THE ACCURACY OF BALLISTIC DENSITY
DEPARTURE TABLES 1934-1972

By
Marvin J. Lowenthal

April 1972

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Destroy this report when it is no longer needed. Do not return it to the originator.
The accuracy of ballistic density departure tables is examined, starting with the earliest available sets in 1934. The extension of the tables (originally developed for the US) to encompass the entire Northern Hemisphere is discussed and the shortcomings of the current climatological regional zones described. New tables, based on current data and used for a more limited geographical area, are shown to be accurate to one half of one percent, hence furnish excellent back-up information when a current sounding is not available for artillery firings. A procedure for minimizing ballistic density errors that accrue between observational periods is also presented.
1. Ballistic Density
2. Air Density
3. Radiosondings
4. Station Elevation
5. Geographical Regions
6. Meteorological Messages
THE ACCURACY OF BALLISTIC DENSITY DEPARTURE TABLES 1934-1972

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April 1972

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ABSTRACT

The accuracy of ballistic density departure tables is examined, starting with the earliest available sets in 1934. The extension of the tables (originally developed for the US) to encompass the entire Northern Hemisphere is discussed and the shortcomings of the current climatological regional zones described.

New tables, based on current data and used for a more limited geographical area, are shown to be accurate to one half of one percent, hence furnish excellent back-up information when a current sounding is not available for artillery firings.

A procedure for minimizing ballistic density errors that accrue between observational periods is also presented.
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INTRODUCTION

One of the basic premises of all firing tables is the "Standard Atmosphere" - one whose properties are enumerated in the "1962 Standard Atmosphere" [1]. With such a density distribution and zero wind throughout, the shell should describe the calculated trajectory given in the firing tables. Those conditions are never encountered on earth, hence corrections for nonstandard conditions are always in order. Artillery Meteorological Sections, organic to the Army Artillery, provide meteorological information on winds and density to permit such corrections.

Accurate, representative, fresh observations are the best method of correction available today. There are, however, occasions when these measurements are not available, and alternate methods are necessary to provide the requisite information. In the case of atmospheric density, such an option is available.

Air density at ground level can always be determined, the only equipment required being a good barometer and wet- and dry-bulb thermometers. If the upper atmospheric density can be inferred from the surface measurement, the ballistic density can be furnished for correction of artillery fire. The vital question is: Is the assumption justified that the surface density is a good predictor of ballistic density aloft?

To test the theory, some 3000 radiosondings from Vietnam and 2000 from the Korean region were examined. Figure 1 shows the mean of ballistic density aloft as a function of surface density over a wide range of surface values.* Values of ballistic density are expressed as a percent

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*The values on the abscissa of Figure 1 represent ±0.2% on each side of the central value.
of standard with the ordinate being the normal NATO zones (Table I).

TABLE I

BALLISTIC ZONES, STANDARD ATMOSPHERE

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Height at Top of Zone (Meters)</th>
<th>Temperature (°K)</th>
<th>Density (g/m³)</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>Surface</td>
<td>288.2</td>
<td>1225.0</td>
</tr>
<tr>
<td>1</td>
<td>200</td>
<td>287.5</td>
<td>1213.3</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
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<td>1184.4</td>
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<tr>
<td>3</td>
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<td>283.3</td>
<td>1139.2</td>
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<td>280.0</td>
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<td>2000</td>
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<td>1032.0</td>
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<tr>
<td>6</td>
<td>3000</td>
<td>271.9</td>
<td>957.0</td>
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<td>7</td>
<td>4000</td>
<td>265.5</td>
<td>863.4</td>
</tr>
<tr>
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<td>10</td>
<td>8000</td>
<td>242.7</td>
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<td>229.8</td>
<td>467.0</td>
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</tr>
<tr>
<td>14</td>
<td>16000</td>
<td>216.7</td>
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</tr>
<tr>
<td>15</td>
<td>18000</td>
<td>216.7</td>
<td>142.3</td>
</tr>
</tbody>
</table>

As can be seen, the ballistic densities are a function of the surface value. The various curves do not intersect, although the spread between the extremes continually decreases. The values all seem to trend toward a common value at some higher level, beyond the limit of our data. The curves do show that ballistic density is more independent of the surface value at the high line numbers than at low line numbers. Thus, a climatological value of ballistic density at high altitudes (line 10 or above) would have a very small error for all ranges of surface density. This is most fortunate since the range errors due to an incorrect assessment of ballistic density are greatest for high maximum ordinates.

In absolute values, the ballistic density curves exhibit a behavior similar to that of the Standard Atmosphere (Figure 2). Again, the convergence of all curves is seen with increasing altitude. It should be noted that the curves for the tropical region (Vietnam) converge toward a different value than those from the nontropical Korea. This difference indicates that one set of values is not satisfactory for the entire
globe, regional tables being required to minimize the error in the estimate of the ballistic density.

It has been shown, therefore, that a surface density measurement can be used to categorize ballistic densities aloft for a particular geographic region; that a climatological value of ballistic density becomes more accurate with increasing height (at least to the levels of interest for conventional tube artillery); and that all geographic localities cannot be accurately described by a single set of values.

Once the feasibility of the procedure is established, it is necessary to determine the accuracy of such a technique. It may well be that even though the mean curves of ballistic density do not intersect, the dispersion within each set is so great that the probability of an accurate estimate from the mean is quite small. This involves, of course, the study of the standard deviations of density for any given surface value. A portion of the computer analysis for both Vietnam and Korea is shown in Table 11.
The standard deviations (STD) are seen to be of the order of 0.5% or less, which suggests that ballistic density aloft can be estimated to about 1/2% from a measurement of surface density alone. Since density is calculated from measurements of pressure and temperature from the radiosonde, the accuracy of the density measurement is dependent upon the accuracy of the pressure and temperature sensors. By use of current specification criteria, the error of the density determination is found to be 0.3 - 0.4% at low line numbers and greater than 0.5% at high line numbers. An assessment of ballistic densities to about 1/2 to 1% by a surface measurement alone is an excellent back-up system that is always available where a radiosonde observation of upper air density cannot be made.

Historically, the use of density tables for the assessment of ballistic densities was the standard procedure before radiosondes were available. The earliest manual available for this report was TR-1236-I, "Meteorological Message for the Artillery," 1934. At that time it was stated:

"It is impractical to actually measure the temperature, pressure, and moisture content of the atmosphere at various heights, compute the ballistic densities, and get the computed data to the Artillery without the elapse of considerable time. As atmospheric conditions are continually changing, there must be as little delay as possible between the times that meteorological observations are made and the times that the completed reports are available to the Artillery. No attempt is made, therefore, to determine air densities from observations made at various heights above the ground. Instead, the air density is determined near the ground at the meteorological station and the air density is assumed to decrease at a definite rate in height above the ground." [2]

Three decades of technological advances have, of course, made it possible to obtain an atmospheric measurement of temperature and density and transmit it to the Fire-Direction Center. The other statements concerning atmospheric variability and the necessity for fresh meteor information are as true today as when the original words were written.

The tables given in TR 1236-1 were meant to apply to the US only (page 24) and values of ballistic density were given for station elevations near sea level, and at 1000 and 2000 feet above mean sea level. For higher elevations, the data given in the table for stations located 2000 feet above sea level may be used [3].

[3] Ibid., pp 82, 92.
The data, in part, are shown in Figure 3. The plot shows recognition of (1) the convergence of all values of ballistic density at higher levels, (2) the difference in behavior of ballistic densities with increasing station elevation.

The main sources of error in the tables are (1) use of the values at 2000 feet for stations at higher elevations, and (2) assumption that one set of values will suffice for the entire US. To correct these errors, a study was carried out by the Signal Corps General Development Laboratory, Fort Monmouth, New Jersey, to revise and extend the tables [4]. The results of this study appeared in Sep 1942 as Change 4 to Technical Manual TM 4-240 which superseded TR 1236-I in Dec 1941.

As a result of that study, the US and its Western Hemisphere territories (Alaska, Hawaii, Canal Zone, Puerto Rico, etc.) were divided into six geographical regions**: The boundaries of the regions in the US were

**Region 7, the Pacific Northwest, is not mentioned in Ch 4 to TM 4-240, 17 Sep 1943. It first appears in TM 20-240 which superseded TM 4-240 in Nov 1944. No data are available on the introduction of this new region.
the same as shown in the current FM 6-16. Appendix I, Change 4 to TM 4-240 dated 17 Sep 1943, extended the valid geographical limits of the density regions to include the entire Northern Hemisphere. Figure 4 below, a reproduction p 10, shows the revisions.

Figure 4. Meteorological Table (from TM 4-240)
Figure 5. Time zones and climatological regions of the Northern Hemisphere.

The departure tables themselves have been only slightly modified through the years, the greatest change occurring with the issuance of FM 6-16 in May 1961, owing to the change in units from yards to meters, and the adoption of a new standard surface density, 1225 g/m\(^3\) instead of the older value of 1203.4 g/m\(^3\).

Some of the ballistic density profiles from TR 1236-1 and the revision are shown in Figure 6. Surface densities 5% above the mean and 5% below the mean were chosen to emphasize the difference in high and low surface density conditions.***

***Prior to 1961 (FM 6-16) the mean surface density of a meteorological station at sea level was 103%. Currently, the standard mean surface density for a sea-level station is 100%.
The profiles show wide variations between regions, in the example, as high as 3%. The profile from TR 1236-I falls between the extremes and is not coincident with any single region - nor is it the mean of all regional values. For 98% and 108%, the old profile is closest to that of Region 5, the Alaskan area. The figure clearly shows that a geographical area the size of the US cannot be adequately described with a single density profile. Thus, it is difficult to comprehend why almost the entire Eurasian Continent from the Baltic Sea all the way to the Sea of Japan was considered a single region, or why the Northern Siberian regions were adjudged similar to Eastern US rather than the Polar Regions of North America, or why the mountainous Western US is similar to the lands around the northern borders of the Mediterranean.

The revised tables do show, however, that the profiles are dependent upon the elevation of the station, since a given surface density may represent mean conditions, above-normal surface density, or abnormally low surface density depending on whether the station in question is at sea level or at an elevated location. The behavior of the profile of 100% surface density at a sea-level station, at 1000', and at 2000' above MSL is shown in Figure 7 for both old and revised tables, Region 5 being used since it appears closest to the values in TR 1236-I.
The figure clearly indicates that ballistic density aloft decreases most rapidly in the cases of high surface densities and increases aloft when surface densities are low. Meteorologically, this is known as compensation, where lighter air overlays dense surface air, and below-normal densities at the surface gradually disappear aloft to be replaced with higher-than-normal densities. This emphasizes again the fact noted earlier, that at some altitude the atmospheric density is independent of surface density, and at this level horizontal density gradients are minimal.\textsuperscript{1}

The distinction between atmospheric density and ballistic density should be emphasized here. The atmospheric density at any height is dependent on the pressure and temperature of the atmosphere at that elevation. The pressure at that height is the weight of the column of air above that level, irrespective of conditions below.

The ballistic density at any line "n" is the sum of the weighted densities from the surface up to line "n" in question, where the weighting

\textsuperscript{1}This is the so-called isopycnic level of the atmosphere. In middle latitudes, it occurs near 8 km and was first discovered by A. Wagner in 1910 and more closely investigated by F. Linke in 1919.
function is described by

\[ j=n \]
\[ \sum a_j = 1 \]
\[ j=1 \]

and listed in Table III.

**TABLE III**

DENSITY WEIGHTING FACTORS, IN % (from FM 6-16)

<table>
<thead>
<tr>
<th>Height at Top of Zone #</th>
<th>Line # Meters</th>
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<th>2</th>
<th>3</th>
<th>4</th>
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<th>6</th>
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<td>5</td>
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</tr>
</tbody>
</table>

Hence, while the atmospheric density at the isopycnic level is nearly constant around the globe, the ballistic density at the same level will vary somewhat, although far less at that level than at lower altitudes, since the weighting factors are greatest at higher line numbers, as noted earlier.

It is undoubtedly true that each air mass has a distinctive density structure, since the source region determines the temperature and moisture content of the air mass. With cold frontal passages, the air mass changes and density changes at low levels (generally line 5 and below) are greater than normal for a short period of time. It would thus seem logical to categorize surface densities in terms of air mass, or relation to other meteorological factors (highs, lows, position in relation to a cold front, etc.). However, this technique would require trained meteorologists for each metro section, a luxury the current Army Table of Distribution and Allowances (TDA) cannot afford. Hence, the simplistic
climatological approach, that determines the values of ballistic density aloft from the surface density without regard to other meteorological conditions, was adopted.

The ultimate success of such tables - accurate assessment of ballistic density aloft - will be dependent on restriction of geographic extent of the regions to insure uniformity of conditions, adequate sample of data on which to base the tables, and diurnal or season breakdowns where required.

As an example, note the curves in Figures 8 and 9 that show the differences in density for night and day conditions for high and low line numbers.

![Graph showing differences in density for night and day conditions for high and low line numbers.](image-url)
In both the tropical Vietnam and temperate Korean regions, there is a significant difference in upper air density for a given surface condition. Hence, separate tables are needed for nighttime and daytime, a fact realized in the first revision of the old tables in TR 1236-1. However, significant changes in day and night sounding are noted in all lines, not only in line 4 as promulgated in the revised tables ([4], pp 3-4).

Similarly, plotting the densities as a function of season reveals the necessity for constructing density tables on a seasonal rather than an annual basis. In Vietnam (Figure 10), the difference between summer and winter is significant at all levels above line 2, whereas a combination spring-fall table could be developed that would fit these transitional seasons.
Figures 11 and 12 show the seasonal density profiles for Korea. The contrast between summer and winter could not be depicted, since there were no corresponding surface densities (lowest surface density in winter 102%, highest in summer, 100%). There is, however, sufficient spread to make separate seasonal tables worthwhile.
The third factor, geographical extent of a region, is harder to define. For the Vietnam area, the density profiles from the most northerly station (near 16°50' N) and the most southerly one (near 10°20' N) were so nearly alike that it was obvious that all of South Vietnam could be included in a single region. Data were not available from North Vietnam to delineate the northern extent of the region. It is most probable that the tables can be extended to 20°N and possibly to 25°N. The westward extension to the Bay of Bengal is also probable but must await further checking with data from upper air stations in that area.

For the Korea Region, the ballistic densities in the southern portion (near Pusan) differ markedly from those in the north (around Pyongyang). The separation is most pronounced in winter (Figure 13) where variations of more than 1% are possible between the extreme limits of the Korean area.
The one-sigma values shown as vertical lines between the northern and southern curves indicate that the differences are significant between lines 3 and 7. For that reason, three separate tables were constructed - one for the southern islands to 36°N; the second valid between 36°N and the DMZ; and the third to be used from the DMZ to 40°N.

Construction of the Tables

Once the differential criteria for the tables are established, the actual construction largely depends on the availability of the data. Where large numbers of Artillery Metro Sections are in operation as in South Vietnam, the problem is simplified. In fact, zone and ballistic densities are available on Forms DA 6-57 and DA 6-59 for processing by computer. Frequency distributions of upper air densities as a function of surface density are produced, and tables are constructed for surface density steps of 0.5%. Abrupt changes in slope of the profiles are avoided both from zone to zone for a given surface density and for a given zone as the surface density varies. Such smoothing rarely changes the tabular values by more than 0.2 - 0.3%, much less than the standard deviations of the values themselves.

Where sources other than Artillery Metro Sections are used, radiosonde data are available only at mandatory or significant levels. In those cases, the densities must be computed from the sounding (see Appendix). After this has been completed, the procedure follows that is stated above.
for the Vietnam data. Recognizing that variations in station altitude cause the greatest difference in density profiles (see Figures 3 and 7), the original tables in TR 1236-1 specified density factors for sea level, 1000' above MSL, and 2000' above MSL. A similar procedure, incorporating metric rather than English units, was followed for the new Vietnam tables as sufficient data were available. Tables were constructed for stations whose altitudes lay between sea level and 200 meters†† Another set was valid for those stations in the range of elevation 200 to 450 meters, the next 450 - 650 meters, etc. This procedure has the advantage of allowing for discrimination between density profiles due to elevation while keeping the number of tables within reason. In Vietnam, five sets of tables covered all operational station elevations.

The question naturally arises: Can a density profile be constructed for an elevated station if a nearby sea-level station is available? A special series of Vietnam flights was made every three hours for a week at a pair of locations separated by approximately 20 kilometers, one 30 meters above sea level, the other 770 meters above sea level. These flights were ideal for determining if ballistic densities from the station near sea level could be used to construct a density profile for the elevated location.

A simplistic approach was attempted. The difference between the surface densities of the two stations was computed and the difference subtracted from the ballistic densities of the sea-level station message. The resulting values were compared with the actual ballistic densities from the higher station. The results are shown in Table IV below.

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Density error in percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.3</td>
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<tr>
<td>2</td>
<td>.4</td>
</tr>
<tr>
<td>3</td>
<td>.5</td>
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<tr>
<td>4</td>
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<td>5</td>
<td>.6</td>
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<td>6</td>
<td>.7</td>
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<td>7</td>
<td>.7</td>
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<tr>
<td>8</td>
<td>.8</td>
</tr>
<tr>
<td>9</td>
<td>.9</td>
</tr>
</tbody>
</table>

It is readily seen that the error increases with height and becomes greater than 0.5% above line five. More complex schemes were tried. The most successful involved subtracting the sum of the difference in

††It is believed that by presenting the density for each zone as a tabular value, rather than a departure from the mean (which involves algebraic addition before the ballistic density is obtained), arithmetic errors will be reduced.
surface densities and one tenth of the line number from each ballistic density at sea level to give the corresponding ballistic density at 770 meters. The results of the computations are shown in Table V.

TABLE V
ERROR IN BALLISTIC DENSITY

<table>
<thead>
<tr>
<th>Line No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>.3</td>
<td>.4</td>
<td>.5</td>
<td>.6</td>
<td>.4</td>
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<td>.6</td>
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</tr>
<tr>
<td>error in percent</td>
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</tr>
</tbody>
</table>

The errors for high line numbers have been reduced, but the results are still not encouraging.

Neither of the two techniques is as accurate as the results obtained using the new climatological density tables as shown in Table VI.

TABLE VI
DENSITY ERRORS USING CLIMATOLOGICAL TABLES

<table>
<thead>
<tr>
<th>Line No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>.2</td>
<td>.2</td>
<td>.3</td>
<td>.3</td>
<td>.4</td>
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<td>.4</td>
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<td>.4</td>
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<tr>
<td>error in percent</td>
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</tr>
</tbody>
</table>

The mean error is seen to be less than 0.5% in all cases and does not continuously increase with height. It appears, therefore, that ballistic densities from a climatological table are superior to values obtained from extrapolating density profiles from a sea-level station.

It has been shown that climatological tables are feasible, seem to be reasonably accurate in one sample case investigated, and are easy to use. It remains to be seen whether they are accurate for a much larger sample and whether they are better or worse than the existing tables.

Since only the data from Artillery Metro Sections in Vietnam are available for direct comparison of tabular and actual ballistic densities, the analysis will be confined to those data. It should be noted that the absolute value of the error is the important factor in the application of the ballistic density to an actual firing. The resulting effects from a shell whose trajectory is 150 meters too short, for example, are
just as bad as those from a shell whose trajectory is 150 meters too long. The algebraic error is a measure of the amount of bias in the tables and indicates whether the tables could be improved by changing the individual values to eliminate the bias. Ideally the algebraic error should be zero, but, owing to the inherent inaccuracies in the determination of density, small biases of a few tenths of a percent appear. These, however, are insignificant.

Tables VII and VIII show sample distribution of errors for the new Vietnami tables. Shown are the errors for the summer daytime and the winter nighttime tables for stations below 200 meters elevation.

<table>
<thead>
<tr>
<th>MONTHS</th>
<th>ELEVATION</th>
<th>0 TO 200 METERS</th>
<th>48 HOURS TO 1440 HOURS</th>
</tr>
</thead>
<tbody>
<tr>
<td>+12</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>+11</td>
<td>0</td>
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<td>+10</td>
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<tr>
<td>+9</td>
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<td>-7</td>
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<td>-8</td>
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<tr>
<td>-10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Tables VII and VIII**

**DISTRIBUTION OF ERRORS**

**Vietnam Tables**

**TABLE VII**

It can be seen that the mean absolute error is 0.5% or less for all lines in both seasons and the bias error is not significant. All seasons and altitude ranges are similar, although the number of data points decreases markedly with increasing elevation.

Table IX shows a corresponding error distribution using the density departure tables in FM 6-16 for region 6. Since the older tables are in-

**2005 DATA POINTS**

* Values in this column represent density errors X 10

**DISTRIBUTION OF ERRORS**

**NEW VIETNAM TABLES**

**TABLE VIII**

**TABLE IX**
tended for use in all seasons, there is no differentiation for summer and winter, and the tables include all data for all months.

It is obvious that the absolute errors are greater and that there is a significant bias in many zones. Note especially line II, which has an average error of more than 2%. It can be seen that fewer than 2000 of the 4334 values in line II have errors less than 2.0%, all the others are off range as far as the table goes. It is all the more surprising that line 10 is excellent, with mean errors as low as lines 1 and 2, and no significant bias!

If one wishes to compare the tables on a yearly basis, the seasonal errors can be combined to give an annual figure. Table X shows the breakdown into four categories of error.

Cases with errors from 0 to 0.5% can be classified excellent, those from 0.6% to 1.0% can be called fair; those from 1.0% to 1.5%, poor; and greater than 1.5% would be disastrous.
TABLE X

ANNUAL DISTRIBUTION OF DENSITY ERRORS (SEA-LEVEL STATIONS)

<table>
<thead>
<tr>
<th>Percent Error</th>
<th>New Tables</th>
<th>Old Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Day</td>
<td>Night</td>
</tr>
<tr>
<td>0 - 0.5</td>
<td>72.6%</td>
<td>77.1%</td>
</tr>
<tr>
<td>0.6 - 1.0</td>
<td>22.4%</td>
<td>19.2%</td>
</tr>
<tr>
<td>1.1 - 1.5</td>
<td>4.3%</td>
<td>3.3%</td>
</tr>
<tr>
<td>&gt; 1.5</td>
<td>0.7%</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

We see that the number of cases of very poor assessment of ballistic density when the new tables are used is less than one-tenth the number with the old tables; the probability of obtaining an excellent ballistic density has increased by more than 20% and an acceptable density profile is obtained in 95 of every 100 cases. There is no doubt that the tables offer an excellent back-up system for obtaining ballistic densities in Vietnam in the absence of a direct radiosonde observation.

The lack of a direct measurement may be due to outage of equipment, communication difficulties, or it may be that it is the period between scheduled balloon releases, usually every six hours in Vietnam.

Earlier studies have shown that the variation of ballistic density in a 1-to-8-hour period is not significant [5]. Should the tables be used when the latest metro message is 3, 4, 5, or 6 hours old?

To answer this question one must examine the diurnal variation of density. The special 3-hourly data from Vietnam (mentioned earlier) are ideal for this analysis. Even though the period is short, about one week, the changes in density throughout the day will give some clue as to those periods in which the temporal variation is greatest. Figures 14 and 15 show the density for the various ballistic zones throughout the day for the station near sea level and the elevated location. The shape of both the curves is the same although the amplitude of the density fluctuation is greater at the lower station. About 2100Z (0500LST), well before sunrise, the surface and ballistic densities decrease rapidly, fluctuating between 0300Z and 0900Z (1100-1700LST) to a minimum a little before 0900Z. The decrease over this 6-hour period is greater than 3.0% at sea level and more than 2.0% at elevated stations.

**FIG. 14** DIURNAL VARIATION OF BALLISTIC DENSITY
VIETNAM STATION 142080 ELEVATION 30 METERS
GMT + GREENWICH MERIDIAN TIME (Z)

**FIG. 15** DIURNAL VARIATION OF BALLISTIC DENSITY
VIETNAM STATION 140080 ELEVATION 770 METERS
GMT + GREENWICH MERIDIAN TIME (Z)
Thus, the ballistic density obtained from a sounding made at 0000Z and used at 0500Z (before a new measurement is customarily made) would be in error by more than 1%. Similarly, a sounding made at 0600Z and used at 1100Z would contain an error almost as great, whereas the ballistic densities from a 1200Z sounding used at 1700Z and from a 1800Z sounding used at 2300Z would be in error by less than 1%. While these values are for the tropics, similar curves, although with different amplitudes, obtain for the other climatic regions. This means that the time of day is the determining factor for the time variability of ballistic density for periods of less than 6 hours.

To decrease the error in ballistic density in the interval between soundings, the curves of Figures 14 and 15 may be used to correct the earlier measurement. Even though the daily curves of ballistic density variation may not coincide absolutely, the trend is the same. Thus a diminishing of the 0000Z ballistic density by 1% for use at 0300Z would give a much better estimate of the density than the use of the uncorrected 0000Z value. A table could be constructed giving such corrections for each measurement hour of the day and the time of usage.

This could be supplied to the Fire-Direction Center to decrease the range error due to improper ballistic density corrections. A sample of such a table for one location in Vietnam is shown in Table XI. While both location and altitude are different from Figures 14 and 15, the trend is the same.

Corrections for Ballistic Density -- Line 1
Station 11405Z (Vietnam)

<table>
<thead>
<tr>
<th>Standing Time (LT)</th>
<th>00</th>
<th>01</th>
<th>02</th>
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</tr>
</tbody>
</table>

: apply appropriate correction to measured Ballistic Density to get corrected value for use at firing time.

TABLE XI

23
CONCLUDING REMARKS

In this report, the development of ballistic density departure tables has been traced over the last three decades from their earliest beginnings, where elevation of the battery was the only parameter considered, through the promulgation of climatic regions where geography and time of day were added, to the latest tables that are constructed for a limited geographical area far smaller than the previous regions in FM 6-16.

In addition, the possibilities for minimizing interdiurnal density variations have been pointed out, as well as impossibility of predicting ballistic densities at elevated locations with sufficient accuracy for Artillery purposes from measurements at sea level.

The new tables constructed from individual radiosondings have been shown to be much more accurate than the older ones derived mainly from mean values at selected locations.

Use of the tables can effect a considerable savings in expendables, as wind measurements can be made every two hours by tracking a balloon alone (without the more costly radiosonde), since the density can be corrected by the tables and associated techniques.
APPENDIX

COMPUTER DETERMINATION OF METRO MESSAGES
FROM RADIOSONDE DATA

By

Hope S. Perlman
U. S. Army Electronics Command

Computer reduction of radiosonde flights to give thermodynamical quantities is common where a large number of flights must be handled or where checking of manual operations is required in data repositories. Less common is the computer determination of NATO or computer meteorological messages since ballistic meteorology is less extensively researched than pure synoptic meteorology.

Secondly, the recent change in the format of the computer met message has necessitated a change in the reported quantities. That, in turn, requires a program change in any computer routine that produces the artillery met messages.

This paper outlines the procedure for computer determination of ballistic and computer met messages, with especial detail given to the evaluation of the two new elements of the computer message - mean virtual temperature and pressure at the midpoint of the zone.

Procedures

Since data for research purposes is most often obtained from World Data Centers, the analysis begins with the data in the usual card (or tape) format of the mandatory pressure levels - the customary way in which data are archived. It must be remembered that heights on the cards are expressed above mean sea level (MSL). Since artillery met data are concerned with heights above ground, the elevation of the station must be subtracted from all given heights of the standard pressure levels. That is the reason for Steps 1 and 3(c) below. A complete computer program is available upon request.

Method of Solution

1. Read in surface height \( S_z \) of station.

2. Read in complete data for one flight. This will include \( n \) sets of pressure \( P \), height \( Z \), temperature in \(^\circ C(T)\) and relative humidity in \( %H \).
3. Adjust data and number of levels (n) as follows:
   a. If \( H < 0 \), set it equal to zero.
   b. Eliminate any level where temperature is missing.
   c. Subtract \( S_Z \) from all values of \( Z \).

4. Calculate virtual temperature, in °K, \((T_V)\) at each level, as follows:
   \[
   T_V = \frac{P(T+273)}{P - (.0037812)(H)(6.105)\exp\left(\frac{25.22T}{T+273} - 5.31 \ln \frac{T+273}{273}\right)}
   \]  
   (1)

5. Print out adjusted data \((P, Z, T, H, T_V)\) for each level.

6. Set up ballistic zone top heights \((ZLT)\) and standard temperatures \((ZLTMS)\) and densities \((ZLDNS)\) at these heights.

7. Print all surface information.

8. For each zone \((ZL)\) compute \(ZZTM\), zone temperature and \(ZZPR\), pressure at the top of the zone, using intermediate data level values of \(P, Z\) and \(T_V\), indexed from 0, the bottom of the zone, to \(d\), the top of the zone.
   a. Obtain a starting value of \(T_{vd}\), virtual temperature at the top of the zone, by interpolating linearly, using values of
      \(T_{vd+1}, T_{vd-1}, Z_{d+1}, Z_d, \) and \(Z_{d-1}\).
   b. \(ZZTM\) is calculated by selecting a starting value \((1/2\) the sum of the maximum and minimum of the \(T_V\) values within the zone), and moving it toward the maximum or minimum until
      \[
      1/2 \sum_{i=0}^{d-1} (Z_{i+1} - Z_i) (T_{Vi} + T_{Vi+1} - 2ZZTM)^{0*}
      \]  
      (2)
   In practice, the process is continued until two successive values of \(ZZTM\) differ by less than .1°K.
   c. \(P_d\) is found by solving the transcendental equation
      \[
      29.27 ZZTM \ln \left(\frac{P_0}{P_d}\right) = Z_n - Z_o
      \]  
      (3)

* The rationale for Equation (2) is found in Appendix I.
d. A new value of $T_{vd}$ is computed by interpolating linearly, using $T_{vd-1}$, $T_{vd+1}$, $P_{d-1}$, $P_d$, and $P_{d+1}$.

e. Steps b, c, and d are repeated until two successive values of ZZTM, calculated in step d differ by .2°K or less.

f. $ZZPR = P_d$  \hfill (4)

9. For each zone level, compute

a. $ZZ TMP_{ZL}$ zone temperature (% of standard)

$$ZZ TMP_{ZL} = \frac{100(ZZ TM_{ZL})}{ZZ LM_{ZL}} \hfill (5)$$

b. $ZZ PR_{ZL}$, pressure at the midpoint of the zone, is found by solving the following transcendental equation:

$$29.27(ZZ TM_{ZL}) \ln \left(\frac{ZZ PR_{ZL}}{ZZ PR_{ZL-1}}\right) = 1/2 \left(Z_d - Z_0\right) \hfill (6)$$

c. $ZZ DN_{ZL}$, zone density

$$ZZ DN_{ZL} = 348.384 \frac{ZZ PR_{ZL}}{ZZ TM_{ZL}} \hfill (7)$$

d. $ZZ DN_{ZL}$, zone density in percent of standard

$$ZZ DN_{ZL} = \frac{100(ZZ DN_{ZL})}{ZZ LNS_{ZL}} \hfill (8)$$

10. For each zone, calculate ballistic weighted temperature ($ZZ BT_{ZL}$) and ballistic weighted density ($ZZ BD_{ZL}$), using temperature weighting factors (TWF) and density weighting factors (DWG) for that zone.
APPENDIX I

CALCULATION OF ZONE TEMPERATURE

Given: $T_{V_0'}$, $Z_0'$, $T_{V_1}$, $Z_1$ ---- $T_{V_d'}$, $Z_d'$, where the zero subscript denotes the bottom of the zone and the d subscript the top of the zone.

Find: ZZTM, the zone temperature, such that the areas to the right of ZZTM equal the areas to the left of ZZTM, or ZZTM is correct to .1°K.

Analysis:

1. Between any successive reading of $T_{V_i}$, $Z_i$ and $T_{V_{i+1}}$, $Z_{i+1}$ the figure is either a trapezoid or 2 triangles.
   a. If a trapezoid, Area = 1/2 h (a+b)
      
      \[= \frac{1}{2} (Z_{i+1} - Z_i) \left[ (T_{V_{i+1}} - ZZTM) + (T_{V_i} - ZZTM) \right] \]
      
      \[= \frac{1}{2} (Z_{i+1} - Z_i) (T_{V_i} + T_{V_{i+1}} - 2ZZTM) \]
      
      and will be positive or negative depending on whether $T_{V_i}$ and $T_{V_{i+1}}$ are greater or less than ZZTM.
   b. If two triangles, the area equals area of triangle
      \[(T_{V_i}, Z_i) (T_{V_{i+1}}, Z_{i+1}) (T_{V_{i+1}}, Z_i)\]
      
      minus area of rectangle
      \[(ZZTM, Z_i) (ZZTM, Z_{i+1}) (T_{V_{i+1}}, Z_{i+1}) (T_{V_{i+1}}, Z_i)\]
or \( \frac{1}{2} (Z_{i+1} - Z_i) (T_{vi} - T_{v_i+1}) = (Z_{i+1} - Z_i) (Z_{TM} - T_{v_i+1}) \)

*the area of the trapezoid equals the area of the two triangles, and is

\[
\frac{1}{2}(Z_{i+1} - Z_i) (T_{vi} + T_{v_i+1} - 2Z_{TM})
\]

2. \( Z_{TM} \) is calculated by selecting a starting value of \( Z_{TM} \) (half the sum of the maximum and minimum values of the \( T_{vi}'s \)) and moving it \( \Delta T \) toward \( T_{vi} \) min until

\[
\frac{1}{2} \sum_{i=0}^{d-1} (Z_{i+1} - Z_i) (T_{vi} + T_{v_i+1} - 2Z_{TM}) \approx 0.
\]
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