELY3+DELY3
SP0D=1.2471976/C2
SP0D=0.321816616/C2
SP0D=0.78539816/C2
SP0D=0.34363323/AL0/C2/AL0
SP0D=0.17453293/C2
RE=1
21 RE=REVN(NRE)
NEC1=1
22 ECA=ECAST(NEC1)
NAT=NATS(NEC1)
IF(NAT)=31,24,24
23 NAFI=NAT
J1=NAFI
GO TO 29
24 NAFI=NA
J1=NAT
25 CC=CE N1,1,1
26 EAS(J1)=ESASTA(NEC1,J1)
EAS=ESASTA(N)
NA=1
NA1=0
NA2=1
32 DP0=1
34 DX=(THT(IPD)-THN(IPD))/HM*I.017453293
DELY3=1.0/DX/DX
DELY3=1.0/DY/DY
SPD=SP0D/DX
IF(NA)=30,27,30
27 IF(NPUN)=28,30,29
28 IF(NPUN)=29,30,30
29 C1=THT(IPD)-THN(IPD)
C2=(180.0-THN(IPD))/C1
WRITE(1,114)ALD,C1,C2,RE,NPUN
NPUN=NPUN+1
30 C1=0.017453293*TAN(IPD)
C2=SENC1,1
C1=COS(C1)
C1=ECASC1*PRLD(IPD)
C2=SSA*C2*PRLD(IPD)
ECASQR(C1+C2*C2)
IFEL=ECAS/115,13,18
15 WRITE(1,140)IPD
IF(NA=NAFI)16,17,17
16 NA=NAFI
GO TO 443
17 WRITE(16,134)
GO TO 485
18 ATP=TANV(C1,C2)
C1=ATP+180.0
KRB=1
KRE=1
KBE=1
C2=THN(IPD)
C3=C2+360.0
C4=THN(IPD)
C5=C4+360.0
35 IF(C1=360.0)37,36,36
36 C1=C1-360.0
GO TO 35
37 IF(ATP+C4)39,38,38

-203-
30 IF (CL = C4) G0, 43, 43
31 IF (ATP = C2) 40, 40, 44
32 IF (ATP = C4) 44, 41, 41
33 IF (C2 = C1) 46, 42, 42
34 IF (C1 = C6) 45, 43, 43
35 KBE = 0
36 GO TO 49
37 MBE = 0
38 IF (CL = C4) 46, 49, 49
39 IF (CL = C2) 47, 47, 48
40 IF (CL = C5) 48, 49, 49
41 KRE = 0
42 D13 = 0, 0
43 CSJ = C2-ATP
44 LP = C4-C2
45 C3 = +0.017453293#CSJ
46 DON 60 J = 1, MB
47 C1 = COS(C3)
48 C2 = SIN(C3)
49 CSJ = C1
50 CSJ = C2
51 C4 = ECC#C1
52 C5 = ECC#C2
53 C6 = I#C4
54 C7 = C5#C9
55 C8 = SQRT(C6)
56 C9 = C8#C8
57 IF (RE) 50, 50, 51
58 D1 = 1, 0
59 D2 = 1, 0
60 D3 = 0, 0
61 D4 = 0, 0
62 D5 = 0, 0
63 D6 = 0, 0
64 D7 = 1, 0
65 D12 = 1, 0
66 GO TO 56
67 D8 = RE#C6
68 D0 '55 X = M, NRET
69 IF (D0 = RHI(K)) 52, 52, 53
70 D9 = RHI(K) - RHI(K-1)
71 D6 = (RHI(K) - D8) / D9
72 D7 = (D0 - RHI(K-1)) / D9
73 D1 = D6 * XG3(K-1) + D7 * XG4(K)
74 D12 = D6 * CFR(K-1) + D7 * CFR(K)
75 D14 = D6 * XG4(K-1) + D7 * XG4(K)
76 C2 = SQRT(D1)
77 D3 = RE * D6 * XG1(K-1) + D7 * XG1(K)
78 D4 = RE * D6 * XG2(K-1) + D7 * XG2(K)
79 D5 = RE * D6 * XG3(K-1) + D7 * XG3(K)
80 D6 = RE * D6 * XG5(K-1) + D7 * XG5(K)
81 D7 = D14
82 D11 = C9#D2
83 H32(J) = U11
84 GO TO 56
85 CONTINUE
86 D11 = C9#D2
87 M32(J) = U11
GO TO 318
317 A(I,1-I)+C7+G8
318 B(I,J)=0L
C(I)=U2
319 CONTINUE
320 DO 322 I=1,N
CJ=321 K=1,N
321 AK(I,K)=A(I,K)+B(J,1)*D(J-1,1,K)
322 CONTINUE
323 CALL MATINV(AK,N,GK,J,DET,ID)
GO TO (324,323,1,1D)
323 WRITE(6,139)IPD
GO TO 425
324 DO 325 I=1,N
325 C=AK(I,K)
326 CONTINUE
327 CALL PRESS(J51,JF1,N,PR)
328 IF(NR) 341,344,344
341 WRITE(6,117)NI
WRITE(6,118)
DO 342 I=1,N
342 WRITE(6,119),sx(I),EX(I),TS(I),TH(I)
343 WRITE(6,120)
344 DO 347 J=J0,JF
CJ=321(J)
345 IF(J,1)C4=EC
346 WRITE(6,121)(P(J,1),I=1,N)
347 CONTINUE
348 CALL SIMP(N,FR,FT)
349 FR=SP+DFR
350 FT=SP+DFT
351 BRDA=FR-FT
352 BTDA=FT-FT
353 FR=EC+FR
354 FT=EC+FT
355 SMFV=SORT(FR*FR+FT*FT)
356 IF(SMFV) 423,423,349
357 SMFV=L+0/SMFV
358 ATT=TANV(FR,FT)
359 CFP1(10)=3.1415927+13+1.5*EC*FT
360 C1=0.0745329*ATP
361 CATP=CATP(C1)
362 SATP=SIN(C1)
363 FXP(10)=FR+CATP+FT+SATP
364 FYP(10)=FT+CATP-FT+SATP
365 IF(KRB) 351,362,351
366 IF(N/21#2-N) 352,353,353
357 QXS=16.0*P(2,N)-4.0*P(3,N)
358 J1=N-2
359 GO T0 354
360 QXS=2.0*P(3,N)-6.0*P(2,N)
361 J1=N-1
362 DO 355 I=1,1,J2
363 QXS=QXS+32.0*P(2,1)-8.0*P(3,1)+16.0*P(2,11)-4.0*P(3,11)
364 J1=N+1
365 DO 355 I=1,N+1
366 GO T0 354
367 QXS=QXS+32.0*P(2,1)-8.0*P(3,1)+16.0*P(2,11)-4.0*P(3,11)
368 J1=N+1
369 GO T0 354
370 QXS=QXS+32.0*P(2,1)-8.0*P(3,1)+16.0*P(2,11)-4.0*P(3,11)

-207-
186 IF(NNP=314) 387,331,301
187 JI=0
188 IF(NPM) 384,392,384
189 WRITE(6,122)INIT,ERR
190 WRITE(6,121)DO 389 I=1,N
191 WRITE(6,121)I,SX(I),EX(I),THS(I),THE(I)
192 IF(NPM) 390,392,392
193 WRITE(6,120)DO 391 J=1,M
194 WRITE(6,121)(P(J,II),I=1,N)
195 DO 393 J=1,M
196 IF(J=1) THEN 400
197 WRITE(6,121)(J=1,JF1)
198 C1=AF1(J)
199 C2=AF2(J)
200 C3=AF3(J)
201 KJ=KI(J)
202 IF(KJ) 394,394,395
203 IF(IN=1) 399,399,400
204 WRITE(6,121)(J=1,JF1)
205 C1=AF1(J)
206 C2=AF2(J)
207 C3=AF3(J)
208 MJ=KI(J)
209 IF(MJ) 405,405,406
210 IF(J1=1) THEN 411
211 IF(IN=1) 410,410,411
212 WRITE(6,121)(J=1,JF1)
213 C1=AF1(J)
214 C3=AF3(J)
215 C1=AF1(J)
216 MJ=KI(J)
217 IF(MJ) 405,405,406
218 IF(J1=1) THEN 411
219 IF(IN=1) 410,410,411
220 WRITE(6,121)(J=1,JF1)
413 CONTINUE
CALL P355(JS1,JF1,N,-1)
DO 415 JS,JF
C1=H32(JJ)
C2=AF9(JJ)
DO 414 JS,JF
414 E(J,J)=E(J,J)/C1+C2*(P(J,J))
415 CONTINUE
CALL SIMP(N,C1,C2)
DRDA=SPD*C1
TDTA=SPD*C2
DO 419 JS,JF1
C1=AF3(JJ)
IF(JJ) 416,417,416
416 JS,JF1
417 DO 418 JS,JF
418 F(J,J)=C1
419 CONTINUE
CALL P355(JS1,JF1,N,-1)
PRI 421 JS,JF
C1=H32(JJ)
DO 420 JS,JF
-420 E(J,J)=E(J,J)/C1
421 CONTINUE
CALL SIMP(N,C1,C2)
BRDE=SPD*C1
TDTE=SPD*C2
C1=DRDE=CATP*OTDE*SATP
C2=DRDA=CATP*TDTA*SATP
SXPJ(IPD)=C1*CATP=C2*SATP
SYPJ(IPD)=C1*SATP=C2*CATP
C1=BRDE=CATP*BTDE*SATP
C2=BRDA=CATP*TDTSATP
BXPJ(IPD)=C1*CATP=C2*SATP
BYPJ(IPD)=C1*SATP=C2*CATP
C1=BRDE=CATP*BTDE*SATP
C2=BRDA=CATP*TDTSATP
BYPJ(IPD)=C1*CATP=C2*SATP
BYPJ(IPD)=C1*SATP=C2*CATP
C1=IF(SXPJ(IPD)-C1)*SYXP(IPD)-SXPJ(IPD)*BYPJ(IPD)
L=SYXP(IPD)*BYPJ(IPD)/(BYPJ(IPD)/SYPJ(IPD)).
C2=(SXPJ(IPD)-C1)*SYXP(IPD)-SYPJ(IPD))/
L(BXPJ(IPD)*BYPJ(IPD)-BYPJ(IPD)*BXPJ(IPD))
C1=C2/C1
C3=SQR(TABS(C2))
C4=SMP*C1
C1=C1/39,478418
IF(NP) 422,425,422
422 WRITE(6,126)
WRITE(6,127)
WRITE(6,128)
WRITE(6,129)
WRITE(6,123)
GO TO 423
423 CFPJ(IPD)=3,1415927#D13
CXL(IPD)=1.5707963*(1.0+6C*4C1)
QXT(IPD)=QXL(IPD)
QZT(IPD)=.0.0
FXP(IPD)=.0.0
FYR(IPD)=.0.0
SXXP(IPD)=.0.0
SYX(IPD)=.0.0
SYY(IPD)=.0.0
BXXP(IPD)=.0.0
BYR(IPD)=.0.0
BYY(IPD)=.0.0
IF(NPAD=1) 424,425,426
426 WRITE(6,129)
WRITE(6,130)
WRITE(6,123)CFP(IPD),QXL(IPD)
IF(NPAD=1) 427,428
428 IF(IP=1) 427,428
427 IF(IP=1) 427,428
426 FXB=5.0
FYB=0.0
SXXB=0.0
SYXB=0.0
SYYB=0.0
BXXB=.0.0
BYYB=0.0
IF(NPAD=1) 427,428
428 C1=QXT(IPD)-QXL(IPD)
GO TO 429
429 C1=QXT(IPD)-QXL(IPD)
430 C1=0.0
431 CBT=CBT+C1
CXLH=QXLH+QXL(IPD)
C2TH=Q2TH+Q2T(IPD)
PCFW=PCFW+CFP(IPD)
FXB=FXB+FXP(IPD)
FYB=FYB+FYP(IPD)
SXXB=SXXB+SXXP(IPD)
SYXB=SYXB+SYX(IPD)
SYYB=SYYB+SYY(IPD)
BXXB=BXXB+BXBP(IPD)
BYYB=BYYB+BYBP(IPD)
432 BLYB=BLXB+BLX(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
QB1=QB1+Q2T(IPD)
HERE 211

WRITE (6,131) ESA, FY8
WRITE (6,139)
WRITE (6,125) FXB, RFCW, QZTB; QBT, QXLR, SMFV
WRITE (6,127)
WRITE (6,123) SXH, SXYB, SXYB, SYX8, BXYB, BXYB, SYVR
1440 IF (NAI = NA01) 442,441,441
1441 WRITE (6,134)
1442 GO TO 475
1443 ESA = ESAS (NAI)
1444 IF (FXB) 446,445,447
1445 FXB = 1.0
1446 IF (NAI) 447,452,447
1447 IF (CVLA-ABS (FY8/FXB)) 448,475,475
1448 IF (NAI) 449,449,450
1449 IF (NAI) 451,451,450
1450 NA2 = 0
1451 IF (FY8*X (1)) 453,475,452
1452 X (1) = FY8
1453 NA1 = 1
1454 ESA = (Y (1) + ESA) / 2.0
1455 IF (ESA = Y (2)) 456,456,456
1456 IF (X (2) * X (3)) 457,457,458
1457 X (1) = X (2)
1458 Y (1) = Y (2)
1459 X (2) = ESA
1460 X (3) = FY8
1461 IF (X (1) * X (2)) 462,462,463
1462 X (3) = X (2)
1463 IF (Y (1)) 464,464,460
1464 X (1) = FY8
1465 C1 = Y (2) - Y (1)
1466 Y (2) = C2
1467 C3 = Y (3) - Y (1)
1468 C4 = X (1) - X (2) / (C2*C3)
1469 C5 = (X (1) - X (2)) / (C1*C3)
1470 C3 = C4 + C5

-212-
WRITE(10,114) BXXB,BYXB,BYXB,BYXB
NPUNI=NPUNI+1
485 NE1=NECI+1
IF(INEC=NECI) 486,22,22
486 NE1=NE1+1
IF(INEC=NECI) 487,21,21
487 NLDO-NLDI+1
IF(INLC=NLDI) 488,20,20
488 IF(INPI) 489,12,489
489 STOP
98 FORMAT(5E15.7)
100 FORMAT(72H1)

101 FORMAT(12I5)
102 FORMAT(1PE14.6)
103 FORMAT(1PE14.6)
104 FORMAT(1HO,11THCIRC,DIV AX*DIV NO/LD NO*REYN NO*PAD N

IFNECA-NECI)
105 FORMAT(16,1110)
106 FORMAT(1HO,27THCONV,USUM,A CONV,USUM,RUP)
107 FORMAT(1HO,10H/LD RATIO)
108 FORMAT(1HO,16HREYNOLDS NUMBERS)
109 FORMAT(1HO,17HEC*COS(ATT) DATA)
110 FORMAT(1HO,13HEC*COS(ATT) =1PE13.6,14H NO.E*SIN(A)=13).
111 FORMAT(2H LIST OF ECC*SIN(ATT))
112 FORMAT(1HO,21HBEARING GEOMETRY DATA)
113 FORMAT(1HO, M INLET ANGLE OUTLET ANGLE PRELOAD ANGLE PRLO-X)
114 FORMAT(5E13.5,128)
115 FORMAT(5E13.5,115)
116 FORMAT(1HL,19HBEARING PAD NO.,12)
117 FORMAT(///1HO,21HRUPTURE ITERATION NO.,12)
118 FORMAT(1HO,50H I BEGIN INDEX END INDEX BEGIN ANGLE E

119 FORMAT(13,1PE15.7)
120 FORMAT(1HO,21HPRESSURE DISTRIBUTION)
121 FORMAT(1PE12.6,1PE11.4)
122 FORMAT(22H RUPTURE ITERATION NO.,9H ERROR=1PE13.6)
123 FORMAT(1PE15.7)

124 FORMAT(1HO,9HSDOMPERFELD NO ECC.RATIO ATT*ANGLE CALC.*AT

125 FORMAT(1HO,12H F=R/5W F=T/5W)
126 FORMAT(1HO,7H DFR=6X,7HDF=6X,7HDF=6X,7HDF=6X,7HDF=6X,7HDF=6X,7HDF=6X,7HDF=6X

127 FORMAT(1HO,6X,7HCMX/SW,7HCMX/SW,7HCMX/SW,7HCMX/SW,7HCMX/SW,7HCMX/SW,7HCMX/SW,7HCMX/SW

128 FORMAT(1HO,6X,7HCMN/MVOLIR/C)2 CMW**2/FW FREQ/W (FREQ

129 FORMAT(1HO,58HCMN/MVOLIR/C)2 CMW**2/FW (FREQ

130 FORMAT(1HO,15HPAD IS UNLOADED)
131 FORMAT(1HO,28H I/SMR/C*F/W) (Q-X/NDLC)IN (Q-Z/NO

132 FORMAT(1HO,33HBEARING RESULTS,ATT,ANGLE IT=NO.,12)
133 FORMAT(1HO,40HNUMBER OF ATT,ANGLE ITERATIONS TOO LARGE

134 FORMAT(1HO,6X,9HREYN NO. ECC*SIN(ATT) ECC*SIN(A

135 FORMAT(1HO,12H CIRC F-X/SW CMN/MVOLIR/C)12 FREQ/W (FREQ

136 FORMAT(1HO,72H SUMMERFELD NO F-X/W F-Y/W (R/C

-214-
110 (F/W) CMW#2(W)
138 FORMAT(IHJ,5X,6HCKXY/W,9X,6HCKXY/W,9X,6HCKYY/W,9X,6HCKXY/W,9X,6HCKYY/W)
    18X/4,8X,THCMBXY/W,8X,THCMBXY/W,8X,THCMBXY/W,8X,THCMBXY/W
139 FORMAT(IHJ,26H/MATRIX IS SINGULAR,PAK NO.,12)
140 FORMAT(IHJ,41H/ECCENTRICITY RATIO GREATER THAN 1,PAK NO.,12)
141 FORMAT(IHJ,34H/MINIMUM FILM THICKNESS FOR PAK NO.,1)
142 FORMAT(IHJ,12,4)
143 FORMAT(17,912)
END

510B/T.C SUMPT DECK
SUBROUTINE SUMPT(M,N,KRB,KRE,MP,JS,JF,JS1,JF1,ER=1)
    DIMENSION D(37,13,13),G(37,13,13),B(37,13,13),E(37,13,13),F(37,13,13),
    IP(37,13),ALFW(37,13),ALFM(37,13),H32(37,13),THA(37,13),AS1(37,13),
    2AG4(37),CS(37),SS(37),8T(37),K1(37),JSX(13),JEX(13),SX(13),EX(13),
    SMS(13),THE(13)
    COMMON DO,4,E,F,P,ALFW,ALFM,H32,THA,AG4,CS,SS,AT,K1,JSX,
    1JEX,SX,EX,TH5,THE,DX,D2,CSII,ALFM
M=N
N=N
KRB=KRB
KRE=KRE
MP=MP
JS=JS
JF=JF
JF1=JF1
JS1=JS1
ECC=ER
N1=M+1
M=M+1
NC=N+1
N2=NC+2
SER1=1.0
SER2=0.0
SER3=1.0
SER4=0.0
OR=0.1
JS=1
JS1=2
JF=M
JF1=M
DO 502 J=1,M8
BT(IIJ)=1.0
MIIJ=0
DO 501 I=1,N
ALFM(I,J)=1.0
501 ALFW(I,J)=1.0
502 CONTINUE
C1=M8
CD 503 I=1,N
THS(IJ)=CSII
THE(J)=CSII
ALPH
JSX(IJ)=2
503 JEX(IJ)=M
DO 502 I=1,N
J1=NC-1
K7=I-1
C1=K7
Y(4)=CI+OZ
K7=5
IF(J1=11) 509, 508, 507
507 K5=1
GO TO 510

-215-
EF(C1.+C5. 549,556,547
547 IF(C6+Y1) 535,548,548
548 571+C1
C7=V1
C6=THC*K+K6
KZ=1
GO TO 557
549 K4=1
C9=C6+0.5+DR
GO TO 556
550 K4=1
03=(C1+C3-C5)/DR+0.0/DR
04=(C3-C1)/OR
IF(03) 552,551,552
551 05=(-C5/04
GO TO 555
552 06=04/03*0.5
07=SQR((A8-2-C5/03))
IF(06) 553,554,554
553 09=06+07
GO TO 555
554 09=06+07
555 C6=C6+05
GO TO 556
556 IF(C6+Y1) 557,558,558
557 K4=0
Y1=07
THC=C5
IF(K5) 559,560,558
558 Z1=C6
Y1=0.0
GO TO 551
559 03=Y1+Y1
03=(C6-Z2)/D3
04=0.0
ZI=03*0D*0Z+Z2
GO TO 561
560 05=Y1+Y1
06=04+02
07=(Z3-Z2)/06
08=(C6-Z2)/08
04=05*06
03=(07+08)/03
04=08*06=07+05/04
Z1=0Z*0D*03+0A*22
561 IF(Z1+DXX) 565,566,562
562 K1(K1)=0
JP=KJ
563 XX=XX+1
JP=JP+XX1
Z1=Z1+DX
Z2=Z2+DX
C6=C6+DX
IF(Z1) 564,564,563
564 Z1=Z1+DX
Z2=Z2+DX
C6=C6+DX
XX=XX+1
JP=JP+XX1
565 IF(K5) 566,566,580
566 IF(Z1-Z2) 568,567,567
-218-
SUBROUTINE PRESSIJITJ2T5C)

DIMENSION D(37,13),G(37,13),B(37,13),E(37,13),F(37,13),
1P(37,13),ALFAP(37,13),ALFAM(37,13),AG1(37,13),AG2(37,13),
2AG4(37,13),CS(37,13),SS(37,13),BT(37,13),KI(37,13),JK(37,13),JSX(37,13),JEX(13),
JEX2(13),EX(13),
COMMON D,G,B,E,F,P,ALFAP,ALFAM,AG1,AG2,CS,SS,BT,KI,JSX,
JEX,SS,EX,THS,THE,DX,DZ,CSII,ALPH,
J1=J1T,
J2=J2T,
NC=NCT,
N2=NZT,
CC 503 J=J1,J2
CC 502 I=I,NC
CC 501 K=K,NC

501 CC=C+{(F(J,K)=B(J,K)*E(J-1,K))*G(J)}
502 CC=C
503 CONTINUE

504 CC=C+{(F(J,K)=E(J,K))}

IF(NZ) 505,506,507
505 IF(C) 506,506,507
506 C=J,0
507 CC=C+1
508 CONTINUE

END

$1BFTE SUNSIM DECK
SUBROUTINE SIMP(NCT,FRT,FTT)
DINMENSION O(37,13,13),G(37,13,13),B(37,13),E(37,13),F(37,13),
1F(37,13),ALFAM(37,13),ALFAMH(37,13),H(37,13),THA(37),ABG(37),
20A(37),C(3),SS(37),BT(37),K(37),JSX(13),JEX(13),SX(13),EX(13),
THS(13),THE(13)
COMMON D,G,B,E,F,P,ALFAM,ALFAMH,TMA,ABG,C,SS,BT,K,JSX,
JEX,SX,STH,SHE,DX,DX,CSII,ALPH
DINMENSION SA(13),ST(13)
NC=NCT
DO 710 I=1,NC
J=JSX(I)
JE=JEX(I)
AS=ALFAM(J)
AE=ALFAM(J)
NX=JE-JS
J2=JE-1
C1=0(JS,1)
C2=(1.0-AE)*E(JE,1)
C3=C2*CS(JE)
C4=C2*SS(JE)
1F(KK/2)*2-KK) 704,703,703
703 C1=(1.0+AS)*C1
C3=C3+C1*CS(JS)
C4=C4+C1*SS(JS)
J1=JS+1
GO TO 708
704 C5=(1.0+AS)*(3.0-AS)/4.0
1F(AS=0.0) 705,705,706
705 C6=0.0
C5=C5/3.0+4.0
GO TO 707
706 C6=1.0+AS
C6=C6/AS*6+2
C6=C6*C6
707 C5=(1.0+C5)*E(JS+1,1)
C6=C6*C6
C3=C3+C6*CS(JS)-C5*C5*CS(JS+1)
C4=C4+C6*SS(JS)+C5*SS(JS+1)
J1=JS+2
708 DO 709 J=J1,J2,2
C3=C3+4.0*E(J,1)*CS(J)-2.0*E(J,1)*CS(J+1)
709 C4=C4+4.0*E(J,1)*SS(J)+2.0*E(J,1)*SS(J+1)
SR(J)=C3
710 ST(I)=C6
Cl=SR(NC)
C2=ST(INC)
1F(KC/2)*2-NC) 711,712,712
711 J1=NC-2
FR=4.0*C1
FF=4.0*C2
GO TO 713
712 J1=NC-1
FR=2.0*C1
FF=2.0*C2
713 DO 714 J=1,J1+2
FR=FR+8.0*SR(J)+4.0*SR(J+1)
714 FT=FT+8.0*ST(I)+4.0*ST(I+1)
FR=FR
PTT=PT
RETURN
END
$FTC MAINVR DECK

-221-
SUBROUTINE MATHINV(A, N, L, DETERM, IOM)

DIMENSION A(13,13), B(13,1)
DIMENSION INDEX(13,1)
EQUIVALENCE (IROW, JROW), (ICOLM, JCOLM), (AMAX, T, SWAP)

INITIALIZATION

M=N
N=N
DO 8 I=1,N
  K1=1
  K2=1
  DO 6 J=1,N
      IF(A(I,J)) 3,4,3
      3 K1=0
      4 IF(A(I,J)) 5,6,5
      5 M2=0
      6 CONTINUE
      IF(K1+K2) 8,8,7
      7 ID=2
      DETERM=0.0
      GO TO 740
      8 CONTINUE
      10 DETERM=1.0
      15 DO 20 J=1,N
      20 INDX(J,3)= 0
      30 DO 550 I=1,N

SEARCH FOR PIVOT ELEMENT

40 AMAX=0.0
45 DO 105 J=1,N
    IF(INDX(J,3)=1) 60, 105, 60
    60 DO 100 K=1,N
        IF(INDX(K,3)=1) 80, 100, 715
        80 IF(AMAX-ABS(A(J,K))) 85,100,100
        85 IROW=J
        90 ICOLM=K
        AMAX=ABS(A(J,K))
        100 CONTINUE
        105 CONTINUE
INDEX(ICOLM,3) = INDEX(ICOLM,3) +1
260 INDEX(I,1)=IROW
270 INDEX(I,2)=ICOLM

INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL

130 IF (IROW-ICOLM) 140, 310, 140
140 DETERM=-DETERM
150 DO 200 L=1,N
160 SWAP=A(IROW,L)
170 A(IROW,L)=A(ICOLM,L)
200 A(ICOLM,L)=SWAP
IF(M) 310, 310, 210
210 CO 25G L=1, N
220 SWAP=B(IROW,L)
230 R(IROWN,L)=S(ICOLUML,L)
250 B(ICOLUML,L)=SWAP
C
DIVIDE PIVOT RCW BY PIVOT ELEMENT
C
310 PIVOT =A(ICOLUML,I)
DETERM=DETERM*PIVOT
330 A(ICOLUML,I)=A(I)
340 DO 150 L=1,N
350 A(ICOLUML,L)=A(ICOLUML,L)/PIVOT
355 IF(M) 380, 380, 360
360 DO 370 L=1,N
370 B.ICOLUML,L)=B.ICOLUML,L)/PIVOT
C
REDUCE Rynes-PIVOT ROWS
C
380 DO 550 L=1,N
390 IF(LL=ICOLUML) 400, 550, 400
400 T=A(,L,I)
420 A(LL,ICOLUML)=0.0
430 DO 450 L=1,N
450 A(LL,L)=A(LL,L)-A(ICOLUML,L)*T
455 IF(M) 550, 550, 460
460 DO 500 L=1,N
500 B(LL,L)=B(LL,L)-B.ICOLUML,L)*T
550 CONTINUE
C
INTERCHANGE COLUMNS
C
600 DO 710 I=1,N
610 L=L+1
620 IF (INDEX(L,1)=INDEX(L,2)) 630, 710, 63
630 JROW=INDEX(L,1)
640 JCOLUMN=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=K,JROW)
670 AIK,JROW)=AIK,JCOLUMN)
700 AIK,JCOLUMN)=SWAP
705 CONTINUE
710 CONTINUE
720 DO 730 K=1,N
730 IF(INDEX(K,3) -1) 715,720,715
715 ID =2
GO TO 740
720 CONTINUE
730 CONTINUE
740 CONTINUE
C
LIST CARD OF PROGRAM
END
C
SUBFC ARCLU KOCK
C
ARCTAN ROUTINE: TANY GREATER OR EQUAL ZERO LESS THAN 360
FUNCTION TANV(CSS,SNM)
CS=CSS
SM=SNM
IF(CS) 57,53,54
50 IF(SM) 53,51,52
51 TANV=0.0
GO TO 61
52 TANV=90.0
GO TO 61
53 TANV=270.0
-233-
<table>
<thead>
<tr>
<th>MRLAJE BEARING, M=0.5</th>
<th>M=0.6</th>
<th>M=10</th>
<th>M=80</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>1.0</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.0</td>
<td>2000.0</td>
<td>10000.0</td>
<td>30000.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.02</td>
<td>10</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.05</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.25</td>
<td>0.3</td>
<td>0.35</td>
<td>0.4</td>
<td>0.43</td>
<td>0.43</td>
<td>0.43</td>
<td>0.43</td>
</tr>
<tr>
<td>10.0</td>
<td>110.0</td>
<td>0.5</td>
<td>60.0</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>150.0</td>
<td>230.0</td>
<td>0.5</td>
<td>180.0</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>250.0</td>
<td>350.0</td>
<td>0.5</td>
<td>300.0</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>THREE LOBE BEARING, ( M = 5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>CIRC.DIV</td>
<td>AX.DIV</td>
<td>NO.L/D</td>
<td>NO.REVN</td>
<td>NO.PAD</td>
<td>NO.COSA</td>
<td>NO.A-1/10</td>
<td>NO.RUP.L/PAD</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>CONV.LIM.ATT.A</td>
<td>CONV.LIM.RUP.&gt;::</td>
<td>1,00000E-04</td>
<td>1,00000E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L/D RATIOS</td>
<td>9.00000E-01</td>
<td>1.00000E-00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REYNOLDS NUMBERS</td>
<td>0.</td>
<td>2.000000E 03</td>
<td>1.000000E 04</td>
<td>3.000000E 04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ECC*COS(ATT) DATA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ECC*COS(ATT)</td>
<td>2.000000E-02</td>
<td>NO.EsSIN(A)* 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LIST OF ECC*SIN(ATT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.</td>
<td>5.000000E-02</td>
<td>1.000000E-01</td>
<td>1.400000E-01</td>
<td>2.000000E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.900000E-01</td>
<td>3.000000E-01</td>
<td>3.500000E-01</td>
<td>4.000000E-01</td>
<td>4.500000E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BEARING GEOMETRY DATA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INLET ANGLE</td>
<td>OUTLET ANGLE</td>
<td>PRELOAD</td>
<td>ANGLE PRLD-X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000000E 01</td>
<td>1.000000E 02</td>
<td>5.000000E 01</td>
<td>6.000000E 01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000000E 02</td>
<td>2.300000E 02</td>
<td>5.000000E 01</td>
<td>1.800000E 02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.500000E 02</td>
<td>3.500000E 02</td>
<td>5.000000E 01</td>
<td>3.000000E 02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L/D</td>
<td>HEYN.NO.</td>
<td>ECC*COS(ATT)</td>
<td>ECC*SIN(ATT)</td>
<td>ECC*RATIO</td>
<td>ATT.ANGL</td>
<td>1/S</td>
<td>F-X/S W</td>
</tr>
<tr>
<td>5.000000E 01</td>
<td>0.</td>
<td>0.00000E 02</td>
<td>3.000000E 02</td>
<td>3.600000E 02</td>
<td>5.900000E 01</td>
<td>1.000000E 01</td>
<td>2.800000E 08</td>
</tr>
<tr>
<td>1/S*(R/CIF/(W)</td>
<td>(Q-Z/NDLC)SIDE</td>
<td>(Q/NDLC)TOTAL</td>
<td>(Q-X/NDLC)CIRC</td>
<td>F-X/S W</td>
<td>CHM/HDL(R/C)12</td>
<td>FREQ/W</td>
<td>(FREQ/W)**2</td>
</tr>
<tr>
<td>2.9009359E 01</td>
<td>3.3327824E 01</td>
<td>3.0327824E 01</td>
<td>2.8777793E 00</td>
<td>3.193139E 01</td>
<td>5.9000043E 01</td>
<td>4.8104757E 01</td>
<td>2.3140676E 01</td>
</tr>
<tr>
<td>CXXXY/SW</td>
<td>CXXXY/SW</td>
<td>CHYY/SW</td>
<td>CXXXY/SW</td>
<td>CHYY/SW</td>
<td>CHYY/SW</td>
<td>CHYY/SW</td>
<td>CHYY/SW</td>
</tr>
<tr>
<td>5.9947423E 00</td>
<td>0.000000E 00</td>
<td>-7.173985E 00</td>
<td>4.5513619E 00</td>
<td>1.5259333E 01</td>
<td>-4.5704956E 01</td>
<td>-4.1995123E 01</td>
<td>1.3793705E 01</td>
</tr>
<tr>
<td>SOMMERFELD NO</td>
<td>F-Y/W</td>
<td>F-Y/W</td>
<td>(R/CIF/(W)</td>
<td>CHM**2/W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1206475E 00</td>
<td>0.000000E 00</td>
<td>-6.3926949E 00</td>
<td>9.3203356E 00</td>
<td>6.0000044E 01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CXXXY/W</td>
<td>CXXXY/W</td>
<td>CHYY/W</td>
<td>CHXXY/W</td>
<td>CHXXY/W</td>
<td>CHXXY/W</td>
<td>CHXXY/W</td>
<td></td>
</tr>
<tr>
<td>1.7497387E 01</td>
<td>2.1242683E 01</td>
<td>-2.2430524E 01</td>
<td>1.4930504E 01</td>
<td>4.4711175E 01</td>
<td>-1.4263292E 00</td>
<td>-1.3136395E 00</td>
<td>4.3198054E 01</td>
</tr>
<tr>
<td>MINTTHUMM FILM THICKNESS FOR PAD NO.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5.1674E 01</td>
<td>4.7891E 01</td>
<td>4.8274E 01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L/D</td>
<td>HEYN.NO.</td>
<td>ECC*COS(ATT)</td>
<td>ECC*SIN(ATT)</td>
<td>ECC*RATIO</td>
<td>ATT.ANGL</td>
<td>1/S</td>
<td>F-X/S W</td>
</tr>
<tr>
<td>5.000000E 01</td>
<td>0.</td>
<td>2.000000E 03</td>
<td>2.000000E 03</td>
<td>3.2777690E 02</td>
<td>3.0596325E 02</td>
<td>5.6004219E 01</td>
<td>4.4820513E 01</td>
</tr>
<tr>
<td>1/S*(R/CIF/(W)</td>
<td>(Q-Z/NDLC)SIDE</td>
<td>(Q/NDLC)TOTAL</td>
<td>(Q-X/NDLC)CIRC</td>
<td>F-X/S W</td>
<td>CHM/HDL(R/C)12</td>
<td>FREQ/W</td>
<td>(FREQ/W)**2</td>
</tr>
<tr>
<td>LD</td>
<td>HEYN, NO.</td>
<td>ECC=SIN(ATT)</td>
<td>ECC=RAT(ATT)</td>
<td>ATT, ANGLE</td>
<td>1/B</td>
<td>F/Y/SW</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-----------</td>
<td>--------------</td>
<td>--------------</td>
<td>------------</td>
<td>-------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>1/S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/S</td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**MINIMUM FILM THICKNESS FOR PAD NO.**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4858E-01</td>
<td>4.7895E-01</td>
<td>4.8039E-01</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>LD</th>
<th>HEYN, NO.</th>
<th>ECC=SIN(ATT)</th>
<th>ECC=RAT(ATT)</th>
<th>ATT, ANGLE</th>
<th>1/B</th>
<th>F/Y/SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td>1/S</td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
</tbody>
</table>

**MINIMUM FILM THICKNESS FOR PAD NO.**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4858E-01</td>
<td>4.7895E-01</td>
<td>4.8039E-01</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>LD</th>
<th>HEYN, NO.</th>
<th>ECC=SIN(ATT)</th>
<th>ECC=RAT(ATT)</th>
<th>ATT, ANGLE</th>
<th>1/B</th>
<th>F/Y/SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td>1/S</td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
</tbody>
</table>

**MINIMUM FILM THICKNESS FOR PAD NO.**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4858E-01</td>
<td>4.7895E-01</td>
<td>4.8039E-01</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>LD</th>
<th>HEYN, NO.</th>
<th>ECC=SIN(ATT)</th>
<th>ECC=RAT(ATT)</th>
<th>ATT, ANGLE</th>
<th>1/B</th>
<th>F/Y/SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td>1/S</td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(O-XDLC) (O-XDLC)</td>
<td></td>
<td></td>
<td></td>
<td>1/B</td>
<td></td>
</tr>
</tbody>
</table>

**MINIMUM FILM THICKNESS FOR PAD NO.**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4858E-01</td>
<td>4.7895E-01</td>
<td>4.8039E-01</td>
</tr>
</tbody>
</table>
NUMBER OF ATT. ANGLE ITERATIONS TOO LARGE

<table>
<thead>
<tr>
<th>L/D</th>
<th>KEY No.</th>
<th>ECC=COG(ATT)</th>
<th>ECC=SIN(ATT)</th>
<th>ECC/RATIO</th>
<th>ATT. ANGLE</th>
<th>1/5</th>
<th>F-Y/SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000000E-00</td>
<td>J.0000000E+00</td>
<td>2.0000000E+00</td>
<td>4.4919224E-02</td>
<td>4.30887E-02</td>
<td>6.5053142E-01</td>
<td>5.2634884E-00</td>
<td>1.4978993E-03</td>
</tr>
<tr>
<td>1/5*(R/C+F/W)</td>
<td>O/L/N(DLC)</td>
<td>I/O(N/DLC)</td>
<td>TOTAL</td>
<td>I/O(N/DLC)</td>
<td>I/R</td>
<td>F-Y/SW</td>
<td>CHN/N/DLC(R/C)</td>
</tr>
<tr>
<td>3.4654656E-02</td>
<td>2.0612097E-01</td>
<td>2.6170027E-01</td>
<td>2.7381068E-01</td>
<td>5.2634882E-00</td>
<td>4.9557544E-00</td>
<td>4.8837462E-01</td>
<td>5.345977E-01</td>
</tr>
<tr>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
</tr>
<tr>
<td>5.310699E-01</td>
<td>9.2900129E-01</td>
<td>-9.4469126E-01</td>
<td>4.1735998E-01</td>
<td>2.145264E-02</td>
<td>-3.4455467E-00</td>
<td>2.9079985E-01</td>
<td>1.860495E-02</td>
</tr>
<tr>
<td>SCHRANFELD NO</td>
<td>F-X/W</td>
<td>F-Y/W</td>
<td>F/X*(F/C)</td>
<td>F/Y*(F/W)</td>
<td>CHN**2/V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4998887E-01</td>
<td>9.2900129E-01</td>
<td>2.7121052E-04</td>
<td>6.5010939E-01</td>
<td>3.7108946E-01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
<td>C/KY/SW</td>
</tr>
<tr>
<td>1.0199668E-01</td>
<td>1.6620274E-01</td>
<td>-1.7983004E-01</td>
<td>7.8405694E-00</td>
<td>4.0762441E-01</td>
<td>-6.9242303E-01</td>
<td>-5.6299137E-01</td>
<td>5.6407648E-01</td>
</tr>
</tbody>
</table>

MINIMUM FILM THICKNESS FOR PAD NO.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4827E-01</td>
<td>4.761E-01</td>
<td>4.702E-01</td>
</tr>
</tbody>
</table>
This appendix describes the computer program PNO144: "The Performance and Stability of a Hybrid Journal Bearing with Flexible, Damped Support." It gives the detailed instructions for using the program, for preparing the input data and a listing of the Fortran instructions is also given. The analysis for the program is contained in Appendices V and VI. The program calculates the load carrying capacity, the flow, the 8 dynamic coefficients and the critical journal mass at the onset of instability. The program applies to a purely hydrostatic, a purely hydrodynamic or a hybrid bearing with a compressible lubricant. The bearing is cylindrical with a single row of feeder holes in the center plane of the bearing, and the support for the bearing has flexibility, damping and mass.

The analysis assumes that the bearing operates with a small eccentricity ratio

**COMPUTER INPUT**

An input data form is given in the back of this appendix for quick reference when preparing the computer input. In the following, the more detailed instructions are given.

**Table of Vena Contracta Coefficients (5E14.6)**

This table consists of 5 cards with a total of 25 values of the vena contracta coefficient \( C_D \) for the feeder hole restrictor. \( C_D \) is defined through eq. (E-31), Appendix V. The program assumes \( C_D = 1 \) when the feeder hole is choked, and for unchoked conditions, \( C_D \) is specified by the present input list. The dimensionless flow through the restrictor is a function of the pressure ratio across the restrictor:
where \( \dot{m} \) is the mass flow, lbs/sec/inch, \( \Delta T \) is the product of the gas constant and the total temperature, inch\(^2\)/sec\(^2\), \( a \) is the orifice radius, inch, \( P_s \) is the supply pressure, psia, \( P' \) is the pressure downstream of the orifice, psia, and \( k \) is the ratio of specific heats (\( k = 1.4 \) for air).

The vent contracts coefficient \( C_p \) is the ratio between the actual flow (made dimensionless) and the ideal isentropic flow such that \( C_p \leq 1 \) (in general, \( 0.6 < C_p \leq 1 \)). The 25 input values of \( C_p \) are taken at those pressure ratios which result from subdividing the range \( \left( \frac{2k}{k+1} \right)^{\frac{2(k-1)}{k}} \leq \frac{P'}{P_s} \leq 1 \) into 25 equal parts. The first value of \( C_p \) applies to the first pressure ratio after \( \frac{P'}{P_s} = \left( \frac{2k}{k+1} \right)^{\frac{2(k-1)}{k}} \), and the last value of \( C_p \) is taken at \( \frac{P'}{P_s} = 1 \).

**Card (5814,6)**

This card contains one item, the ratio of specific heats, \( k \). For air, \( k = 1.4 \).

**Card (72R)**

Any descriptive text may be given to identify the calculation.

**Card (815)**

This card is the "control" card which describes the subsequent input data. The card has 8 values:

1. \( \text{NVL} \) gives the number of pressure ratios, \( \frac{P'}{P_s} \), in the designated list. The maximum value of NVL is 25. For a purely hydrodynamic bearing, set NVL = 0 (then NLB \( \neq 0 \)).
2. \( NLSL \) gives the number of restrictor coefficients, \( A_i \), in the designated list. The maximum value of \( NLSL \) is 25. For a purely hydrodynamic bearing, set \( NLSL = 0 \) (when \( NLSL = 0 \), \( NVL \) must be zero).

3. \( NLB \) gives the number of compressibility numbers, \( A \), in the designated list. The maximum value is 25. For a purely hydrostatic bearing, set \( NLB = 0 \) (then \( NVL \neq 0 \)).

4. \( NEP \) gives the number of eccentricity ratios, \( E \), in the designated list. The maximum value is 25 (\( NEP \neq 0 \)).

5. \( MNS \) gives the number of values in the list of frequency ratios (when \( NLB = 0 \), the list contains values of the squeeze number instead of the frequency ratio). The maximum value of \( MNS \) is 95 (\( MNS \neq 0 \)).

6. \( NT \). As described in Appendix E, the program calculates the pressure in the gas film by numerical integration. To this end, the length between the admission plane and the end of the bearing is subdivided into increments. \( NT \) specifies the number of subdivisions. \( NT \) should be large for high values \( E \) and/or high values of the squeeze number \( d \). A typical value for \( NT \) is 10 to 15 for moderate \( A \) or \( d \) values (say up to \( A \leq 5 \) and \( d \leq 5 \)). The maximum value for \( NT \) is 30.

7. \( INT \). If \( INT = 0 \), the bearing is rigidly supported and no input data can be given for the three support parameters. If \( INT = 1 \), the bearing is flexibly supported.

8. \( INP \). If \( INP = 0 \), the program returns to read in more input after completion of the calculations for the present set of input data. A new set of input data starts with the identification card (i.e., with the card (72H)). The table of vena contracta coefficients and the card with the ratio of specific heats are not to be repeated.
If INF = 1, the present set of input data is the last set.

Card (X14.6)

This card contains 4 values:

1. \( L/D \), the ratio between the total bearing length and the journal diameter.

2. \( \lambda \), the source correction factor. \( \lambda \) is given by eq. (E-50), Appendix V, or in approximate form:

\[
\lambda = 1 + \frac{2}{n} \cdot \log_e \left( \frac{D}{n} \right)
\]

where \( n \) is the number of feeder holes, \( d \) is the feeder hole diameter (not the orifice diameter), \( D \) is the journal diameter and \( L \) is the total bearing length. Typically, \( \lambda = 1.3 \) to 1.5.

3. \( \delta \), the inherent compensation factor. Even when the feeder holes are provided with orifices, an additional flow restriction normally occurs where the gas leaves the feeder hole and enters the bearing film:

![Figure 63: Feeding Hole](image)

The orifice area is \( \pi a^2 \) where \( a \) is the orifice radius. The "curtain" area at the rim of the feeder hole has the area \( \pi dC \) where \( d \) is the feeder hole diameter and \( C \) is the radial bearing clearance. The ratio between the two areas is called the inherent compensation factor:

\[
\delta = \frac{\pi a^2}{\pi dC}
\]

When there is no orifice, \( \delta = \infty \) and the bearing is called inherently compensated (for the inherently compensated bearing, set \( \delta = 1000 \) or some other large value in the input). If the bearing is purely orifice restricted, set \( \delta = 0 \).
4. \( \frac{\nu_c}{\pi D L C} \), the ratio between the combined volume of all feeder holes and the volume of the gas film. Here, \( D \) is the journal diameter, \( L \) is the total bearing length, \( C \) is the radial bearing clearance, \( n \) is the number of feeder holes and \( \nu_c \) is the "capacitative" volume of one feeder hole. The effect of a feeder hole volume is to introduce a time lag between a change in the downstream feeder hole pressure and the corresponding change in the flow through the hole, see eq. (E-28), Appendix V. This time lag tends to reduce the damping capacity of the bearing and, in this way, adversely affects the stability margin. At present, there is no method to determine what this "capacitative" volume is, but it is suggested to set \( \nu_c \) equal to the volume of the feeder hole below the orifice, i.e. set \( \nu_c = \frac{2}{3} d^2 L \) in fig. 63: where \( L \) is the length of the feeder hole below the orifice. For an inherently compensated bearing in which there are no orifices, \( \nu_c \) may be set equal to zero. However, it must be remembered that \( \nu_c \), properly interpreted, gives a measure of the time lag between pressure change and flow change. If there are other factors than the feeder hole volume which contributes to such a time lag, their effect must be reflected in the value for \( \nu_c \) according to eq. (E-28).

List of Supply Pressure Ratios (5E14.6)

This list consists of up to five cards with values of the ratio between the supply pressure \( p_s \), psia, and the ambient pressure \( p_a \), psia. In total, there are NVL values (item 1, control card), maximum 25 values. In the analysis in Appendix V, the pressure ratio is denoted by the symbol \( V \). The ratio must be greater than 1. If NVL = 0, the list is omitted in the input and the bearing is purely hydrodynamic.

List of Restrictor Coefficients (5E14.6)

This list consists of up to five cards, giving the values of the restrictor coefficient \( \Lambda_s \):

\[
\Lambda_s = \frac{6 \mu a^2 \sqrt{RT}}{p_s c_s^2 \sqrt{1+\delta}}
\]

where \( \mu \) is the viscosity of the gas, lbs. sec/inch, \( RT \) is the product of the gas constant and the total temperature, inch^2/sec^2, \( p_s \) is the supply pressure.
pressure, psi, \( C \) is the radial clearance, inch, \( n \) is the number of feeder holes, 
\( a \) is the orifice radius, inch, and \( d \) is the inherent compensation factor. In the
inherently compensated bearing where there is no orifice, the restrictor coefficient
becomes:

\[
\text{Inherently Compensated Bearing: } \Lambda_s = \frac{6\mu \omega \sqrt{RT}}{P_a C^2}
\]

where \( d \) is the feeder hole diameter, inch.

In the purely hydrodynamic bearing, where \( NVL = 0 \) and, therefore, \( NLSL = 0 \), this
list is omitted. For \( NLSL = 9 \), give a total of \( NLSL \) values of \( \Lambda_s \), maximum 25.

List of Compressibility Numbers (5E14.6)

This list consists of up to five cards, giving a total of \( NLB \) values of the com-
pressibility number \( \Lambda \) (see item 3, control card). \( \Lambda \) is defined as:

\[
\Lambda = \frac{6\mu \omega}{P_a} \left( \frac{R}{C} \right)^2
\]

where \( \mu \) is the viscosity of the gas, lbs/sec/inch\(^2 \), \( P_a \) is the ambient pressure,
psi, \( \omega \) is the angular speed of the journal, radians/sec, \( R \) is the journal radius,
inches and \( C \) is the radial clearance, inch. A maximum of 25 values can be given.

For the purely hydrostatic bearing, set \( NLB = 0 \) and omit this list.

List of Eccentricity Ratios (5E14.6)

This list consists of up to five cards, giving a total of \( NEF \) values of the
eccentricity ratio \( \epsilon \) (see item 4, control card). \( \epsilon \) is defined as:

\[
\epsilon = \frac{E}{C}
\]

where \( E \) is the distance between the journal center position and the bearing center,
inches, and \( C \) is the radial clearance, inch. \( E \) cannot be zero and the input list
cannot be omitted. A maximum of 25 values can be given.

List of Frequency Ratios or Squeeze Numbers (5E14.6)

This list consists of up to 19 cards, giving a total of \( CNS \) values (item 5,
control card). A maximum of 95 values can be given.

In the hybrid bearing or the purely hydrodynamic bearing (NLB ≠ 0, item 3, control card), the list gives values of the frequency ratio \( \gamma \):

\[
\gamma = \frac{V}{\omega}
\]

where \( V \) is the frequency for the journal center motion, radians/sec, and \( \omega \) is the angular speed of the rotor, radians/sec. \( \gamma \) cannot be zero. The input values of \( \gamma \) are used by the program in searching for the threshold of instability. For each value of \( \gamma \), the program calculates the left hand side of eq. (D-29), Appendix IV (as modified per Appendix VII), which represents the overall effective damping of the gas film and the bearing support. The threshold of instability occurs at those values of \( \gamma \) for which the effective damping is zero. The program uses the values of \( \gamma \) in the same sequence as given in the input. Thus, the input list should start with a small value of \( \gamma \) (say, \( \gamma = 0.0001 \)) and thereafter, give successively larger values of \( \gamma \) with the last value of \( \gamma \leq 0.51 \). Whenever the effective damping changes sign as the program goes through the \( \gamma \) list, the program interpolates to determine accurately the \( \gamma \) values at which the damping is zero.

The very first value of \( \gamma \) is not used in the stability calculation and, therefore, can be any value. It is used as that value for which the program output gives the 8 dynamic coefficients. Thus, it is suggested to set the first value of \( \gamma \) equal to 1.

For the purely hydrostatic bearing (NLB = 0), this input list gives values of the squeeze number \( \delta \):

\[
\delta = \frac{12\mu V}{R^2} \left( \frac{R}{t} \right)
\]

where the symbols have been defined previously. The input is prepared in the same way as described above except that, whereas the range of interest for \( \gamma \) is known to be from 0 to 0.5, the range for \( \delta \) is not known in advance. It is suggested to let \( \delta \) go from a small value (say 0.001) to approximately 100. It should be noted, however, that the program is basically a hybrid bearing program and it is not very efficient in calculating a purely hydrostatic bearing although the answers are correct.
If INT = 0 (item 7, control card), the bearing is rigidly supported and the last input data is the preceding list of frequency ratios (or squeeze numbers). If INT = 1, the bearing is flexibly mounted and it is necessary to specify the support parameters. Schematically, the bearing and its support can be shown as:

![Diagram of bearing and support](image)

Card (315)

This card is the "control" card for the support parameters. It contains 3 items:

1. **NK** gives the number of values of the dimensionless support stiffness in the designated list (1 ≤ NK ≤ 25).

2. **NM** gives the number of values of the dimensionless bearing boundary mass in the designated list (1 ≤ NM ≤ 25).

3. **ND** gives the number of values of the dimensionless support damping in the designated list (ND ≤ 25). If ND = 0, the support is undamped and no input can be given for the support damping.

List of Support Stiffness Values (5E14.6)

This list consists of up to five cards, giving a total of NK values of the dimensionless support stiffness:

\[
\bar{K}_b = \frac{C K_e}{P_a L D}
\]

where \(K_e\) is the support spring coefficient, lbs/inch, \(C\) is the radial bearing clearance, inch, \(L\) is the bearing length, inch, \(D\) is the journal diameter, inch, and \(P_a\) is the ambient pressure, psia. There can be a maximum of 25 values.

-238-
List of Support Mass Values (SE14.6)

This list consists of up to five cards, giving a total of NM values of the dimensionless bearing housing mass:

$$\bar{m} = \frac{\sqrt{\frac{m \omega^3}{P_a L D}}}{\text{lbs. sec/}^2\text{inch}}$$

where $m$ is the bearing housing mass, lbs. sec$^2$/inch, $\omega$ is the angular speed of the journal, radians/sec, and the other symbols are defined above. There can be a maximum of 25 values. The mass may be zero if desired.

List of Support Damping Values (SE14.6)

If ND = 0, this list is omitted and the support has no damping. Otherwise, the list consists of up to five cards, giving a total of ND values of the dimensionless support damping:

$$\bar{B}_o = \frac{B_o \mu L}{\pi (\xi)^3}$$

where $B_o$ is the support damping coefficient, lbs. sec/inch, and the other symbols are defined above.

The support damping has a strong influence on the stability of a flexible mounted bearing. Properly chosen, the damping may vastly improve the stability limit of the bearing or even remove the tendency for instability altogether.

To determine which damping to provide, a whole range of damping values should be tried (say, from $10^{-3}$ to 10). At the same time, the support stiffness should be considerably smaller than the bearing stiffness (of the order of one half or less of the bearing stiffness).

COMPUTER OUTPUT

The first page of output is a repetition of the input for checking purposes. Therefore, follow the output for each case. The output values are identified by text as follows:
L/D: the L/D-ratio

PR.RATIO: the supply pressure ratio \( V = \frac{P}{P_0} \)

LAMBDA-8: the restrictor coefficient \( \Lambda_8 \)

LAMBDA: the compressibility number \( \Lambda \)

ECCENTR: the eccentricity ratio \( \varepsilon \)

LAMBDA-T: \( \Lambda_8 V \)

Q: the dimensionless flow \( q = \frac{\frac{\mu}{P} \sqrt{M}}{V_0} \) where \( P \) is the total mass flow to the bearing, lbs/sec/inch, and the other symbols have been defined previously. Thus, knowing \( Q \), the bearing gas consumption can be determined.

U-O: the dimensionless orifice flow \( m_o = \frac{q}{\Lambda_8 V^3} \) which is given by the right hand side of eq. (1-31), Appendix V.

PSI: \( \psi \) the dimensionless rate of change of flow with changing downstream pressure (defined by eq. (1-60), Appendix V).

ORIF. PR. RATIO: the pressure ratio \( \frac{P}{P_0} \) across the feeder hole (\( P \) is the pressure downstream of the feeder hole).

W/PALD: \( W/P_{aLD} \), the dimensionless bearing load where \( W \) is the bearing load, lbs.

W/DPLD: \( W/(P_0 - P_{a} \) LD

ATT. ANG: the attitude angle \( \theta \), degrees.

FB/PALD: \( \xi = \frac{F_r}{P_{a} \) LD , the dimensionless radial component of the bearing reaction, eq. (1-19), Appendix V.
$f_r = \frac{F_r}{P_a L D}$, the dimensionless tangential component of the bearing reaction, eq. (E-120), Appendix V.

$K_r = \frac{C K_r}{P_a L D}$, the dimensionless radial spring coefficient, eq. (E-131), Appendix V.

$\bar{B}_r = \frac{C \omega B_r}{P_a L D}$, the dimensionless radial damping, eq. (E-131), Appendix V.

$K_t = \frac{C K_t}{P_a L D}$, the dimensionless radial cross-coupling spring coefficient, eq. (E-131), Appendix V.

$\bar{B}_t = \frac{C \omega B_t}{P_a L D}$, the dimensionless radial cross-coupling damping, eq. (E-131), Appendix V.

$\bar{K}_{tt} = \frac{C K_{tt}}{P_a L D}$, the dimensionless tangential cross-coupling spring coefficient.

$\bar{B}_{tt} = \frac{C \omega B_{tt}}{P_a L D}$, the dimensionless tangential cross-coupling spring coefficient.

$\bar{B}_{tt} = \frac{C \omega B_{tt}}{P_a L D}$, the dimensionless tangential damping.

EFF. STIFF-1: the major dimensionless, effective stiffness $\bar{K}_9$ (eq. (D-24), Appendix IV, with plus in front of the square root instead of minus.

EFF. STIFF-2: the minor dimensionless, effective stiffness $\bar{K}_9$, eq. (D-24), Appendix IV.

EFF. DAMP-1: the major dimensionless, effective damping $\sqrt{\bar{B}_9}$ (eq. (D-25), Appendix IV, with plus in front of the square root instead of minus.

EFF. DAMP-2: the minor dimensionless, effective damping $\sqrt{\bar{B}_9}$, eq. (D-25) Appendix IV.
FREQ/W: \( \gamma = \gamma' \), the frequency ratio.

\( (\text{RAMO2}): \quad M' = \left( \frac{K_0 \gamma B + K_0 \gamma B}{(\gamma B + (1+\gamma) \gamma B)} \right) \), see eq. (D-28), Appendix IV.

\( \text{CM2/FAOD}: \quad \Gamma = C M \omega^2 / P_a L D \), the dimensionless journal mass.

\( \text{CM2/DFLD}: \quad C M \omega^2 / (P_0 - P_a) L D \), dimensionless expression for the journal mass.

\( \text{WPA/ME2L (R-C)}): \quad M P_a / \mu^2 L (E)^5 \), dimensionless expression for the journal mass.

\( \text{ERROR}: \quad \text{the left hand side of eq. (D-29), Appendix IV.} \)

\( \text{PEDEST. STIFFN.}: \quad \Gamma = C K_0 / P_a L D \), the dimensionless support stiffness.

\( \text{PEDEST. DAMPING}: \quad \Gamma = B_0 / \mu^2 L (E)^5 \), the dimensionless support damping.

\( \text{EFF. STIFFNESS}: \quad \text{the minor dimensionless, effective stiffness} \quad \frac{\Gamma}{\Gamma} \), eq. (D-24), Appendix IV.

\( \text{EFF. DAMPING}: \quad \text{the negative of the minor dimensionless, effective damping} \quad \gamma \Gamma \)

In each combination of \( \frac{\alpha}{\beta}, \nu, A, A, \varepsilon \), there is a separate output which begins by specifying the appropriate values of \( \frac{\alpha}{\beta}, \nu, A, A, \), and then gives the corresponding bearing flow, the load carrying capacity, the attitude angle, the dynamic coefficients and the four effective coefficients as identified by the labels explained above. The dynamic and the effective coefficients are calculated for the first frequency ratio given in the input data. Thereafter, follow the result of the stability calculation for a rigid support where eqs. (D-28) and (D-29) \((\text{RAMO2} \text{AND ERROR})\) are listed for the specified values of the frequency ratio. The threshold of instability occurs when the "error" is zero and the program determines this by interpolation. For the threshold value of the frequency
ratio, the dimensionless journal mass is given in three different forms, identified by the labels explained above.

When the bearing is flexibly supported, the preceding results are followed by the calculations of the stability of the flexibly supported bearing. First, the dimensionless support stiffness is given together with the corresponding threshold value of the dimensionless journal mass for an undamped support. Thereafter, for each specified value of the dimensionless support damping is a 6 column list with the results of the calculations. The first column gives the frequency ratio \( \gamma \), the second column gives \( \bar{M} \gamma^2 \), the third column gives the dimensionless minor effective bearing stiffness, the fourth column gives the dimensionless minor effective bearing damping and the sixth column gives the "ERROR". When an instability threshold is found, the corresponding dimensionless journal mass is given in three different forms in columns 3, 4, and 5.

For any given support damping, there may be up to three instability thresholds. At a threshold, the "error" must be very small (in theory it should be zero). This condition should be used to "weed out" a false root which frequently occurs. At such a false root, \( \bar{M} \gamma^2 \) goes to plus-minus infinity on either side of the frequency ratio value and the corresponding dimensionless journal mass will tend to be very large (in theory, it is infinite). The solution should be ignored. It should be noted, however, that there are proper solutions where the dimensionless journal mass also is very large which must not be ignored.

**SAMPLE CALCULATION**

In back of this Appendix is shown the input for calculating the stability of a flexible supported bearing \((L/D = 1, \varepsilon = 0.02, \lambda_r = 0.7, \lambda = 1.5, \delta = 1000, 4 \text{ values of } \frac{P_s}{P_e}, 7 \text{ values of } \lambda, 3 \text{ values of support stiffness and 15 values of support damping})\). Also shown are the first couple of pages of output. The results are most conveniently plotted on logarithmic paper with the support damping as abcissa and the dimensionless journal mass as ordinate for fixed values of the supply pressure ratio and the support stiffness. The
compressibility number is used as a parameter. A typical plot is shown in figure 27. From such a plot it can be determined what support damping to provide to improve the stability limit of the bearing. A more detailed discussion is given in the body of the report.
Table of Vena Contracta Coefficients (5E14,6)

Give 25 values of the orifice vena contracta coefficient $C_D$

Card (5E14,6)

$k$, the ratio of specific heats

Card (72R)

Text

Card (8I5)

1. NVL  Number of $P/P_s$ -values in input (NVL ≤ 25). If NVL = 0, the bearing is purely hydrodynamic (NLB ≠ 0).

2. NLSL  Number of $A_2$ -values in input (NLSL ≤ 25). When NLSL = 0, NVL must be zero.

3. NLB  Number of $A$-values in input (NLB ≤ 25). If NLB = 0, the bearing is purely hydrostatic (NVL ≠ 0).

4. NEP  Number of $\varepsilon$ -values in input (NEP ≤ 25).

5. MNS  If NLB ≠ 0, MNS gives number of $\phi$ -values in input. If NLB = 0, MNS gives number of $\phi$ -values in input (MNS ≤ 95).

6. MT  Number of finite difference increments (MT ≤ 30).
7. INF 0: Bearing support is rigid. 1: Bearing support is flexible.

8. INF 0: More input follows. 1: Last set of input.

Card (5E14.6)

1. L/D, the length-to-diameter ratio.

2. \( \lambda \), the source correction factor (see eq. (E-50), Appendix V).

3. \( \delta = \frac{a^2}{dC} \), the inherent compensation factor.

4. \( \frac{nC}{\pi DL C} \), the feeder hold volume ratio.

List of Supply Pressure Ratios (5E14.6)

Give NVL values of \( V = \frac{P_s}{P_a} \). Omit the list when NVL = 0.

List of Restrictor Coefficients (5E14.6)

Give NLSL values of \( \lambda_s \). Omit the list when NVL = 0 (and, hence, NLSL = 0).

List of Compressibility Numbers (5E14.6)

Give NLB values of \( \lambda \). Omit the list when NLB = 0.

List of Eccentricity Ratios (5E14.6)

Give NIF values of \( \varepsilon \) (\( \varepsilon \neq 0 \))

List of Frequency Ratios or Squeeze Numbers

If NLS \( \neq 0 \) (hybrid or hydrodynamic bearing), give NNS values of the frequency ratio \( \varepsilon \).
If NLB = 0 (hydrostatic bearing), give MiNS values of the squeeze number $\phi$.

Note: If INT = 0, omit the following input data.

Card (815)

1. NK  Number of support stiffness values in input ($1 \leq NK \leq 25$)
2. NM  Number of support mass values in input ($1 \leq NM \leq 25$)
3. ND  Number of support damping values in input ($ND \leq 25$)

List of Support Stiffness Values (5E14.6)

Give NK values of the dimensionless support stiffness.

List of Support Mass Values (5E14.6)

Give NM values of the dimensionless support mass (may be zero).

List of Support Damping Values (5E14.6)

Give ND values of the dimensionless support damping. Omit the list when ND = 0.
C  MECHANICS TECHNOLOGY INC. JORGENSEN W. LUND 6-22-1966
C
C PNC44. HYBRID JOURNAL BEARING STABILITY WITH FLEX-DAMPED SUPPORT
C
DIMENSION : FRI(31) + VFE(31) + VFR(31) + EPS(25) + FRT(31) + FRJ(31)
DIMENSION : FJE(31) + VCF(26) + BSIG(25)
DIMENSION : FF(99) + EF(99) + PDML(25) + PDLM(25) + PJML(25)
DIMENSION : VLSL(99) + FLSL(99) + FLBL(99) + GST(99) + PZ2(1)

199 = PZ2(99)
Y1 = 0.0
READ(5,301) (VCF(1), I = 2, 26) READ(5,301) ADC
VCF(1) = 1.0
WRITE(6,310)
WRITE(6,810) (VCF(1), I = 2, 26)
WRITE(6,315)

READ(5,300)
READ(5,301) (VLST(I), I = 1, NVL) READ(5,301) FLDL, PLS, CMP, FDVL
WRITE(6,300)
WRITE(6,303)
WRITE(6,304) (NLB, NEP, MNS, INT, INP
WRITE(6,309)
WRITE(6,810) (PRC, CMP, FDVL, ADC
IF(NVNL) 15 = 14, 15

VLST(1) = 2.0
FLSL(1) = 0.0
GO TO 20

WRITE(6,305)
READ(5,301) (VLST(I), I = 1, NVL) WRITE(6,810) (VLST(I), I = 1, NVL)
WRITE(6,306)
READ(5,301) (FLSL(I), I = 1, NLSL)
WRITE(6,810) (FLSL(I), I = 1, NLSL)
IF(NL) 20 = 19, 20

FLBL(I) = 1.0
GO TO 22

WRITE(6,307)
READ(5,301) (FLBL(I), I = 1, NLB)
WRITE(6,810) (FLBL(I), I = 1, NLB)

WRITE(6,316)
READ(5,301) (EPSL(I), I = 1, NEP)
WRITE(6,810) (EPSL(I), I = 1, NEP)
WRITE(6,308)
READ(5,301) (BSIG(I), I = 1, MNS)
WRITE(5,810) (BSIG(I), I = 1, MNS)
IF(INT) 24 = 29, 24

READ(5,302) (NK, NM, ND)
READ(5,301) (PDML(I), I = 1, NK)
READ(5,301) (PDLM(I), I = 1, NM)
IF(ND) 29 = 29, 27

READ(5,301) (PDL(I), I = 1, ND)

EX6 = 2.0 / (ADC + 1.0)
EX1 = 1.0 / (ADC - 1.0)
EX7 = (EX6 + EX1) * SQRT(1 - EX6 * ADC)
EX4 = 2.0 / EX1
EX1 = ADC * EX1
CRP = EX6 + EX1
CI = SQRT(1 + EX1)

-248-
FXN=·; I·TX6
X=1·; AOC
IT=(·; -CRP) /50#0
CX=CRP
DO 51 I=1#50
CX=CX+DLT
C5=CX**EX1
51 QST(1)*C1*C5*SQRT (1.0-CX/C5)
C1=1.0+CMP*CMP
CMP=0.5/C1
C1=MT
DLT=FLD/C1
DL2=DLT*DLT
DL2=0.5*DL2
SFAC=1.5707963/C1
MT1=MT+1
DO 285 NV=1#NV
V=VLST(NV)
VMW=V-1.0
V2=V*V
DO 284 NLS=1#NLSL
FLAS=FLSL(NLS)
FLAT=FLAS#V
C1=FLAT#V*FLD*PRC
CX=SQRT (1.0+C1*EX7)/V
IF (CRP-CX) 152#151#151
151 Q=FLAT#V*EX7
SLP=0.0
PZ=V*CX
VC=1.0
DVC=0.0
GO TO 200
152 C2=V*CRP
C2=C2+C2-1.0-C1*EX7
C6=DLT+DLT
CX=CRP+C4
KS=0
L=2
M=2
154 VC=VC(F(M)
155 C5=CX*CX*V2-1.0-C1*VC*QST(L)
IF (KS) 161#158#161
158 C6=C5*C2
IF (C6) 160#160#159
159 L=L+2
M=M+1
C2=C5
C5=VC(F(M-1)
DVC=(VC-C5)/C4
VC=(VC+C5)/2.0
CX=CX-DIT
GO TO 155
161 C1=C4*C4
C1=2.0*(C2+C7-2.0*C5)/C1
C3=(C7-C2)/C4
IF (C1) 163#162#163
-249-
162 C6=C5/C3
GO TO 166
163 C4=C5*C3/C1
   C6=SQRT (C4*C4-C5/C1)
   IF (C4) 164,165,166
164 C6=C6
165 C6=C6-C4
166 CX*CX+C6
167 PZ=Z*CX
   Q=(PZ*PZ-1.0)/FLD/PRC
   VC=VC*C6*DC
   CA=VC*FLAT*V
   C4=0/C4
   C5=DC/VC;
   C6=CF**EX1
   SL1=VC/2.0*FLAT/PZ*(C5*C4+EX4*C6/C4*(EX6-C6/CX;))
200 PZSQ=1.0+Q*FLD
   PSC=PZ
   PZ=SQRT(PZSQ)
   C4=SL1*FLD*PRC
   C6=1.0*DC
   FDV1=Can*DDV1/FLD*PSC*FLD*DLT
   SLP=C4*SL1
   BCZ=(CMP*Q-SLP*PZSQ)*DLT
   BCZ1=DLT*SLP
   IF (NVLI) 192,191,192
191 FLWZ=0.0
   SPZ=1.5707963
   GO TO 193
192 FLWZ=Q/(FLAT*V)
   SPZ=(PZSQ*PZ-1.0)/FLD*1.0471976/Q
193 PZ211=PZSQ
   C1=1.0+Q*FLD
   CZ=Q*DLT
   PZN(I)=PZ
   DO 201 I=2,MT1
   C1=C1-C2
   PZ2(I)=C1
201 PZN(I)=SQRT (C1)
   DO 283 NL=1,NL9
       FLA=FLBL(NL)
       IF (NL9) 198,197,198
197 FLA=0.0
   FLX=1.0
   GO TO 199
198 FLX=2.0*FLA
   FLA2=FLA*FLA
199 FLFZ=FLA/PZ
   PR=DL2H*PZSQ
   PE=0.0
   RR=DL2H
   RE=DL2H*FFZ
   TFR(1)=0.0
   TFE(1)=0.0
   VFR(1)=1.0
   VFE(1)=0.0
   C1=0.0
   C2=0.0
   C3=0.0/PZ
   C4=0.0
   DO 205 I=2,MT1
TFRI(I)=TFR(I-1)+PR+BCZ
TFE(I)=TFE(I-1)+PE
VFR(I)=VFR(I-1)+RR+BC21
VFE(I)=VFE(I-1)+RE
C5=PZN(I)
IF (I=MT1) 203, 202, 202
202 C5=2.0*C5
C6=1.0-TFR(I)
C7=TFE(I)
C8=VFR(I)
C9=VFE(I)
GO TO 204
203 C6=FLA/C5
PR=PR+DL2*(TFR(I)-C6*TFE(I)-C5*C5)
PE=PE+DL2*(TFE(I)+C6*TFR(I))
RR=RR+DL2*(VFR(I)-C6*VFE(I))
RE=RE+DL2*(VFE(I)+C6*VFR(I))
204 C1=C1+TFRI(I)/C5
C2=C2+TFE(I)/C5
C3=C3+VFR(I)/C5
205 C4=C4+VFE(I)/C5
C5=C8*C8+C9*C9
HZR=(C6*C8-C7*C9)/C5
HZE=(C6*C9-C7*C8)/C5
SFZR=SFACT*(C1+C3*HZR-C4*HZE)
SFZE=SFACT*(C2+C3*HZE+C4*HZR)
DO 206 I=1, MT1
TFRI(I)=TFRI(I)-HZR*VFR(I)-HZE*VFE(I)/PZN(I)
206 TFE(I)=TFE(I)-HZR*VFR(I)+HZE*VFE(I)/PZN(I)
FRSW=SFZ-SFZR
FTSW=-SFZE
DO 282 NE=1,NPE
EPS=EPSLINE
MC=MNS
GAM=0.0
EP2=EPS*EPS
E7=1.0-EP2
C1=SQRT(E7)
C2=1.0+C1
E1=E1+C2
E2=E1+C2
E3=E1+C1
E4=E3*EPS
E5=E4*EPS/C2
E6=2*0/(C1+E7)-E3
E7=E4*EPS/E7+E5/C1
FRS=E4*FRSW
FTS=E2*FTSW
WPLD=SQRT(FRS*FRS+FTS*FTS)
WPLD=WPLD/VMW
IF (FRS) 230, 226, 230
IF (FTS) 227, 228, 229
226 IF (FRS) 230, 226, 230
227 ANGZ=90.0
GO TO 232
228 ANGZ=0.0
GO TO 232
229 ANGZ=90.0
GO TO 232
230 ANGZ=FTS/FRS
ANGZ=57.295780*ATAN(ANGZ)
IF (FRS) 231, 232, 232
-251-
231 ANGZ*ANGZ+180+0
232 WRITE(6+311)
   WRITE(6+810)FLD*V*FLAS*FLA*EPS
   WRITE(6+312)
   WRITE(6+810)FLAT*Q*FLWZ*SLP+CX
   WRITE(6+313)
   WRITE(6+810)WPLD*WDPLD*ANGZ*FRS*FTS
   FRS=FRS/EPS
   FTS=FTS/EPS
   MS=2
   KC=0
   LC=0
   KK=0
   DO 281 NS=1*MNS
   SIG=BSIG(NS)
   SIG2=SIG*SIG
   IF INS-2) 250 550, 250
   WRITE(6+316)
250 KS=0
   SIG2=SIG*SIG
   AS=BCZ
   FDV2=FLX*SIG*FDV1
   AS1=FDV2*PZSQ
   BS=1+0
   DO 251 I=1+MT1
   FRR(I)=-PZ2(I)
   FJR(I)=0+0
   251 FJE(I)=-0+0
   252 PR=DL2H*FRR(I)
   PE=0+0
   QR=DL2H*FJR(I)
   QE=DL2H*FJE(I)
   RR=DL2H
   RE=DL2H*FLFZ
   SR=DL2H*FLX/PZ*SIG
   SE=0+0
   TR=TR+PR+AS
   TE=TE+PE
   UR=UR+QR+AS1
   UE=UE+QE
   VR=VR+RR+BCZ1
   VE=VE+RE
   WR=WR+SR+FDV2
   WE=WE+SE
A1=PSNI(I)
IF (I-MT1) 254,253,253
253 A1=2+0
GO TO 255
254 A2=FLA/A1
A3=FLX/A1*SIG
PR=PR+DL2*(TR-A2*TE-A3*UR+FRR(I))
P0=P0+DL2*(TR-A2*TE-A3*UE)
QR=QR+DL2*(TR-A2*UE-A3*TR+FJR(I))
Q0=Q0+DL2*(TR-A2*UR-A3*TE+FJE(I))
RR=RR+DL2*(TR-A2*VE-A3*WR)
RE=RE+DL2*(VR-A2*WE-A3*WE)
SR=SR+DL2*(VR-A2*WE-A3*VR)
SE=SE+DL2*(WE-A2*WR-A3*VE)
255 C1=C1+TR/A1
C2=C2+TE/A1
C3=C3+UR/A1
C4=C4+UE/A1
C5=C5+VR/A1
C6=C6+VE/A1
C7=VR/A1
C8=WE/A1
TR=BS-TR
A1=TR+VR+TE-VR+UE-VR-UE
A2=TR+VR-TE-VR+UE-VR
A3=TR+VR-TE+VR+UE
A4=TR+VR-TE-VR-UE
A5=TR-VR+TE+VR-UE
A6=TR-VR+TE-VR+UE
A7=A5+A6
H2R=(A1*A5+A2*A6)/A7
H2JR=(A3*A5+A4*A6)/A7
H2JE=(A4*A5-A3*A6)/A7
IF (INP) 401,402,403
401 WRITE(6,810)A1,A2,A3,A4,A5
WRITE(6,810)A6,A7,A8,A9
402 G2RR=SFRACT*(C1+H2R*C5+H2E*C6+H2JR*C7+H2JE*C8)
G2RE=SFRACT*(C2+H2R*C6+H2E*C5+H2JR*C8+H2JE*C7)
G2JR=SFRACT*(C3+H2R*C7+H2E*C6+H2JR*C8+H2JE*C5)
G2JE=SFRACT*(C4+H2E*C7+H2E*C8+H2E*C5+H2E*C6)
IF (K5) 259,257,259
257 G1RR=G2RR
G1RE=G2RE
G1JR=G2JR
G1JE=G2JE
K5=1
A5=0+0
A51=0+0
B5=0+0
A1=SIG*FLX
DO 258 I=1,1,MT1
FRR(I)=0+0
FJR(I)=A1*FJE(I)
258 FJE(I)=A1*FJE(I)
GO TO 252
259 CRR=E6*SPZ-E7*SFZR-E3*G1RR
CJR=E3*G1JR
DRR=E3*G2RR*FTS
DRJ=E3*G2JR
CTR=E5*SFZE-E1*G1RE

-253-
CTJ=-E1*G1JE
DTR=-E1*G2RE-FRS
DTJ=-E1*G2JE
A1=(CRR+DTR)/2.0
A2=CRJ-DTJ
A3=0.5*A2
A4=DRR+CTR*DRJ
A5=DRJ+CTJ-CRJ+DTJ-DRR*CTR
C3=(A1+A1-A3*A3+A5)/2.0
C4=(A1*(CRJ+DTJ)-A4)/2.0
C4=SORT(C4)
FFC=C3+C4
EFC=(CRR*C4-DTR*C3*A4)/C3
IF(LC) 719+188+719
188 IF(KC) 190+189+190
189 FF(NS)=FFC
EFC(N)=EFC
190 WRT=(A1-A3-CRJ-DTR*CRJ)/A2
C4=WRT
FEJL=(CRR-WRT)+(DTR+WRT)+A5
IF(NS-1) 510,510,511
510 CTR=-CTR
CTJ=-CTJ
DTR=-DTR
DTJ=-DTJ
WRITE(6,321)
WRITE(6,810)CRR,CRR,CRJ,DTR
WRITE(6,322)
WRITE(6,810)CTR,CTJ,DTJ
C5=A3*A3-AFEJL
IF(C3) 512,513,513
512 C1=-1.0
C2=-1.0
C3=-1.0
C4=1.0
GO TO 514
513 C3=SORT(C3)
C4=A3+C3
C5=A3-C3
C3=2.0*C3
C3=(A1*(CRJ-DTJ)-A4)/C3
C2=(CRR+DTR)/2.0
C1=C2+C3
C2=C2-C3
514 WRITE(6,323)
WRITE(6,810)C1,C2,C4,C5
GO TO 281
511 IF(NLB) 195,194,195
194 WRT=WRT
WRT=WRT/WRT
WRT3=WRT/S1G2*288+0
GO TO 196
195 WRT=WRT
WRT=WRT/S1G2
WRT=WRT/S1G2
WRT=WRT/S1G2*72+0
-254-
196 IF(KK) 520+521+520
521 WRITE(6*804)SIG,WRIT1+FEJL
   GO TO 522
520 WRITE(6*803)SIG,WRIT2+WRIT3+WRIT4+FEJL
   KK=0
522 IF (KC) 273+260+254
260 IF(MS=2) 262+262+261
261 C1=1*FEJL
   IF (C1) 263+273+262
262 Y1=FEJL
   X1=SIG
   W1=WRT
   MS=MS+1
   IF(MNS=MS) 571+281+281
571 IF(INT) 699+282+699
263 Y3=FEJL
   W3=WRT
   X3=SIG
   KC=1
   DSIG=X3-X1
   SIG=SIG-0.5*DSIG
   GO TO 250
264 Y2=FEJL
   W2=WRT
   X2=SIG
   C3=X3-X2
   C4=X2-X1
   C5=X3-X1
   C6=(Y3-Y2)/C5
   C7=(Y1-Y2)/C5
   C1=C7/C4
   C2=C6+C7
   C4=C4*C6-C3*C7
   D6=(W3-W2)/C5
   D7=(W1-W2)/C5
   A1=D6+D7
   A2=C4+D6-C3*D7
   A3=W2
   IF(C5) 266+265+266
265 C3=Y2/C4
   GO TO 271
266 C4=C4/C5+0.5
   C5=C4+C4-Y2/C5
   C5=SQR(C5)
   IF(C4) 269+270+270
269 C5=-C5
270 C3=-C4+C5
271 SIG =X2+C3
   WRIT=(A1*C3+A21*C3+A3
   KC=1
   WRITE(6*810)SIG,WRIT
   KK=1
   GO TO 250
273 KC=0
   FFC1=FFC
   EFC1=EFC
   FKAP=C4
   GAM=SIG
GAM2=SIG2
MC=NS
IF(INT) 290, 282, 699
290 Y1=Y3
W1=W3
X1=X3
GO TO 281
699 IK=1
700 PDK=PDKL(IK)
PDKQ=PDK*PDK
IF(GAM) 703, 704, 703
703 C1=FKAP+PDK
C1=FKAP/C1*PDK/GAM2
WRITE(6,811)PDK+C1
704 IF=1
701 PDM=PDML(IM)
PDM1=1.0+PDM
PDM2=PDM1#PDM1
IF(PDM) 705, 706, 705
705 IF(GAM) 698, 706, 698
698 C1=(PDK+FKAP/PDM11/PDM#0.5
C2=PDK/PDM#FKAP
C3=C1-C1-C2
C3=SORT (C3)
C2=1 C1-C3/GAM2
C1=C1-C3/GAM2
WRITE(6,814)
WRITE(6,802)PDM,PDK,C1+C2
706 IF(ND) 741, 741, 707
707 ID=1
702 PDP1=PDL(ID)
PDP=PDP1/12.0*FLA
PDPQ=PDP*PDP
WRITE(6,812)PDP1
WRITE(6,314)
NC=2
JC=0
LC=1
718 FFC=FF(NC)
EFC=EF(NC)
X=BSIG(NC)
719 C2=XX
C1=C2#PDPQ
C4=PDP#X
S3=CA=PDM1*FFC
S3=(C4*EFC-PDK#FFC)/S3
C3=PKD-PDM1*S3
C3=C3*C3+C1
ERR3=FFC-C4*S3/C3*S3
IF(NL8) 501, 502, 501
502 RT1=S3
RT2=S3/VMW
RT3=S3/C2 *288.0
GO TO 503
501 RT1=S3/C2
RT2=RT1/VMW
RT3=RT1/FLA2*72.0
503 IF(KK) 523, 524, 523
524 WRITE(6,804)X*S3,ERR3
WRITE(6,805)EFC,FFC
GO TO 525

-256-
523 WRITE(6*803)X*S3,RT1,RT2,RT3,ERR3
      WRITE(6*805)EFC,FFC
      KK=0
525 IF(JC) 760,728,720
520  JC=0
         GO TO 733
    728 IF(NC-21) 729,729,730
    729 X3=X
         Y3=ERR3
         NC=NC+1
         GO TO 718
    730 X1=X3
         X3=X
         Y1=Y3
         Y3=ERR3
         C3=Y1+Y3
         IF(C3) 731,720,733
    731 JC=-1
         X=(X1+X3)/2.+0
         SIG=X
         GO TO 250
    733 NC=NC+1
         IF(MC-NC) 740,734,718
    734 IF(GAM) 735,718,735
    735 X=GAM
         FFC=EFC1
         EFC=EFC
         GO TO 719
    760 X2=X
         Y2=ERR3
         C3=X3-X2
         C4=X2-X1
         C5=X3-X1
         C6=(Y3-Y2)/C5
         C6=C6/C3
         C7=(Y1-Y2)/C5
         C7=C7/C4
         C5=C6+C7
         C4=C4*C6-C3*C7
         IF(C5) 762,761,762
    761 C3=-Y2/C4
         GO TO 765
    762 C4=C4/C5*0.5
         C5=C4*C4-Y2/C5
         C5=SQR(C5)
         IF(C4) 763,764,764
    763 C5=-C5
    764 C3=-C4+C5
    765 X=X2+C3
         SIG=X
         JC=1
         KK=1
         GO TO 250
    740 ID=ID+1
         IF(ND-ID) 741,702,702
    741 IF(M-IM) 742,701,701
    742 IF(NK-IK) 281,700,700
281 CONTINUE
282 CONTINUE
283 CONTINUE
284 CONTINUE
285 CONTINUE
   IF(INP) 508,18,508
10 WRITE(6,315)
   GO TO 13
508 STOP
300 FORMAT(72H1)
   1
301 FORMAT(5E14.6)
302 FORMAT(815)
303 FORMAT(6HO MO*V7X5HNL-56X5HN*LOGLAM6X4HN*EP7X5HN*SIG6X4HDIVS8X3HINP)
304 FORMAT(1X4+7XI4+7XI4+7XI4+7XI4+7XI4+7XI4)
305 FORMAT(16HOPRESSURE RATIOS)
306 FORMAT(14HOLAMBDA-S LIST)
307 FORMAT(12HOLAMBDA LIST)
308 FORMAT(26HOLIST OF FREQUENCY RATIOS)
309 FORMAT(13HO OR*RA*FCT5X8HINH*COMP6X8HFEED*VOL4X11HSP*HE*RATIO)
310 FORMAT(28HOVENA CONTRACTA COEFFICIENTS)
311 FORMAT(11H15X3HL/D9X8HPR*RATIO6X8HLAMBDAD-7X6HLMBDAD7X8HECCENTR.)
312 FORMAT(12HO LAMBDA=T9X1H02X3HM=011X3HPS16X13HORIF PR RATIO)
313 FORMAT(1HO4X6HW/PALD6X6HW/PALD7X7HTAT=ANG7X7HP/R/PALD7X7HT/PALD)
314 FORMAT(11HO FREQ/W7X8H(S*GAM125X9HCW2/PALD5X9HCW2/PALD3X14HMP
1A/MU2L(R/C)54X5HE.decor2/29X13HEFF*STIFFNESS2X11HEFF*DAMPING)
315 FORMAT(1H1)
316 FORMAT(20HODECENTRICITY RATIOS)
318 FORMAT(11HO FREQ/W7X8H(S*GAM125X9HCW2/PALD5X9HCW2/PALD3X14HMP
1A/MU2L(R/C)54X55ERROR)
321 FORMAT(55HO CKXX/PALD CWXX/PALD CKXY/PALD CWXY/PALD)
322 FORMAT(55HO CKXX/PALD CWXX/PALD CKYY/PALD CWXY/PALD)
323 FORMAT(55HO EFF*STIFF-1 EFF*STIFF-2 EFF*DAMP-1 EFF*DAMP-2)
324 FORMAT(+1PE14.6)
325 FORMAT(61PE14.6)
326 FORMAT(+1PE14.6)
327 FORMAT(+1PE14.6)
328 FORMAT(+1PE14.6)
329 FORMAT(+1PE14.6)
810 FORMAT(51PE14.6)
811 FORMAT(16HPEDESTAL*STIFFN*+1PE13.6,14H MCW**2/PALD=+1PE13.6)
812 FORMAT(120HPEDESTAL DAMPING=1PE13.6)
814 FORMAT(14HO PEDEST.MASS5X9HEPED*STIFF6X9HMMASS RT.16X9HMMASS RT.2)
END

-258-
| 0.9985 | 0.9955 | 0.9915 | 0.9831 | 0.9824 |
| 0.9776 | 0.9726 | 0.9675 | 0.9623 | 0.9576 |
| 0.9516 | 0.946 | 0.9402 | 0.9341 | 0.9276 |
| 0.9206 | 0.913 | 0.9046 | 0.895 | 0.8836 |
| 0.869 | 0.8477 | 0.8103 | 0.736 | 0.6 |

**HYBRID-HYDROSTATIC RING BEARING, PS/PA**

*HYBRID-HYDROSTATIC RING BEARING, PS/PA* **1.25**

*HYBRID-HYDROSTATIC RING BEARING, PS/PA** *1.25* **7-7-1967**
## Hybrid-Hydrostatic Ring Bearing, PS/PA = 1.25, 7-7-1967

<table>
<thead>
<tr>
<th>NO. V</th>
<th>N. L-S</th>
<th>N. LAM</th>
<th>N. EP</th>
<th>N. SIG</th>
<th>DIVS</th>
<th>INP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>33</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OR. RA. FCT</th>
<th>INH. COMP</th>
<th>FEED. VOL</th>
<th>SP. HE. RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.500000E 00</td>
<td>1.000000E 03</td>
<td>0.25</td>
<td>1.407000E 00</td>
</tr>
</tbody>
</table>

### Pressure Ratios

1.250000E 00

### \( \Lambda \) List

- 7.000000E-01

### \( \Lambda \) List

- 3.000000E-01 1.000000E 00 2.000000E 00 5.000000E 00 1.000000E 01
- 3.000000E 01 1.000000E 02

### Eccentricity Ratios

2.000000E-02

### List CF Frequency Ratios

| 1.000000E 00 1.000000E-05 | 5.000000E-05 10.000000E-05 | 5.000000E-04 |
| 10.000000E-04 3.000000E-03 | 6.000000E-03 10.000000E-03 | 3.000000E-02 |
| 6.000000E-02 3.000000E-01 | 1.500000E-01 2.000000E-01 | 2.500000E-01 |
| 3.000000E-01 3.500000E-01 | 4.000000E-01 4.200000E-01 | 4.400000E-01 |
| 4.940000E-01 4.960000E-01 | 4.970000E-01 4.980000E-01 | 4.990000E-01 |
| 5.000000E-01 5.010000E-01 | 5.100000E-01 | |
IBJOB VERSION 5 HAS CONTROL.

IBJOB MTI

SIBLR MTI

11/21/66 MTI 0000

VENA CONTRACTS COEFFICIENTS

<table>
<thead>
<tr>
<th>Coefficient 1</th>
<th>Coefficient 2</th>
<th>Coefficient 3</th>
<th>Coefficient 4</th>
<th>Coefficient 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.690000E-01</td>
<td>8.477000E-01</td>
<td>8.105000E-01</td>
<td>7.360000E-01</td>
<td>6.000000E-01</td>
</tr>
<tr>
<td>L/C</td>
<td>PR.RATIO</td>
<td>LAMBA-S</td>
<td>LAMH0</td>
<td>ECCENTR.</td>
</tr>
<tr>
<td>-----</td>
<td>-----------</td>
<td>---------</td>
<td>-------</td>
<td>----------</td>
</tr>
<tr>
<td>1.00000E-01</td>
<td>1.25000E-00</td>
<td>7.00000E-01</td>
<td>3.00000E-01</td>
<td>2.00000E-02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LAMBA-T</th>
<th>Q</th>
<th>M-O</th>
<th>PSI</th>
<th>ORIF.PR.RAT.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.75000E-01</td>
<td>2.76214E-01</td>
<td>7.32536E-01</td>
<td>3.00000E-01</td>
<td>2.00000E-02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>W/PALD</th>
<th>W/DPLD</th>
<th>ATT.ANG</th>
<th>FR/PALD</th>
<th>FT/PALD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.02286E-03</td>
<td>0.89146E-01</td>
<td>4.37088E-01</td>
<td>1.46225E-01</td>
<td>3.79788E-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CKXX/PALD</th>
<th>CWXX/PALD</th>
<th>CKXY/PALD</th>
<th>CWXY/PALD</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.66624E-02</td>
<td>1.37786E-01</td>
<td>6.78915E-02</td>
<td>1.39059E-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CKYY/PALD</th>
<th>CWYY/PALD</th>
<th>CKXY/PALD</th>
<th>CWXY/PALD</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.78333E-02</td>
<td>1.35032E-01</td>
<td>8.66159E-02</td>
<td>1.39778E-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EFF.STIFF-1</th>
<th>EFF.STIFF-2</th>
<th>EFF.DAMP-1</th>
<th>EFF.DAMP-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00000E-01</td>
<td>7.79274E-01</td>
<td>2.04624E-01</td>
<td>7.09214E-02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FREC/W</th>
<th>(s*GAM/2)</th>
<th>CMW2/PALD</th>
<th>CMW2/DPLD</th>
<th>MPA/MU2L(R/C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00000E-05</td>
<td>6.62143E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.00000E-05</td>
<td>6.62147E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.00000E-05</td>
<td>6.62147E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.00000E-04</td>
<td>6.62147E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.00000E-04</td>
<td>6.62147E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.00000E-03</td>
<td>6.62147E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.00000E-03</td>
<td>6.62147E-02</td>
<td>-4.93339E-03</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| PEDESTAL STIFFNESS | 5.00000E-02 | MWC/W2/PALD | 1.00000E-01 |
| PEDESTAL CAMPING | 2.00000E-01 |

<table>
<thead>
<tr>
<th>FREC/W</th>
<th>(s*GAM/2)</th>
<th>CMW2/PALD</th>
<th>CMW2/DPLD</th>
<th>MPA/MU2L(R/C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00000E-05</td>
<td>4.999998E-02</td>
<td>-4.94973E-04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

-262-
<table>
<thead>
<tr>
<th>5.000000E-05</th>
<th>4.999992E-02</th>
<th>7.313440E-02</th>
<th>6.989497E-02</th>
<th>-9.009510E-03</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.000000E-05</td>
<td>4.999983E-02</td>
<td>7.313385E-02</td>
<td>6.988941E-02</td>
<td>-4.502197E-03</td>
</tr>
<tr>
<td>5.000000E-04</td>
<td>4.999917E-02</td>
<td>7.312765E-02</td>
<td>6.982681E-02</td>
<td>-9.01618E-02</td>
</tr>
<tr>
<td>10.000000E-04</td>
<td>4.999834E-02</td>
<td>7.312078E-02</td>
<td>6.975726E-02</td>
<td>-4.503558E-02</td>
</tr>
<tr>
<td>3.000000E-03</td>
<td>4.999501E-02</td>
<td>7.309331E-02</td>
<td>6.947903E-02</td>
<td>-1.499771E-02</td>
</tr>
<tr>
<td>6.000000E-03</td>
<td>4.998998E-02</td>
<td>7.305239E-02</td>
<td>6.906166E-02</td>
<td>-7.487353E-01</td>
</tr>
<tr>
<td>1.000000E-03</td>
<td>4.998320E-02</td>
<td>7.299818E-02</td>
<td>6.850510E-02</td>
<td>-4.830333E-01</td>
</tr>
<tr>
<td>3.000000E-04</td>
<td>4.997999E-02</td>
<td>7.273359E-02</td>
<td>6.572138E-02</td>
<td>-1.478163E-01</td>
</tr>
<tr>
<td>6.000000E-04</td>
<td>4.989048E-02</td>
<td>7.235728E-02</td>
<td>6.154291E-02</td>
<td>-7.259640E-00</td>
</tr>
<tr>
<td>1.000000E-04</td>
<td>4.980264E-02</td>
<td>7.189382E-02</td>
<td>5.596558E-02</td>
<td>-4.235936E-00</td>
</tr>
<tr>
<td>1.500000E-05</td>
<td>4.966756E-02</td>
<td>7.137624E-02</td>
<td>4.898833E-02</td>
<td>-2.700306E-00</td>
</tr>
<tr>
<td>2.000000E-05</td>
<td>4.948962E-02</td>
<td>7.092734E-02</td>
<td>4.200385E-02</td>
<td>-1.901803E-00</td>
</tr>
<tr>
<td>2.500000E-05</td>
<td>4.923927E-02</td>
<td>7.054729E-02</td>
<td>3.501268E-02</td>
<td>-1.303757E-00</td>
</tr>
<tr>
<td>3.000000E-05</td>
<td>4.885526E-02</td>
<td>7.023618E-02</td>
<td>2.801635E-02</td>
<td>-9.775532E-01</td>
</tr>
<tr>
<td>3.500000E-05</td>
<td>4.818385E-02</td>
<td>6.999411E-02</td>
<td>2.101588E-02</td>
<td>-6.177224E-01</td>
</tr>
<tr>
<td>4.000000E-05</td>
<td>4.669986E-02</td>
<td>6.982114E-02</td>
<td>1.401231E-02</td>
<td>-2.789018E-01</td>
</tr>
<tr>
<td>4.200000E-05</td>
<td>4.544253E-02</td>
<td>6.977129E-02</td>
<td>1.121625E-02</td>
<td>-1.610079E-01</td>
</tr>
<tr>
<td>4.400000E-05</td>
<td>4.307060E-02</td>
<td>6.973254E-02</td>
<td>8.407920E-03</td>
<td>-6.730940E-02</td>
</tr>
<tr>
<td>4.600000E-05</td>
<td>3.628871E-02</td>
<td>6.970487E-02</td>
<td>5.605391E-03</td>
<td>-1.036437E-02</td>
</tr>
<tr>
<td>4.800000E-05</td>
<td>6.73318E-02</td>
<td>6.968825E-02</td>
<td>2.802725E-03</td>
<td>2.012720E-03</td>
</tr>
<tr>
<td>4.700000E-06</td>
<td>2.536681E-02</td>
<td>6.969517E-02</td>
<td>4.204676E-03</td>
<td>1.668095E-03</td>
</tr>
<tr>
<td>4.675185E-06</td>
<td>2.920510E-02</td>
<td>6.951166E-02</td>
<td>5.344673E-01</td>
<td>1.197531E-06</td>
</tr>
<tr>
<td>4.850000E-06</td>
<td>1.978179E-01</td>
<td>6.969735E-02</td>
<td>4.551807E-03</td>
<td>-2.239775E-03</td>
</tr>
<tr>
<td>4.825000E-06</td>
<td>1.140688E-00</td>
<td>6.986585E-02</td>
<td>2.102247E-03</td>
<td>-2.382574E-04</td>
</tr>
<tr>
<td>4.827751E-06</td>
<td>1.270730E-03</td>
<td>6.968693E-02</td>
<td>2.452388E-03</td>
<td>-2.735917E-07</td>
</tr>
<tr>
<td>4.900000E-06</td>
<td>9.598949E-02</td>
<td>6.968413E-02</td>
<td>1.401367E-03</td>
<td>-9.24167E-03</td>
</tr>
<tr>
<td>4.925000E-06</td>
<td>8.616499E-02</td>
<td>6.968361E-02</td>
<td>1.219090E-03</td>
<td>-1.277892E-02</td>
</tr>
<tr>
<td>4.940000E-06</td>
<td>7.901617E-02</td>
<td>6.968319E-02</td>
<td>8.408194E-04</td>
<td>-1.672000E-02</td>
</tr>
<tr>
<td>4.960000E-06</td>
<td>7.543107E-02</td>
<td>6.968294E-02</td>
<td>5.605491E-04</td>
<td>-2.105237E-02</td>
</tr>
<tr>
<td>4.970000E-06</td>
<td>7.369077E-02</td>
<td>6.968278E-02</td>
<td>4.204111E-04</td>
<td>-2.336124E-02</td>
</tr>
<tr>
<td>FREC/W</td>
<td>(S+GAM1Z</td>
<td>CMW2/PALD</td>
<td>CMW2/DPLD</td>
<td>MPA/MUZL/R/C1S</td>
</tr>
<tr>
<td>-------</td>
<td>-----------</td>
<td>-----------</td>
<td>-----------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1.000000E+05</td>
<td>4.999996E-02</td>
<td>7.313444E-02</td>
<td>5.98497E-02</td>
<td>1.801912E-00</td>
</tr>
<tr>
<td>2.000000E+05</td>
<td>4.999996E-02</td>
<td>7.313444E-02</td>
<td>5.98497E-02</td>
<td>1.801912E-00</td>
</tr>
<tr>
<td>3.000000E+05</td>
<td>4.999996E-02</td>
<td>7.313444E-02</td>
<td>5.98497E-02</td>
<td>1.801912E-00</td>
</tr>
<tr>
<td>4.000000E+05</td>
<td>4.999996E-02</td>
<td>7.313444E-02</td>
<td>5.98497E-02</td>
<td>1.801912E-00</td>
</tr>
<tr>
<td>5.000000E+05</td>
<td>4.999996E-02</td>
<td>7.313444E-02</td>
<td>5.98497E-02</td>
<td>1.801912E-00</td>
</tr>
<tr>
<td>6.000000E+05</td>
<td>4.999996E-02</td>
<td>7.313444E-02</td>
<td>5.98497E-02</td>
<td>1.801912E-00</td>
</tr>
</tbody>
</table>

**PECESTAL CAMPING= 5.000000E-01**

-264-
<table>
<thead>
<tr>
<th>PSAM</th>
<th>CMWZ/PALD</th>
<th>CMWZ/PLD</th>
<th>MPA/MIUZI(R/C)</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.968825E-02</td>
<td>2.802725E-03</td>
<td>6.968585E-02</td>
<td>2.10247E-03</td>
<td>-4.91053E-02</td>
</tr>
<tr>
<td>6.968413E-02</td>
<td>1.401367E-03</td>
<td>6.968361E-02</td>
<td>1.12149E-03</td>
<td>-4.93043E-02</td>
</tr>
<tr>
<td>6.968319E-02</td>
<td>8.408194E-04</td>
<td>6.968294E-02</td>
<td>5.605491E-04</td>
<td>-5.35435E-02</td>
</tr>
<tr>
<td>6.968278E-02</td>
<td>4.204111E-04</td>
<td>6.968274E-02</td>
<td>2.82722E-04</td>
<td>-6.44433E-02</td>
</tr>
<tr>
<td>6.968276E-02</td>
<td>1.401352E-04</td>
<td>6.968270E-02</td>
<td>0.</td>
<td>-6.891187E-02</td>
</tr>
</tbody>
</table>

**PEDESTAL DAMPING** = 2.000000E 00

<table>
<thead>
<tr>
<th>PSAM</th>
<th>CMWZ/PALD</th>
<th>CMWZ/PLD</th>
<th>MPA/MIUZI(R/C)</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.968825E-02</td>
<td>2.802725E-03</td>
<td>6.968585E-02</td>
<td>2.10247E-03</td>
<td>-4.91053E-02</td>
</tr>
<tr>
<td>6.968413E-02</td>
<td>1.401367E-03</td>
<td>6.968361E-02</td>
<td>1.12149E-03</td>
<td>-4.93043E-02</td>
</tr>
<tr>
<td>6.968319E-02</td>
<td>8.408194E-04</td>
<td>6.968294E-02</td>
<td>5.605491E-04</td>
<td>-5.35435E-02</td>
</tr>
<tr>
<td>6.968278E-02</td>
<td>4.204111E-04</td>
<td>6.968274E-02</td>
<td>2.82722E-04</td>
<td>-6.44433E-02</td>
</tr>
<tr>
<td>6.968276E-02</td>
<td>1.401352E-04</td>
<td>6.968270E-02</td>
<td>0.</td>
<td>-6.891187E-02</td>
</tr>
</tbody>
</table>
REFERENCES


This volume treats three special bearing types selected for study because of their favorable stability characteristics and, hence, their potential for use in high speed rotating machinery applications. The three bearing types are:

a. The Three Lobe Journal Bearing
b. The Floating Sleeve Bearing with an Incompressible Lubricant
c. The Floating Sleeve Bearing with a Compressible Lubricant.

In the floating sleeve bearings, the ring is prevented from rotating but is otherwise free to move. The ring is floated by pressurizing the outer film of the bearing. In the case of a compressible lubricant, the inner film is pressurized as well.

The volume gives extensive design data in form of charts and tables from which the bearing dimensions can be obtained for a given application. Data are given for bearing flow, friction power loss and the speed at which hydrodynamic instability sets in. In addition, two computer programs accompany the volume, and instructions and listings of the programs are included. The programs may be used to obtain data for cases not covered by the presented design data.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lobed Bearings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lubrication</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid Film</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floating Ring</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrodynamic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrostatic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotor-Bearing Dynamics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stability</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical Speed</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>