RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS. VOLUME II. STRUCTURAL SYSTEM IDENTIFICATION FROM SINGLE-POINT EXCITATION

William C. Flannelly, et al

Kaman Aerospace Corporation

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November 1972

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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS

VOLUME II
STRUCTURAL SYSTEM IDENTIFICATION FROM SINGLE-POINT EXCITATION

By
William G. Flannelly
Alex Berman
Nicholas Giansante

November 1972

EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

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KAMAN AEROSPACE CORPORATION
BLOOMFIELD, CONNECTICUT

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This program was conducted under Contract DAAJ02-70-C-0012 with Kaman Aerospace Corporation.

This report contains the theoretical derivation and the presentation of a methodology for system identification of structures. Computer experiments were run to verify this methodology.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. Arthur J. Gustafson, Technology Applications Division.
The parameters in Lagrange’s equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data obtained by forcing the structure at a single point. In conjunction with the mobility data, it is also necessary that the approximate system natural frequencies be known. Thus, using only a minimum amount of impedance-type test data without the use of an intuitive mathematical model, the equations of motion for the complete structure may be obtained. Further, the eigenvector or mode shape, generalized mass, stiffness, and damping associated with each natural frequency are also determined.

A digital computer program was generated to numerically test the aforementioned theory. Computer experiments were conducted to test the sensitivity of the theory to errors in the simulated test data and to determine the practicality of the theory.
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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS

Volume II
Structural System Identification From Single-Point Excitation

Final Report

Kaman Report R-1001-2

By
William G. Flannelly
Alex Berman
Nicholas Giansante

Prepared by
Kaman Aerospace Corporation
Bloomfield, Connecticut

for

EUSTIS DIRECTORATE
U.S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

Approved for public release; distribution unlimited.
The work presented in this report was performed by Kaman Aerospace Corporation under Contract DAAJ02-70-C-0012 (Task 1F162204AA4301) for the Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia. The program was implemented under the technical direction of Mr. Joseph H. McGarvey of the Reliability and Maintainability Division* and Mr. Arthur J. Gustafson of the Structures Division.** The report is presented in four volumes, each describing a separate phase of the basic theory of structural dynamic testing using impedance techniques.

Volume I presents the results of an analytical and numerical investigation of the practicality of system identification using fewer measurement points than there are degrees of freedom. The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data. Volume II describes the method of system identification wherein the necessary impedance data are experimentally determined by applying a force excitation at a single point on the structure. Volume III presents a method of determining the free-body dynamic responses from data obtained on a constrained structure. Volume IV describes a method of obtaining the equations for the combination of measured mobility matrices of a helicopter and its subsystems. The response of the combination of a helicopter and its subsystems is determined from data based on the experimental results of the main system and subsystems separately.

*Division name changed to Military Operations Technology Division.

**Division name changed to Technology Applications Division.
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LIST OF SYMBOLS

C        influence coefficient
D        damping
F        force
\tilde{F}        force phasor
G        structural damping coefficient
I        imaginary operator (i = \sqrt{-1})
K        stiffness
\kappa        modal stiffness, generalized stiffness
M        mass
\mathcal{M}        modal mass, generalized mass
R        residual, defined in text
S        modal mobility ratio, defined in text
Y        displacement mobility, \partial y/\partial f
[\phi]        matrix of modal vectors

BRACKETS

[ ], ( )        matrix
\mathcal{N}        diagonal matrix
{ }        column or row vector

SUPERSCRIPTS

(q)        q-th iteration
*        modal parameter
R        real
LIST OF SYMBOLS (Continued)

I    imaginary
T    transpose
-1   inverse
-T   transpose of the inverse
+    pseudoinverse, generalized inverse, generalized reciprocal

SUBSCRIPTS

( ) a subscripted index in parentheses means the index is held constant

i    modal index
j    degree of freedom index, generalized coordinate index
k    degree of freedom index, generalized coordinate index

OTHER INDICES

N    number of degrees of freedom
Q    number of modes
P    number of forcing frequencies
J    number of generalized coordinates
JxP  capital letters under matrices indicate the number of rows and columns respectively
.
    a dot over a quantity indicates differentiation with respect to time
INTRODUCTION

The success of a helicopter structural design is highly dependent on the ability to predict and control the dynamic response of the fuselage and mechanical components. Conventionally, this involves the formulation of intuitively based equations of motion. Ideally, this process would reduce the physical structure to an analytical mathematical model which would predict accurately the dynamic response characteristics of the actual structure. Obviously, the creation of such an intuitive abstraction of a complicated real structure requires considerable expertise and inherently includes a high degree of uncertainty. Structural dynamic testing is required to substantiate the analytical results. The analysis is modified until successful correlation is obtained between the analytical predictions and the test results.

This report describes the theory of structural dynamic testing using impedance techniques as applied to a mathematical model having fewer degrees of freedom than the structure it represents. The test information is obtained with single point excitation of the model. Reference 1 describes the method of obtaining a model directly from test measurements for a hypothetical structure which has the same number of degrees of freedom as the mathematical model. In reality, the number of degrees of freedom of a physical structure is infinite; therefore, the usefulness of model identification, necessarily with a finite number of degrees of freedom, using impedance testing techniques depends on the ability to simulate the real structure with a small mathematical model. Reference 2 illustrates the method of obtaining a model, using impedance testing techniques, that is comprised of less degrees of freedom than the physical structure it approximates. That method required measured mobility data obtained at selected points of the structure with the force input applied at each of the prescribed locations. The present theory is similar to that of Reference 2 except that the excitation is applied at only one point on the model, thereby substantially reducing the mobility data essential to the analysis.

The process of deriving the equations of motion from test data is referred to as system identification. The only input information required in this theory is measured mobility data obtained with the excitation at only one point on the model and the approximate natural frequency of each
mode. This information can be readily obtained from impedance testing of the actual structure over the frequency range of interest yielding the second order, structurally damped linear equations of motion.

System identification theories of any practical engineering significance must be functional with a reasonable degree of experimental error. In this report, a series of computer experiments incorporating experimental errors was documented. This report presents an extension of the analysis derived in Reference 2 whereby an identified model with a finite number of degrees of freedom, obtained from impedance type testing with excitation at only one point on the structure, simulates the actual structure wherein the number of degrees of freedom is infinite.
THEORY

DERIVATION OF THE SINGLE-POINT ITERATION PROCESS

As indicated in References 1 and 2, the mobility of a structure is given by

\[ [Y_\omega] = [\phi][Y_{i(\omega)}^*][\phi]^T \]  (1)

With excitation at station \( k \), the responses at station \( j \), including \( J_c \), are obtained. These provide the \( k \)-th column of the mobility at a particular forcing frequency \( \omega_1 \):

\[ \{Y_{j(k)}\} = \sum_{i=1}^{N} Y_{i(1)}^* \phi_{ki}[\phi]_i = [\phi]\{Y_{i1}^*\phi_{ki}\} \]  (2)

\( 1 \leq j \leq J \), \( 1 \leq i \leq N \)

This represents a column of mobility values, each of which is the response at a point of interest on the structure with excitation at station \( k \) and at forcing frequency \( \omega_1 \).

Similarly, with the exciter remaining at station \( k \), the \( k \)-th column of the mobility at another frequency, \( \omega_2 \), can be obtained:

\[ \{Y_{j(k)2}\} = \sum_{i=1}^{N} Y_{i(2)}^* \phi_{ki}[\phi]_i = [\phi]\{Y_{i2}^*\phi_{ki}\} \]  (3)

The mobility columns represented by (2) and (3) may be combined into one matrix:

\[ [\{Y_{j(k)1}\}{Y_{j(k)2}}] = [\phi][\{Y_{i1}^*\phi_{ki}\}{Y_{i2}^*\phi_{ki}}] \]

\( J \times 2 \)

\[ = [\phi][\phi_{ki}][\{Y_{i1}^*\}{Y_{i2}^*}]] \]

\( J \times N \) \( N \times N \) \( N \times 2 \)  (4)

In general, for \( P \) forcing frequencies \( (1 \leq p \leq P) \),

\[ [Y_{j(k)p}] = [\phi][\phi_{ki}][Y_{ip}] \]

\( J \times P \) \( J \times N \) \( N \times N \) \( N \times P \)  (5)
If \( J > P \), Equation (5) is a set of more equations than unknowns for which there is no solution. Equation (5) can then be written as

\[
[Y_j(k)p] = [\Phi] [\Phi_{ki}] [Y_{ip}^*] + [R_{jp}]
\]

where \( R_{jp} \) is the residual associated with the \( j \)-th station and the \( p \)-th forcing frequency.

As described in References 1 and 2, the imaginary displacement mobility contains significant information relating to modes associated with natural frequencies in proximity to the forcing frequency. As shown in Reference 3, accurate estimates of the modal vectors may be obtained by considering only the effects of modes proximate to the forcing frequency. Therefore the analysis will employ only \( Q \) modes, where \( Q \) is less than \( N \). Consider the imaginary displacement mobility

\[
[Y_I^i(k)p] = [\Phi] [\Phi_{ki}] [Y_{ip}^*] + [R_{jp}]
\]

The dominant element in each row of the \([Y_{ip}^*]\) matrix will be the modal mobility measured at the forcing frequency in proximity to a particular natural frequency. Normalizing the rows of the aforementioned matrix on the largest element yields

\[
[Y_I^i]_{ip} = (-Y_{ip}^*)_{in}
\]

where \( Y_{ip}^* \) is the maximum value of the \( i \)-th row. Equation (7) may be rewritten, incorporating Equation (8):

\[
[Y_I^i(k)p] = [\Phi] [\Phi_{ki}] [Y_{ip}^*]_{ip} + [R_{jp}]
\]

The \([S_{ip}]\) matrix can be evaluated by considering the expression for the imaginary displacement modal mobility.
Therefore from Equation (8),

\[ g_i^2 + (1 - \frac{\omega_n^2}{\Omega_i^2}) \]

Because \( g_i \), the structural damping coefficient of the i-th mode, is generally quite small, typically of the order 5 percent, the \([S]\) matrix can be accurately estimated by assuming \( g_i = 0 \), thus, requiring knowledge of only the forcing frequencies and the natural frequencies. It will be shown that an accurate estimate of \( S \) is not necessary, although helpful, as iterations will converge on the best values in \( S \) in the least-squares sense.

The matrix Equation (9) has no solution. An approximation to a solution may be defined as that which makes the Euclidian norm of the matrix of residuals a minimum. This, as will be proved later, is given through use of the pseudo-inverse.

Equation (9) will be solved utilizing matrix iteration techniques using \([S]^{(0)}\) as a first estimate. As indicated in the following sections, the modal vector matrix with respect to which the Euclidian norm of the residuals is a minimum is given by

\[ \phi^{(1)} = \left[ Y_j(k)p \right] \left[ S_i^{(0)} \right]^+ \left[ \frac{1}{\phi_k Y_i} \right] \]

where \( [S]^{(0)} \) is defined as the generalized inverse or pseudoinverse of \([S]^{(0)}\) and is given by
\[ [S_{ip}^{(0)}]^+ = [S_{ip}^{(0)}]^T ([S_{ip}^{(0)}][S_{ip}^{(0)}])^{-1} \]

where

\[ [S_{ip}^{(0)}][S_{ip}^{(0)}] = [I_L] \]

It follows then that

\[ \left[ Y^I_{j(k)p} \right] = \left[ \phi^{(1)} \right]_{ki} Y^*_i \left[ S_{ip}^{(1)} \right] + \left[ R_{jp}^{(0)} \right] \]

in which the Euclidian norm of \( R_{jp}^{(0)} \) is a minimum with respect to \( \phi^{(1)} \).

Using \( \phi^{(1)} \), a matrix \( S_{ip}^{(1)} \) can be found to give an equation

\[ \left[ Y^I_{j(k)p} \right] = \left[ \phi^{(1)} \right]_{ki} Y^*_i \left[ S_{ip}^{(1)} \right] + \left[ R_{jp}^{(1)} \right] \]

such that the Euclidian norm of \( R_{jp}^{(1)} \) is a minimum with respect to \( S_{ip}^{(1)} \). This is given by

\[ S_{ip}^{(1)} = \left[ \frac{1}{\phi_{ki} Y^*_i} [\phi^{(1)}]^+ \left[ Y^I_{j(k)p} \right] \right] \]

where

\[ [\phi]^+ = ([\phi]^T[\phi])^{-1} [\phi]^T \text{ and } [\phi]^+[\phi] = [I_R] \]

It is apparent from the first cycle of the iteration, by comparing Equations (11) and (15), that the process consists of alternately dealing with the left and right identity matrices. At each successive iteration, a solution is found that minimizes the Euclidian norm of the residual matrix with respect to the newly found matrix of either \( S \) or \( \phi \).
In simplified notation, the $q$-th iteration becomes

\[ [\phi(q^\prime)] = [Y^I][S(q-1)^\prime] + \frac{1}{\phi_{ki}Y_{in}} \]  

(18)

and

\[ [S(q^\prime)] = \frac{1}{\phi_{ki}Y_{in}} [\phi(q^\prime)]^\prime [Y^I] \]

The next iteration is

\[ [\phi(q+1)] = Y^I[S(q^\prime)] + \frac{1}{\phi_{ki}Y_{in}} \]

\[ [S(q+1)] = \frac{1}{\phi_{ki}Y_{in}} [\phi(q+1)]^\prime [Y^I] \]  

(19)

This is the basic algorithm used in the matrix iteration procedure.
DETERMINING THE MODAL PARAMETERS

From Equation (6) of the previous section, one column, which
is at a particular forcing frequency, \( p \), with the excitation
at station \( k \), can be written as

\[
\{ Y_j(kp) \} = [\phi] [\phi_{ki}] \{ Y^*_i(p) \} + \{ R_j(p) \} \tag{20}
\]

The number of modes, \( Q \), included in Equation (20) cannot
be greater than the number of points of interest on the
specimen, \( J \), and generally will be much less since only those
modes which have significant effect on the mobility at the
forcing frequency, \( \omega_p \), will be considered. Ordinarily, the
number of modes used will not be greater than 3 or 4 for any
given forcing frequency, and these will be the modes in the
vicinity of the forcing frequency in question.

The real and imaginary modal mobilities are calculated from

\[
\{ Y^*_i(p) \} = \left[ \frac{1}{\phi_{ki}} \right] [\phi]^+ \{ Y^*_i(p) \} \tag{21}
\]

and

\[
\{ Y^*_i(p) \} = \left[ \frac{1}{\phi_{ki}} \right] [\phi]^+ \{ Y^*_i(p) \} \tag{22}
\]

From Reference 1 the real displacement mobility can be
calculated as

\[
Y^*_i\omega_p = \frac{1}{K_i} \frac{1 - \omega_p^2/\Omega_i^2}{g_i^2 + (1 - \omega_p^2/\Omega_i^2)^2} \tag{23}
\]

and the imaginary modal mobility by

\[
Y^*_i\omega_p = \frac{-g_i}{K_i} \frac{1}{g_i^2 + (1 - \omega_p^2/\Omega_i^2)^2} \tag{24}
\]

The real modal impedance can be written as
Substituting Equations (23) and (24) into (25) yields

\[ Z^*_{i \omega_p} = K_i - \frac{Y^*_{i \omega_p}}{(Y^*_{i \omega_p})^2 + (Y^*_{i \omega_p})^2} \]  

(26)

From Equation (26) it is observed that the modal impedance is a linear function of the square of the forcing frequency.

The forcing frequency at which the modal impedance becomes zero is, therefore, the natural frequency. From a least-squares analysis of modal impedance as a function of forcing frequency squared, proximate to the natural frequency, the generalized stiffness of the i-th mode and the natural frequency of the i-th mode can be calculated.

The generalized mass associated with the i-th mode is given by

\[ m_i = \frac{K_i}{\Omega_i^2} \]  

(27)

The structural damping coefficient may be determined from

\[ g_i = \left( \frac{\omega_p^2 - 1}{\Omega_i^2} \right) \frac{\frac{Y^*_{i \omega_p}}{\Omega_i}}{\Omega_i^2} \]  

(28)
EQUATIONS OF MOTION

There are two basic types of dynamic mathematical models describing structures. The conventional type, covering as many modes as degrees of freedom, is called "Complete Models" and is considered in References 1 and 2. The other type labelled "Incomplete Models" considers fewer modes than points of interest on the structure and was first described in Reference 5. Using the methods described herein, it is possible to identify either a complete model or a form of incomplete model.

Incomplete Models

Consider a rectangular identified modal matrix which has J rows indicating the points of interest on the structure and Q columns representing the modes being considered where J > Q. The influence coefficient matrix for the incomplete model is given by

\[ [c_{inc}] = [\phi][\frac{1}{K_i}][\phi]^{T} \]  \hspace{2cm} (29)

The above matrix, similar to all incomplete model parameter matrices, is singular, being of rank Q and order J. The mass, stiffness and damping matrices for the incomplete model are

\[ [m_{inc}] = [\phi]^{+T}[m_1][\phi]^{+} \]

\[ [K_{inc}] = [\phi]^{+T}[K_1][\phi]^{+} \]  \hspace{2cm} (30)

\[ [d_{inc}] = [\phi]^{+T}[g_1K_1][\phi]^{+} \]

The classical modal eigenvalue equation has the analogous incomplete form

\[ [c_{inc}][m_{inc}][\phi_i] = \frac{1}{\Omega_i^{2}}[\phi_i] \]  \hspace{2cm} (31)

Complete Models

For the complete model the identified modal vector matrix is square, having the same number of degrees of freedom as mode shapes; that is, J = Q. The influence coefficient matrix is given by
\[ [c] = [\phi][1/\kappa_i^*][\phi]^T = \sum_{i=1}^{N} \frac{1}{\kappa_i^*} [\phi_i^*][\phi_i]^T \]  

The mass, stiffness and damping matrices for the complete model are

\[ [m] = [\phi]^{-T}[\mu_i^*][\phi]^{-1} \]
\[ [k] = [\phi]^{-T}[\kappa_i^*][\phi]^{-1} \]
\[ [d] = [\phi]^{-T}[\sigma_i^*][\kappa_i^*][\phi]^{-1} \]  

as indicated in Reference 1.

**Full Mobility Matrix**

The full mobility matrix of either complete or incomplete models is given by

\[ [Y] = [\phi][Y_i^*][\phi]^T \]  

where for the complete model the [\phi] matrix is square, having J columns and J rows. However, in the case of the incomplete model the modal matrix [\phi] is rectangular, having J rows and Q columns, where J > Q.
PROOF THAT THE PSEUDOINVERSE MINIMIZES THE NORM OF THE RESIDUALS

Take the transpose of Equation (9) and write the equation for one column of the transpose of the mobility matrix:

\[
[Y^T_{\text{in}}]_p = [S_{ip}]^T[\phi]_i + [R_{jp}]^T
\]

\[
\{y(jk)p\} = [S_{ip}]^T[\phi(j)i] + \{r(j)p\}
\]

\[
\{r(j)p\} = \{y(jk)p\} - [S_{ip}]^T[\phi(j)i]
\]

\[
\{r(j)p\}^T\{r(j)p\} = \{y(jk)p\}^T\{y(jk)p\} - \{y(jk)p\}^T[S_{ip}]^T[\phi(j)i] -
\]

\[
[\phi(j)i]^T[S_{ip}]\{y(jk)p\} + \{\phi(j)i\}^T[S_{ip}]^T[S_{ip}]^T[\phi(j)i]
\]

Equation (37) is, of course, a scalar product and it is recognized that the derivative of a scalar with respect to a vector is a vector; in other words, Equation (36) is a vector in p-dimensional space and Equation (37) is its dot product on itself - that is, its length squared. We wish to find the vector \(\{\phi\}\) which makes the length of the residuals vector a minimum.

Take the partial derivative of Equation (37) with respect to \(\{\phi(j)i\}^T\) and set equal to zero to obtain the modal vector for which the Euclidean norm of the residuals is a minimum:

\[
0 = -2[S_{ip}]^T[y(jk)p] + 2[S_{ip}]^T[S_{ip}]^T[\phi(1)i]
\]

or

\[
\{\phi(1)i\} = ([S_{ip}]^T[S_{ip}]^T)^{-1}[S_{ip}]^T[y(jk)p]
\]

and

\[
\{\phi(1)i\}^T = \{y(jk)p\}^T[S_{ip}]^T([S_{ip}]^T[S_{ip}]^T)^{-1}
\]
as the inverted matrix is symmetrical. Equation (39) is any row in Equation (12). The sum of the minimum Euclidian norms of the rows of a matrix is, by definition, the minimum Euclidian norm of the matrix, and it therefore follows from Equation (39) that

\[ \Phi^{\dagger} = [Y_j(k)p][S_{ip}]^T([S_{ip}][S_{ip}]^T)^{-1} \]

which is given by Equations (12) and (13). Q.E.D. The basic observation which makes the above proof of the pseudoinverse possible should be credited to Klosterman, Reference (4).

To show that the [S] matrix obtained using the pseudoinverse of [\Phi] minimizes the norm of the residual, write the equation for a column of Equation (9):

\[ \{Y_j(k)p\} = [\Phi]\{S_{i}(p)\} + \{R_j(p)\} \]

\[ \{R_j(p)\} = \{Y_j(k)p\} - [\Phi]\{S_{i}(p)\} \]  \hspace{1cm} (40).

\[ \{R_j(p)\}^T\{R_j(p)\} = \{Y_j(k)p\}^T\{Y_j(k)p\} - \{Y_j(k)p\}^T[\Phi]\{S_{i}(p)\} \]

\[ - \{S_{i}(p)\}^T[\Phi]^T\{Y_j(k)p\} + \{S_{i}(p)\}^T[\Phi]^T[\Phi]\{S_{i}(p)\} \]  \hspace{1cm} (41)

Set \[ \frac{\partial\{R_j(p)\}^T\{R_j(p)\}}{\partial\{S_{i}(p)\}^T} = 0 \] and solve for \( \{S_{i}(1)p\} \)

\[ \{S_{i}(1)p\} = ([\Phi]^T[\Phi])^{-1}[\Phi]^T\{Y_j(k)p\} \]  \hspace{1cm} (42)

or

\[ \{S_{i}\} = ([\Phi]^T[\Phi])^{-1}[\Phi]^T\{Y_j(k)p\} \]  \hspace{1cm} (43)

which is the same as Equation (16). Q.E.D.
PROOF THAT ITERATIONS USING THE PSEUDOINVERSE OF S AND φ
CONVERGE MONOTONICALLY ON MINIMUM SUM OF RESIDUAL SQUARES

In the q-th iteration, where q is odd,

\[ [Y_j^I(k)p] = [\phi(q-1)] [S_i^{(q-1)}] + [R_j^{(q-1)}] \]

(44)

\[ [\phi(q)] = [Y_j^I(k)p] [S_i^{(q-1)}] = [\phi(q-1)] + [R_j^{(q-1)}] [S_i^{(q-1)}] \]

(45)

because \([S][S]^+ = [I_L]\). Then

\[ [Y_j^I(k)p] = [\phi(q)] [S_i^{(q-1)}] + [R_j^{(q)}] \]

(46)

Substitute Equation (45) into Equation (46):

\[ [Y_j^I(k)p] = [\phi(q-1)] [S_i^{(q-1)}] + [R_j^{(q-1)}] [S_i^{(q-1)}]^+ [S_i^{(q-1)}] \]

\[ + [R_j^{(q)}] \]

(47)

or

\[ [Y_j^I(k)p] = [Y_j^I(k)p] - [R_j^{(q-1)}] + [R_j^{(q-1)}] [S_i^{(q-1)}]^+ [S_i^{(q-1)}] \]

\[ + [R_j^{(q)}] \]

Therefore

\[ [R_j^{(q)}] = [R_j^{(q-1)}] ([I_L] - [S_i^{(q-1)}]^+ [S_i^{(q-1)}]) \]

(48)

The p-th row of \([R_j^{(q)}]\) is

\[ (R_j^{(q)})^T = (R_j^{(q-1)})^T ([I_L] - [S_i^{(q-1)}]^+ [S_i^{(q-1)}]) \]

\[ (R_j^{(q)})^T (R_j^{(q)}) = (R_j^{(q-1)})^T ([I_L] - [S_i^{(q-1)}]^+ [S_i^{(q-1)}])([I_L] - [S_i^{(q-1)}]^+ [S_i^{(q-1)}]) \]

\[ - [S_i^{(q-1)}]^+ [S_i^{(q-1)}])^T (R_j^{(q-1)}) \]
But \([I] - [S_{ip}^{(q-1)}]^+ [S_{ip}^{(q-1)}]\) is symmetrical and, from Equation (13),

\[[S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^+ = [I_L].\] Therefore,

\[
{R_j(p)}^T {R_j(p)} = {R_j(p)}^T {R_j(p)}
\]

\[- {R_j(p)}^T [S_{ip}^{(q-1)}]^+ [S_{ip}^{(q-1)}] {R_j(p)}\] (49)

\[[S_{ip}^{(q-1)}] \text{ is maximally ranked in its rows, of rank } Q \text{ where } 1 \leq i \leq Q. \] Therefore \([S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^T\) and its square root \(([S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^T)^{1/2}\) are nonsingular of rank \(Q\) and symmetrical. Now, \([S_{ip}^{(q-1)}]^+ [S_{ip}^{(q-1)}]\) is real, symmetric and singular. It is known that a real symmetric matrix \([A]\) of rank \(Q\) is positive semidefinite if and only if there exists a matrix \([C]\) of rank \(Q\) such that \([A] = [C]^T [C]\). Let

\[([S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^T)^{-1/2} [S_{ip}^{(q-1)}] = [C], \text{ rectangular of rank } Q.\]

\[ [S_{ip}^{(q-1)}]^T ([S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^T) - \frac{T}{2} ([S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^T)^{-1} [S_{ip}^{(q-1)}] \]

\[= C^T C = [S_{ip}^{(q-1)}]^T ([S_{ip}^{(q-1)}][S_{ip}^{(q-1)}]^T)^{-1} [S_{ip}^{(q-1)}] \]

\[= [S_{ip}^{(q-1)}]^+ [S_{ip}^{(q-1)}] \] (50)

Therefore \([S_{ip}^{(q-1)}]^+ [S_{ip}^{(q-1)}]\) is positive semidefinite and

\[{R_j(q-1)}^T [S_{ip}^{(q-1)}]^+ [S_{ip}^{(q-1)}] {R_j(p)}\] in Equation (49) must be a nonnegative number. But the first term on the right side and the left side of Equation (49) are also necessarily nonnegative. Therefore
\[
\begin{align*}
\{R_j(p)\}^T \{R_j(p)\} &< \{R_j(p-1)\}^T \{R_j(p-1)\} \text{ and} \\
\sum_{j=1}^{J} \sum_{p=1}^{P} (R_j(p))^2 &< \sum_{j=1}^{J} \sum_{p=1}^{P} (R_j(p-1))^2
\end{align*}
\] (51)

For the alternate calculation, \( q \) odd
\[
[S_{iP}^{(q)}] = [\Phi(q)]^+ [Y_{j(k)}^I]_{iP}
\] (18)

But \([Y_{j(k)}^I]_{iP} = [\Phi(q)] [S_{iP}^{(q-1)}] + [R_{jP}^{(1)}]_{iP}\), so

\[
[S_{iP}^{(q)}] = [S_{iP}^{(q-1)}] + [\Phi(q)]^+ [R_{jP}^{(1)}]_{iP}
\] (52)

Substituting \([S_{iP}^{(q)}]\) for \([S_{iP}^{(q-1)}]\), we obtain

\[
[Y_{j(k)}^I]_{iP} = [\Phi(q)] [S_{iP}^{(q)}] + [R_{jP}^{(q+1)}]
\] (53)

From Equations (46) and (53),

\[
[Y_{j(k)}^I]_{iP} = [Y_{j(k)}^I]_{iP} - [R_{jP}^{(q)}] + [\Phi(q)] [\Phi(q)]^+ [R_{jP}^{(q)}] + [R_{jP}^{(q+1)}]
\]

or

\[
[R_{jP}^{(q+1)}] = ([I] - [\Phi(q)] [\Phi(q)]^+) [R_{jP}^{(q)}]
\] (54)

Compare Equation (54) to Equation (48).
Consider a column of Equation (54) \( \{R_j^{(q+1)}\} \). Because of Equation (18),

\[
\delta \{R_j^{(q+1)}\}^T \{R_j^{(q+1)}\} \delta \{S_i(p)\} = 0
\]

\[
\{R_j^{(q+1)}\}^T \{R_j^{(q+1)}\} = \{R_j^{(q)}\}^T ([I] - [\phi(q)][\phi(q)]^+) T ([I] - [\phi(q)][\phi(q)]^+) \{R_j^{(q)}\}
\]

\[
- [\phi(q)][\phi(q)]^+ \{R_j^{(q)}\} = \{R_j^{(q)}\}^T \{R_j^{(q)}\}
\]

\[
- \{R_j^{(q)}\} [\phi(q)][\phi(q)]^+ \{R_j^{(q)}\}
\]

because \([\phi]^+ [\phi] = [I]_R\) (Equation 15) and \([\phi(q)][\phi(q)]^+\) is symmetrical. Now \([\phi(q)][\phi(q)]^+ = [\phi(q)][[\phi(q)]]^T - \frac{T}{2}\)

\(([[\phi(q)]]^{-1/2} [\phi(q)]^T + [\phi(q)]\) and \([\phi(q)]\) is necessarily maximally column ranked. Therefore, \([\phi(q)][\phi(q)]^+\) is positive semi-definite. The left side of Equation (55) is the positive difference between two positive numbers, and it follows that

\[
\sum_{j=1}^J \sum_{p=1}^P (R_j^{(q+1)})^2 < \sum_{j=1}^J \sum_{p=1}^P (R_j^{(q)})^2
\]

Equation (56) shows that the Euclidian norm of residuals with odd index \(q\) is less than the norm of residuals of index \(q-1\); Equation (56) shows that the norm of residuals of index \(q+1\) is less than the norm of residuals of index \(q\). Equations (51) and (56) show that it is immaterial whether \(q\) is odd or even.

\[
\sum_{j=1}^J \sum_{p=1}^P (R_j^{(q+1)})^2 < \sum_{j=1}^J \sum_{p=1}^P (R_j^{(q)})^2 < \sum_{j=1}^J \sum_{p=1}^P (R_j^{(q-1)})^2
\]

Equation (57) covers a complete iteration cycle. Q.E.D.

17
NOTE ON THE DERIVATIVE OF A SCALAR WITH RESPECT TO A VECTOR

Let \([S]\) be a square matrix of order \(R\)

\[
{x}^T [S] (y) = \sum_{i=1}^{R} \sum_{j=1}^{R} S_{ij} x_i y_j
\]

\[
{y}^T [S]^T (x) = \sum_{i=1}^{R} \sum_{j=1}^{R} S_{ji} y_i x_j
\]

\[
\frac{\partial {x}^T [S] (y)}{\partial {x}^T} = \sum_{j=1}^{R} S_{ij} y_j = [S] (y)
\]

\[
- \frac{\partial {y}^T [S] (y)}{\partial y^T} = \sum_{i=1}^{R} S_{ij} x_i = [S]^T (x)
\]

\[
\frac{\partial {x}^T [S] (y)}{\partial y^T} = \frac{\partial {y}^T [S]^T (x)}{\partial y^T} = \sum_{i=1}^{R} \sum_{j=1}^{R} S_{ij} y_i x_j = [S]^T (x)
\]

\[
\frac{\partial {x}^T [S] (x)}{\partial x^T} = [S] (x) + [S]^T (x) = ([S] + [S]^T) (x)
\]
IDENTIFIED GENERALIZED MASSES

Typical generalized mass identifications are shown in Tables I through VI. Table VII describes the various models for which data is presented in Tables I through VI. Table VIII presents a lumped mass description of the twenty-point specimen which was used to generate the simulated experimental data. The model stations used in the various models refer to the corresponding stations in the twenty-point specimen. Table I presents results for model 5C, which are typical of the results obtained for other five-point models. Data are presented for conditions of zero experimental error and for simulated experimental displacement mobility data recorded with a random error of +5 percent and a bias error of +5 percent. For the cases involving error, the random displacement error was computed using a uniformly distributed probability density function. This error was applied to both the real and imaginary components of the displacement mobility data. Table I presents the effects of random number, the seed used in generating the random error. The results indicate the method is extremely insensitive to measurement errors as applied herein.

Table II shows results for several different five-point models. It is apparent that no outstanding differences exist among the models considered. The results for the twenty-point specimen, the simulated actual structure, are also given in the table for comparison. The generalized mass distribution associated with each of the models is in excellent agreement with the twenty-point results.

Tables III and IV present results for the nine-point models studied. Again, the calculations of the generalized masses for the various nine-point models under consideration are in agreement with the simulated structure.

Tables V and VI describe the results of the computer experiments conducted employing the twelve-point models. The calculations produced acceptable results except for identification of the generalized masses of the 10th and 11th modes. The generalized masses associated with these models are extremely small in comparison to the remaining modal generalized masses. Further, the mode shape of the 10th mode indicates lack of response at all points of interest on the structure other than the first station. Therefore, the effect of the 10th mode is difficult to evaluate in the calculation of the generalized parameters.
<table>
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<th>294</th>
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<td>+5%</td>
<td>+5%</td>
<td>+5%</td>
<td>0</td>
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<td>+5%</td>
<td>+5%</td>
<td>+5%</td>
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<td>-</td>
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<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
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<td></td>
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<td>8.560</td>
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<td>.471</td>
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<td>.495</td>
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<td>1.000</td>
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<td>1.022</td>
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* Model 5C

** From 20 x 20 Specimen
<table>
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<tr>
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<th>5C</th>
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<td>Bias Disp. Error</td>
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<td>Random Error Seed</td>
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<th>Mode</th>
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<tr>
<td>3</td>
<td>.494  .494  .494  .493  .495</td>
</tr>
<tr>
<td>4</td>
<td>1.048 1.047 1.050 .994 1.087</td>
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<tr>
<td>5</td>
<td>.653  .653  .651  .629  .630</td>
</tr>
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** From 20 x 20 Specimen
### Table III. Identification of Generalized Masses, 9 x 9 Model* of 20 x 20 Specimen

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<th>1**</th>
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<td>Bias Disp. Error</td>
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<td>+5%</td>
<td>+5%</td>
<td>0</td>
</tr>
<tr>
<td>Random Error Seed</td>
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<td>5</td>
<td>13</td>
<td>421</td>
<td>1094</td>
<td>-</td>
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<th>Station (In.)</th>
<th>Mode</th>
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<tr>
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<td>3</td>
<td>.504  .462  .472  .467  .483  .495</td>
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<td>160</td>
<td>4</td>
<td>1.094  .975 1.042 1.053 1.095 1.087</td>
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<tr>
<td>220</td>
<td>5</td>
<td>.631  .659  .551  .577  .610  .630</td>
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<td>6</td>
<td>.761  .717  .786  .674  .646  .743</td>
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<td>400</td>
<td>8</td>
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<td>460</td>
<td>9</td>
<td>.813  .713  .787  .860  .719  .786</td>
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* Model 9A

** From 20 x 20 Specimen
## TABLE IV. IDENTIFICATION OF GENERALIZED MASSES,
9 X 9 MODEL OF 20 X 20 SPECIMEN

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<td>±5%</td>
<td>±5%</td>
<td>0</td>
</tr>
<tr>
<td>Bias Disp. Error</td>
<td>+5%</td>
<td>+5%</td>
<td>+5%</td>
<td>0</td>
</tr>
<tr>
<td>Random Error Seed</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>-</td>
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<td><strong>Generalized Masses (Lb·Sec²/In.)</strong></td>
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* From 20 x 20 Specimen
### TABLE V. IDENTIFICATION OF GENERALIZED MASSES, 12 X 12 MODEL* OF 20 X 20 SPECIMEN

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<td>4</td>
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* Model 12B

** From 20 x 20 Specimen
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* From 20 x 20 Specimen
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Computer experiment 309 yielded a negative 10th generalized mass. All computer experiments that failed in this respect gave drastically unrealistic values of generalized mass. Ordinarily, using different stations or forcing frequencies produced proper identification of all modes.
RESPONSE FROM IDENTIFIED MODEL

Figures 1 through 12 portray typical real and imaginary acceleration mobility response obtained from the various models considered in the present study. In each instance, the exact curve represents the simulated experimental data for the twenty-point structure, obtained with zero error. Figures 1 and 2 provide the effect of random number seed for a typical five-point model. Figures 3 and 4 present the results obtained for one of the nine-point models considered in the investigation. Figures 5 and 6 show the effect of the random error seed on a twelve-point model. All computer experiments which incorporated error used a +5 percent random and a +5 percent bias on the real and imaginary displacement mobility data.

Figures 7, 8, 9, 10, 11 and 12 present the reidentified acceleration mobility, both real and imaginary, for typical five-, nine-, and twelve-point models respectively. The models varied in that different spanwise masses were considered. Some of the models employed in the study are given in Table VII showing the various points of interest for each model. For each model, the computer experiments were executed using the same random number seed and the aforementioned errors were incorporated. As evidenced by the figures, the various models provided acceptable reidentification of the twenty-point specimen simulated experimental displacement mobility data.
Figure 2. Effect of Error on Five-Point Model
Identification of Imaginary Acceleration Response; Driving Point at Hub.
Figure 3. Effect of Error on Nine-Point Model
Identification of Real Acceleration Response; Driving Point at Hub.
Figure 4. Effect of Error on Nine-Point Model Identification of Imaginary Acceleration Response; Driving Point at Hub.
Figure 5. Effect of Error on Twelve-Point Model Identification of Real Acceleration Response; Driving Point at Hub.
Figure 6. Effect of Error on Twelve-Point Model Identification of Imaginary Acceleration Response; Driving Point at Hub.
Figure 7. Effect of Model on Five-Point Model Identification of Real Acceleration Response; Driving Point at Hub.

CASE SEED ERROR
295 13 YES
292 13 YES

CASES WITH ERROR
±5% RANDOM, 5% BIAS
Figure 8. Effect of Model on Five-Point Model Identification of Imaginary Acceleration Response; Driving Point at Hub.
Figure 10. Effect of Model on Nine-Point Model Identification of Imaginary Acceleration Response; Driving Point at "ubs.

Acceleration, Imaginary (G/lb-force)
Figure 11. Effect of Model on Twelve-Point Model Identification of Real Acceleration Response; Driving Point at Hub.
Figure 11 - Continued.

ACCELERATION, REAL (G/IN-FOUCF)
Figure 12. Effect of Model on Twelve-Point Model Identification of Imaginary Acceleration Response; Driving Point at Hub.
CONCLUSIONS

1. Single-point excitation of a structure yields the necessary mobility data to satisfactorily determine the mass, stiffness and damping characteristics for a mathematical model having less degrees of freedom than the linear elastic structure it represents.

2. The method does not require an intuitive mathematical model and uses only a minimum amount of impedance-type test data.

3. The eigenvector or mode shape associated with each natural frequency is also determined in the analysis.

4. Computer experiments using simulated test data indicate the method is insensitive to the level of measurement error inherent in the state of the measurement art.

5. A fully populated mass matrix should be assumed for an accurate analytical model of a real structure.
LITERATURE CITED


A digital computer program was designed for computer experiment to investigate the proper physical interpretation of identified parameters for use in helicopter engineering. The program was written for the IBM 360/40 operating system using FORTRAN IV language. A flow chart indicating the program logical procedure is shown in Figure 13. A description of the input cards and a program source listing are included in this appendix.
Figure 13. Flow Chart of Computer Program.
Figure 13 - Continued.
Figure 13 - Concluded.
DESCRIPTION OF INPUT CARDS

Note: All integer variables must be right justified with no decimal point.

Tape, Card Reader and Printer Assignments

1  Card Reader
3  Printer (On Line)
13 Tape Assignment. Contains displacement mobility data for all degrees of freedom, with no error for specified frequencies.

All input data must be in the following units:

Mass - Lb-Sec²/In.
Stiffness - Lb/In.
Frequencies - Hz
### INPUT STRUCTURAL DYNAMICS PROGRAM STIDN

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<th>IP1</th>
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<td>IP1=0 Print Full Mobility Matrix, Real and Imaginary at Each Specified Frequency</td>
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<td>IP1=1 Print Only Diagonal Elements and Row of Mobility Matrix, Real and Imaginary at Each Specified Frequency</td>
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<td>11-20</td>
<td>IP2</td>
<td></td>
<td>IP2=1 Print Full Acceleration Amplitude in G's and Phase Angle in Degrees at Each Specified Frequency</td>
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<td></td>
<td>IP2=2 Print Only Diagonal Elements and Row of Acceleration Amplitude in G's and Phase Angle in Degrees at Each Specified Frequency</td>
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<td>41-50</td>
<td>PCTI</td>
<td>Random Error Applied to Imaginary Mobilities Uniform Between - And + PCTI</td>
</tr>
<tr>
<td></td>
<td>51-60</td>
<td>PCTBI</td>
<td>Bias Error Applied to Imaginary Mobilities</td>
</tr>
<tr>
<td></td>
<td>61-70</td>
<td>IZ</td>
<td>Random Number Seed</td>
</tr>
<tr>
<td></td>
<td>71-80</td>
<td>IA</td>
<td>Print Control</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IA = 0  Displacement Mobilities Printed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IA ≠ 0  Acceleration Mobilities Printed</td>
</tr>
<tr>
<td>Card No. 4</td>
<td>1-10</td>
<td>NPHI</td>
<td>Number of Modes Desired</td>
</tr>
</tbody>
</table>
The following cards (5-8 inclusive) are repeated NPHI Times

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Columns</th>
<th>Field(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1-10</td>
<td>NQ</td>
<td>Number of Modes to be Calculated at Each Natural Frequency (Usually 2 or 3)</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>NP</td>
<td>Number of Forcing Frequencies Used in Calculating the Number of Modes</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>OMF</td>
<td>Forcing Frequencies Used in Calculating the NQ Modes (NP Forcing Frequencies). Ten Columns Per Value, 8 Values Per Card. Format (8F10.4). Hertz</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>INDX</td>
<td>The Number of Each Forcing Frequency Used. (Frequencies are Stored on Tape 13)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S</td>
<td>Matrix Used in Iteration for Mode Shape (Format 8F10.4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HZ</td>
<td>Frequencies at Which Reidentification of Mobilities is to be Calculated. Ten Columns Per Value, 8 Values Per Card (Format 8F10.4) Hertz</td>
</tr>
<tr>
<td>10</td>
<td>1-10</td>
<td>IC</td>
<td>Control on Subsequent Cases</td>
</tr>
</tbody>
</table>
C STRUCTURAL DYNAMICS --- SINGLE POINT FORCING

C

INTEGER HEAD(20), HT(7)
DIMENSION S(20,21), YI(20,21), JSW(20),
AY(20,21), PHIA(20,21), UA(20), G(20,21), S(20,21),
SM(20,21), SI(20,21), ZSII(20,21),
CANS(20), ADS(20), OMNS(20), YRTI(20,100), YIT(20,100),
DIMENSION PHIN(20,21), YINI(20,21), PHIM(20,21),

DIMENSION SI(20,21), YI(20,21), PHIA(20,21), UA(20), G(20,21),
SM(20,21), SI(20,21), ZSII(20,21),
CANS(20), ADS(20), OMNS(20), YRTI(20,100), YIT(20,100),
DIMENSION PHIN(20,21), YINI(20,21), PHIM(20,21),

DIMENSION SI(20,21), YI(20,21), PHIA(20,21), UA(20), G(20,21),
SM(20,21), SI(20,21), ZSII(20,21),
CANS(20), ADS(20), OMNS(20), YRTI(20,100), YIT(20,100),
DIMENSION PHIN(20,21), YINI(20,21), PHIM(20,21),

LOGICAL TORF
DATA HT/EXEC', 'TA', 'S', 'NUL', 'ATED', ' TES', 'T /
NJ=NUMBER OF GENERALIZED COORDINATES
N=NUMBER OF DEGREES OF FREEDOM
NK=NUMBER OF NODES
NP=NUMBER OF FORCING FREQUENCIES
NK=FORCE INPUT STA
REMIND 14
READ (1,140) IP1,IP2,NROW,NN,NJ,NK,LIM,NFF
READ (1,140) (KEEP(I),I=1,NJ )
1DO READ (1,120) ATOL,PTOL,PCTR,PCTI,PCTBI,I2,IA
IX=I2+1
READ (1,140 ) NPHI
REMIND 16
READ (13) NCOL,HT,HEAD,NF,N0,
DO 110 I=1,NF
110 READ (13) HZ(I),((YRT(I,L),YIT(I,L))),I=1,N0
120 FORMAT (16I10.6,2I10)
WRITE (3,130) PCTR,PCTBI,IZ
130 FORMAT (8110)
WRITE (3,130) PCTR,PCTBI,IZ
140 FORMAT (6F6.3,10X,MAX RAND ERROR ON KE',
ON REAL =F6.3, OF ELEMENT*/T10,4 ON IMAGINARY
y=6.3,2I10,11X,ON IMAGINARY='F6.3,15X,SEED='15/111
141 FORMAT (11I10)
150 FORMAT (18F10.4)
NP=60
WRITE (3,170) (KEEP(I),I=1,NJ )
WRITE (3,180) KEEP(NK)
160 FORMAT (11F10.4)
FORCE INPUT IS AT STA '13///'
170 FORMAT (11F10.4,STATIONS USE) '///(20,10I5)
PTOL=PTOL/100.
DO 600 NN=1,NPHI
600 READ (1,140) NN,NP
READ (1,150) (OMF(I),I=1,NP)
DO 180 I=1,NP
180 OMFR(I)=OMF(I)*6.2A3189
2MNP 48
OMFR(I)=OMF(I)*6.2A3189
2MNP 48
190 FORMAT (11F10.4)
READ (1,140) (INDEX(I),I=1,NP )
1MNP 50
C READ INITIAL S MATRIX (ROWWISE)
DO 190 I=1,NQ
190 READ (1,150) ( SI(I,J),J=1,NQ)
2MNP 53
CALL REDI (YR,YI,NP,NJ,KEEP,IXA,YAT,YIT)
1MNP 54
ICY=1
C
C
56
WRITE (3,200) (CMF(I),I=1,NP)
200 FORMAT (15H15,15H,15H)              1NP 56
WRITE (3,210)
210 FORMAT (15H15,15H)                  1NP 57
WRITE (3,210)
211 FORMAT ('15T50,'REAL MOBILITY MATRIX'/) 1NP 58
CALL MOUT2 (YR,NJ,NP)                  1NP 59
WRITE (3,220)                          1NP 60
220 FORMAT ('15T50, 'IMAGINARY MOBILITY MATRIX'/) 1NP 61
CALL MOUT2 (YI,NJ,NP)                  1NP 62
IF (PCTR.NE.O.OR.PCTBR.NE.O.OR.PCTI.NE.O.OR.PCTBI.NE.O) CALL ERRM1       1NP 63
A (YR,YI,PCTR,PCTBR,PCTI,PCTBI,NJ,NP,I)  1NP 64
WRITE (3,230)                          1NP 65
230 FORMAT ('15T50, 'MOBILITY MATRICES WITH ERROR REAL,IMAGINARY') 1NP 66
CALL MOUT2 (YR,NJ,NP)                  1NP 67
CALL MOUT2 (YI,NJ,NP)                  1NP 68
C                                           1NP 69
C NORMALIZE IMAGINARY MOBILITY              1NP 70
C                                           1NP 71
C                                           1NP 72
C                                           1NP 73
C                                           1NP 74
C ITERATE FOR MODE SHAPE AND S MATRIX      1NP 75
240 CALL PSEUDO (S,NQ,NP,SM)             1NP 76
WRITE (3,250) ITC                       1NP 77
250 FORMAT ('S ITERATION=14')             1NP 78
CALL MOUT2 (S,NQ,NP)                    1NP 79
C                                           1NP 80
C CALL MMPY (YI,SM,NJ,NP,NQ,PHI)          1NP 81
C                                           1NP 82
C                                           1NP 83
C                                           1NP 84
C                                           1NP 85
C                                           1NP 86
C                                           1NP 87
C                                           1NP 88
C                                           1NP 89
C                                           1NP 90
C                                           1NP 91
C                                           1NP 92
C                                           1NP 93
C                                           1NP 94
C                                           1NP 95
C                                           1NP 96
C                                           1NP 97
C                                           1NP 98
C                                           1NP 99
C                                           1NP 100
C CALL MMPY (PHIA,YI,NQ,NJ,NP,SI)         1NP 101
C                                           1NP 102
C CALL TRAN (SI,SM,NQ,NP)                  1NP 103
C CALL ANORM (SM,ST,NP,NO)                 1NP 104
C CALL TRAN (ST,SI,NP,NO)                  1NP 105
C CHECK CONVERGENCE OF S MATRIX             1NP 106
DO 300 I=1,NQ                               1NP 107
DO 300 J=1,NP                               1NP 108
DEL = SI(I,J)-SI(J,I)                        1NP 109
IF (ABS(DEL)-ATOL) .GT. 300,300,280         1NP 110
300 IF (SI(I,J)) 290,310,290                 1NP 111
290 IF (ABS(DEL/SI(J,I))-PTOL) .GT. 300,300,310 1NP 112
310 CONTINUE
GO TO 360
310 IF (ITC = IMS) 320,320,340
320 ITC = ITC + 1
   DO 330 J = 1, NP
      DO 330 I = 1, NQ
   330 S(I,J) = S(I,J)
   GO TO 240
340 WRITE (3,350)
350 FORMAT (T10, 'MAXIMUM NUMBER OF S MATRIX ITERATIONS EXCEEDED,' 
      'JOB TERMINATED')
   GO TO 870
360 WRITE (13,260)
   CALL MOUT2 (PHI M,NJ,NQ )
   WRITE (3,370)
370 FORMAT (T10,'CONVERGED S MATRIX')
   CALL MOUT2 (SI,NQ, NP )
   C CALCULATE MODAL MOBILITY
C SM*Y* REAL SI*Y* IMAG
   CALL PSEUDO (PHI M,NJ,NQ,PHIN )
   CALL MMPY (PHI M,YR,NQ,NJ,NP, SM )
   CALL MMPY (PHI M,YI,NQ,NJ,NP, SI )
   WRITE (3,380)
380 FORMAT (T10, 'MODAL MOBILITIES, REAL, IMAGINARY')
   CALL MOUT2 (SM,NQ, NP )
   CALL MOUT2 (SI,NQ, NP )
   C CALCULATE MODAL IMPEDANCE
C DO 390 I=1,NQ
390 WRITE (3,150) PHI MINK,I
   DO 390 J = 1, NP
      LUN = PHI MINK(I)/S(I,J)* S(I,J)* S(I,J)* SM(I,J)
      ZSR(I,J) = S(I,J)+CON
   390 ZSI(I,J) = S(I,J)+CON
   WRITE (3,400)
400 FORMAT (T10,' MODAL IMPEDANCE REAL, IMAGINARY')
   CALL MOUT2 (ZSR,NQ, NP )
   CALL MOUT2 (ZSI,NQ, NP )
   C LEAST SQUARES ANALYSIS ON MODAL IMPEDANCE AS FUNCTION
   C OF FORCING FREQUENCY SQUARED
C NL=NP/NQ
C ANL=NL
C NLC=NL
C K=1
   DO 420 K = 1, NQ
      SUM = 0.
      SUMA = 0.
      SUMB = 0.
      SUMC = 0.
   420 DO 410 I = KJ,NLC
SUM = OMFS(I) + SUM
SUMA = ZSR(K) + SUM
SUMB = OMFS(I) + OMFS(I) + SUMB

410 SUMC = OMFS(I) * ZSR(K) + SUMC
DET = ANL * SUMB - SUM * SUM
X = (SUMA * SUMB - SUMC * SUM) / DET
XB = (ANL * SUMA - SUMC * DET)

KJ = IC + 1
NLC = NLC + (K + 1)
OMNC(K) = SQRT(ABS(XA/XB))
AKSR(K) = -XB * OMNC(K) * OMNC(K)
AMS(K) = XB
OMNS(K) = OMNC(K) + OMNC(K)

420 CONTINUE
L = 1
DO 430 I = 1, NL
ADSK(I) = (OMFS(L)/OMNS(I) - 1.0) * SL(I,L) * AKSR(I) / SM(I,L)
OMNC(I) = OMNC(I) / 6.28318

430 L = L + 1
C
IF ( MM .NE. 1 ) GO TO 450
SUM = 0.
DO 440 I = 1, NL
440 SUM = 2 * SUM + 1, I = SUM
G( 1 ) = SUM / AKSR( 1 ) * ANL
OMN( 1 ) = OMNC( 1 )
ADS( 1 ) = ADSR( 1 )
AMS( 1 ) = AMSR( 1 )
AKS( 1 ) = AKSR( 1 )
WRITE (14) (PHI(I, MM ), I = 1, NJ)
GO TO 480

450 DO 460 I = 1, NJ
460 PHI(I, MM ) = PHI(I, 2)
SUM = 0.
NJ = NJ + 1
NZ = Z + NL
DO 470 I = NJ, NL
470 SUM = 2 * SUM + 2, I = SUM
G( MM ) = SUM / AKSR( 2 ) * ANL
OMN( MM ) = OMNC( 2 )
ADS( MM ) = ADSR( 2 )
AMS( MM ) = AMSR( 2 )
AKS( MM ) = AKSR( 2 )
WRITE (14) (PHI(I, MM ), I = 1, NJ)

490 WRITE (3,540) MM, OPN(MM), AMS(MM), AKS(MM), ADS(MM)
WRITE (3,490) (OMN(I), I = 1, NPHI)
WRITE (3,510) (AMS(I), I = 1, NPHI)
WRITE (3,510) (AKS(I), I = 1, NPHI)

490 FORMAT (*.CALCULATED NATURAL FREQUENCIES, CYCLES/SEC* /
*IPINE13.4)

510 FORMAT (*.CALCULATED GENERALIZED STIFFNESS*/IPINE13.21)

510 FORMAT (*.CALCULATED GENERALIZED MASS*/IPINE13.21)

ENDINO 14
DO 520 J = 1, NPHI

59
UMMS(I,J)=OM(I,J)*OMM(I,J)
USQ(I,J)=G1(I,J)*G1(I,J)
523 HEAD (14) (PHI(I,J),I=1,NJ)
NQ=NPHI
CALL ANORM (PHI,PHI,NJ,NQ)
WRITE (3,530)
530 FORMAT ('14,550','*NORMAL MODES*')
CALL IMOUT2 (PHI,NJ,NQ)
END PSEUDO (PHI,NJ,NQ,PHIA)

IDENTIFICATION OF MASS, STIFFNESS AND DAMPING MATRICES
CALL TRN (PHIA,PHIA,NJ)
543 FORMAT ('14,550','*NORMAL PARAMETERS *')
CALL IMOUT2 (PHIA,NJ,NJ)
DO 560 J=1,NJ
DO 560 K=1,NQ
SUM=0.
SUMM=0.
SUMD=0.
DO 550 I=1,NQ
ACON=PHI(K,J)*PHIM(I,J)
SUM=ACON/ABS(I)+SUM
SUMM=ACON/ABS(I)+SUMM
550 SUM=ACON/ABS(I)+SUMM
ST(1,J)=SUM/J
SM(1,J)=SUM
560 S1(1,J)=SUMD
CALL INVRS (SM,NJ,JSR)
WRITE (3,580)
570 FORMAT ('14,550','*IDENTIFIED MASS MATRIX*')
CALL IMOUT2 (ZSR,NJ,NJ)
WRITE (3,580)
580 FORMAT ('14,550','*IDENTIFIED INFLUENCE COEFFICIENT MATRIX*')
CALL IMOUT2 (STh,NJ,NJ)
CALL INVRS (STh,NJ,JSR)
WRITE (3,590)
590 FORMAT ('14,550','*IDENTIFIED STIFFNESS MATRIX*')
CALL IMOUT2 (ZSR,NJ,NJ)
WRITE (3,600)
600 FORMAT ('14,550','*IDENTIFIED DAMPING MATRIX*')
CALL INVRS (SZ,NJ,JSR)
CALL IMOUT2 (ZSR,NJ,NJ)
SUM=0.
DO 610 I=1,NQ
WRITE (3,620) G I(1)
610 FORMAT ('14,550','*IDENTIFIED DAMPING MATRIX*')
CALL INVRS (SZ,NJ,JSR)
CALL IMOUT2 (ZSR,NJ,NJ)
SUM=0.
DO 610 I=1,NQ
WRITE (3,650) GS
620 FORMAT ('14,550','*IDENTIFIED DAMPING MATRIX*')
CALL INVRS (SZ,NJ,JSR)
CALL IMOUT2 (ZSR,NJ,NJ)
SUM=0.
DO 610 I=1,NQ
WRITE (3,650) GS

```
650 FORMAT ('"" AVG STRUCTURAL DAMPING='FB.4')
   IF (NNF.EQ.0) GO TO 650
640 KEAU (1,150) (HZ(I),I=1,NNF)
   NF=NNF
   GO TO 660
650 IF (NNF.EQ.0) GO TO 870
660 TORF=NROD.GT.0.AND.NROD.LE.NQ
   DO 750 L=1,NF
   CON=HZ(I)+HZ(L)
   CALL MOBPHI (G,GSQ,CON,AMS,OMS,YA,YI,PHIM,NQ,NJ)
670 IF (IP1) 690,680,730
660 IF (IP2.NE.0) CALL MATAMP (HZ(L),YR,YI,NQ)
   IF (IP2.NE.0) GO TO 700
   WRITE (3,690) HZ(L)
690 FORMAT ('"'40, 'REAL MOBILITY, IMAGINARY MOBILITY
   FREQ =''F10.2')
   A = Hertz''/
   GO TO 720
700 WRITE (3,710) HZ(L)
710 FORMAT ('"'40, 'ACCELERATION AMPLITUDE IN G's, PHASE IN DEG.
   FREQ =''F10.2')
   A = Hertz''/
720 CALL MOUT2 (YR,NQ,NQ)
   CALL MOUT2 (YI,NQ,NQ)
   GO TO 750
730 DO 740 I=1,NQ
   DPR(I,I)=YR(I,I)
   DPI(I,I)=YI(I,I)
   IF (.NOT. TORF) GO TO 740
   TRIL(I,I)=YR(NROW,I)
   TRIU(I,I)=YI(NROW,I)
740 CONTINUE
750 CONTINUE
760 IF (IP1) 870,870,760
760 IF (IP2.NE.1) GO TO 780
680 CALL AMP (HZ,DPR,DPI,NF,NQ)
   IF (.NOT. TORF) CALL AMP (HZ,TR,TR,NF,NQ)
690 WRITE (3,810)
700 CONTINUE
710 FORMAT ('"'40, 'DRIVING POINT RESPONSE, AMP IN G's AND PHASE IN
   DEGREES'"
   GO TO 810
720 WRITE (3,790)
730 FORMAT ('"'40, 'DRIVING POINT MOBILITY, REAL AND IMAGINARY'"
   GO TO 830
740 FORMAT ('"'40, 'ACCELERATION MOBILITY'"
750 CALL YOUT (HZ,DPR,NF,NQ,IA)
   WRITE (3,820)
760 FORMAT ('"'"
   CALL YOUT (HZ,DPI,NF,NQ,IP2,IA)
   IF (.NOT. TORF) GO TO 870
   IF (IP2.NE.1) GO TO 840
770 WRITE (3,830) NROW
780 FORMAT ('"'40, 'TRANSFER RESPONSE, ROW '15,' AMP IN G's AND PHAS
   AE IN DEG'"
   GO TO 860
790 WRITE (3,850) NROW
800 FORMAT ('"'40, 'TRANSFER MOBILITY, ROW '15,' REAL AND IMAG'
   GO TO 860
```

IF (IA .NE. 0) WRITE (3,800)
800 CALL YOUT (HZ,TR,NF,NQ,0,IA)
WRITE (3,820)
CALL YOUT (HZ,TI,NF,NQ,IP2,IA)
870 CONTINUE
REWIND 13
CALL EXIT
END
SUBROUTINE TRN ( A, B, NR, NC )

C B=TRANSPOSE OF MATRIX A
C A=UNDISTURBED MATRIX
DIMENSION A(20,21), B(20,21)
DO 100 I=1, NR
DO 100 J=1, NC
130 B(J,I) = A(I,J)
RETURN
END
SUBROUTINE INVRS (B,N,A)

A = INVERSE OF B  \[B UNDETERMINED\]

DIMENSION A(20,21),D(20,21),IRW(21),ICOL(21),B(20,21)

DO 100 I=1,N
DO 100 J=1,N
100 A(I,J)=B(I,J)
M=N+1
DO 110 I=1,N
IROW(I)=I
110 ICOL(I)=I
DO 120 K=1,N
AMAX= A(K,K)
DO 130 I=K,N
DO 130 J=K,N
IF(AABS(A(I,J))-ABS(AMAX))130,120,120
120 AMAX= A(I,J)
IC=I
JC=J
130 CONTINUE
KI=ICOL(K)
ICOL(K)=ICOL(IC)
ICOL(IC)=I
KI=IROW(K)
IROW(K)=IROW(JC)
IROW(JC)=KI
IF(AMAX)160,140,160
140 WRITE (3,150)
150 FORMAT(' SOLUTION OF EXISTING MATRICE NOT POSSIBLE')
GO TO 330
160 DO 170 J=1,N
E=A(K,J)
A(K,J)=A(IC,J)
170 A(IC,J)=E
DO 180 I=1,N
E=A(I,K)
A(I,K)=A(I,JC)
180 A(I,JC)=E
DO 210 I=1,N
IF(I-K)200,190,200
190 A(I,M)=1.
GO TO 210
200 A(I,M)=0.
210 CONTINUE
FVT=A(K,K)
DO 220 J=1,M
220 A(K,J)=A(K,J)/FVT
DO 230 I=1,N
IF(I-K)230,250,230
230 AMULT=A(I,K)
DO 240 J=1,M
240 A(I,J)=A(I,J)-AMULT*A(K,J)
250 CONTINUE
DO 260 I=1,N
260 A(I,K)=A(I,M)

64
DO 290 I=1,N
DO 270 L=1,N
1F(1ROW(I)L)270,280,270
270 CONTINUE
280 DO 290 J=1,N
290 D(L,J)=A(I,J)
DO 320 J=1,N
DO 300 L=1,N
1F(1COL(J)L)300,310,300
300 CONTINUE
310 DO 320 I=1,N
320 A(I,L)=D(I,J)
330 RETURN
END
SUBROUTINE MNPY (A,B,N1,N2,N3,C)
C = A * B
A (N1 X N2) B (N2 X N3) C (N1 X N3)
REAL A(20,21),B(20,21),C(20,21)
DO 100 I=1,N1
   DO 100 J=1,N3
      C(I,J)=0.
      DO 100 K=1,N2
         C(I,J)=C(I,J)+A(I,K)*B(K,J)
   100 RETURN
END
SUBROUTINE MOUT2 (A,M,N)

REAL A(20,100)

ID=MIN0(N,10)

WRITE (3,100) (I,1=1,10)

FORMAT (/15,10I12)

WRITE (3,100)

DO 110 I=1,10

110 WRITE (3,120) I,(A(I,J),J=1,10)

120 FORMAT (15,5X,1P10E12.4)

IF (ID-N) 130,170,170

130 ID=MIN0(N,20)

WRITE (3,100) (I,I=11,10)

WRITE (3,100)

DO 140 I=1,10

140 WRITE (3,120) I,(A(I,J),J=11,10)

IF (ID-N) 150,170,170

150 WRITE (3,100) (I,I=21,10)

WRITE (3,100)

DO 160 I=1,10

160 WRITE (3,120) I,(A(I,J),J=21,N)

170 RETURN

END
SUBROUTINE ANORM (PHI, PHIN, NR, NC)
DIMENSION PHI(20, 21), PHIN(2N, 21)

DO 120 I=1, NC
A0 = PHI(I, I)
DO 110 J=2, NR
IF (ABS(A0).LE.ABS(PHI(J, I))) A0 = PHI(J, I)
100 CONTINUE

DO 110 J=1, NR
PHIN(J, I) = PHI(J, I) / A0
110 CONTINUE
RETURN
END
SUBROUTINE ERRMU (A,B,PCTR,PCTJR,PCTI,PCTBI, NJ,NP,IX)

C A BIAS ERROR, PCTB (RATIO) ON AMPLITUDE, AND A UNIFORM RANDOM ERROR HAVING A +/- MAXIMUM OF PCT (RATIO) ON AMPLITUDE.

C

USES RANDU

DIMENSION A(20,21), B(20,21)

IF(PCTR) 110, 100, 110

130 IF(PCTBR) 110, 130, 110

110 DO 120 I=1,NJ

DO 120 J=1,NP

CALL RANDU (IX, IY, VFL)

IX=IX

E=1.0+2.0*PCTR*(VFL-0.5)+PCTBR

A(I,J)=A(I,J)*E

CALL RANDU (IX, IY, VFL)

IX=IX

E=1.0+2.0*PCTI *(VFL-0.5)+PCTBI

B(I,J)=B(I,J)*E

110 RETURN

END
SUBROUTINE KANOU (IX, IV, YFL)

C THIS SUBROUTINE IS FROM SSP VERS. II

IV = IX + 65249
IF (IV) 100, 110, 10
100 IV = IV + 2147483647
110 YFL = IV
   YFL = YFL - 4294967296
RETURN
END

SUBROUTINE REDI (YR, VI, NP, NJ, KEEP, IND, YRT, YIT)

C REDUCES DISPLACEMENT MOBILITY DATA TO MATRIX OF NJ SPECIMEN
C COORDINATES AND FORCING FREQUENCIES YR(NJ*NP)
DIMENSION VR(20, 21), VI(20, 21), KEEP(20), INDX(20)
DIMENSION YRT(20, 100), YIT(20, 130)
DO 120 I = 1, NP
   DO 120 J = 1, NJ

VR (J, I) = YRT(KEEP(J), INDX(I))
120 VI(J, I) = YIT(KEEP(J), INDX(I))

RETURN
END
SUBROUTINE YOUT (OMH, A, NINC, ND, IAMP, IA)
C
C IF IA NOT = 0 USE ACCELERATION ABILITY
C
REAL OMH(100), A(100, 2N)
C IF ( IA ) = 100, 120, 100
100 CON = 6.283185 * 6.283185
DO 110 I = 1, NINC
OMH(I) = OMH(I) + CON
DO 110 J = 1, ND
110 A(I, J) = A(I, J) * CM
C
C ID = MNO (ND, 10)
130 IL = MNO (NINC, 45)
C
140 WRITE (3, 150) (I, I = J1, 10)
150 FORMAT (15, *HERTZ', 16, 9112)
WRITE (3, 160)
C
160 FORMAT (1X)
170 IF (NAMP) = 170, 170, 200
DO 180 I = 11, IL
180 WRITE (3, 190) OMH(I), (A(I, J), J = J1, IL)
190 FORMAT (1X, F9.3, 1P10E12.4)
GO TO 230
C
C DO 210 I = 11, IL
210 WRITE (3, 220) OMH(I), (A(I, J), J = J1, IL)
220 FORMAT (1X, F9.3, 1P10F12.2)
230 IF (IL - NINC) > 240, 260, 260
240 WRITE (3, 250)
250 FORMAT ('1'/*)
C
C IL = NINC
GO TO 140
C
260 IF (I = ND) = 270, 280, 280
270 J1 = 11
ID = ND
WRITE (3, 220)
GO TO 130
C
RETURN
END
SUBROUTINE AMP (OMH, A, B, NINC, NR)

CONVERTS A + IB IN DISPLACEMENT UNITS
TO AMP (IN A) IN G'S AND PHASE (IN B) IN DEG
EACH ROW IS AT A FREQUENCY OMH(1) IN HERTZ

DIMEN: OMH(100), A(100,201), B(100,201)

DO 210 I=1,NINC
OM=OMH(I)*0.01626
OMR=OMH(I)*6.283185
DO 210 J=1, NR
R=A(I,J)
C=B(I,J)
A(I,J)=SQRT(R*R+C*C)*OM*OMR
IF(R) 140,100,140
100 IF(C) 110,120,130
110 B(I,J)=270.
GO TO 210
120 B(I,J)=0
GO TO 210
130 B(I,J)=90.
GO TO 210
140 P=ATAN(ABS(C/R))*57.2958
IF(R) 150,150,180
150 IF(C) 160,161,170
160 B(I,J)=180.+P
GO TO 210
170 B(I,J)=180.-P
GO TO 210
180 IF(C) 190,190,200
190 B(I,J)=360.-P
GO TO 210
200 B(I,J)=P
210 CONTINUE
RETURN
END
SUBROUTINE CINV (A, B, N, C, D)

C
DIMENSION A(20, 21), B(20, 21), C(20, 21), D(20, 21), E(20, 21)

C+I = INVERSE OF A+I*0   I=SQR(-1)

C
C
  0 ASSUMED NON SINGULAR

CALL INVRS(B, N, C)
CALL MMPY(C, A, N, N, N, E)
CALL MMPY(A, E, N, N, N, C)
DO 100 I = 1, N
  DO 100 J = 1, N
    C(I, J) = C(I, J) + B(I, J)
100 CALL INVRS(C, N, D)
CALL MMPY(E, D, N, N, N, C)
DO 110 I = 1, N
  DO 110 J = 1, N
    D(I, J) = -D(I, J)
110 RETURN

END
SUBROUTINE MOBPHI ( G, GSQ, CON, AMS, VNS, VR, Y1, PHIM, NO, NJ )
C
CALCULATES VR AND Y1 USING MODAL MODULUS AND MODE SHAPE
DIMENSION G(20), GSQ(20), AMS(20), VNS(20), VR(20, 20), Y1(20, 20), PHIM(20, 20)
AYS(20), YS1(20), ONS(20)
DO 100 I=1, NO
CONA=CON/ONS(1)
COND=1./(CON*AMS(10)) * 39.478413
CONC=CONA-1.
COND=CONA*CONB/(CONC*CONC*GSO(11))
YSR(11)=CONC*COND
100 YSI(11)=G(11)*COND
DO 120 J=1, NJ
DO 120 K=1, NO
SUMR=0.
SUMI=0.
DO 110 I=1, NO
ACON=PHIM(11)*PHIM(11)
SUMR=YSR(11)*ACON*SUMR
110 SUMI=YSI(11)*ACON*SUMV
VR(11)=SUMR
Y1(11)=SUMI
RETURN
END
SUBROUTINE PSEUDO (A,NR,NC,C)

C = PSEUDOINVERSE OF A UNDISTURBED
A IS A RECTANGULAR MATRIX OF MAXIMAL RANK (NR X NC)
NR .GT. OR .LT. NC
C = -1
C = (A*A) A OR A*(AA*)
NR,NC MAY NOT EXCEED 25
REAL A(20,21),B(20,21),C(20,21)

UO 100 I=1,NR
DO 100 J=1,NC
100 B(I,J)=A(I,J)
IF(NR-NC).LT.100,110,130
110 CALL INVRS (A,NR,C)
GO TO 140

C = A*A
CALL INVRS (C,NR,A)
CALL MPY (B,A,NC,NR,C)
GO TO 140

C = A*A
CALL INVRS (C,NC,A)
CALL MPY (A,B,NC,NC,NR,C)
RESTORE A
RETURN

140 DO 150 I=1,NR
DO 150 J=1,NC
150 A(I,J)=B(I,J)
RETURN
END