AD NUMBER

AD809330

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov’t. agencies and their contractors; Critical Technology; DEC 1966. Other requests shall be referred to Air Force Flight Dynamics Lab., Attn: FDTR. Wright-Patterson AFB, OH 45433.

AUTHORITY

AFFDL ltr, 12 Mar 1973
TRANSIENT ANALYSIS OF HEAT CONDUCTION THROUGH A SLAB BY INFINITE SERIES

THOMAS N. BERNSTEIN
ROBERT M. ENGLE, JR.

TECHNICAL REPORT AFFDL-TR-68-109

DECEMBER 1968

This document is subject to special export controls and each transmission to foreign governments or foreign nationals may be made only with prior approval of the Theoretical Mechanics Branch, Structures Division, Air Force Flight Dynamics Laboratory (FDRL), Wright-Patterson Air Force Base, Ohio 45433.
NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Copies of this report should not be returned to the Research and Technology Division unless return is required by security considerations, contractual obligations, or notice on a specific document.
Best Available Copy
TRANSIENT ANALYSIS OF HEAT CONDUCTION THROUGH A SLAB BY INFINITE SERIES

THOMAS N. BERNSTEIN
ROBERT M. ENGLE, JR.
FOREWORD

This report was prepared by Thomas N. Bernstein and Robert M. Engle, Jr., of the Theoretical Mechanics Branch, Structures Division, Air Force Flight Dynamics Laboratory. The work was conducted in house under Project No. 1467, "Structural Analysis Methods," Task No. 146702, "Thermoelastic Structural Analysis Methods," and was administered by the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. Mr. Robert M. Bader is the Project Engineer administering Project No. 1467.

This report covers research conducted from July 1964 to July 1966. The manuscript was released by the authors in September 1966 for publication as a technical report.

This technical report has been reviewed and is approved.

FRANCIS J. FANK, JR.
Chief, Theoretical Mechanics Branch
Structures Division
ABSTRACT

The exact solution to the problem of conduction of heat through a slab is developed. The solution, formulated in terms of an infinite series, allows arbitrary initial conditions and time-dependent boundary conditions. The solution is programmed in FORTRAN IV for the IBM 7094 II computer. Several check problems were solved and the results were compared with those obtained from a finite difference heat transfer program.
# CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. MATHEMATICAL FORMULATION</td>
<td>1</td>
</tr>
<tr>
<td>A. Boundary Conditions</td>
<td>1</td>
</tr>
<tr>
<td>B. Steady State Solution</td>
<td>2</td>
</tr>
<tr>
<td>C. Transient Solution</td>
<td>3</td>
</tr>
<tr>
<td>D. Modification of Solution for Time Dependent Boundary Conditions</td>
<td>6</td>
</tr>
<tr>
<td>E. Solution of Initial Condition Problem</td>
<td>7</td>
</tr>
<tr>
<td>F. Unsteady State Solution</td>
<td>7</td>
</tr>
<tr>
<td>G. Transient Solution</td>
<td>9</td>
</tr>
<tr>
<td>H. Complete Solution for the General Problem</td>
<td>9</td>
</tr>
<tr>
<td>III. APPLICATIONS</td>
<td>10</td>
</tr>
<tr>
<td>IV. COMPUTER PROGRAM FOR SERIES TRANSIENT ANALYSIS OF SLAB HEAT TRANSFER (STASH)</td>
<td>11</td>
</tr>
<tr>
<td>A. Description</td>
<td>11</td>
</tr>
<tr>
<td>B. Input</td>
<td>13</td>
</tr>
<tr>
<td>C. Sample Problem</td>
<td>19</td>
</tr>
<tr>
<td>D. Restrictions</td>
<td>21</td>
</tr>
<tr>
<td>E. Output</td>
<td>22</td>
</tr>
<tr>
<td>V. CONCLUSIONS</td>
<td>26</td>
</tr>
<tr>
<td>APPENDIX I COMPUTER PROGRAM SOURCE LISTING</td>
<td>27</td>
</tr>
<tr>
<td>APPENDIX II EIGENVALUE SUBROUTINES</td>
<td>41</td>
</tr>
<tr>
<td>APPENDIX III RESULTS OF CHECK PROBLEMS</td>
<td>45</td>
</tr>
</tbody>
</table>
ILLUSTRATIONS

FIGURE | PAGE
--- | ---
1. Geometrical Representation | 2
2. Simplified Flow Chart of Transfer of Information | 12
3. Input Data Format | 17
4. Sample Problem Data Deck | 20
5. Sample Output (Case 4) Showing Display of Input Data | 23
6. Sample Output (Case 4) Showing Eigenvalue Solution | 24
7. Sample Output (Case 4) Showing Intermediate Print | 25
8. Temperature Profiles (Case 1) | 50
9. Temperature Profiles (Case 2) | 51
10. Temperature Profiles (Case 3) | 52
11. Temperature Profiles (Case 4) | 53
12. Temperature Profiles (Case 5) | 54

TABLES

TABLE | PAGE
--- | ---
I. Systems of Units Stored Internally | 16
II. Comparison of Data for Case 1 | 46
III. Comparison of Data for Case 2 | 47
IV. Comparison of Data for Case 3 | 48
V. Comparison of Data for Case 4 | 49
VI. Comparison of Data for Case 5 | 49
## SYMBOLS

<table>
<thead>
<tr>
<th><strong>MATH SYMBOL</strong></th>
<th><strong>FORTRAN SYMBOL</strong></th>
<th><strong>PHYSICAL DEFINITION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( A, A_0, A_L )</td>
<td>CP</td>
<td>Coefficients of Steady State Solution</td>
</tr>
<tr>
<td>( B, B_0, B_L )</td>
<td>DETK</td>
<td>Coefficients of Steady State Solution</td>
</tr>
<tr>
<td>( C_0, C_1, C_2, C_n )</td>
<td>FO, FL</td>
<td>Constants of Integration</td>
</tr>
<tr>
<td>( C_p )</td>
<td></td>
<td>Specific heat</td>
</tr>
<tr>
<td>( D )</td>
<td></td>
<td>Determinant of ( K_{ij} )'s</td>
</tr>
<tr>
<td>( f(x), F_x )</td>
<td>FOFX</td>
<td>Boundary condition constants</td>
</tr>
<tr>
<td>( K )</td>
<td></td>
<td>Initial conditions</td>
</tr>
<tr>
<td>( K_{ij} )</td>
<td></td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>( L )</td>
<td></td>
<td>Boundary condition indicators</td>
</tr>
<tr>
<td>( n )</td>
<td>NTERMS</td>
<td>Length</td>
</tr>
<tr>
<td>( N )</td>
<td></td>
<td>Summation index</td>
</tr>
<tr>
<td>( t )</td>
<td>T</td>
<td>Particular value of ( n )</td>
</tr>
<tr>
<td>( T )</td>
<td>TEMP</td>
<td>Time</td>
</tr>
<tr>
<td>( T_s, T_x )</td>
<td></td>
<td>Temperature</td>
</tr>
<tr>
<td>( T_T, T_{T^1}, T_{T^2}, T_{T^3} )</td>
<td></td>
<td>Steady state solutions</td>
</tr>
<tr>
<td>( T_{IC} )</td>
<td></td>
<td>Transient solutions</td>
</tr>
<tr>
<td>( T )</td>
<td></td>
<td>Solution of initial condition problem</td>
</tr>
<tr>
<td>( S )</td>
<td></td>
<td>Complete problem solution</td>
</tr>
<tr>
<td>( x )</td>
<td>x</td>
<td>Cross-sectional area of slab</td>
</tr>
<tr>
<td>( X(x) )</td>
<td></td>
<td>Distance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Assumed solution</td>
</tr>
<tr>
<td>MATH SYMBOL</td>
<td>FORTRAN SYMBOL</td>
<td>PHYSICAL DEFINITION</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>$\gamma_n$</td>
<td>$\gamma_n$</td>
<td>Repetitive term in solution</td>
</tr>
<tr>
<td>$Z$, $zn$</td>
<td>$Zn$</td>
<td>Eigenvalues</td>
</tr>
<tr>
<td>$\infty$</td>
<td></td>
<td>Infinity</td>
</tr>
<tr>
<td>$\alpha = \kappa/\rho C_p$</td>
<td>$\alpha$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>$\beta$, $\beta_n$</td>
<td>$\beta_n$</td>
<td>Eigenvalues</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\lambda$</td>
<td>Dummy time variable</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\pi$</td>
<td>Density</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\rho$</td>
<td>Boundary condition</td>
</tr>
<tr>
<td>$\phi(t)$</td>
<td>$\phi(t)$</td>
<td>time functions</td>
</tr>
<tr>
<td>$\phi_i(t)$</td>
<td>$\phi_i(t)$</td>
<td>Assumed solution</td>
</tr>
<tr>
<td>$\Phi(t)$</td>
<td>$\Phi(t)$</td>
<td></td>
</tr>
</tbody>
</table>

**Subscripts**

- IC
- L, O
- P
- S
- T
- $0, 1, 2, 3, j, \bar{u}, N$

**Superscripts**

- Primed denote differentiation

---

**SYMBOLS (Cont'd)**

---

viii
SECTION I
INTRODUCTION

The conduction of heat through a slab is governed by the following partial differential equation:

\[
\frac{\partial}{\partial x} \left[ K \frac{\partial T}{\partial x} \right] = \rho C_p \frac{\partial T}{\partial t}
\]

For constant thermal diffusivity, this equation simplifies to

\[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]

The exact solution to this equation can be formulated in terms of an infinite series. This report develops the exact solution for arbitrary initial conditions and time dependent boundary conditions. The solution has been programmed in FORTRAN for an IBM 7094 computer and the source program listing is contained in Appendix I.

SECTION II
MATHEMATICAL FORMULATION

A. BOUNDARY CONDITIONS

The general solution of Equation (2) must satisfy arbitrary initial and time dependent boundary conditions which can be expressed in the following form:

\[
T(x,i) = f(x) \quad t = 0 \quad (3)
\]

\[
K_0 \frac{\partial T}{\partial x} + K_{t2} T = F_0 \phi_0 (\lambda) \quad x = 0 \quad (4)
\]

\[
K_{x1} \frac{\partial T}{\partial x} + K_{x2} T = F_L \phi_L (\lambda) \quad x = L \quad (5)
\]
where the $k_{ij}$'s are constants. Selecting different values of these coefficients dictates the mode of heat transfer present at the boundary. By various combinations of constants, the imposition of surface temperature, convection, heat flux or insulation is possible. A more detailed discussion on the interpretation of boundary conditions is contained in Section III, "Applications."

\[ k_{11} \frac{dT}{dx} + k_{12} T = \phi \]

\[ \phi = k_{21} \frac{dT}{dx} + k_{22} T \]

**Figure 1. Geometrical Representation**

The application of D'Alambert's superposition theorem to account for the time dependent boundary conditions necessitated breaking the solution into a transient portion satisfying initial conditions, and a steady state plus transient with zero initial conditions.

Since the restriction has been placed on the boundary conditions that the time dependency can be expressed as a product of a time function and one of the standard boundary conditions, it is therefore possible to solve the equations neglecting the time variation and then modify the solution to account for $\phi$.

The problem is first simplified by breaking the solution into two parts: a steady state portion, $T_s(x, \omega)$, satisfying the arbitrary boundary condition, and a transient portion, $T_t(x, t)$, satisfying the initial temperature distribution and homogeneous boundary conditions.

**B. STEADY STATE SOLUTION**

For steady state conditions we note that $\frac{dT}{dt}=0$ and Equation (3) simplifies to

\[ \frac{dT}{dx} = 0 \]  \hspace{1cm} (6)

The solution of this equation is found directly by integration with the result;

\[ T_s = Ax + B \]  \hspace{1cm} (7)

We now impose the arbitrary boundary conditions;

\[ k_{11} \frac{dT}{dx} + k_{12} T = T_0 \] \hspace{1cm} (8)

\[ x = 0 \]
\[ K_{21} \frac{\partial T}{\partial x} + K_{22} T = F_L \quad x = L \]  

(9)

Substituting Equation (7) into (6) and (9) yields

\[ K_{11} A + K_{12} B = F_0 \]  

(10)

\[ K_{21} A + K_{22} (A L + B) = F_L \]  

(11)

The constants of integration, A and B, can now be evaluated from Equations (10) and (11). In order to keep these expressions in general form, the solution is accomplished by Cramer's rule with the result

\[ A = \frac{K_{12} F_L - K_{22} F_0}{K_{12} K_{22} L - (K_{11} K_{22} - K_{12} K_{21})} \]  

(12)

\[ B = \frac{(K_{21} + K_{22} L) F_0 - K_{11} F_L}{K_{12} K_{22} L - (K_{11} K_{22} - K_{12} K_{21})} \]  

(13)

C. TRANSIENT SOLUTION

A product form of solution is assumed for Equation (3), and designated by \( \Phi(x) \). Substitution into Equation (2) yields

\[ \frac{\Phi''}{\Phi} = \frac{1}{\alpha} \frac{x}{\Phi}' \]  

(14)

Rearranging

\[ \frac{\Phi''}{\Phi} = \frac{\Phi'}{\alpha \Phi} \]  

(15)

which requires that each of these functions be equivalent to some, as yet-constant, by setting this constant equal to \(-\beta^2\) results in two ordinary differential equations of the form

\[ \Phi' + \alpha \beta^2 \Phi = 0 \]  

(16)

\[ \chi'' + \beta^2 \chi = 0 \]  

(17)

Equation (6) has the exponential form of solution

\[ \Phi = C_0 e^{-\alpha \beta^2 t} \]  

(18)

whereas Equation (17) is satisfied by

\[ \chi(x) = C_1 \cos \beta x + C_2 \sin \beta x \]  

(19)
The solution to Equation (2) is then

$$T(x,t) = \left[ C_1 \cos \beta x + C_2 \sin \beta x \right] \left[ C_0 \ e^{-\alpha \beta^2 t} \right]$$  \hspace{1cm} (20)

This transient solution must satisfy the initial temperature distribution and homogeneous boundary conditions as follows:

$$T(x,0) = f(x) \hspace{1cm} t = 0 \hspace{1cm} (3)$$

$$K_{12} \frac{\partial T}{\partial x} + K_{12} T = 0 \hspace{1cm} x = 0 \hspace{1cm} (21)$$

$$K_{22} \frac{\partial T}{\partial x} + K_{22} T = 0 \hspace{1cm} x = L \hspace{1cm} (22)$$

Note first that the constant $C_0$ can be eliminated since its effect can be included in $C_1$ and $C_2$.

To evaluate the remaining constants substitute Equation (20) into Equations (3), (21) and (22).

Substitution of Equation (20) into (21) yields

$$K_{12} C_1 + K_{11} \beta C_2 = 0 \hspace{1cm} (23)$$

from which we obtain

$$C_1 = -\frac{K_{11} \beta}{K_{12}} \ C_2 \hspace{1cm} (24)$$

At this point it becomes necessary to impose the artificial restriction that $K_{12} \neq 0$, in order that calculations performed on the computer remain bounded.

Substitution of Equation (20) into (22) yields

$$\left[ K_{22} \cos \beta L - K_{12} \sin \beta L \right] C_1 + \left[ K_{21} \beta \cos \beta L + K_{22} \sin \beta L \right] C_2 = 0 \hspace{1cm} (25)$$

To obtain a nontrivial solution for $C_1$ and $C_2$, the determinant of their coefficients must be set equal to zero. This yields the following transcendental equation.

$$\tan z = \frac{DL}{\frac{D L \ z}{K_{21} K_{11} z^2 + K_{22} K_{12} L^2}} \hspace{1cm} (26)$$

where

$$z = \beta L \hspace{1cm} (27)$$

and

$$D = K_{11} K_{22} - K_{12} K_{21} \hspace{1cm} (28)$$
Equation (26) has infinitely many solutions (eigenvalues), and we shall denote these by \( Z_n \), where \( n = 0, 1, 2, \ldots \). The remaining constant \( C_2 \) is evaluated by substituting our solution into Equation (3) in order that the initial temperature distribution be satisfied. It is obvious at this point that, in general, arbitrary functions for the initial temperature distribution can not be satisfied using only one value of \( Z_n \) and \( C_2 \). We are thus required to expand our initial condition and our solution in an infinite series. Then, the coefficient \( C_2 \) becomes \( C_n \) and its evaluation proceeds as follows. The total solution at this point can be expressed

\[
T(x,t) = Ax + B + \sum_{n=0}^{\infty} C_n \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right] e^{-\alpha \beta_n^2 t} \tag{29}
\]

Imposing the problem initial condition yields

\[
T(x,0) = f(x) = Ax + B + \sum_{n=0}^{\infty} C_n \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right] \tag{30}
\]

Rearranging

\[
f(x) - (Ax+B) = \sum_{n=0}^{\infty} \tilde{C}_n \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right] \tag{31}
\]

Since the sines and cosines form a complete set of orthogonal functions, the \( C_n \)'s can be evaluated by multiplying both sides of Equation (31) by

\[
\left[ \sin \beta_N x - \frac{K_H \beta_N}{K_{12}} \cos \beta_N x \right] \text{ and integrating from zero to } L.
\]

Thus,

\[
\int_0^L \left[ f(x) - (Ax+B) \right] \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right] dx = \\
\int_0^L \sum_{n=0}^{\infty} C_n \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right] \left[ \sin \beta_N x - \frac{K_H \beta_N}{K_{12}} \cos \beta_N x \right] dx \tag{32}
\]

By orthogonality this integration produces nontrivial results only in the case of \( n = N \). Therefore

\[
C_n = \frac{\int_0^L \left[ f(x) - (Ax+B) \right] \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right] dx}{\int_0^L \left[ \sin \beta_n x - \frac{K_H \beta_n}{K_{12}} \cos \beta_n x \right]^2 dx} \tag{33}
\]
The denominator of $C_n$ can be evaluated directly with the result:

$$
\int_0^L \left[ \sin \beta_n x - \frac{K_{II} \beta_n}{K_{I2}} \cos \beta_n x \right]^2 dx = \frac{1}{2 \beta_n} \left\{ z_n \left[ \frac{K_{II} \beta_n}{K_{I2}} \right]^2 + 1 \right\} + \sin z_n \cos z_n \left[ \frac{K_{II} \beta_n}{K_{I2}} \right] - 1 + 2 \left( \frac{K_{II} \beta_n}{K_{I2}} \right) \sin^2 z_n \right\}
$$

Collecting the formulations required for problem evaluation leads to the expression of the solution for time independent boundary conditions in the form

$$
T(x, t) = Ax + B + \sum_{n=0}^{\infty} \int_0^L \left[ f(x) - (Ax + B) \right] \left[ Y_n(x) \right] dx
$$

where

$$
A = \left[ \frac{K_{I2} F_I - K_{I2} F_0}{K_{II} K_{I2} - D} \right]
$$

$$
B = \left[ \frac{K_{II} + K_{I2} L} {K_{II} K_{I2} - D} \right]
$$

$$
Y_n(x) = \left[ \sin \beta_n x - \frac{K_{II} \beta_n}{K_{I2}} \cos \beta_n x \right]
$$

$$
\beta_n = Z_n / L
$$

and

$$
\tan = Z_n = \frac{D L Z_n}{K_{I2} K_{II} Z_n^2 + K_{II} K_{I2} L^2}
$$

D. MODIFICATION OF SOLUTION FOR TIME DEPENDENT BOUNDARY CONDITIONS

Duhamel's superposition integral is now applied to the solution, Equation (33), to account for the time varying boundary conditions. F. B. Hildebran gives the solution in the form:

$$
\mathbf{T}(x,t) = T_0(x, \alpha \phi (z) + \left\{ \phi (z) + \int_0^t \alpha \beta_n \phi \phi' (\lambda) \ d\lambda \right\} \beta_T (x)
$$

This formulation is based on certain limitations, however, which must be eliminated. The first is the assumption of zero initial conditions. This restriction is eliminated by considering a


6
separate problem, possessing the given initial conditions and homogeneous boundary conditions. The solution of this problem is added to Equation (41). The second simplification utilized to obtain Equation (41) was to hold one boundary at zero and consider the remaining boundary to vary with time. For our problem, both boundaries can vary with time so we use the superposition principle once again by varying first one boundary condition and then the other, with the remaining boundary held at zero. The two results are then added. Note that \( f(x) = 0 \) for both these solutions.

**E. SOLUTION OF INITIAL CONDITION PROBLEM**

For the given initial condition, Equation (3), and the homogeneous boundary conditions, Equations (21) and (22), a "zero" steady state solution is obtained from Equation (7), i.e., \( A = 0, B = 0 \). Employing the given initial condition in Equation (35) then yields the desired result.

\[
T^*_C (x, t) = \sum_{n=1}^{\infty} \frac{\int_0^L f(x) \left[ y_n(x) \right] \, dx}{\int_0^L \left[ y_n(x) \right]^2 \, dx} \left[ y_n(x) \right] e^{-\alpha \beta_n^2 t},
\]

**F. UNSTEADY STATE SOLUTION**

The steady state solution, \( T_* (x, \infty) \), employed in Equation (35) is modified to \( T_* (x, \infty) \phi (t) \) in Equation (41). This result can be viewed as the steady state solution to a problem with our boundary conditions, if those conditions were "frozen" at the instant \( t \), and remained constant as \( t \to \infty \). Since our boundary conditions vary continuously with time, it is not possible to reach a steady state condition. This explains the title employed for this section of the report. \( T_* (x, \infty) \phi (t) \) can be obtained immediately from Equation (7), using boundary condition Equations (4) and (5) in place of (3) and (9). The result is

\[
T_* \phi (t) = \psi_0 = A z + B
\]

where

\[
A = \frac{K_{21} F_{H1} \phi_{H1} (t) - K_{22} F_0 \phi_0 (t)}{K_{12} K_{22} L - D}
\]

\[
B = \frac{(K_{21} F_{L} + K_{22} L) F_0 \phi_0 (t) - K_{12} F_{L} \phi_{L} (t)}{K_{12} K_{22} L - D}
\]

**G. TRANSIENT SOLUTION (TIME VARIABLE BOUNDARY CONDITIONS)**

The transient solution for boundary condition Equations (4) and (5) must be evaluated in two parts as indicated in Section II D. First, consider the conditions

\[
K_{11} \frac{\partial T}{\partial x} + K_{12} T = F_0 \phi_0 \quad (A)
\]

and

\[
K_{21} \frac{\partial T}{\partial x} + K_{22} T = 0 \quad (22)
\]
Equations (44) and (45) for $\mathcal{A}$ and $\mathcal{B}$ are modified by letting $F_L = 0$, with the result

$$\mathcal{A}_0 = -\frac{K_{12} F_L \phi_0(t)}{K_{12} K_{22} L - D}$$  \hspace{1cm} (47)$$

$$\mathcal{B}_0 = \frac{(K_{21} + K_{22} L) F_L \phi_0(t)}{K_{12} K_{22} L - D}$$  \hspace{1cm} (48)$$

The transient solution is obtained by substituting these results into the transient solution in Equation (35) with the result

$$T_{T_0} = \sum_{n=0}^{\infty} \int_0^L \left[ \mathcal{B}_0 - \mathcal{A}_0 \right] \gamma_n(x) \, dx \frac{\gamma_n(x)}{\int_0^L \left[ \gamma_n(x) \right]^2 \, dx} e^{-\alpha \beta_n^2 \bar{t}}$$  \hspace{1cm} (49)$$

Similarly for the conditions

$$K_{11} \frac{\partial T}{\partial x} + K_{12} T = 0$$  \hspace{1cm} (35)$$

$$K_{21} \frac{\partial T}{\partial x} + K_{22} T = F_L \phi_L(t)$$  \hspace{1cm} (36)$$

and letting $F = 0$ in Equations (41) and (45), with the result

$$\mathcal{A}_L = \frac{K_{12} F_L \phi_L(t)}{K_{12} K_{22} L - D}$$  \hspace{1cm} (50)$$

$$\mathcal{B}_L = \frac{-K_{11} F_L \phi_L(t)}{K_{12} K_{22} L - D}$$  \hspace{1cm} (51)$$

we obtain from Equation (35) the transient solution

$$\mathcal{T}_{T_L} = \sum_{n=0}^{\infty} \int_0^L \left[ \mathcal{A}_L \gamma_n(x) - \mathcal{B}_L \right] \gamma_n(x) \, dx \frac{\gamma_n(x)}{\int_0^L \left[ \gamma_n(x) \right]^2 \, dx} e^{-\alpha \beta_n^2 \bar{t}}$$  \hspace{1cm} (52)$$
H. COMPLETE SOLUTION FOR THE GENERAL PROBLEM

The general form of the complete solutions was expressed by Equation (41). Collecting the solutions obtained in Equations (42), (43), (49), and (52), and substituting into Equation (41), yields the final result,

\[
T_0^2(x,t) = T_0 + \left\{ \phi_0(x) + \int_0^t \alpha \beta_n^2 \lambda \phi_0'(\lambda) d\lambda \right\} T_\alpha(x,t)
\]

\[
+ \left\{ \phi_L(x) + \int_0^t \alpha \beta_n^2 \lambda \phi_L'(\lambda) d\lambda \right\} T_L(x,t) + T_{1c}(x,t)
\]  

(53)
SECTION III
APPLICATIONS

The generalized boundary conditions utilized in the mathematical formulation can be specialized to handle a number of physical problems. For example, take Equation (5)

\[ K_{21} \frac{\partial T}{\partial x} + K_{22} T = F_L \phi_L (\lambda) \]  

Appropriate choices of the indicators lead to the following:

a. Prescribed constant surface temperature.
Let \( K_{21} = 0, K_{22} = 1, \phi_L (\lambda) = 1, F_L = \) applied temperature

b. Prescribed constant heat flux.
Let \( K_{21} = K_S, K_{22} = 0, \phi_L (\lambda) = 1, F_L = \) applied flux

c. Insulated boundary.
Let \( K_{21} = K_S, K_{22} = 0, \phi_L (\lambda) = 1, F_L = 0 \)

d. Linear heat transfer at the surface (convection).
Let \( K_{21} = K_S, K_{22} = -hS, \phi_L (\lambda) = 1, F_L = -hST_c \)

This results in a boundary condition equation of the form

\[ K_S \frac{\partial T}{\partial x} = hS (T - T_c) \]

where \( h \) is the usual convective heat transfer coefficient per unit area.

e. Sign Convention.

The sign convention is such that a positive sign indicates flux into the body.

The boundary conditions described in "a" and "b" above may be arbitrarily varied with time by applying the appropriate time function, \( \phi_L (\lambda) \).
SECTION IV
COMPUTER PROGRAM FOR SERIES TRANSIENT
ANALYSIS OF SLAB HEAT TRANSFER (STASH)

A. DESCRIPTION

The program described below was written to solve for the temperature distribution in a one-dimensional rod with arbitrary initial conditions and time-varying boundary conditions. STASH is coded in FORTRAN IV for the IBM 7044-7094 II Direct Coupled-System. Fifteen subprograms make up the program, each of which has a specific task to perform. These subprograms are listed below:

- MAIN: reads in data, sets up calculations, and prints the results.
- SOLVE 1: solves the eigenvalue equation for positive values of DETK (see Appendix II).
- SOLVE 2: solves the eigenvalue equation for negative values of DETK.
- SOLVE 3: solves the eigenvalue equation for a zero value of DETK.
- SOLVE 4: solves the eigenvalue equation for DETK infinite.
- FINT: Simpson's rule integration routine.
- FUNCX: sets up the integrand for the x integral.
- FUNCT: sets up the integrand for the λ integral.
- TABIN: reads in tabular data, if present.
- INTERP: performs linear interpolation on tabular data.
- PHIO: defines the time-varying boundary condition at x = 0.
- PHI: defines the time-varying boundary condition at x = L.
- PHIPRO: defines the derivative of the time-varying boundary condition at x = 0.
- PHIPNL: defines the derivative of the time-varying boundary condition at x = L.
- FOFX: defines the initial conditions in the rod.

Figure 2 is a simplified flow chart depicting the transfer of information between the subprograms discussed above.
Figure 2. Simplified Flow Chart of Transfer of Information
There are basically two types of input data to STASH, the physical parameters and the problem parameters. The physical parameters are the characteristics of the rod and the conditions to which it is subjected. The problem parameters are the accuracy parameters, the calculation controls and the print controls.

The rod is characterized by four basic quantities: length, mass density, thermal conductivity and specific heat. The boundary conditions are identified by the indicators $K_{11}$, $K_{12}$, $K_{21}$, $K_{22}$. The magnitude of the boundary conditions is characterized by the indicators $F_0$ and $F_L$. The magnitude of the initial conditions is similarly characterized by the indicator $F_x$.

Since the solution may be required at any station on the rod for any given point in time, it is convenient to specify a length increment and a time increment as input parameters. A final time is also specified to terminate calculations. The accuracy of the solution is basically governed by three factors: the accuracy of the eigenvalues, the number of terms in the series portion of the solution and the number of intervals taken in the numerical integration routine. In the interest of maximum flexibility, each of these quantities was made an input parameter.

Information may be input to the program in two basic forms. The first type is the data card. There will always be seven cards in the data deck. If the tabular data option is used there may be many more. The second type of input consists of FORTRAN IV statements which may be inserted into the subprograms defining the initial and boundary conditions on the rod. An example problem using both types of input is given in Section V.C. Detailed instructions on inputting the data cards are given below, in the following format:

(1) Card number and contents

(2) Program name for contents

(3) Format of card input referenced to format statement number

(4) Description of each variable on the card

### 1. Data Cards

**Card 1** Intermediate print options

```
JPRINT(1), JPRINT(2), JPRINT(3)
```

```
5600  FORMAT(3H1)
```

**JPRINT(1)** — prints series portion of solution term by term if a one is entered. If no print is desired enter a zero.

**JPRINT(2)** — prints unsteady state portion of the solution if a one is entered. If no print is desired enter a zero.

**JPRINT(3)** — prints solution for eigenvalues if a one is entered. If no print is desired enter a zero.
Card 2  Title Card

IUNIT, TITLE

1 FORMAT (2, 13A6)

IUNIT — an indicator which prints out the system of units to be used in the problem. See Table I for a list of systems presently contained in the program.

TITLE — any alphanumeric information through column 80.

Card 3  Physical Parameters (all must be in consistent units)

L, K, RHO, CP, DELTAX, DELTAT, TIMEF

2 FORMAT (7E10.0)

L     length of the rod
K     thermal conductivity
RHO   mass density
CP    specific heat
DELTAX increment of length (100 increments maximum)
DELTAT increment of time
TIMEF final time (initial time is zero)

Card 4  Boundary Condition Indicators

K11, K12, K21, K22

3 FORMAT (4E10.0)

K11 indicator for $\frac{\partial T}{\partial x} \text{ at } x = 0$
K12 indicator for $T \text{ at } x = 0$
K21 indicator for $\frac{\partial T}{\partial x} \text{ at } x = L$
K22 indicator for $T \text{ at } x = L$

Card 5  Function Multiplying Factors

FO, FL, FX

4 FORMAT (3E10.0)

FO coefficient on function $\phi_0(t)$
FL coefficient on function $\phi_L(t)$
FX coefficient on function $f(x)$
<table>
<thead>
<tr>
<th>UNIT</th>
<th>LENGTH</th>
<th>MASS</th>
<th>TIME</th>
<th>WEIGHT</th>
<th>TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INCH</td>
<td>SLUG</td>
<td>SEC</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>2</td>
<td>INCH</td>
<td>SLUG</td>
<td>HR</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>3</td>
<td>INCH</td>
<td>SLUG</td>
<td>MIN</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>4</td>
<td>FOOT</td>
<td>SLUG</td>
<td>SEC</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>5</td>
<td>FOOT</td>
<td>SLUG</td>
<td>HR</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>6</td>
<td>FOOT</td>
<td>SLUG</td>
<td>MIN</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>7</td>
<td>INCH</td>
<td>POUND</td>
<td>SEC</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>8</td>
<td>INCH</td>
<td>POUND</td>
<td>MIN</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>9</td>
<td>INCH</td>
<td>POUND</td>
<td>HR</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>10</td>
<td>FOOT</td>
<td>POUND</td>
<td>SEC</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>11</td>
<td>FOOT</td>
<td>POUND</td>
<td>MIN</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
<tr>
<td>12</td>
<td>FOOT</td>
<td>POUND</td>
<td>HR</td>
<td>POUND</td>
<td>FAHRENHEIT</td>
</tr>
</tbody>
</table>
Card 6  Calculation Parameters

\[ \text{NTERMS, NSTEPX, NSTEPT, NTAB(1), NTAB(2), NTAB(3), NTAB(4), NTAB(5)} \]

6 FORMAT (3(5I1))

\[ \begin{align*}
\text{NTERMS} & \quad \text{number of terms in the series portion of the solution (100 maximum)} \\
\text{NSTEPX} & \quad \text{number of intervals for the } x \text{-integration} \\
\text{NSTEPT} & \quad \text{number of intervals for the } \lambda \text{-integration} \\
\text{NTAB(1)} & \quad \text{flag for table 1} \\
\text{NTAB(2)} & \quad \text{flag for table 2} \\
\text{NTAB(3)} & \quad \text{flag for table 3} \\
\text{NTAB(4)} & \quad \text{flag for table 4} \\
\text{NTAB(5)} & \quad \text{flag for table 5}
\end{align*} \]

If NSTEPX or NSTEPT is zero the program sets the value of the respective integral to zero.

Card 7  Eigenvalue Solution Parameters

\[ \text{LIMIT, ITERMX} \]

6 FORMAT (2E10.0,15)

\[ \begin{align*}
\text{LIMIT} & \quad \text{difference between two successive iterations} \\
& \quad \text{necessary to define convergence to a root.} \\
\text{ITERMX} & \quad \text{maximum number of iterations to be made in} \\
& \quad \text{searching for each eigenvalue.}
\end{align*} \]

If no tabular data is to be used, this is the last card in the data deck. If tabular data is to be an input, however, the following format will be used.

Card 1  Table Number and Comments

\[ \text{NTABLE, COMMENTS} \]

\[ \text{FORMAT (B8, 20A, 55H } ) \]

\[ \text{NTABLE} \quad \text{table number} \]

\[ \text{COMMENTS} \quad \text{any alphanumeric information in columns 25 through 60.} \]
Card 2 Tabular Data (2 to 50 data cards per table)
INDVAR, DEPVAR, COMMENTS
FORMAT (6X, 2E10.0, 55H  
INDVAR independent variable
DEPVAR dependent variable
COMMENTS any alphanumeric information in columns 26 through 80
Card 3 End of Table
N
FORMAT (15)
N negative of table number
There are five tables provided in the program, which are assigned as follows:
  Table 1 PHIO
  Table 2 PHIL
  Table 3 PHIPRO
  Table 4 PHIPRL
  Table 5 FOFX

Figure 3. shows a symbolic data deck. A sample problem is generated in detail in Section
IV C.

2. Subprogram Input Cards

If the tabular data option is not used STAGY calculates the required functions internally using
FORTRAN statements as loaded in the subprograms at compilation time. The affected sub-
programs are:

FUNCTION PHIO
FUNCTION PHIL
FUNCTION PHIPRO
FUNCTION PHIPRL
FUNCTION FOFX

As the initial or boundary conditions change, cards containing the functional statement of the
variation must be inserted. Since all the above functions have an associated multiplying factor
it is convenient to use normalized functional statements in the subprogram. Thus, if we entered the following card in the FUNCTION FOFX

\[ FOFX = 1.0 \]

we would obtain the initial condition

\[ f(x) = FX \]

The program (Appendix I) contains subprogram statements corresponding to the following initial and boundary conditions,

\[ \phi_0(t) = \text{const} \]
\[ \phi_L(t) = \text{const} \]
\[ \phi'_0(t) = 0 \]
\[ \phi'_L(t) = 0 \]
\[ f(x) = \text{const} \]

C. SAMPLE PROBLEM

Consider a rod, ten inches long, having the following properties:

\[ K = 10 \text{ Btu/hr-ft-°F} \]
\[ \rho = 500 \text{ lbm/ft}^3 \]
\[ a_p = 0.1 \text{ Btu/lbm} \]

We wish to obtain a time history of the temperature distribution through the rod which is subject to the following boundary conditions

\[ T(0, t) = t \]
\[ T(L, t) = 0 \]

The initial conditions imposed on the rod are:

\[ T(x, 0) = 0 \]

For purposes of illustration the intermediate print option will be carried out on card one.

The first task in setting up the problem is to decide upon a system of units to employ. Since the length of the rod is given in inches, we shall choose the inch as the unit of length. For a transient study a small time unit is desirable, hence, the second becomes the unit of time. The remainder of the system of units is determined by the temperature and mass units. Going now to Table I, we obtain the correct value of UNIT. This indicator along with a suitable title becomes card two in Figure 4.
### FORTRAN CODING FORM

<table>
<thead>
<tr>
<th>STATEMENT NUMBER</th>
<th>FORTRAN STATEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TABLE NO. 1 PHIO VS. TIME</td>
</tr>
<tr>
<td>2</td>
<td>TABLE NO. 3 PHIVR VS. TIME</td>
</tr>
<tr>
<td>3</td>
<td>END OF TABLE</td>
</tr>
</tbody>
</table>

**Example 4. Sample Problem Data Deck**
Card three contains the physical parameters of the system in the system of units called for by the indicator, UNIT. These parameters are:

\[
L = 10 \text{ in.} \quad K = 0.002314 \text{ Btu/in.-sec}^{-F} \quad \text{RHO} = 0.2395 \text{ lb/in.}^3
\]

\[
CP = 0.10 \text{ Btu/lb m}\quad \text{DELTAX} = 0.5 \text{ in.} \quad \text{DELTAT} = 100 \text{ sec} \quad \text{TIME} = 1000 \text{ sec}
\]

Since we have only temperature boundary conditions we set K11 = K21 = 0 and K12 = K22 = 1. This information is entered on card four.

The functions defining our initial conditions are

\[
\phi_0(t) = 0, \quad \phi_L(t) = 0, \quad f(x) = 0, \quad \phi_0'(\lambda) = 1, \quad \phi_L'(\lambda) = 0
\]

We can take advantage of the functions already stored in the program by using \( F_L \) and \( F_\lambda \) to zero out the appropriate functions. We then can use \( F_0 = 1 \) to bring in the other boundary condition. This is shown on card five.

The calculation parameters are entered on card six. A series solution containing twenty-five terms is completely adequate for this problem. Since the initial condition function is zero, we set NSTEPX equal to zero. For a Simpson's rule integration we can use fifty steps should suffice for NSTEP. Our choice of multiplying factors on card five enabled us to do much of the function calculation internally. However, to eliminate recompiling any portion of the program we used tables to define the functions \( \phi_0(t) \) and \( \phi_L'(\lambda) \). Thus we set NTAB(1) and NTAB(3) equal to one and the rest equal to zero.

For a solution with temperature boundaries only, the eigenvalues become simply \( \pi n \). Thus, the eigenvalue parameters have little meaning. However, for a more complex case they would have a significant effect on the solution so these parameters should be made as stringent as required. Typical values are entered on card seven.

Our choice not to recompile any subprocedures leads to the use of tables one and three. The first card in each table is a table designation number. The following cards contain data points. The last card of each table contains the negative of the table designation number and is a flag signalling the end of the table.

This, then is the data deck for the sample problem. The assembled deck is shown in Figure 4.

D. RESTRICTIONS

Certain restrictions must be adhered to in order for the solution to be successful. Violation of these restrictions will usually produce an error message from the computer program.

a. A consistent set of units must be employed. An indicator is provided on the title card which will label the system of units on the output. If this indicator is omitted, the following error message is printed.

**SYSTEM OF UNITS NOT SPECIFIED. UNIT NOT ENTERED OR ZERO.**

This message merely informs the user of this omission. Execution of the problem is not terminated.
b. \( K_{11} \) and \( K_{12} \) cannot be zero simultaneously. Similarly, \( K_{21} \) and \( K_{22} \) cannot be zero simultaneously. These situations lead to an undefined boundary. The error message below results from this case.

**BOTH INDICATORS AT ONE BOUNDARY ARE ZERO**

c. \( K_{12} \) cannot be zero. This is a somewhat artificial restriction imposed by the formulation of the problem. In a physical sense it prevents the possibility of the unsolvable Neumann problem. If \( K_{12} \) is zero the following error message is printed:

**FORMULATION DOES NOT PERMIT \( K_{12} \) TO BE ZERO.**

d. The number of integration intervals must be even. This restriction arises from the computer formulation of Simpson's rule. If an odd number is entered, the following error message results:

**NUMBER OF INTEGRATION INTERVALS IS NOT EVEN**

e. The computer program generates an error message if the time increment or the length increment is zero or negative. This message reads:

**TIME OR LENGTH INCREMENT IS ZERO OR NEGATIVE**

f. The initial time for each program is zero.

g. The program uses even increments of time and length. The maximum number of length increments is one hundred.

**E. OUTPUT**

The output generated by STASH consists of two segments: the input data display and temperature profiles which are always generated; and the intermediate print which is controlled by the first card in the data deck. After reading the data, STASH prints it out along with suitable titles and headings as shown in Figure 5. If the eigenvalue solution is requested a table of the eigenvalues and iterations is printed as shown in Figure 8. Intermediate print options giving the values of the series and the unsteady state produce output as shown in Figure 7 for each station along the slab at each time step. Figure 7 also shows the form of the temperature profiles as generated at each time step.
CASE 4  INSULATED BOUNDARY

PHYSICAL CONSTANTS TO DEFINE THE PROBLEM

<table>
<thead>
<tr>
<th>SYSTEM OF UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LENGTH</td>
</tr>
<tr>
<td>INCH</td>
</tr>
<tr>
<td>0.0999999E+02</td>
</tr>
</tbody>
</table>

BOUNDARY CONDITION INDICATORS FROM THE DIFFERENTIAL EQUATION

<table>
<thead>
<tr>
<th>R11</th>
<th>K12</th>
<th>K21</th>
<th>K22</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0999999E+01</td>
<td>0.0999999E+01</td>
<td>0.0999999E+01</td>
</tr>
</tbody>
</table>

MULTIPLYING FACTORS FOR BOUNDARY AND INITIAL CONDITION FUNCTIONS

<table>
<thead>
<tr>
<th>K1(0)</th>
<th>K11</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0999999E+04</td>
<td>0.0999999E+04</td>
</tr>
</tbody>
</table>

CALCULATION PARAMETERS

<table>
<thead>
<tr>
<th>NUMBER OF TERMS IN THE SUMMATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0</td>
</tr>
</tbody>
</table>

EIGENVALUE SOLUTION PARAMETERS

<table>
<thead>
<tr>
<th>ACCURACY LIMIT</th>
<th>NUMBER OF ITERATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0999999E+04</td>
<td>200.0</td>
</tr>
</tbody>
</table>

ALPHA

| 0.7973091SE-01 |

DEIK

| -0.0999999E+01 |

Figure 5: Sample Output (Case 4) Showing Display of Data.
<table>
<thead>
<tr>
<th>ROOT NO</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15767962E+01</td>
</tr>
<tr>
<td>2</td>
<td>0.47123838E+01</td>
</tr>
<tr>
<td>3</td>
<td>0.76538145E+01</td>
</tr>
<tr>
<td>4</td>
<td>0.10995574E+02</td>
</tr>
<tr>
<td>5</td>
<td>0.14137166E+02</td>
</tr>
<tr>
<td>6</td>
<td>0.17278759E+02</td>
</tr>
<tr>
<td>7</td>
<td>0.20420359E+02</td>
</tr>
<tr>
<td>8</td>
<td>0.23561844E+02</td>
</tr>
<tr>
<td>9</td>
<td>0.26703537E+02</td>
</tr>
<tr>
<td>10</td>
<td>0.29845123E+02</td>
</tr>
</tbody>
</table>

Figure 6. Sample Output (Case 4) Showing Eigenvalue Solution
### SERIES PARTION OF SOLUTION

<table>
<thead>
<tr>
<th>TEMP</th>
<th>SUMMATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11627815E-03</td>
<td>-0.11927431E-03</td>
</tr>
<tr>
<td>0.1302856E-03</td>
<td>-0.11927402E-03</td>
</tr>
<tr>
<td>-0.84559987E-13</td>
<td>-0.11927392E-03</td>
</tr>
<tr>
<td>-0.21439305E-31</td>
<td>-0.11927392E-03</td>
</tr>
<tr>
<td>0.</td>
<td>-0.11927392E-03</td>
</tr>
<tr>
<td>0.</td>
<td>-0.11927392E-03</td>
</tr>
<tr>
<td>0.</td>
<td>-0.11927392E-03</td>
</tr>
<tr>
<td>0.</td>
<td>-0.11927392E-03</td>
</tr>
<tr>
<td>0.</td>
<td>-0.11927392E-03</td>
</tr>
</tbody>
</table>

### UNSTEADY STATE PARTION OF SOLUTION

<table>
<thead>
<tr>
<th>TEMP</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09999999E 02</td>
<td>-0.09999999E 04</td>
</tr>
</tbody>
</table>

**TEMPERATURE DISTRIBUTION ALONG THE ROD AT 1000 INTERVALS**

- 0.09999999E 04
- 0.36229942E 03
- 0.42559927E 03
- 0.33279926E 03
- 0.32343399E 03
- 0.40002189E 03
- 0.78022243E 03
- 0.77502173E 03
- 0.76633632E 03

---

**Figure 7.** Sample Output (Case 4) Showing Intermediate Print
CONCLUSIONS

Several classes of problems were run to check out the program. The results were compared with a finite-difference heat transfer program (LTA) which was developed by Lockheed Aircraft Corporation. These results were examined for accuracy and speed of convergence.

The intermediate print feature of the program was used to determine the convergence. Examination showed that convergence was quite rapid, usually less than ten terms, for constant boundary conditions, away from time zero. More terms were needed in the vicinity of zero time to produce convergence. For time-varying boundary conditions, the series does not converge very quickly. However, a study of the solution convergence showed that a twenty term series using one hundred integration steps obtained results within one percent of the LTA solution for times exceeding one hundred seconds. At smaller times a fifty term series with one hundred integration steps was required.

Appendix III contains the results of five check problems compared with the results from the LTA finite difference program.

No comparison is made at zero time since the program obtains these values from the initial conditions rather than from a series calculation. The curves are plotted from data generated by the program. Case I is the sample problem detailed in Section IV C. All other cases used the same physical parameters.
APPENDIX I

COMPUTER PROGRAM SOURCE LISTING
AFFDL-TR-86-109

$8BJCB STASH PAP
LIBCIC MAIN P94/2, XR7
C
A GENERAL SOLUTION TO THE ONE-DIMENSIONAL HEAT TRANSFER PROBLEM
WITH TIME-DEPENDENT BOUNDARY CONDITIONS AND ARBITRARY INITIAL
CONDITION

VERSION 2

VERSION 2 CALCULATES ZERO TIME USING BOUNDARY AND INITIAL CONDITIONS
AND BOUNDARY CONDITION FUNCTIONS TO PERMIT THE USE OF NORMALIZED
FUNCTIONS

EXTERNAL TFUNC, XFUNC
DIMENSION TEMPR(100), TITLS(13), EIGEN(100)
DIMENSION JPK(41), NTA(5)
REAL K11, K12, K121, K122
REAL LIMIT, L, LAMBDA, NUMSS, N野55, KPHI, KPHIO, KPHII, K11, K12

1, K21, K22, K, KTERM1, KTERM2
COMMON T, X, LAMBDA, KTERM1, KTERM2, AMPMA, BETAN, EXPUL, CONX, S0N5, DELTA
COMMON/DUTCH, L, LTERMX, K11K21, K12K22, LIMIT
COMMON/PRINT, PRNT

FORMAT STATEMENTS
1 FORMAT(I12, 13A6)
2 FORMAT(E10.0)
3 FORMAT(E10.0)
4 FORMAT(E10.0)
5 FORMAT(I15-511)
6 FORMAT(E10.0-15)
7 FORMAT(I11-29X, 13A6)
20 FORMAT(I10, 8X, 6HLEN, 12X, THERMAL, 11X, THDENSITY, 10X, BHSPECIFIC, 8X)
111, 6HLENGTH, 13X, 4HTIME, 14X, 5HFINTL/24X, 12HCONDUCTIVITY, 20X, 4HNEAT
2, 11X, 9HINCREMENT, 9X, 9HINCREMENT, 12X, 4HTIME
22 FORMAT(I10, 3X, E15.3, E15.3, 4X, E15.3, E15.3, 2X, E15.3, E15.3)
30 FORMAT(I10, 3X, 60BBOUNDARY CONDITION INDICATORS FROM THE DIFFERENT
11AL EQUATION//36X, 3HKL1, 16X, 3HKL2, 16X, 3HKL1, 16X, 3HKL2, 16X
31 FORMAT(I14, 55X, 15HMULTIPLYING FACTORS/6X, 3MFORD/4X, 50BOUNDARY ANGST
10 INITIAL CONDITION FUNCTIONS/30X, 4HF01, 30X, 4HF1, 30X, 4HF2, //22STASHO42
24X, 2(E15.8-10X), E15.8)
33 FORMAT(I10, 29X, 4(E15.9-4X))
66 FORMAT(I14, 51X, 30HEIGENVALUE SOLUTION PARAMETERS/42X, BHACCURACY, 3STASHO45
*0X, 8HNUMBER OF F44X, 5HLIMIT, 31X, 10HITERATIONS/39X, E15.8, 26X, 15)
120 FORMAT(I28X, 4HINC, 14X, 4HSLUG, 14X, 3MHN1, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO48
130 FORMAT(I28X, 4HINC, 14X, 4HSLUG, 14X, 3MHN1, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO50
140 FORMAT(I28X, 4HINC, 14X, 4HSLUG, 15X, 2HHRR, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO52
150 FORMAT(I28X, 4HFCUT, 14X, 4HSLUG, 14X, 3MHN1, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO54
160 FORMAT(I28X, 4HFCUT, 14X, 4HSLUG, 14X, 3MHN1, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO56
170 FORMAT(I28X, 4HFOOT, 14X, 4HSLUG, 15X, 2HHRR, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO58
180 FORMAT(I28X, 4HINC, 14X, 5HPOUND, 13X, 3MSEC, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO60
190 FORMAT(I28X, 4HINC, 14X, 5HPOUND, 13X, 3MSEC, 16X, 5HPOUND, 13X, 10HFARHENHEAT
117) STASHO62

28
AFFDL-TR-6b-100

303 FORMAT(1HA,54X,22HCALCULATION PARAMETERS//26X,15HNUMBER OF TERMS,1SSTASH063
15X,19HNUMBER OF INTERVALS,13X,19HNUMBER OF INTERVALS//25X,10MIN THEASTASH064
2 SUMMATION,14X,21I-FOR THE X INTEGRATION,11X,21I-FOR THE Y INTEGRATION,STASH065
3CH//30X,15,38X,15,28X,15) STASH066
332 FORMAT(1HA,63X,5HALPHA) STASH067
1002 FORMAT(26X,5El5.8) STASH068
1003 FORMAT(1X,El5.8) STASH069
1100 FORMAT(28X,4HINC,14X,5HPOUND,14X,5HPOUND,13X,10FAHRENHEITSTASH070
1110 FORMAT(2X,4HFCOT,14X,5HPOUND,13X,3HSEC,16X,5HPOUND,13X,10FAHRENHEITSTASH071
1120 FORMAT(28X,4HFCOT,14X,5HPOUND,13X,3HMIN,16X,5HPOUND,13X,10FAHRENHEITSTASH072
1130 FORMAT(28X,4HFCOT,14X,5HPOUND,14X,2HHR,16X,5HPOUND,13X,10FAHRENHEITSTASH073
1140 FORMAT(1HA,45X,4HPHYSICAL CONSTANTS TO DEFINE THE PROBLEM//59X,1SSTASH074
14SYSTEM OF UNITS//27X,6LENGTH,13X,4HAMM,14X,4HTIME,14X,6HWEIGHT,STASH075
213X,1HTIME) STASH076
2002 FORMAT(7X,4HTIME,22X,42HTIME DISTRIBUTION ALONG THE ROD AT STASH081
1,IP3,3,10H INTERVALS) STASH082
3003 FORMAT(1HA:) STASH083
3333 FORMAT(1HO:58X,E15.8) STASH084
4004 FORMAT(1HA:63X,4HDETK) STASH085
4444 FORMAT(1HO:58X,E15.8) STASH086
5000 FORMAT(511) STASH087
6100 FORMAT(1HL,25X,2HIF,E15.8,2UL,2HXX,E15.8/45X,26H%RIES PORTION OFSTASH088
# SOLUTION/14X,11IN,16X,4HTERA,19X,9HSUMMATION//) STASH089
6110 FORMAT(10X,15,10X,E15.8) STASH090
6200 FORMAT(54X,3HUNSTEADY STATE PORTION OF SOLUTION//35X,1HXX,25X,STASH091
#4HTEMP//)) STASH092
6210 FORMAT(28X,F15.8,20X,E15.8) STASH093
ERROR=0 STASH094
CC INTERMEDIATE PRINT OPTIONS STASH095
CC READ(5,5000) PRINT STASH096
CC JPRINT=1,PRINT INTERMEDIATE CALCULATIONS, JPRINT=0,DO NOT PRINT. STASH097
CC JPRINT=0,FIXED SIZE PORTION OF THE SOLUTION TERM BY TERM STASH098
CC JPRINT(2)=1,STEADY STATE PORTION OF THE SOLUTION STASH099
CC JPRINT(3)=1,SOLUTION FOR THE EIGENVALUES STASH100
CC READ (5,1)UNIT,ITITLE(I),ITITLE(J) STASH101
CC READ(5,2)K,RRP,CO,DELTA,DEL,TIME,STASH102
CC READ(5,3)K11,K12,K21,K22 STASH103
CC READ(5,4)FO,FL,FY STASH104
CC READ(5,5) NTERMS,NSTEP,NSTEP,NTAB STASH105
CC READ(5,6) LIMIT,ITERMX STASH106
CC PRINT 1MPUT DATA STASH107
CC TITLE STASH108
CC WRITE (6,21)ITITLE(I),ITITLE(J) STASH109
CC SYSTEM OF UNITS STASH110
CC WRITE(6,1140) STASH111
CC IF(UNIT.EQ.0) GO TO 5500 STASH112
CC GO TO (102,103,1,4,105,106,107,108,109,110,111,112,113,114)UNIT STASH113
102 WRITE(6,120) STASH114

29
GO TO 9999
103 WRITE(6,130) STASH125
GO TO 9999
104 WRITE(6,140) STASH126
GO TO 9999
105 WRITE(6,150) STASH127
GO TO 9999
106 WRITE(6,160) STASH128
GO TO 9999
107 WRITE(6,170) STASH129
GO TO 9999
108 WRITE(6,180) STASH130
GO TO 9999
109 WRITE(6,190) STASH131
GO TO 9999
110 WRITE(6,1100) STASH132
GO TO 9999
111 WRITE(6,1110) STASH133
GO TO 9999
112 WRITE(6,1120) STASH134
GO TO 9999
113 WRITE(6,1130) STASH135
GO TO 9999
5500 WRITE(6,5550) STASH136

C PHYSICAL CONSTANTS

9999 WRITE(6,20) STASH137
WRITE(6,2211,K,RH,G,CP,BETA,T,TIME) STASH139
C BOUNDARY CONDITIONS

C WRITE(6,30)
WRITE(6,3111,K11,K12,K21,K22) STASH140
C MULTIPLYING FACTORS

C WRITE(6,31)FU,F2,FX STASH141
C CALCULATION PARAMETERS

C WRITE(6,303)INTERM,NSTEP,NSTEP T
C EIGENVALUE SOLUTION PARAMETERS

C WRITE(6,661) LIMIT,IERROR
C TEST FOR TABULAR DATA

CO 50 I=1,5
IF(INTAB(I).NE.0) CALL YASIA(1)
50 CONTINUE
C IF TABLES ARE USED, THEY ARE ASSIGNED AS FOLLOWS
C No.1 FUNCTION PHI1(I)
C No.2 FUNCTION PHII(I)
C No.3 FUNCTION PHIPRO(LAMBDA)
C No.4 FUNCTION PHIPOL(LAMBDA)
C No.5 FUNCTION FOFX(X)
C ERROR CHECKS ON INPUT DATA

C

C NUMBER OF INTEGRATION STEPS MUST BE EVEN OR ZERO.
C IF ZERO THE PROGRAM SETS THE INTEGRAL EQUAL TO ZERO.
C

C IF(NSTEPX.EQ.0) GO TO 248
C IF(MOD(NSTEPX,2).EQ.0) GO TO 248
WRITE (6,5553)
IERROR=IERROR+1

C 248 IF(NSTEPX.EQ.0) GO TO 249
C IF(MOD(NSTEPX,2).EQ.0) GO TO 249
WRITE (6,5553)
IERROR=IERROR+1

C 249 IF((DELTAX.GT.0.)
* .AND.  
* (DELTAT.GT.0.)
* GO TO 250
WRITE (6,5551)
IERROR=IERROR+1

C 250 IF(TIMEF.GT.0.) GO TO 251
WRITE (6,5552)
IERROR=IERROR+1

C K12 CANNOT BE ZERO DUE TO A RESTRICTION IN THE FORMULATION.
C

C 251 IF (K12.NE.0.) GO TO 252
WRITE (6,5555)
IERROR=IERROR+1

C CHECK FOR UNDEFINED BOUNDARY CONDITIONS.
C

C 252 IF(((K11.NE.0.).OR.(K12.NE.0.))
* .AND.
* ((K21.NE.0.).OR.(K22.NE.0.)))
* GO TO 253
WRITE (6,44)
IERROR=IERROR+1

C

C 253 IF(IERROR.GT.0) STOP
C
WRITE (6,333)
PI=3.1415926
ALPHA=(RHO*CP)
WRITE (6,3333) ALPHA
DET=K11*K22-K12*K21
WRITE (6,4004)
WRITE (6,4444) DETK

C C DETERMINE CONSTANTS FOR USE IN DO LOOPS
C
C CENOM=K12*K22+L-DETK
C CON01=K11*K12+L
C CON02=K11+2
C CON03=(K12+L)**2
C CON04=FL
C CON05=K11*K21
C CON06=K12*K22*(L**2))
C CON07=DET+L
C CON08=FL
C CON09=K12+FL

C

31
C CETERMINF EIGENVALUES FOR SERIES SOLUTION
C
IF(K11K2Ę. "O. O.", AND.(K12K22.EG.0.) )Go TO 400
IF(DET. .2. >300.100
100 CALL SGL'EI(DET,EIGEN,NTERMS)
GO TO 900
200 CALL SOLVE2(DET,EIGEN,NTERMS)
GO TO 900
300 CALL SOLVE3(DET,EIGEN,NTERMS)
GO TO 900
400 CALL SOLVE4(DET,EIGEN,NTERMS)
C C SET UP INDICES FOR DO LOOPS
C
900 ATEMPX=(T1FF/DELTAT)+1.5
ATEMPX=(L/DELTAX)+1.5
WRITE(6,3003)
T=DELTAT
GO 199 NT=1,NTEMP
T=+DELTAT
X=-DELTAX
DO99NX=1,NTEMP
IF(EQ.0.1) GO TO 299
X=X+DELTAX
SERIES=0.
IF(P=PRINTUL).NE.0) WRITE(6,6100) TX
C C SET UP LOOP TO GENERATE SERIES SUMMATION FOR TRANSIENT SOLUTION
C
C9991=1,NTERMS
ZN=EIGEN(I)
9 BETA=N2*ZN/L
ZM2=ZN**2
ZDEN=N2*DENCM
SNX=SN(BETA*X)
CBNX=COS(BETA*X)
ZN=SNI(ZN)
CNX=COS(ZN)
K10N4=K1+1*BETA
K11BNZ=K11BN*2
ZN2SZN=ZN2*SN
EXPPLU=EXP(1(-ALPHA*(BETA)*+2*+T))
SUNCON=BEATM/(K12+SNX*K11BN+CBNX)/ZM/(K11BN2+CONO8)+(K11BN2-
ICONOH8)+SNX+K11BNZ*2.*K11BN4+12*(ZM+2))
KTERM1=(CONO1*ZN2SZN*CONO7+ZN2SZN+CONO3*ZN)1+SN+SN2)/infeldm)
ICONO4
KTERM2=(ICONO5+ZN2SN+CONO6+(ZM-ZN)+CONO7-ZN*(1.-SN2))/infeldm)*
ICONO8
KPHI=KTERM1*PHIL(0)
KPS=KTERM2*PHIL(0)
KPHI=EXPPLU*(KPS+KPHI)
IF(NSTEP.EQ.0) GOTO 399
TIN=PI*10.4,T,NSP,T,TFUNC)
GO TO 999
399 TIINI=0.
$\text{PI} = 3.1415926/
\text{SLOPE } = \text{DEK}/(K1K2K2*}L1$
\text{IF(JPRINT(3),NE.0) WRITE(6,6300)}
CO999 I=1,ITERMS
N=1
\text{IF(K1K21.EQ.0.1) GO TO 801}
\text{GO TO 799}
801 \text{IF(ABS(SLOPE)+L.T.1.0) GO TO 800}
799 \text{Z0}=(Z+\text{NN}-1.0)*4(PI/2.)
\text{GO TO 890}
800 \text{Z0}=(Z+\text{NN}-1.0)*4(PI/2.)
850 \text{BOUND=0.0}
\text{GO 99 J=1,ITERMS}
U=DEK*L*Z0/(K1K21+Z0+2*K1K22*}L*2)
\text{IF(ABS(SLOPE)+L.T.1.0) GO TO 860}
Z1=(N-1.0)*PI+1STAN(U)
\text{GO TO 870}
860 Z1=N*N*(1+ATAN(U))
870 \text{IF(ABS(Z1-Z0)+L.T.LIMIT) GO TO 9}
\text{IF(BOUND.EQ.(Z1-Z0)) GO TO 90}
\text{BOUND=Z1-Z0}
\text{Z0=Z1}
\text{IF(JPRINT(3),NE.0) WRITE(6,6310) I,J,U,Z1}
99 \text{CONTINUE}
\text{WRITE(6,111) I,ITERMS}
\text{GO TO 999}
9 \text{EIGEN(J)=Z1}
\text{GO TO 999}
90 \text{EIGEN(J)=(Z1+Z0)}/.02.
999 \text{CONTINUE}
\text{RETURN}

C \text{ERROR MESSAGES}
11 \text{FORMAT} (10X,12HROOT NUMBER +13,22HOLD NOT CONVERGE AFTER,15,11H ITERATIONS)\text{END}
66 SITBFC SOLY2 294/2,XRT:
\text{SUBROUTINE SOLY2(DEK,EIGEN,ITERMS)}
C \text{SOLVES TAN(Z)=}0L+Z/(K1+K2+Z+2*K22*K12*L+2)
C \text{DIMENSION EIGEN(ITERMS)}
C DIMENSION JPRINT(3)
C DIMENSION NTAB(5)
REAL K1K21,K1K22,L,LIMIT,NN
REAL LIMIT,L,K,LAMBDAX,NUMASS,NUMSS,KFPHI,KFPHIO,KEPHI,KEPHII,K11,K12
L,K21,K22,L,KERK1,KERK2
COMMON TX,LAMBDAX,KERK1,KERK2,ALPHA,BETA,EXPXUL,CBXY,SNXY,DELTA,TAB
C \text{FORMAT} (10X,12HROOT NO.,5X,9HTESTS)\text{END}
C \text{RATIONAL SOLY2 SOLUTION FOR EIGENVALUES//26X,THROOT NO.,5X,9HTESTS}646
C \text{FORMAT} (31,NE.0) WRITE(6,6300)
CO999 I=1,ITERMS
N=1
Z0=(N+1)*PI
850 \text{BOUND=0.0}
34
AFFDL-TR-66-109

DO 99 J=1,ITERMX
  U=DET(K+L+Z0)/(K11K21+Z0**2+K12K22+L**2)
  Z1=NNP+1,ATAM(U)
  IF(ABS(Z1-Z0).LT.LIMIT) GO TO 9
  IF(BOUND.EQ.(Z1-Z0)) GO TO 99
  DOUND=Z1-Z0
  Z0=Z1
  IF(JPRINT(3).NE.0) WRITE(6,6101).I,J,U,Z1
  CONTINUE
  WRITE(6,11) I,ITERMX
  STOP
9  EIGEN(I)=Z1
  GO TO 999
99  EIGEN(I)=(Z1+Z0)/2.0
999 CONTINUE
  RETURN
  C          ERROR MESSAGES
  C
  11 FORMAT(10X,12HROOT NUMBER I3,I2HDO NOT CONVERGE AFTER I5,I1H IRTATIONS)
  END
$IBFIC SOLV3 P94/2,X77
  SUBROUTINE SOLVE3(DETK,EIGEN,ITERMX)
  C
  SOLVES TAN(Z)=0.0
  C
  DIMENSION EIGEN(ITERMX)
  DIMENSION JPRINT(3)
  DIMENSION NTERMS(5)
  REAL K11,K21,K12K22,L,LIMIT,NN
  REAL LIMIT,XX,LAMBDA,NUMSS,NUKS3,KFPI,KFPHIO,KFPHIL,XX11,XX12
  L=K22+K12,K11,KTERM1,KTERM2
  COMMON T1,X,LAMBDA,KTERM1,KTERM2,ALPHA,BETA,EXPHUL,CBXN,SENK,DELTA)
  COMMON/ROOTS/ITERMX,K11K21,K12K22,LIMIT
  COMMON/PRINTJPRINT
  6300 FORMAT(10X,12HOLVE FOR EIGENVALUES//26X,7HROOT NO.49X,1H2LSTAS473
  6320 FORMAT(10X,12HOLVE FOR 13,49X,E15.8)
  C
  IF(JPRINT(3).NE.0) WRITE(6,6300)
  999 CONTINUE
  P=TAN
  0
  C
  $IBFIC SOLV4 P94/2,X77
  SUBROUTINE SOLVE4(DETK,EIGEN,ITERMX)
  C
  SOLVES TAN(Z)= INFINITY
  C
  DIMENSION EIGEN(ITERMX)
  DIMENSION NTERMS(5)
  DIMENSION JPRINT(3)
  REAL K11K21,K12K22,L,LIMIT,NN
  REAL LIMIT,XX,LAMBDA,NUMSS,NUKS3,KFPI,KFPHIO,KFPHIL,XX11,XX12
  L=K22+K12,K11,KTERM1,KTERM2
  35
COMMON T,X,LAMBDA,KTERM1,KTERM2,ALPHA,BETA,H,EXP,MUL,D,CMP,SNX,D,DELTA,TSH497
I,F,DELTA,K11,K12,K12,K12,F,f,NTAB
COMMON/KOETS/HINTERM1,K11,K12,K12,K12,LIMIT
COMMON/PRINT/PRINT
0300 FORMAT(1H1,X6,5X,24SOLUTION FOR EIGENVALUES/26X,THROU H,NO,49X,17X)TSH500
6320 FORMAT(5X,13,4X,15.8)
CAYA P13.14536
IF (PRINT(3).NE.0) WRITE(6,6300)
C 999 I=1,INTERMS
C I=1(2,NN-1)-PI/2.0
C EIGEN1=21
C IF (PRINT(3).NE.0) WRITE(6,6320)1,1Z1
C 999 CONTINUE
C RETURN
C END

**$IBFTC SIMPS WK4/2, XM7
REAL FUNCTION FINT(A,B,NN,F)
C
C SIMPSON METHOD
A=LOWER BOUND
B=UPPER BOUND
NN=NUMBER OF INTERVALS, MUST BE EVEN
F=FUNCTION TO BE INTEGRATED, FUNCTION NAME MUST BE DECLARED
C
REAL LIMIT,T,X,LAMBDA,HUMSE,NUMS,F,KPHI,KPHID,KPHIK,K11,K12
I,K11,K12,S,KTERM1,KTERM2
COMMON T,X,LAMBDA,KTERM1,KTERM2,ALPHA,BETA,H,EXP,MUL,CBNX,SNX,D,INTERM5
XX,DELTA,K11,K12,K12,K12,F,F,x,NTAB
FX=0
IC=A/A1/FX
SUMA=0.0
SUMB=0.0
P=NN-1
C010J=1/4*P
F=J
XX=FX+H
10 SUMA=SUMA+F(XX)
XX=XX+H
SUMB=SUMB+F(XX)
20 CONTINUE
FINT=(3.0+A1*F(A)+F(B)+2.0*SUMA+2.0*SUMB)
RETURN
END

**$IBFTC FUNCX WK4/2, XM7
FUNCTION XFNCX(XX)
C
C THIS FUNCTION SETS UP THE INTEGRAND FOR THE INTEGRAL
C
C DIMENSION NTAB(5)
REAL LIMIT,T,X,LAMBDA,HUMSE,NUMS,F,KPHI,KPHID,KPHIK,K11,K12
I,K11,K12,S,KTERM1,KTERM2
COMMON T,X,LAMBDA,KTERM1,KTERM2,ALPHA,BETA,H,EXP,MUL,CBNX,SNX,D,INTERM5
XX,DELTA,K11,K12,K12,K12,F,F,x,NTAB
BNX=BETA+UX
BNX=EXP(MUL*(K12*SINHBNX)-K11*BETA+COS(SINBNX))+FX+FOFX
C

RETURN
END

*TABLE FUNCT L994/2,XR7
FUNCTION TFUNCT(LAPROA)

THIS FUNCTION SETS UP THE INTEGRAND FOR THE LAMBD A INTEGRAL

REAL X,L,K,LAPROA,NUXSS,NUYSS,KFPHI,KFPRD0,KFPHIK=K12,K12
1,K21,K22,N,KTERM,KTERM
COMMON T,X,LAPROA,KTERM,KTERM2,AEPHA,BETAN,EXPMH,CONX,SNX,DELTA
1X,DELTA,K11,K12,K21,K22,FX,NTAB
TERM1=KTERM*PHRIL(1AMD0A)
TERM2=KTERM2*PHRPRO(1AMDOA)
TFUNCT(KTERM+KTERM2)*EXP((ALPHA*(BETAN**2)0*1AMDOA-1))
RETURN
END

*TABLE FUNCT L994/2,XR7
SUBROUTINE TAIN(1)

READS IN TABULAR DATA

1 FORMAT(5210,0.55H
= )
2 FORMAT(1HA,6IX,9I(TABLE NO.,I2)
= )
3 FORMAT(1HA,50X,34H(MODE,P*33N) DEPENDENT/SIX) VARIABE
4 FORMAT(48X,15.8,10X,15.8)

REAL INVAR
CINSSXH:NATAB(5)
COMMON/PDATAB/INVAR,NEVAR
READ(5,I) MABLE(1)
WRITE(6,2) KTABLE(1)
WRITE(6,3) DO 150 J=1,50
READ(5,L) INVAR(I,J),DEPVAR(I,J)
WRITE(6,4) TABLE(1)
150 CONTINUE
WRITE(6,5) TABLE(1)
100 RETURN
50 WRITE(6,6)
STOP

ERROR MESSAGES

5 FORMAT(1HA,6IX,9I(TABLE NO.,I2-25H CONTAINS MORE THAN 50 POINTS)
6 FORMAT(1HA,48X,27HERROR IN TABULAR INPUT DATA)
END

*TABLE ENTERP L994/2,XR7

REAL FUNCTION INTERP(TX,1)

LINEAR INTERPOLATION FOR TABULAR DATA

CINSSXH:NATAB(5)
DIMENSION INVAR(5,501),DEPVAR(5,501)
COMMON/PDATAB/INVAR,NEVAR
REAL INVAR
CO 10 J=1,50

37
IF (TX-INDVAR(I,J)) .GT. 20,30,10
    CONTINUE
    RETURN
10  WHILE (I,J) 1
    CALL EXIT
20  INTERP = DEPVAR(I,J-1) + (DEPVAR(I,J) - DEPVAR(I,J-1)) * (TX-INDVAR(I,J-1))
    GO TO 100
30  INTERP = DEPVAR(I,J)
100  RETURN
C
ERROR MESSAGES
C
99  FORMATION, MARGUMENT EXCEEDS EXTENT OF TABLE NO.,12)
   RETURN
C
$IBFC PHD T 94/2,XX7
FUNCTION PHD(T)
C
THIS FUNCTION CALCULATES THE INSTANTANEOUS VALUE OF THE
TIME-VARYING BOUNDARY CONDITION AT X=0.
C
THE FUNCTION PHD(T) MAY BE LOADED INTO THE PROGRAM
C
AS AN ANALYTICAL EXPRESSION OR AS POINT DATA IN
C
TABULAR FORM.
C
REAL INTERP
REAL LIMIT, K, LAMBDA, NUMSS, KFPHI, KFPHIO, KFPHIL, KII, K12, K11
COMMON T, X, LAMBDA, KTER, KTERM2, ALPH, BETAN, EXPML, CBNS, SBNS, DELTAS
DIMENSION NTAB(5)
C
IF (NTAB(1) .NE. 0) GO TO 100
C
PHIO = ANY FUNCTION OF TIME
C
PHIO = 1.0
C
100  PHIO = INTERP(T,1)
      RETURN
   END
C
$IBFC PHIL 94/2,XX7
FUNCTION PHIL(T)
C
THIS FUNCTION CALCULATES THE INSTANTANEOUS VALUE OF THE
TIME-VARYING BOUNDARY CONDITION AT X=L.
C
THE FUNCTION PHIL(T) MAY BE LOADED INTO THE PROGRAM
C
AS AN ANALYTICAL EXPRESSION OR AS POINT DATA IN
C
TABULAR FORM.
C
REAL INTERP
REAL LIMIT, L, LAMBDA, NUMSS, KFPHI, KFPHIO, KFPHIL, KII, K12, K11
COMMON T, X, LAMBDA, KTER, KTERM2, ALPH, BETAN, EXPML, CBNS, SBNS, DELTAS
DIMENSION NTAB(5)
C
IF (NTAB(2) .NE. 0) GO TO 100
C
PHI = ANY FUNCTION OF
C
PHI = 1.
C
100  PHI = INTERP(T,2)
      RETURN
   END
RETURN
FND

*DFIC DERIV0 P94/2, XR7
FUNCTION PHIPG(LAMBDA)
C
THIS FUNCTION CALCULATES THE INSTANTANEOUS VALUE OF THE
DERIVATIVE OF THE TIME-VARYING BOUNDARY CONDITION AT X=0.
C
THE FUNCTION MAY BE LOADED ANALYTICALLY OR AS POINT
DATA IN TABULAR FORM.
C
REAL INTERP
REAL LIMIT, L, LAMBDA, NUXSS, NUX55, KFPHI, KFPHIO, KFPHIL, K11, K12
I, K21, K22, N, KTERM1, KTERM2
COMMON T, X, LAMBDA, KTERM1, KTERM2, ALPHAB, BETAN, EXPMUL, CBNX, SBNX, DLTASTASH69
C
C
RETURN

100 PHIPG=INTERP(LAMBDA, 3)
RETURN
END

*DFIC DERIV0 P94/2, XR7
FUNCTION ALPR(LAMBDA)
C
THIS FUNCTION CALCULATES THE INSTANTANEOUS VALUE OF THE
DERIVATIVE OF THE TIME-VARYING BOUNDARY CONDITIONS AT X=L.
C
THE FUNCTION MAY BE LOADED ANALYTICALLY OR AS POINT DATA
IN TABULAR FORM.
C
REAL INTERP
REAL LIMIT, L, LAMBDA, NUXSS, NUX55, KFPHI, KFPHIO, KFPHIL, K11, K12
I, K21, K22, N, KTERM1, KTERM2
COMMON T, X, LAMBDA, KTERM1, KTERM2, ALPHAB, BETAN, EXPMUL, CBNX, SBNX, DLTASTASH69
C
C
RETURN

100 ALPR=INTERP(LAMBDA, 4)
RETURN
END

*DFIC FX - P94/2, XR7
FUNCTION CX( )
C
THIS FUNCTION COMPUTES THE INITIAL CONDITIONS OF THE PROG.
C
THESE INITIAL CONDITIONS MAY BE LOADED INTO THE PROGRAM
ANALYTICALLY OR AS POINT DATA IN TABULAR FORM.
C
REAL INTERP
REAL LIMIT, L, LAMBDA, NUXSS, NUX55, KFPHI, KFPHIO, KFPHIL, K11, K12
I, K21, K22, N, KTERM1, KTERM2
COMMON T, X, LAMBDA, KTERM1, KTERM2, ALPHAB, BETAN, EXPMUL, CBNX, SBNX, DLTASTASH69
XI.CEIAT,K11,K12,K21,K22,FX,XIAB
C DIMENSION TABS(5)
IF(TABS(N).NE.0) G0 TO 100
C
F0FX= ANY FUNCTION OF X
C
F0FX=1.
55 RETURN
100 F0FX=INTERP(xx,5)
RETURN
END

STASH45
STASH46
STASH47
STASH48
STASH49
STASH50
STASH51
STASH52
STASH53
STASH54
STASH55
APPENDIX II

EIGENVALUE SUBROUTINES
APPENDIX II
EIGENVALUE SUBROUTINES

The solution of equation 53 depends on values of $\beta_n$ and which are derived from the positive eigenvalues of Equation 40.

$$\tan z_n = \frac{D L Z_n}{K_{21} K_{11} Z_n^2 + K_{22} K_{12} L^2}$$

(40)

Since we are seeking positive values of $z_n$, the sign of the left-hand side of the equation may be associated with the parameter, $D$. Thus three formulations are possible corresponding to $D$ being positive, negative or zero. A fourth possibility is that of the denominator going to zero. The last two solutions are trivial. If $D$ is zero we have

$$\tan z_n = 0$$

The solution to this equation is merely

$$z_n = n \pi$$

(54)

If the denominator of equation 40 goes to zero we have

$$\tan z_n = \infty$$

The solution to the equation is

$$z_n = \frac{(2n - 1) \pi}{2}$$

(55)

However, if $D$ has a value other than zero the equations are solved by an iterative process. In these cases the eigenvalue subroutine has been programmed to ignore the root at $(0,0)$ since this produces a trivial solution. The procedure then is outlined below for a positive value of $D$.

We know that the solution lies between

$$\left[ (n-1) \pi, \frac{(2n-1) \pi}{2} \right], \quad n = 1, 2, 3, \cdots$$

For a first approximation to the root, $z_{n,0}$, we shall choose

$$z_{n,0} = \frac{(2n - 1) \pi}{2}$$

(56)

We then write two equations

$$u_{n,m} = \frac{D L Z_{n,m-1}}{K_{21} K_{11} Z_{n,m-1}^2 + K_{22} K_{12} L^2}$$

(57)

$$z_{n,m} = (n - 1) \pi + \tan^{-1} (u_{n,m})$$

(58)

Where the subscripts $n$ and $m$ refer to the $m$th iteration toward the $n$th root. The root is then determined to any desired LIMIT of accuracy by writing
The basis is made in the program to print out the iteration steps should any trouble occur. The basic difference in the solution for a negative value of D arises in the first approximation, \( z_{n,0} \). For negative values of D Equation 56 becomes

\[
Z_{n,m} - Z_{n,m-1} < \text{LIMIT}
\]  

Equation 59

Provision is made in the program to print out the iteration steps should any trouble occur. The basic difference in the solution for a negative value of D arises in the first approximation, \( z_{n,0} \). For negative values of D Equation 56 becomes

\[
Z_{n,0} = (n + 1) \pi
\]

Equation 60

and the solution proceeds as before with Equation 58 becoming

\[
Z_{n,m} = n \pi + \tan^{-1}(u_{n,m})
\]

Equation 61
APPENDIX III

RESULTS OF CHECK PROBLEMS
## TABLE II

**COMPARISON OF DATA FOR CASE I**

<table>
<thead>
<tr>
<th>L</th>
<th>TEMP @ t=0</th>
<th>TEMP @ t=50</th>
<th>TEMP @ t=500</th>
<th>TEMP @ t=1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STASH</td>
<td>LTA</td>
<td>STASH</td>
<td>LTA</td>
</tr>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>50.00</td>
<td>500.00</td>
<td>1000.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00</td>
<td>37.39</td>
<td>455.95</td>
<td>930.71</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00</td>
<td>27.45</td>
<td>414.85</td>
<td>586.38</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
<td>19.79</td>
<td>376.56</td>
<td>300.87</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00</td>
<td>13.99</td>
<td>340.92</td>
<td>207.04</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00</td>
<td>9.76</td>
<td>307.74</td>
<td>740.04</td>
</tr>
<tr>
<td>3.0</td>
<td>0.00</td>
<td>6.55</td>
<td>276.88</td>
<td>681.66</td>
</tr>
<tr>
<td>3.5</td>
<td>0.00</td>
<td>4.37</td>
<td>248.18</td>
<td>571.82</td>
</tr>
<tr>
<td>4.0</td>
<td>0.00</td>
<td>2.84</td>
<td>221.48</td>
<td>20.03</td>
</tr>
<tr>
<td>4.5</td>
<td>0.00</td>
<td>1.80</td>
<td>196.61</td>
<td>470.11</td>
</tr>
<tr>
<td>5.0</td>
<td>0.00</td>
<td>1.12</td>
<td>173.43</td>
<td>421.90</td>
</tr>
<tr>
<td>5.5</td>
<td>0.68</td>
<td>0.68</td>
<td>151.77</td>
<td>375.26</td>
</tr>
<tr>
<td>6.0</td>
<td>0.40</td>
<td>0.41</td>
<td>131.43</td>
<td>330.03</td>
</tr>
<tr>
<td>6.5</td>
<td>0.23</td>
<td>0.24</td>
<td>112.26</td>
<td>286.04</td>
</tr>
<tr>
<td>7.0</td>
<td>0.13</td>
<td>0.13</td>
<td>94.38</td>
<td>243.15</td>
</tr>
<tr>
<td>7.5</td>
<td>0.07</td>
<td>0.07</td>
<td>77.27</td>
<td>201.19</td>
</tr>
<tr>
<td>8.0</td>
<td>0.04</td>
<td>0.04</td>
<td>60.91</td>
<td>160.02</td>
</tr>
<tr>
<td>8.5</td>
<td>0.02</td>
<td>0.02</td>
<td>45.16</td>
<td>119.47</td>
</tr>
<tr>
<td>9.0</td>
<td>0.01</td>
<td>0.01</td>
<td>29.86</td>
<td>79.48</td>
</tr>
<tr>
<td>9.5</td>
<td>0.00</td>
<td>0.00</td>
<td>14.85</td>
<td>39.61</td>
</tr>
<tr>
<td>10.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
TABLE III
COMPARISON OF DATA FOR CASE 2

<table>
<thead>
<tr>
<th>L</th>
<th>TEMP @ t=0</th>
<th>TEMP @ t=50</th>
<th>TEMP @ t=500</th>
<th>TEMP @ t=1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>100.00</td>
<td>150.00</td>
<td>600.00</td>
<td>1100.00</td>
</tr>
<tr>
<td>0.5</td>
<td>100.00</td>
<td>137.38</td>
<td>555.95</td>
<td>1030.71</td>
</tr>
<tr>
<td>1.0</td>
<td>127.45</td>
<td>127.46</td>
<td>514.85</td>
<td>964.38</td>
</tr>
<tr>
<td>1.5</td>
<td>119.79</td>
<td>119.80</td>
<td>476.56</td>
<td>900.87</td>
</tr>
<tr>
<td>2.0</td>
<td>113.99</td>
<td>114.00</td>
<td>440.92</td>
<td>840.02</td>
</tr>
<tr>
<td>2.5</td>
<td>109.70</td>
<td>109.70</td>
<td>407.74</td>
<td>781.66</td>
</tr>
<tr>
<td>3.0</td>
<td>106.58</td>
<td>106.59</td>
<td>376.88</td>
<td>725.55</td>
</tr>
<tr>
<td>3.5</td>
<td>104.37</td>
<td>104.38</td>
<td>348.18</td>
<td>671.82</td>
</tr>
<tr>
<td>4.0</td>
<td>102.84</td>
<td>102.85</td>
<td>321.48</td>
<td>620.03</td>
</tr>
<tr>
<td>4.5</td>
<td>101.80</td>
<td>101.81</td>
<td>296.61</td>
<td>570.11</td>
</tr>
<tr>
<td>5.0</td>
<td>101.11</td>
<td>101.13</td>
<td>273.43</td>
<td>521.40</td>
</tr>
<tr>
<td>5.5</td>
<td>100.68</td>
<td>100.68</td>
<td>251.77</td>
<td>475.76</td>
</tr>
<tr>
<td>6.0</td>
<td>100.40</td>
<td>100.40</td>
<td>231.48</td>
<td>430.03</td>
</tr>
<tr>
<td>6.5</td>
<td>100.23</td>
<td>100.24</td>
<td>212.40</td>
<td>386.04</td>
</tr>
<tr>
<td>7.0</td>
<td>100.13</td>
<td>100.13</td>
<td>194.38</td>
<td>343.15</td>
</tr>
<tr>
<td>7.5</td>
<td>100.07</td>
<td>100.07</td>
<td>177.27</td>
<td>301.19</td>
</tr>
<tr>
<td>8.0</td>
<td>100.04</td>
<td>100.04</td>
<td>160.91</td>
<td>260.02</td>
</tr>
<tr>
<td>8.5</td>
<td>100.02</td>
<td>100.02</td>
<td>145.14</td>
<td>219.47</td>
</tr>
<tr>
<td>9.0</td>
<td>100.01</td>
<td>100.01</td>
<td>129.84</td>
<td>179.37</td>
</tr>
<tr>
<td>9.5</td>
<td>100.00</td>
<td>100.00</td>
<td>114.85</td>
<td>139.60</td>
</tr>
<tr>
<td>10.0</td>
<td>100.60</td>
<td>100.00</td>
<td>109.00</td>
<td>109.00</td>
</tr>
</tbody>
</table>
### TABLE IV
Comparison of data for case 5

<table>
<thead>
<tr>
<th>L</th>
<th>TEMP @ t=0</th>
<th>TEMP @ t=50</th>
<th>TEMP @ t=500</th>
<th>TEMP @ t=1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STASH</td>
<td>LTA</td>
<td>STASH</td>
<td>LTA</td>
</tr>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>50.00</td>
<td>50.00</td>
<td>500.00</td>
</tr>
<tr>
<td>0.5</td>
<td>5.00</td>
<td>42.38</td>
<td>43.38</td>
<td>460.95</td>
</tr>
<tr>
<td>1.0</td>
<td>10.00</td>
<td>37.46</td>
<td>37.46</td>
<td>434.85</td>
</tr>
<tr>
<td>1.5</td>
<td>15.00</td>
<td>34.79</td>
<td>34.89</td>
<td>391.56</td>
</tr>
<tr>
<td>2.0</td>
<td>20.00</td>
<td>33.99</td>
<td>34.00</td>
<td>360.92</td>
</tr>
<tr>
<td>2.5</td>
<td>25.00</td>
<td>34.70</td>
<td>34.70</td>
<td>32.74</td>
</tr>
<tr>
<td>3.0</td>
<td>30.00</td>
<td>36.58</td>
<td>36.59</td>
<td>306.88</td>
</tr>
<tr>
<td>3.5</td>
<td>35.00</td>
<td>39.37</td>
<td>39.38</td>
<td>283.18</td>
</tr>
<tr>
<td>4.0</td>
<td>40.00</td>
<td>42.84</td>
<td>42.85</td>
<td>261.48</td>
</tr>
<tr>
<td>4.5</td>
<td>45.00</td>
<td>46.80</td>
<td>46.81</td>
<td>241.61</td>
</tr>
<tr>
<td>5.0</td>
<td>50.00</td>
<td>51.11</td>
<td>51.13</td>
<td>223.43</td>
</tr>
<tr>
<td>5.5</td>
<td>55.00</td>
<td>55.68</td>
<td>55.68</td>
<td>206.77</td>
</tr>
<tr>
<td>6.0</td>
<td>60.00</td>
<td>60.40</td>
<td>60.41</td>
<td>191.48</td>
</tr>
<tr>
<td>6.5</td>
<td>65.00</td>
<td>65.23</td>
<td>65.24</td>
<td>177.40</td>
</tr>
<tr>
<td>7.0</td>
<td>70.00</td>
<td>70.13</td>
<td>70.13</td>
<td>164.38</td>
</tr>
<tr>
<td>7.5</td>
<td>75.00</td>
<td>75.07</td>
<td>75.07</td>
<td>152.27</td>
</tr>
<tr>
<td>8.0</td>
<td>80.00</td>
<td>80.04</td>
<td>80.04</td>
<td>140.91</td>
</tr>
<tr>
<td>8.5</td>
<td>85.00</td>
<td>85.02</td>
<td>85.02</td>
<td>130.16</td>
</tr>
<tr>
<td>9.0</td>
<td>90.00</td>
<td>90.01</td>
<td>90.01</td>
<td>119.86</td>
</tr>
<tr>
<td>9.5</td>
<td>95.00</td>
<td>95.00</td>
<td>95.00</td>
<td>109.85</td>
</tr>
<tr>
<td>10.0</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

48
<table>
<thead>
<tr>
<th>L</th>
<th>TEMP @ t=0</th>
<th>TEMP @ t=50</th>
<th>TEMP @ t=100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>0.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>1.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>1.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>2.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>2.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>3.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>3.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>4.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>4.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>5.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>5.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>6.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>6.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>7.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>7.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>8.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>8.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>9.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>9.5</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>10.0</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
</tbody>
</table>
### TABLE V

**Comparison of Data for Case 4**

<table>
<thead>
<tr>
<th>L</th>
<th>Temp @ t=0</th>
<th>Temp @ t=300</th>
<th>Temp @ t=1800</th>
<th>Temp @ t=3600</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STASH</td>
<td>LTA</td>
<td>STASH</td>
<td>LTA</td>
</tr>
<tr>
<td>0.0</td>
<td>100.00</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
</tr>
<tr>
<td>2.0</td>
<td>899.95</td>
<td>899.90</td>
<td>994.85</td>
<td>994.40</td>
</tr>
<tr>
<td>3.0</td>
<td>710.26</td>
<td>709.99</td>
<td>985.06</td>
<td>984.72</td>
</tr>
<tr>
<td>4.0</td>
<td>625.68</td>
<td>625.30</td>
<td>980.06</td>
<td>978.98</td>
</tr>
<tr>
<td>5.0</td>
<td>550.27</td>
<td>550.05</td>
<td>976.72</td>
<td>976.13</td>
</tr>
<tr>
<td>6.0</td>
<td>486.38</td>
<td>486.14</td>
<td>973.37</td>
<td>972.66</td>
</tr>
<tr>
<td>7.0</td>
<td>435.25</td>
<td>434.97</td>
<td>970.67</td>
<td>970.04</td>
</tr>
<tr>
<td>8.0</td>
<td>397.98</td>
<td>397.52</td>
<td>968.08</td>
<td>967.10</td>
</tr>
<tr>
<td>9.0</td>
<td>375.51</td>
<td>375.02</td>
<td>967.49</td>
<td>966.99</td>
</tr>
<tr>
<td>10.0</td>
<td>367.70</td>
<td>367.56</td>
<td>967.08</td>
<td>966.99</td>
</tr>
</tbody>
</table>

### TABLE VI

**Comparison of Data for Case 5**

<table>
<thead>
<tr>
<th>L</th>
<th>Temp @ t=0</th>
<th>Temp @ t=300</th>
<th>Temp @ t=1800</th>
<th>Temp @ t=3600</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STASH</td>
<td>LTA</td>
<td>STASH</td>
<td>LTA</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>129.76</td>
<td>129.08</td>
<td>416.41</td>
<td>416.34</td>
</tr>
<tr>
<td>2.0</td>
<td>266.86</td>
<td>265.45</td>
<td>833.21</td>
<td>833.08</td>
</tr>
<tr>
<td>3.0</td>
<td>418.26</td>
<td>416.26</td>
<td>1250.77</td>
<td>1250.58</td>
</tr>
<tr>
<td>4.0</td>
<td>590.98</td>
<td>588.36</td>
<td>1669.46</td>
<td>1669.22</td>
</tr>
<tr>
<td>5.0</td>
<td>791.36</td>
<td>788.17</td>
<td>2089.65</td>
<td>2089.92</td>
</tr>
<tr>
<td>6.0</td>
<td>1025.20</td>
<td>1021.51</td>
<td>2511.51</td>
<td>2511.17</td>
</tr>
<tr>
<td>7.0</td>
<td>1297.34</td>
<td>1293.43</td>
<td>2934.41</td>
<td>2934.04</td>
</tr>
<tr>
<td>8.0</td>
<td>1612.46</td>
<td>1608.04</td>
<td>3351.52</td>
<td>3351.12</td>
</tr>
<tr>
<td>9.0</td>
<td>1973.83</td>
<td>1968.31</td>
<td>3789.29</td>
<td>3789.57</td>
</tr>
<tr>
<td>10.0</td>
<td>2381.11</td>
<td>2376.43</td>
<td>4220.90</td>
<td>4220.48</td>
</tr>
</tbody>
</table>

|    | STASH      | LTA           |
|    | 431.70    | 431.69        |
|    | 863.41    | 863.39        |
|    | 1299.14   | 1299.12       |
|    | 1726.81   | 1726.28       |
|    | 2158.72   | 2158.66       |
|    | 2539.54   | 2539.54       |
|    | 3022.45   | 3022.45       |
|    | 3654.47   | 3654.47       |
|    | 3856.47   | 3856.47       |
Figure 8. Temperature Profiles (Case 1)
Figure 9. Temperature Profiles (Case 2)
Figure 10. Temperature Profiles (Case 3)
Figure 11. Temperature Profiles (Case 4)
Figure 12. Temperature Profiles (Case 5)
TRANIENT ANALYSIS OF HEAT CONDUCTION THROUGH A SLAB BY INFINITE SERIES

The exact solution to the problem of conduction of heat through a slab is developed. The solution, formulated in terms of an infinite series, allows arbitrary initial conditions and time-dependent boundary conditions. The solution is programmed in FORTRAN IV for the IBM 7094 II computer. Several check problems were solved and the results were compared with those obtained from a finite difference heat transfer program.
### Thermal Analysis
- Mathematical Modeling
- Digital Computer Analysis
- Arbitrary Boundary Conditions
- Infinite Series
- Computer Programming
- Case Equations
- Transient
- Conduction
- FORTRAN

### Unclassified

**INSTRUCTIONS**

1. **ORIGINATING ACTIVITY**: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2. **REPORT SECURITY CLASSIFICATION**: Enter the overall security classification of the report. Indicate whether the document is to be in accordance with appropriate security regulations.

3. **GROUP**: Indicate the group number. Enter the group number. Also, enter the group number. Where applicable, show that original manuscripts have been used for Group 3 and Group 4 as authorized.

4. **REPORT TITLE**: Enter the complete report title in capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capital letters immediately following the title.

5. **DESIGNATION**: Enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

6. **AUTHORS**: Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If necessary, show rank and branch of service. The name of a principal author should be an absolute minimum requirement.

7. **REVIEW DATE**: Enter the date of the review by the person, by whom, to whom the report was submitted.

8. **TOTAL NUMBER OF PAGES**: The total page count should include all pages of the report, not including covers, tables, figures, appendices, etc.

9. **NUMBER OF REFERENCES**: Enter the total number of references cited in the report.

10. **CONTRACT OR GRANT NUMBER**: If applicable, enter the number of the contract or grant under which the report was written.

11. **PROJECT NUMBER**: Enter the project number on which the report was written.

12. **REPORT NUMBER**: Enter the official report number which the document bears. It should be identified and controlled by the originating activity. It is a number not to be duplicated in this report.

13. **OTHER REPORT NUMBERS**: If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

14. **AVAILABILITY/LIMITATION NOTICES**: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

   - (1) "Qualified requesters may obtain copies of this report from DDC."
   - (2) "Fees for reproduction and dissemination of this report by DDC are authorized."
   - (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through...
   - (4) "Other qualified users shall request through...
   - (5) "All distribution of this report is controlled. Qualified DDC users shall request through...

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

**SUPPLEMENTARY NOTES**: Use for additional explanatory notes.

**SPONSORING MILITARY ACTIVITY**: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include all values.

**ABSTRACT**: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere as the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the security classification of the information in that paragraph, represented as (U), (T), or (S).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

**KEY WORDS**: Key words are typically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment, model designations, issue, name, military project code name, geographic locations, may be used as key words but will be followed by an indication of technical context. The assignment of topic, index, and subject index is optional.