<table>
<thead>
<tr>
<th>TO:</th>
<th>Approved for public release; distribution is unlimited.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FROM:</td>
<td>Distribution authorized to U.S. Gov't. agencies and their contractors; Critical Technology; DEC 1966. Other requests shall be referred to Space Systems Division, ATTN: SSTRT, Los Angeles AFB, CA 90009-2960. This document contains export-controlled technical data.</td>
</tr>
<tr>
<td>AUTHORITY</td>
<td>SAMSO ltr dtd 24 Jan 1972</td>
</tr>
</tbody>
</table>
DYNAMIC PERFORMANCE OF A SUBLIMING SOLID REACTION JET

Prepared by
H. Greer
Applied Mechanics Division
El Segundo Technical Operations

and

D. J. Griep
Electronics Division
El Segundo Technical Operations

Laboratory Operations
AEROSPACE CORPORATION
El Segundo, California

December 1966

Prepared for
BALLISTIC SYSTEMS AND SPACE SYSTEMS DIVISIONS
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California
FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-1001 and documents research carried out from March 1965 through May 1966. This report was forwarded to Lt J. B. Beaver, SSTRT, on 17 November 1966 for review and approval.

The authors would like to acknowledge Mr. F. Cox for his assistance and contributions regarding the sea level and vacuum experiments conducted in the Aerospace Guidance and Control Laboratory.

Information in this report is embargoed under the U. S. Export Control Act of 1949, administered by the Department of Commerce. This report may be released by departments or agencies of the U. S. Government to departments or agencies of foreign governments with which the United States has defense treaty commitments. Private individuals or firms must comply with Department of Commerce export control regulations.

Approved

W. F. Radcliffe, Director
Engineering Sciences Subdivision
Applied Mechanics Division

E. Levin, Director
Guidance and Control Subdivision
Electronics Division

D. D. King, Director
Electronics Research Laboratory
Laboratories Division

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Approved

John B. Beaver, Lt USAF
Program Element Coordinator for Aerospace Propulsion
ABSTRACT

The pulsed propulsive performance of a low-thrust subliming-solid ammonium carbamate reaction jet is analyzed and compared with the results of laboratory experiments. Tests of the 15 millipound thruster were conducted at sea level using a 1.68 to 1 expansion ratio nozzle and at vacuum using a 48 to 1 expansion ratio nozzle. The transient processes, which dominate the short-pulse or limit-cycle mode of thruster operation, are formulated and show good correlation with the data. The apparatus, procedures, and techniques required to obtain accurate test results for a low-thrust, dynamic mode of operation are described. Impulse bit size, gas consumption, and specific impulse are characterized in terms of thruster geometry, gas properties, and command pulse width to provide a systematic basis for design. Limitations and design criteria necessary for successful spacecraft installation are discussed. A comparison is made between the performance of the subliming solid reaction jet system and the performance of other propellants.
FIGURES

1. Schematic of Experimental Apparatus .......................... 3
2. Equilibrium Vapor Pressure Subliming Ammonium Carbamate .......................... 6
3. Typical Pulse Pressure History ........................................ 8
4. Chamber Pressure Rise Transients .................................... 10
5. Chamber Pressure Decay Transients .................................... 13
6. Dynamic Impulse Bit Correlation ...................................... 14
7. Dynamic Gas Consumption Correlation ................................. 15
8. Pulsed Specific Impulse Correlation ................................... 18
9. Thrust Stand Assembly .................................................. 23
10. Correlation of Thrust and Chamber Pressure .......................... 25
11. Schematic of Command and Computer System .......................... 26
12. Impulse Bit Size Distribution .......................................... 28
NOMENCLATURE

\[ A \quad = \quad \text{area} \]
\[ a_1 \quad = \quad A_o a_1^{1/2} \frac{(2/\gamma)^{1/2}}{2V_c} \]
\[ a_2 \quad = \quad (A_t a_1^{1/2} V_c) [2/(\gamma + 1)]^{(\gamma + 1)/2(\gamma - 1)} \]
\[ a^* \quad = \quad \text{acoustic velocity} \]
\[ B \quad = \quad \text{equivalent orifice-to-nozzle area ratio} = a_1/a_2 \]
\[ \frac{A_o}{A_t} \left( \frac{2}{\gamma} \right)^{1/2} \left( \frac{\gamma + 1}{2} \right)^{\gamma+1} \left( \frac{\gamma - 1}{2(\gamma - 1)} \right) \]
\[ C_d \quad = \quad \text{nozzle discharge coefficient} \]
\[ C_f \quad = \quad \text{thrust coefficient} \]
\[ C_t \quad = \quad \text{thrust correlation factor} \]
\[ C_v \quad = \quad \text{exhaust velocity coefficient} \]
\[ D \quad = \quad \text{diameter} \]
\[ F \quad = \quad \text{thrust level} \]
\[ g \quad = \quad \text{proportionality constant in Newton's second law} \]
\[ h \quad = \quad \text{heat of sublimation} \]
\[ I \quad = \quad \text{specific impulse} \]
\[ I_{\text{eff}} \quad = \quad \text{effective system specific impulse} \]
\[ I_{\text{tot}} \quad = \quad \text{total impulse} \]
\[ K_d \quad = \quad \text{empirical decay time integration factor} \]
\[ K_n \quad = \quad \text{choked nozzle flow factor} = a^* \left[ \frac{2}{(\gamma + 1)} \right]^{(\gamma + 1)/2(\gamma - 1)}_{\text{RT}} \]
\[ K_r \quad = \quad \text{empirical rise time integration factor} \]
\[ M = \text{Mach number} \]
\[ m = \text{molecular weight} \]
\[ N_r = \text{Reynolds number} \]
\[ P = \text{pressure} \]
\[ R = \text{universal gas constant} \]
\[ T = \text{temperature} \]
\[ V = \text{volume} \]
\[ v = \text{specific volume} \]
\[ W = \text{weight flow rate} \]
\[ w = \text{weight} \]
\[ \gamma = \text{pressure ratio transformation} = (1 - P_c/P_g)^{1/2} \]
\[ \gamma_o = (1 - P_{a}/P_g)^{1/2} \]
\[ a = \text{sublimation coefficient = ratio of actual to maximum sublimation rates} \]
\[ \gamma = \text{ratio of specific heats} \]
\[ e = \text{nozzle expansion ratio} \]
\[ \eta = \text{efficiency} \]
\[ \theta = \text{time} \]
\[ \xi = T_c/T_r \]
\[ \rho = \text{density} \]
\[ \sigma = \text{working stress} \]
\[ \tau = \text{dimensionless time factor} = a \xi \theta \]
SUBSCRIPTS:

\( a \) = ambient
\( c \) = chamber or command
\( d \) = decay
\( e \) = nozzle exit
\( f \) = final
\( i \) = initial
\( n \) = nozzle
\( o \) = orifice
\( p \) = subliming solid propellant
\( r \) = reservoir or rise
\( s \) = steady state
\( t \) = throat
I. INTRODUCTION

There are numerous applications for thrust systems in the $10^{-6}$ to $10^{-1}$ lb range in satellite missions. Previous operational systems have utilized cold gas as the propellant. Cold gas systems, although fully developed and operational, have weight and reliability disadvantages associated with the necessity of storing a low density gas at high pressure.

In a subliming solid reaction jet propulsion system, the propellant is stored as a solid in a thin wall tank. The pressure in the tank is regulated by controlling heat input to the subliming solid. The remainder of the system typically consists of a filter, a propellant valve, and a nozzle. Thrust is produced by opening the valve so that the subliming vapor flows through the nozzle. Heat of sublimation is supplied to the solid to replenish the vapor which is used. Since combustion and high pressures are not involved in the use of the subliming solid, no reaction chambers, pressure regulators, or high pressure storage tanks are required. Although thrusts ranging from $10^{-6}$ to $10^{-1}$ lb may be produced by subliming solid propulsion systems, the input heat required for sublimination is a limiting factor at the higher thrust levels.

The pulsed or propulsion performance of low thrust reaction jets using various cold gases has been previously reported (Refs. 1-4). Although steady-state performance of subliming solid reaction jets was reported (Ref. 5), an attempt to determine pulsed performance was unsuccessful (Ref. 6) (due to corrosion and leakage of the sublimation chamber); thus no information on the dynamic performance of a subliming solid reaction jet is presently available.

In this document, performance and design relationships for a low thrust subliming solid jet reaction system are analytically derived and are correlated with the results of sea level and vacuum experiments conducted in the

---

18 July 1966
Aerospace Guidance and Control Laboratory using a 15 mlb thruster (shown schematically in Fig. 1). From these relationships a simplified means of calculating dynamic impulse bit, gas consumption, and specific impulse is developed. Limitations and design criteria necessary for successful spacecraft installation are discussed.
Fig. 1. Schematic of Experimental Apparatus

A. THRUST STAND (DETAIL 1)
B. SUBLIMATION CHAMBER
C. QUICK DISCONNECT
D. PRESSURE GAGE
E. ENVIRONMENTAL OVEN
F. THERMOCOUPLE
G. SOLENOID VALVE
H. PRESSURE TRANSDUCER
J. LVDT POSITION TRANSDUCER (THRUST PICKUP)
K. THRUST CHAMBER NOZZLE
L. DAMPING FLUID
M. CANTILEVER BEAM AND PNEUMATIC FEED

DETAIL 1, THRUST STAND
II. ANALYTICAL APPROACH

A. GENERAL DISCUSSION

A good subliming solid propellant should produce low molecular weight products, provide adequate pressure for thrusting at reasonable temperatures, and should generally have a low heat of sublimation. Other investigations (Refs. 5-8) have found that subliming ammonium salts appear attractive for propulsion applications. In the present study ammonium carbamate, \( \text{NH}_2\text{CO}_2\text{NH}_4 \), was selected as a typical subliming dissociating solid for analysis and testing. It is assumed that the dissociation process proceeds fully as

\[
\text{NH}_4\text{CO}_2\text{NH}_2 + h \rightarrow 2\text{NH}_3 + \text{CO}_2
\]  

requiring a heat of sublimation \( h \) (Ref. 7) of 877 Btu/lb. The molecular weight \( m \) of the products is 26 and the ratio of specific heats \( \gamma \) is 1.31.

An experimental curve is presented of the steady-state vapor pressure \( P_{ps} \) produced by subliming dissociating ammonium carbamate as a function of the temperature of the gas in the sublimation chamber (see Fig. 2). Good agreement was obtained between the data from Ref. 5 and the data measured in the Aerospace Guidance and Control Laboratory. It should be noted that about 30 minutes were required for the vapor pressure to stabilize at each test temperature shown. This equilibrium pressure time lag is kinetically determined by bond breakage occurring at the surface of the solid crystals. The sublimation kinetics may be characterized by a sublimation coefficient \( a \) which is defined (Ref. 5) as the ratio of the actual rate of sublimation to the maximum equilibrium sublimation rate per unit of solid surface area at a specified temperature. It is necessary to know both \( P_{ps} \) and \( a \) in
Fig. 2. Equilibrium Vapor Pressure Subliming Ammonium Carbamate
order to size the total propellant surface area \( A_p \) so that the design thrust level is not kinetically limited.  

**B. PRESSURE TRANSIENTS**

The reaction jet performance is based on a dynamical analysis of the thrust chamber pressure history. In general, the chamber pressure history is composed of steady-state and transient (startup and shutdown) phases illustrated in Fig. 3. In the following analysis, it is shown that the transient phases cause performance nonlinearities. Design relationships for predicting and minimizing these nonlinearities are developed.

During startup, the flow rate through the solenoid supply valve orifice \( W_o \) exceeds that through the discharge nozzle \( W_n \), causing the chamber pressure to build up. The expression reported (Ref. 4) for the compressible isentropic pressure buildup can be extended to include the nonadiabatic case where the thrust chamber temperature \( T_c \) does not equal the reservoir temperature \( T_r \). If \( \xi = T_c / T_r \), then the compressible expression for the nonadiabatic pressure buildup is

\[
\frac{dP_c}{d\theta} = 2a_1 \xi \left( \frac{\gamma}{\gamma - 1} \right)^{1/2} \left[ \left( \frac{P_c}{P_r} \right)^{2/\gamma} - \left( \frac{P_s}{P_r} \right)^{(\gamma+1)/\gamma} \right]^{1/2} - 2a_2 \xi^{1/2} P_c \tag{1}
\]

For steady-state operation, \( dP_c/d\theta = 0 \), and \( P_c/P_r = P_s/P_r < 1 \). This ratio is a design variable and is determined by the selection of \( A_o \) and \( A_t \).

Equation (1) cannot be solved directly, but may be simplified using one of the following assumptions: (1) that the thrust nozzle flow is zero during startup; (2) that both nozzles are always choked; (3) that the flow through the orifice occurs at constant density. A comparison between the results obtained using each of these three assumptions and measured data was made (Ref. 4).

\[ A_p = \frac{F}{\sigma I (P_s - P_c)^{(gm/R)1/2}} \]
Fig. 3. Typical Pulse Pressure History
It was concluded that the best correlation was obtained over a wider range of variables when assumption 3 was used. Thus, assuming constant density orifice flow, Eq. (1) becomes

\[
\frac{dP}{d\theta} = 2a_1 \xi P_r \left(1 - \frac{P_c}{P_r}\right)^{1/2} - 2a_2 \xi^{1/2} P_c
\]  

(2)

Equation (2) can be integrated directly, producing

\[
-2\xi^{1/2} = \ln(-y^2 - \xi^{1/2} B y + 1) +
\]

\[
\frac{2\xi^{1/2} B^{1/2}}{(B^2 + 4)^{1/2}} \ln \left(\frac{-2y - \xi B^{1/2} (B^2 + 4)}{(-2y - \xi^{1/2} B y + 1)^{1/2}}\right)_{y_o}^{y}
\]

(3)

Assuming \( \xi B^2 \gg 4 \), Eq. (3) is solved explicitly for \( \frac{P_c}{P_s} \) to obtain

\[
\frac{P_c}{P_s} = 1 - \left(y_o + \xi^{1/2} B\right)^2 e^{-2\xi^{1/2} \tau} + 2\xi^{1/2} B \left(y_o + \xi^{1/2} B\right) e^{-\xi^{1/2} \tau} - \xi B^2
\]  

(4)

In the laboratory experiments the complete pneumatic system, including the sublimation chamber, lines, valve, and thrust stand, were all electrically heated to the same temperature so that \( \xi = 1 \). In Fig. 4 solutions of Eq. (4) are presented for \( y_o = 0.87 \) (sea level) to 1.0 (vacuum), and \( \xi^{1/2} B = 10 \).

\[^3\text{This implies that } A_o/A_l > 1 \text{ and affects the accuracy of Eq. (3) in the region where } \frac{P_c}{P_s} \text{ approaches unity, cf. Refs. 3 and 4.}\]
Fig. 4. Chamber Pressure Rise Transients
Note that the time required to reach steady-state operation \( \left( \frac{P_c}{P_s} = 1 \right) \) is reduced by designing the jet reaction chamber with a small chamber volume \( V_c \) and a high orifice to nozzle area ratio \( A_o / A_t \).

From Eq. (4) the rise time \( \theta_r \), or the time required for the pressure in the thrust chamber to reach steady-state \( \left( \frac{P_c}{P_s} = 1 \right) \), is expressed by

\[
\theta_r = \frac{-1}{\xi^{1/2} a_2 \gamma} \ln \left( \frac{\sqrt[1/2]{B}}{\gamma_o + \sqrt[1/2]{B}} \right)
\]  

(During shutdown, the orifice valve is closed and the gas accumulated in the thrust chamber discharges through the nozzle. The instantaneous chamber pressure during an isentropic shutdown was given (Ref. 4) as

\[
\frac{P_c}{P_s} = \left[ 1 - a_2 \left( \gamma - 1 \right) \right]^{2(\gamma - 1)}
\]  

with the corresponding decay time (time required to discharge from \( P_s \) to \( P_a \)) given by

\[
\theta_d = \left[ \left( \frac{P_s}{P_a} \right)^{1/2(\gamma - 1)} - 1 \right] a_2 (\gamma - 1)
\]  

However, for an isothermal expansion and decay (which might occur using a heated thrust chamber),

\[
\frac{dP_c}{d\theta} = 2a_2 \xi^{1/2} P_c
\]
so that Eq. (6) becomes

$$P_c / P_s = e^{-2a_\xi \xi^{1/2}}$$

(9)

and Eq. (7) becomes

$$\theta_d = -\frac{1}{2a_\xi^{1/2}} \ln(P_a / P_c)$$

(10)

A comparison between the isentropic and the isothermal chamber pressure decay transients is shown in Fig. 5. It is noted that there is very little difference between the isothermal and the isentropic decay pressures. Since one-dimensional isentropic flow is a convenient standard for comparing nozzle performance, this assumption was used.

C. SYSTEM CHARACTERISTICS

The thrust and nozzle flow transients are readily determined from the chamber pressure history. The dynamic impulse bit size (total impulse per pulse) and the dynamic gas consumption (gas consumed per pulse) are obtained by integrating the rise transient, the steady-state value (if $\theta_c > \theta_r$), and the decay transient of thrust and of gas flow. The thrust correlation factors and nozzle discharge coefficients listed in Table I (and discussed under losses) were used in the transient analysis to calculate impulse bit size and gas consumption as functions of command pulse $\theta_c$ (defined as the time the valve is open). The results are shown in Figs. 6 and 7, both at sea level and in a vacuum. Note that in the region of small command pulses ($\theta_c < \theta_r$), both impulse and gas consumption are nonlinear functions of command pulse. This nonlinearity, or deviation from the ideal square pulse-wave (instantaneous rise and decay), is produced by the rise and decay transients. For minimum impulse bit size (gas consumption) at any given command pulse, these transients
Fig. 5. Chamber Pressure Decay Transients

SUBLIMING AMMONIUM CARBAMATE

V_c = 0.183 in.³
D_0 = 0.040 in.
D_f = 0.018 in.
T_c = 250°F (SEA LEVEL)
T_c = 300°F (VACUUM)

- ISENTROPIC
- ISOTHERMAL

CHAMBER PRESSURE, P_c (psia)

TIME, \( \theta \) (msec)
SUBLIMING AMMONIUM CARBAMATE

\( V_c = 0.183 \text{ in.}^3 \)
\( D_0 = 0.040 \text{ in.} \)
\( D_f = 0.018 \text{ in.} \)

SEA LEVEL
- \( T_c = 250^\circ \text{F} \)
- \( P_c = 60 \text{ psia} \)
- \( \epsilon = 1.68 \)

VACUUM
- \( T_c = 300^\circ \text{F} \)
- \( P_c = 45 \text{ psia} \)
- \( \epsilon = 48 \)

ANALYTICAL
- ISENTROPIC

- \( \text{EQ. II} \)

IDEAL SQUARE PULSE-WAVE

**Fig. 6. Dynamic Impulse Bit Correlation**
Fig. 7. Dynamic Gas Consumption Correlation
Table I. Subliming Solid Performance Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Sea Level</th>
<th>Vacuum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Gas Temperature, °F</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>Average Gas Pressure, psia</td>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td>Nozzle Exit Pressure(^a)</td>
<td>6 psia</td>
<td>2480µ</td>
</tr>
<tr>
<td>Nozzle Exit Temperature(^a), °F</td>
<td>-11</td>
<td>-310</td>
</tr>
<tr>
<td>Reynolds Number at Nozzle Throat</td>
<td>28,000</td>
<td>19,000</td>
</tr>
<tr>
<td>Effective Nozzle Expansion Ratio</td>
<td>1.6</td>
<td>45</td>
</tr>
<tr>
<td>Ideal Specific Impulse, sec</td>
<td>55</td>
<td>99</td>
</tr>
<tr>
<td>Nozzle Residence Time, µsec</td>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>Measured Steady-State Thrust, Fg, lb</td>
<td>0.0145</td>
<td>0.0170</td>
</tr>
<tr>
<td>Measured Steady-State Weight Flow, Ws, (10^{-4}) lb/sec</td>
<td>2.78</td>
<td>1.72</td>
</tr>
<tr>
<td>Thrust Correlation Factor, C(_t), %</td>
<td>87</td>
<td>77</td>
</tr>
<tr>
<td>Discharge Coefficient, C(_d), %</td>
<td>90</td>
<td>78</td>
</tr>
<tr>
<td>Impulse Efficiency, C(_v), %</td>
<td>95</td>
<td>99</td>
</tr>
<tr>
<td>Measured Specific Impulse, sec</td>
<td>52</td>
<td>96</td>
</tr>
<tr>
<td>Estimated Experimental Accuracy, %</td>
<td>@(\theta_c = 20) msec</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>@(\theta_c = 200) msec</td>
<td>4.8</td>
</tr>
</tbody>
</table>

\(^a\) Isentropic perfect gas expansion
should be minimized or possibly designed to effectively complement each other; i.e., the sum of the effects of the decay transient and of the rise transient approaches the ideal square pulse-wave. Equations (4) and (6) provide the designer with relationships between the variables so that such design tradeoffs may be made.

Pulsed specific impulse is shown in Fig. 8 as a function of \( \theta_c \). For vacuum operation with command pulses greater than 20 msec, specific impulse is independent of command pulse duration, although at sea level a nonlinearity is produced by the effects of transient operation at non-optimum nozzle pressure ratios.

D. PERFORMANCE LOSSES

The tests in this study were made using a 15 deg half angle conical aluminum nozzle with a measured throat diameter of 0.188 in. Two nozzle expansion ratios were used. The sea level tests were performed using a 1.68 to 1 expansion ratio nozzle and the vacuum tests were performed using a 48 to 1 expansion ratio nozzle. The sublimation chamber pressure at sea level ranged from 55 to 65 psia and was controlled by the heat input from the 250°F environmentoven and by the discharge demand based on the thruster duty cycle. The steady-state vapor pressures shown in Fig. 2 could not be maintained under flowing conditions. As previously mentioned, the rate of sublimation is determined by the amount of propellant surface area (provided the sublimation mechanism does not change over the temperature range, e.g., surface spallation). Since the rate of vaporization (or \( a \)) is low (Ref. 5), the ratio of propellant surface area to nozzle throat area becomes important and should be a large number (typically > 10^4) in order to minimize variation of pressure with duty cycle. In the test setup the propellant surface area was insufficient to maintain constant pressure during thrusting. The vacuum tests were conducted using a different duty cycle than at sea level so that the sublimation chamber pressure ranged from 40 to 50 psia in a 300°F environment. The ambient pressure in the vacuum chamber ranged from 300 to 1000 μ (depending also on the thruster duty cycle).
**Fig. 8. Pulsed Specific Impulse Correlation**
Perfect gas isentropic flow relationships were used to calculate the ideal performance characteristics listed in Table I. The throat Reynolds number (20,000 to 30,000) indicates that operation is in the continuum flow regime and that boundary layer viscous effects are not particularly severe. The values of discharge coefficient $C_d$ and thrust correlation factor $C_t$ are based on thrust and flow measurements obtained in the Aerospace Guidance and Control Laboratory. The sea level value of $C_d$ was 0.90 and the vacuum value of $C_d$ was 0.78. The sea level value of $C_t$ was 0.87 and the vacuum value of $C_t$ was 0.77. For comparison, using unheated (cold) gases, the average value (Ref. 4) of $C_d$ was 0.85 and the average for $C_t$ was 0.80.

The isentropic impulse efficiency $C_V$ is equal to the ratio of $C_t/C_d$ and was 95 and 99 percent, respectively, for sea level and for vacuum conditions in the present tests. The average value of $C_V$ reported in Ref. 4 was 91 percent. The high values of $C_V$ obtained during the present tests using ammonium carbamate are attributed to the following four effects: (1) a slight performance improvement (2 to 3 percent) due to some isothermal expansion in the heated nozzle; (2) possible presence of recondensed ammonium carbamate in the pneumatic system; (3) greater uncertainties in the measurements due to pressure and temperature fluctuation; and (4) uncertainties in the thermodynamic properties of the expanding dissociated sublimated gases.

E. TRANSIENT PERFORMANCE APPROXIMATIONS

Three approximate relationships were developed (Ref. 4) to simplify preliminary design calculations of the dynamic impulse bit, the dynamic gas consumption, and the pulsed specific impulse. These expressions are:

**Impulse Bit**

For $\theta_c \geq \theta_r$:

$$I_{tot} = C_i A_t P_s [K_r \theta_r + (\theta_c - \theta_r) + K_d \theta_d]$$

(4) The isothermal impulse efficiency was 70% both at sea level and in vacuum.
For \( \theta_c \leq \theta_r \):

\[
I_{\text{tot}} = C_f A P_s \left( K_r \theta_r + K_d \theta_d \right) \left( \theta_r / \theta_c \right)
\]  \hspace{1cm} (11)

**Gas Consumption**

For \( \theta_c \geq \theta_r \):

\[
w = K_n A P_s \left[ K_r \theta_r + \left( \theta_s - \theta_r \right) + K_d \theta_d \right]
\]  \hspace{1cm} (12)

For \( \theta_c \leq \theta_r \):

\[
w = K_n A P_s \left( K_r \theta_r + K_d \theta_d \right) \left( \theta_r / \theta_c \right)
\]

**Specific Impulse** (vacuum or steady-state at sea level)

\[ I = C_f / K_n \]  \hspace{1cm} (13)

where

- \( K_r \) = empirical rise time integration factor
- \( K_d \) = empirical decay time integration factor
- \( K_n \) = choked nozzle flow factor

\[
= a \left[ 2 / \left( \gamma + 1 \right) \right] \left( \gamma + 1 \right) / 2 \left( \gamma - 1 \right) / RT
\]

It has been found (Ref. 4) that for vacuum operation \( K_r = 0.69 \) and \( K_d = 0.0685 \). The sea level rise time integration factor \( K_r \) is the same as for vacuum. However, a decay time integration factor \( K_d = 0.48 \) should be used to provide good correlation at sea level. The results using this simplified technique of approximating the transient performance show good correlation with the more exact analysis and with the measured performance (Figs. 6-8).
III. EXPERIMENTAL PROGRAM

The experimental apparatus shown in Fig. 1 includes a gas-generation and pneumatic system, a thrust stand, and an electronic command and computer system.

The gas-generator and pneumatic system consists of a sublimation chamber (13.45 in. $^3$ capacity), a pressure gauge, a thermocouple temperature monitor, and a quick-disconnect coupler between the sublimation chamber and the thrust stand assembly. The heat of sublimation was supplied by locating the sublimation chamber in an electrically heated environmental oven. Preliminary tests indicated that the supply lines, solenoid valve, and thrust stand should be heated at least to the same temperature (a positive temperature gradient) as the sublimation chamber for trouble-free, non-clogging operation. An unexplained hysteresis between the clogging and unclogging temperature was also observed. Tests indicated that because of the limited vaporization rate of the solid, and differences in duty cycle and in the test configuration, a sublimation chamber gas temperature of 250°F was required at sea level and of 300°F in vacuum to establish the desired operating pressure and to minimize fluctuations during pulsed operation.

A pressure regulator was not used during the tests because of materials compatibility and clogging problems. The pressure in the sublimation chamber was maintained at 55 to 65 psia during the sea level tests and at 40 to 50 psia during the vacuum tests by proper adjustment of the time interval between pulses (a 2-sec interval for a 20-msec command pulse and 10 sec for 200 msec). It is noted that these sublimation chamber pressures are lower than the stabilized values shown in Fig. 2 for the same temperature. The rate of sublimation is determined by the heat flux and by the propellant surface area. The long (0.5-hr) stabilization time required to generate the steady-state sublimation pressures of Fig. 2 was due to insufficient propellant area in the
sublimation chamber. Continuous depletion of sublimed gas from the chamber by the pulsing thruster at a rate greater than the sublimation rate therefore resulted in sublimation chamber pressures which are somewhat lower than the stabilized values.

The complete apparatus was positioned inside the environmental oven during the sea level tests so that the lines and thruster stand were also heated to 250°F. In the vacuum tests, the gas flow lines and thrust stand were heated by auxiliary electric heaters to 300°F, measured by a bimetallic contact-thermometer mounted on the thrust stand in the vacuum chamber.

The sublimation chamber was accurately weighed before and after a test to determine that mass consumed during the test. A typical test consisted of several thousand pre-programmed command pulse and requires about 1.5 hr to run. Each data point is an average of three tests.

The thrust measurement apparatus, Fig. 9, has a nozzle mounted on the end of a short length of tubing to serve the dual function of a pneumatic feed line and a cantilever beam spring. When thrust is produced, the beam is displaced an amount proportional to the thrust magnitude. The deflection of the beam is sensed by a position transducer, and an electrical output proportional to the position (thrust level) is generated. Viscous damping is provided through the shearing action of a flat plate moving through a viscous fluid. A high natural frequency of the cantilever beam and nozzle assembly (250 cps) is necessary to provide adequate dynamic response for the thrust transients.

The natural frequency of the thrust measurement fixture was verified experimentally by two methods. One method was to displace the beam by a discharge of gas from the pneumatic system through the nozzle. The natural frequencies of the fixture, both with and without damping fluid, were displayed on the oscilloscope and photographed. The second method was to position a coaxial solenoid valve so that high pressure air would discharge through the

5 An additional thermal stabilization time of about 1.5 hr is required before each test to insure that all components are properly heated.
valve and impinge on the thrust fixture nozzle block. The resulting displacement of the fixture was monitored on the oscilloscope. By adjusting the high pressure air supply valve the peak cantilevered beam displacement was set equal to the displacement experienced during thrust measurement. The thrust stand dynamic response is more than adequate for the rise times encountered, as evidenced by the correlation of thrust and pressure traces in Fig. 10.

The thrust stand was calibrated in two ways. One calibration involved the use of a Scherr-Tumico force gauge and provided a scale factor in terms of volts per pound of force. The other calibration was made by orienting the thrust stand so that the beam was subjected to a 1 g loading. Test weights were then placed on the cantilever beam, and a correlation of force versus electrical output (deflection) was made. Both methods were in excellent agreement.

A schematic of the command and computer system is shown in Fig. 11. A preset counter or clock was used to fix the repetition rate of the command pulses, and a digital counter recorded the number of cycles or pulses applied to the solenoid valve. The output of the thrust transducer was amplified, demodulated, and fed to the voltage-to-frequency converter. The output of the converter was fed into a reversible counter which is used to integrate the thrust profile and thus obtain the total impulse per pulse (impulse bit size). The reversible counter computes the impulse bit including drift offset during a time increment $\Delta t$. Then, the counter totals the integrated offset and drift for an equal time interval $\Delta t$ and obtains the difference between the two values to provide a direct readout of total impulse. The use of a reversible counter provides improved accuracy over the two separate counters previously used (Ref. 3) for the same function. This is because the reversible counter eliminates any errors due to differences in timing sensitivity between individual counters.

A pressure transducer with a 1200 cps acoustic frequency was used to monitor pressure.
Fig. 10. Correlation of Thrust and Chamber Pressure

\[ P_c = 40 \text{ psia} \]

\[ F = 17 \text{ mlb} \]

50 msec
Fig. 11. Schematic of Command and Computer System
The specific impulse was obtained by dividing the total impulse by the gas consumed as measured over a large number of command pulses (>1000). The solenoid valve closing time was adjusted to equal the valve opening time by valve driver circuitry modifications (Ref. 9). This configuration resulted in a valve open time equal to the electrical command pulse width, with a 3.5 msec delay relative to the command pulse.

To complete the experimental evaluation, the repeatability of impulse bit size was examined. Eighteen hundred 20-msec command pulses were applied to the solenoid valve, and the total impulse of each pulse was recorded. A distribution plot of these data is shown in Fig. 12. The spread and character of these data is a result of the thermal drift of the entire pneumatic assembly. The pressure variations observed in the supply tank during typical operating conditions are due to changes in sublimation rate (caused by heat flux variations) and directly affect the impulse bit size repeatability.

An estimate of the experimental error in measured specific impulse was made. This estimate included the effects of: mass measurement errors, limited dynamic response of the thrust fixture, calibration errors, and electronic errors. The largest error arises from the uncertainty in measuring propellant weight. An uncertainty of 2.5 percent was associated with the weighing of 4 g of propellant typically consumed during each specific impulse measurement consisting of 2000 command pulses. The second largest error was due to the inaccuracy in thrust stand calibration, i.e., inaccuracies in the correlation between the displacement (thrust) of the cantilevered beam and the measured output voltage. This calibration uncertainty was estimated to be 4.1 percent. A third major source of error was produced by limitations of the thrust stand dynamics. This error was estimated to be 1.5 percent at a 20 msec command pulse width and 0.5 percent at a 200 msec command pulse width. Electronic errors such as drift and timing were insignificant over intervals of 2000 command pulses. A worst case estimate of the average experimental error was obtained by RSS (root-sum-square) of these measurement uncertainties. As indicated in Table I, the estimated average experimental error is 5 percent for command pulse widths from 20 to 200 msec.
Fig. 12. Impulse Bit Size Distribution
IV. PERFORMANCE COMPARISONS

Although specific impulse is the most important single measure of propulsive performance for system design, consideration also must be given to the propellant density and associated tankage weight. The effective system specific impulse is therefore defined (Ref. 10) as the ratio of delivered total impulse to the combined propellant and minimum spherical tankage weight, or

\[ I_{\text{eff}} = \frac{I_k}{1 + 1.5vP(p/\sigma)_t} \]  

(14)

where

- \( P \) = propellant storage pressure
- \( K_s \) = a system factor to account for electric heater and minimum gauge weights = 0.87 for stored gases and 0.80 for stored liquids and solids (based on actual spacecraft designs)
- \( v \) = the propellant specific volume
- \( (p/\sigma)_t \) = the ratio of tank density to working stress = 3.25 x 10^{-5} \text{ ft}^{-1} for titanium with a safety factor of 2

Equation (14) was used to evaluate the effective system specific impulse of \( H_2, NH_3, N_2, \) Freon-14, and Freon-12 propulsion systems. The condensable gases \( NH_3 \) and Freon-12 were assumed stored as liquids at their room temperature vapor pressures of 130 and 85 psia, respectively. The other gases were assumed to be stored at 3500 psia. Solid ammonium carbamate was assumed to be heated by an electric resistance heater to a temperature of 300°F, producing a dissociated gas at a sublimation pressure of 45 psia (as in the laboratory tests). To provide an equal basis for comparison, the other five gases were also assumed to be heated to 300°F. No correction for
possible partial isothermal expansion effects (2 to 3 percent in impulse) was assumed. The results, listed in Table II for a thrust level of about $10^{-2}$ lb, show that due to its low molecular weight the effective system specific impulse of liquid NH$_3$ is nearly 20 percent greater than solid ammonium carbamate. However, the system specific impulse of subliming ammonium carbamate is twice that of N$_2$, Freon-14, or Freon-12 (all with $I_{eff} \approx 30$ sec), and 5 times that of H$_2$ (due to its low bulk density). It should be noted that while the weight of the heater was included in this effective specific impulse comparison, the weight of the electric power source was not included and must be considered in a spacecraft design application. It requires from 1 to 3 watts/mlb of thrust to heat the gases to 300°F. When the heat of vaporization is included, about 10 watts/mlb are needed for the subliming solid, and 6 watts/mlb for NH$_3$, cf. Table II. If long thruster lines are used downstream of the sublimation chamber (or boiler), then line heaters are required to maintain a gaseous propellant (to prevent flow clogging), and this additional heater and power source weight should also be considered. However, the final selection of one propellant over another would depend on the results of a detailed design study for a specific mission.
Table II  Performance Comparison of Various Gases

\( (T_c = 300^\circ F, \epsilon = 48, F = 15 \text{ mlb}) \)

<table>
<thead>
<tr>
<th>Gas</th>
<th>( H_2 )</th>
<th>( NH_3 )</th>
<th>( N_2 )</th>
<th>Freon-14</th>
<th>Freon-12</th>
<th>( NH_2CO_2NH_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of Specific Heats, ( \gamma )</td>
<td>1.40</td>
<td>1.31</td>
<td>1.40</td>
<td>1.22</td>
<td>1.14</td>
<td>1.31</td>
</tr>
<tr>
<td>Molecular Weight, ( m )</td>
<td>2</td>
<td>17</td>
<td>28</td>
<td>88</td>
<td>121</td>
<td>26</td>
</tr>
<tr>
<td>Characteristic Velocity, ( c^* ), fps</td>
<td>6350</td>
<td>2220</td>
<td>1690</td>
<td>1010</td>
<td>870</td>
<td>1800</td>
</tr>
<tr>
<td>Ideal Specific Impulse, sec</td>
<td>332</td>
<td>121</td>
<td>90</td>
<td>58</td>
<td>53</td>
<td>98</td>
</tr>
<tr>
<td>Delivered Specific Impulse, sec(^a)</td>
<td>314</td>
<td>116</td>
<td>81</td>
<td>56</td>
<td>45</td>
<td>96</td>
</tr>
<tr>
<td>System Effective Specific Impulse, sec(^b)</td>
<td>14</td>
<td>91</td>
<td>30</td>
<td>34</td>
<td>34</td>
<td>77</td>
</tr>
<tr>
<td>Power Required to Heat to 300(^\circ)F watts/mlb(^c)</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

\(^a\) Measured at \( T_c = 70^\circ F \) and adjusted to 300\(^\circ\)F (except for \( NH_2CO_2NH_4 \)).

\(^b\) Does not include weight of electric power supply. \( H_2, N_2, \) and Freon-14 are assumed to be stored as gases at 3500 psia with \( \rho_h = 3.25 \times 10^{-5} \text{ ft}^{-1} \) in Eq. (14).

\(^c\) Includes heat of vaporization
V. CONCLUSIONS

Accurate predictions of impulse bit size, gas consumption, and specific impulse can be made using the gas dynamical relationships given for the transient pressure histories. A simplified approach to transient performance determination was also developed to rapidly evaluate design tradeoffs for preliminary design purposes.

Both the gas consumed and the impulse bit size are nonlinear functions of command pulse width for small pulses but become linear when steady-state chamber pressure is reached. The nonlinearity, or deviation from the ideal square pulse-wave, is due to the rise and decay transients. For minimum impulse bit size (and gas consumption) at a given command pulse, the rise and decay transients should be minimized or possibly designed so that the sum of the effects of the decay transient and of the rise transient approaches an ideal square pulse-wave.

The pulsed vacuum specific impulse is independent of command pulse-width for pulses > 20 msec and is equal to the steady-state value. At sea level, variable thrust coefficients and unchoked nozzle flows, caused by chamber pressure rise and decay transients, lower specific impulse and make it a nonlinear function of command pulse.

No performance losses due to the presence of condensed substances in the nozzle flow could be detected. However, all parts of the pneumatic system had to be maintained at least as hot as the sublimation chamber, i.e., a positive temperature gradient, to prevent recondensation of the subliming solid from clogging the system. A slight gain of 2 to 3 percent in specific impulse efficiency of the subliming solid compared to other cold gases is attributed to partial isothermal expansion produced by this requirement for a heated nozzle.

The observed time lag required to reach equilibrium sublimation pressure is due to kinetic molecular reactions occurring at the solid surface.
It is necessary to know the sublimation rate (or coefficient of sublimation) as well as the equilibrium pressure to size the total propellant surface area so that sufficient sublimation can occur to minimize large variations in thrust level with duty cycle. This is a significant factor to be considered in the design of a subliming solid engine.

The results of a performance comparison between the subliming solid system and systems using five other gases (all heated to the same temperature and at a thrust of $10^{-2}$ lb) show that subliming solids are competitive and possibly even advantageous if sufficient heating power is available. The subliming solid heating power requirement of at least 10 watts/mlb of thrust is mitigated at very low thrust levels or if the duty cycle and/or installation is such that a weight penalty for the power source is not incurred. In general, the final selection of one propellant system over another must depend on the results of a detailed design study for a specific application.
REFERENCES


The pulsed propulsive performance of a low-thrust subliming-solid ammonium carbamate reaction jet is analyzed and compared with the results of laboratory experiments. Tests of the 15 millipound thrust were conducted at sea level using a 1.63 to 1 expansion ratio nozzle and at vacuum using a 48 to 1 expansion ratio nozzle. The transient processes, which dominate the short-pulse or limit-cycle mode of thruster operation, are formulated and show good correlation with the data. The apparatus, procedures, and techniques required to obtain accurate test results for a low-thrust dynamic mode of operation are described. Impulse bit size, gas consumption, and specific impulse are characterized in terms of thruster geometry, gas properties, and command pulse width to provide a systematic basis for design. Limitations and design criteria necessary for successful spacecraft installation are discussed. A comparison is made between the performance of the subliming solid reaction jet system and the performance of other propellants.
**KEY WORDS**

- Low thrust propulsion
- Reaction control jets
- Jet reaction dynamics
- Attitude control propulsion
- Transient impulse analysis
- Subliming solid propulsion
- Ammonium carbamate propulsion
- Subliming ammonium carbamate
- Theoretical performance of warm gases
- Warm gas propulsion
- Warm gas reaction jets
- Electric chemical reaction jets
- Low pulse width performance
- Spacecraft vernier motor performance

---

**Abstract (Continued)**