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PHYSICAL MEASUREMENTS OF PROJECTILES

Elizabeth R. Dickinson

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BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND
PHYSICAL MEASUREMENTS OF PROJECTILES.

Elizabeth R. Dickinson

Department of the Army Project BRL-503-03-001
Ordnance Research and Development

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PHYSICAL MEASUREMENTS OF PROJECTILES

ABSTRACT

Standard techniques for the measurement of projectiles are described in detail. Such measurements include: dimensions, weight, center of gravity, moments of inertia, static and dynamic unbalance, yaw-major axis relation.

The techniques described give the accuracy required for routine measurements. In a few cases, more accurate methods are given for use with experimental projectiles.
INTRODUCTION

Recently the author was requested to review the physical measurements section of the Ordnance Proof Manual for possible revision. This indeed appeared desirable. Since the present section was published in 1943, considerable improvements have been made in the apparatus and techniques used for obtaining the physical characteristics of projectiles.

The text follows the form of the present Manual and is intended as a suggested replacement of the present section.

The material is based on techniques developed by the Free Flight Aerodynamics Branch of the Exterior Ballistics Laboratory, BRL. Substantially similar apparatus and techniques are used by the Physical Test Laboratory, D&PS.

Acknowledgement is made to Mr. James McKinley, Physical Test Laboratory, D&PS, for the sections on radiographic and ultrasonic inspection, bullet pull measurements, and dynamic unbalance.

STANDARD TECHNIQUES FOR MEASUREMENT OF PROJECTILES

SECTION I

In order to determine the aerodynamic characteristics of a projectile, certain basic physical measurements are required. These are: Dimensions, weight, center of gravity, axial and transverse moments of inertia. In addition, dynamic unbalance, eccentricity (often called static unbalance), the yaw-major axis relation, surface finish measurements, and bullet pull measurements are sometimes required. Radiographic and ultrasonic inspection are also discussed briefly.

In general, the techniques described below are designed to give the accuracy required for routine measurements. In the cases of center of gravity and of moment of inertia determinations, alternative methods are also described which will give an accuracy of 0.1%. In most cases, such accuracy is superfluous; however, because it is sometimes required in connection with work on experimental shells, it was deemed advisable to include the material in this manual.

Dimensions. The following dimensions of conventional, spinning projectiles should be measured: (Figure I)

a. Overall length, including fuse

b. Front bourrelet diameter

c. Rear bourrelet diameter at rear bearing surface behind the rotating band.
In the case of experimental projectiles, other critical dimensions may be required, such as nose angle, boattail angle, body diameter, etc. Routine measurements are made by means of vernier and micrometer calipers. If greater accuracy is desired, optical measurement by means of a contour measuring projector is recommended.

Within recent years, interest in finned missiles has necessitated the development of a technique for measuring fins. Some parameters, such as fin span and maximum fin thickness, are readily measured by conventional methods. Other parameters, however, such as fin wedge angle or radius, cant, etc., present problems in measurement which cannot be accurately solved by the use of standard measuring equipment. It was necessary to have designed and built a special fin contour measuring machine, the details of which will not be elaborated upon here. In brief, the missile is held in a fixture which permits measurable alignment, both linear and angular, in all three planes. A two dimensional tracing is then made on a glass plate, which can be magnified and measured by an optical projector. Any number of these tracings may be made, with any desired alignment of the fins with respect to the missile body. Hence a complete physical picture of the fins can be obtained.

Figure 10 is a photograph of two records made by this instrument. At the right end of each will be seen the reference line. The upper record shows a series of contours of the same fin, the root being at the left end, the tip, next to the reference line. The lower record also shows the contour of a fin at different sections; this fin, however, was canted, as can be seen by its relation to the reference line.

Weight. Projectiles should be weighed on appropriate balances, in order to obtain the requisite accuracy. The weight of small arms projectiles is recorded in grains; that of other projectiles in pounds.

Center of Gravity. The distance from the base of a projectile, to its center of gravity, is measured by either of the two following methods.

a. Method I. This technique is used for all projectiles which can be handled without the use of a crane, ranging from cal.50 to 155mm projectiles (see Figures 2 & 7). The apparatus used is a center of gravity balance, consisting of a V-yoke with two knife-edge trunnions, a counterweight at the bottom (for adjusting sensitivity) and a pointer on the top. This yoke, in use, is placed on a support, in front of a scale. Although the divisions on the scale can be any arbitrary number of degrees apart, the spacing must be even. The yoke is clamped on the projectile at some arbitrary point, estimated to be near the center of gravity; the distance of the knife-edges from the base of the projectile is measured and recorded; the yoke is then placed on the support; the deflection of the pointer on the scale is observed and recorded. The yoke is then arbitrarily shifted on the projectile and reclamped. A second measurement and deflection are recorded. Distance from the base is then plotted against deflection. The point at which this line crosses the zero deflection point, is the distance of the center of gravity from the base of the projectile.
The actual dimension measured each time is from the base of the projectile to the face of the yoke (this being an easy measurement to make). The distance from the face of the yoke to the knife-edges is a previously determined constant.

Although it would at first appear to be simpler and quicker to balance a projectile directly on a knife-edge, and measure the distance to the balance point; in actual practice, such a procedure is more arduous and time consuming than the method described above. The reason for this is that by the method described above, it is not necessary to find the exact balance point. If an extremely accurate center of gravity determination is required, it is advisable to measure the distance from the base of the projectile to the face of the yoke in the following way: place the projectile, base down, with yoke attached, on a surface plate. With a vernier height gage, take four measurements, approximately equidistant around the projectile, from the surface plate to the face of the yoke. The average of these four measurements is plotted against deflection. Still greater accuracy can be attained by plotting three or four positions, rather than two. (Figure 3)

b. Method II. This technique is used for all large projectiles, including bombs. It can be used for small projectiles, but due to the disproportionate ratio of projectile weight to apparatus weight, in small sizes, the method described above is to be preferred. The apparatus required in this procedure (see Figure 4) is a beam (or trough) supported at each end by a line contact. Sharp knife-edges are impractical for this purpose, because of the weight involved. However, angle iron, with a machined radius along the bottom edge, will give the proper line contact. One end of the beam (or trough) has a vertical stop, the inner edge of which is in the same plane as the line contact of the supporting angle iron. End A (see Figure 4) of the beam is placed on a platform scale, and B on an adjustable support so that the beam is level. The projectile is then placed on the beam, with the base of the projectile against the stop. The center of gravity of the projectile is computed from the following equations:

\[ L_0 = \frac{L_2 W_2 - L_1 W_1}{W_0} \]

where: 
- \( L_0 \) = distance of center of gravity of projectile from its base
- \( L_1 \) = distance of center of gravity of beam from support B
- \( L_2 \) = distance between supports A and B
- \( W_0 \) = weight of projectile
- \( W_1 \) = weight of beam
- \( W_2 \) = reading on scale with projectile in position on beam

The quantities \( L_1, L_2 \) and \( W_1 \) (hence the product \( L_1 W_1 \)) are constants for any given beam.
The center of gravity of the beam itself is easily determined in the same way. Referring to the same diagram (Figure 4):

\[ L_1 = \frac{L_2 W_2}{W_1} \]

where: \( W_2 \) = reading on scale with beam only, in place.

It should be remembered that the accuracy of the center of gravity determination is directly dependent upon the distance \( L_2 \). Hence, this measurement should be made with the greatest possible precision.

Moments of Inertia

a. Moment of inertia is a measure of the effectiveness of mass in rotation. If a projectile is a solid of revolution, it has two principal moments of inertia: (1) about its longitudinal axis, called the axial moment of inertia and (2) about any axis through the center of gravity, perpendicular to the longitudinal axis, called the transverse moment of inertia. Theoretically, only the latter needs to be measured for non-spinning, fin stabilized projectiles. However, experience shows that most fin stabilized projectiles do spin; hence their axial moments of inertia should be determined.

b. These moments of inertia are obtained from the measured periods of oscillation on a torsion pendulum by using the following equations:

\[ I = kT^2 + i \]

in which \( I \) = the moment of inertia of the projectile
\( k \) = the torsional constant of the wire
\( T \) = the period of torsional oscillation
\( i \) = the moment of inertia of the projectile holder

In order to determine the moment of inertia of a projectile, therefore, one must determine not only its period of oscillation, but also the wire constant and moment of inertia of the holding device. These values are measured by means of test masses, a set of which is shown in Figure 5(c). A test mass is a simple geometric shape, whose moment of inertia can be easily calculated. It must be within 5% of the weight of the projectile in order to eliminate effects of weight variation on the effective wire constant. Because there are two unknowns, \( k \) and \( i \), two equations (hence two test masses) will be necessary for their determination:

\[ I_1 = kT_1^2 - i \]
\[ I_2 = kT_2^2 - i \]

where subscript 1 refers to the mass with the larger moment of inertia; subscript 2 refers to the mass with the smaller moment of inertia. The
masses should be so designed that their moments bracket the estimated moment of the projectile. Solving these equations for \( k \) and \( i \) one obtains:

\[
k = \frac{I_1 - I_2}{T_1^2 - T_2^2}
\]

\[
i = \frac{T_2^2}{T_1^2 - T_2^2} \cdot \frac{I_1 - I_2}{T_1^2 - T_2^2}
\]

Test masses are most easily designed as cylinders of various sizes, either solid or hollow, superimposed on each other (Figure 5,c). The formulae used in the design are as follows:

- **Solid Cylinder**
  \[
  I_{\text{Axial}} = \frac{WR^2}{2}
  \]
  \[
  I_{\text{Transverse}} = \frac{W(3R^2 + h^2)}{12}
  \]

- **Hollow Cylinder**
  \[
  I_{\text{Axial}} = \frac{WR^2}{2}
  \]
  \[
  I_{\text{Transverse}} = \frac{W[R^2 + r^2]}{12} \left[ (R^2 + r^2) + \frac{h^2}{12} \right]
  \]

where:
- \( W = \) weight
- \( R = \) outside radius
- \( r = \) inside radius
- \( h = \) length

One end of the axial masses must be the same diameter as that of the base of the projectile; the center of the transverse masses must be the same diameter as the projectile at its center of gravity. These conditions are necessary in order that the same fixtures can be used to hold both projectile and masses. The photographs in Figure 5 show the apparatus used for obtaining moments of inertia of both large and small projectiles.

The torsion pendulum method of obtaining the moments of inertia is employed for all sizes of projectiles. The suspension itself is a straight length of tungsten wire, with a fixture at each end. Figure 6 is a diagram of a torsion pendulum suspension designed to give an accuracy of 0.1%. A suspension for routine measurements would eliminate the graduated disc and the mirror. The top fixture fits into a hole in a rigidly held, level plate. In operating the apparatus, the oscillation is set up by rotating and returning this top fixture.

The wire itself should be relatively short, ranging in length from \( 12" \) to \( 24" \) only, for all types of projectiles. In diameter, the range is from \( .004" \) up to the diameter necessary to hold the projectile weight, the
tensile strength of tungsten being in the neighborhood of 500,000 lbs./in.$^2$

At the bottom of the wire is a fixture to which the projectile holder is attached.

Fixtures for holding projectiles in axial moment of inertia measurements are of two types. The first is a metal cup, which fits over the projectile, and is held in place by set screws. This cup, which may be slotted for finned missiles, is customarily placed over the base of the projectile, which is then oscillated nose down. Some projectiles have bases of such a contour, that a base cup holder is impractical. If made long enough, however, this same type of holder can be fitted down over the nose, and tightened against the body of the projectile; which is then oscillated base down.

If a cup holder is impossible to use either on the base or the nose, a dummy fuze can be screwed into the projectile and used as a holder. If the moment of inertia of the projectile, complete with fuze, is required; the moment of inertia of the latter can be determined independently. It must be remembered that the entire set-up must be symmetrical. The projectile must be centered in the holder; the tapped hole, in the holder, by which the projectile is held to the suspension, must also be centered. If these conditions are not met, the true moment of inertia about the longitudinal axis of the projectile, will not be obtained. A projectile in position for determining its axial moment of inertia is shown in Figure 5(b).

Transverse holders are always of a double-V type, though often modified to adapt themselves to unusual shapes. The same holder is often used for both center of gravity and transverse moment of inertia determinations (see Figure 7). In use as a transverse moment of inertia holder, the projectile is placed in the fixture, so that the knife-edges are in a line with the center of gravity of the projectile; the clamping device is tightened, and the holder screwed into place on the suspension.

After the projectile is attached to the suspension, for either axial or transverse moment of inertia measurement, the pendulum is brought to rest, and a chalk mark made on the projectile. The whole set-up is then steadied so that there is no vibration or swinging motion. The projectile is set into torsional oscillation by twisting the top holder and returning it to its zero position. The total oscillation should be approximately $90^\circ$ ($45^\circ$ each way from the position of rest). This amplitude of $45^\circ$ is necessary for two reasons: (1) the same amplitude must be used from projectile to projectile in order to obtain comparable results; (2) $45^\circ$ has been found, by extensive experimentation, to be the optimum amplitude, giving experimental results most nearly approaching theoretical calculated values.

The operator looks at the projectile through a telescope, and starts a stop watch when the chalk mark first passes the cross hairs of the telescope. After the requisite number of complete oscillations, the watch is stopped at the passage of the chalk mark across the cross hairs.
For axial moment of inertia determinations, five complete periods of oscillation are timed; for transverse, three periods are timed. This procedure is repeated three times in each case, starting the oscillation each time by twisting the top suspension fixture, and starting the timing operation when the amplitude of oscillation has damped to 45°. These times are recorded and averaged.

For example:

**Axial**

11.94 seconds: time for 5 periods
11.92 seconds: time for 5 periods
11.95 seconds: time for 5 periods
35.81 seconds: time for 15 periods
2.387 seconds: time for 1 period = T
5.698 = T²

**Transverse**

18.01 seconds: time for 3 periods
18.03 seconds: time for 3 periods
17.99 seconds: time for 3 periods
54.03 seconds: time for 9 periods
6.003 seconds: time for 1 period = T
36.014 = T²

Let us assume that we have used two transverse test masses whose moments of inertia are 2 lb-in² and 5 lb-in², and whose periods of oscillation were 5 seconds and 7 seconds.

Then (from the equations on page 9):

\[ k = \frac{5 - 2}{49 - 25} = .1250 \]

\[ i = \frac{(25)(5) - (49)(2)}{49 - 25} = 1.1250 \]
The transverse moment of inertia of the projectile is then computed by the equation given on page 9:

\[ I_B = kT^2 - i \]
\[ I_B = (1.125)(36.0h) - 1.125 \]
\[ I_B = 3.380 \text{ lb-in}^2 \]

Experience has shown that the so-called constants \( k \) and \( i \) remain constant only within a limited period of time. Why this should be so is not entirely clear. A partial answer is that temperature change results in a change in the wire constant. Temperature differences also cause dimensional changes and therefore changes in weight distribution of the test masses. In other words, the moment of inertia of a given test mass at 60° F. is different from its moment of inertia at 80° F. However, in the equations used for the determination of \( k \) and \( i \), the moments of inertia of the test masses are used as constants. Due to the form of the equation, all errors in the system become a part of the value assigned to \( i \).

Therefore, if the test masses and all the projectiles to be measured can be swung on the pendulum within a period of approximately four hours, the three masses need to be timed only as described above. However, if the time extends beyond four hours, the group of three masses should again be oscillated and timed. A schedule of operations might be as follows:

0800-1200: 3 test masses (axial), 10 projectiles (axial)
1300-1700: 3 test masses (axial), 10 projectiles (axial)

The \( k \) and \( i \) derived from the morning measurements would be used for determining the moments of inertia of the projectiles measured in the morning; the \( k \) and \( i \) derived from the afternoon measurements would be used for determining the moments of inertia of the projectiles measured in the afternoon. The foregoing procedure results in an accuracy of measurement in the order of 1%.

It should be remembered that the difference in weight of projectiles and masses should not exceed 5%; and that the amplitude of oscillation should be 15°.

Described below is a technique for determining moments of inertia to an accuracy of 0.1%. Although two test masses, and two equations are all that are necessary from a mathematical point of view; due to the errors involved in any measurement, greater accuracy can be obtained by means of three test masses. If the estimated moment of inertia of the projectile to be measured is designated as \( I \); the three test masses should be so designed that their moments are approximately \( \frac{2I}{3} \), \( I \) and \( \frac{4I}{3} \).
should be repeated that the weights of the masses should be within 5% of
the weight of the projectile. With these masses, three equations, in
only two unknowns, can be set up, and a solution obtained by the least
squares method, (Appendix II, pp. 33ff). Because such a method is best
used by placing the observed quantity on the right hand side of the
equation, the original equation is changed to the form:

\[ I \cdot kr = r^2 \]

where:

\[ i = \frac{1}{k} \]

\[ k = \frac{1}{k} \]

For this technique, a fixture is used on the end of the wire (Figure 6)
which serves three purposes: (1) to trigger the timing device, (2) to
hold the projectile and (3) to regulate the amplitude of oscillation.

(1) For triggering purposes a first-surface, plane mirror is let
into one face of the fixture. At a distance of approximately 18" (de-
pending on the lens system) a photo-electric circuit is set up. Included
in this circuit is a light source, whose beam is focused on the mirror.
As the pendulum oscillates, this beam is reflected back to the photo-cell,
which in turn triggers an electronic cycle counter. In the trigger circuit
is a relay mechanism which can be pre-set so that a given number of cycles
can be timed. At the first passage of the light beam across the photo-
cell, the cycle counter is started and continues to run until stopped by
the photo-cell circuit after the required number of passages of the beam.

(2) A portion of this bottom fixture is threaded, so that the axial
and transverse projectile holders can be screwed into place on the pendulum.

(3) This fixture is also provided with a disc, marked off in degrees
(see Figure 6). The purpose of this disc, is to enable the operator to
obtain the correct amplitude of oscillation, when measuring moments of
inertia.

The entire suspension system should be enclosed, or at least so
located that drafts of air cannot influence the oscillation, or set up a
secondary swinging motion.

After the projectile is attached to the suspension, for axial or
transverse moment of inertia measurement, the pendulum is brought to rest,
with the mirror in line with the photo-cell, and the whole set-up steadied
so that there is no vibration, or swinging motion. The projectile is set
into torsional oscillation by twisting the top holder and returning it to
its zero position. The operator then looks at the disc through a tele-
scope, and turns on the trigger circuit when the total oscillation is 90°
(±5° each way from the position of rest). It is necessary for consistent
results that the amplitude of oscillation be the same for the projectile and test masses. The choice of a half-amplitude of $45^\circ$ was based on experiments with the apparatus described, indicating that the results were more consistent with half-amplitudes near $45^\circ$ than with substantially larger or smaller oscillations.

Eccentricity. The displacement of the center of gravity of a projectile from its longitudinal axis, is known as eccentricity, or static unbalance. Although the determination of this physical characteristic is not required on all projectiles, it is occasionally needed. It is possible to measure this distance from the axis, and also to locate its angular displacement from a reference mark on the projectile body, by means of the apparatus shown in Figure 6.

Four reference marks $90^\circ$ apart, should be lightly scribed on the surface of the projectile. The more accurately this is done, the greater the accuracy which will be achieved in the final result. The projectile is then placed in the modified V, and the counterweight adjusted so that a maximum of $5\%$ of the total weight of the projectile will be on the pan of the balance. As fine a balance as possible should be used to assure maximum accuracy. The projectile should then be weighed in four positions, $90^\circ$ apart, by aligning the reference marks on the projectile, with the reference mark on the V. Let the weight at $0^\circ$ = $a$; at $180^\circ$ = $b$; at $90^\circ$ = $c$; at $270^\circ$ = $d$.

Let \( \frac{H}{2} = C \)

where \( H \) = the distance between the fulcrum and the support on the balance pan, in inches.

\( M \) = the total weight of the projectile, expressed in the same units as $a$, $b$, $c$, $d$ above.

then \( e = C \sqrt{(a-b)^2 + (c-d)^2} \) inches

The angular displacement of the center of gravity from $0^\circ$ reference mark on the projectile is obtained from the equation:

\[ \tan \theta = \frac{c-d}{a-b} \]

For example:

Let $a = 7$ grams; $b = 5$ grams; $c = 16$ grams; $d = 10$ grams; $H = 6$ inches;

\( M = 600 \) grams.

Then \( C = \frac{3}{600} = .005 \)
\[
\begin{align*}
\tan \phi &= \frac{(16-10)/(7-5)}{6/2} = 3 \\
\phi &= 71^\circ 34' \\
\end{align*}
\]

Care must be used in assigning the proper signs to the quantities \((a-b)\) and \((c-d)\), for the quadrant in which \(\phi\) lies, depends upon the signs of their algebraic differences (See Appendix III, pp 362).

Yaw-Major Axis Relation. The yaw of a projectile is the angle between its axis and the direction of motion of its center of gravity. This is determined in two ways: (1) by measurement of photographs of the projectile in free flight; and (2) by measurement of the major axis of the holes made by the projectile as it is fired through yaw cards set at right angles to the trajectory.

The measurement of yaw by the first method is a special computing technique outside of the scope of this paper. Measurement of yaw by the second method, however, is directly related to the physical dimensions of the projectile, by what is known as the yaw-major axis relation.

This relation may be established in one of three ways: (1) by mathematical calculation, (2) by measuring the projectile, and (3) by measuring the drawing of the projectile. The first method will not be discussed here. The apparatus shown in Figure 9, is that used for measuring the projectile in order to establish the yaw-major axis relation. On a horizontal plate are mounted a rotating V-block, whose sides are parallel to the V, and two parallel vertical plates which can be moved toward or away from each other, maintaining their parallelism.

If it is necessary to determine the yaw from only a few cards, the following procedure is followed. The major axis of each hole is recorded. For each yaw-card measured, the vertical plates are set so that the distance between them (as measured by means of a vernier caliper) is that of the major axis of the hole. The projectile is then placed in the V-block, and the block rotated until the projectile touches both plates. Contact may be determined either visually, or electrically. The smaller angle between the plane of the plates and the face of the block is then measured with a protractor and is the angle of yaw corresponding to the major axis of the hole in the yaw-card.

If an extensive yaw program is to be fired, a slightly different procedure is followed. A series of yaw-major axis relations is tabulated at the beginning of the program. A projectile is placed in the V-block, and the block is rotated from \(0^\circ\) to \(45^\circ\) (or some other limits as determined by the proof director), by intervals of \(1^\circ\) (or some other interval as determined by the proof director). For each setting of the V-block, the vertical plates are brought into contact with the projectile, and the distance between the plates is measured. Thus angle of yaw is tabulated and graphed against major axis. When the yaw cards are measured, the angle can be immediately read from the graph or interpolated from the table.
An optical, rather than a mechanical, approach to the problem is essentially the same as that described above, but will result in greater accuracy for small arms projectiles. The projectile is placed on the rotating stage of a contour measuring projector with its longitudinal axis parallel to a vertical line on the screen of the projector. The stage is rotated, as was the V-block in the mechanical system. The stage is then traversed until the projectile touches the line (see position 1 in Figure 9), and a reading taken. Again the stage is traversed until the other side of the projectile touches the line (position 2 in Figure 9). The difference between these readings is the distance corresponding to the major axis of the hole, for the given angle of rotation, hence that angle of yaw.

The method of establishing the yaw-major axis relation by measuring the drawing of the projectile is, again, basically the same as the methods described above. Two parallel lines are drawn, or two parallel rulers placed in such a way on the drawing that they touch the contour on opposite sides. The distance between corresponds to the major axis; the angle between the axis of the projectile and either of the parallel lines is the angle of yaw.

Surface Finish Measurement. It is known that surface roughness affects the aerodynamic performance of projectiles, especially drag. However, its effect on other aerodynamic characteristics is unknown at the present, and no definite criteria have been established. In experimental work, measurement of surface roughness is sometimes required. Two types of measuring devices are available; one gives a root mean square record; the other, a true profile, with magnifications up to 5000 X (Figure 11). Interpretation of these records and correlation with aerodynamic performance are still in the early stages of ballistic research. For these reasons, measurement of surface finish is not part of the routine measurement procedure.

Dynamic Unbalance.

(1) Objects such as motor armatures that are in perfect balance statically when checked on level ways may be quite unbalanced when rotated at various speeds. Such unbalances quite often set up vibrations which may ultimately destroy or damage the motor. Similarly dynamic unbalance in a projectile in flight seriously affects its exterior ballistics. The ballistician is therefore interested in correlating dynamic unbalance of projectiles with actual range firings. Measurement of the unbalance of projectiles is made on a dynamic balancing machine prior to firing; sometimes the unbalanced condition is corrected depending on the nature of the test.

(2) In order to measure dynamic unbalance, it is necessary to rotate the projectile in the dynamic balancing machine. Any unbalance will set up centrifugal forces causing a vibration of the machine. This vibration is measured either by (a) creating an artificial unbalance in the machine which exactly counter-balances the unbalance in the projectile, causing the vibration to cease; Figure 12 or by (b) having suitable
electrical pick-ups on the machine, the meter reading of the pick up current being the measure of unbalance in the part.

**Bullet Pull Measurements.**

(1) The force required to de-bullet a projectile from its cartridge case considerably affects the interior ballistics of a projectile. Crimping of the case to the projectile should be accomplished to give a uniform de-bulleting force in order to maintain a uniform velocity from round to round, this becomes increasingly important as a gun becomes worn. Acceptance specifications of complete rounds of ammunition require that this bullet pull force be verified as falling within specified limits.

(2) Bullet pull is measured by inserting the complete round of ammunition in a universal testing machine, a lower clamping device holds the bottom of the cartridge case and an adjustable sensitive cross head grips the projectile body between the rotating band and bourrelet. Motion of the sensitive cross head at specified strain rates produces hydraulic pressure in the loading cylinder of the machine, load indication read directly on the dial of the test machine is a function of this hydraulic pressure.

(3) In actual practice, hydraulic and pneumatic grips operated by foot pedals are used to simplify and speed up insertion and removal of the test rounds. The operator for safety reasons conducts the debulleting test by remote control from a barricaded room.

**Radiographic Inspection.** Soundness of explosive or inert filler of projectiles, alignment of fuse components, and wall thickness are frequently checked by radiography. Radiography is also useful in determining seating of rotating band. Industrial radiographic equipment is ordinarily used; the usual technique is to take exposures in two planes 90° apart.

b. **Ultrasonic Inspection.**

(1) Ultrasonics is a relatively new method of inspection. It is used not only as an inspection device such as for checking the soundness of armor piercing shot; it is also used to measure wall thickness of projectiles, cartridge cases, and bomb and rocket bodies. A commercial ultrasonic instrument the Reflectoscope operates by sending high frequency pulsating sound waves in the material to be tested through a quartz crystal search unit. The waves pass through the crystal and are reflected back from the other side. The elapsed time between sending and return is registered and shown on the screen of a cathode ray tube as vertical indication along a line of square wave markers. Internal defects cause a difference in elapsed time and result in deviations from the line, thus internal defects are shown. The Reflectoscope can also be used for measuring thickness by testing from one side only by establishing calibration markings on the scope with materials of known thickness.
TECHNIQUES FOR MEASUREMENT OF AIRCRAFT BOMBS

SECTION II

Dimensions. The following dimensions of bombs should be measured to the requisite accuracy:

a. Body diameter
b. Fin span
c. Overall length (exclusive of nose fuse)
d. Body length
e. Fin length

Other dimensions, such as position of lugs, fuse dimensions, fin assembly dimensions, should be measured as required.

Weight. The following weights should be measured to the requisite accuracy:

a. Body - empty
b. Adapter - booster
c. Complete body assembly, as dropped
d. Fin assembly
e. Nose fuse
f. Tail fuse
g. Complete bomb, as dropped.

Center of Gravity. Both the center of gravity of the complete assembly and of the body assembly alone, are usually required. These measurements are made as described in Method II on page 7. If it is desired to record the center of gravity as distance from the nose of the body, either the position of the bomb in the trough may be reversed (Figure 4) or the distance of the center of gravity from the base may be subtracted from the appropriate overall length.

Moments of Inertia. If a bomb were without lugs and fuses, it would have its mass distributed symmetrically about the longitudinal axis of the body and fin assembly, and one principle axis of rotation would coincide with this longitudinal axis as in the case of artillery projectiles. The measurements and computations of moments of inertia of bombs are made as if the asymmetric portions of the complete assembly were without weight. The resultant moments of inertia deviate so little
from the true principal moments of inertia, that they are the measurements conventionally used.

The technique by which these measurements are made, is described on pages 8, 9, above. An elaborate timing system is not necessary, however; due to the long periods of oscillation, the method explained on page 8 is always adequate.

ELIZABETH R. DICKINSON

ELIZABETH R. DICKINSON
APPENDIX I

Figures

Figure 1  Typical projectile
Figure 2  Center of gravity apparatus for small projectiles
Figure 3  Graph: Center of gravity determination
Figure 4  Center of gravity apparatus for large projectiles
Figure 5  Moment of inertia apparatus
Figure 6  Torsion pendulum suspension
Figure 7  Center of gravity, and transverse moment of inertia holder
Figure 8  Eccentricity apparatus
Figure 9  Yaw-major axis apparatus
Figure 10 Tracings of fin contour
Figure 11 Surface finish record
Figure 12 Dynamic Balance Apparatus
APPENDIX I

A - FUZE
B - OGIVE
C - BOURRELET, FRONT
D - BODY
E - BAND, DRIVING
F - GROOVE, CRIMPING
G - BOURRELET, REAR
H - BOATTAIL

FIG. 1
CENTER OF GRAVITY DETERMINATION
C.G. 105 MM M1 HOWITZER PROJECTILE

C.G. FROM BASE = 7.590 - .375 = 7.225"
APPENDIX I

CENTER OF GRAVITY APPARATUS FOR LARGE PROJECTILES

FIG. 4
APPENDIX I

a. Small Torsion Pendulum

b. Large Torsion Pendulum

c. Set of Test Masses

FIG. 5  Moment of Inertia Apparatus
APPENDIX I

TOP FIXTURE

LEVEL RIGID PLATE

TUNGSTEN WIRE

DISC GRADUATED IN DEGREES

MIRROR

BOTTOM FIXTURE

CROSS SECTION OF FIXTURES

TOP FIXTURE

BOTTOM FIXTURE

TORSION PENDULUM SUSPENSION

FIG. 6
APPENDIX I

Either a pointer (for C.G.), or the suspension fixture (for moment of inertia) may be screwed into this hole.

Knife edge replaceable by a counterweight for C.G. determination.

Adjustable elevator for accommodating projectiles of various calibers.

Center of gravity and moment of inertia holder.

Fig. 7
APPENDIX I

PROJECTILE OF WEIGHT (M)

COUNTERWEIGHT

FULCRUM

REFERENCE MARK ON "VEE"

ECCENTRICITY APPARATUS FIG. 8
APPENDIX I

A - MECHANICAL APPROACH

B - OPTICAL APPROACH

POSITION 1

YAW - MAJOR AXIS

APPARATUS

FIG. 9
FIG. 10 Fin Contour Records
Fig. 11

Surface Finish Record
APPENDIX I

COMPUTATION OF UNBALANCE

Let the following sketch represent the relative positions of the pivot points with respect to the compensator on the balancing machine:

\[ \begin{array}{c}
L.H \quad R.H \quad W \\
| a | b | c |
\end{array} \]

where L.H. = Left hand plane containing pivot point.
R.H. = Right hand plane containing pivot point.
W = Plane passing through compensator.
WL = Compensator reading (inch ounces).

Then the unbalance in the L.H. plane can be computed by taking moments about pivot R.H. as follows: unbalance in L.H. (inch ounces) = \( \frac{WL \times b}{a} \), and the unbalance in the R.H. plane by taking moments about pivot L.H. as follows: unbalance in R.H. (inch ounces) = \( \frac{WL \times c}{a} \). The unbalance at the periphery in each plane is then unbalance (inch ounces) \( \frac{\text{shell's radius at that location}}{a} \).

The longitudinal plane including the compensator's major axis and the pivot point in question will locate the unbalance relative to the shell base time reference position.

Fig. 12
Determination of k and l by the least squares method.

The least squares method of determining the most probable values of unknowns, given a greater number of equations than there are unknowns, is explained below by means of a numerical example.

Let the moments of inertia of the three test masses be 2 lb-in$^2$, 3 lb-in$^2$ and 4 lb-in$^2$ and their periods of oscillation 5 seconds, 6 seconds and 7 seconds. The equations are then set up in tabular form using the coefficients of the unknowns thus:

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k I</td>
<td>$\bar{r}^2$</td>
</tr>
<tr>
<td>(1)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(2)</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>(3)</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>(4)</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>(5)</td>
<td>29</td>
<td>354</td>
</tr>
<tr>
<td>(6)</td>
<td>[3]</td>
<td></td>
</tr>
</tbody>
</table>

Lines (1), (2) and (3), in the foregoing, represent the three equations; e.g. (1), $1 \bar{r} + 2 \bar{k} = 25$.

Line (4) represents the sum of the three equations (each coefficient is theoretically multiplied by the coefficient of $\bar{r}$ in its own equation; but this being 1, the sum results) $3 \bar{r} + 9 \bar{k} = 110$.

Line (5) represents the sum of the three equations when each coefficient is multiplied by the coefficient $k$ in its own equation. In other words equation (1) is multiplied by 2 (Col. B, line 1) to give $2 \bar{r} + 4 \bar{k} = 50$ equation (2) is multiplied by 3 (Col. B, line 2) to give $3 \bar{r} + 9 \bar{k} = 108$ equation (3) is multiplied by 4 (Col. B, line 3) to give $4 \bar{r} + 16 \bar{k} = 196$

The sum: $9 \bar{r} + 29 \bar{k} = 354$
Thus lines (4) and (5) represent a reduction of the three original equations, to two simultaneous equations:

\[3 \bar{i} + 9 \bar{k} = 110\]
\[9 \bar{i} + 29\bar{k} = 354\]

Because the coefficient of \(i\) in the second equation, will always be the same as the coefficient of \(k\) in the first equation, it is not necessary to record it.

The figure in brackets at the beginning of line (6) is the quotient resulting from dividing the coefficient of \(k\) by that of \(i\) in line (4): \(9/3 = 3\). Line (6) represents line (4) multiplied by this quotient and then subtracted from line (5) thus:

\[
(5) \text{ represents: } 9 \bar{i} + 29\bar{k} = 354
\]
\[
\text{Line (4) } \times 3 \quad 9 \bar{i} + 27\bar{k} = 330
\]
\[
\text{Difference} \quad 0 + 2\bar{k} = 2\bar{h}
\]

Because the coefficient of \(i\) will always be zero, it is not necessary to record it.

Thus line (6) represents a reduction of the three original equations, to one equation, with one unknown, which is immediately solved, line (7):

\[2\bar{k} = 2\bar{h}; \quad \bar{k} = \frac{2\bar{h}}{2} = 12\]

Substituting this value in line (4), we obtain line (8):

\[3 \bar{i} + 9\bar{k} = 110\]
\[\text{Since } \bar{k} = 12\]
\[3 \bar{i} + (9)(12) = 110\]
\[3 \bar{i} = 110 - (9)(12)\]
\[\bar{i} = \frac{110 - (9)(12)}{3} = 6.667\]

As a check on the numerical accuracy of the computation, substitutions are made in line (5), remembering that it represents the equation \(9 \bar{i} + 29\bar{k} = 354\),
\[(9)(6.667) + (29)(12) = 354\]

Though at first reading this computation may seem long and complicated, in reality it goes very quickly on a computing machine. The increased accuracy obtained is well worth the trouble of learning this technique, because experimental errors are reduced to a minimum by giving a truer overall trend.
When this method is used, employing three test masses, an accuracy of 0.05% can be obtained. These masses should be so designed that their weight is within 5% of the weight of the projectile. Also, if the estimated moment of inertia of the projectile is \( I \); the moments of the three masses should be approximately \( \frac{2I}{3} \), \( I \) and \( \frac{4I}{3} \). 
APPENDIX III

DIAGRAMMATIC EXPLANATION - ECCENTRICITY DETERMINATION

\[ \tan \phi = \frac{C - D}{A - B} \]

**EXAMPLE**

**LET:**
- \(A = 7\)
- \(B = 5\)
- \(C = 16\)
- \(D = 10\)

\[ \tan \phi = \frac{6}{2} \]

\(\phi = 71^\circ 34'\)

**CASE I**

- \(A - B: \text{POSITIVE}\)
- \(C - D: \text{POSITIVE}\)

\(\therefore \phi \text{ LIES IN QUADRANT I}\)

**CASE I**
**APPENDIX III**

**DIAGRAMMATIC EXPLANATION ECCENTRICITY DETERMINATION**

\[ \tan \phi = \frac{C-D}{A-B} \]

**EXAMPLE**

**LET:**

A = 5
B = 7
C = 16
D = 10

\[ \tan \phi = \frac{6}{-2} = -3 \]

\[ \phi = 108^\circ 26' \]

**CASE II**

A - B : NEGATIVE
C - D : POSITIVE

\[ \therefore \phi \text{ LIES IN QUADRANT II} \]
APPENDIX III

DIAGRAMMATIC EXPLANATION ECCENTRICITY

DETERMINATION

\[ \tan \phi = \frac{c - d}{a - b} \]

EXAMPLE

LET: \[ A = 5 \]
\[ B = 7 \]
\[ C = 10 \]
\[ D = 16 \]

\[ \tan \phi = \frac{-6}{-2} = 3 \]
\[ \phi = 251^\circ 34' \]

CASE III

A - B : NEGATIVE
C - D : NEGATIVE

\[ \therefore \phi \text{ LIES IN QUADRANT III} \]
APPENDIX III

DIAGRAMMATIC EXPLANATION ECCENTRICITY DETERMINATION

\[ \tan \phi = \frac{C-D}{A-B} \]

**EXAMPLE**

**LET:**
- \( A = 7 \)
- \( B = 5 \)
- \( C = 10 \)
- \( D = 16 \)

\[ \tan \phi = \frac{-6}{2} = -3 \]

\( \phi = 288^\circ 26' \)

**CASE IV**

- \( A - B \): POSITIVE
- \( C - D \): NEGATIVE

\( \therefore \phi \) LIES IN QUADRANT IV

**CASE IV**
APPENDIX III

COMPLETE GEOMETRIC DIAGRAM OF ECCENTRICITY DETERMINATION

LET: A = 5, B = 7, C = 16, D = 10, H = 6, M = 600

THEN E (ECCENTRICITY) = 0.032 in.

\( \phi = 108^\circ 26' \)

\[
\tan \phi = \frac{C-D}{A-B} = -3
\]

\[
R = C \sqrt{(A-B)^2 + (C-D)^2} = 0.032\text{ in.}
\]
APPENDIX III

Thus it is seen that the determination of \( e \) tells us that the center of gravity in this plane, is at a distance of 0.032 inches from the longitudinal axis of the projectile; or somewhere on a circle whose radius is 0.032 inches. The determination of \( \phi \) tells us that the center of gravity in this plane is on a line which makes an angle of 108°26' with the 0° reference line. Hence, the center of gravity (marked X on the diagram) is at the intersection of the circle and the line: a point 0.032 inches from the longitudinal axis, 108°26' from the 0° reference mark.