POLARIZED RADIANCE. VOLUME II. POLARIZED SPECTRAL EMITTANCE FROM 4 TO 14 MICROMETERS

J. R. Maxwell, et al

Environmental Research Institute of Michigan

Prepared for:
Ballistic Research Laboratories

May 1974
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Volume II of this report provides the Ballistic Research Laboratories with a description of a model for predicting emission polarization from paints. The report includes a description of measurements that were made along with graphs of measurement data. Validation of the model shows that model predictions agree with measurement to within the measurement accuracy 70% of the time, thus allowing degree of polarization to be calculated for smooth-surfaced materials in the thermal IR region for any \( \lambda \) (from about 7 to 15\( \mu m \)) and any polar angle, with (Continued on reverse side)
Block 20. Abstract

only a measurement of spectral emittance from 7 to 15μm at any near-normal incidence required.

A listing of model parameters appropriate to the sample paints supplied by BRL is included along with a document explaining the use of the computer programs and listings of the programs as well.
The work reported herein, covering the period 10 April 1972 to 31 December 1972, was carried out by the Infrared and Optics Division of the Environmental Research Institute of Michigan (formerly the Willow Run Laboratories of The University of Michigan), Ann Arbor, Michigan. The work was performed under Contract DAAD05-72-C-0216 for the Army Ballistic Research Laboratories, and was done in three parts, each of which represent one volume.

The three volumes are:

I  - Polarized Bidirectional Reflectance With Lambertian or Non-Lambertian Diffuse Component.

II - Polarized Spectral Emittance From 4 to 14 μm.

III - Wavelength Dependence of Polarized Bidirectional Reflectance.

The internal number for volume II of this report is 192500-1-T(II)
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1. INTRODUCTION

Most materials on the earth's surface emit a large portion of their radiated energy in the thermal-infrared wavelength region. If irregularities of the surface of a natural target are very large compared to the wavelength of emitted radiation, total emissivity measurements are the prime source of information. However, if the surface irregularities of a target are small compared to the wavelength of emitted radiation, the surface tends to be more specular and the electric vectors of this emitted radiation will vibrate in preferential directions, giving rise to an observable emission polarization. For relatively smooth surfaces, therefore, polarization measurements are an additional source of information. Since the surfaces of military targets are generally smoother and have more ordered geometric patterns than naturally occurring materials, polarization is a potentially important parameter for target discrimination.

The purpose of this report is to describe a phenomenological model for predicting emission polarization of paints using as input only a single, near-normal reflectivity spectrum.

2. MODEL DESCRIPTION

For the case of an emitting polished surface, that portion of the emitted radiance at some angle $\theta$ to the target surface normal that has its electric vector vibrating perpendicular to the plane of emission (containing the surface normal and the propagation vector), divided by half the radiance emitted by a blackbody at the same temperature, is called the perpendicular component $E_\perp(\theta)$ of the emissivity. [Note: All the emissivity components are wavelength dependent, but the $\lambda$ notation will be suppressed.]

Likewise, the parallel emissivity component $E_\parallel(\theta)$ defines that portion of radiation emitted with electric vector vibration parallel to the plane of emission. The total emissivity $E(\theta)$ is half the sum of $E_\perp(\theta)$ and $E_\parallel(\theta)$.

The degree of polarization for emission can be defined as:

$$P_E = \frac{[E_\parallel(\theta) - E_\perp(\theta)]}{[E_\parallel(\theta) + E_\perp(\theta)]}.$$

(1)
The relationship between the emission polarization components and reflection polarization components can be found from a consideration of the reflection experiment where source (at $\theta_i$) and observed (at $\theta_r$) are located on opposite sides of the target surface normal at an angle $\theta = \theta_i = \theta_r$. Let $R_\perp$ and $R_\parallel$ be the reflectivity components for radiation with electric vectors vibrating perpendicular and parallel, respectively, to the plane of incidence containing the incident and reflected rays. With the constraint that $\theta = \theta_i = \theta_r$, implying that the plane of emission coincides with the plane of incidence, Kirchhoff's law dictates (for equilibrium conditions) that

$$E(\theta)_{\perp} = 1 - R(\theta)_{\perp}, \quad (2a) \quad E(\theta)_{\parallel} = 1 - R(\theta)_{\parallel}. \quad (2b)$$

The reflectivity flux components are functions only of $\theta_i = \theta_r = \theta$, and the complex index of refraction of the target material is given by $N = n - ik$. Expressions for $R_{\perp}$ and $R_{\parallel}$ in terms of $u_e$, $n$, and $k$ are as follows [1]:

$$R_{\perp}(\theta) = \frac{[(a - \cos \theta)^2 + b^2]}{[(a + \cos \theta)^2 + b^2]} \quad (3a)$$

$$R_{\parallel}(\theta) = \frac{[(a - \cos \theta)^2 + b^2]}{[[(a + \cos \theta)^2 + b^2]} \times \frac{[(a - \sin \theta \tan \theta)^2 + b^2]}{[[(a + \sin \theta \tan \theta)^2 + b^2]} \quad (3b)$$

where

$$a^2 = \frac{1}{2} \left( n^2 - k^2 - \sin^2 \theta + \left[ 4n^2k^2 + (n^2 - k^2 - \sin^2 \theta)^2 \right]^2 \right)$$

and

$$b^2 = \frac{1}{2} \left( -n^2 + k^2 + \sin^2 \theta + \left[ 4n^2k^2 + (n^2 - k^2 - \sin^2 \theta)^2 \right]^2 \right)$$

From Eqs. (1), (2), and (3), therefore, it is possible to express the degree of emission polarization $P_E$ as a function of $\theta$, $n$, and $k$. 

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For materials which exhibit reststrahlen bands in the thermal-IR wavelength region, \( n \) and \( k \) are strongly wavelength-dependent in that region. The wavelength dependence of \( P_E \) for a given material can be calculated from Eqs. (1), (2), and (3) if the dispersion curves of \( n \) and \( k \) are known. Once these dispersion curves have been determined as continuous functions of wavelength for a given paint, it is then possible to calculate the emission polarization for all \( \theta \) as a function of \( \lambda \). This will be calculated for several O.D. paints.

One means for determining the indices of refraction is the classical oscillator fitting method. The complex index of refraction \( N = n - ik \) can be calculated as a function of frequency \( \nu \), according to a classical oscillator model of crystals [2], from the following equations:

\[
2nk = \sum_{j} 4\pi p_j \nu_j \left[ \frac{\nu_j \nu_j}{(\nu_j^2 - \nu^2)^2 + \gamma_j^2 \nu_j^2} \right],
\]

\[
\eta^2 - k^2 = c_{\infty} + \sum_{j} 4\pi p_j \nu_j \left[ \frac{\nu_j^2 - \nu^2}{(\nu_j^2 - \nu^2)^2 + \gamma_j^2 \nu_j^2} \right], \quad (4)
\]

where \( p_j, \gamma_j, \) and \( \nu_j \) are the strength, width, and frequency, respectively, of the \( j \)th lattice oscillator and \( c_{\infty} \) is the high frequency dielectric constant. If these oscillator parameters are known, \( n \) and \( k \) are calculable from Eq. (4). If they are unknown, they can be estimated by the following procedure. Initial guesses for the oscillator parameters are assumed, (see Appendix D) and a normal incidence Fresnel reflectivity curve \( R(\nu) \) is calculated from the equation:

\[
R(\nu) = \left\{ \frac{[n(\nu) - 1]^2 + k^2(\nu)}{[n(\nu) + 1]^2 + k^2(\nu)} \right\}
\]

(5)
A computer program, which employs an IBM scientific subroutine, [see Appendix F] compares the theoretical reflectivity curve calculated from Eq. (5) with an experimental normal incidence spectral reflectivity curve for a smooth surface of the specimen, changes the oscillator parameters in Eq. (4), calculates a new theoretical curve, and reiterates until a good fit is made between theoretical and experimental curves. With the oscillator parameters of the best-fitting theoretical curve, the indices can then be calculated from Eq. (4). (See Appendix D for use of Oscillator Program.)

3. MEASUREMENTS

The measurements in this program consisted of two phases. The first phase consisted of a spectral scan of the emittance from each sample with a normal angle of observation. The second phase consisted of angular scans of the emittance with receiver polarizer set alternately perpendicular and parallel to the receiver incidence plane. In this latter case, each scan took place at a particular wavelength.

In the following paragraphs we provide a brief description of the instrument which was used for these measurements, followed by data which resulted.

3.1 The Instrument

The field infrared spectroradiometer (FISR) which was utilized is a double-beam instrument with output proportional to the radiance difference between the objects filling the fields of the two Herschelian reflecting telescopes. The measurement of the difference is accomplished by a reflective chopper which alternately samples the radiation from the two telescopes. After being chopped, the radiation from the two telescopes is imaged at a common focal plane at which a field stop slit is placed, limiting the FOV to a 0.6' x 2.5° field. The fore-optics just described are illustrated in Figure 1. The chopped energy passing through the field stop slit is refocused, by a 90° off-axis ellipsoid mirror, onto an entrance aperture which lies immediately in front of the circular variable-filter (CVF) monochromator.
Figure 1. SCHEMATIC OF FFR FORE-OPTICS
The CVF then transmits a narrow spectral band of this energy to a 45° off-axis ellipsoid mirror, which refocuses it onto the detector.

The generated ac detector signal is amplified and then synchronously demodulated. The synchronous demodulator transforms those components of the signal which have the particular frequency of the chopper modulation into a proportional dc signal, while rejecting those components with different frequency (i.e., noise). Thus, at the output a dc signal is presented which has a value at any point in time proportional to the energy difference between the two optical beams at a particular wavelength determined by the CVF position. Mechanical rotation of the CVF provides a spectral scan. The output voltage is recorded by using a digital voltmeter as an analog-to-digital converter and printing on a digital paper-tape printer, and the wavelength is registered simultaneously by printing the digital output of a shaft encoder mechanically linked to the CVF.

The spectrometer portion of the system utilized a circular variable filter for spectral dispersion in the range from 2.5 to 14.5μm. The CVF has the advantage of providing more efficient energy transmission than a prism disperser, allowing faster spectral scan rates or narrower spectral resolution for a given energy input.

In order to achieve high system sensitivity, an indium antimonide and a mercury doped germanium detector are used to cover the wavelength regions 2.5 to 5.5μm and 4.5 to 14.5μm, respectively. These detectors are readily interchangeable, so one may be replaced by the other during the course of scanning from 2.5 to 14.5μm without stopping the instrument.

Two temperature-controlled blackbody calibration sources are provided for insertion at the entrance optics in either or both of the two channels. In the normal mode of operation, a blackbody remains in one channel, and the other channel, having previously been calibrated, views the target of interest. In this way the system can be used to measure absolute radiance. Alternatively, if the target temperature is known, or if a black surface at the same temperature is available, the spectral emittance of the surface can be determined. These blackbodies are only suitable for the spectral
range 4.5 to 14.5μm. Calibration sources below 4.5μm have not as yet been implemented. However, for these reflective wavelengths (less than 4.5μm), a calibrated reflectance panel may be inserted in the reference beam and the instrument then used to make relative reflectance measurements.

Supplementary instrumentation has been developed to permit the basic FISR equipment to be used for obtaining directional polarized emissivity data. Two similar samples maintained at different known temperatures are separately viewed through each port of the FISR, and the difference in radiance values is measured. From this measured difference, with the available temperature information, the emissivity can be calculated. The sample temperatures are maintained by thermally controlled water baths. The sample holder is designed to be operated either at a fixed viewing angle or by revolving and permitting a scan of the sample surface.

Complete details of the FISR can be found in reference 4.

3.2 Spectral Emittance

Figures 2, 3 and 4 show the spectral emittance data in plotted form. In each case there are many scans, some of which cover different spectral regions than others. Each scan number has its own symbol so it can be roughly identified on the plot. Shown also are curves representing both accuracy and precision of the measurements. Accuracy is determined with respect to both systematic and random errors. Precision is determined with respect to noise in the recorded voltage. (Reference 4 provides a complete discussion of determination of accuracy and precision in this context.)

Note that for all three samples, there is an apparent dip in the spectrum between 9.5μm and 10μm. This dip is interpreted to be a reststrahlen band. Structure of this sort arises from the wavelength dependence of the indices of refraction. (In section 1, we have described how one calculates the indices of refraction by use of a classical oscillator fitting method.)
Note also that in Figure 3 there is an apparent discrepancy in that scans numbers 3 and 4 are translated upward with respect to scans 2 and 23. This discrepancy appears to be due to a combination of calibration error and difficulty of maintaining accurate measurement of environment temperature during the measurements. The problem is still being analyzed. In the validation (see Section 4) scans 3 and 4 were used. In view of the apparent difficulty it is not clear that these were the most appropriate.
3.3 Polarized Emittance vs. Angle

Figures 5 through 16 show the variation for each sample at a variety of wavelengths of relative polarized emittance. In each case the curve which represents parallel polarization is a plot of \( e_{||}(\theta) = \frac{E_{||}(\theta)}{E_{||}(0)} \) and that which represents perpendicular polarization is \( e_{\perp}(\theta) = \frac{E_{\perp}(\theta)}{E_{\perp}(0)} \).

Therefore, for each angle, degree of polarization is extracted from the data by equation (1), where substituting \( e_{||}(\theta) \) and \( e_{\perp}(\theta) \) for \( E_{||}(\theta) \) and \( E_{\perp}(\theta) \) leaves \( P_e \) unchanged, since \( E_{||}(0) = E_{\perp}(0) = E(0) \). [See Appendix A]. Comparison of model predictions with selected measurement data from this group is described in Section 4.
FIGURE 5. Relative Polarized Emittance vs. Polar Angle
AO-2017 \( \lambda = 9.3 \) μm
Figure 6. Relative Polarized Emittance vs. Polar Angle

AO-2017 = 10.0 mm
FIGURE 7. Relative Polarized Emittance vs. Polar Angle
AO-2022 $\lambda = 5.3\mu m$
FIGURE 8. Relative Polarized Emittance vs. Polar Angle
AO-2022 λ = 8.3μm
FIGURE 9. Relative Polarized Emittance vs. Polar Angle
AO-2022 \( \lambda = 9.7 \mu \text{m} \)
FIGURE 10. Relative Polarized Emittance vs. Polar Angle
AO-2022 λ = 10.6μm

POLAR ANGLE θ 25
FIGURE 11. Relative Polarized Emittance vs. Polar Angle
AO-2022   $\lambda = 12.0\mu m$
FIGURE 12. Relative Polarized Emittance vs. Polar Angle
AO-2023 $\lambda = 5.3\mu m$
FIGURE 13. Relative Polarized Emittance vs. Polar Angle
AO-2023  \( \lambda = 8.3 \mu m \)
FIGURE 14. Relative Polarized Emittance vs. Polar Angle
AO-2023   $\lambda = 9.7\mu$m
FIGURE 15. Relative Polarized Emittance vs. Polar Angle
AO-2023 $\lambda = 10.6\mu m$
4. MODEL VALIDATION

Figure 17 shows results of the emittance model for an O. D. paint, calculated last year as part of the Target Signature Analysis Program of ERIM, for the U. S. Air Force. In this figure the degree of polarization of the emittance of an O. D. paint sample was calculated as a function of wavelength for the case of no reflective input other than clear sky radiance. The target was assumed to be at a temperature \( T = 300^\circ K \) and the angle of observation was chosen to be \( \theta = 70^\circ \). In this theoretical treatment, the paint was assumed to be a perfectly specular surface, with negligible volume and multi-surface reflection. A normal-incidence reflectivity curve was fit with the classical oscillator model discussed above to find values for the complex index of refraction \( N = n - ik \) as a function of wavelength. These indices were then fed into Fresnel equations for \( R_{||} \) and \( R_{\perp} \) (Eq. 3) from which \( E_{||}(\theta) \) and \( E_{\perp}(\theta) \) were calculated from Eq. (2). Emission polarization was then calculated from Eq. (1).

The degree of polarization of most naturally-occurring materials is low in the thermal IR, compared to the polarization of painted surfaces. The only known measurements of emission polarization from natural materials are given in Table I, which shows the degree of polarization for a few natural targets measured broadband (approximately 8 to 14\( \mu \)m) under clear, humid sky conditions [3]. The highest degree of polarization is approximately 2% for these materials, as compared with approximately 9% for an O.D. paint sample measured at the same angle (\( \theta = 75^\circ \)) under the same conditions. Had the measurements been in a 2\( \mu \)m-wide band centered near 9.8\( \mu \)m, Figure 17 predicts that for the O.D. paint the degree of polarization would have been appreciably higher. The increase in the degree of polarization in the reststrahlen band for target materials may be useful in the discrimination of such targets from backgrounds like water, which also can have a high degree of polarization.
THEORETICAL PERCENT POLARIZATION AND NORMAL EMISSIVITY VERSUS WAVELENGTH FOR O.D. PAINT (Sample A01792)

KEY: ▲ Measured Normal Emissivity
    □ Calculated Normal Emissivity (Connected by solid line)
    ○ Calculated Percent Polarization for θ = 70°, Neglecting Sky Radiance
    □ Calculated Percent Polarization for θ = 70°, and Target Temperature = 300°K, Including Clear Sky Radiance

WAVELENGTH (μm)

FIGURE 17

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TABLE I
POLARIZATION FIELD MEASUREMENTS

(August 5, 1970, Clear Sky Conditions, Willow Run Airport)

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<th>Name</th>
<th>$\theta$ (Measured from average macroscopic surface normal)</th>
<th>$L_{\parallel} \left( \frac{\text{mw}}{\text{cm}^2 \text{sr}} \right)$</th>
<th>$L_{\perp} \left( \frac{\text{mw}}{\text{cm}^2 \text{sr}} \right)$</th>
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</table>

$L_{\parallel}$ = detected radiance from target with electric vector vibrating parallel to plane of emission

$L_{\perp}$ = detected radiance from target with electric vector vibrating perpendicular to plane of emission
For the natural materials in Table I, the degree of polarization generally increased with increasing observation angle for $\theta > 50^\circ$, but the measured polarization was so slight for $\theta < 50^\circ$ that it fell below noise levels of the radiometer system. As for angular dependence of polarization from O. D. paint, Table II gives measured and calculated values for the degree of polarization of the emittance for $\theta$ from 10° to 70° for a wavelength of 9.83\(\mu m\), the center of the reststrahlen band. For $\theta > 40^\circ$ the emission polarization of paints can be quite large compared with that of natural target materials.

The good agreement between calculated results and the measured values shown in Table II were encouraging enough to justify further validation with respect to other O.D. paint samples. Under this BRL contract, three O.D. painted surfaces were studied using an ERIM spectrometer which enabled us to measure spectral radiance distributions as well as polarization dependent radiance as a function of observation angle. The samples are labeled 2017, 2022, and 2023.

Compared with the earlier sample (1792), these three O.D. paints exhibited less pronounced reststrahlen bands; hence, their degree of polarization is neither as large or as wavelength dependent. Table III shows the measured and calculated (from the above model) degree of polarization for the three O.D. paints 2017, 2022, and 2023 at angular increments of 10° for wavelengths of 10.6\(\mu m\), 12.0\(\mu m\), 9.7\(\mu m\), 8.3\(\mu m\), and 5.3\(\mu m\). No calculations were made for the 5.3\(\mu m\) wavelength primarily because the data were very noisy below 7\(\mu m\).

As can be observed from the "Model Result" and "Measured" columns, the model always predicts a higher degree of polarization than is measured, owing to the assumptions in the model that the paint surfaces are perfectly smooth and that the paint layer is optically thick. The middle column under each paint sample shows the product of multiplying the sample result times a constant factor of 0.75. The physical reason for choosing a constant
correction factor is that the surface roughness should affect radiation from all wavelengths and angles about the same, if the roughness is on a much different scale of magnitude than the wavelength variation (5µm – 12µm) used for the calculations of this model. The magnitude of the constant factor, however, was chosen solely on an empirical basis. As shown in Table III, the product of the model result and the 0.75 factor is within experimental error of the polarization measurements for 6 of 6 measurements on O.D. paint 2017, for 22 of 28 measurements on O.D. paint 2022, and for 15 of 27 measurements on O.D. paint 2023. The calculation of experimental error is explained in Appendix A. Overall, these calculated results for three paint samples and 4 wavelengths fell within experimental error for 43 of 61 measurements, or 70% of the time. On this basis, the model (including the 0.75 constant multiplicative factor) has been verified for degree of polarization calculations.

Figures 18, 19, and 20 are plots of relative polarized emittance versus polar angle at λ = 10.6µm for O.D. paints 2017, 2022, and 2023 respectively. They show how the parallel and perpendicular components of emittance calculated by the model compared with experimental measurements. No multiplicative or other factors have been employed to alter the oscillator model results in these figures. The relative polarized emittances are

\[ e_{||}(\theta) = \frac{E_{||}(\theta)}{E(0)} \quad \text{and} \quad e_{\perp}(\theta) = \frac{E_{\perp}(\theta)}{E(0)}. \]

Figure 18 shows a good correlation between theoretical and observed relative polarized emittances for paint 2017, where 4 unpolarized, near-normal spectral emittance curves were averaged to produce the input for the oscillator model. Figure 19 shows what can happen when the unpolarized spectral emittance curves have measurement discrepancies. An average of the test two curves was used as oscillator model input to produce the dashed line. The two worst curves were used to produce the results shown by the rectangles, and the circles show what happens when all four curves are averaged prior to application of the
TABLE II

COMPARISON OF THEORETICALLY CALCULATED AND EXPERIMENTALLY MEASURED DEGREE OF POLARIZATION FOR EMITTANCE FOR $\lambda = 9.83\mu m$

(O.D. Paint A01792)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Calculated Degree of Polarization</th>
<th>Measured Degree of Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>0.58%</td>
<td>0.25%</td>
</tr>
<tr>
<td>20°</td>
<td>2.37%</td>
<td>1.06%</td>
</tr>
<tr>
<td>30°</td>
<td>5.45%</td>
<td>3.47%</td>
</tr>
<tr>
<td>40°</td>
<td>10.03%</td>
<td>7.50%</td>
</tr>
<tr>
<td>50°</td>
<td>16.35%</td>
<td>12.56%</td>
</tr>
<tr>
<td>60°</td>
<td>24.71%</td>
<td>20.25%</td>
</tr>
<tr>
<td>70°</td>
<td>35.44%</td>
<td>32.46%</td>
</tr>
</tbody>
</table>
### TABLE III

**COMPARISON OF THEORETICALLY CALCULATED AND EXPERIMENTALLY MEASURED DEGREE OF POLARIZATION FOR O.D. PAINTS**

<table>
<thead>
<tr>
<th>O.D. Paint A02017</th>
<th>O.D. Paint A02022</th>
<th>O.D. Paint A02023</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>λ = 10.6μm</strong></td>
<td><strong>λ = 12.0μm</strong></td>
<td><strong>λ = 9.7μm</strong></td>
</tr>
<tr>
<td><strong>λ = 8.3μm</strong></td>
<td><strong>λ = 5.3μm</strong></td>
<td></td>
</tr>
<tr>
<td>θ (°)</td>
<td>Model Result</td>
<td>Multiplied by 0.75</td>
</tr>
<tr>
<td>10°</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>20°</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>30°</td>
<td>3.3</td>
<td>2.5</td>
</tr>
<tr>
<td>40°</td>
<td>6.1</td>
<td>4.6</td>
</tr>
<tr>
<td>50°</td>
<td>10.2</td>
<td>7.7</td>
</tr>
<tr>
<td>60°</td>
<td>15.9</td>
<td>11.9</td>
</tr>
<tr>
<td>70°</td>
<td>23.7</td>
<td>17.8</td>
</tr>
</tbody>
</table>

| λ = 10°           | Model Result      | Multiplied by 0.75| Measured (%)    | Model Result      | Multiplied by 0.75| Measured (%)    | Model Result      | Multiplied by 0.75| Measured (%)    |
| 10°               | 0.3               | 0.2               | <0.3 ± 1.9      | 0.3               | 0.2               | <0.3 ± 1.9      | 0.6               | 0.5               | <0.5 ± 2.0      |
| 20°               | 1.1               | 0.8               | 0.7 ± 1.9       | 1.8               | 1.4               | 1.6 ± 2.0       |
| 30°               | 2.6               | 2.0               | 2.6 ± 1.9       | 4.1               | 3.1               | 3.3 ± 2.0       |
| 40°               | 8.4               | 6.3               | 7.5 ± 1.9       | 12.0              | 9.0               | 6.6 ± 2.0       |
| 50°               | 13.3              | 10.0              | 12.0 ± 2.0      | 17.6              | 13.2              | 10.2 ± 2.1      |
| 60°               | 20.0              | 15.0              | 18.0 ± 2.2      | 24.7              | 18.5              | 15.7 ± 2.3      |
| 70°               | 22.6              | 17.2              | 21.7 ± 2.4      | 25.3              | 19.0              | 18.6 ± 2.4      |
| 10°               | 0.3               | 0.2               | <0.3 ± 2.0      | 0.3               | 0.2               | <0.3 ± 2.0      | 0.5               | 0.5               | <0.5 ± 2.0      |
| 20°               | 1.7               | 1.3               | 1.3 ± 2.0       | 2.7               | 2.0               | 2.4 ± 2.0       |
| 30°               | 3.8               | 2.9               | 2.9 ± 2.0       | 5.9               | 4.4               | 4.6 ± 2.0       |
| 40°               | 6.9               | 5.2               | 5.4 ± 2.0       | 10.0              | 7.5               | 4.6 ± 2.0       |
| 50°               | 11.0              | 8.3               | 9.7 ± 2.0       | 14.7              | 11.0              | 8.0 ± 2.0       |
| 60°               | 16.2              | 12.2              | 14.6 ± 2.1      | 19.8              | 14.9              | 12.5 ± 2.1      |
| 70°               | 22.6              | 17.2              | 21.7 ± 2.4      | 25.3              | 19.0              | 18.6 ± 2.4      |

| λ = 10°           | Model Result      | Multiplied by 0.75| Measured (%)    | Model Result      | Multiplied by 0.75| Measured (%)    |
| 10°               | 0.4               | 0.3               | <0.3 ± 1.8      | 0.8               | 0.6               | 1.0 ± 1.9      |
| 20°               | 1.8               | 1.4               | 0.7 ± 1.8       | 3.3               | 2.5               | 1.6 ± 1.9      |
| 30°               | 4.2               | 3.2               | 2.0 ± 1.8       | 7.1               | 5.3               | 3.5 ± 1.9      |
| 40°               | 7.4               | 5.6               | 3.5 ± 1.8       | 11.3              | 8.6               | 5.6 ± 1.9      |
| 50°               | 11.3              | 8.5               | 6.3 ± 1.8       | 15.9              | 11.9              | 9.8 ± 1.9      |
| 60°               | 15.4              | 11.6              | 9.5 ± 1.9       | 20.0              | 15.0              | 15.6 ± 2.0     |
| 70°               | 19.6              | 14.7              | 15.0 ± 2.1      | 23.8              | 17.9              | -              |

| λ = 10°           | Model Result      | Multiplied by 0.75| Measured (%)    |
| 10°               | 0.2               |                  |
| 20°               | 0.8               |                  |
| 30°               | 1.8               |                  |
| 40°               | 3.2               |                  |
| 50°               | 5.5               |                  |
| 60°               | 9.0               |                  |
| 70°               | 13.9              |                  |
FIGURE 18. Model-Measurement Comparison—Relative Polarized Emittance vs. Polar Angle  AO-2017 $\lambda = 10.6\mu m$

- Experimental
- Theoretical
FIGURE 19. Model-Measurement Comparison-Relative Polarized Emittance vs. Polar Angle
AO-2022 \( \lambda = 10.6 \mu m \)

- Experimental
- Theoretical using best 2 scans of spectral emittance data.
- Theoretical using worst 2 scans of spectral emittance data.
- Theoretical using all 4 scans of spectral emittance data.
FIGURE 20. Model-Measurement Comparison—Relative Polarized Emittance vs. Polar Angle
AO-2023  \( \lambda = 10.6 \mu m \)

- \( e_{\text{e}} \) = Experimental
- \( e_{\text{i}} \) = Theoretical
oscillator model. This points out the need for better spectral emittance measurements as inputs to the model. Figure 20 shows the poorer results with paint 2023. The oscillator model was applied twice to the average of 4 spectral emittance curves, with almost the same results. The poorer agreement in Figure 20 is obvious also in the smaller than expected calculated degree of polarization results of Table III for paint 2023.

5. CONCLUSIONS

The oscillator model for calculating polarized components of spectral emittance and degree of polarization in the thermal infrared region for smooth-surfaced targets has produced theoretical values that are within experimental measurement accuracy 70% of the time. The accuracy of the model could be improved with better measurements of unpolarized, near-normal spectral emittance values. Such measurements could be improved by utilizing a parabolic reflectometer in the reflection mode instead of FISR, in the emission mode because of the greater absolute accuracy afforded by the former. FISR measurements would still probably be preferred for the relative polarized emittance versus polar angle measurements, because of the induced polarization inherent to the parabolic reflectometer. The model itself could probably be improved by accounting for thin-film transmission of paint on metal, but this might not be required after better spectral emittance measurements are available.

Although the O.D. paints used in this investigation are similar in spectral emittance features, the oscillator model should work at least as well for other smooth-surfaced materials which exhibit marked dispersion (high λ-dependence of the spectral emittance) in the thermal IR region, such as teflon, mylar, and possibly phenolics.

The most important feature of this model is that the degree of polarization can be calculated for smooth-surfaced materials in the thermal IR region for any λ (from about 7 to 15μm) and any polar angle, with only a measurement of spectral emittance from 7 - 15μm at near-normal incidence required. This will save considerable time and money on experimental measurements.
APPENDIX A
DETERMINATION OF EXPERIMENTAL ERROR IN MEASUREMENT OF DEGREE OF POLARIZATION

The equation for degree of polarization can be written as:

\[ P_E(\theta) = \frac{E_{||}(\theta) - E_{\perp}(\theta)}{E_{||}(\theta) + E_{\perp}(\theta)} = \frac{e_{||}(\theta) - e_{\perp}(\theta)}{e_{||}(\theta) + e_{\perp}(\theta)} \]  

(A-1)

where \( e_{||}(\theta) = \frac{E(\theta)}{E(\theta)} \) = relative polarized emittance, parallel component

\( e_{\perp}(\theta) = \frac{E(\theta)}{E(\theta)} \) = relative polarized emittance, perpendicular component

All of the above and following parameters are wavelength and angle dependent, but the \( \lambda \) and \( \theta \) notation will be suppressed here.

The error in \( P_E \) due to errors in measured values of \( e_{||} \) and \( e_{\perp} \), respectively, can be represented by:

\[ \Delta P_{||} = \left( \frac{1}{e_{||} + e_{\perp}} - \frac{e_{||} - e_{\perp}}{(e_{||} + e_{\perp})^2} \right) \Delta e_{||} = \frac{2e_{\perp}}{(e_{||} + e_{\perp})^2} \Delta e_{||} \]  

(A-2)

\[ \Delta P_{\perp} = \left( -\frac{1}{e_{||} + e_{\perp}} - \frac{e_{||} - e_{\perp}}{(e_{||} + e_{\perp})^2} \right) \Delta e_{\perp} = -\frac{2e_{||}}{(e_{||} + e_{\perp})^2} \Delta e_{\perp} \]  

(A-3)

The total error in \( P_E \) can be described as:

\[ \Delta P_E = \sqrt{(\Delta P_{||})^2 + (\Delta P_{\perp})^2} = \left[ \frac{2}{(e_{||} + e_{\perp})^2} \right] \sqrt{(\Delta e_{||})^2 + (\Delta e_{\perp})^2} \]  

(A-4)

Let \( \sigma_p \) be the precision error in the measurement of unpolarized spectral emittance. It will be assumed that \( \Delta e_{||} \) and \( \Delta e_{\perp} \) are equal and that the addition of a polarizer into the optical system increases the precision.
error by a factor of \( \sqrt{2} \), such that

\[ \Delta e_\parallel = \Delta e_\perp = \sqrt{2} \sigma_p \]  

(A-5)

When equation (A-5) is substituted into equation (A-6), the resultant error in \( P_E \) from error measurements in \( e_\parallel \) and \( e_\perp \) is:

\[ \Delta P_E = \frac{2\sigma_p \sqrt{e_\parallel^2 + e_\perp^2}}{(e_\parallel + e_\perp)^2} = 2.828 \sigma_p \frac{\sqrt{e_\parallel^2 + e_\perp^2}}{(e_\parallel + e_\perp)^2} \]  

(A-6)

The \( \Delta P_E \) is multiplied by 100 in Table III to get \( \% \Delta P_E \). It can be observed from equation (A-6) that \( \Delta P_E \) is not a strong function of the values of \( e_\parallel \) and \( e_\perp \), and that \( \Delta P_E \approx 2.828\sigma_p \) for small \( \theta \), where \( e_\parallel \) and \( e_\perp \) are near 1.0 in value.
APPENDIX B

DOCUMENTATION FOR CLASSICAL

OSCILLATOR FITTING PROGRAM

The oscillator fitting program is designed to compute oscillator parameters from which effective indices of refraction can be calculated, according to Eq. (1) of the text. Initial estimates of the number of oscillators responsible for spectral features of the specimen and the strengths (S), frequency positions (NU = \( \frac{\lambda}{10,000} \), for \( \lambda \) in \( \mu \text{m} \)), and widths (GAMMA) of each oscillator must be made. The dielectric constant at infinite frequency (E) must also be made known. [Note: Here E = E\(_\infty\).]

To begin with one assumes at least one oscillator is located at each reflectance maximum in the infrared reflectance spectrum. This determines the initial number of oscillators and their positions (NU). The initial values of E, S, and GAMMA can be taken to be equal to the values determined for the O.D. paint 2017 in Table B-1. Table B-1 gives the result of the oscillator fitting program for the three O.D. paints listed in Table 1 of the text.

The program will refine the position, strength, and shape of the given oscillators, but it will not add or delete oscillators. However, if an oscillator is grossly superfluous, the strength will be decreased, and/or the width will be increased, to such an extent as to make its contribution negligible.

The program returns an error parameter before supplying the new oscillator parameters. If the error parameter is 0, convergence (under the input EPS and EST) has taken place, otherwise it has not. Whether or not convergence has taken place, it is often advisable to look at the oscillator parameters and the emittance values calculated from them to decide if additional oscillators should be added and/or subtracted, or whether the limiting conditions (EPS, EST) should be changed. If the theoretical and experimental reflectance curves do not quantitatively match, i.e. do not peak and trough near the same wavelengths, addition or subtraction of an oscillator is called for. In general, two things are

*See last page of this Appendix for definitions.

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necessary to judge a good fit: a sum of the differences squared (between theoretical and experimental spectral reflectance) less than about $10^{-2}$ and a qualitative match between peaks and valleys of experimental and theoretical spectral reflectance curves. For low-contrast materials (small reststrahlen bands), a good quantitative fit is usually not a sufficient criterion for successful fitting, as seen by the results for sample 2023 in Table B-1 and Table 1 of the text. (See discussion in text of Appendix C.)
TABLE B-1

EFFECTIVE OSCILLATOR PARAMETERS FOR POINTS RESULTING FROM CLASSICAL OSCILLATOR FITTING PROGRAM

<table>
<thead>
<tr>
<th>O. D. Paint Sample</th>
<th>Oscillator Parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>S</td>
<td>NU</td>
<td>GAMMA</td>
</tr>
<tr>
<td>A02017</td>
<td>1.040</td>
<td>0.5421</td>
<td>993.7</td>
<td>0.3234</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1053</td>
<td>1373.9</td>
<td>0.4264</td>
</tr>
<tr>
<td>A02022</td>
<td>1.051</td>
<td>0.0866</td>
<td>787.8</td>
<td>0.1190</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5716</td>
<td>988.4</td>
<td>0.4168</td>
</tr>
<tr>
<td>A02023</td>
<td>0.728</td>
<td>1.976</td>
<td>828.0</td>
<td>0.7384</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2456</td>
<td>1022.6</td>
<td>4.683</td>
</tr>
<tr>
<td>CARD #</td>
<td>FORMAT</td>
<td>VARIABLES(s)</td>
<td>DESCRIPTION</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20A4</td>
<td>A</td>
<td>Any title up to 80 characters in length (including blanks).</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>I5, E15.5, and F10.0</td>
<td>LIMIT, EPS, EST</td>
<td>LIMIT = number of iterations desired (30 is a good choice); EPS = test value for the expected absolute error (see the write-up on FMFP; despite what it says 0.05 has worked reasonably well); EST = estimated value of F(x) (also see Appendix F).</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>I5</td>
<td>N</td>
<td>Three times the number of cards of type 4, plus 1 (i.e., total number of parameters needed to describe all the oscillators with card types 4 and 5).</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3F10.0</td>
<td>S, NU, GAMMA</td>
<td>S = strength of the oscillator; NU = location of the oscillator (in wave numbers); GAMMA = width of oscillator.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>F10.0</td>
<td>E</td>
<td>Dielectric constant at infinite frequency.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>I5</td>
<td>M</td>
<td>Number of points used to describe the curve (i.e., no. of cards of type 7).</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2F10.0</td>
<td>WIN(I), RIN(I)</td>
<td>WIN(I) = wave number for the data point; RIN(I) = percent emittance at WIN(I). Use as many of these as necessary to thoroughly delineate the curve (at least 45-50).</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C

DOCUMENTATION FOR PROGRAM 'EMISPOL'.

EMISPOL is a straightforward program, in which the oscillator parameters from the previous program are input parameters. Output is degree of polarization (PERPOL) and \( e_{//}(\theta) \) and \( e_{\perp}(\theta) \), relative emittance components.

Where reststrahlen bands are small, it is important to note that the index of refraction as determined by the oscillator program is not reliable. If band structure is not pronounced it is difficult to determine suitable oscillator parameters. On the other hand, the emissivity is then not very spectrally dependent, which means that it is reasonable to regard the refractive index as constant over the same wavelength region.

Under these circumstances it is valid to by-pass the oscillator program and feed a constant value for refractive index into EMISPOL. Doing so requires a slight modification of EMISPOL. Immediately after statement 600 (see EMISPOL listing in Appendix E), the program calls in a subroutine which calculates \( n \) and \( k \) given the final oscillator parameters. This subroutine (and hence the oscillator program) can be by-passed by substituting a statement which supplies values for \( n \) and \( k \). Values can be determined from the Brewster angle as described in Volume I of this report.
<table>
<thead>
<tr>
<th>CARD #</th>
<th>FORMAT</th>
<th>NAME</th>
<th>DESCRIPTION AND/OR COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10A4</td>
<td>ITITLE</td>
<td>Any title information, up to 80 characters.</td>
</tr>
<tr>
<td>2</td>
<td>I3</td>
<td>K</td>
<td>No. of Llamdas (wavelengths) to be provided (a maximum of 185; see card 10).</td>
</tr>
<tr>
<td>3</td>
<td>F5.3</td>
<td>EPS</td>
<td>E from Program OSSC.</td>
</tr>
<tr>
<td>4</td>
<td>I2</td>
<td>NPTS</td>
<td>No. of Oscillators (a maximum of 8).</td>
</tr>
<tr>
<td>5</td>
<td>7F8.2</td>
<td>FR(I)</td>
<td>NU values from Program OSSC.</td>
</tr>
<tr>
<td>6</td>
<td>7F8.4</td>
<td>S(I)</td>
<td>S values from Program OSSC.</td>
</tr>
<tr>
<td>7</td>
<td>7F8.4</td>
<td>G(I)</td>
<td>GAM values from Program OSSC.</td>
</tr>
<tr>
<td>8</td>
<td>I3</td>
<td>M</td>
<td>No. of AI's (a maximum of 10) (see card 11).</td>
</tr>
<tr>
<td>9</td>
<td>I3</td>
<td>T</td>
<td>Temperature of the object which is emitting (in degrees K).</td>
</tr>
<tr>
<td>10</td>
<td>10F8.3</td>
<td>LAMDA(J)</td>
<td>Lambda's at which emittance polarization is calculated for each AI(N).</td>
</tr>
<tr>
<td>11</td>
<td>F10.6</td>
<td>AI(N)</td>
<td>Angles of incidence (1 per card).</td>
</tr>
</tbody>
</table>
APPENDIX D
PROGRAM LISTING - OSCILLATOR

DIMENSION RFC(13,600),C1(602),D2(602)
DIMENSION W(300),RIN(300)
DIMENSION LABEL(6)
DIMENSION LBL(9),LBM(2)
REAL XS(25),H(27)
REAL X(25),G(25),D(20),DATA(250)
DATA LBL/36H ALPHA
DATA LABEL/4H R
REAL XS(25),H(27)
REAL X(25),G(25),D(20),DATA(250)
DATA LBL/36H ALPHA
DATA LABEL/4H R

C ***********************************************************************
C 548 FORMAT(20A4,15,E15.5,F10.0)
C 55 FORMAT(15)
C 470 FORMAT(5X,*INITIAL GUESSES FOR OSCILLATOR PARAMETERS ARE */ )
C 471 FORMAT(1A,16X,INU,14X,IGAMMA',/(5X*E12.5,8XE12.5)
C 472 FORMAT(3X, E(INF)= ' ,F12.4)
C 1766 FORMAT(' CARD COUNT IS ' ,15)
C 2000 FORMAT(' REFLECTIVITY BASED ON INITIAL GUESSES'*/)
C 555 FORMAT(2F10.0)
C 29 X(I)=SQR(X(I))
C 30 X(I)=X(I)**2
C 10 CONTINUE
C 1112 FORMAT(5X,* SUM OF THE DIFFERENCES Squared IS ' ,E12.4)
C CALL FUNCTION(W,WIN,KIN)

51
EXTERNAL FUNCT
IF(LIMIT .EQ. 0) GO TO 250
CALL FMP(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,8)

250 CONTINUE
DO 19 I=1,N
19 XS(I)=X(I)**2
C
WRITE(6,9745)
9745 FORMAT(1H1)
C
WRITE(6,473)F,IER
473 FORMAT(3X,/ 'FITTING HAS BEEN COMPLETED WITH F(X) = ',E15.8,/
  ' AND ERROR PARAMETER = ',15// ' FITTED VALUES FOR OSCILLATOR, /
  ' PARAMETERS ARE */
C
WRITE(6,471)XS(I),I=1,N
WRITE(6,472)XS(N)
WRITE(6,487)
487 FORMAT(1H1,3X*ENERGY R(EXP) R(FITTED) DIFFERENCE*/
ZLT=0
DO 13 J=1,M
YY=FUNX(N,X,WIN(J))
Z=YY-RIN(J)
ZLT=ZLT+Z**2
C
WRITE(6,16)WIN(J),RIN(J),YY,Z
16 FORMAT(1X,4F12.5)
13 CONTINUE
C
WRITE(6,1112)ZLT
END

SUBROUTINE FUNCT(N,WIN,RIN)
DIMENSION WIN(300),RIN(300)
RETURN
ENTRY FUNCT(N,X,VAL,GRAD)
REAL X(25),GRAD(25)
REAL W(10),S(10),G(10)
REAL P(300),Q(300),V(300)
COMPLEX Z(300),H(300),M(300)
NN=N/3
DO I=1,NN
L=3*(I-1)
S(I)=X(L+1)
W(I)=X(L+2)
1 G(I)=X(L+3)
EINF=X(N)
DO J=1,NN
W=I/H(J)**2
G2=G(J)**2
DO 2 J=1,M
WW=WIN(I)**2
2 D(I,J)=S(J)*CHPLX(1.-WW**2,-WW*G2)
D(I,J)=EINF*E2
DO 4 J=1,NN
4
\[
\begin{align*}
E(I) &= E(I) + S(J) \cdot D(I, J) \\
E(I) &= \text{CEXP}(1.5 \cdot \text{CLOG}(E(I))) \\
\text{IF} (\text{REAL}(E(I)).LT.0.) E(I) = -E(I) \\
P(I) &= \text{CABS}(E(I)) \cdot 2 - 2 \cdot \text{REAL}(E(I)) \cdot 1. \\
Q(I) &= P(I) + 4 \cdot \text{REAL}(E(I)) \\
V(I) &= P(I) \cdot Q(I) - \text{RI}(I) \\
\text{VAL} &= 0 \\
\text{DO} 33 \ I = 1, M \\
\text{VAL} &= \text{VAL} + V(I) \cdot 2 \\
\text{WRITE}(6, 76) \text{ VAL} \\
\text{FORMAT}(1X, E14.5) \\
\text{DO} 7 \ I = 1, M \\
Z &= \text{CCNJG}(E(I)) \\
H(I) &= 8 \cdot V(I) \cdot 5 \cdot (Q(I) \cdot (Z - 1.) - P(I) \cdot (Z + 1.)) / E(I) / Q(I) \cdot 2 \\
\text{DO} 6 \ J = 1, N \\
L &= 3 \cdot (J - 1) \\
\text{GRAC}(L+1) &= 0 \\
\text{GRAC}(L+2) &= 0 \\
\text{GRAC}(L+3) &= 0 \\
W2 &= G(J) / W(J) \cdot 2 \\
W3 &= -G(J) \cdot 2 / W(J) \cdot 3 \\
W5 &= -2 / W(J) \cdot 5 \\
\text{DO} 6 \ I = 1, M \\
\text{GRAC}(L+1) &= \text{GRAD}(L+1) + \text{REAL}(H(I) \cdot D(I, J)) \\
Z &= H(I) \cdot D(I, J) \cdot 2 + \text{WIN}(I) \\
\text{GRAC}(L+2) &= \text{GRAD}(L+2) + \text{REAL}(Z \cdot \text{CMPLX}(\text{WIN}(I) \cdot W5, W3)) \\
\text{GRAC}(L+3) &= \text{GRAD}(L+3) - \text{AIMAG}(Z) \cdot W2 \\
\text{CONTINUE} \\
\text{GRAC}(N) &= 0 \\
\text{DO} 7 \ I = 1, M \\
\text{GRAC}(N) &= \text{GRAD}(N) + \text{REAL}(H(I)) \cdot \text{EINF} \\
\text{RETURN} \\
\text{END} \\
\text{FUNCTION} \ \text{FUNX}(N, X, WIN) \\
\text{REAL} \ X(25) \\
\text{REAL} \ W(10), C(10), S(10) \\
NN = N / 3 \\
\text{DO} 1 \ I = 1, NN \\
L &= 3 \cdot (I - 1) \\
S(I) &= X(L+1) \\
W(I) &= X(L+2) \\
G(I) &= X(L+3) \\
\text{CONTINUE} \\
\text{COMPLEX} \ E \\
E &= X(N) \cdot 2 \\
\text{DO} 2 \ J = 1, NN \\
E &= E \cdot S(J) \cdot 2 / \text{CMPLX}(1. -(\text{WIN} / W(J) \cdot 2) \cdot 2, -(\text{WIN} \cdot G(J) \cdot 2 / W(J) \cdot 2) \\
\text{CONTINUE} \\
E &= \text{CEXP}(1.5 \cdot \text{CLOG}(E)) \\
\text{IF} (\text{REAL}(E).LT.0.) E = -E \\
\text{FUNX} &= \text{CABS}(E-1.) / (E+1.) \cdot 2 \\
\text{RETURN} \\
\text{END} 
\end{align*}
\]
APPENDIX E

PROGRAM LISTING - EMISPOL

DIMENSION LAM(185), AI(185), AK(185), EL(185), ER(185), NR(185), NL(185), ON(185), ITITLE(10), NSKY(9,185)
DIMENSION FR(8), S(8), G(8), EE(185)
DIMENSION PERPOL(185)
COMMON FR, S, G
COMMON NPTS, EPS
REAL LAMDA, NSKY, NTAR, NR, NL
INTEGER T
INTEGER SKY

C INPUT
100  READ (5,5) ITITLE
      5 FORMAT (10A4)
      WRITE (6,6) ITITLE
      6 FORMAT (1HL,10A4)
      READ (5,10) K
      10 FORMAT (13)
      READ (5,12) EPS
      12 FORMAT (F5.3)
      READ (5,21) NPTS
      21 FORMAT (12)
      READ (5,20) (FR(I), I=1,NPTS)
      20 FORMAT (8F8.2)
      READ (5,25) (S(I), I=1,NPTS)
      25 FORMAT (9F8.4)
      READ (5,40) M
      40 FORMAT (13)
      READ (5,60) T
      60 FORMAT (13)
      READ (5,610) (LAMDA(I), I=1,K)
      610 FORMAT (10F8.0)
      N=0
      645 N=N+1
      READ (5,650) AI(N)
      650 FORMAT (10F6.6)
      AI(N)=90.0-AI(N)
      DO 660 I=1,K
      NSKY(I,1)=0.0
      IF (N.GE.M) GO TO 600
      GO TO 645
      660 CONTINUE

C CALCULATIONS
600 DO 700 JJ=1,K
      CALL AAN(K,LAMDA(JJ),AI(JJ),AK(JJ))
      700 CONTINUE
      DO 200 II=1,M
      AAII=AI(II)*3.1416/180.
      200 DO 150 JJ=1,K
      QQ=SQRT(AAII*AI(JJ)**2+(AI(JJ)**2-AK(JJ)**2)**2)
      150 A=0.5*(AI(JJ)**2-AK(JJ)**2-(SQRT(AAII*AI(JJ)**2)**2)+Q)
      B=0.5*(-AK(JJ)**2+AI(JJ)**2-(SQRT(AAII*AI(JJ)**2)**2)+Q)
      RR=(SQRT(A)+CSTAII)**2+Q)
      1 (RR*(CSTAII)**2+B)**2
      RL=RR*(CSTAII)**2+B)**2
      1 (RL*(CSTAII)**2+B)**2
      NTAR=1.19E4/(LAMDA(JJ)**9)*EXP(1.4388E4/(LAMDA(JJ)*T))-1.0)
      ER(JJ)=1.0-KR
      EL(JJ)=1.0-KL
      EE(JJ)=5*(ER(JJ)+EL(JJ))
DE(JJ) = EL(JJ) - ER(JJ)
NR(JJ) = (RK*NSKY(II,JJ)/2.0 + (ER(JJ)*NTAR/2.0)
NL(JJ) = (EL*NSKY(II,JJ)/2.0 + (EL(JJ)*NTAR/2.0)
DN(JJ) = NL(JJ) - NR(JJ)
PERPOL(JJ) = (EL(JJ) - ER(JJ))/(EL(JJ) + ER(JJ))
150 CONTINUE
AI(I) = AI(I)*180./3.1416
WRITE (6,70) AI(I), T, (LAMDA(I), ANL(I), AK(I), NSKY(I), ER(I), EL(I)
1, DE(I), NR(I), NL(I), DN(I), EE(I), PERPOL(I), L=1,K)
2, ', PERPOL, '/ (F6.2,2X,F6.4,2X,F6.4,2X,F6.4,1X,F10.4,1X,F10.4,1X,F10.4,1X
3, F10.4,1X, E9.3,1X, E9.3,1X, E10.4,1X, E10.4,1X)
200 CONTINUE
GO TO 100
END

SUBROUTINE ANAK(LAMDA, ANL, AK)
DIMENSION FR(8), S(8), G(8)
COMMON FR, S, G
COMMON NPTS, EPS
REAL LAMDA
F = 1000.0/LAMDA
AE = 0.
BB = 0.
DO 110 KK = 1, NPTS
Q = (S(KK) + (FR(KK))**2)/( (FR(KK))**2 - F**2) + G(KK)*FR(KK)*F**12)
AE = CQ + (FR(KK))**2 - F**2) + BB
BB = CQ + FR(KK)*F*QC + BB
110 CONTINUE
AA = AE + EPS
AN = SQRT(.5*(AA + SQRT(AA**2 + 4.*(BB**2))))
AK = BB/AN
RETURN
END
APPENDIX F

PROGRAM LISTING AND INSTRUCTIONS FOR IBM SCIENTIFIC
SUBROUTINE - FMFP

(This material in this Appendix is a direct copy of an
IBM Scientific Subroutine as it appears in IBM Manual
GH20-0205-4 of System/360 Scientific Subroutine Package,
Version III, Programmer's Manual.)
Extremum of Functions

Subroutines FMFP and DFMFP


It is assumed that the function \( f \) of the \( n \) variables \( x_1, \ldots, x_n \) (abbreviated as argument vector \( x \)) may be computed together with its gradient vector \( g(x) \) for any point \( x \). The generalized Taylor expansion for functions of several variables is

\[
 f(x+u) = f(x) + g(x) \cdot u + \frac{1}{2} u^T G(x) u + \text{higher terms}
\]

where \( g \) is the gradient vector and \( G \) the matrix of second order partial derivatives. Vectors are assumed to be column vectors; \( u^T \) means transpose of vector \( u \). It is assumed that in the neighborhood of the required minimum \( x_{\text{min}} \) the function is approximated closely by the first three terms of its Taylor expansion, giving

\[
 f(x) = f(x_{\text{min}}) + \frac{1}{2} (x - x_{\text{min}})^T G(x_{\text{min}}) (x - x_{\text{min}})
\]

since \( g(x_{\text{min}}) = 0 \). Then the gradient is seen to be approximately \( g(x) = G(x_{\text{min}}) (x - x_{\text{min}}) \).

Assume now that the symmetric matrix \( G \) is positive definite. Then the following equation holds true:

\[
 x - x_{\text{min}} = G^{-1} (x_{\text{min}} - g(x))
\]

which would allow \( x_{\text{min}} \) to be calculated in one step if \( G^{-1} (x_{\text{min}}) \) were available.

To approach \( G^{-1} (x_{\text{min}}) \), a method of successive linear searches in \( G \)-conjugate directions is used. Starting with the identity matrix \( G(0) = I \), a sequence of symmetric matrices \( G(i) \) is generated which tends to \( G^{-1} \). At the \((i+1)^{\text{st}} \) iteration step a linear search is made in direction \( h(i) = -G(i) g(i) \), where \( g(i) \) is an abbreviation for \( g(x(i)) \). By means of the linear search the minimum of \( y(t) = f(x(i) + t \cdot h(i)) \) is determined, giving argument \( x(i+1) = x(i) + t_i \cdot h(i) \).

The argument of the minimum \( x(i+1) \) on the line through \( x(i) \) in direction \( h(i) \) is determined by the relation that scalar product \( (g(i+1), h(i)) = 0 \).

Now:

\[
 x^{(n)} = x^{(1)} + \sum_{i=1}^{n-1} t_i h^{(i)}
\]

and:

\[
 g^{(n)} = g^{(1)} + \sum_{i=1}^{n-1} t_i G h^{(i)}
\]
Therefore:

Scalar product \( (g^{(n)}, h^{(l)}) = \sum_{j=1}^{n-1} t_j (g^{(l)}, h^{(l)}) \)

Suppose now that the vectors \( h^{(0)}, h^{(1)}, \ldots, h^{(n-1)} \) are G-conjugate, satisfying \( (Gh^{(l)}, h^{(l)}) = 0 \) for \( i \neq j \). Then \( g^{(n)}, h^{(l)} = 0 \), and since \( h^{(0)}, h^{(1)}, \ldots, h^{(n-1)} \) form a basis, \( g^{(n)} = 0 \) and \( x^{(n)} = x_{\text{min}} \). This shows that the minimum is located at the \( n \)th iteration for a quadratic function when using successive linear searches for G-conjugate directions.

For the generation of G-conjugate directions, start with \( h^{(0)} = -g^{(0)} \) and calculate successive directions \( h^{(l)} \) by means of \( h^{(l)} = -G^{(l)} g^{(l)} \), where \( G^{(l)} \) is modified to \( G^{(l+1)} \) so that \( h^{(l)} \) is an eigenvector of the matrix \( G^{(l+1)} \) with eigenvalue 1. This ensures that \( G^{(l)} \) approaches \( G^{-1} \) as \( x^{(l)} \) approaches \( x_{\text{min}} \). An easy calculation shows:

\[
G^{(l+1)} = G^{(l)} + dx \cdot dx^T \frac{G^{(l)}dg \cdot dg^T G^{(l)}}{dx^T dg \cdot dg^T G^{(l)}dg}
\]

with \( dg = g^{(l+1)} - g^{(l)} \)

\[
dx = x^{(l+1)} - x^{(l)}
\]

where all vectors are regarded as column vectors, and superscript \( T \) means transpose of column vector—that is, row vector.

The strategy adopted for termination of the successive linear searches is as follows:

1. If the function value has not decreased in the last iteration step, the search for the minimum is terminated provided the gradient is already sufficiently small; otherwise, the next step is in the direction of steepest descent.

2. If the argument vector and the direction vector change by very small amounts, and at least \( n \) iterations are performed, the minimization is terminated again.

3. If the number of iterations exceeds an upper bound furnished by the user, further calculation is bypassed, and an error code is set to 1 indicating poor convergence.

Mathematics—Extremum of Functions
4. If one of the successive linear searches indicates that no constrained minimum exists, further calculation is bypassed again, and the error code is set to 2 indicating that it is likely that no minimum exists.

The $i$th term $G^{(i)}$ is reset to the identity matrix if there is indication that the current $G^{(i)}$ is not positive definite, or if the formula for $G^{(i+1)}$ breaks down due to zero divisors.

The linear search technique used in subroutines FMFP and DFMFP is as follows. For a given argument vector $x$ and a vector $h$ defining a direction through $x$, a local minimum of the function $y(t) = f(x + th)$ is determined. This means that a value $t_m$ must be determined such that 

$$y'(t_m) = 0.$$ 

Given $y'(t) = \text{scalar product } (g(x + th), h)$, therefore $y(t)$ and $y'(t)$ may be calculated for any value of $t$, and:

$$y(0) = f(x) \text{ and } y'(0) = (g(x), h)$$

In case $y'(s_2) = 0$, $t_m$ is set equal to $s_2$ and $x_m = x + s_2$, $h$ is used as argument of a constrained minimum on the line through $x$ with direction $h$. In the second and third case a minimum lies between the points $x_1 = x + s_1 \cdot h$ and $x_2 = x + s_2 \cdot h$, that is, $t_m$ must be in the interval $(s_1, s_2)$.

The argument of the minimum is obtained by means of cubic interpolation using the function and derivative values at the points $x_1$, $x_2$. Let $x_3 = x + s_3 \cdot h$ be the argument of the minimum of the third degree interpolation polynomial. Then:

$$s_3 = s_2 - \alpha(s_2 - s_1) = s_1 + (\alpha - 1)(s_1 - s_2)$$

with:

$$\alpha = \frac{y'(s_2) + z}{y'(s_2) - y'(s_1) + 2w}$$

and:

$$z = 3 \frac{y'(s_2) - y'(s_1)}{s_2 - s_1} + y'(s_1) + y'(s_2)$$

$$w = \sqrt{[s^2 - y'(s_1) \cdot y'(s_2)]}$$

If $f(x_3) \leq f(x_1)$ and $f(x_3) \leq f(x_2)$, $x_m$ is set equal to $x_3$ and used as argument of the wanted minimum along the given line. Otherwise the interval $(x_1, x_2)$ is reduced by replacing $x_1$ by $x_3$ if $f(x_3) \leq f(x_1)$ and $(g(x_1), h) < 0$, and by replacing $x_2$ by $x_3$ in all other cases. Then the interpolation process is repeated for this new reduced interval.
SUBROUTINE FMFP

PURPOSE

TO FIND A LOCAL MINIMUM OF A FUNCTION OF SEVERAL VARIABLES

BY THE METHOD OF FLETCHER AND POWELL

USAGE

CALL FMFP(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H)

DESCRIPTION OF PARAMETERS

FUNCT = USER-WRITTEn SUBROUTINE CONCERNING THE FUNCTION TO

RE MINIMIZED. IT MUST BE OF THE FORM

SUBROUTINE FUNCT(ARG,VAL,GRAO) RETURN

AND MUST SERVE THE FOLLOWING PURPOSE

FOR EACH N-DIMENSIONAL ARGUMENT VECTOR ARG, FUNCTION VALUE AND GRADIENT VECTOR MUST BE COMPUTED

AND ON RETURN, STORED IN VAL AND GRAD RESPECTIVELY

N = NUMBER OF VARIABLES

X = VECTOR OF DIMENSION N CONTAINING THE INITIAL

ARGUMENT WHERE THE ITERATION STARTS. ON RETURN,

X HOLDS THE ARGUMENT CORRESPONDING TO THE

COMPUTED MINIMUM FUNCTION VALUE

F = SINGLE VARIABLE CONTAINING THE MINIMUM FUNCTION

VALUE ON RETURN, I.E. F(X).

G = VECTOR OF DIMENSION N CONTAINING THE GRADIENT

VECTOR CORRESPONDING TO THE MINIMUM ON RETURN.

I.E. GRAD(X). ON RETURN.

EST = IS AN ESTIMATE OF THE MINIMUM FUNCTION VALUE.

EPS = TEST VALUE REPRESENTING THE EXPECTED ABSOLUTE ERROR.

A REASONABLE CHOICE IS 10**(-6), I.E.

NOWHAT GREATER THAN 10**(-6), WHERE D IS THE

NUMBER OF SIGNIFICANT DIGITS IN FLOATING POINT

REPRESENTATION.

LIMIT = MAXIMUM NUMBER OF ITERATIONS.

IER = ERROR PARAMETER

IER = 0 MEANS CONVERGENCE WAS OBTAINED.

IER = 1 MEANS NO CONVERGENCE IN LIMIT ITERATIONS

IER = 1 MEANS ERRORS IN GRADIENT CALCULATION

IER = 1 MEANS LINEAR SEARCH TECHNIQUE INDICATES

IT IS LIKELY THAT THERE EXISTS NO MINIMUM

H = WORKING STORAGE OF DIMENSION N*(N+7)/2.

REMARKS

1) THE SUBROUTINE NAME REPLACING THE DUMMY ARGUMENT FUNCT

MUST BE DECLARED AS EXTERNAL IN THE CALLING PROGRAM.

2) IFK IS SET TO 2 IF , 44, OR IN ONE OF THE COMPUTED

OPTIMIZATIONS, THE FUNCTION WILL NEVER INCREASE WITHIN

A TOLEVRABLY NARROW RANGE OF ARGUMENT.

IFK = 2 MAY OCCUR ALSO IF THE INTERVAL WHERE F

INCREASES IS SMALL AND THE INITIAL ARGUMENT WAS

RELATIVELY FAR AWAY FROM THE MINIMUM SUCH THAT THE

MINIMUM WAS OVERLAPPED. THIS IS DUE TO THE SEARCH

TECHNIQUE WHICH DOUBLES THE STEPSIZE UNTIL A POINT

IS FOUND WHERE THE FUNCTION INCREASES.

SUMMARY OF THE SUBPROGRAMS REQUIRED

FUNCT
A1 C METHOD
A2 C
A3 C THE METHOD IS DESCRIBED IN THE FOLLOWING ARTICLE
A4 C N. Fletcher and M.J.D. Powell, A RAPID DESCENT METHOD FOR
A5 C MINIMIZATION
A6 C
A8 C
A9 C
70 C SUBROUTINE FMPF(FUNCT,N,X,F,G,EST, EPS,LIMIT, IEN,H)
71 C
72 C DIMENSIONED DUMMY VARIABLES
73 C DIMENSION N(1),X(1),G(1)
74 C
75 C COMPUTE FUNCTION VALUE AND GRADIENT VECTOR FOR INITIAL ARGUMENT
76 C CALL FUNCT( X,F,G)
77 C
78 C
79 C IF(N)=0
80 C KNUT=0
81 C NEN=0
82 C N3=N2+N
83 C N31=N3+1
84 C KKN=1
85 C DN=6 J=1, N
86 C H(K)=1
87 C NEN=1
88 C IF(NJ95.5,2
89 C DN 3 L=1, NJ
90 C KLER=L
91 C H(KL)=0
92 C KKN=1
93 C
94 C START ITERATION LOOP
95 C 5 KNUT=KNUT +1
96 C
97 C SAVE FUNCTION VALUE, ARGUMENT VECTOR AND GRADIENT VECTOR
98 C OLDVF
99 C ON 9 J=1, N
100 C KKN+J
101 C H(K)=G(J)
102 C KKN+N
103 C H(K)=X(J)
104 C DETERMINE DIRECTION VECTOR H
105 C KJ+N3
106 C T=0
107 C ON 8 L=1, N
108 C T=T+G(L)/H(K)
109 C IF(T<0) G(J)=7.7
110 C KKN+N+1
111 C G0 TO 8
112 C 7 KKN+1
113 C 0 CONTINUE
114 C 9 H(J)=T
115 C
116 C CHECK WHETHER FUNCTION WILL DECREASE STEPPING ALONG H.
117 C DYS0
118 C HMRN=0
119 C GMNR=0
120 C
CALCULATE DIRECTIONAL DERIVATIVE AND VALUES FOR DIRECTION

VECTOR H AND GRADIENT VECTOR G.

DO 10 J=1,N

HNN=GNNM*AMB*(H(J))

GNN=GNNM*AMB*(G(J))

10 CONTINUE

REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTIONAL DERIVATIVE APPEARS TO BE POSITIVE OR ZERO.

IF(DY)I1,51,12

REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTION.

11 IF(HNN/GNNM-FP(N))I1,51,12

SEARCH MINIMUM ALONG DIRECTION H.

SEARCH ALONG H FOR POSITIVE DIRECTIONAL DERIVATIVE.

12 FY=P*(EST-FL)/DY

13 AMB=ALPHA.

14 USE ESTIMATE FOR STEPSIZE ONLY IF IT IS POSITIVE AND LESS THAN 1, OTHERWISE TAKE 1 AS STEPSIZE.

15 IF(ALPHA)I15,15,13

16 IF(ALPHA)I15,15,15

17 AMB=AMB*(15,15,15

18 ALPHA=ALPHA.

SAVE FUNCTION AND DERIVATIVE VALUES FOR OLD ARGUMENT.

19 FX=FY

20 DY=DY

STEP ARGUMENT ALONG H.

21 DO 17 I=1,N

22 X(I)=X(I)+AMB*AMH(I)

23 CONTINUE

CALL FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT.

24 FY=F

25 CUMMULATIVE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT. TERMINATE SEARCH, IF DY IS POSITIVE, IF DY IS ZERO THE MINIMUM IS FOUND.

26 DY=0.

27 IF(ALPHA)I15,15,13

28 IF(ALPHA)I15,15,15

29 AMB=AMB*(15,15,15

30 CALF=ALPHA.

31 TERMINATE SEARCH ALSO IF THE FUNCTION VALUE INDICATES THAT A MINIMUM HAS BEEN PASSED.

32 IF(FY-FX)I19,56,22

33 TERMINATE IF THE CHANGE IN ARGUMENT SETS VERY LARGE.

34 IF(HNNM*AMB<16,15,21

35 IF(HNNM*AMB<16,15,21

36 END OF SEARCH LUMP.

37 IF(HNNM*AMB<16,15,21

38 TERMINATE IF THE CHANGE IN ARGUMENT SETS VERY LARGE.

39 IF(HNNM*AMB<16,15,21

40 IF(HNNM*AMB<16,15,21

63
LINEAR SEARCH TECHNIQUE INDICATES THAT NO MINIMUM EXISTS

RETURN

INTERPOLATE CURVILINERLY IN THE INTERVAL DEFINED BY THE SEARCH ABOVE AND COMPUTE THE ARGUMENT X FOR WHICH THE INTERPOLATION POLYNOMIAL IS MINIMIZED

IF X = 0,

IF |F(X)| > |F(Y)|, RETURN

IF X < 0,

IF |F(X)| > |F(Y)|, RETURN

IF X > 0,

IF |F(X)| > |F(Y)|, RETURN

TERMINE, IF THE VALUE OF THE FUNCTION AT X IS LESS THAN THE FUNCTION VALUES AT THE INTERVAL END, OTHERWISE REDUCE

THE INTERVAL BY CHOOSING ONE END-POINT EQUAL TO X AND REPEAT

THE INTERPOLATION, WHICH END-POINT IS CHOSEN DEPENDS ON THE VALUE OF THE FUNCTION AND ITS GRADIENT AT X

CALL FUNCTION(N,X,F,G)

IF |F(X)| > |F(Y)|, RETURN

IF X < 0,

IF |F(X)| > |F(Y)|, RETURN

IF X > 0,

IF |F(X)| > |F(Y)|, RETURN

IF X = 0,

IF |F(X)| > |F(Y)|, RETURN

IF X < 0,

IF |F(X)| > |F(Y)|, RETURN

IF X > 0,

IF |F(X)| > |F(Y)|, RETURN

TERMINE, IF FUNCTION HAS NOT DECREASED DURING LAST ITERATION

IF |F(X)| > |F(Y)|, RETURN

IF X < 0,

IF |F(X)| > |F(Y)|, RETURN

IF X > 0,

IF |F(X)| > |F(Y)|, RETURN

TERMINE, IF FUNCTION HAS NOT DECREASED DURING LAST TWO CONSECUTIVE ITERATIONS

GO TO 2

GO TO 252

GO TO 251

GO TO 205

GO TO 204
241 K3x+C
242 37 H(K)=f(J)-f(K)
243 C
244 C TEXT LENGTH OF ARGUMENT DIFFERENCE VECTOR AND DIRECTION VECTOR EQUATION.
245 C IF AT LEAST N ITERATIONS HAVE BEEN EXECUTED, TERMINATE. IF
246 C BOTH ARE LESS THAN EPS
247 C IF(K=0)
248 C IF(MOUNT=N)>2,39,39
249 C 39 T0
250 C 0
251 C 40 J=1,4
252 C K=K+J
253 C M(K)
254 C K=K+N
255 C T=AP3(N(K))
256 C 40 Z=P3(M(K))
257 C IF(MOUNT=EPB)>1,41,42
258 C 41 IF(T=FPB)>55,56,42
259 C C TERMINATE, IF NUMBER OF ITERATIONS WOULD EXCEED LIMIT
260 C 42 IF(MOUNT=LIMIT)>93,50,50
261 C 42 PREPARE UPDATING OF MATRIX M
262 C 03 ALFA=0.
263 C 47 J=1,4
264 C K=J+N
265 C 0
266 C 46 H(J)=W
267 C 46 ON TO 46
268 C 45 K=K+1
269 C 46 CONTINUE
270 C 47 M(J)=W
271 C 47 ALFA=0.
272 C 47 REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF RESULTS
273 C ARE NOT SATISFACTORY
274 C IF(Z=ALFA)=1,46
275 C UPDATE MATRIX M
276 C 46 K=31
277 C DO 49 L=L+1
278 C K=L+2
279 C 49 H(J)=W
280 C 49 M(J)=W
281 C 49 K=K+1
282 C END OF ITERATION LOOP
283 C 50 IF(N)
284 C NU CONVERGENCE AFTER LIMIT ITERATIONS
285 C RETURN
286 C 51 DO 92 J=1,N
287 C RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS
288 C 92 DO 92
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>301</td>
<td>K=2+J</td>
<td>Call FUNCT(N,X,P,G)</td>
</tr>
<tr>
<td>302</td>
<td>X(J)=X(K)</td>
<td>Call FUNCT(N,X,P,G)</td>
</tr>
<tr>
<td>303</td>
<td>CALL FUNCT(N,X,P,G)</td>
<td>Repeat search in direction of steepest descent if derivative</td>
</tr>
<tr>
<td>304</td>
<td>C</td>
<td>Fails to be sufficiently small</td>
</tr>
<tr>
<td>305</td>
<td>C</td>
<td>If (GNRM-EP5)55,55,55</td>
</tr>
<tr>
<td>306</td>
<td>C</td>
<td>Test for repeated failure of iteration</td>
</tr>
<tr>
<td>307</td>
<td>C</td>
<td>IF (NLR)56,56,56</td>
</tr>
<tr>
<td>308</td>
<td>C</td>
<td>IER=1</td>
</tr>
<tr>
<td>309</td>
<td>C</td>
<td>GOTO 1</td>
</tr>
<tr>
<td>310</td>
<td>53</td>
<td>RETURN</td>
</tr>
<tr>
<td>311</td>
<td>54</td>
<td>END</td>
</tr>
</tbody>
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