A two-dimensional axial symmetric cloud dynamics model, which utilizes the Boussinesq approximation form of the equations of motion, was used to study downwash-induced fog clearings. The flow produced by constraining the vorticity to a prescribed value in a small region near the centerline of the model is similar to that observed to occur below a hovering helicopter. Several numerical experiments are discussed which show the efficacy of...
20. (continued)

different hovering altitudes on the clearing of fog. The major feature affecting the size of clearing by helicopter downwash is the size of the primary downwash volume. Clearings near the surface occurred only when the flight level was low enough for the primary downwash to reach the ground. The primary downwash size was shown to be greatly affected by the temperature difference between the air in the primary wake and its environment. As the wake temperature was increased, the downwash penetration distance was greatly reduced, resulting in smaller clearings. Turbulent mixing generated at the edge of the primary wake was found to be a significant factor in extending the volume of clearing.
A two-dimensional axial symmetric cloud dynamics model, which uses the Boussinesq approximation form of the equations of motion, was used to study downwash-induced fog clearings. The flow produced by constraining the vorticity to a prescribed value in a small region near the centerline of the model is similar to that observed to occur below a hovering helicopter. Several numerical experiments are discussed which show the efficacy of different hovering altitudes on the clearing of fog. The major feature affecting the size of clearing by helicopter downwash is the size of the primary downwash volume. (Modified author abstract)
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G. D. HAMILTON

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USING HELICOPTERS TO CLEAR FOG:
A NUMERICAL STUDY

by

DAVID B. JOHNSON, EDWARD H. BARKER
and
PAUL R. LOWE

MARCH 1974
Qualified requestors may obtain additional copies from the Defense Documentation Center. All others should apply to the National Technical Information Service.
I. INTRODUCTION

Fog constitutes a considerable hazard to military operations. Several studies (e.g., Kunkel, Silverman and Weinstein, 1973) have shown that the direct application of heat energy to fog offers a workable, cost-effective procedure for fog dispersal. Other work (e.g., Tag, 1971) has illustrated the usefulness of seeding with hygroscopic materials as a means of fog clearing.

A major drawback to the above two approaches is the requirement for considerable logistical support. This is particularly true of the heat procedure which is best suited as a fog dispersal system at military air installations. The hygroscopic seeding approach also requires considerable support in the form of equipment, personnel and other resources. Further, the seeding procedure has the added problem of difficulty in targeting the seeding material in order to produce clearing in a precise location.

The resources necessary to implement the above procedures frequently are not available in military operations. The availability of helicopters in many military operations, however, provides the opportunity to use another fog dispersal technique. Through this technique, holes may be cleared in fog or stratus by the downwash (wake) from a hovering or slowly moving helicopter.

The first documented case of fog clearing by helicopter is reported by Nordquist and Dickson (1972). In this case, which occurred in West Germany in the spring of 1961, a U. S. Army UH-73 helicopter was used to clear ground fog from a runway to a length and width sufficient for the recovery of three fixed wing aircraft. There are several other documented cases of fog dispersal by means of helicopter downwash, some in combat situations.
The first research study relative to this procedure, was conducted by the U. S. Army Cold Regions Research and Engineering Laboratories (CRREL) (Hicks, 1965) in Greenland. Since then, several field research studies have been conducted. Notable among these have been the U. S. Air Force Cambridge Research Laboratory's (AFCRL) experiments in clearing holes in stratus banks in the vicinity of Eglin AFB in 1968 (see Plank and Spatola, 1969, and Plank, 1969), and Air Force field studies applying the technique to ground and steam fogs in the vicinity of Smith Mountain Lake, Virginia, (also, Plank, 1969). Perhaps the most useful field studies of the helicopter technique were conducted in a joint effort by, the U. S. Army, Atmospheric Sciences Laboratory (ASL), CRREL and AFCRL in which the technique was applied to valley radiation fogs in West Virginia (see Plank, Spatola and Hicks, 1970). Also joint Naval Weapons Center (NWC) and ASL experiments have been conducted on relatively deep coastal advection fogs at Arcata, California (Nordquist, 1972).

From the results of these several field tests, it was concluded that the procedure of using helicopters could be useful under certain conditions. Unfortunately, the number of field tests was insufficient to delineate the conditions of applicability necessary to place the helicopter procedure on an operational basis. The expense of gathering personnel and equipment, added to a lack of suitable fog situations, prohibits a sufficient number of cases. However, another avenue of investigation was available -- numerical modeling.

In 1972, at the request of the United States Marine Corps Development and Educational Command, Quantico, Virginia, the Environmental Prediction Research Facility undertook to develop a mathematical computer model for the purpose of making realistic simulations of the helicopters and the effect of helicopter downwash on existing fogs under a variety of atmospheric
conditions. The employment of such a model makes it possible to study a variety of cases, sufficient for the differentiation of those conditions under which the procedure is or is not useful. Further, modeling techniques can produce information at a very small fraction of the cost of field experiments. This report presents results and conclusions from the first phase of the modeling studies. The study, to date, has been jointly funded by the Marine Corps Development and Educational Command, and the Environmental Prediction Research Facility.
2. THE MODEL

While studies have been conducted in which downwash wakes have been simulated (see Shupe, 1972) there has been, until now, no effort to simulate the downwash together with its effect on fog in realistic atmospheric situations. However, two such studies have been recently undertaken. Besides the work reported here, another study of this problem has been underway by W. S. Nordquist at the Army's Atmospheric Sciences Laboratory. Nordquist's work has been based on simulations using a one-dimensional model, the results of which have not yet been published.

The experiments reported here are based on simulations using a two-dimensional model derived from the cumulus dynamics model developed at The Rand Corporation (Murray, 1971; and Murray and Koenig, 1972). The model utilizes axial symmetry and is based on the Boussinesq form of the equations of motion. Two-dimensionality is required for a study of this nature for two reasons. Firstly, it is necessary in order to simulate the interactive effects of the downwash wake and the less active environmental air. Secondly, it is absolutely necessary to be able to simulate the outflow along the ground at the bottom of the downwash wake (outwash).

The mathematical description of the basic model as well as those modifications needed to simulate the downwash are detailed in Appendix A.
3. RESULTS

3.1 FLOW FIELDS COMPUTED USING MODEL

Hover-state parameters as indicated by Table 1 were used to design the heat, water and momentum impulses required for the simulation of CH-46 and CH-53 helicopters. Examples of the flow fields generated by the two helicopters after one minute of hover time are shown in Figures 1 and 2.

Table 1. Helicopter hover-state parameters (from Plank, Spatola and Hicks, 1970b).

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>Units</th>
<th>HELICOPTER TYPE</th>
<th>CH-46</th>
<th>CH-53B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Kg (pounds)</td>
<td></td>
<td>8,618 (19,000)</td>
<td>15,876 (35,000)</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>m</td>
<td></td>
<td>15.24</td>
<td>22.02</td>
</tr>
<tr>
<td>Mass flux of air across rotor(s) at standard gravity and air density</td>
<td>Kg/sec</td>
<td></td>
<td>4300</td>
<td>5963</td>
</tr>
<tr>
<td>Maximum downwash velocity at standard gravity and air density</td>
<td>m/sec</td>
<td></td>
<td>19.6</td>
<td>26.0</td>
</tr>
<tr>
<td>Average fuel consumption rate and power setting*</td>
<td>gm/sec</td>
<td>%</td>
<td>150-50%</td>
<td>290-50%</td>
</tr>
<tr>
<td>a. In normal cruise condition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. In free-air hover condition</td>
<td></td>
<td></td>
<td>250-75%</td>
<td>375-80%</td>
</tr>
<tr>
<td>Assumed engine efficiency*</td>
<td>%</td>
<td></td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>Rotor type</td>
<td></td>
<td></td>
<td>Tandem</td>
<td>Single</td>
</tr>
</tbody>
</table>

*With JP-4 fuel,
  a. heat of combustion is 10,220 cal/gm
  b. water produced by combustion is 1.27 gm/gm
Figure 1. Flow generated by CH-46 helicopter at flight level 370 m. Isotachs are labeled in units of m sec\(^{-1}\).

Figure 2. Flow generated by CH-53 helicopter at flight level 370 m. Isotachs are labeled in units of m sec\(^{-1}\).
Figure 3. Flow generated by CH-46 helicopter at flight level 90 m. Isotachs are labeled in units of m sec$^{-1}$.

Figure 3 illustrates the flow field resulting when the downwash created by the perturbation used in Figure 1 reaches the model's lower boundary.

Profiles of vertical velocity along the centerline of a CH-53 downwash for different values of eddy viscosity ($v_m$) along with the observed profile of Plank et al. (1969) are shown in Figure 4. An eddy viscosity of $5 \text{ m}^2 \text{ sec}^{-1}$ produces the least overall error. The maximum downwash velocities near the helicopter are slightly underestimated, while the velocities further beneath the helicopter are slightly overestimated.
Figure 4. Computed velocity along centerline of downwash for several values of eddy viscosity ($\nu_m$) compared with that observed by Plank et al, 1969.
3.2 EFFECT OF HELICOPTER HEIGHT ON CLEARING

Figure 5 presents a summary of a series of experiments in which the altitude of the helicopter was varied. The size of a given clearing is defined as the minimum radius of that clearing. For the CH-46 simulations, a flight level of 210 m (~100 m above the fog) resulted in the largest clearing during the first four minutes; whereas a flight level of 130 m (~30 m above the fog) eventually resulted in the largest clearing. During the CH-53 simulations, a similar pattern occurred. While a higher flight level of 290 m resulted in the largest clearing during the first four minutes, a flight level of 210 m eventually resulted in the largest clearing.

In general, a low helicopter flight level will produce flow similar to that shown in Figure 6, undercutting the fog, but not producing effective clearings. In some cases a breakthrough occurs to produce a dramatic clearing after three to four minutes (see Figure 7). The best clearings are produced by a relatively narrow range of hover altitudes which allow the primary downwash to reach the ground with sufficient force to be broadened by the surface induced divergence, but which do not produce major undercutting. While not producing a dramatic breakthrough, this method of clearing quickly forms a moderate sized opening in the fog. As the helicopter height is increased, the size of clearing produced reduces until the helicopter wake is too weak to penetrate the fog (see Figure 8).

3.3 EFFECT OF DOWNWASH STRENGTH ON CLEARING

Figure 5 illustrated the effect of height on clearing for two strengths of downwash. For lower helicopter heights, the clearings produced (see Figure 9) are almost identical; the stronger downwash producing openings of slightly larger diameter. Helicopter heights near the upper limit for fog penetration show considerably more differences; the stronger downwash being able to produce clearings from greater altitudes (see Figure 10).
Figure 6. Example of downwash-induced clearing, (a) after 2 minutes and (b) after 4 minutes, when CH-46 helicopter was too low. (Velocity vectors are drawn for every third grid point and are scaled in such a manner as to show the characteristics of flow in regions of light wind - logarithmically scaled.)
Figure 7. Example of a CH-46 hover altitude which produced a dramatic clearing, (a) after 4 minutes, (b) after 5 minutes.
Figure 8. Examples of downwash-induced clearing when CH-46 helicopter was too high, (a) flight level 290 m, (b) flight level 370 m.
Figure 9. Time sequences of clearing by (a) CH-46 and (b) CH-53 helicopters hovering at 130 m.
Figure 10. Simulations of (a) CH-46 and (b) CH-53 helicopters after 3 min hovering at 370 m.
3.4 EFFECT OF HELICOPTER ENGINE HEATING ON CLEARING

Heat produced by jet engines has been investigated as a means of dissipating fog (Appleman and Coons, 1970). Three experiments were performed which included source terms for heat and water vapor produced by the helicopter engines. In one experiment all heating was suppressed; in the second, normal engine heating was included; and, in the third, engine heating was increased by a factor of six. In the case of heat produced by one helicopter, the departure of virtual temperature from its basic state directly below the helicopter increased by 0.9°C over that which occurred with no engine heating. An increase of the engine heating by a factor of six increased the departures of virtual temperature directly below the helicopter by 4°C over that which occurred with no engine heating.

The results of these runs (Figures 11, 12 and 13) show decreasing effectiveness as engine heating is increased. Although the heated downwash would be more effective when mixed with the fog, the additional heat increases the buoyancy, thus restricting the downward and horizontal penetration of the downwash, and resulting in less effective clearings.

3.5 EFFECT OF STABILITY ON CLEARING

Three experiments were run to test the role of atmospheric stability within a fog on the helicopter's ability to produce a clearing. For these experiments the lapse rate ranged from isothermal to a 10°C inversion (Figure 14). The results of these experiments show that increasing the atmospheric stability decreases the size of the clearing (see Figures 15, 16, and 17). The buoyancy of a parcel of air being displaced downward in the helicopter wake increases with increasing stability. This increased buoyancy limits the size of the primary downwash and restricts its penetration, resulting in smaller clearings than
Figure 11. Clearing produced by CH-46 helicopter without engine heat.

Figure 12. Clearing produced by CH-46 helicopter with normal engine heat.
Figure 13. Clearing produced by CH-46 helicopter when engine heat was increased six times normal engine heating.

computed for less stable conditions. The effect of increased stability of clearing parallels the effect of increased engine heating of the downwash in that the buoyancy of the downwash was enhanced in both cases.

3.6 EFFECT OF FOG LIQUID WATER CONTENT ON CLEARING

A series of experiments were performed to investigate the effect of the fog's liquid water content on clearing. The initial thickness of the fog was the same in each experiment, but during the course of the experiment the fogs with higher liquid water content diffused into the surrounding air, and resulted in slightly deeper fogs. The experiments show that, although the major features of the clearing remain essentially the same, the difficulty of clearing increases with liquid water content. The increased liquid water content tends to slow the production of a given sized clearing by increasing the volume of dry air which must be mixed into the fog to evaporate the liquid water.
Figure 14. Temperature and dewpoint sounding for the three experiments depicting effect of stability within the fog.

Figure 15. Clearing produced in an isothermal fog layer (sounding A).
Figure 16. Clearing produced in a fog with a 2.5°C temperature increase from base to top (sounding B).

Figure 17. Clearing produced in a fog with a 10°C temperature increase from base to top (sounding C).
Figures 18 and 19 illustrate the results of two of these experiments. After one minute the clearings are almost identical, the fog with a lower liquid water content having a slightly larger opening. At five minutes, however, the lower liquid content fog has been cleared through the undercut fog while in the higher liquid content fog this stage has not been reached.

3.7 EFFECT OF FOG DEPTH ON CLEARING

Three experiments were conducted to examine the effect of fog depth on clearing. Except for the depth of the fog, the initial conditions used for the experiments were identical. Experiment results show that in general, the deeper the fog, the more difficult it is to clear. Fogs deeper than about 100 m are difficult to clear effectively since any broadening of the primary downwash by the surface cannot extend to the fog top (see Figures 20 and 21). The clearing illustrated by Figure 20 is of additional interest in that the diameter of clearing at the surface is wider than observed in a similar experiment (Figure 21) with a shallower fog. The greater penetration was caused by evaporative cooling in the downwash, which cooled the wake and reduced its buoyancy. An experiment in which the helicopter hovered within the fog merely caused mixing in the fog and failed to produce a useful clearing.

3.8 EFFECT OF DOWNWASH ON A DUST CLOUD

In order to determine the importance of fog droplet evaporation through mixing of dry air into foggy air, the effect of advection alone must be considered. In order to make this comparison, one of the experiments was rerun while modifying the thermodynamic equation to simulate a dust cloud instead of a fog. A comparison of the results of this experiment (Figure 22) with those in Figure 23 show that mixing of dry air into foggy air as a means of subsaturating the air is an important process in extending the clearing beyond the
Figure 18. Clearing produced in a fog with a liquid water content of .20 gm kg\(^{-1}\), (a) after 1 minute, (b) after 5 minutes.
Figure 19. Clearing produced in a fog with a liquid water content of 0.40 gm kg⁻¹, (a) after 1 minute, (b) after 5 minutes.
Figure 20. Clearing produced in a fog after 5 minutes with initial depth of 200 m.

Figure 21. Clearing produced in a fog after 5 minutes with initial depth of 95 m.
Figure 22. Clearing produced in a dust cloud with a dust concentration of .20 gm kg\(^{-1}\), (a) after 1 minute, (b) after 4 minutes. (Isopleths of concentration are in gm kg\(^{-1}\).)
Figure 23. Clearing produced in a fog with a water concentration of .20 gm kg\(^{-1}\), (a) after 1 minute, (b) after 4 minutes.
primary downwash volume. In the dust cloud, the clearing radius was equal to the downwash radius; whereas in the fog the clearing radius reached nearly twice that of the downwash.

3.9 EVALUATION

The usefulness of numerical modeling in the design of techniques for fog modification by helicopter has been demonstrated. An obvious major advantage of this type of study over field experiments is the very low cost of conducting numerical experiments (about $75 for a 5-minute simulation). Additionally, the results are much more easily analyzed and diagnosed. Although an absolutely exact description of the helicopter's downwash is not possible in this type of model, the results attained can be used to limit the endless number of possibilities which would otherwise exist.

Future modeling experiments will include the effects of dispensing desiccant-type seeding agents into the downwash (a pilot study indicates that design of improved techniques is possible using the computer). Another useful series of experiments will be a study of the wake effects of a moving helicopter. This will aid in determining the most efficient helicopter flight altitude and speed for different fogs.
4. SUMMARY

The major feature affecting the size of clearing produced by helicopter downwash is the size of the primary downwash volume. In order to produce a surface clearing, the helicopter must be low enough for the primary downwash to reach the ground, thus the downwash becomes wider and, therefore, more effective. Modeling results show that the most effective hovering altitude for the CH-53 is 290 m and 210 m for the CH-46 for fog approximately 100 m thick. Liquid water content or fog thickness does not greatly affect the dimensions of the primary downwash. However, temperature departure of the downwash from that of its environment greatly affects the size of the primary downwash. The penetrative ability of the downwash is greatly diminished as its temperature relative to the inactive environment increases. This is true whether the increase is the result of engine heat or atmospheric stability. In other words, the more buoyant the downwash, the smaller the depth of penetration.

The experiments producing dramatic clearings involve more than the size of the primary downwash. In these instances the secondary circulations, which form around the primary downwash, mix dry air into foggy air and thereby evaporate the fog (see point A, Figure 18). The increase in the size of clearing resulting from the mixing action of the secondary circulations is demonstrated by the much larger clearing generated in fog compared to the clearing generated in suspended dust of the same depth and concentration.

The most dramatic clearings produced in this study occurred when the helicopter was relatively low and undercut the fog, eventually resulting in dissipation of the undercut "lip" by the secondary circulation. However, this type of clearing takes a relatively long time to break through and it is unknown whether such a clearing would be observed in the field where ambient winds are most likely to greatly disguise and suppress the secondary circulations.
REFERENCES


REFERENCES (CONT.)


Tag, P.M., 1971: Results generated from a one dimensional warm fog model which simulates hygroscopic seeding. NAVWEARSCMFAC Tech. Paper No. 11-71, 63 pp.
APPENDIX

MODEL DESCRIPTION

The numerical model used for this study is an adaptation of the warm precipitation version of the RAND cumulus dynamics model (Murray, 1971; and Murray and Koenig, 1972). The model utilizes axial symmetry and is two-dimensional.

The equation of motion with the Boussinesq approximation and the continuity equation are the basis of the model where

\[ \frac{\partial \vec{v}}{\partial t} = -\vec{v} \cdot \nabla \vec{v} - \frac{1}{\rho_m} \nabla p' + g \left( \frac{T'}{T_{vm}} - q_c \right) \hat{k} + \nu_m \nabla^2 \vec{v} + \vec{f} \quad (A.1) \]

\[ \nabla \cdot \vec{v} = 0 \quad (A.2) \]

Variables undefined in the text are defined in Table A-1.

Buoyancy

\[ B = g \left( \frac{T'}{T_{vm}} - q_c \right) \quad (A.3) \]

depends on the departure of virtual temperature from its basic state and the weight of suspended mass (i.e., liquid water). The mechanical forcing term, \( \vec{f} \), is that generated by the helicopter.

Combination of Eq. (A.2) and the curl of Eq. (A.1) in a cylindrical coordinate system (assuming axial symmetry and no rotation about the symmetric axis) yields the vorticity tendency equation

\[ \frac{\partial \eta}{\partial t} = - \left[ \frac{\partial}{\partial r} (u\eta) + \frac{\partial}{\partial z} (w\eta) \right] - \frac{\partial B}{\partial r} + \nu_M \left\{ \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r\eta) \right] \right. \]

\[ + \frac{\partial^2 \eta}{\partial z^2} \left\} \right. + \frac{\partial F}{\partial z} \frac{\partial F}{\partial r} \quad (A.4) \]

Horizontal and vertical components of the wind field are defined in terms of the Stokes stream function, \( \psi \), where
Table A-1. Symbols not defined in text.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>buoyancy</td>
</tr>
<tr>
<td>C</td>
<td>rate of condensation</td>
</tr>
<tr>
<td>(c_p)</td>
<td>specific heat of dry air at constant pressure</td>
</tr>
<tr>
<td>(c_{pv})</td>
<td>specific heat of water vapor at constant pressure</td>
</tr>
<tr>
<td>(c_w)</td>
<td>specific heat of liquid water</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>k</td>
<td>vertical unit vector</td>
</tr>
<tr>
<td>L</td>
<td>latent heat of condensation</td>
</tr>
<tr>
<td>(p')</td>
<td>departure of air pressure from its value at initial time for the given altitude</td>
</tr>
<tr>
<td>(q_c)</td>
<td>mixing ratio of suspended water to dry air</td>
</tr>
<tr>
<td>(q_s)</td>
<td>saturation value of (q_v)</td>
</tr>
<tr>
<td>(q_v)</td>
<td>mixing ratio of water vapor to dry air</td>
</tr>
<tr>
<td>(R_d)</td>
<td>gas constant for dry air</td>
</tr>
<tr>
<td>r</td>
<td>radial coordinate</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>(T'_v)</td>
<td>departure of temperature from its value at initial time for the given altitude</td>
</tr>
<tr>
<td>(T'_v)</td>
<td>departure of virtual temperature from its value at initial time for the given altitude</td>
</tr>
<tr>
<td>(T_{vm})</td>
<td>mean value of virtual temperature at initial time over the whole region</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>u</td>
<td>radial component of wind</td>
</tr>
<tr>
<td>v</td>
<td>velocity of air parcel</td>
</tr>
<tr>
<td>w</td>
<td>vertical component of wind</td>
</tr>
<tr>
<td>z</td>
<td>vertical coordinate</td>
</tr>
<tr>
<td>(e)</td>
<td>ratio of molecular weights of water vapor and dry air</td>
</tr>
<tr>
<td>(n)</td>
<td>azimuthal component of vorticity</td>
</tr>
<tr>
<td>(v_M)</td>
<td>eddy diffusion coefficient for momentum</td>
</tr>
<tr>
<td>(v_p)</td>
<td>eddy diffusion coefficient for temperature</td>
</tr>
<tr>
<td>(v_v)</td>
<td>eddy diffusion coefficient for water vapor</td>
</tr>
<tr>
<td>(v_c)</td>
<td>eddy diffusion coefficient for cloud water</td>
</tr>
<tr>
<td>(\rho)</td>
<td>air density</td>
</tr>
<tr>
<td>(\rho_c)</td>
<td>vapor density due to combustion</td>
</tr>
<tr>
<td>(\rho_m)</td>
<td>mean value of air density at initial time</td>
</tr>
<tr>
<td>(\rho_d)</td>
<td>density of dry air at a given altitude at initial time</td>
</tr>
<tr>
<td>(\psi)</td>
<td>stream function</td>
</tr>
</tbody>
</table>
\[ u = - \frac{1}{r} \frac{\partial \psi}{\partial z} \quad (A.5) \]

\[ w = \frac{1}{r} \frac{\partial \psi}{\partial r} , \]

which is related to the tangential component of vorticity,

\[ \eta = - \left( \frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} \right) = - \frac{1}{r} \left[ r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \frac{\partial^2 \psi}{\partial z^2} \right] . \quad (A.6) \]

Assuming that the velocity field, and consequently the vorticity field, does not change with time in the immediate vicinity of the helicopter (extending from the flight level to 30 m immediately below), allows a simple treatment of this region. For vorticity to be invariant in the region of the helicopter with time, the vorticity source term,

\[ S_{\eta} = \frac{\partial F}{\partial z} - \frac{\partial F}{\partial r} , \quad (A.7) \]

must exactly balance the loss due to the other terms in Eq. A.4. Therefore, once the vorticity field is established to that which corresponds to the type of helicopter being simulated, its value is maintained throughout the period of simulated mechanical forcing.

Outside the region of constrained vorticity the source term vanishes and the vorticity balance depends on the remaining terms in Eq. A.4. The choice of a proper value for the eddy viscosity coefficient allows a controlled diffusion of momentum similar to observed helicopter-generated flow fields. The value of eddy viscosity most suitable for all experiments conducted was found to be 5 m\(^2\) sec\(^{-1}\).

Model thermodynamics are described by

\[ \frac{dT'}{dt} = \frac{LC + Q_H - gw}{c_p + q_v c_{pv} + (q_c + q_p)c_w} - \frac{dT}{dt} + \nabla T \cdot \nabla T' , \quad (A.8) \]
where \( Q_H \) is the heat released by the helicopter engine; and, \( C \) is the rate of condensation. From conservation of energy, \( Q_H \) is defined as

\[
Q_H = (1 - E) S_f G/V , \tag{A.9}
\]

where \( E \) is the engine efficiency (30\%), \( S_f \) is the fuel consumption rate, \( G \) is the heat of combustion \((10.22 \cdot 10^6 \text{ cal/kg})\), and \( V \) is volume of air affected by the heat increase.

Water is accounted for by partitioning it into vapor and suspended cloud liquid. Therefore, the governing equations for conservation of water are

\[
\frac{dq_v}{dt} = -C + v_v \nabla^2 q_v + S_v , \tag{A.10}
\]

\[
\frac{dq_c}{dt} = C + v_c \nabla^2 q_c . \tag{A.11}
\]

\( S_v \) is the moisture source resulting from the combustion of fuel,

\[
S_v = \frac{1}{\rho_d} \left( \frac{\partial \rho_c}{\partial t} \right) , \tag{A.12}
\]

where

\[
\frac{\partial \rho_c}{\partial t} = \varepsilon S_f / V
\]

where \( \varepsilon \) is the quantity of water vapor produced per gram of fuel during combustion.

As derived by Murray (1970), the condensation rate is defined by

\[
C = \begin{cases} 
\frac{1}{\varepsilon L} & \text{if } q_v = q_s \\
\frac{c_p T - L q_s}{c_p R d T^2} & \text{if } q_v < q_s
\end{cases}
\]

\[
C = 0 \quad \text{if } q_v < q_s .
\]
The eddy diffusion coefficient chosen for both heat and water substance was 2 m²/sec.

BOUNDARY AND INITIAL CONDITIONS

The numerical model treats the computations at the center-line as though they have axial symmetry. The remaining limits of the computational domain are treated as rigid, free-slip boundaries. The modeled volume (see Figure A-1) for a 10-meter grid spacing is a cylinder 350 m radius and 500 m in height.

Figure A-1. Computational grid. \( H \) is the value of \( K \) at flight level.
The region of constrained vorticity previously discussed is shown within the heavy boundary in Figure A-1, its radius is two mesh-lengths and its thickness is three. The shear between the downwash and its environment is assumed to occur along the J=2 column and, therefore, the strength of the downwash is determined by the strength of vorticity assigned to this column (in the constrained region). The upstream points from this "vorticity source", the J=3 column and the top point of the J=2 column, are constrained to zero. At the model's centerline, as on the other points surrounding the computational domain, vorticity is kept at zero.

Initially, the air is assumed at rest and horizontally homogeneous. Liquid water is introduced at the lower levels of the model to simulate fog. Figure A-2 illustrates the initial temperature and dewpoint sounding for the model domain. The depth of the fog is indicated by the position of the dot-dash line in Figure A-2. The liquid water content was constant at 0.2 g/kg throughout the fog.

Figure A-2. Temperature and dewpoint sounding.