CONTACTLESS MEASURING OF VIBRATIONS IN THE ROTOR BLADES OF TURBINES

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* ye initially, after vowels, and after я, ё; е elsewhere. When written as ё in Russian, transliterate as ye or е. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.
FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

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CONTACTLESS MEASURING OF VIBRATIONS IN THE ROTOR BLADES OF TURBINES


INTRODUCTION

Measuring various quantities in relative motion on a rotating rotor wheel of an axial compressor or turbine is an extremely complex problem. The most common and obvious solution here is to convert the measured quantities into electrical quantities and transmit the signal to stationary devices by means of a slip ring.

There is, however, another solution to this problem, which is a special case of the discrete-phase method: instantaneously measuring the value of a certain quantity by means of a stationary sensor at the moment that a certain point on the rotor wheel passes by it. Since information concerning the measured quantity is received in a registering device not directly, but discretely - with the rotation rate of the rotor - then it is natural that in this case we cannot get an impression of all of the processes in the rotor wheel. According to the theory of V. A. Kotel'nikov [9], when the signal is quantized its original shape can be recovered completely only when the quantizing frequency is less than twice the frequency of the highest harmonic component of the signal. In a number of cases in
practice, where complete signal recovery is not required (for example, when we are studying only the amplitude of the vibrations), a certain loss of information is permissible and the indicated condition need not be met.

In measuring vibration amplitudes in the blades of the rotor wheel the discrete-phase method can be achieved by means of a device which was specially designed for these purposes, which is called the cathode ray amplitude register, abbreviated as ELURA [ЭЛУРА].

In the present article, using this device as an example, we discuss the principles and describe the possibilities of the discrete-phase measuring method. Additional information on the ELURA can be found in [1-4].

1. MEASURING NONMULTIPLE BLADE VIBRATIONS

Let us first examine the use of the discrete-phase method to measure the amplitudes of bending vibrations in blades, where the frequency of the vibrations is not a multiple of the number of revolutions of the rotor wheel. Figure 1 shows a diagram of the device and the arrangement of the impulse transmitters. The three impulse transmitters, which are mounted on the stationary parts of the turbomachine, generate electrical impulses at the moment that certain rotor parts pass by them.

Transmitter \( A_0 \) is mounted near the turbine shaft, to which a steel pin has been attached. An electrical impulse is generated in transmitter \( A_0 \) at the moment that the pin passes by it. Through amplifier 1 this impulse synchronizes back-sweep generator 2, which controls the horizontal movement of the cathode-ray beam across the screen of the cathode-ray tube. Thus, with each revolution of the rotor wheel the beam will travel horizontally from left to right across the screen and return to its original position. Let us call the line which is traced by the beam as it travels across the screen the base line.
Fig. 1. Diagram of ELURA: 1 - amplifier, 2 - back-sweep generator, 3 - amplifier-shaper, 4 - line generator, 5 - intensifier pulse generator of beam, 6 - base line, 7 - vertical lines, 8 - light marks, 9 - blurred light marks.

Transmitter \( D_n \) is mounted near the so-called blade markers, which can take the form of pins or holes drilled in the disk of the rotor wheel or any other rotor part. The angle between the blade markers should be equal to the angular pitch of the rotor blades. Each blade marker generates an electrical impulse in transmitter \( D_n \) as it passes by. The impulses from transmitter \( D_n \) enter line generator 4 through amplifier-shaper 3. As the impulse is received from the transmitter this generator forces the electron beam to move along the vertical as well as the horizontal. Consequently, the vertical movement of the beam begins at the moment when the blade marker is near transmitter \( D_n \). After each impulse from transmitter \( D_n \) the beam makes a vertical line on the screen and quickly moves back down. As a result of the action of the two generators - the reverse and line generators - the pattern shown in Fig. 1a appears on the screen of the cathode-ray tube. The number of vertical lines is equal to the number of blade markers and usually corresponds to the number of blades.

The impulses which are sent from transmitter \( D_n \), which is mounted on the housing above the blades (on the periphery of the rotor wheel), enter through the amplifier-shaper the intensifier.
pulse generator of the beam 5, which at the moment that the tip of the blade passes by transmitter \( D_\Pi \), forms a bright glowing point - marker 8 - on each line traced by the beam (Fig. 1b). Distance \( x \) from base line 6 to the glowing dot is proportional to time interval \( \tau \) between the moment that the blade marker passes transmitter \( D_\Pi \) and the moment that the blade tip passes transmitter \( D_\Omega \) on radius \( R \). In turn,

\[
\tau = \frac{v \cdot R}{u} \quad \text{and} \quad x = \frac{v_p}{u} \cdot \varphi \cdot R,
\]

where \( \varphi \) is the angular distance between transmitters \( D_\Pi \) and \( D_\Omega \); \( v_p \) - the rate of the vertical movement of the beam across the screen; \( u \) - the tip speed of the blade.

If the blades do not vibrate, then when the rotor wheel has a constant rotation rate the angular distance \( \varphi \) for each blade will remain constant and, consequently, in each revolution of the rotor wheel \( x \) will also be constant and the light markers will lie at the same place on each line. In the case that blade vibrations develop with a frequency which is not a multiple of the revolutions, then the markers begin to appear at different distances from the end of the lines. In practical terms, as a result of the afterglow effect on the screen, this will cause the markers to blur into tiny bands - glowing lines 9. Calculations show that when all vibration frequencies have the same probability, then at a probability of 0.99 and a relative error which does not exceed 1% the maximal deviations in a blade to one side or another will be established in at least 100 revolutions of the rotor (Fig. 1c):

\[
l = x_{\max} - x_{\min} = \frac{v_p}{u} \cdot 2A,
\]

where \( A \) is the amplitude of blade tip vibrations around the periphery.

Thus, if we observe or photograph the blurring in the markers we can measure the amplitude of vibrations in all blades of the rotor wheel.
Figures 2a and 2b show photographs of the images on the screen of the ELURA cathode-ray tube. In the first photograph the markers are not blurred, i.e., there are no blade vibrations. In the second photo, which was made under natural blade vibrations in the rotor wheel, the vibration amplitudes of the individual blades reach a significant level. One notices immediately that some blades virtually do not vibrate, and that the amplitude scatter reaches a factor of 5, 6, or above. Hence it follows that in the case of natural vibrations measuring strain in a limited number of blades may not provide a reliable measurement of vibration amplitudes or stresses, since at the present time predicting in advance the blades which will have the highest amplitudes is very difficult. (More about this problem in [6, 7].)

The above concepts referred to bending vibrations in blades. With another transmitter arrangement (Fig. 3) we can ... so measure torsional vibrations. In this case transmitters $A_h$ and $A_n$ are placed above the leading and trailing edges of the rotor blades. Impulses generated by the blades passing by the transmitter mounted above the leading edge trigger a driven sweep, while impulses from the trailing edges trigger the impulse intensifier generator. The distance between the base line and the marker is equal to the time between the moments that the blades pass the transmitters over the leading and trailing edges; the blurring of the lines is proportional to the amplitudes of torsional blade vibrations.
Fig. 3. Arrangement of transmitters for measuring torsional vibrations in rotor blades.

Fig. 4. Transmitter arrangement for measuring amplitude of blade vibration velocity.

In addition to measuring the amplitudes of displacements in the blade tips, it is possible, if we mount still another transmitter \( A_{n2} \) on the periphery of the rotor wheel, to measure the amplitude of the rate of vibrations on the blade tips. In fact, if the distance between the transmitters is equal to \( L \) (Fig. 4) and if the vertical driven sweep is triggered by transmitter \( A_{n1} \) and the marker made by transmitter \( A_{n2} \), then the distance between the marker and the base line will be equal to

\[
x = v_p \frac{L}{v_u},
\]

where \( v_u \) is the absolute speed of the blade tip around the periphery.

Distance \( L \) is here assumed to be rather small as compared to \( v_u / 4f \), where \( f \) is the vibration frequency of the blades.

If the blade does not vibrate, then its peripheral speed \( v_u = u_k \). If vibrations develop, then the maximal speed of the blades become equal to

\[
v_{u, \text{max}} = u_k + \omega,
\]

and the minimal to
\[ v_{u \min} = u_k - A\omega, \]

where \( \omega = 2\pi f \) is the angular velocity of blade vibrations.

Thus, the blurring of the marker is

\[ l = x_{\text{max}} - x_{\text{min}} = v_l 2 \frac{A\omega}{u_k^2 - (A\omega)^2} \]

and from it we can determine the amplitude of the velocity of blade tip vibrations in the rotor wheel \( A\omega \). If in this case the amplitude of blade vibrations \( A \) have already been measured by the system described earlier, then we can also determine the angular velocity and frequency of the blade vibrations. Quantity \( (A\omega)^2 \) in the denominator of expression (1) can be ignored, in which case we will finally get

\[ l \approx 2 \frac{v_l A\omega}{u_k^2 - A\omega}. \]

Stresses in the root section of the blade are proportional to the amplitude of the vibration velocity at the end of the leading edge of the blade [10]. The proportionality factor is not greatly dependent on the shape of the vibrations. This makes it relatively simple to determine stresses in the blade from the measured amplitude of vibration velocity \( A\omega \).

The discrete-phase method of measuring blade vibrations in rotor wheels of compressors and turbines when used with information about the nature of vibrations in these blades enables us to take other important measurements. Thus, for example, it has been theoretically and experimentally established that the blade vibrations in the rotor wheel, regardless of their nature, are synchronous at rather high amplitude values.

By using this property we can measure the phase shift in the movement of neighboring blades. For this purpose the stage is prepared with three working transmitters (not including reverse transmitter \( A_0 \)), which are arranged according to the system shown
in Fig. 5. The measurement can be taken by two ELURA units simultaneously or by one unit in series with the transmitter switch. In the latter case the transmitter pair $A_n$ and $A_{n1}$ is connected to device No. 1 according to the usual system for measuring displacement amplitudes. Transmitters $A_{n2}$ and $A_{n1}$ are attached to device No. 2 in such a way that the impulse from transmitter $A_{n2}$ will trigger lines and the impulse of transmitter $A_{n1}$ will leave the marks. In this transmitter arrangement, as will be shown below, the amplitudes of displacements in the ends of neighboring blades relative to one another are measured, and these depend on the amplitudes of vibrations in these blades and the phase shifts between their vibrations. Thus, after measuring the amplitudes of relative displacements and the amplitudes of absolute displacements in each blade, we can determine the unknown phase shifts.

![Fig. 5. Arrangement of transmitters for measuring amplitudes of vibrations in neighboring blades relative to one another.](image)

Now let us examine the method more thoroughly. Figure 5 shows a wheel at moment in time $\tau_j$, when blade $A_j$ passes under transmitter $A_{n2}$. At this moment a line is triggered. Within a certain time $\tau$ the neighboring blade $A_{j+1}$ approaches transmitter $A_{n1}$ and a marker appears on the line. For this time interval we can write

$$\tau = \frac{1}{u_k} [L + y_j(\tau) - y_{j+1}(\tau_j + \Delta\tau)],$$

where $t$ is the pitch of the blades; $L$ - transmitter base; $y_j(\tau)$ and $y_{j+1}(\tau_j + \Delta\tau)$ - deviations in tips of two neighboring blades under transmitters $A_{n2}$ and $A_{n1}$, respectively; the distance between the marker and the base line is equal, however, to

$$x = M [L + y_j(\tau) - y_{j+1}(\tau_j + \Delta\tau)],$$ \hspace{1cm} (2)
where $M = v_p/u_w$ is the measuring scale. Since base $L$ and pitch $t$ are constant for each pair of blades, let us study the transformation of only the last two terms in expression (2).

Let the blades vibrate synchronously according to the harmonic law:

$$y = Ae^{i(\omega t + \phi)}$$

where $i$ is an imaginary unit. Then

$$y_j(t_j) - y_{j+1}(t_j + \Delta t) = e^{i(-\phi_j + \phi_{j+1})} \times$$

$$\times (A_j - A_{j+1}e^{i(\omega \Delta t + \phi_j + \phi_{j+1})}) = B_{ji+1}e^{i(\omega \Delta t + \phi_j + \phi_{j+1})},$$

where $\phi_j + \phi_{j+1} - \phi_j$ is the phase shift of the vibrations of neighboring blades.

The light part of expression (3) is the vector $B_{ji+1}e^{i(\omega \Delta t + \phi_j + \phi_{j+1})}$, which rotates at an angular velocity of $\omega$ and has a length equal to the geometric amplitude difference of vibrations in neighboring blades. When the blades vibrate at a frequency which is not a multiple of the number of revolutions, then vector $B_{ji+1}e^{i(\omega \Delta t + \phi_j + \phi_{j+1})}$ occupies various positions, and thus the distance $x$ from the bottom of the line to the marker will be different in each revolution. Here the position where $\omega t_j + \phi_j = 0$, corresponds to the maximal distance, $\omega t_j + \phi_j = \pi$ - to the minimal. Thus, the length of the bright band on the line, which constitutes a series of markers in several revolutions of the wheel, will be proportional to quantity $2B_{ji+1}$. Actually $x_{\max} = M[t - L + B_{j,(j+1)}]$; $x_{\min} = M[t - L - B_{j,(j+1)}]$ hence

$$l = x_{\max} - x_{\min} = 2MB_{ji+1}.$$  

Now, from the results of measuring amplitudes $A_1$, $A_2$, ..., $A_z$ (device No. 1) and $B_{1,2}$, $B_{2,3}$; ... $B_{z1}$ (device No. 2) we can determine the angle $[\omega \Delta t + \phi_j,(j+1)]$ graphically from the triangle shown in Fig. 6 or according to the formula

$$B_{ji+1}^2 = A_j^2 + A_{j+1}^2 - 2A_jA_{j+1}\cos(\omega \Delta t + \phi_j,(j+1)),$$

from which

$$\phi_j,(j+1) = \pm \arccos \frac{A_j^2 + A_{j+1}^2 - B_{ji+1}^2}{2A_j} - \Delta \omega.$$  

(4)
In equation (4) we see that the magnitude of the phase shift is determined accurate to the sign.

To estimate quantity \( \omega \Delta t \) let us take advantage of the fact that generally

\[
t-L \gg y_j(t_1) - y_{j+1}(t_1 + \Delta t).
\]

Since

\[
\tau \approx t - L u_k,
\]

then

\[
\omega \Delta t \approx \frac{\omega}{u_k} t - L = \frac{\omega}{u} \frac{t - L}{D_k} = \frac{\omega}{u} \frac{t}{D_k} (1 - \frac{L}{t}).
\]

Considering that \( t = \frac{\pi D}{z} \), we finally get

\[
\omega \Delta t \approx 2 \frac{\omega}{u} \frac{\pi}{z} (1 - \frac{L}{t}).
\]

Here \( \Omega \) is the angular velocity of the wheel; \( D \) - the diameter of the wheel; \( z \) - the number of blades.

In low-frequency vibrations quantity \( \omega \Delta t \) is generally several degrees. Thus, for example, when \( \omega / \Omega = 2 \), \( L = 0.8 t \) and \( z = 30 \), we get

\[
\omega \Delta t = 2 \cdot 2 \cdot \frac{\pi}{30} \cdot 0.2 \cdot 0.081 \approx 5^\circ.
\]

Measuring the mutual vibrations of neighboring blades relative to one another can be useful in more ways than merely determining the phase shift in the vibrations of individual blades. From the results of simultaneous measurements of absolute and relative vibrations in neighboring blades it is possible, for example, to
discover torsional vibrations in the rotor. It is obvious that when
torsional vibrations are present in the rotor the amplitudes of
visible blade vibrations will be the same, while the amplitude of
relative vibrations in all blades will be equal to zero.

As an example Fig. 7a shows the images on the ELURA screen of
vibration amplitudes of blades in a model compressor, 7b - amplitudes
and their relative vibrations. On the basis of these photographs
it was decided that torsional vibrations were present in the rotor.
Measurements taken by other methods confirmed this deduction.

Fig. 7. Photographs of images on screen of
ELURA of vibration amplitudes in neighboring
blades in tests on compressor model: a) ab-
solute; b) relative to each other.

Until now we have been studying measurements of blade vibrations
whose frequencies are not multiples of the number of revolutions per
second of the rotor wheel. These types of vibrations include
natural, stall, and resonance vibrations in blades resulting from
a rotating stall. However, there is one more very common form of
vibrations - these are resonance vibrations from a stationary
irregularity in the flow, whose frequencies are strict multiples of
the circular frequency. Thus, in each revolution of the rotor wheel
the vibrating blade arrives at a given point on the periphery in
exactly the same phase and exactly the same position. Markers
corresponding to the moment when the blade tip passes the sensor
will lie at the same point on the screen of the cathode-ray tube of
the ELURA in each revolution of the rotor wheel, which does not
enable us to detect resonance vibrations in the blades.
We must take special measures in this case to be able to use the ELURA. Below a method is outlined which enables measuring resonance vibrations in blades using this device by employing stationary impulse transmitters, just as in the case of nonmultiple vibrations.

2. MEASURING MULTIPLE BLADE VIBRATIONS

In order to measure resonance vibrations in blades a special method [1, 4] must be used. The turbomachine is brought to revolutions corresponding to the beginning of resonance excitation in the blades. Then, after gradually increasing the rotation speed of the rotor, a camera is used to establish the sequence of instantaneous positions of the blade tip in a single revolution on one frame.

This recording of multiple blade tip positions ceases when the revolutions depart from the resonance range.

When there is a change in the rotation speed, i.e., the excitation frequency, the phase shift between the movement of the blade and the generating force, which is stationary with respect to the housing, will change in keeping with the phase-frequency characteristic of the vibrating system. Thus, in each revolution of the rotor the blade will pass the measuring place in different phases, i.e., different instantaneous positions of the blade tip will be registered. Now let us show that the distance between the extreme positions of the blade will be equal to its maximal amplitude as it passes through resonance.

Figure 8 shows the scan line of the housing above the wheel and the harmonic of the generating force \( P_\theta = P \cos \omega t \), which is stationary with respect to the housing. Here also we see the reaction of the blade - the movement of its tip \( y \):

\[
y = A \cos (\omega t - \psi),
\]
where $\psi$ is the phase shift between the reaction of the blade and the generating force.

Blade movements measured in the place where transmitter $A_n$ is placed will obviously be

$$y = A \cos (\phi - \psi).$$

Here angle $\phi_n$ between the position of the maximum harmonic of the generating force and the location of the transmitter is not known in advance and can be anywhere between 0 and $2\pi$. In measuring the number of revolutions of the rotor, i.e., the excitation frequency, amplitude $A$ and phase shift $\psi$ will change according to the following law (Fig. 9a and b):

$$A = A_0 \frac{1}{\sqrt{(1 - \bar{\omega}^2 + (\omega + \Delta \omega)^2)}}$$

$$\phi = \arctan \frac{\bar{\omega}}{1 - \bar{\omega}^2},$$

where $A_0$ is static movement under the influence of force $P$, $\bar{\omega}$ - the ratio of excitation frequency to the natural blade frequency, $\delta$ - the logarithmic vibration decrement.

Fig. 8. Change in generating force $P = P \cos \omega t$ and blade displacement $y = A \cos (\omega t - \psi)$ around periphery.

If we represent the frequency characteristics of (5) and (6) in the form of a vector diagram (Fig. 9c) with a radius vector length equal to amplitude $A$ and a polar angle equal to phase shift $\psi$, then the fixed instantaneous deformation $y$ of the blade will be expressed as the length of the projection of vector $A$ onto the line $x-x$, which is drawn at an angle of $\phi_n$:

$$y = A \cos (\phi_n - \psi).$$
Fig. 9. Blade characteristics: a) amplitude-frequency; b) phase-frequency; c) vector diagram.

If there is a change in the number of revolutions, i.e., if the excitation frequency changes, then radius vector \( \mathbf{A} \) will describe a virtually closed curve. The projection of this curve in direction \( x-x \) will represent the geometrical location of points, from which the distances to the pole 0 will be the fixed instantaneous displacements in the blade tip. The length of projection \( \mathbf{M'M''} \) will be equal to the distance between the extreme positions registered for the blade. Now let us show, first, that the length of projection \( \mathbf{M'M''} \) is equal to the maximal (resonance) amplitude \( A_{\text{max}} \) and, second, that this line does not depend on the location of the transmitter, i.e., on angle \( \phi_n \). For this reason we will write the hodograph equation for the vector in polar coordinates.

From formula (16) we get

\[
\sin \psi = \frac{-\omega \xi}{\sqrt{(1 - \omega^2) + (\omega^2 \pi^2)}}.
\]

If we compare expressions (15) and (17), then we find

\[
A = A_0 \frac{\pi}{\xi} \frac{1}{\omega} \sin \psi.
\]  

(7)

In the case of weak damping, which is characteristic of turbo-machine blade vibrations (\( \delta \leq 0.3 \)), the hodograph is described almost entirely in very small deviations of the excitation frequency from
the natural blade frequency. If we set $\bar{\omega} = 1$ in equation (7), then we get the equation for a periphery with a diameter equal to $A_0 \pi / \delta$:

$$A = A_0 \frac{\pi}{\delta} \sin \phi.$$ 

Since the projection of the periphery in any direction is equal to its diameter, then the length of $M'M''$ does not depend on angle $\phi_n$, i.e., on the place of fixation. On the other hand, according to expression (5) the maximal amplitude (when $\bar{\omega} = 1$) is equal to the diameter of the hodograph of vector $A$. Thus, a second position is shown which confirms that $M'M'' = A_{\text{max}}$.

The described method of measuring the amplitudes of resonance vibrations is finding practical application and is in good agreement with strain gage measurements. As an example Fig. 10 uses photographs of images on the screen of the ELURA as "pass-by" resonance vibrations are being measured with respect to revolutions. However, there is a broad class of compressors and turbines rigidly connected to a-c engines and generators which have a strictly constant rotation speed. It is impossible, of course, to use the described method to measure the amplitudes of resonance vibrations in the blades of their rotor wheels.

Fig. 10. Photographs of images on screen of ELURA during measuring of "by-pass" resonance vibrations in revolutions.

In order to measure resonance vibrations when the rotor has a constant rotation speed we can take advantage of the fact that the blades arrive in different vibration phases at the different points on the periphery. To determine the parameters of these vibrations it is sufficient to measure the deviation of a blade from its neutral position in two or three points around the periphery, which is achieved by placing the transmitters on a single plane of rotation.
with the shift of one relative to another at a certain known geometric angle $\alpha$. The result for each measurement can be written as follows (Fig. 11):

for the first transmitter $y_1 = A \sin (\varphi_n - \psi)$;

for the second transmitter $y_2 = A \sin (\varphi_n - \psi + \alpha \nu)$;

for the third transmitter $y_3 = A \sin (\varphi_n - \psi + 2 \alpha \nu)$,

where $\nu$ is the ratio of the angular speed of the blade vibrations to the angular speed of rotation of the rotor wheel; $\alpha \nu$ - the phase angle between transmitters.

![Fig. 11. Transmitter arrangement for measuring resonance vibrations in constant rotation speed.](image)

In a case where the multiple $\nu$ is known in advance, it is sufficient to take measurements in two points on the periphery. Unknown quantities $A$ and $\phi_n - \psi$ are then determined from the system

$$ y_1 = A \sin (\varphi_n - \psi); \quad y_2 = A \sin (\varphi_n - \psi + \alpha \nu), $$

from which

$$ A = \frac{y_1}{\sin \alpha \nu} \sqrt{1 + \frac{y_2^2 y_1^2}{-2(y_2/y_1) \cos \alpha \nu}}. $$

An example of measuring the resonance vibrations of the blades of the rotor wheel in the axial compressor stage by two pairs of transmitters is presented in Fig. 12.
Finally, there is one more method of measuring resonance vibrations which has found practical application. It consists of a device designed to create a nonuniformity, which is slowly rotated ahead of the studied compressor [5]. Here the strict multiplicity of the blade vibration frequency and the circular frequency is destroyed and resonance vibrations are measured just as in the case of nonmultiple vibrations of other types.

This method provides the best results, although, unfortunately it can only be used in research, where the circular irregularity is created artificially by special devices.

3. CONCLUSION

The discrete-phase method for contactless measuring of blade vibrations in a rotor wheel, despite the short time that it has been used, can be highly recommended on the basis of experience. The ELURA which is based on it is used to measure bending and torsional blade vibrations in cases where these vibrations can be either nonmultiples of the circular frequency (natural vibrations, stall vibrations, etc.) or multiples of it (resonance vibrations).
have been worked out to measure the vibration amplitudes of blades, the velocity amplitudes of their vibrations, frequencies, and also the phase shift of vibrations in individual blades. Studies [6-8] present data for the instantaneous distribution of amplitudes over all blades in the ring in the presence of natural vibrations. It is very difficult to obtain such data using strain gage methods.

The possibility of measuring vibrations simultaneously in all rotor blades, unlimited work reserves, the possibility of various combinations in placing the transmitters - all of these advantages of the contactless method will no doubt expand the research front and enable new data on the behavior of both individual blades and groups of rotor blades in compressors and turbines.

In a number of cases it is desirable to use standard strain gages along with the ELURA. These make it possible to measure strain in a certain part of the blade. This can be particularly important in testing blades with hinged attachment.

The possibilities of using the discrete-phase method to study and measure vibrations in rotor blades is not at all limited to the examples which we have studied. Furthermore, ELURA, despite its universality and great potential, is not the only means of achieving this method. In a number of cases it might be more convenient to record in advance the coded time intervals between the signals of the impulse transmitters on magnetic tape in order to subsequently process measurement results on the electronic computer.

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