CONTUR-A FORTRAN IV SUBROUTINE FOR THE PLOTTING OF CONTOUR LINES

George W. Hartwig, Jr.

Ballistic Research Laboratories

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CONTUR-A FORTRAN IV SUBROUTINE FOR
THE PLOTTING OF CONTOUR LINES.

by

George W. Hartwig, Jr.

March 1973

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In engineering or scientific data analysis or computations it frequently becomes necessary to examine data which is a single valued function of two independent variables. One convenient method of displaying this type of data is with contour plots. This report describes an efficient algorithm for construction of contour lines and the implementation of this algorithm as a FORTRAN IV subroutine, CONTUR.
FORTRAN IV Subroutine
Contour Lines
Calcomp Plots
Computer Graphics

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George W. Hartwig, Jr.
Applied Mathematics Laboratory

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ABSTRACT

In performing engineering or scientific data analysis or computations it frequently becomes necessary to examine data which is a single valued function of two independent variables. One convenient method of displaying this type of data is with contour plots. This report describes an efficient algorithm for construction of contour lines and the implementation of this algorithm as a FORTRAN IV subroutine, CONTUR.
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I. INTRODUCTION

In performing engineering or scientific data analysis it frequently becomes necessary to examine data which is a single valued function of two independent variables. A common and useful technique for displaying such data is through the use of contour plots. When three independent variables are involved any method of graphic display is cumbersome, but the plotting of contours in two dimensions for several values of the third independent variable may be the most practical alternative.

Organizations heavily involved in scientific computation utilizing digital computers frequently need to reduce results into an easily comprehensible form such as contour plots. Accordingly, it is highly desirable that an easy to use subroutine for contour plotting be available for the organization's computer users. CONTUR is such a subroutine, written in FORTRAN IV, and hence, is compatible with many digital computers in use today. The subroutine described herein was designed to work in conjunction with the California Computer Products, model 780 digital, incremental plotting system and the associated plotting subroutines in use at BRL. However, with simple modifications, CONTUR may be used with other forms of graphic display equipment.

II. ALGORITHM

The data for which contours are to be drawn is assumed to be a discrete tabulation of the single valued function

\[ Z = f(x,y) \]  

for \( x, y \), in the range over which contours are desired. For a fixed \( Z \), \( Z = Z_0 \), Eq. (1) may be written

\[ Y = g(x,z_0). \]  

9

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In this form the curve is called a contour and in general a different contour would occur for each value of \( Z_0 \). Usually the function, \( f(x,y) \) is not known, the data arising either from experiment or by numerical approximation techniques. Hence, the explicit expression as a function of \( x \) and \( Z_0 \), Eq. (2), is not available and a numerical procedure for determining the contours is necessary.

The algorithm described below represents a significant simplification of the algorithm described by James Downing [1].

The algorithm is derived by focusing attention on four adjacent data points \( Z_{i,j}', Z_{i+1,j}', Z_{i,j+1}' \) and \( Z_{i+1,j+1}' \) where the corresponding independent variables have the values \( (X_{i, j}, Y_{i, j}) \), \( (X_{i+1, j}, Y_{i+1, j}) \) etc. Assuming the data contains \( I \) points in the \( x \) direction and \( J \) points in the \( y \) direction, the algorithm must be applied to \( N = (I-1)(J-1) \) cells.

Within such a cell, Figure 1, the center point is located and assigned a \( Z \) value equal to the average of the four \( Z \) values at the corners. These five points are then connected with line segments which are in turn numbered one through eight in a clockwise direction.

![Figure 1. A Typical Cell.](image-url)
Each segment is then tested to see if the required contour intersects with it, in the following manner. Starting with segment one, the contour value $Z_o$ is subtracted from the end points.

$$T_1 = Z_{1,j} - Z_o$$

$$T_2 = Z_{1+1,j} - Z_o$$

If the quantity

$$\Delta = T_1 \cdot T_2$$

is greater than zero, the entire segment is either above or below $Z_o$, if $\Delta$ equals zero, either $Z_{1,j}$ or $Z_{1+1,j}$ is equal to $Z_o$, and if $\Delta$ is less than zero the contour intersects the segment. In this last case the point of intersection, $x_o$ is found by linear interpolation (see Figure 2) with $x_o$ given by

$$x_o = (Z_o - Z_{1,j})(x_{1+1,j} - x_{1,j})/(Z_{1+1,j} - Z_{1,j}) \cdot x_{1,j}.$$  

The x and y values of this intersection are then stored in temporary arrays PX and PY.

![Figure 2. The Interpolation Scheme.](image)
The procedure is then repeated for segments two through eight. When segment eight is completed, the points stored in PX and PY are plotted and the next set of points are considered.

Before the ordered pairs (PX, PY) can be plotted successfully there are several conditions which must be tested for and if present, properly handled. (1) If all four of the cell’s corner points are equal to \( Z_0 \), no points should be plotted. (2) When the contour intersects segment eight, the PX and PY arrays must be reordered. The reason for this becomes obvious when one remembers that the segments are tested in a clockwise direction. For instance, assume CONTUR finds intersections on segments one, seven and eight. Plotting these points as originally stored would result in an extraneous line being drawn. See Figure 3. By simply rearranging the points so that they are stored seven, eight, one, the correct contour is drawn.

![Figure 3. Error Condition 2.](image)

(3) Provision is also made for the case where two contours of the same value pass through the cell. This occurs only when two opposite \( Z \) values are greater than \( Z_0 \) and the other two points are less than \( Z_0 \). By noting if the center point, \( Z_2 \), is greater than or less than \( Z_0 \), the paths taken by the contours are specifically known and are plotted as a special case. See Figure 4.
III. THE SUBROUTINE

CONTUR is accessed through the statement

CALL CONTUR (Z, X, Y, IS, IY, DZ, NZ, IZ).

Z, X, Y are the arrays containing the values $Z_{ij}$, $X_i$, and $Y_j$, respectively. IX and IY are the number of points in the X dimension and Y dimension. The subroutine requires that Z be of dimensions (IZ, IY). DZ(NZ) is a one dimensional array containing the Z values at which contours are desired. NZ is the number of these Z values. The declared number of rows in the Z array is IZ.

Since the subroutine uses just four data points at a time, requiring no knowledge of where it has been or where it is going, enormous amounts of data can be handled by reloading the Z array and calling CONTUR several times with different portions of the data.
The computer time required by CONTUR depends on the size of the \( Z \) array, the number of values at which contours are desired and the smoothness or irregularity of the data, with time increasing for large arrays, large numbers of contour values and irregular data. For some typical times see the examples of contour output.

In order to keep the subroutine as efficient and machine independent as possible, no labeling of contours is done, nor are any borders or titles plotted in CONTUR. The user must initialize the plot routines and set scales prior to calling CONTUR. PLTCCD is a predefined subroutine on the Ballistic Research Laboratories BRLESC computers that generates input data for the Cal Comp 780 digital, incremental plotting system, and must be replaced for use of CONTUR on other computer systems.

BRLESC users should note that the positioning on the plotter page and scales used by CONTUR are determined by the latest reference to PLTCCS. [1] Thus, it may be necessary to reset the plotting scales before calling CONTUR.

IV. CONTUR INPUT VARIABLES

\[ Z(IZ, IY) \] - is a two dimensional array containing the functional values of the data.

\[ X(IX) \] - is a one dimensional array containing the values of one of the independent variables.

\[ Y(IY) \] - is a one dimensional array containing the values of the other independent variable.

\[ IX \] - is the number of elements in the \( X \) array.

\[ IY \] - is the number of elements in the \( Y \) array.

\[ DZ(NZ) \] - is a one dimensional array containing the \( z \) values at which contours are desired.

\[ NZ \] - is the number of elements in the \( Z \) array.

\[ IZ \] - the number of declared rows in the \( Z \) array.
V. PLTCCD INPUT VARIABLES

N=1,L=0 - signifies that a line plot is to be drawn.

PX(I),PY(J) - are arrays containing the X and Y coordinates to be plotted. The first point plotted is PX(I),PY(J).

K - is the number of data pairs to be plotted

M=0 - causes the subroutine to start a new curve with the point (PX(I), PY(J)).

M=1 - causes the subroutine to continue the curve plotted by the previous PLTCCD entry.
REFERENCES


APPENDIX A

Examples of CONTUR Output

Figure A-1 is a three dimensional graph of the function

\[ Z = \sin(x + y)/(1 + (x - y)^2) \]  

as plotted by the subroutine GRAF3D(3). Although this plot is interesting and demonstrates general trends, it is virtually impossible to retrieve any useful quantitative information from it. Figure A-2 is a contour plot as drawn by CONTUR of the same data. The contour lines are at values of .1, .4, .6, and .9. The Z array contains 3600 points and CONTUR required 14.4 seconds on the BRLESC II computer to generate the curves.

Figures A-3 and A-4 represent experimental data. Again the contour plot is the more analytically useful, although not as esthetically pleasing as the 3-D plot. The PR01 subroutine (4) was used in this case to generate the three dimensional plot.

Figures A-5,6,7 are included to demonstrate the results of allowing the data grid to become too coarse. Figure A-5 is the 3-D representation of

\[ Z = |\sin(\sqrt{x^2 + y^2})/(\sqrt{x^2 + y^2})| \quad -20 \leq x, y \leq 20. \]

Both Figures A-6 and A-7 are contour plots of the above function with contour lines drawn at Z values of .1, .4, .6, and .9. In Figure A-6 the grid contains 10000 points and the representation is accurate. The grid in Figure A-7 contains only 2500 points and the interpolation scheme is no longer sufficiently accurate to portray the function correctly. The BRLESC I computer required 60.6 seconds to generate A-6 and 19.2 seconds for A-7.
A-1. 3-D Plot of \( \sin(x + y)/(1 - (x - y)^2) \)
A-2. Contours of $Z = \text{SIN} \left( \frac{x}{1 + (x - Y)^2} \right)$
A-3. 3-D Plot of Experimental Data
A-4. Contours of Experimental Data
A-5. 3-D Plot of $Z = |\sin \left( \sqrt{\frac{x^2 + y^2}{x^2 + y^2}} \right) |$
A-6. Dense Grid Contours of $Z = \mid \sin \left( \frac{\sqrt{x^2 + y^2}}{x^2 + y^2} \right) \mid$
A-7. Coarse Grid Contours of $Z = \left| \sin \left( \sqrt{\frac{x^2 + y^2}{\sqrt{x^2 + y^2}}} \right) \right|$
APPENDIX B

Flow Chart of Contur

ENTER

\[ \text{NNX} = \text{IX}-1 \]
\[ \text{NNY} = \text{TY}-1 \]

DO 2 \( J = 1, \text{NNY} \)

DO 1 \( I = 1, \text{NNX} \)

DO 100 \( \text{ICONT} = 1, \text{NZ} \)

CONTINUE

A

F

100

CONTINUE

1

CONTINUE

2

CONTINUE

RETURN

25
Are all 4 points equal to the contour value?

NO

Does the contour intersect line segment 1?

NO

YES

Find X & Y values at intersection interpolating if necessary. Then store in PX & PY and set flag.

Does the contour intersect line segment 2?

NO

YES

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.

Does the contour intersect line segment 3?

NO

YES

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.

Does the contour intersect line segment 4?

NO

YES

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.
Does the contour intersect line segment 5?

Does the contour intersect line segment 6?

Does the contour intersect line segment 7?

Does the contour intersect line segment 8?

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.

Find X & Y values at intersection interpolating if necessary. Store in PX & PY and set flag.
Did the contour cross 2 more line segments?

89

YES → E

NO → D

90

Did 6 or more flags get set?

YES

NO → Was the flag for line segment 8 set?

YES

Reorder the PX and FY arrays so that the data is plotted consecutively.

200

Plot segment of contour.

300

Arrange data so that the two contour segments are in correct order and plot them.

Clear PX & FY arrays and initialize flags.

F
SUBROUTINE CONTR (Z, X, Y, IX, IY, DZ, NZ, IZ) 

DIMENSION X(IX), Y(IY), Z(IX, IY), PX(B), PY(B), KCHK(B), DZ(NZ) 

THIS SUBROUTINE PLOTS NZ CONTOURS AT DZ VALUES.

X AND Y ARE ONE DIMENSIONAL ARRAYS OF LENGTH IX AND IY, RESPECTIVELY.

Z IS A TWO DIMENSIONAL ARRAY OF SIZE (IX, IY).

DZ IS A ONE DIMENSIONAL ARRAY OF LENGTH NZ IN WHICH THE Z VALUES AT WHICH CONTOURS ARE DESIRED ARE PLACED.

THIS VERSION OF CONTR WAS COMPLETED IN FEBRUARY 1973.

IC=0
NNX=IX-1
NNY=IY-1
DO 2 J=1,NNY
DO 1 I = 1,NNX
DO 100 ICOUNT=1,NZ
Z0 = DZ(1COUNT)

IF(ALL CUR DATA POINTS ARE EQUAL TO ZO, DO NOT PLOT ANY LINES FOR THIS CELL.)

TEST SEGMENT 1 FOR AN INTERSECTION WITH THE CONTOUR LINE.

T1=Z(I,J)-Z0
T2=Z(I+1,J)-Z0
D=T1*T2
IF(D)10,11,19
10 IC=IC+1
PX(IC)=-T1*X(I+1)+X(I)+Z0/2
PY(IC)=Y(J)
KCHK(I)=1
GOTO 19

11 IF(T1<0) GOTO 13

TEST SEGMENT 2 FOR AN INTERSECTION WITH THE CONTOUR LINE.

T3=.25*(Z(I,J)+Z(I+1,J)+Z(I,J+1)+Z(I+1,J+1))-Z0
D=T1*T3

13 IC=IC+1
PX(IC)=X(I+1)
PY(IC)=Y(J)
KCHK(I)=1
GOTO 19

19 IC=IC+1

This page is reproduced from the best available copy.
PX( IC ) = - T1 * ( X2 - X(I) ) / ( Z2 - Z(I, J) ) + X(I)
PY( IC ) = - T1 * ( Y2 - Y(J) ) / ( Z2 - Z(I, J) ) + Y(J)
KCHK(2) = 1
GOTO 29
23 IF( T1 > NE. J ) GOTO 29
   IC = IC + 1
   PX( IC ) = X2
   PY( IC ) = Y2
   KCHK(2) = 1
   C TEST SEGMENT 3 FOR AN INTERSECTION WITH THE CONTOUR LINE.

29 T2 = Z(I, J+1) - ZC
   D = T1 * T2
   IF( D ) 30, 33, 35
30 IC = IC + 1
   PX( IC ) = X(I)
   PY( IC ) = - T1 * ( Y(J+1) - Y(J) ) / ( Z(I, J+1) - Z(I, J) ) + Y(J)
   KCHK(3) = 1
   GOTO 39
33 IF( T2 > NE. J ) GOTO 39
   IC = IC + 1
   PX( IC ) = X2
   PY( IC ) = Y(J+1)
   KCHK(3) = 1
   C TEST SEGMENT 4 FOR AN INTERSECTION WITH THE CONTOUR LINE.

39 T1 = Z(I+1, J) - ZC
   D = T1 * T3
   IF( D ) 40, 45, 49
40 IC = IC + 1
   PX( IC ) = X(I+1)
   PY( IC ) = - T1 * ( Y2 - Y(I+1) ) / ( Z2 - Z(I+1, J+1) ) + Y(I+1)
   KCHK(4) = 1
   GOTO 59
45 IF( T2 > NE. J ) GOTO 59
   IC = IC + 1
   PX( IC ) = X(I+1)
   PY( IC ) = Y(J+1)
   KCHK(5) = 1
   C TEST SEGMENT 5

49 T2 = Z(I+1, J+1) - ZC
   D = T1 * T2
   IF( D ) 50, 53, 59
50 IC = IC + 1
   PX( IC ) = X(I+1)
   PY( IC ) = - T1 * ( X2 - X(I) ) / ( Z2 - Z(I+1, J) ) + X(I)
   KCHK(5) = 1
   GOTO 59
53 IF( T2 > NE. J ) GOTO 59
   IC = IC + 1
   PX( IC ) = X(I+1)
   PY( IC ) = Y(J+1)
   KCHK(5) = 1
   C TEST SEGMENT 6

59 D = T2 * T3
   IF( D ) 60, 69, 69
60 IC = IC + 1
   PX( IC ) = - T2 * ( X2 - X(I+1) ) / ( Z2 - Z(I+1, J+1) ) + X(I+1)
   PY( IC ) = - T2 * ( Y2 - Y(J+1) ) / ( Z2 - Z(I+1, J+1) ) + Y(J+1)
   KCHK(6) = 1
69  T1=T2  
   T2=Z(I+1,J)-Z2  
   D=T1*T2  
   IF(D)70,79,79  
70  IC=IC+1  
   PX(IC)=X(I+1)  
   PY(IC)=-(I+1)*Y(J+1)-(Y(I+1,J)+Z(I+1,J+1)*Y(J+1)  
   KCHK(7)=1  

79  D=T2*T3  
   IF(D)80,99,89  
80  IC=IC+1  
   PX(IC)=-T3*(X(I+1)-X2)/(Z(I+1,J)-Z2)+X2  
   PY(IC)=-T3*(Y(J)-Y2)/(Z(I+1,J)-Z2)+Y2  
   KCHK(8)=1  
89  IF(IC.GE.2) GOTO 90  
   GOTO 201  

90  IF(IC.GE.6) GOTO 300  
   IF(KCHK(9).NE.1) GOTO 200  
   DO 101 M=1,IC  
   IF(KCHK(M).NE.1) GOTO 101  
   IC=IC+1  
   PX(M)=PX(1)  
   PY(M)=PY(1)  
   IC=IC-1  
   DO 102 M=1,IC  
   PX(M)=PX(M+1)  
   IF((MCD(1,2).EQ.1).AND.KCHK(1).EQ.1) GOTO 200  
102 CONTINUE  
   GOTO 201  

200 CALL PLTCCD (1,3,PX(1),PY(1),IC,C)  
   GOTO 201  

300 IF(Z(I,J),GT.ZC.AND.Z(I+1,J+1),GT.0.0) GOTO 301  
   IF(Z2.GT.Z0) GOTO 303  
302 N=IC+1  
   PX(N)=PX(1)  
   PY(N)=PY(1)  
   CALL PLTCCD (1,0,PX(5),PY(5),3,0)  
   CALL PLTCCD (1,0,PX(2),PY(2),3,0)  
   GOTO 31:  
303 CALL PLTCCD (1,0,PX(1),PY(1),3,0)  
   CALL PLTCCD (1,0,PX(4),PY(4),3,0)  
   GOTO 31:  
301 IF(Z2.GT.ZC) GOTO 302
GOTO 303
310 CONTINUE
C CLEAR WORKING ARRAYS AND INITIALIZE FLAGS
C
201 IC=0
DC A MN=1,B
PX(MN)=1.
PY(MN)=0.
8 KCHK(MN)=0
100 CONTINUE
1 CONTINUE
2 CONTINUE
RETURN
END