IGNITION OF A DROP OF FUEL

by

G. A. Varshavskiy, D. V. Fedoseyev, and
A. D. Frank-Kamenetskiy

Country: USSR

This document is a rendition of the original foreign text without any analytical or editorial comment.

Approved for public release; distribution unlimited.
The point of ignition and ignition delay period are determined for a drop of radius \( R_0 \) placed in a medium with temperature \( T_0 \), assuming that no combustion of fuel vapors occurs before the moment of ignition, \( Nu=2 \), the time of thermal and concentration relaxation is infinitesimal, the Lewis-Semenov criterion is equal to one and that all heat physical quantities are independent of temperature.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ROLE</td>
<td>WT</td>
<td>ROLE</td>
</tr>
</tbody>
</table>

Ignition
Ignition Test
Chemical Reaction
Combustion

Subject Code: 07
Country Code: UR
ENGLISH TITLE: Ignition of a Drop of Fuel

FOREIGN TITLE: Vosplameneniye Kapli Topliva

AUTHOR: G. A. Varshavskiy, D. V. Fedoseyev, A. D. Frank-Kamenetskiy


Translated for FSTC by Albert L. Peabody
LEO KANNER ASSOCIATES

NOTICE

The contents of this publication have been translated as presented in the original text. No attempt has been made to verify the accuracy of any statement contained herein. This translation is published with a minimum of copy editing and graphics preparation in order to expedite the dissemination of information. Requests for additional copies of this document should be addressed to Department A, National Technical Information Service, Springfield, Virginia 22151. Approved for public release; distribution unlimited.
The basic quantities usually determined during the process of ignition are the point of ignition and the ignition delay period. We will seek these quantities for a drop of radius \( r_0 \), placed in a medium with temperature \( T_m \) under the following assumptions: 1) combustion of fuel vapors does not occur until the moment of ignition (similar to the assumption of O. M. Todes in the unstable theory of thermal ignition); 2) \( Nu=2 \), which is generally incorrect, since at the initial moment in time \( Nu \approx 0 \), after which it rapidly approaches its stable value of 2; 3) the times of thermal and concentration reaction are infinitesimal (that is the problem is solved in the quasistable statement); 4) there is similarity between the fields of excess concentration and temperature (i.e., the Lewis-Semenov criterion is equal to one); 5) all heat physical quantities are independent of temperature.

The quantity of heat used to heat the drop is significantly greater than the quantity of heat used for evaporation as long as the drop is far from the point of equilibrium evaporation. Therefore, heating of the drop can be analyzed with some approximation without considering evaporation. Assuming that the temperature of the drop is constant through its radius but variable with time, we produce, considering the assumptions made,

\[
\frac{T}{T_0} = 1 - \left( 1 - \frac{T}{T_0} \right) \exp \left( - \frac{3\sigma_ho}{kT_0} \right).
\]

Here \( T_0 \) is the initial temperature of the drop, \( T_1 \) is the (instantaneous) temperature of the drop, \( k \) is the heat conductivity coefficient, \( c_p \) and \( \rho \) are the heat capacity and density of the drop, \( t \) is time. In order to produce the distribution of temperature in space, we shall use the fundamental solution of the stable heat conductivity equation for the spherical case:

\[
\frac{T_r - T_i}{T_0 - T_i} = \frac{r_r}{r_i}.
\]
Considering equations (1) and (2), we produce, in the quasistable approximation

\[ \frac{\dot{T}}{T} = 1 - \frac{r_1}{r_2} \theta_\phi \exp \left( - \frac{3\alpha r}{\dot{r}_2 \theta_\phi} \right), \]  \hspace{1cm} (1.3)

where

\[ \theta_\phi = \left( 1 - \frac{r_2}{r_1} \right). \]

According to the fourth assumption made above

\[ \frac{T - T_1}{T_2 - T_1} = \frac{P_1 - P_1^0}{P_0^1}. \]

Here \( P_1 \) is the partial pressure of fuel vapors, a function of the coordinates and time, while \( P_{10} \) is the partial pressure of the fuel vapors on the surface of the drop, determined by the saturated vapor pressure at temperature \( T_1 \). Therefore, the following expression can be produced for the distribution of fuel vapor pressure in space at any moment in time:

\[ P_1 = \frac{H}{R T_2} \frac{r_1}{r} \exp \left\{ \frac{H}{R T_1} \left[ 1 - \frac{H}{R T_2} \exp \left( - \frac{3\alpha r}{\dot{r}_2 \theta_\phi} \right) \right] \right\}. \]  \hspace{1cm} (1.4)

Here \( H \) is the molar heat of evaporation of the fuel, a function of temperature and pressure, \( P \) is the total pressure, \( T_2 \) is the boiling point of the liquid.

Ignition will occur at the point of the local maximum of the rate of the chemical reaction. In all of the following, we will analyze the second order chemical reaction. Actually, at infinity the temperature is great, but the fuel vapor pressure is insufficient. The level of temperature near the surface of the drop is insufficient. Furthermore, the greatest number of collisions leading to formation of the end products of the chemical reaction will occur with the stoichiometric ratio. Therefore, we can write a system of two equations defining the ignition of evaporating liquid fuel:

\[ \frac{\partial W}{\partial r} = 0, \]  \hspace{1cm} (1.5)

\[ P_1 = \phi P_0, \]  \hspace{1cm} (1.6)

where \( P_2 \) is the partial pressure of the oxidizer (and inert gases);

\( \phi \) is the stoichiometric coefficient,

the rate of the chemical reaction.

For a second order reaction, we can approximately produce, in place of what (1.5) and (1.6)
where \( E \) is the activation energy of the chemical reaction.

Substituting the distributions of temperature and pressure found earlier into (1.7), we can find the ratio of the radius of ignition to the radius of the drop in the \( t \to 0 \) approximation:

\[
\frac{r_i}{r_1} = \frac{E}{RT_e} \left( 1 - \frac{T_1}{T_2} \right). \tag{1.9}
\]

Following certain calculations, (1.3) leads us to the expression

\[
\frac{H}{RT_e} - \frac{H}{RT_e} \left[ 1 - \frac{3T_2}{2RT_e} \right] = \ln \frac{\tau_0}{RT_e} \tag{1.10}
\]

under the condition that \( \frac{E\theta}{RT} \gg 2 \). From this we can find the ignition delay time.

\[
\tau_0 = \frac{r_i}{H} \left[ 1 - \frac{1}{1 - \frac{3T_2}{2RT_e} \left( \frac{H}{RT_e} \ln \frac{\tau_0}{RT_e} \right) \right] \tag{1.10}
\]

It follows from (1.10) that as temperature \( T_\infty \) is increased, the ignition delay period decreases and as \( T_\infty \to \infty, \tau_0 \to 0 \). As the pressure in the system increases, the magnitude decreases, which in turn decreases \( \tau_0 \). Thus, the relationship with pressure is concealed in the heat of evaporation. Formula (1.10) can be significantly simplified if we assume that

\[
\frac{RT_e}{8H} \ln \frac{\tau_0}{RT} \ll 1.
\]

Actually, for octane, for example, \( T_2 = 400^\circ K, H = 10^6 \) cal/mol, \( \phi \approx 1, \) \( E = 20000 \) cal/mol. Then where \( \phi_0 \approx 1 \) and \( T_\infty = 1100^\circ K \) we have

\[
\frac{RT_e}{H} \ln \frac{F_0}{RT_\infty} = 0.17 \ll 1. \text{ In this case}
\]

\[
\tau_0 = \frac{r_i}{H} \left( T_2 - T_1 \right).
\]

-3-
If we assume $T_o=300^\circ K$, we find using (1.11) for octane with $2r_0=0.1$ cm, $c_p=0.5$ cal · g · deg, $\rho=0.8$ g/cm$^3$, $k=8.10^{-5}$ cal/cm·sec·deg $r*=0.44$ sec and $r_1/r_o=8.6$.

We can see from the expressions produced that the ignition delay period depends strongly on droplet size. The dependence on temperature is weaker than that usually observed experimentally, which may be related to the assumption of independence of $c_p$ and $k$ on temperature.

We note that in the quasistable approximate analysis presented above, the results produced are independent of the preexponential factor of the chemical reaction. This is explained by the use of conditions (1.5) and (1.6).

For comparison with the approximate solution produced, the system of equations of diffusion and heat conductivity was numerically calculated in the form

\begin{align*}
\frac{\partial c_1}{\partial t} &= D \left( \frac{\partial^2 c_1}{\partial r^2} + \frac{2}{r} \frac{\partial c_1}{\partial r} \right) + \kappa_1 c_1 c^2 - \frac{E}{RT} \\
\frac{\partial T}{\partial t} &= \alpha \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) - \frac{q}{\rho c_p R^2 T^2}
\end{align*}

with the initial conditions

\begin{align*}
T(r>r_o, 0) &= T_0 \\
c_1(r, 0) &= 0
\end{align*}

1.13

and boundary conditions

\begin{align*}
T(\infty, t) &\to T_0 \\
c_1(\infty, t) &\to 0. \\
T_s &= T_0 \left[ 1 - \delta_s \exp \left( -\frac{3\kappa_1}{r^2_o r^2_0} \right) \right] \\
c_{10} &= \frac{p_s}{R T_s} \exp \left[ \frac{H}{K} \left( \frac{1}{T_s} - \frac{1}{T_1} \right) \right].
\end{align*}

1.14

Here $c_1$ is the fuel concentration, $c_{10}$ is the concentration of fuel of the surface of a drop, $c_2$ is the concentration of oxidizer, $k_o$ is the preexponential factor, $q$ is the heat value of the fuel.

The following values of dimensional quantities were selected for calculation:

\begin{align*}
&\begin{array}{c}
 r_0=10^{-1} \text{ cm; } T_0=300^\circ \text{K; } T_2=400^\circ \text{K; } T_\infty = 1600^\circ \text{K; }
\end{array} \\
&\begin{array}{c}
 E=4.10^4 \frac{\text{cal}}{\text{mol}}; \\
 k_o=10^{10} \frac{\text{cm}^3}{\text{mol} \cdot \text{sec}}; \\
 a=D=0.1 \frac{\text{cm}^2}{\text{sec}}; \\
 k=8.10^{-5} \frac{\text{cal}}{\text{cm} \cdot \text{sec} \cdot \text{deg}}.
\end{array}
\end{align*}
Calculation was performed by the grids method using the M-20 computer. The calculations showed that the increase in temperature begins at $t=1.385$ sec at 9 radii from the center of the drop. Then the temperature begins to increase significantly, and the temperature maximum approaches the drop.

If we compare the values of $r_1$ and $r_1$ produced with the values calculated according to the approximate formulas (1.9) and (1.11), we produce:

\[
\frac{r_1}{r_0} = 10.6 \\
\tau = 2.5 \text{ sec.}
\]

The correspondence between approximate and precise calculation must be considered quite satisfactory.