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### AUTHORITY

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CALCULATION OF OPTIMUM
SUPERSONIC DELTA WINGS

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Aerodynamics Group Engineer

No. of Equations 285

CONVAIR-ASTRONAUTICS
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1.0 SUMMARY

This paper presents the distributed and total drag-due-to-lift in closed form by means of linearized supersonic wing theory for delta wings having arbitrary warp described by a symmetrical ten-term power series expansion for which the loading was reported on previously by the author 1-3 and others 4-7. For both sonic and supersonic leading edges one hundred drag contributions are derived, ninety of which are interference terms induced by the interaction of each loading with each of the non-corresponding downwashes. For subsonic leading edges additional terms are included to account for the suction forces induced by the leading edge pressure singularities.

The mean chord shape required to deflect under aeroelastic load to the desired shape computed herein has been programmed for the IBM 704 Digital Computer by the Dynamics Group.
2.6 INTRODUCTION

With the advent of supersonic operational aircraft during the past decade there has been increasing demand for improved procedures for predicting the distributed and integrated force and moment characteristics of wings, bodies, and complete aircraft configurations.

In regard to warped triangular wings, the steady-state forces (excluding drag) and moments have been fully developed and reported on by the author and others for a ten-term power-series approximation to the actual downwash. With the exception of a limited amount of additional terms by Roper, the excessive labor involved has precluded the analytical consideration of terms more than the first ten of the power series. It is recognized, however, that not only is it possible to obtain any desired number of solutions by means of high-speed digital computing equipment, but that some organizations have already, or are in the process of accomplishing this. For example, at Convair-San Diego the complete design problem has been mechanized for highly generalized planforms comparable to the method discussed herein. The Forth Worth Division of Convair utilizes power series expansions for the downwash which involve oblique Mach line coordinates.

Several noteworthy contributions were made to the development of low-drag delta wings particularly of the subsonic leading edge variety. Baldwin warped subsonic leading-edge delta wings to support some non-singular specified distributions of lift. Tucker presented a method to warp subsonic leading-edge wings. The assumed non-singular pressure distribution was replaced by four terms of a power series, the constants of which were chosen to satisfy the design lift, pitching-moment and the condition of nearly elliptic span loading. Tsien studied subsonic leading-edge delta wings of the conical family cambered to give minimum pressure drag with and without full leading edge suction. Rodriguez, Lagerstrom and Graham extended the drag reduction procedure.
developed by Graham, whereby use is made of orthogonal loadings. Strand applied the Graham technique to the problem of sonic leading-edge delta wings. Using four terms of a Legendre polynomial representation for the downwash, the method produced nearly 75% drag reduction from the untrimmed flat plate. Grant approximated Jones' criterion of constant downwash in the combined forward and reverse-flow fields by using Lagrangian multipliers to combine four non-singular loadings on delta wings having subsonic leading edges. To date the best wings were developed in a recent paper by Doris Cohen. Generally, six terms of a power-series expansion for the downwash were used to determine the shape of subsonic, sonic and supersonic leading-edge delta wings. For the subsonic leading edges the results include the effect of full-leading edge suction thereby leading to greater reductions than the results of previous methods. For sonic leading edges gains of 0.97 were reported for the series considered. A similar treatment was concurrently reported on by Fergain and Vallée. At Convair-Fort Worth Stewart and Danby and Stanell developed procedures for determining the shape of wings having nearly minimum drag due to lift.

In order to achieve the leading-edge suction it is necessary, however, to bend the wing leading edge in the direction of the streamlines to avoid separation. Leading edge conical camber may be used to maintain satisfactory characteristics at subsonic speeds for wing shapes designed to operate at supersonic conditions. The optimum leading edge conical camber is a function of lift coefficient and Mach Number and movable leading-edge controls may be in order.

None of the previous methods for drag reduction have considered the large trim drag at supersonic speeds for tailless aircraft. In addition to the possible requirement for maintaining straight hinge-lines a further drag savings can be realized if the constraint of trimmer flight is applied at the design lift coefficient.
At subsonic and low supersonic speeds (where the component of Mach Number normal to the leading edge is subsonic) a suction force may be realized provided that leading-edge separation is eliminated. Boyd, Mizotsky and Wetzel published a method to determine the required leading-edge modification for some restricted flat delta wings. Recently, Falk developed a design procedure to determine the required modification for given warped (as well as flat) delta wings utilizing slender-body theory.

The current paper presents the formulas in closed form for the one hundred distribute and integrated drag contributions resulting from a ten-term power-series expansion for the downwash on delta wings having subsonic, sonic and supersonic leading edges. With the formulas included for the suction forces for subsonic leading edges one may thereby compute the drag for off-design conditions for warped delta wings with undeflected elevons at supersonic speeds when the known mean chord line is expressible by the chosen power series. This procedure has been programmed for the IBM 704 Digital Computer for the determination of the mean chord shape, drag polar and pitching moment for pointed tip wings having swept trailing edges (e.g., arrow and diamond wings). The set-up did not include design for subsonic leading edges since this case is of little practical competitive interest. The drag polar for off-design conditions where the closed form equations for the mean chord shape are modified by either or both elevon deflection and leading-edge camber (not described by the chosen power series for the downwash) are computed by numerical means by an IBM 704 program prepared by the Dynamics Group. This latter procedure requires only the design shape with the numerical modifications. Furthermore, this IBM 704 program will compute the required shape which under aerelastic load will deflect to the shape predicted herein.

The procedure for determining the mean chord line for delta wings trimmed for level flight at design lift coefficient with two straight hinge-lines is described in the text. The detailed procedure for some illustrative examples are included in the Appendix for
the option of the reader. In order to make the drag expression more tractable for the minimization procedure, the interference drag terms were eliminated at the design condition by constructing orthogonal loadings \(^1\). (The evaluation of the design drag coefficient thereby requires only ten known orthogonal terms or merely two Lagrangian multipliers instead of one hundred drag terms as required for off-design conditions.)

Several delta wings having sonic and supersonic leading edges were designed and compared to the flat plate value. All designs were carried out for the ten-term power-series expansion for the downwash. For a sonic leading edge delta wing at \(C_L = .136\), about 9.5\% drag reduction from the untrimmed flat plate was found without specifying the static margin and with only one straight hinge-line. The penalties mentioned previously nullify the gain, however. A second sonic leading edge wing designed for trimmed level flight indicated about 46\% drag reduction from the trimmed flat plate. The third wing of this group designed for trimmed level flight with two straight hinge-lines resulted in about 33\% less drag than the trimmed sonic leading-edge flat-plate delta wing.

Similar studies were carried out for delta wings having supersonic leading edges. For the supersonic leading-edge delta wing at design \(C_L = .937\), the untrimmed case resulted in about 3.5\% drag reduction from the untrimmed flat plate. The second wing of this group designed for trimmed level flight resulted in about 31\% drag reduction from the trimmed flat plate. The supersonic leading-edge delta wing designed for trimmed level flight with two straight hinge-lines resulted in 39\% less drag than the trimmed flat plate.

No illustrative examples were carried out for wings having subsonic leading edges for lack of interest. However, the procedure outlined for the sonic and supersonic cases applies and all the quantities required for application are included. It is pointed out that the current procedure can include leading-edge suction at the option of the reader.

The conical functions derived herein for the section drag parameters were used to set-up an IBM 704 program to treat pointed tip wings having supersonic swept trailing edges.
The effect on drag reduction of including additional terms up to ten taken one at a time is illustrated for the sonic leading-edge delta wing without restraints.

The drag-due-to-lift is quite sensitive to the shape of the mean chord line. For this reason it is suggested that the next logical step in this direction would be to include flexibility in the drag reduction process. The extension could be made by using a conservative structure and replacing the resulting elastic warp by a power series similar to the one used for the rigid wing downwash.
3.0 SYMBOLS

3.1 FRIED STREAM CONDITIONS

V \quad \text{Velocity}

M \quad \text{Mach Number}

\beta \quad (M^2 - 1)^{1/2}

\mu \quad \text{Mach angle, } \arcsin (1/M)

\rho \quad \text{Mass density of air}

q \quad \text{Dynamic pressure of free stream } (1/2) \rho V^2

3.2 WING GEOMETRY

b/2 \quad \text{Semispan}

c \quad \text{Local chord}

c_r \quad \text{Root chord}

c_{av} \quad \text{Average chord, } c_r/2

\bar{c} \quad \text{Mean aerodynamic chord, } (2/3) c_r

S \quad \text{Wing Area}

\alpha \quad \text{Wing half apex angle}

\phi \quad \text{Angle between trailing edge and stream axis}

m \quad \beta \tan \phi

m_o \quad \beta \tan \alpha

x, y, z \quad \text{Cartesian coordinates of system of axes with origin at leading edge of root chord}

\alpha, \alpha(x', y') \quad \text{Wing angle of attack in stream direction, radians}

\alpha_l \quad (x')^r (y')^s

x', y', z' \quad x, y, z \text{ non-dimensionalized by } c_r, b/2, c_r, \text{ respectively}

\alpha_{rs} \quad \text{Constants of proportionality}
3.3 ANALYSIS PARAMETERS

- \( \phi \) velocity potential
- \( \phi_x \) horizontal perturbation velocity \( (= \partial \phi / \partial x) \)
- \( \phi_{xx}, \phi_{yy}, \phi_{zz} \) second partial derivatives of velocity potential with respect to \( x, y, z \) respectively
- \( \phi_n(x, y) \) velocity potential normal to leading edge
- \( v_n \) velocity normal to leading edge
- \( x_n, y_n \) coordinates perpendicular and parallel to leading edge
- \( w \) upwash velocity \( (= \partial \phi / \partial z) \)
- \( t = y/x \)
- \( \theta \) \( \arccos \left( \frac{1}{m} \right) \)
- \( \theta_1 = \arccos \left( \frac{(1-m^2)/(m^2-t)} \right) \)
- \( \theta_2 = \arccos \left( \frac{(1+m^2)/(m+t)} \right) \)
- \( \theta_3 = \arccos \left( \frac{m}{t} \right) \)
- \( \theta_4 = \arccos \left( \frac{1}{t} \right) \)
- \( x_{T,E.} \) value of \( x \) at trailing edge
- \( \Delta p' \) lifting pressure coefficient
- \( K, E \) complete elliptic integrals of the first and second kinds, respectively, with modulus \( (1-m^2)^{1/2} \)

- \( A_i, a_i, b_i \) functions of \( m, K, E \) defined in Table I, reference 3, with additional results tabulated in section 6.2 herein

- \( c_i, e_i, f_i, k_i \) span load parameter, chord load parameter and section drag coefficients, respectively
L, D, M wing lift, drag and pitching moment, respectively

C_L, C_D, C_M wing lift coefficient, L/qS; wing drag coefficient, D/qS; wing pitching moment, M/qS, respectively

E_l(t); F_l(t); conical functions upon which the following coefficients depend, respectively: velocity potential and span loading;

G_l(t); H_l(t); pressure; pitching moment; and drag

g_l(t); h_l(t)

T * functions of m upon which the suction drags depend

q_T a number used to indicate percentage of full leading edge suction assume:

C_D suction drag coefficient

G_n, G_{n,k} functions used to obtain suction drag (see section 4.3)

G_n

X_{i,k} orthogonal weighting numbers

\lambda_{l,j} C_{D_l,j} + C_{D_{l,i}}

a_{kn} weighting numbers chosen to minimize the drag

\omega_{kn} functions of X_{i,k} and a_{kn} [equation (4.42)]

\bar{a}_{kn}, \bar{\omega}_{kn} a_{kn}/\beta C_{L_d} and \omega_{kn}/\beta C_{L_d}, respectively

C_{L_d} design lift coefficient

\Gamma(Y'), \Gamma_l functions required to satisfy geometric boundary conditions

C_{M_m} pitchin{g}-moment coefficient about a fraction x_m of the M.A.C.

x_m point of zero pitching moment, percent of M.A.C.
Lagrangian numbers

Equations describing restraints

Geometric quantities relating shape and position of geometric boundary conditions (see figure 1)
4.0 ANALYSIS

4.1 Background

In order to compute the shape of wings having reduced drag due to lift at supersonic speeds as well as the drag of the resulting configurations, it is necessary to know the pressure distribution. The pressure distribution, however, depends upon the downwash over the wing. The procedure used herein is to assume that the wing shape is expressible in terms of the power series

\[
\frac{w}{V} = -\alpha (x', y') = - \sum_{r,s} \alpha_{rs} (x')^r | y' |^s
\]  

(4.1)

for \( r + s \leq c \) where the primed quantities are non-dimensionalized by \( e_r \) and \( h_2 \), respectively. Solutions are obtained to the linearized equation for the velocity potential:

\[
\beta^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = 0
\]  

(4.2)

for the boundary conditions of equation (4.1).

It has been shown in reference 3 that for each term of the boundary conditions, equation (4.1), there is associated a potential, \( \phi_x / \alpha_{rs} \), and its corresponding horizontal perturbation velocity, \( \phi_x / \alpha_{rs} \). Therefore, for convenience, \( \dagger \) one may write the total potential and total horizontal perturbation velocity in the form

\[

\frac{w}{V} = -\alpha (x', y') = - \sum_{r,s} \alpha_{rs} (x')^r | y' |^s
\]

(4.1)

\[
\beta^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = 0
\]

(4.2)

for the boundary conditions of equation (4.1).

\( \dagger \) The change in notation from a double summation to a single one represents, at best, an abbreviation to make the following work more readable. For example, this analysis has been restricted to \( r + s \leq 2 \) and so equation (4.1) may be expanded to ten terms. For each of these "i" terms there is an effective potential given by \( \beta^2 \) such that the total potential is the sum of each contribution. Since all \( \dagger \) the aerodynamic quantities are related to the potential the remaining definitions are consistent.
\[ \phi^* = \sum_i \phi_i^* \]  \hspace{1cm} (4.3)

\[ \phi_i^* = \sum_i (\phi_i^*)^6 \]  \hspace{1cm} (4.4)

where \( i \) is now related to \( rs \) as shown in Table I.

**TABLE I**

Relation Between the Single Index, \( i \), and the Double Index, \( rs \)

<table>
<thead>
<tr>
<th>( i )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( rs )</td>
<td>00</td>
<td>01</td>
<td>02</td>
<td>03</td>
<td>04</td>
<td>05</td>
<td>06</td>
<td>07</td>
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By linear theory the lifting pressure distribution is given by

\[ \frac{\Delta p}{l} = \frac{4}{V} \phi \]

and it follows that the total lifting pressure coefficient may be written in terms of each of the contributions corresponding to the downwash series:

\[ \left( \frac{\Delta p}{q} \right)_i^s = \sum \left( \frac{\Delta p}{q \alpha} \right)_i^s \alpha^i \]  \hspace{1cm} (4.6)

The details involved with the determination of the quantities \( \left( \phi / \alpha \right)_i^s \), \( \left( \phi / \alpha \right)_i^s \), and \( \left( \Delta p / q \alpha \right)_i^s \) are presented both in analytical and graphical form for the first ten terms \((r + s = 3)\) of the downwash for subsonic, sonic and supersonic leading edges in reference 5.
4.2 Aerodynamic Characteristics

The lift, moment and drag coefficients are defined, respectively, by:

\[ C_L = \frac{1}{S} \int \int \frac{\Delta p}{\frac{1}{2} \rho v^2} \, dS \quad (4.7) \]

\[ C_M = -\frac{1}{S} \int \int x \frac{\Delta q}{q} \, dS \quad (4.8) \]

\[ C_D = \frac{1}{S} \int \int \frac{\Delta \tau}{\rho v^2} \frac{dz}{dx} \, dS \quad (4.9) \]

Substituting equation (4.6) into equations (4.7) to (4.9) results in

\[ C_{L*} = \sum_i \left( C_{L/\alpha} \right)_i \alpha_i \quad (4.10) \]

\[ C_{M*} = \sum_i \left( C_{M/\alpha} \right)_i \alpha_i \quad (4.11) \]

\[ C_{D*} = \sum_i \alpha_i \sum_j \alpha_j C_{D/\alpha} \quad (4.12) \]

since

\[ \frac{dz}{dx} = -\alpha (x, \beta) \quad (4.13) \]

The form of the drag coefficient results from the substitution of two power series for the lifting pressure coefficient and the slope, respectively, into equation (4.9).
Thus $C_{D_{1,1}^*}$ is the contribution of the pressure coefficient $(\Delta p/q_{\alpha})^*$ induced by $\alpha_i$ acting upon the downwash $\alpha_j$.

The functions $(C_{L_{1,1}^*})$ and $(C_{M_{1,1}^*})$ are presented in both analytical and graphical form in reference 3 for subsonic, sonic and supersonic leading-edge delta wings.

The basic drag contributions were derived as follows:

$$C_{D_{1,1}^*} = \int_0^1 C_{D_{1,1}^*} \frac{c}{c_{a v}} \, dx' \quad (4.14)$$

where

$$C_{D_{1,1}^*} = \frac{c}{c_{a v}} = 2 \int_{x'}^{x_{T.E.}} \left(\frac{\Delta p}{q_{\alpha}}\right)_{\alpha_i} \left(\frac{dz'}{dx'}\right)_{\alpha_i} \, dx' \quad (4.15)$$

with $\frac{dz'}{dx'}$ now considered to be equal to $(x')^{-1}$ for $j' = 0$, where the primes on $r$ and $s$ signify that the loading $(rs)$ may interfere with the downwash $(r's')$.

Equation (4.14) reduces to

$$C_{D_{1,1}^*} = \frac{c}{c_{a v}} = \frac{1}{\pi} \left(\frac{2 \, e \, r}{\beta \, b}\right)^{s+s'} (x')^{-\kappa} H_{1,1}(t) \quad (4.16)$$

where

$$\kappa = 1 + n + n' + s + s'$$

$$x' = \left(\frac{m}{m - t}\right)^{\alpha_0}$$

$$\beta = \left(\frac{m_0 - m}{m / m_0}\right)^{-1}$$

$$\frac{2 \, e \, r}{\beta \, b}$$
Equation (4.16) is valid for all triangular wings. The functions $H_{i,j}(t)$ are presented in closed form in the Appendix, section 5.1, Tables A, B and C for subsonic, sonic and supersonic leading edges, respectively.

Substituting equation (4.16) into equation (4.15) there follows

$$C_{D_{i,j}}^* = \frac{8}{\pi} \left( \frac{m_0 - m}{m_0 m} \right)^{1+\gamma} m_0^{2+\kappa} \int_0^m \frac{(m - t)^{-2+\kappa}}{H_{i,j}(t)} dt \quad (4.15)$$

which, for delta wings ($m_0 = \infty$) reduces to

$$C_{D_{i,j}}^* = \frac{8}{\pi} \left( \frac{1}{m} \right)^{1+\gamma} \int_0^m \frac{m}{H_{i,j}(t)} dt \quad (4.19)$$

The functions $C_{D_{i,j}}^*$ are presented in the Appendix, section 5.3, Tables A, B, C and D, for subsonic, sonic and supersonic leading edges, and the limiting case, $m = 0$, respectively.

For wings with swept trailing edges equation (4.15) may be readily evaluated with the aid of high-speed computing equipment, a procedure currently being programmed at Convair-San Diego.

4.9 Suction Drag Coefficients

The suction drag coefficient is defined by

$$C_{D_s} = -\frac{b/2}{S} \int_0^{\gamma} \frac{G_0^2}{\gamma n} dy \quad (4.20)$$
where

\[
G_n^\prime = \ln \left( \frac{x_n}{y_n} \right) \quad (4.21)
\]

\[
3n = \sqrt{1 - m^2 \cos \epsilon} \quad (4.22)
\]

and \( v_n \) is the velocity normal to the leading edge. \( x_n \) and \( y_n \) are the components of \( x, y \) normal to and parallel to the leading edge, respectively.

\[
v_n = \frac{\partial}{\partial x_n} \cdot (x_n, y_n) \quad (4.23)
\]

From equation (4.4) it follows that

\[
G_n^\prime = \sum_k G_{n,k} \quad (4.24)
\]

where \( G_{n,k} \) is the contribution of the \( k^{th} \) term of the downwash to \( G_n \) and \( G_n^\prime \) is the value of \( G_n \) corresponding to \( i \) terms of the downwash.

The suction drag coefficients which are presented in the Appendix, section 8.4, for various terms of the downwash equation may be expressed by

\[
C_D^{(n)} = \sum_{i=0}^{n} \sum_{j=0}^{m} \frac{1}{T_i, j} \quad (4.25)
\]
where the superscript \((n)\) refers to the highest number of terms of the power series for the downwash and \(T_{1,j}\) are functions of \(m = \tan \epsilon\). Uncited references indicate the amount of leading-edge suction likely to develop depends upon \(m\) and the nose shape. The parameter \(0 \leq q_T \leq 1.0\) may be used empirically in this regard.

### 4.4 Drag Reduction

#### 4.4.1 Orthogonal Loading

The minimization of the drag is greatly facilitated by eliminating the interference drag. A set of orthogonal loadings, \((\Delta p/q)(k)\), \(\alpha(k)\), are constructed from the basic loadings, \((\Delta p/q)_1\), \(\alpha_i\), by

\[
\alpha(k) = \sum_{i=0}^{n} X_{ik} \alpha_i \tag{4.26}
\]

\[
(\Delta p/q)(k) = \sum_{i=0}^{n} X_{ik} (\Delta p/q)_1 \tag{1.27}
\]

where \(n\) is the number of terms \((0, 1, 2 \ldots 9)\) of the downwash power series used and

\[
\alpha_i = (x')^r (y')^s \quad ; \quad r, s \geq 0 \tag{1.25}
\]

\[
(\Delta p/q)_1 = \left(\frac{\Delta \rho}{\rho}\right)_{rs} \quad \text{(reference 9)} \tag{4.29}
\]

\(X_{ik}\) are weighting numbers chosen to satisfy the orthogonality requirement that the inner drag product disappears.
\[ \int_{s} \left( \frac{\Delta p}{q} \right)_{j} \alpha_{j} + \left( \frac{\Delta p}{q} \right)_{j} \alpha_{j}^{l} \right] dS = 0 \quad (\ast.30) \]

The \( X_{lk} \) are obtained from

\[
\sum_{i=0}^{k} X_{ik} \lambda_{i,i} = 0, \quad j = 0, 1, 2 \ldots k-1 \quad (4.51)
\]

\[
\lambda_{i,j} = \lambda_{j,i} + C_{i,j}^{*} + C_{i,j}^{*} \quad (4.52)
\]

A new loading \( (\Delta p/q, \alpha) \) is constructed from the orthogonal set with the aid of the weighting numbers \( a_{kn} \) where

\[
\alpha = \sum_{k=0}^{n} a_{kn} \alpha^{(k)} \quad (4.33)
\]

\[
\frac{\Delta p}{q} = \sum_{k=0}^{n} a_{kn} \left( \frac{\Delta p}{q} \right)^{(k)} \quad (4.34)
\]

Substituting equations (4.33) and (4.34) into equation (4.9) results in

\[
C_{D}^{*} = \sum_{k=0}^{n} a_{kn} 2 (C_{D})^{(k)} \quad (4.35)
\]
where the orthogonal drag coefficients are found by

\[
\begin{align*}
(C_D^o)^{(k)} &= \frac{X_{ik}}{2} \sum_{l=0}^{k} X_{l,k} X_{l,k} \lambda_l, k \\
\end{align*}
\]

(4.36)

4.4.2 Drag Minimization for Specified Lift

Following the results of Graham\(^{15}\), one may write for the minimum drag coefficient

\[
\frac{C_D}{\beta C_L u^2} = \sum_{k=0}^{n} \bar{a}_{kn} \frac{2}{\lambda} (\gamma^*)^{(k)}
\]

(4.37)

where

\[
\bar{a}_{kn} = \left( \frac{n}{\mu C_L d} \right) \left( \frac{C_l^*}{C_L} \right)^{(k)} / \left( \frac{C_D^*}{C_D} \right)^{(k)}
\]

(4.38)

From equations (4.7) and (1.27) it follows that

\[
(C_L^*)^{(k)} = \sum_{l=0}^{k} X_{l,k} C_L^*(k)
\]

(4.39)

The mean chord line may be determined from
where \( z \) and \( x \) are non-dimensionalized by \( c \), and \( y \) is non-dimensionalized by \( b/2 \).

Integrating equation (4.39) there results

\[
- \frac{1}{\beta C_{L_d}} \frac{dz'}{dx'} = \frac{\alpha(x', y')}{\beta C_{L_d}} = \sum_{k=0}^{n} \alpha_{kn}^{(k)}
\]

(4.40)

where \( z' \) and \( x' \) are non-dimensionalized by \( c \), and \( y' \) is non-dimensionalized by \( b/2 \).

Integrating equation (4.39) there results

\[
- \frac{z'}{\beta C_{L_d}} = \int \left( \sum_{k=0}^{n} \alpha_{kn}^{(k)} \right) dx' + \Gamma(y')
\]

(4.41)

where \( (y') \) is used to satisfy geometric boundary conditions. Since \( \alpha^{(k)} \) can be represented by a series in terms of the basic downwashes, \( \alpha_i' \), equation (4.41) can be made more tractable by means of the substitution

\[
\tau_{kn} = \sum_{i=k}^{n} X_{kd} \tilde{a}_{ln}
\]

(4.42)

which results in

\[
- \frac{\tau'}{\beta C_{L_d}} = \int \left( \sum_{k=0}^{n} \tau_{kn} \alpha_k^{(k)} \right) dx' + \Gamma(y')
\]

(4.43)
- \frac{z'}{C_L d} = \left( \phi_0 + \phi_1 \gamma + \phi_2 \gamma^2 + \phi_3 \gamma^3 \right) x' \\
+ \frac{1}{2} \left( \phi_0 + \phi_4 \gamma + \phi_7 \gamma^2 \right) x'^2 \\
+ \frac{1}{3} \left( \phi_5 + \phi_8 \gamma \right) x'^3 + \frac{1}{4} \psi_9 x'^4 \right] + \Gamma(y') \quad (4.44)

4.4.3 Drag Reduction with Specified Static Margin at Design Lift

This problem is readily treated with the aid of Lagrange's constant multipliers. The static margin is specified in terms of the pitching-moment about the leading edge apex by means of the transfer formula

$$C_{M_{x=0}} = C_{M_{x=0}} + \left( \frac{1+2 x_m}{2} \right) C_L \quad (4.45)$$

where $x_m$ is the point at which zero moment is desired as a given fraction of the mean aerodynamic chord. From equations (4.7), (4.8) and (4.34) there results

$$\frac{C_{L}}{C_L d} = \sum_{k=0}^{n} \bar{\alpha}_{kn} (C_{L}^{(k)}) \quad (4.46)$$

$$\frac{C_{M_{x=0}}}{C_L d} = \sum_{k=0}^{n} \bar{\alpha}_{kn} (C_{M}^{(k)}) \quad (4.47)$$
where

\[
(C^M_{L})^{(k)} = \sum_{i=0}^{n} \chi_{ik} C^*_M
\]  \hspace{1cm} (4.43)

Define

\[
\bar{D} = \frac{C_D}{\beta C^*_{Ld}} + \sum_{i=1}^{l} \Omega_i \varphi_i
\]  \hspace{1cm} (4.49)

(\text{where } l = 2)

\[
\varphi_1 = \sum_{k=0}^{n} \tilde{a}_{kn} (C^*_{L})^{(k)} - 1 = 0
\]  \hspace{1cm} (4.50)

\[
\varphi_2 = \sum_{k=0}^{n} \tilde{a}_{kn} (C^*_M)^{(k)} + \frac{1+2 \chi_m}{2} = 0
\]  \hspace{1cm} (4.51)

The condition for the drag to be a minimum for the specified lift and static margin is satisfied when

\[
\frac{\partial \left( C_D / \beta C^*_{Ld} \right)}{\partial \tilde{a}_{kn}} + \sum_{i=1}^{l} \Omega_i \frac{\partial \bar{D}}{\partial \tilde{a}_{kn}} = 0
\]  \hspace{1cm} (4.52)
The $n + 3$ unknowns $(a_{kn}, \Omega_i)$ can be determined by matrix methods using the $n + 3$ equations (4.50), (4.51) and (4.52). For the simple case under consideration where $f = 2$, an alternate approach is to relate $a_{kn}$ to $\Omega_i$, determine $\Omega_i$ from the two boundary equations and then compute $a_{kn}$. Thus, from equations (4.52) and (4.35) there results

$$\sum_{i=1}^{2} \Omega_i \frac{\partial a_{kn}}{\partial \Omega_i} = \frac{\sum_{i=1}^{2} \Omega_i}{2 (C_D^*)^{(k)}}$$ (4.52)

which is substituted into equations (4.50) and (4.51):

$$\Omega_1 \sum_{k=0}^{n} \left[ \frac{(C_L^*)^{(k)}}{(C_D^*)^{(k)}} \right]^2 + \Omega_2 \sum_{k=0}^{n} \frac{(C_L^*)^{(k)} (C_M^*)^{(k)}}{(C_D^*)^{(k)}} = -2.0 \quad (4.54)$$

$$\Omega_1 \sum_{k=0}^{n} \frac{(C_L^*)^{(k)} (C_M^*)^{(k)}}{(C_D^*)^{(k)}} + \Omega_2 \sum_{k=0}^{n} \left[ \frac{(C_M^*)^{(k)}}{(C_D^*)^{(k)}} \right]^2 = 1 + 2 \times \frac{m}{m} \quad (4.55)$$

The summations in equations (4.54) and (4.55) are determined from the previously computed quantities $(C_L^*)^{(k)}$, $(C_M^*)^{(k)}$, $(C_D^*)^{(k)}$.

The total drag at the design condition may be determined with little effort from equation (4.35) or by the procedure indicated by Stancil. Omitting the details involved in deriving the results on the basis of the current analysis using orthogonal loads,
The mean chord line is determined from equation (4.44) when a new set of $\Xi_{kn}$ are computed for the values of $\bar{a}_{kn}$ obtained in this section. The weighting numbers $X_{lk}$ obtained in section 4.4.2 are the same for the present section.

4.4.4 Drag Reduction with Specified Static Margin and Two Straight Hinge-Lines

This problem follows directly from the previous problem when additional relations between $a_{kn}$ are obtained which describe the desired boundary conditions. This results in additional functions $\varphi_1 (i = 3)$ for equation (1.52).
The solution for this problem first involves the determination of \( n + 1 + 1 \)
constants \((a_{kn}, \Omega_1)\) for which there are a like number of equations. The
values for \( \Omega_{kn} \) are determined from equation (4.42) using the proper \( a_{kn} \).

If the two straight hinge-lines are described by (see Figure 4.1)

\[
\begin{align*}
\frac{Z'}{\beta L_d} &= \delta_1 + \beta y' \quad \text{(at } x' = n_1 y') \\
&= \delta_2 + \beta y' \quad \text{(at } x' = m_1) 
\end{align*}
\]  

then the arbitrary function \( \Gamma(y') \) consistent with the assumed conditions is

\[
\Gamma(y') = \Gamma_0 + \Gamma_1 y' + \Gamma_2 y'^2 + \Gamma_3 y'^3 + \Gamma_4 y'^4
\]  

where

\[
\begin{align*}
\Gamma_0 &= 0 = \delta_3 - m_1 (\psi_{0n} + \frac{1}{2} m_1 \psi_{2n} + \frac{1}{3} m_1 \psi_{3n} + \frac{1}{4} m_1 \psi_{4n}) \\
\Gamma_1 &= \delta_0 - n_1 \psi_{0n} = \delta_2 - m_1 (\psi_{1n} + \frac{1}{2} m_1 \psi_{4n} + \frac{1}{3} m_1 \psi_{5n}) \\
\Gamma_2 &= -n_1 (\psi_{1n} + \frac{1}{2} n_1 \psi_{2n}) = -m_1 (\psi_{3n} + \frac{1}{2} m_1 \psi_{4n}) \\
\Gamma_3 &= -m_1 \psi_{3n} = -n_1 (\psi_{3n} + \frac{1}{2} n_1 \psi_{5n}) \\
\Gamma_4 &= 0 = n_1 (\psi_{6n} + \frac{1}{2} n_1 \psi_{8n} + \frac{1}{3} n_1 \psi_{9n} + \frac{1}{4} n_1 \psi_{10n})
\end{align*}
\]
Figure 4.1

- \frac{Z}{\beta C_{lo}}

1. \( S + S_{x} \)

2. \( S + S_{y} \)

\( O, x' \)

\( m_{b}, \frac{m_{b}}{n_{b}} \)

\( y' \)

\( x' = m \)

\( x' = n \)

\( D \)

\( z', 0 \)

\( c_{n} \)

\( \epsilon \)

\( \theta \)

\( \gamma' \)

\( A \)

\( B \)

\( C \)

\( D \)

\( E \)

\( F \)

\( G \)

\( H \)

\( I \)

\( J \)

\( K \)

\( L \)

\( M \)

\( N \)

\( O \)

\( P \)

\( Q \)

\( R \)

\( S \)

\( T \)

\( U \)

\( V \)

\( W \)

\( X \)

\( Y \)

\( Z \)
The equations for the intersection of the two hinge-lines is

\[ (\delta_2 - \delta_0) = \frac{n_1}{m_1} (\delta_n - \delta_1) \]  \hspace{1cm} (4.64)

From equations (4.63) and (4.64) one can eliminate \( r \), \( (r_1 - 6) \), and \( (r_2 - r_0) \) thereby resulting in four relations for \( a_{kn} \) which are consistent with equations (4.60) and (4.61). These relations were written in the form

\[ \psi_5 \equiv \sum_{k=0}^{n} C_k a_{kn} = 0 \]  \hspace{1cm} (4.65)

\[ \psi_4 \equiv \sum_{k=0}^{n} D_k a_{kn} = 0 \]  \hspace{1cm} (4.66)

\[ \psi_6 \equiv \sum_{k=0}^{n} E_k a_{kn} = 0 \]  \hspace{1cm} (4.67)

\[ \psi_7 \equiv \sum_{k=0}^{n} F_k a_{kn} = 0 \]  \hspace{1cm} (4.68)

where \( C_k \), \( D_k \), \( E_k \), \( F_k \) are known functions of \( m_1 \), \( n_1 \), \( X_{lk} \) given in section 8.5 of the Appendix. Thus, the \( n + 1 + 1 \) unknowns, \( \pi_{kn} \), \( \Omega_i \) are found from the \( n + 1 + 1 \) equations.
The total drag is computed from equation (4.35) or (4.56) where $R_i = 0$
for $i \geq 3$. The mean chord line is found from equations (4.44) and (4.63).

### 4.4.5 Drag Reduction for Wings Having Subsonic Leading Edges

When leading edge suction is neglected the previous techniques are applicable. When complete leading edge suction is included [Equation (4.25)] the drag relation becomes

$$
\frac{C_D}{L_d} = \frac{2}{C^{2}} \sum_{k=0}^{n} \bar{a}_k \left( C_D^{(k)} \right)
$$

$$
\sum_{j=0}^{n} \bar{a}_{jn} \sum_{j=0}^{n} \bar{a}_{jn} T_{1,j} + \frac{L_d}{C^{2}}
$$

where $q_T$ represents the percentage of full leading edge suction one may expect. Substituting Equation (4.71) into (4.52) results in the system of $n + 1$ equations.
\[ 2 \left( C_s \right)^{(k)} \delta_{kn} + q_T X_{kk} \left\{ \sum_{p=0}^{k} X_{pl} T_{p,k} \sigma_{ln} + \sum_{p=0}^{1} X_{pl} T_{p,k} \sigma_{ln} \right\} + \sum_{i=k+1}^{n} \sum_{j=1}^{l} \Omega_{ij} \frac{\partial}{\partial \alpha_{kn}} = 0 \] (4.72)

In the \( n + 1 + 1 \) unknowns \( \delta_{kn} \) and \( \Omega_{ij} \), which may be solved by the previously established method compatible with the additional \( \phi \) equations.

The mean chord line is determined from equation (4.44) from the \( \gamma_{kn} \) which were computed in terms of the \( \delta_{kn} \) which resulted from the solution of equations (4.72) and the equations for \( \phi \).

The drag polars are computed from equation (4.71) which includes leading edge suction.

4.5 Span-Loading

When the shape of the wing has been determined the span-loading for any leading edge condition may be obtained from

\[ \frac{C_n}{\bar{C}} \frac{c}{c} = \frac{8}{\pi} \frac{\partial}{\partial \beta} \sum_{l=0}^{n} \left( \frac{m - m}{m - m} \right)^{s} (x') \left( T_{l,n} \right) \left( \gamma_{ln} \right) \left( \beta \right) \] (4.73)
where \((x')_{T.E.}\) = 1 for the delta wing and for the wing with swept trailing edge

\[
(x')_{T.E.} = \left(\frac{m}{m_o - m}\right) \frac{y'}{m_o} + 1 \quad (4.74)
\]

The \(\frac{y}{m}\) were obtained from the minimization process and the \(E_i(t)\) may be obtained from reference 3 (Tables II A), B), C) for subsonic, sonic and supersonic leading edges, respectively, or from Figures 5.1 to 5.20).

4.6 Chord-Loading

Consistent with the total load induced by the various basic loadings, \((\Delta p/\eta)_1\), the chord-loading is defined as

\[
\frac{C_L y}{C_{L_d} b/2} = 2 \int_{i=0}^{\infty} \frac{\beta d}{\beta} x' \sum_{i=0}^{n} \left(\int_{0}^{1} \left(\frac{\Delta p}{\eta}\right) dy'\right) \left(\frac{m_o - m}{m m_o}\right) \left(\frac{m_o - m}{m m_o}\right) dy'
\]

which reduces to

\[
\frac{C_L y}{C_{L_d} b/2} = 2 \int_{0}^{1} \left(\frac{\Delta p}{\eta}\right) dy'
\]

for delta wings.
In reference 3 it was shown that

\[ \left( \frac{\Delta p}{\rho} \right)_{i} = \frac{4}{\pi} \left( \frac{m_{2} - m_{0}}{m_{0}} \right)^{2} (s')^{r+b} F_{2}(t) \] (4.77)

where the \( F_{2}(t) \) are given therein (see Tables III A), B), C) for subsonic, sonic, and supersonic leading edges, respectively, or Figures 5.21 to 5.40).

Since \( \Delta p / \rho \) have integrable singularities at the leading edge for subsonic and sonic leading edges the results of equations (4.76) and (4.77) are presented analytically:

\[ \frac{c_{f} \frac{y}{b/2}}{c_{L_{d}}} = \sum_{i=0}^{n} \frac{1}{\ln \left( \frac{c_{f} \frac{y}{b/2}}{i} \right)} \] (4.78)

where \( \left( \frac{c_{f}}{b/2} \right) \) are given in Table II for \( m \leq 1 \).

For the supersonic leading edge case one can plot the pressure distributions [equation (4.77)] for any given chordwise position and graphically integrate equations (4.75) or (4.76) since there are no singularities.
### TABLE II

Chord Loading Functions for Subsonic and Sonic Leading Edges

<table>
<thead>
<tr>
<th>i</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_x y^* / b/2 i$</td>
<td>$4x^i$</td>
<td>$2x^{i^2}$</td>
<td>$4x^{i^2}$</td>
<td>$(4/3)x^{i^3}$</td>
<td>$2x^{i^3}$</td>
</tr>
<tr>
<td>$m = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $C_x y^* / b/2 i$ | $2πm / E$ | $-2mA_4 / A_1 x^{i^2}$ | $3π ma_6 / A_1 x^{i^2}$ | $2πmA_5 / A_2 x^{i^3}$ | $-2mA_8 / 3A_2 x^{i^3}$ |
| $m < 1$ |        |        |        |        |        |

<table>
<thead>
<tr>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x^{i^3}$</td>
<td>$x^{i^4}$</td>
<td>$(4/3)x^{i^4}$</td>
<td>$2x^{i^4}$</td>
<td>$4x^{i^4}$</td>
</tr>
</tbody>
</table>

| $2πmA_6 / A_2 x^{i^3}$ | $-mA_6 / A_3 x^{i^4}$ | $5πa_6 C_{32} / 2A_3 x^{i^4}$ | $mA_9 / 2A_3 x^{i^4}$ | $-15πa_6 C_{42} / 2A_3 x^{i^4}$ |
5.0 DISCUSSION

The analytic means and procedure for minimizing the drag-due-to-lift of triangular wings has been outlined in the previous section. To aid the engineer in carrying out similar investigations some illustrative examples for sonic (M = 2.0) and supersonic (M = 2.5) leading-edge delta wings are presented in section 9.0. The procedure is spelled out for the sonic leading-edge delta and the results are tabulated for the supersonic leading-edge design conditions.

Since there are six wings to be discussed the following designations are used:

Wing Warp 

**56:** \( C_{L_d} = .097, M_d = 2.5 \) (m = 1.523)

**5a:** \( C_{L_d} = .097, M_d = 1.5, C_{M.363} = 0 \)

**5a:** \( C_{L_d} = .097, M_d = 1.5, C_{M.363} = 0 \)

Two straight hinge lines (Figure 4.1)

**5b:** \( C_{L_d} = .136, M_d = 2.0 \) (m = 1.0)

**5c:** \( C_{L_d} = .136, M_d = 2.0, C_{M.363} = 0 \)

**5d:** \( C_{L_d} = .136, M_d = 2.0, C_{M.363} = 0 \)

Two straight hinge lines

These lift coefficients permit the configurations to maintain level flight at the assumed design Mach numbers. The boundary conditions for the two straight hinge lines are defined in Figure 4.1.
A comparison of the drag polars with the trimmed and untrimmed flat plate is presented in Figure 5.1. These curves are trimmed only at the design lift for the "a" and "b" types. Figures 5.1 (a) and (b) present the results for configurations designed for the supersonic and sonic leading edges, respectively. As expected, less drag reduction is realized as the number of restraints are increased in all cases. The effect of Mach number on the drag polars is illustrated in Figures 5.2 to 5.3 for the supersonic and sonic leading edge design conditions, respectively.

The pitching moment curves are presented in Figure 5.4 at each design Mach number. The effect of Mach number for the various wing designs is indicated in Figures 5.5 and 5.6.

The effect of twist and camber on the span loadings at the design condition are shown in Figure 5.7 with the flat plate span loading for the supersonic and sonic leading edges. It is observed that all the warped wings deviate from the elliptic span-loading of the flat plate. (An elliptic span-loading is a sufficient but not a necessary condition and the drag reduction results mainly from the marked improvement in the chord loading.)

The requirement for two straight hinge-lines (α-type wing) is accompanied by a penalty in both drag and bending moment as indicated. Figures 5.8 and 5.9 show the effect of Mach number on the span loading for each wing at M = 2.0 and 2.1.

The chord loadings resulting at the design condition are compared to the flat plate loading in Figure 5.10. It is observed that the effect of warp generally tends to modify the undesirable triangular flat plate loadings in some manner toward the desired elliptic shape. The effect of Mach number on the chord loadings is illustrated in Figures 5.11 and 5.12 for each wing.

The shapes of the wings designed for each restraint are shown in Figure 5.13 for each of the leading-edge conditions.

* This is substantiated by Fenain and Vallee and presented the span-loading for a Germain "type sonic leading-edge Delta wing. The results herein are in perfect agreement with the Fenain-Vallee computation.
The effect on drag reduction of each additional term in the power-series expansion for the downwash, equation (4.1), is shown in Figure 5.14 for the sonic leading-edge delta wing and the supersonic leading-edge delta wing.
Figure 5.1(a). Comparison of Drag Polars for Supersonic Leading Edge Deltas for Various Design Restrictions

\( \delta_e = 0 \) for warped wings
Figure 5.1(b). Comparison of Drag Polars for Sonic Leading Edge Deltas for Various Design Restrictions
(\( \delta_e = 0 \) for warped wings)
Figure 5. §(a). Effect of Mach Number on Drag Polars
for Supersonic Leading Edge Delta Wing $\frac{a}{\delta}$
Figure 3.2(b). Effect of Mach Number on Drag Polars for Supersonic Leading Edge Delta Wing $\beta_a$
Figure 5.2(c). Effect of Mach Number on Drag Polars for Supersonic Leading Edge Delta Wing

\[ \alpha \]
Figure 5.3(h). Effect of Mach Number on Drag Polars for Cone Lending Edge Delta Wing **[3]**.
Figure 5. Effect of Mach Number on Drag Polars for Sonic Leading Edge Delta Wing "2".

\[ \frac{C_d}{C_l} = \frac{2M}{\sqrt{1 + \frac{2}{M^2}}} \]

\[ 8 = 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \]
Figure 6.4(a). Pitching Moment Coefficient
Comparisons for Supersonic Leading Edge Delta Wings for Various Design Restrictions

\[ C_{L_d} = 0.087 \]
\[ M_d = 2.5 \]
\[ \alpha = 0^\circ \]

RESULTS GOOD FOR \( M = 2.0 \) (\( M = 1.0 \))

\[ C_{L_d} \]

\[ \alpha \]

\[ \beta \]

\[ \gamma \]

\[ C_{M_{36\circ}} \]

\[ +0.01 \to +0.05 \]
Figure 5.4(b). Pitching Moment Coefficient Comparisons for Sonic Leading Edge Delta Wings for Various Design Restrictions

\[ C_{L_d} = 0.136 \]

\[ M_d = 2.0 \]

\[ \delta_c = 0^\circ \]

Results good for \( M = 2.5 (m = 1.323) \)
Figure 5.5(a). Effect of Mach Number on
Pitching Moment Coefficient for Supersonic
Leading Edge Delta Wing "B"
Figure 5.5(b): Effect of Mach Number on Pitching Moment Coefficients for Supersonic Leading Edge Wing \( \beta_a \).

- \( C_{L_d} = 0.057 \)
- \( M_d = 2.5 \)
- \( \delta_e = 0^\circ \)

\( C_L \) versus \( \Theta_m \) for \( M = 1 \):

Untrimmed Flat Plate.
Figure 5.5(c): Effect of Mach Number on Pitching Moment Coefficient for Supersonic Leading Edge Delta Wing \( \alpha_d \)

- \( C_{L_d} = 0.087 \)
- \( M_0 = 2.3 \)
- \( \theta_0 = 0^\circ \)
Figure 5.6-(a). Effect of Mach Number on Pitching Moment Coefficient for Sonic Leading Edge Delta Wing "S"
Figure 5.6(b). Effect of Mach Number on Pitching Moment Coefficient for Sonic Leading Edge Delta Wing "β".

\[ C_{L_d} = 1.38 \]
\[ M_d = 2.0 \]
\[ \delta_e = 0^\circ \]
Figure 5.6. Effect of Mach Number on Pitching Moment Coefficient for Sonic Leading Edge Deformable Wing. 

\[ C_{l_{d}} = 0.136 \]

\[ M_{c} = 2.0 \]

\[ \delta_{c} = 0^\circ \]
Figure 8.7(a). Comparison with Flat Plate of Span Loading at Design Lift for Supersonic Leading Edge Deltas with Various Design Restrictions.

\[ C_{Ld} = 0.87 \]

\[ M_d = 2.5 \]

\[ \alpha_c = 0^\circ \]
Figure 3.7(b). Comparison with Flat Plate of Span Loading at Design Lift for Sonic Leading Edge
Delta Wings with Various Design Restrictions

\[ C_{Ld} = 0.136 \]

\[ M_d = 2.0 \]

\[ \beta_e = 0^\circ \]
Figure 5.7(b). Comparison with Flat Plate of Span Loading at Design Lift for Sonic Leading Edge.

Delta Wings with Various Conditions.

\[ C_{Ld} = 13 \]
\[ M_d = 2.0 \]
\[ \delta_e = 0^\circ \]
Figure 5.8(a). Effect of Mach Number on Span Loading of Supersonic Leading Edge Delta Wing "δ_a"

\[ C_{L_a} = 0.08 \]

\[ M_d = 2.5 \]

\[ \delta = 0^\circ \]

\[ M \]

\[ M = 1.325 \]

\[ 1.0 \]

\[ 2.0 \]
Figure 5.8(b). Effect of Mach Number on Span Loading of Supersonic Leading Edge Delta Wing "B".
Figure 5.8(c). Effect of Mach Number on Span Loading of Supersonic Leading Edge Delta Wing $\alpha_n$.

\begin{align*}
C_{Ld} &= 0.087 \\
M_0 &= 2.5 \\
\delta_e &= \delta^* \\
\end{align*}

\begin{align*}
\frac{C_{Ld}}{C_{Ld_{ref}}} &= 0.8 \\
M &= 1.325, 2.5 \\
M &= 1.0, 2.0 \\
\end{align*}
Figure 5.9(h). Effect of Mach Number on Span Loading of Sonic Leading Edge Delta Wing "O".
Figure 5, 9(b). Effect of Mach Number on Span Loading of Sonic Leading Edge Delta Wing.  

- $C_{L_d} = 0.36$  
- $M_d = 2.0$  
- $\delta_0 = 0^\circ$
Figure 5.9(d). Effect of Mach Number on Span Loading
for Sonic Leading Edge Delta Wing "D".

\[ \frac{C_{h}}{C_{L_{A,av}}} \]

\[ M_{d} = 2.0 \]

\[ \delta = 0^\circ \]
Figure 5.10(a). Comparison with Flat Plate of Chord Loadings at Design Lift for Supersonic Leading Edge Delta Wings with Various Design Restrictions.

\[ C_{L_d} = 0.087 \]
\[ M_d = 2.5 \]
\[ \alpha = 0^\circ \]
Figure 5.10(c). Comparison with Flat Plate of Chord Loading at Design Lift for Sonic Leading Edge Delta Wings with Various Design Restrictions.
Figure 5.11(a). Effect of Mach Number on Chord Loading of Supersonic Leading Edge Delta Wing 

\[ C_{L_d} = 0.027 \]

\[ M_d = 2.5 \]

\[ \alpha_c = 0^\circ \]
Figure 5.11(b). Effect of Mach Number on Chord Loading of Supersonic Leading Edge Delta Wing "a"
Figure 5.11(c). Effect of Mach Number on Chord Loading of Supersonic Leading Edge Delta Wing \( \alpha = \alpha_a \). 

- \( C_{Ld} = 0.087 \)
- \( M_d = 2.5 \)
- \( \delta_e = 0^\circ \)

Flat Plate
Figure 5.12 (a). Effect of Mach Number on Chord Loading of Sonic Leading Edge Wing $^{16}$. 

$C_{L_d} = 0.136$

$M_d = 2.0$

$\gamma_e = 0^\circ$. 

Flat plate
Figure 5, 12(b). Effect of Mach Number on Chordal Loading of Sonic Leading Edge Delta Wing "B".
Figure 5.12(c). Effect of Mach Number on Chord Loading of Sonic Leading Edge Delta Wing "α".

\[ C_{ld} = 0.136 \]
\[ \bar{M} \; \Delta = 0 \]
\[ \delta_c = 0^\circ \]
Figure 5.13(a): Comparison of Mean Chord Lines for Supersonic Leading Edge Designs for Various Design Restrictions.

\[ C_{Ld} = 0.087 \]

\[ M_d = 2.5 \]
Figure 5.13(b). Comparison of Mean Chord Lines for Sonic Leading Edge Designs for Various Design Restrictions.

\[ C_{p_{11}} = 0.136 \]

\[ M_{ao} = 2.0 \]
Figure 5.14. Relative Effect on Drag Reduction of Increasing Number of Terms of Power Series Expansion for Downwash.
6.0 CONCLUSIONS

(1) Maximum drag reductions of the order of 10% are possible depending upon the leading edge condition for untrimmed warped delta wings at supersonic speeds. Delta wings, however, commonly use elevons as trimming devices although they are inefficient. For trimmed delta wings, however, very significant reduction in drag-due-to-lift is realized by proper warping for both sonic and supersonic leading edges. (No investigation was made for subsonic leading edge designs.) The drag reductions were 46% and 33% for the trimmed sonic leading wings with one (β wing) and two straight hinge lines (α wing) respectively, and 51% and 39% for the corresponding supersonic leading edge designs (β and α respectively).

(2) The drag reduction procedures lead to marked forward shift in chord loading from the triangular flat plate loading for the trimmed wings (α, β). Very little change is observed for the untrimmed (δ) cases. However,

(3) the span loading does not retain the elliptic shape indicated by flat plates having sonic or subsonic leading edges. The trimmed wings appear to be affected more adversely than the untrimmed type wings. The trimmed wings with one straight hinge line (β-type) indicate greater inboard shift of load than the untrimmed wings (δ-type) for sonic and supersonic leading edge designs whereas a significant outward shift in load is observed for trimmed wing types having two straight hinge lines (α). (It is pointed out, however, that the current approximation agrees excellently with the loading for the absolute minimum drag wing computed by Fenain and Vallée.)

(4) Without trim as an aerodynamic restraint the shapes resulting from the current procedure for sonic and supersonic leading edge designs results in the desirable bending of the leading edge into the free stream. The imposed condition
of trim counteracts this feature and requires large positive angles of attack in the immediate vicinity of the root chord.

(5) Preliminary results (not shown) indicate that the trimmed drag increases significantly below and above the design condition.

(6) The Dynamics Group has programmed a procedure to determine the shape required to deflect to the design shape under aeroelastic load. This IBM 704 program has been arranged to provide the input required for the design of sweptback wings having streamwise tips.

(7) The Theoretical Aerodynamics Group has had programmed the procedure to obtain the input required to design pointed tip wings with swept supersonic trailing edges.
7.0 REFERENCES


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8.0 APPENDIX OF FUNCTIONS

8.1 Tabulated Functions $H_i(t)$

In general

$$H_{i,0}(t) = E_i(t) \quad (8.1)$$

$$H_{i,1}(t) = t E_i(t) \quad (8.2)$$

$$H_{i,2}(t) = G_i(t) \quad (8.3)$$

$$H_{i,3}(t) = t^2 E_i(t) \quad (8.4)$$

$$H_{i,4}(t) = t G_i(t) \quad (8.5)$$

$$H_{i,5}(t) = g_i(t) \quad (8.6)$$

$$H_{i,6}(t) = t^3 E_i(t) \quad (8.7)$$

$$H_{i,7}(t) = t^2 G_i(t) \quad (8.8)$$

$$H_{i,8}(t) = t g_i(t) \quad (8.9)$$

$$H_{i,9}(t) = h_i(t) \quad (8.10)$$

$E_i(t)$, $G_i(t)$ for $0 \leq i \leq 9$ are given in Tables II, A, B, C, and III, A, B, C, of reference 3, respectively, for subsonic, sonic and supersonic leading edges. The functions $g_i(t)$ and $h_i(t)$ are given below.
A. Subsonic Leading Edges, \( m < 1 \)

\[
g_0(t) = \frac{\pi}{2m E} (m^2 + 2t^2)^{\frac{1}{2}} m^2 - t^2
\]  
(3.11)

\[
g_1(t) = \frac{1}{4m A_1} \left[ m (2m a_1 + A_{35} t^2 + A_{45} t^4) O \right]
\]  
(8.12)

\[
g_2(t) = \frac{\pi a_o}{4m A_1} \left[ m (2m^3 + t^2) \right] (m^2 - t^2 + t^4 O)
\]  
(8.13)

\[
g_3(t) = \frac{\pi a_0}{30m A_2} \left[ 12m^4 a_2 + m^2 (a_2 - 5c_3) t^2 \right]
\]  
(8.14)

\[
g_4(t) = \frac{1}{180m A_2} \left[ \left( 108m^4 a_{11} + m^2 (9a_{11} - 15m c_9 + 10m A_2) t^2 \right) \right]
\]  
(8.15)

\[
g_5(t) = \frac{\pi}{15m A_2} \left( 6m^4 a_7 + m^2 c_{30} t^2 - 2c_{30} t^4 \right) m^2 - t^2
\]  
(8.16)
\[
g_{6}(t) = \frac{1}{120m} \left[ 4 \alpha_{4}^{4} + 2 \alpha_{4}^{2} \alpha_{1}^{2} t^{2} \right]
\]
\[
+ \left( 3 \alpha_{4} + 2 \alpha_{1}^{2} - 2 \alpha_{3}^{3} \right) \frac{t^{4}}{3} \]
\[
+ \left( 3 \alpha_{4} + 2 \alpha_{1}^{2} - 2 \alpha_{3}^{3} \right) \frac{t^{6}}{3} \theta_{3}^{2}
\]

(8.17)

\[
g_{7}(t) = \frac{\pi}{180m} \left[ 8 \alpha_{13}^{4} + 2 \alpha_{13}^{2} - 9 \alpha_{13}^{4} \right] \frac{t^{2}}{2} \]
\[
+ \left( 6 \alpha_{13} - 3 \alpha_{3}^{2} - 4 \alpha_{11}^{2} \right) \frac{t^{4}}{4} \]
\[
+ \left( 6 \alpha_{13} - 3 \alpha_{3}^{2} - 4 \alpha_{11}^{2} \right) \frac{t^{6}}{6} \theta_{3}^{2}
\]

(8.18)

\[
g_{8}(t) = \frac{1}{144m} \left[ \alpha_{4}^{4} + \alpha_{4}^{2} t^{2} \right] \frac{t^{2}}{2} \]
\[
+ \left( 72 \alpha_{4}^{4} + 4 \alpha_{4}^{2} t^{2} \right) \frac{t^{4}}{4} \theta_{3}^{2}
\]

(8.19)

\[
g_{9}(t) = \frac{\pi}{60m} \left[ 8 \alpha_{9}^{4} + 2 \alpha_{9}^{2} \left( 2 \alpha_{9} + 9 \alpha_{9} \right) t^{2} \right]
\]
\[
+ \left( 2 \alpha_{9} + 4 \alpha_{9} \right) \frac{t^{4}}{4} \]
\[
+ \left( 2 \alpha_{9} + 4 \alpha_{9} \right) \frac{t^{6}}{6} \theta_{3}^{2}
\]

(8.20)
\[ h_0(t) = \frac{\pi}{8m^3 E} \left[ m \left( 2m^2 + 3t^2 \right) m^2 - t^2 + 3t^4 \Omega_3 \right] \quad (8.21) \]

\[ h_1(t) = \frac{1}{30m A_1} \left[ 12m^4 a_1 + m^2 (a_1 + 5m A_{36}) t^2 \right. \]
\[ + 2(a_1 + 5m A_{36}) t^4 \left. \right] m^2 - t^2 \quad (8.22) \]

\[ h_2(t) = \frac{\pi a_2}{4m A_1} \left[ (2m^4 + m^2 t^2 + 2t^4) m^2 - t^2 \right] \quad (8.23) \]

\[ h_3(t) = \frac{\pi}{120m A_2} \left[ m \left[ 40m^4 a_2 + 2m^2 (a_2 - 6A_{36}) t^2 \right. \right. \]
\[ + 3(a_2 - 6A_{36}) t^4 \left. \right] m^2 - t^2 \]
\[ + 3(a_2 - 6A_{36}) t^6 \Omega_3 \right] \quad (8.24) \]

\[ h_4(t) = \frac{1}{32m A_2} \left[ m \left[ 16m^4 a_{11} + 2m^2 (a_{11} - m c_9 + m A_2) t^2 \right. \right. \]
\[ + 3(a_{11} - m c_9 + m A_2) t^4 \left. \right] m^2 - t^2 \]
\[ + \left[ 8m^3 A_2 + 3(a_{11} - m c_9 + m A_2) t^4 \right] t^2 \Omega_3 \right] \quad (8.25) \]
\[
\begin{align*}
  h_5(t) &= \frac{-\pi}{120 m A_2} \left[ 10m^4 a_1 + 2m^2 \left( a_1 - 3c_{20} \right) t^2 \right. \\
  &\quad + 3 \left( a_7 - 3c_{20} \right) t^4 \sqrt{m^2 - t^2} + 3 \left( a_7 - 3c_{20} \right) t^6 \Theta_3 \right] \\
  h_6(t) &= \frac{-1}{3150 m A_3} \left[ 900m^6 a_4 + 36m^4 \left( 2a_4 + 7m e_1 \right) t^2 \right. \\
  &\quad + m^2 \left( 96a_4 + 56m e_1 - 35m^3 c_5 \right) t^4 \left. \\
  &\quad + 2 \left( 96a_4 + 56m e_1 - 35m^3 c_5 \right) t^6 \sqrt{m^2 - t^2} \right] \\
  h_7(t) &= \frac{-2\pi a_6}{4725 m A_3} \left[ 900m^6 a_{13} + 9m^4 \left( 8a_{13} - 21m e_3 \right) t^2 \right. \\
  &\quad + m^2 \left( 96a_{13} - 42m^2 c_3 - 35m^2 c_{11} \right) t^4 \left. \\
  &\quad + 2 \left( 96a_{13} - 42m^2 c_3 - 35m^2 c_{11} \right) t^6 \sqrt{m^2 - t^2} \right] \\
  h_8(t) &= \frac{1}{75,600 m A_3} \left[ 3600m^6 a_{15} + 60m^5 \left( 2a_{15} - 7f \right) t^2 \right. \\
  &\quad + m^2 \left( 160a_{15} + 55m f_3 + 550m f_4 + 2016m A_3 \right) t^4 \right. \\
\end{align*}
\]
\[ + 2 \left( 160 a_{15} + 38m f_3 + 556m f_4 + 2616m A_3 \right) t^6 \left( m^2 - t^2 \right) \]
\[ + 30,240m^6 A_3 t^2 \Theta_3 \]
\[ \frac{2 \pi a_6}{35m A_3} (20m a_9 + m f_1 t^2 + 3m^2 f_2 t^4 + 6 f_2 t^6) \left( m^2 - t^2 \right) \]
(8.29)

\[ h_g(t) = \frac{2 \pi a_6}{35m A_3} (20m a_9 + m f_1 t^2 + 3m^2 f_2 t^4 + 6 f_2 t^6) \left( m^2 - t^2 \right) \]
(8.30)

B. Sonic Leading Edges, \( m = 1 \)

\[ g_0(t) = \frac{2}{3} (1 + 2t^2) \sqrt{1 - t^2} \]
(8.31)

\[ g_1(t) = \frac{1}{12} \left( (2 + 7t^2) \sqrt{1 - t^2} + 7t^4 \Theta_4 \right) \]
(8.32)

\[ g_2(t) = \frac{1}{3} \left( (2 + 7t^2) \sqrt{1 - t^2} + t^4 \Theta_4 \right) \]
(8.33)

\[ g_3(t) = \frac{4}{225} \left( 1 + 19t^2 + 32t^4 \right) \sqrt{1 - t^2} \]
(8.34)

\[ g_4(t) = \frac{1}{225} \left( (27 + 45t^2 + 92t^4) \sqrt{1 - t^2} + 7t^2 \Theta_4 \right) \]
(8.35)

\[ g_5(t) = \frac{4}{75} \left( 11 + 3t^2 + 6t^4 \right) \sqrt{1 - t^2} \]
(8.36)

\[ g_6(t) = \frac{1}{420} \left( (8 + 82t^2 + 147t^4) \sqrt{1 - t^2} + 147t^2 \Theta_4 \right) \]
(8.37)
\[ g_7(t) = \frac{2}{945} \left[ (16 + 206 t^2 + 165 t^4) \frac{1}{1 - t^2} + 105 t^6 \right] \] (8.32)

\[ g_6(t) = \frac{1}{1690} \left[ (144 - 22 t^2 + 287 t^4) \frac{1}{1 - t^2} + 7 (120 + 41 t^2) t^2 \right] \] (8.39)

\[ g_5(t) = \frac{2}{15} \left[ (80 + 22 t^2 + 21 t^4) \frac{1}{1 - t^2} + 21 t^6 \right] \] (8.40)

\[ h_0(t) = \frac{1}{4} \left[ (2 + 2 t^2) \frac{1}{1 - t^2} + 3 t^4 \right] \] (8.41)

\[ h_1(t) = \frac{2}{15} \left[ (1 + 3 t^2 + 6 t^4) \frac{1}{1 - t^2} \right] \] (8.42)

\[ h_2(t) = \frac{4}{15} \left[ (2 + t^2 + 2 t^4) \frac{1}{1 - t^2} \right] \] (8.43)

\[ h_3(t) = \frac{1}{180} \left[ (4 + 46 t^2 + 69 t^4) \frac{1}{1 - t^2} + 69 t^6 \right] \] (8.44)

\[ h_4(t) = \frac{1}{450} \left[ (48 + 86 t^2 + 129 t^4) \frac{1}{1 - t^2} + (40 + 43 t^4) t^2 \right] \] (8.45)

\[ h_5(t) = \frac{1}{180} \left[ (88 + 26 t^2 + 39 t^4) \frac{1}{1 - t^2} + 39 t^6 \right] \] (8.46)

\[ h_6(t) = \frac{4}{3675} \left[ (15 + 144 t^2 + 227 t^4 + 454 t^6) \right] \] (8.47)
\[
\begin{align*}
\frac{h_7(t)}{t} &= \frac{16}{11025} \left( 20 + 241 t^2 + 124 t^4 + 246 t^6 \right) \left( 1 - t^2 \right)^{1/2} \\
\frac{h_6(t)}{t} &= \frac{2}{11025} \left( 405 t^2 + 725 t^4 + 1576 t^6 \right) \left( 1 - t^2 \right)^{1/2} \\
&+ 2205 t^2 \left( 1 - t^2 \right)^{1/4} \\
\frac{h_9(t)}{t} &= \frac{16}{1875} \left( 100 + 29 t^2 + 27 t^4 + 54 t^6 \right) \left( 1 - t^2 \right)^{1/2}
\end{align*}
\]

C. Supersonic Leading Edges, \( m > 1 \)

\[
\begin{align*}
g_0(t) &= \frac{1}{2} \left[ \frac{1}{3m} \left( m^2 - 1 \right) O_1 + \left( m^2 + t^2 \right) O_2 \right] \\
&+ 2m \left( m^2 - 1 \right) t \left( 1 - t^2 \right)^{1/2} \\
g_1(t) &= \frac{1}{2} \left[ \frac{3}{12m} \left( m^2 - 1 \right) \left( 6m^2 + 2 \right) t \right] \left( 1 - t^2 \right)^{3/2} \\
&+ \left( m^2 - 1 \right) k_{18} t^4 O_4 - \left( 3m^4 - 4m^5 t + k_{17} t^4 \right) O_1 \\
&- \left( 3m^4 + 4m^5 t + k_{17} t^4 \right) O_2
\end{align*}
\]
\[ g_2(t) = \frac{1}{12m (m-1)} \left( \left( \frac{2m^2}{m-1} \right)^{2/3} (3m - 2)^{2/3} (1 - t)^2 \right) \]

\[ + (3m^2 a_{29} + 4m^3 t - c_{14} t^4) \Theta_1 + (3m^2 a_{29} - 4m^3 t - c_{14} t^4) \Theta_2 \]

\[ + 4 \left( m - 1 \right)^{3/2} t^4 \Theta_4 \]

\[ g_2(t) = \frac{1}{60m (m-1)} \left( \left( \frac{2m^2}{m-1} \right)^{5/2} (6m a_{15} - 4m^2 t + 10m a_{19} t^2 - k_{21} t^5) \Theta_1 \right) \]

\[ + (6m^2 a_{15} + 15m^4 t + 10m a_{19} t^2 + k_{21} t^5) \Theta_2 \]

\[ - 2m m^{-1} \left( 1 m^4 + m^2 k_{19} t^2 - k_{21} t^5 \right) \Theta_4 \]

\[ g_4(t) = \frac{1}{100m^3 (m-1)} \left( \frac{2m^2}{m-1} \right)^3 (18m a_{35} + 15m a_{36} t + 10m a_{36} t^2) \]

\[ - k_{43} t^5 \Theta_1 + 3 (18m a_{35} - 15m a_{36} t + 10m a_{36} t^2 + k_{43} t^5) \Theta_2 \]

\[ + 2m m^{-1} \left( 18m a_{35} - m^2 k_{44} t^2 - k_{45} t^4 \right) \left( 1 - t^2 \right) \]

\[ + 60m (m-1)^{3/2} t^2 \Theta_4 \]

\[ g_6(t) = \frac{1}{60m (m-1)^{3/2}} \left( \left( \frac{2m^2}{m-1} \right)^{5/2} (2m a_{13} - 5m^4 t + 10m^5 t^2 - c_{16} t^4) \Theta_1 \right) \]
\[ g_6(t) = \frac{1}{360m^2(m-1)} \left( \frac{2}{2^{1/2}} \left( 2m^2 \right)^{1/2} + \frac{3}{2^{1/2}} \right) (1 - t^2) + \frac{6(1 + t)}{2^{1/2}} k_{27} t^6 \] (5.56)

\[ g_6(t) = 3(10 m a_6 t - 20 m a_2 t^2) + 45 m a_6 t^2 - 20 m a_2 t^3 + k_{21} t^6 \] (5.57)

\[ g_7(t) = \frac{1}{360m^3(m-1)} \left( \frac{5}{2^{1/2}} \right) (2m^2 + 10m^2 t + 9m a_7 t^2 \right) \] (5.58)
\[ \begin{align*}
\frac{g_y(t)}{720m^3 / (m-1)^{7/2}} & \quad \frac{1}{2} \cdot \left( m^{-1} t \right)^{7/2} \left( 120m^4 + f_1 t^4 \right) t^2 O_4 \\
+ & \quad m^2 \left( m^{-1} \right)^{5/2} \left( 40m^4 a_{24} + 2m^2 f_1 t^3 + 10m^2 t^2 + f_9 t^6 \right) O_1 \\
- & \quad 6 \left( 10m^2 a_{24} - 12m^4 a_{41} + 15m^4 a_{45} t^2 - 100m^7 t^3 + f_9 t^6 \right) O_1 \\
- & \quad 6 \left( 10m^2 a_{24} - 12m^4 a_{44} t + 15m^4 a_{45} t^2 \\
+ & \quad 100m^7 t^3 + f_9 t^6 \right) O_2 \\
\end{align*} \] (4.59)

\[ \begin{align*}
\frac{g_y(t)}{560m^3 / (m-1)^{7/2}} & \quad \frac{1}{2} \cdot \left( 10m^6 a_{42} + 36m^5 a_{24} t - 45m a_{25} t^2 \\
+ & \quad 20m a_{26} t^3 - k_4 t^6 \right) O_1 + 3 \left( 10m a_{22} - 36m a_{24} t \\
- & \quad 45m a_{25} t^2 - 20m a_{26} t^3 - k_4 t^6 \right) O_2 \\
+ & \quad 48 \left( m^{-1} t \right)^{7/2} 6 O_4 + 2m^2 t^2 \left( 10m^4 a_{26} \right) \\
+ & \quad m^2 k_{12} t^2 - k_4 t^4 \left( 1 - t^2 \right) \\
\end{align*} \] (5.60)

\[ \begin{align*}
\frac{g_y(t)}{4m^3 \left( m^{-1} t \right)^{4/3} + \Theta_2} & \quad \frac{1}{3} \cdot \left( m^{-1} t \right)^{4/3} \left( 1 - t^4 \right) O_4 \\
+ & \quad \frac{m^2 t^2 \left( m^{-1} t \right)^{2/3} \left( 1 - t^2 \right)}{1 - \left( m^{-1} t \right)^{2/3} O_4} \\
\end{align*} \] (9.11)
\begin{align*}
 h_1(t) &= \frac{1}{60m} \left( \frac{2}{m-1} \right)^{1/2} \left[ 2m \left( m^{-1} - 1 \right) \left( 12m^4 + m^2 \alpha_{c_1} - 1 \right) t^2 \\
 &+ k_{\alpha 1} (4m^4 - t^2 - 3m^5 t + k_{\alpha 0} t^3) \Theta_1 \right] \\
 &- 3 (4m^5 + 5m^6 t - k_{\alpha 0} t^5) \Theta_2 \\
 &= \frac{1}{20m} \left( \frac{2}{m-1} \right)^{1/2} \left[ (4m^5 + 5m^6 t - k_{\alpha 0} t^5) \Theta_1 \right] (8.62)
\end{align*}

\begin{align*}
 h_2(t) &= \frac{1}{60m} \left( \frac{2}{m-1} \right)^{1/2} \left[ (4m^5 + 5m^6 t - k_{\alpha 0} t^5) \Theta_1 \right] \\
 &+ (4m^5 + 5m^6 t - k_{\alpha 0} t^5) \Theta_2 \\
 &+ 2m \left( \frac{2}{m-1} \right) \left( 4m^5 + 5m^6 t - k_{\alpha 0} t^5 \right) \Theta_2 \\
 &= \frac{1}{20m} \left( \frac{2}{m-1} \right)^{1/2} \left[ (4m^5 + 5m^6 t - k_{\alpha 0} t^5) \Theta_1 \right] (8.63)
\end{align*}

\begin{align*}
 h_3(t) &= \frac{1}{120m} \left( \frac{2}{m-1} \right)^{1/2} \left[ (10m^5 + 72m^6 t + 15m^6 a_{19} t^2 \right. \\
 &- k_{32} t^6) \Theta_1 \\
 &+ (10m^5 + 72m^6 t + 15m^6 a_{19} t^2 - k_{32} t^6) \Theta_2 \\
 &+ 2m \left( \frac{2}{m-1} \right) \left( 10m^5 + 72m^6 t + 15m^6 a_{19} t^2 - k_{32} t^6 \right) \Theta_2 \\
 &+ 2 \left( \frac{2}{m-1} \right) k_{35} t^6 \Theta_4 \\
 &= \frac{1}{60m} \left( \frac{2}{m-1} \right)^{1/2} \left[ (10m^5 + 72m^6 t + 15m^6 a_{19} t^2 - k_{32} t^6) \Theta_1 \right] (8.64)
\end{align*}
\[
\begin{align*}
\tag{4.65}
h_4(t) &= - \frac{1}{4} \frac{2}{480m^m (m^{-1})^{5/2}} \left[ 12 \left( 10m^m + 8m^m a_{33} + 5m^m a_{38} - 5m^m a_{30} t - 3m^m a_{30} t^2 \right) \right. \\
&\quad \left. \left. - k_{50}^6 t^6 \right) O_1 + 12 \left( 10m^m - 5m^m a_{35} + 5m^m a_{30} t - k_{30}^6 t^2 \right) \theta_2 \right] \\
&\quad + 2 \sqrt{m^{-1}} \left[ (50m^m a_{35} - 2m^m k_{51} t^2 - 3m^m t^4) \right] \sqrt{1 - t^2} \\
&\quad + 3 (m^{-1})^{5/2} (40m^m k_{43} t^4)^2 \theta_4 \\
\tag{4.66}
h_5(t) &= - \frac{1}{5} \frac{2}{126m^m (m^{-1})^{5/2}} \left[ (11m^m a_{30} - 24m^m a_{18} + 45m^m t^2 \right. \\
&\quad \left. - k_{47}^5 t^5 \right) O_1 + (10m^m a_{30} + 24m^m a_{18} + 45m^m t^2 - k_{17}^6 t^6 \right) \theta_2 \right] \\
&\quad + 2 \sqrt{m^{-1}} \left[ (10m^m a_{31} - 2m^m a_{51} t^2 + k_{40}^6 t^4) \right] \sqrt{1 - t^2} \\
&\quad + 2 (m^{-1})^{5/2} k_{49}^6 t \theta_4 \\
\tag{4.67}
h_6(t) &= - \frac{1}{3} \frac{2}{840m^m (m^{-1})^{7/2}} \left[ 2m \sqrt{m^{-1}} \left( 20m^m a_{22} + 2m^m k_{37} t^2 \right) \\
&\quad + m^m k_{38}^4 + k_{39}^6 t^4 \right] \sqrt{1 - t^2 - 3 \left( 20m^m a_{24} - 70m^m a_{23} t \right. \\
&\quad \left. + 84m^m a_{26} t^2 - 35m^m a_{27} t^2 + k_{36}^6 \right) O_1 \right] \\
&\quad - 3 (20m^m a_{24} + 70m^m a_{23} t + 84m^m a_{26} t^2 + 70m^m a_{27} t^2 \right) \\
\end{align*}
\]
\[ h_7(t) = \frac{1}{840m} \left( \frac{1}{2} - \frac{7}{2} \right) \left( 20m a_{37}^7 + 1050m^8 t + 84m a_{38}^7 t^2 \right) + 35m a_{39}^4 t^3 - 3k_{58}^7 t_0 + (20m a_{37}^7 - 1050m^8 t \right) \\
+ 84m a_{25}^7 t^2 - 35m a_{25}^7 t^3 + 3k_{58}^7 t_0 + 2m \left( \frac{2}{m - 1} \right) (20m a_{40}^6 + 2m k_{59}^2 t^2) \\
+ m^2 k_{60}^4 t^4 + k_{61}^6 \left( \sqrt{1 - t^2} \right) \]  

\[ h_8(t) = \frac{1}{3780m} \left( \frac{1}{5} - \frac{7}{2} \right) \left( - 43 (60m a_{25}^7 - 70m a_{44}^7 t \right) \\
+ 84m a_{45}^7 t^2 - 323m^8 t^3 + k_{54}^7 t_0 + 43 (60m a_{25}^7 + 70m a_{44}^7 t + 84m a_{45}^7 t^2 \right) \\
+ 525m^8 t^3 - k_{54}^7 t_0 + 2m \left( \frac{2}{m - 1} \right) (900m a_{42}^6 - 18m^4 k_{55}^2 t^2) \\
+ m^2 k_{56}^4 t^4 + k_{57}^6 \left( \sqrt{1 - t^2} + 15120m^5 \left( \frac{2}{m - 1} \right) t^2 \right) \]
\[ h(t) = \frac{1}{840m^{2}(m^{-1})^{7/2}} \left[ \left(20m^{7}a_{22} + 70m^{6}a_{24}t\right) + 3(20m^{7}a_{32} - 70m^{6}a_{24}t - 84m^{7}a_{25}t^{2} - 35m^{6}a_{26}t^{3}) \right. \]
\[ \left. \left.f_{5}t^{7}O_{2} + 2m^{4}m^{-1}(20m^{6}a_{33} + 2m^{4}a_{6}t^{2}) + m^{2}f_{7}t^{4} + \left[f_{8}t^{6}\right]^{2} \right] \right] \]

Tabulated Abbreviations

See Table I, Reference 3, for \( A_{1}, b_{1}, c_{1}, e_{1}, f_{1} \) not listed here.

\[ A_{49} = \left[ 2m^{2}K(3 - 4m^{2}) - E(12 - 19m^{2} + 5m) \right] \]

\[ A_{37} = \left[ m^{2}K - E(4 - 3m^{2}) \right] \]

\[ A_{36} = \left[ 9m^{4}K^{2} - 2m^{2}KE(13 - 7m^{2}) + E^{2}(32 - 7m^{2} + 12m^{4}) \right] \]

\[ A_{56} = \left[ m^{2}K(7 - 5m^{2}) + 2E(2 - 8m^{2} + 3m^{4}) \right] \]

\[ A_{55} = \left[ 2m^{2}K + E(1 - 4m^{2}) \right] \]
\begin{align*}
A_{40} &= \left[ m^4 K \left( 16 - 57m^2 + 87m^4 \right) + 2m^2 KE \left( 8 - 25m^2 - 13m^4 + 32m^6 \right) \\
&\quad - E^2 \left( 32 - 140m^2 + 215m^4 - 173m^6 + 20m^8 \right) \right] \\
A_{41} &= \left[ m^4 K \left( 236 - 1663m^2 + 2041m^4 - 653m^6 \right) + 2m^2 KE \left( 168 - 145m^2 \\
&\quad + 650m^4 - 1417m^6 + 630m^8 \right) - E^2 \left( 2016 - 7652m^2 + 12345m^4 - 9742m^6 \\
&\quad + 2531m^8 + 100m^{10} \right) \right] \\
A_{42} &= \left[ m^2 K \left( 6 - 9m^2 - 5m^4 \right) - E \left( 12 - 27m^2 + 17m^4 - 10m^6 \right) \right] \\
A_{43} &= \left[ m^2 K - E \left( 5 - 4m^2 \right) \right] \\
A_{44} &= \left[ 2m^2 K \left( 1 - 2m^2 \right) + E \left( 5 - 11m^2 + 8m^4 \right) \right] \\
A_{45} &= \left[ m^4 K \left( 7 - 24m^2 + 8m^4 \right) + 2m^2 KE \left( 7 - 8m^2 + 6m^4 \right) - E^2 \left( 140 - 245m^2 \\
&\quad + 128m^4 \right) \right] \\
A_{46} &= \left[ m^2 K \left( 7 - 9m^2 \right) - 2E \left( 7 - 11m^2 + 3m^4 \right) \right] \\
A_{47} &= \left[ m^2 K \left( 15 - 59m^2 + 16m^4 \right) + E \left( 15 - 57m^2 + 82m^4 - 32m^6 \right) \right] \\
A_{48} &= \left[ m^4 K \left( 128 - 1637m^2 + 1073m^4 - 600m^6 \right) + 2m^4 KE \left( 68 + 55m^2 \\
&\quad - 1315m^4 + 624m^6 \right) - E^2 \left( 168 - 6776m^2 + 11245m^4 - 5809m^6 + 2525m^8 \right) \\
&\quad + 96m^{10} \right] \\
&
\end{align*}
\[
A_{49} = \left[ m^2 K (1 - 6^2 - 6m^4) - E (14 - 51m^2 + 21m^4 - 12m^6) \right]
\]

\[
A_{50} = \left[ m^4 K^2 - 2m^4 KE - E^2 (4 - 5m^2) \right]
\]

\[
A_{51} = \left[ m^4 K^2 (21 - 19m^2) - 8m^2 KE (3 + m^2 + 5m^4) - E^2 (36 - 107m^2 + 77m^4 + 4m^6) \right]
\]

\[
k_1 = 4m^2 - 5
\]

\[
k_2 = 12m^4 - 7m^2 - 2
\]

\[
k_3 = 20m^4 - 47m^2 + 30
\]

\[
k_4 = 6m^6 - 161m^4 + 28m^2 + 4
\]

\[
k_5 = 5m^2 - 6
\]

\[
k_6 = 15m^4 - 8m^2 + 4
\]

\[
k_7 = 10m^4 - 23m^2 + 14
\]

\[
k_8 = 30m^6 - 49m^4 + 12m^2 + 4
\]

\[
k_9 = 70m^6 - 23m^4 + 273m^2 - 112
\]

\[
k_{10} = 21m^8 - 36m^6 + 7m^4 + 72m^2 - 16
\]
\[
\begin{align*}
\kappa_{11} &= 24m^4 - 36m^2 + 35 \\
\kappa_{12} &= 24m^6 - 40m^4 + 11m^2 - 2 \\
\kappa_{13} &= 28m^6 - 95m^4 + 112m^2 - 48 \\
\kappa_{14} &= 420m^6 - 114m^4 + 95m^4 - 160m^2 + 16 \\
\kappa_{15} &= 3m^2 - 4 \\
\kappa_{16} &= 2m - 3 \\
\kappa_{17} &= 4m^2 - 3 \\
\kappa_{18} &= 2m + 6 \\
\kappa_{19} &= 7m^2 - 4 \\
\kappa_{20} &= 6m^4 - 21m^2 + 12 \\
\kappa_{21} &= 26m^4 - 29m^2 + 12 \\
\kappa_{22} &= 12m^4 + 11m^2 - 15 \\
\kappa_{23} &= m^6 + 23m^4 + 27m^2 + 10 \\
\kappa_{24} &= 40m^6 - 84m^4 + 69m^2 - 20 \\
\kappa_{25} &= m^2 + 20
\end{align*}
\]
\[ k_{26} = 0 \]

\[ k_{27} = 10m^6 - 55m^4 + 24m^2 - \ldots \]

\[ k_{28} = 10m^4 + 9m^2 - 10 \]

\[ k_{29} = 10m^4 - 11m^2 + 4 \]

\[ k_{30} = 3m^2 - 4 \]

\[ k_{31} = 2m^4 + 7m^2 - 12 \]

\[ k_{32} = 30m^4 - 47m^2 + 20 \]

\[ k_{33} = 8m^2 - 5 \]

\[ k_{34} = 3m^4 - 16m^2 + 10 \]

\[ k_{35} = 3m^2 + 20 \]

\[ k_{36} = 70m^6 - 161m^4 + 136m^2 - 40 \]

\[ k_{37} = 107m^4 + 100m^2 - 12 \]

\[ k_{38} = 2m^6 - 31m^4 + 104m^2 - 4 \]

\[ k_{39} = 4m^8 + 4m^6 - 27m^4 + 52m^2 - 120 \]

\[ k_{40} = 2m^6 - 35m^4 + 2m^2 - \ldots \]
\[
k_{41} = 20m^4 + 271m^2 - 6
\]
\[
k_{42} = 12m^4 - 11m^2 + 4
\]
\[
k_{43} = 13m^4 - 20m^2 + 9
\]
\[
k_{44} = m^4 + 16m^2 - 8
\]
\[
k_{45} = (2m^6 - 13m^4 + 44m^2 - 24)
\]
\[
k_{46} = 4m^2 - 3
\]
\[
k_{47} = 20m^4 - 29m^2 + 12
\]
\[
k_{48} = m^4 - 16m^2 + 6
\]
\[
k_{49} = 2 + 12
\]
\[
k_{50} = 5m^4 - 12m^2 + 2
\]
\[
k_{51} = m^4 + 26m^2 - 15
\]
\[
k_{52} = m^6 - 6m^4 + 35m^2 - 20
\]
\[
k_{53} = m^4 - 4m^2 + 40
\]
\[
k_{54} = 14m^6 - 39m^4 + 24m^2 - 72
\]
\[
\begin{align*}
  k_{55} &= 174m^6 - 697m^4 - 756m^2 + 36 \\
  k_{56} &= 9m^3 - 24m^6 - 301m^4 + 952m^2 - 360 \\
  k_{57} &= 48m^{10} - 144m^8 + 1494m^6 - 7833m^4 + 9000m^2 - 3240 \\
  k_{58} &= 56m^6 - 124m^4 + 103m^2 - 30 \\
  k_{59} &= 84m^4 + 215m^2 - 44 \\
  k_{60} &= 64m^4 - 79m^2 + 30 \\
  k_{61} &= 40m^6 - 214m^4 + 249m^2 - 96 \\
  k_{62} &= 163m^6 - 563m^4 + 665m^2 - 286 \\
  k_{63} &= 168m^8 - 456m^6 + 377m^4 - 66m^2 - 8
\end{align*}
\]
### 8.3 Tabulated $C_{D_{i,j}}$ Functions

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Report No. ZA-259
Date 30 October 1957
### 8.3 Tabulated Data Functions

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### 8.3 Tabulated $C_D^*$ Functions

#### C. Supersonic Leading Edges

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8.3 Tabulated $\frac{C_D}{A}$ Functions

D. Limiting Case, $m = 0$ ($M = 1.0$)

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\hline
000000  & .20000000 & .25464731 & .40000000 & .14147073 & .16666667 & .21220609 & .33333333 \\
\hline
810000  & .39270000 & .53333333 & .94248000 & .26666667 & .32725000 & .44444444 & .78540000 \\
\hline
416000  & .13090000 & .16666667 & .26180000 & .09523810 & .11220000 & .14285714 & .22440000 \\
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333333  & .20000000 & .26525762 & .44444444 & .14147073 & .17142857 & .22736877 & .38096238 \\
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664000  & .39270000 & .55555556 & 1.04720000 & .26666667 & .33660000 & .47619048 & .39760000 \\
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222222  & .69523810 & .12128063 & .19047619 & .22222222 & .08333333 & .10610305 & .16666667 \\
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555556  & .20000000 & .27283641 & .47619048 & .14147073 & .17500000 & .23873186 & .41666667 \\
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00000   & .39270000 & .57142857 & 1.12200000 & .26666667 & .34361250 & .50000000 & .98175000 \\
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\caption{Table of values}
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## 8.4 Tabulated $T^*_1, \ell$ Functions

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8.4 Tabulated $\frac{T^*}{A}$ Functions

B. Limiting Case, $m = 0$ ($M = 1.0$)

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8.5 Tabulated Geometric Boundary Condition Functions

Note: These functions are not related to any aerodynamic quantities.

\[ A_0 = 12 \]
\[ A_1 = 12 \]
\[ A_2 = 12 + 6 m_1 X_{22} \]
\[ A_3 = 12 + 6 m_1 X_{23} \]
\[ A_4 = 12 + 6 m_1 X_{24} \]
\[ A_5 = 12 + 6 m_1 X_{25} + 4 m_1^2 X_{55} \]
\[ A_6 = 12 + 6 m_1 X_{26} + 4 m_1^2 X_{56} \]
\[ A_7 = 12 + 6 m_1 X_{27} + 4 m_1^2 X_{57} \]
\[ A_8 = 12 + 6 m_1 X_{28} + 4 m_1^2 X_{58} \]
\[ A_9 = 12 + 6 m_1 X_{29} + 4 m_1^2 X_{59} + 3 m_1^3 X_{99} \]
\[
B_0 = -6 n_1
\]
\[
B_1 = -6 n_1 + 6 m_1 X_{11}
\]
\[
B_2 = -6 n_1 + 6 m_1 X_{12}
\]
\[
B_3 = -6 n_1 + 6 m_1 X_{13}
\]
\[
B_4 = -6 n_1 + 6 m_1 X_{14} + 3 m_1^2 X_{44}
\]
\[
B_5 = -6 n_1 + 6 m_1 X_{15} + 3 m_1^2 X_{45}
\]
\[
B_6 = -6 n_1 + 6 m_1 X_{16} + 3 m_1^2 X_{46}
\]
\[
B_7 = -6 n_1 + 6 m_1 X_{17} + 3 m_1^2 X_{47}
\]
\[
B_8 = -6 n_1 + 6 m_1 X_{18} + 3 m_1^2 X_{48} + 2 m_1^3 X_{88}
\]
\[
B_9 = -6 n_1 + 6 m_1 X_{19} + 3 m_1^2 X_{49} + 2 m_1^3 X_{99}
\]
\[
C_0 = 0
\]
\[
C_1 = -2 n_1 X_{11}
\]
\[
C_2 = -2 n_1 X_{12} - n_1^2 X_{22}
\]
\[
C_3 = -2 n_1 X_{13} - n_1^2 X_{23} + 2 m_1 X_{33}
\]
\[
C_4 = -2 n_1 X_{14} - n_1^2 X_{24} + 2 m_1 X_{34}
\]
\[
C_5 = -2 n_1 X_{15} - n_1^2 X_{25} + 2 m_1 X_{35}
\]
\[ C_6 = -2n_1X_{16} - n_1^2X_{26} + 2m_1X_{36} \]
\[ C_7 = -2n_1X_{17} - n_1^2X_{27} + 2m_1X_{37} + m_1^2X_{77} \]
\[ C_8 = -2n_1X_{18} - n_1^2X_{28} + 2m_1X_{38} + m_1^2X_{78} \]
\[ C_9 = -2n_1X_{19} - n_1^2X_{29} + 2m_1X_{39} + m_1^2X_{79} \]
\[ D_0 = 0 \]
\[ D_1 = 0 \]
\[ D_2 = 0 \]
\[ D_3 = -6n_1X_{33} \]
\[ D_4 = -6n_1X_{34} - 3n_1^2X_{44} \]
\[ D_5 = -6n_1X_{35} - 3n_1^2X_{45} - 2n_1^3X_{55} \]
\[ D_6 = -6n_1X_{36} - 3n_1^2X_{46} - 2n_1^3X_{56} + 6m_1X_{66} \]
\[ D_7 = -6n_1X_{37} - 3n_1^2X_{47} - 2n_1^3X_{57} + 6m_1X_{67} \]
\[ D_8 = -6n_1X_{38} - 3n_1^2X_{48} - 2n_1^3X_{58} + 6m_1X_{68} \]
\[ D_9 = -6n_1X_{39} - 3n_1^2X_{49} - 2n_1^3X_{59} + 5m_1X_{69} \]
\begin{align*}
E_6 &= 0 \\
E_1 &= 0 \\
E_2 &= 0 \\
E_3 &= 0 \\
E_4 &= 0 \\
E_5 &= 0 \\
E_6 &= 12 \times X_{66} \\
E_7 &= 12 \times X_{67} + 6 n_1 X_{77} \\
E_8 &= 12 \times X_{68} + 6 n_1 X_{78} + 4 n_1^2 X_{88} \\
E_9 &= 12 \times X_{69} + 6 n_1 X_{79} + 4 n_1^2 X_{89} + 3 n_1^2 X_{99}
\end{align*}
9.0 ILLUSTRATIVE EXAMPLES

9.1 Design Procedure

Inputs: sonic leading edge: "i": m_d = 1, C_{Ld} = .136

"j": m_d = 1, C_{Ld} = .136, m = .92

"a": m_d = 1, C_{Ld} = .136, m = .92

supersonic leading edge: "b": m_d = 1.323, C_{Ld} = .087

"c": m_d = 1.323, C_{Ld} = .087

m = .92, n = 1/1.36

1. At m_d, compute C_{\ast}^{*} from section 4.3. Results presented in section 4.3 B.

2. At m_d, obtain C_{L_{i}}^{*} from tabulated C_{D_{i,j}}^{*}. C_{L_{i}}^{*} = C_{D_{i,j}}^{*}.

3. At m_d, compute C_{M_{i}}^{*} = \frac{2}{3+2n} C_{L_{i}}^{*}.

4. Compute \lambda_{i,j}, equation (4.52). Results presented in Tables IV A and B.

5. At m_d, compute X_{i}, equation (4.51). Results presented in Tables V A and B.
6. At m, compute \((C_L^*)^{(k)}\), \((C_M^*)^{(k)}\), \((C_D^*)^{(k)}\) from equations (4.39), (4.40), and (4.56). Results presented in Table VI.

7. Solve following matrix, the size of which depends upon the number of terms, \(n\), of the series, for "3" class of wings:

\[
\begin{bmatrix}
2(C_D^*)^{(k)} & (C_L^*)^{(k)} & \cdots & (C_M^*)^{(k)} & \cdots & 0 \\
(C_L^*)^{(k)} & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
(C_M^*)^{(k)} & \cdots & \cdots & 0 & \cdots & \cdots \\
\end{bmatrix}
\begin{bmatrix}
\bar{a}_{kn} \\
\alpha \\
\cdots \\
\cdots \\
\cdots \\
\end{bmatrix}
= \begin{bmatrix}
\{0\} \\
\Omega_1 \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
\]

Results for \(\bar{a}_{kn}\) and \(\Omega_1\) presented in Tables VII A) and B) for \(n = 0, 1, \ldots, 2\).

8. Solve following 12 x 12 matrix for "4" class of wings:

\[
\begin{bmatrix}
2(C_D^*)^{(k)} & (C_L^*)^{(k)} & \cdots & (C_M^*)^{(k)} & \cdots & 0 \\
(C_L^*)^{(k)} & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
(C_M^*)^{(k)} & \cdots & \cdots & 0 & \cdots & \cdots \\
\end{bmatrix}
\begin{bmatrix}
\bar{a}_{kn} \\
\alpha \\
\cdots \\
\cdots \\
\cdots \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
\bar{a}_{k1} \\
\bar{a}_{k2} \\
\bar{a}_{k3} \\
\bar{a}_{k4} \\
\end{bmatrix}
\]

Results for \(\bar{a}_{k1}, \bar{a}_{k2}, \bar{a}_{k3}, \bar{a}_{k4}\), \(\Omega_1, \Omega_2, \Omega_3\) for \(k = 0, 1, \ldots, 2\) presented in Table VIII.
9. For wing with two straight hinge-lines only compute \( A_k, B_k, C_k, D_k, E_k, F_k \) from section 3.3. Results tabulated in following section 10.

10. Solve following 16 x 16 matrix for "\( \alpha \)" class of wings:

\[
\begin{bmatrix}
2(C_D^*)^{(k)} & (C_L^*)^{(k)} & (C_M^*)^{(k)} & C_k & D_k & F_k & \lambda_{kn}^i & 0 \\
(C_L^*)^{(k)} & 2(C_L^*)^{(k)} & (C_D^*)^{(k)} & (C_M^*)^{(k)} & D_k & F_k & \lambda_{kn}^i & 0 \\
(C_M^*)^{(k)} & (C_D^*)^{(k)} & 2(C_L^*)^{(k)} & (C_M^*)^{(k)} & C_k & D_k & \lambda_{kn}^i & 0 \\
C_k & D_k & C_k & 1.0 & 0 & 0 & 0 & 0 \\
D_k & F_k & D_k & 1.0 & 0 & 0 & 0 & 0 \\
F_k & E_k & F_k & 1.0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

This matrix is presented as Table VIII. The inputs for steps 7 and 8 may be obtained directly therefrom. Results for \( \alpha_{kn}^i, \Omega_{ij} \) \( (k = 0, 1 \ldots 9, i = 1, 2 \ldots 6) \) presented in Table IX.

11. For subsonic \( m \) (\( \neq m_d \)) only compute \( C_{L, M}^* \) from section 3, 3 and \( \lambda_{ij} \) and \( T_{ij} \) from section 4.4. Results presented in Tables X and XII. These results will be used later to obtain drag polar for subsonic leading edge speeds.

12. For subsonic \( m \) with suction only: compute for each \( k = 0, 1 \ldots 9 \) and \( n = 3 \).
\[ q_{T_{kk}} X_{p0} T_{p,k}^* \quad \tilde{a}_{i0} \quad X_{p0} T_{p,k}^* \]

\[ + \quad X_{p, k+1} T_{k+1, p}^* \quad \tilde{a}_{i, k+1} \]

where \( q_{T_{k}} \) is assumed to be unity. \( T_{p,k}^* \) and \( T_{k,p}^* \) are obtained from Table XII and \( X_{k} \) is obtained from Table V. This computation will result in a contribution to each term of the \( 10 \times 10 \) diagonal matrix of steps 7, 8, and 10. The procedures of steps 7, 8, and 10 should be solved with this change for the three wing types. No subsonic design was carried out.

13. At \( m_d \), \( \nu \), are obtained from equation (4.42). Results are presented in Table XIII.

14. For wings with two straight hinge-lines only, compute

\[ \dot{z}_3 = - \sum_{k=0}^{\infty} A_k \tilde{a}_{k, kn} \]

\[ \dot{z}_1 = 0 (Z_1 = 0 \text{ at } x' = x_1' = 0) \]

\[ \dot{z}_2 = - \dot{z}_0 = \frac{1}{2} \sum_{k=0}^{\infty} B_k \tilde{a}_{k, kn} \]
15. At $m_d$, compute $Z'/\beta C_l L_d$ from equation (4.44). Results are plotted in figures 5.13 (a) and (b).

This completes the design phase. The following procedure is used to obtain drag polars, pitching-moment versus lift, span and chord loadings.

16. Repeat steps (1) - (3) for each value of $m$ for which the drag polars required. Repeat step (11) for $m < 1$.

17. For each wing defined by a set of $T_k$, retain $T_k$ for $k \geq 1$. Compute $y_9$ versus $C_L/C_{L_d}$:

$$\frac{C_L}{C_{L_d}} = \sum_{i=1}^{n=9} y_9 C_l^*$$

where $C_l^*$ were computed in step 16.

18. Compute $C_D/\beta C^2_{L_d}$

$$C_B = \frac{y_9}{C_{D_{0,0}}} C^2_{D_{0,0}} + \sum_{i=1}^{j=9} y_{i,0,0}$$

$$+ \sum_{i=1}^{j=9} \sum_{j=1}^{j=9} y_{i,j}$$
where the bracketed quantity is used only at \( m < 1 \) if suction is considered. The results are shown in figures 5.1 to 5.3.

13. Compute \( \frac{C_{M,365}}{C_{L_d}} \)

\[
\frac{C_{M,365}}{C_{L_d}} = \bar{C}_{M} + 2 \sum_{k=1}^{2} \bar{C}_{M_k} \\
+ \left( \frac{1 + \frac{2x}{m}}{2} \right) \frac{C_L}{C_{L_d}}
\]

Results are shown in figures 5.4 to 5.6.
TABLE III
Tabulated $C_{D_{1}}$ for Supersonic Leading Edge Delta Wing ($m = 1.3229$)

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TABLE VI

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for Sonic and Supersonic Leading Edge Designs

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**TABLE VIII**

Tabulated Input Matrix Suitable for Steps 7, 8 and 10

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\]

\[
\begin{bmatrix}
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\bar{\alpha}_{99}
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### TABLE IX

Tabulated Minimizing Numbers, $\tilde{\alpha}^{\epsilon}_{7,9}$, and Lagrangian Numbers, $\Omega_i$

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