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FREQUENCY-WAVENUMBER ANALYSIS OF SIGNALS AND NOISE
RECORDED AT THE VERTICAL ARRAY AT APACHE, OKLAHOMA

8 September 1967

Prepared For

AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D. C.

By

R. L. Sax
TELEDYNE, INC.

Under

Project VELA UNIFORM

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Nuclear Test Detection Office
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FREQUENCY-WAVENUMBER ANALYSIS OF SIGNALS AND NOISE RECORDED AT THE VERTICAL ARRAY AT APACHE, OKLAHOMA

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AVAILABILITY

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The seismic data analyzed include an Aleutian earthquake, several samples of ambient noise along with the noise occurring before the event, and a 25 second sample of coda following the signal. The signal coda sampled had nearly uniform power. A preliminary interpretation of the noise and signals strongly suggests the importance of energy conversions possibly due to the complex geology at APOK. The geology is known to be very complex and is characterized to first order a thick wedge of low velocity strata imbedded in material of higher propagation velocity. The evidence of conversion is based on the asymmetry of the F-K spectrum of ambient noise with respect to positive and negative wave number. The results indicate a predominance of obliquely incident up-going waves at all frequencies. Further, the signals and coda similarly displayed anomalous down-going pulses attenuated by approximately 6 db with respect to the up-going pulse. Further, the down-going pulse contains lower frequencies than the up-going pulse and has a much lower apparent vertical velocity. The results obtained by mirror imaging the vertical array indicated that a loss of approximately 3 db would be encountered in using a conventional signal model at this site.
Due to the finite dimension of the array and the uneven spacing of sensors, and finite duration of the samples, the response to a sinusoidal wave input can be very complicated and can possibly lead to misinterpretations. For an analysis of the response of a continuous finite array consider the response to sine wave inputs.

\[
p(t,x) = \begin{cases} 
\exp[i\omega (t-x/c)] + \exp[i\omega (t+x/c)] & \text{when } T_1 < t < T_2 \quad X_1 < x < X_2 \\
0 & \text{elsewhere}
\end{cases}
\]

\[
F(\omega, k) = \int_{X_1}^{X_2} \int_{T_1}^{T_2} p(t,x) e^{-i\omega t} e^{-ikx} \, dt \, dx = S(\omega) Y(k)
\]

\[
S(\omega) = \frac{1}{i(\omega_0 - \omega)} \left[ e^{i(\omega_0 - \omega)T_2} - e^{i(\omega_0 - \omega)T_1} \right]
\]

\[
P_T(\omega) = S(\omega) \overline{S(\omega)} = \frac{2 \{1 - \cos [(\omega_0 - \omega) (T_2 - T_1)]\}}{(\omega_0 - \omega)^2}
\]

Let \( Y(k) = Y_1 (\omega_0/c - k) + Y_2 (\omega_0 + k) \)

\[
Y_1(k) = \frac{1}{i(\omega_0/c - k)} \left[ e^{i(\omega_0/c - k) X_2} - e^{i(\omega_0/c + k) X_2} \right]
\]
\[ Y_2(k) = \frac{1}{i(\omega_0/c + k)} \left[ e^{i(\omega_0/c + k) X_2} - e^{i(\omega_0/c + k) X_1} \right] \]

\[ P_X(k) = (Y_1 + Y_2) (\bar{Y}_1 + \bar{Y}_2) = Y_1 \bar{Y}_1 + Y_2 \bar{Y}_2 + (Y_1 \bar{Y}_2 + Y_2 \bar{Y}_1) \]

\[ Y_1 \bar{Y}_1 = \frac{2 \left\{ 1 - \cos \left[ (\omega_0/c - k) (X_2 - X_1) \right] \right\}}{(\omega_0/c - k)^2} \]

\[ Y_2 \bar{Y}_2 = \frac{2 \left\{ 1 - \cos \left[ (\omega_0/c + k) (X_2 - X_1) \right] \right\}}{(\omega_0/c + k)^2} \]

\[ Y_1 \bar{Y}_2 + Y_2 \bar{Y}_1 = \frac{2}{(\omega_0/c - k) (\omega_0/c + k)} \left\{ \cos 2kX_2 + \cos 2kX_1 \right. \]

\[ - \cos \left[ (\omega_0/c - k) X_1 - (\omega_0/c + k) X_2 \right] \]

\[ - \cos \left[ (\omega_0/c - k) X_2 - (\omega_0/c + k) X_1 \right] \}

The total power response of the array is

\[ R(\omega) = P_T(\omega) P_X(k) \]

From this it is clear that the response of the F-K spectrum to a sinusoidal wave of given frequency and wave number is a complicated function of both frequency and wave number and in practice is even much more so due to finite and uneven sampling in space. Thus to make rigorous use of the F-K analysis
as a tool one should compare a model with the observed data, and conclude that the model is adequate only if the F-K spectra qualitatively match the model. In this preliminary report, this is not stressed enough, but as a cautionary aid the response of an impulse for the case of infinite apparent vertical phase velocity is shown with each F-K spectrum, and several synthetic cases are run for simple models of a signal and a model of the ambient noise.

The signal model is generated with 1.25 cps pulse with a .8 second echo at the source and with a receiver echo delayed by using appropriate uphole times obtained from propagation velocities observed at APOK and with a surface reflection coefficient of 0.9. The signal model F-K spectrum is shown on Figure 1. The split peak in the spectrum is due to the source echo which nulls at 1.25 cps. The strip at the bottom shows the array response.

The noise is simulated by taking random numbers from a Gaussian population and passing them through a tuned filter at .25 cps and 2.0 cps to obtain a model of the noise at the surface. The model of the noise at underlying depth is obtained from a stationary Markov chain; for example, the noise at the $i^{th}$ level is taken as a fraction of the noise of the $(i - 1)^{th}$ channel added to a new random realization passed through the tuned filters. An example of this noise model is shown on Figure 2. It is similar to ambient noise observed at APOK (see Figure 3), except that the highly correlated noise peaks observed in the signal band (.7$\leq f \leq 2$.) were not put into the model. In the model the sharp spectral peak at .25 cps (Figure 2) is for highly correlated noise between channels contrasted with that at 2.0 cps where the noise which is uncorrelated between channels, with the results that the peak is spread broadly over all wave numbers.

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AMBIENT NOISE

The ambient noise spectrum derived from a four minute sample is shown on Figure 3. Comparing this with noise generated from a Markovian process, the .25 and 2.0 cps peaks are of similar character suggesting very high correlation between channels for the .25 cps peak and very low correlation for the 2.0 cps noise peak. The principal difference between the observed noise at APOK on Figure 3 and the synthetic noise generated using the extremely simple linear state model is a rotation of the whole pattern toward negative wave numbers. This same effect can be produced by inputing the random function to a process which produces negative delays or lead time equal to X/C representing conversion to up-going waves where the apparent vertical phase velocity C is obtained from the slope of the line shown on Figure 3. The value obtained for C is approximately 12 km/sec corresponding to an incidence angle of about 75°. This suggests the possibility of Stonely waves guided upwards along the thick low-velocity layer, dipping 15°. This possibility is qualitatively consistent with the anomalous signal shown on Figure 5.

Other differences between observed noise on Figure 3 and the model on Figure 2 are the three noise peaks at 1.0 cps, 1.4 cps, and 1.6 cps. The 1.0 and 1.4 peaks appear to be highly correlated between channels; the 1.6 shows low correlation in the noise between adjacent channels. These peaks in the signal band appear to have nearly infinite vertical phase velocity and are probably due to Rayleigh waves, i.e., vertical and possibly also horizontal standing waves trapped in the basin bounded by higher velocity basement complex rocks.
SIGNAL, NOISE PRECEDING SIGNAL, AND CODA

A 30-second noise sample before the arrival of the Aleutian event is shown on Figure 4. Comparing this with the ambient noise samples on Figure 3 we note the same asymmetry in the F-K spectrum. In addition, up-going body waves are indicated by peaks at .85 cps, 1.25 cps, 1.5 cps and 2.0 cps. There is much weaker indication of down-going waves at .85 cps, down by 3 db from the up-going waves. Waves of infinite vertical phase velocity (possibly Rayleigh waves,) are indicated by the series of peaks occurring along the frequency axis at \( k = 0 \). For this sample, these peaks are indicated with approximately the same power as the up-going body waves.

A 6-second sample of the earthquake pulse is shown on Figure 5. The up-going pulse gives spectral peaks at .85 cps, 1.20 cps, and 1.9 cps. The apparent vertical phase velocity is approximately the same as that shown by Figure 4 for the noise preceding the signal. Lower than expected vertical phase velocities suggest departure from the simple model of a pulse and echo based on acoustic log velocities (Figure 1). The apparent velocities are lower by at least fifteen to twenty percent. Also, the down-going earthquake pulse is even more anomalous. The amplitude is down 6 db from that of the up-going pulse; the apparent vertical velocity is very low; and the .85 cps peak down-going phase appears to contain lower frequency. A possible explanation of the anomalous signal can be based on dipping beds.

This may help to explain the anomalous low amplitude down-going reflection. The anomalous apparent vertical velocities may result from forward scattered P-S conversions, especially at the surface, due to anomalously high angle of emergence. Looking again at Figure 5, there appears to be signal peaks at nearly infinite vertical phase velocity.
Although possibly due to the array response, no such effect is observed on the simulated signal on Figure 1 which, of course, uses the same array geometry.

A 30-second sample of the coda following the signal is shown on Figure 6. These show apparent spectral peaks in the signal at .8 cps, 1.1 cps, and 1.35 cps. The overall character of the coda F-K spectrum is more similar to the signal than the noise preceding the signal, but is yet considerably different in detail from that of the signal.

Figures 7, 8, 9, and 10 are processed by taking the mirror image of the vertical array, which aligns the up-going and down-going pulses into a single step-out pattern. This will attenuate conversions to up or down-going P or S waves and amplify normally reflected P pulses and Rayleigh waves. It will also effectively double the aperture of the array for these kinds of waves. Figure 7 for the ambient noise shows body waves down 3 to 6 db from those of apparently infinite vertical phase velocity. Similar results are shown for the noise before the Aleutian event on Figure 8, also showing the noise field to be down considerably lower in the signal event. However, due to the grossly anomalous nature of the signal at APOK, the signal and coda are also down by 6 db compared to 3 db for the noise preceding the signal. Thus a loss of at least 3 db is expected if the signal model at APOK is taken as a normal up-going pulse and echo. For the imaged array, the frequency of F-K spectral peaks appear to be more consistent between the signal and its coda. The imaging techniques appears to eliminate conversions other than a simple echo at the surface.
Figure 1. Simulated signal using acoustic log
propagation velocities measured at APO.
Figure 2. Simulated noise
Figure 3. Ambient noise
Figure 4. Noise sample before the Aleutian Earthquake
Figure 5. Main pulse of the Aleutian Earthquake
Figure 6. Coda following the main pulse of the Aleutian Earthquake
Figure 7. Mirror imaged ambient noise sample
Figure 8. Mirror imaged noise before Aleutian Earthquake
**Figure 9. Mirror imaged main pulse of Aleutian Earthquake**
### Table 1: Scale Factor, Depth, Velocity, and Symbol

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**Figure 10.** Mirror imaged coda of Aleutian Earthquake
The seismic data analyzed include an Aleutian earthquake, several samples of ambient noise along with the noise occurring before the event, and a 25 second sample of coda following the signal. The signal coda sampled had nearly uniform power. A preliminary interpretation of the noise and signals strongly suggests the importance of energy conversions possibly due to the complex geology at APOK. The geology is known to be very complex and is characterized to first order as a thick wedge of low velocity strata embedded in material of higher propagation velocity. The evidence of conversion is based on the asymmetry of the F-K spectrum of ambient noise with respect to positive and negative wave number. The results indicate a predominance of obliquely incident up-going waves at all frequencies. Further, the signals and coda similarly displayed anomalous down-going pulses attenuated by approximately 6 db with respect to the up-going pulse. Further, the down-going pulse contains lower frequencies than the up-going pulse and has a much lower apparent vertical velocity. The results obtained by mirror imaging the vertical array indicated that a loss of approximately 3 db would be encountered in using a conventional signal model at this site.
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