DYNAMIC RESPONSE INDEX MINIMIZATION FOR PERSONNEL ESCAPE SYSTEMS

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On the basis of studies conducted for the U. S. Air Force Aerospace Medical Research Laboratories (1,2) specification of the dynamic response index or DRI, which is a measure of human spinal compression and probability of injury, has been adopted as a new criterion for determining the acceptable tolerance level of humans to the personnel ejection environment. Prior to the adoption of the DRI the maximum acceleration and maximum time rate of change of acceleration were the primary determinants. The specification of the DRI is more realistic, however, in that it relates directly to stresses actually developed within the ejectee.

This study was conducted to determine a more optimum ballistic cycle for the catapult phase of rocket catapult operation in terms of minimization of the DRI for a selected class of acceleration-time output.

THE DYNAMIC RESPONSE INDEX

The dynamic response index arose from an attempt to analytically describe the effect of acceleration on an ejectee in terms of spinal compression, $S$ (ft), and then relate this effect to the probability of injury. A plot of the operational injury rate vs DRI is shown in Figure 1 (3). Experimental studies have led to a second order differential equation relating spinal compression to the longitudinally applied acceleration, $A(t)$. This relation is

$$\frac{d^2 S}{dt^2} + 2\rho \omega \frac{dS}{dt} + \omega^2 S = A(t)$$

(1)
where the damping ratio, $\zeta$, has the value .224 and the natural frequency of the spinal column, $\omega$, equals 52.9 radians/sec for the mean U.S. Air Force flying population (4). The DRI is defined in terms of the maximum spinal compression, $\delta_{\text{max}}$, according to the relation

$$\text{DRI} = \frac{\omega^2}{g} \delta_{\text{max}}$$ (2)

where $g$ is the acceleration due to gravity, 32.2 ft/sec$^2$. The specification limit for the DRI is set at 18 for escape systems temperature conditioned at 70°F (4). This limit corresponds to approximately a 5% probability of injury.

Equation 1 may be restructured to a more convenient form by multiplying through by the factor $\omega^2/g$ to yield the equation

$$\frac{d^2(\text{DR})}{dt^2} + 2\rho \omega \frac{d(\text{DR})}{dt} + \omega^2 \text{DR} = \omega^2 G(t)$$ (3)

where DR is defined as the dynamic response and is equal to $\omega^2 \delta/g$ and $G(t)$ is the applied acceleration in g units, $G(t) = A(t)/g$. In terms of this change of variable, the DRI becomes the maximum value of the dynamic response.

$$\text{DRI} = \text{DR}_{\text{max}}$$ (4)

The form of Equation 3 is particularly advantageous in that the dynamic response and the applied acceleration are of the same order of magnitude and a plot of both of these parameters vs time clearly illustrates the effect of the acceleration on the spinal response.

In an attempt to theoretically investigate the dependence of the DRI on the catapult output and to optimize this output with respect to the DRI and a fixed set of catapult performance parameters, stroke ($S$) and velocity ($V$), an acceleration-time output of the form depicted in Figure 2 was selected for investigation. Although this type of output may not be the optimum from the standpoint of DRI minimization (5), it is a good approximation of the output of current ballistic catapults and that which could be attained with a minimum of system modifications. In addition, being a function of only the four boundary points - $G_0$, $G_{\text{max}}$, $t_F$, and $t_S$ - this form offers considerable flexibility in terms of variation.
of shape with a minimum number of variables. The dependency on these points may be removed in favor of the curve geometry and performance in terms of the velocity, \( V \), and stroke, \( S \), by defining two parameters, \( \Gamma \) and \( \Upsilon \), as indicated.

\[
\Gamma = \frac{G_0}{G_{\text{max}}} \quad (5a)
\]

and

\[
\Upsilon = \frac{t_T}{t_S} \quad (5b)
\]

Employing equations 5a and 5b and integrating the acceleration-time output to determine the catapult velocity and stroke, \( G_{\text{max}} \) and \( t_s \) may be expressed in terms of the curve geometry and performance as indicated.

\[
G_{\text{max}} = \frac{\left[ \Upsilon(1-\Gamma)/3-\Upsilon(1-\Gamma)+1 \right]}{2[1-\Upsilon(1-\Gamma)/2]^2} \left( \frac{v^2}{g_s} \right) \quad (6a)
\]

and

\[
t_s = \frac{2[1-\Upsilon(1-\Gamma)/2]}{[\Upsilon^2(1-\Gamma)/3-\Upsilon(1-\Gamma)+1]} \left( \frac{S}{v} \right) \quad (6b)
\]

Equations 5 and 6 completely specify the catapult acceleration-time output in terms of its geometry and delivered performance.

THEORETICAL RESULTS

Initial efforts (6) were concerned with determining the overall effect of the geometrical parameters and on the dynamic response index. To accomplish this and to provide a uniform basis for comparison, the catapult velocity and stroke were fixed at 50 ft/sec and 3 ft respectively. These are performance parameters characteristic of current rocket catapult operation. The geometrical parameters were varied and the resultant DRI evaluated. A plot of these results is given in Figure 3. For the values of the geometrical parameters considered for this initial overview, the DRI's ranged from a maximum of about 19.2 for a constant or step acceleration (\( \Upsilon = 0 \)) to a minimum of about 14.3 for \( \Gamma = .5 \) and \( \Upsilon = .8 \). A ramp or constantly increasing acceleration is given by the point \( \Gamma = 0, \ Upsilon = 1 \). These three output and their respective dynamic responses are depicted in Figure 4. The large dynamic
overshoot for the step acceleration is characteristic of the under-damped nature ($\phi < 1$) of the spinal response. This range in DRI from 19.2 to 14.3 corresponds to a spread in the injury probability rate of from about 10% to .2%.

Once the overall effect of the geometrical parameters on the dynamic response index was determined, a more detailed analysis was conducted to determine the geometrical parameters required to produce the minimum DRI, $\text{DRI}_m$, for a range of catapult velocities. The stroke was again fixed at 3 feet. The results of this analysis are summarized in Table I. Figure 5 depicts the acceleration-time output which produced minimum DRI's for velocities of 40, 50 and 60 ft/sec.

Contrasted to the above procedure for minimizing the DRI, it is also possible to maximize the catapult velocity with respect to the allowable stroke and DRI specification limit. Interpolation of the data contained in Table I indicates that for a stroke of 3 feet and maximum DRI of 18 a catapult velocity of approximately 58 ft/sec is possible.

**EXPERIMENTAL RESULTS**

Initially, a limited test program was conducted in an attempt to verify the feasibility of producing the type of output required to modulate the DRI. The XM39 rocket catapult which is under development for the U. S. Air Force and which was undergoing charge development tests was selected as the test vehicle. To create the effect of the initial acceleration, $G_0$, booster charges consisting of 4 and 6 grams of finely machined propellant were added to the catapult cartridge. Table II represents the average of two rounds fired at each booster level with zero booster tests used for comparison. Because the main catapult charge used in these tests does not represent the finalized charge, these results are not indicative of the performance of the developed item. These tests did, however, demonstrate the feasibility of modifying the catapult ballistics to produce acceleration-time output required by the theoretical analysis. Figure 6 shows the acceleration-time output for typical 0, 4 and 6 gram booster tests. Only the DRI for the 0 and 4 gram tests could be compared, however, because they produced essentially identical ejection velocities. The average DRI for the zero booster tests was 19.0 and that for the 4 gram tests was 18.0. This represents approximately a 50% decrease in injury probability from 10% to 5% with no sacrifice in performance.

Additional booster tests were conducted utilizing the finalized catapult grain to determine the effect of the addition of various quantities of booster. The test results are listed in Table III. Figure 7 shows the resultant DRI's plotted against ejection
velocity with the theoretical DRI\textsubscript{m} curve superimposed for reference. As is evident from the tabular results, the addition of the booster tended to more nearly optimize the output. The spread between the experimentally obtained and theoretical minimum DRI's decreased with increasing booster charge.

As a direct result of this research, the XM39 rocket catapult currently employs a booster charge in an attempt to morely optimize its output and take advantage of the attendant reduction in DRI.

Conclusions

This paper has outlined a technique which may be employed with minimum modification to existing rocket catapult components to reduce the probability of injury to users of aircraft emergency escape systems. Experimental tests have verified the results of theoretical analyses and have demonstrated the ability to modify the catapult ballistics and thus moderate the dynamic response index.

In addition, this technique may also be employed to upgrade the performance of existing and future aircraft escape systems by permitting maximization of the ejection velocity with respect to the allowable catapult stroke and DRI specification limit.

It may be possible to utilize a technique other than that outlined here to optimize the process of DRI minimization and, thereby, afford the maximum in performance and safety to users of aircraft emergency escape systems. However, this research does represent an initial step towards the achievement of an approach aimed at integrating rocket catapult design and the physiological constraints imposed by specification of the DRI.

References


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Figure 1. Operational Spinal Injury Rate vs Dynamic Response Index
Figure 2. Theoretical Catapult Acceleration-Time Output

TABLE I

THEORETICAL DRI_m RESULTS

<table>
<thead>
<tr>
<th>S (ft)</th>
<th>V (ft/sec)</th>
<th>( \gamma )</th>
<th>( \gamma' )</th>
<th>( \gamma'' )</th>
<th>( \gamma''' )</th>
<th>t_s (sec)</th>
<th>DRI_m</th>
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<tbody>
<tr>
<td>3</td>
<td>40</td>
<td>.47</td>
<td>.57</td>
<td>.68</td>
<td>.78</td>
<td>.169</td>
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<td>3</td>
<td>45</td>
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<td>1.0</td>
<td>1.0</td>
<td>.127</td>
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<td>.38</td>
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<td>2.0</td>
<td>2.0</td>
<td>.118</td>
<td>20.36</td>
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Figure 3. Theoretical DRI vs Output Acceleration-Time Geometry
Figure 4. Theoretical Dynamic Response and Acceleration vs Time
Figure 5. Acceleration-Time Output For Minimum DRI
Figure 6. Experimental Acceleration-Time Output
**TABLE II**

**EXPERIMENTAL CURVE SHAPING TESTS**

<table>
<thead>
<tr>
<th>Booster Charge (grams)</th>
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<th>4</th>
<th>6</th>
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<tbody>
<tr>
<td>Initial Acceleration, $G_0$</td>
<td>0</td>
<td>5.1</td>
<td>7.7</td>
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<tr>
<td>Maximum Acceleration, $G_{max}$</td>
<td>17.5</td>
<td>17.1</td>
<td>20.2</td>
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<tr>
<td>Time to Stroke, $t_s$ (sec)</td>
<td>0.163</td>
<td>0.140</td>
<td>0.125</td>
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<tr>
<td>Catapult Stroke, $S$ (ft)</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Catapult Velocity, $V$ (ft/sec)</td>
<td>52</td>
<td>52</td>
<td>57</td>
</tr>
<tr>
<td>$(G_0/G_{max})$</td>
<td>0</td>
<td>0.30</td>
<td>0.38</td>
</tr>
<tr>
<td>$(t_r/t_s)$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Dynamic Response Index</td>
<td>19.0</td>
<td>18.0</td>
<td>21.9</td>
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**TABLE III**

**EXPERIMENTAL DRI$_m$ TESTS**

<table>
<thead>
<tr>
<th>Booster (grams)</th>
<th>Velocity (ft/sec)</th>
<th>DRI (exp)</th>
<th>DRI$_m$</th>
<th>% DRI$_{exp}$ over DRI$_m$</th>
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<tbody>
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<td>14.9</td>
<td>7</td>
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Figure 7. Experimental $DRI_m$ Results