SATURATION CHARACTERISTICS OF MINIATURE IONIZATION CHAMBERS IN PULSED LINAC EXPOSURES

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ABSTRACT

Theoretical expressions have been derived for the critical applied voltage below which charge collection is incomplete in ionization chambers exposed to pulsed radiation. To test the accuracy of the expressions, a 0.5 cm$^3$ ionization chamber was exposed to 100-nsec pulses in a 35-MeV electron beam from the AFRRI LINAC. Several gases with negligible electron attachment coefficients were used. These included argon, carbon dioxide, nitrogen, and a tissue-equivalent mixture of methane, carbon dioxide and nitrogen. It was found that the theory accurately predicted that, with 1000 volts applied, a charge density of 11 esu per pulse could be completely collected. It was also found that, for an air-filled chamber sufficiently small so that negative charge is transported to the anode by electrons, the derived expressions accurately predict the critical voltage.
I. INTRODUCTION

For an ionizing pulse with a duration short compared to positive ion transport times across the gap of an ionization chamber filled with a nonelectronegative gas, there is a critical applied polarizing voltage below which not all of the charge in the pulse can be collected. This critical voltage is given by

\[ V = 2 \pi q d^2 \]  

where \( q \) is the charge density of one sign liberated in the cavity by the ionizing pulse and \( d \) is the cavity gap width. For cylindrical and spherical geometries "effective" gap widths can be determined by solving Poisson's equation with the boundary condition specified by a positive ion charge density sufficient to shield the anode from the applied field.

In this report, equations will be derived (see section II) which yield expressions for the critical voltages in parallel-plate, infinite cylinder, and spherical geometries. Comparisons of the theoretically derived values with experimental results are made in section IV.

II. THEORY -- SOLUTION OF POISSON'S EQUATION

Assumptions. First, an ionization chamber is exposed to a pulse of radiation that is spatially uniform over the dimensions of the cavity. The duration of the pulse is assumed to be very short compared to positive ion collection times for the chamber. Second, the cavity is completely free of electronegative contaminants so that all negative charge carriers are electrons. The electrons are assumed to have a very large mobility relative to that of the positive ions and the coefficient characteristic of
electron-molecule attachment is assumed to be negligibly small. Third, in the presence of an electric field, ion motion is dominated by field forces.

**Chamber geometry.** Figures 1 and 2 show the geometries used. The high voltage, regardless of its polarity, is always applied at \( r = b \) and the signal lead is always attached at \( r = a \).

![Figure 1. Infinite parallel-plate chamber geometry](image1)

![Figure 2. Infinite cylinder and spherical chamber geometry](image2)
Space charge limit. The quantity of interest is the number of ion pairs created in the cavity. The measured quantity is the number of ion pairs (electrons and positive ions) that reach their respective collecting electrodes. The measured quantity will be equal to the quantity of interest only if, during the very short time it takes to sweep the electrons from the cavity, there is no point in the cavity where the electric field vanishes or reverses itself. Should this happen, electron motion would be stopped at that point by the net positive space charge buildup, and the electrons would begin to recombine with positive ions. The first point at which the electric field will vanish in the presence of a positive space charge is at the anode, so mathematically this limit is expressed by the boundary condition

\[ E_{\text{anode}} = 0 \]  

(2)

Poisson's equation for the residual positive ion space charge is

\[ \nabla \cdot E = \frac{dE}{dr} + \frac{\beta E}{r} = 4\pi q \]  

(3)

where

\[ \begin{align*}
0, & \text{ infinite parallel plates} \\
\beta = 1, & \text{ infinite cylinder} \\
2, & \text{ sphere}.
\end{align*} \]

(4)

By making the transformation, \( u - E/r \), the differential equation is made separable in the variables \( u \) and \( r \) and integration yields

\[ E = \frac{4\pi q}{1 + \beta} \left( r - \frac{\alpha}{r\beta} \right) \]  

(5)

where \( \alpha \) is an integration constant to be determined by application of the boundary condition expressed in equation (2). One has, then
and
\[ E^{-} = \frac{4 \pi q}{1 + \beta} \left( r - \frac{b^{\beta+1}}{r \beta} \right) \]  

where the superscripts on \( E \) indicate the polarity of the applied high voltage. The applied voltages necessary to attain the fields given by equations (6) and (7) are

\[ V_{0}^{+} = \int_{b}^{a} E^{+}(r) \, dr = \int_{b}^{a} \frac{4 \pi q}{1 + \beta} \left( r - \frac{a^{\beta+1}}{r \beta} \right) \, dr \]  
\[ V_{0}^{-} = \int_{b}^{a} E^{-}(r) \, dr = \int_{b}^{a} \frac{4 \pi q}{1 + \beta} \left( r - \frac{b^{\beta+1}}{r \beta} \right) \, dr \]  

The results of integration of equations (8) and (9) are shown in Table I.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>( V_{0}^{+}/2 \pi q )</th>
<th>( V_{0}^{-}/2 \pi q )</th>
</tr>
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<tbody>
<tr>
<td>Infinite parallel plate</td>
<td>( (b - a)^2 )</td>
<td>( (b - a)^2 )</td>
</tr>
<tr>
<td>Infinite cylinder</td>
<td>( b^{2} \ln \left( \frac{b}{a} \right) - \frac{1}{2} \left( b^{2} - a^{2} \right) )</td>
<td>( \frac{1}{2} \left( b^{2} - a^{2} \right) - a^{2} \ln \left( \frac{b}{a} \right) )</td>
</tr>
<tr>
<td>Sphere</td>
<td>( \frac{1}{3 \lambda} \left( a^{3} - 3a b^{2} + 2b^{3} \right) )</td>
<td>( \frac{1}{3 \lambda} \left( b^{3} - 3a^{2} b + 2a^{3} \right) )</td>
</tr>
</tbody>
</table>

For the parallel-plate geometry, one observes that the critical voltage is independent of polarity and is given by the relation derived by Boag.\(^2\) For the cylindrical and spherical geometries, the critical voltage is polarity-dependent. The ratio of the critical voltages at positive and negative polarities is identical to that obtained by Lapsley\(^3\) for chambers exposed to steady-state irradiations.
The entries in Table I are "effective gap widths" and the limiting voltage is given by

$$V_0 = 2\pi q d_{\text{eff}}^2.$$  

(10)

III. EXPERIMENT -- DETERMINATION OF THE EFFICIENCY OF A 0.5 cm$^3$ MAGNESIUM IONIZATION CHAMBER IN A PULSED ELECTRON BEAM

The ionization chamber. Figure 3 shows a cross section of a 0.5 cm$^3$ ionization chamber that was fabricated at Illinois Benedictine College under the direction of Dr. John Spokas. High voltage is applied to the outer wall of the chamber and the signal is taken at the central electrode which is at virtual ground.

![Figure 3. Section of 0.5 cm$^3$ chamber](image)

When the dimensions from Figure 3 are applied to the equations in Table I, one obtains

$$V_0^+ / q \text{ cylinder} = 123 \frac{\text{volt-cm}^3}{\text{esu}}.$$  

(11)
With the chamber operated with positive high voltage applied to the outer wall, the critical voltage below which the efficiency falls below 100 percent is limited by the spherical region of the chamber and is given by equation (13). When a negative high voltage is applied to the outer wall, the cylindrical region limits the critical voltage that is given by equation (12). The chamber was operated with a variety of gases: carbon dioxide, argon, nitrogen, and a tissue-equivalent (TE) gas (64 percent CH₄, 32.8 percent CO₂, 3.2 percent N₂). All of these gases have very small electron attachment coefficients and satisfy the assumption that negative charge is carried to the electrodes by electrons rather than negative ions. The gases were flowed through the chamber at a flow rate of about 1 ft³/h at atmospheric pressure.

It was found that the chamber would arc at an applied voltage considerably below the field strengths obtained by dividing the breakdown voltage for the gas by the gap width. This is due to the fact that the uniform electric field of a parallel-plate geometry is not present and also to the fact that there are large field gradients near the periphery of the sensitive volume of the chamber. In order to stay well below breakdown, the chamber was not operated at applied high voltages above 1000 volts.

To determine the transit time of positive ions, one may estimate

$$\tau \approx \frac{d^2}{2kV}$$ (15)
where $\tau$ is the transit time, $k$ is the positive ion mobility and $V$ is the applied voltage. Positive ion mobilities are on the order of $2 \text{ cm}^2/\text{volt-sec}$. An estimate of the transit time for positive ions for the $0.5 \text{ cm}^3$ chamber with 1000 volts applied is then $15 \mu\text{sec}$.

The radiation field. The radiation field chosen for this experiment was the water-scattered 35-MeV electron beam from the AFRRI LINAC. A 1-cm thick $\text{H}_2\text{O}$ scatterer in an aluminum casing was placed over the beam port; and, with a beam current of 500 mA, a water dose (determined with chemical Fricke dosimetry) of 4.1 water rads/pulse was attained at a point 3 meters from the beam port. To obtain lower values of $q/V$, the chamber was placed in a polyethylene phantom where the dose was measured with Fricke to be 0.5 rad/pulse. A 100-nsec pulse width assures that the energy deposition time is small compared with the transit time of positive ions.

The determination of initial charge density in the chamber from measured water dose. The initial charge density is given by

$$Q/v = \frac{100 e \rho}{W S_{\text{gas}}^{\text{H}_2\text{O}}} D_{\text{H}_2\text{O}}$$

(16)

where

- $Q/v$ = initial charge density (esu/cm$^3$)
- $e$ = electronic charge (esu)
- $D_{\text{H}_2\text{O}}$ = water dose determined by Fricke dosimetry (rads)
- $W$ = average energy required to create an ion pair in the gas (ergs)
- $S_{\text{gas}}^{\text{H}_2\text{O}}$ = ratio of the mass stopping powers of $\text{H}_2\text{O}$ and the fill gas for 33-MeV electrons (it was assumed that the average energy loss in the $\text{H}_2\text{O}$ scatterer was 2 MeV/cm)
- $\rho$ = gas density (g/cm$^3$).
Values of $W$ were taken from Boag\textsuperscript{2} with the exception of that for the tissue-equivalent gas which was measured in this laboratory.\textsuperscript{*} Table II shows the values of $W$ used in this work. The stopping powers were those of Berger and Seltzer\textsuperscript{1} which have been corrected for the density effect.

Table II. Values of $W$ for Electrons in Various Gases

<table>
<thead>
<tr>
<th>Gas</th>
<th>$W$ (eV/ion pair)</th>
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<tbody>
<tr>
<td>Argon</td>
<td>26.2</td>
</tr>
<tr>
<td>$\text{CO}_2$</td>
<td>32.9</td>
</tr>
<tr>
<td>$\text{N}_2$</td>
<td>34.6</td>
</tr>
<tr>
<td>Tissue-equivalent</td>
<td>28.4</td>
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The sensitive volume of the chamber was determined with an independent measurement in the AFRRI $^{60}\text{Co}$ facility with ambient air in the cavity and is given by

$$v = \frac{WS}{86.9 \epsilon \rho} \cdot \left( \frac{\mu_k/\rho}{\mu_k/\rho} \right)_{\text{air}} \cdot \frac{Q_c}{X}$$

(17)

where

$Q_c =$ charge collected for a known exposure, $X$, in the $^{60}\text{Co}$ field ($Q_c$ in coulombs and $X$ in roentgens)

$S =$ ratio of the stopping powers of magnesium and the air for Compton electrons from $^{60}\text{Co}$

$W =$ average energy required to create an ion pair in the air (ergs/ion pair)

\textsuperscript{*} Devine, R. T. and Leonard, B. E., unpublished data
\[
\frac{(\mu_k/\rho)_{\text{air}}}{(\mu_k/\rho)_{\text{Mg}}} = \text{ratio of mass-energy transfer coefficients of air and magnesium for } ^{60}\text{Co photons.}
\]

All charge measurements in \(^{60}\text{Co}\) and LINAC fields were made by integration with operational amplifiers. In the case of the LINAC measurements, a series of pulses were integrated and the charge densities per pulse were determined by dividing by the number of pulses.

**IV. RESULTS AND CONCLUSIONS**

Figures 4 and 5 show the experimental results with the four different gases used. Because of uncertainties in measured sensitive volume of the chambers and in \(D_{\text{H}_2\text{O}}\) as determined with Fricke dosimeters, it was assumed that any experimentally observed efficiency of 0.95 or higher could not be distinguished from unity. The theoretical critical voltage is also shown in Figures 4 and 5, and the agreement with experiment is good. With the chamber operated at negative 1000 volts applied to the outer wall, an initial charge density of 11 esu per pulse can be completely collected. For an initial charge density of 20 esu per pulse, the chamber is about 90 percent efficient and at 100 esu per pulse the efficiency is between 45 and 55 percent and may be a function of which gas is in the cavity.

A smaller chamber made of tissue-equivalent plastic with a volume of 0.05 cm\(^3\) is shown in Figure 6. Boag\(^2\) provides a model for analysis of chambers filled with air but the assumptions required by that model include a chamber with applied field and geometry characteristics such that electrons have ample opportunity to become attached to neutral gas molecules before reaching the anode. The author has demonstrated in another report\(^4\) that, in the case of a chamber this small, even though the attachment
Figure 4. Collection efficiency of 0.5 cm$^3$ chamber (central anode)

Figure 5. Collection efficiency of 0.5 cm$^3$ chamber (outer wall anode)
cross section for the formation of the $O_2^-$ ion is high, the path length traveled by electrons to reach the anode is very short, and most of the negative charge is collected at the anode as unattached electrons. Comparison of these smaller chambers with thermoluminescent and Fricke dosimetry indicates that they respond (with a polarizing voltage of negative 500 volts) linearly with charge density up to 50 esu per pulse. This is just the charge density limit predicted by the model presented in this paper with the assumption that the negative charge carriers are electrons.

![Diagram of chamber](image)

Figure 6. Section of 0.05 cm$^3$ chamber
REFERENCES


Theoretical expressions have been derived for the critical applied voltage below which charge collection is incomplete in ionization chambers exposed to pulsed radiation. To test the accuracy of the expressions, a 0.5 cm$^3$ ionization chamber was exposed to 100 nsec pulses in a 35-MeV electron beam from the AFRRI LINAC. Several gases with negligible electron attachment coefficients were used. These included argon, carbon dioxide, nitrogen, and a tissue-equivalent mixture of methane, carbon dioxide and nitrogen. It was found that the theory accurately predicted that, with 1000 volts applied, a charge density of 11 esu per pulse could be completely collected. It was also found that, for an air-filled chamber sufficiently small so that negative charge is transported to the anode by electrons, the derived expressions accurately predict the critical voltage.