REAL-TIME SIMULATION PROGRAM FOR
DE HAVILLAND (CANADA) "BUFFALO"
AND "TWIN OTTER" STOL TRANSPORTS

R. A. MacDONALD, MEL GARELICK, J. HAAS
TRANSPORTATION SYSTEMS CENTER
55 BROADWAY
CAMBRIDGE, MASS. 02142

TECHNICAL NOTE

Availability is Unlimited. Document may be Released
To the National Technical Information Service,
Springfield, Virginia 22151, for Sale to the Public.

Prepared for
DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
WASHINGTON, D.C. 20590
The contents of this report reflect the views of the Transportation Systems Center which is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policy of the Department of Transportation. This report does not constitute a standard, specification or regulation.
# Abstract

Simulation models of two representative STOL aircraft - the DeHavilland (Canada) "Buffalo" and "Twin Otter" transports - have been generated.

The aircraft are described by means of non-linear equations that will accommodate gross changes in angle of attack, pitch angle, flight path angle, velocity, and power setting. Aircraft motions in response to control inputs and external disturbances are related to Earth-fixed coordinates. The equations are programmed to run in "real time" so that they can be used in conjunction with a manned cockpit simulator. Provisions are made for pilot control inputs to the simulation, and conventional panel display parameters are generated.

The report includes representative simulation results which demonstrate that the simulation is an adequate representation of the two STOL aircraft being modeled.

# Key Words

Aircraft Math Model
STOL Aircraft Stability and Control; Aircraft Simulation

# Distribution Statement

Availability is Unlimited. Document may be Released To the National Technical Information Service, Springfield, Virginia 22151, for Sale to the Public.

# Security Classif. (of this report)
Unclassified

# Security Classif. (of this page)
Unclassified

# No. of Pages
57

# Price
$3.00
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Symbols</td>
<td>i</td>
</tr>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Description of Mathematical Model</td>
<td>3</td>
</tr>
<tr>
<td>A. Definition of Reference Coordinate Frames</td>
<td></td>
</tr>
<tr>
<td>B. Velocity Solutions</td>
<td></td>
</tr>
<tr>
<td>C. Provision for Atmospheric Disturbances (Winds)</td>
<td></td>
</tr>
<tr>
<td>D. Airframe Equations of Motion</td>
<td></td>
</tr>
<tr>
<td>E. Definition of Required Display Quantities</td>
<td></td>
</tr>
<tr>
<td>III. Tabulation of Numerical Data for &quot;Buffalo&quot; and &quot;Twin Otter&quot;</td>
<td>18</td>
</tr>
<tr>
<td>IV. Simulation Program</td>
<td>21</td>
</tr>
<tr>
<td>A. Interface with GAT-1 Cockpit</td>
<td></td>
</tr>
<tr>
<td>B. Definition of Initial Values of Variables</td>
<td></td>
</tr>
<tr>
<td>V. Simulation Results</td>
<td>24</td>
</tr>
<tr>
<td>References</td>
<td>26</td>
</tr>
<tr>
<td>Table I</td>
<td>27</td>
</tr>
<tr>
<td>Figures</td>
<td>35</td>
</tr>
<tr>
<td>Appendix</td>
<td>A1</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Aircraft lift curve slope</td>
<td>rad⁻¹</td>
</tr>
<tr>
<td>AR</td>
<td>Aspect ratio of wing = ( b^2/S )</td>
<td></td>
</tr>
<tr>
<td>B_nm</td>
<td>Elements of A-frame to I-frame transformation matrix</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Wing span</td>
<td>ft</td>
</tr>
<tr>
<td>c</td>
<td>Mean chord of wing</td>
<td>ft</td>
</tr>
<tr>
<td>CD</td>
<td>Aircraft drag coefficient</td>
<td></td>
</tr>
<tr>
<td>C_f</td>
<td>Aircraft parasite drag coefficient</td>
<td></td>
</tr>
<tr>
<td>ΔCD</td>
<td>Aircraft drag coefficient less wing drag coefficient</td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>Aircraft lift coefficient</td>
<td></td>
</tr>
<tr>
<td>C_LO</td>
<td>Trimmed aircraft lift coefficient</td>
<td></td>
</tr>
<tr>
<td>C_m_t</td>
<td>Pitching moment coefficient which may be made variable to shape trim ( \delta_e ) vs ( V_R ) curve (( C_m_t = 0 ) in this report)</td>
<td></td>
</tr>
<tr>
<td>C_m_q</td>
<td>Pitching moment coefficient due to pitch rate</td>
<td></td>
</tr>
<tr>
<td>C_m_α</td>
<td>Pitching moment coefficient due to angle of attack</td>
<td></td>
</tr>
<tr>
<td>C_m_α_δ</td>
<td>Pitching moment coefficient due to angle of attack rate</td>
<td></td>
</tr>
<tr>
<td>C_m_δ_e</td>
<td>Pitching moment coefficient due to elevator deflection</td>
<td></td>
</tr>
<tr>
<td>CL_p</td>
<td>Rolling moment coefficient due to roll rate</td>
<td></td>
</tr>
<tr>
<td>CL_β</td>
<td>Rolling moment coefficient due to sideslip angle</td>
<td></td>
</tr>
<tr>
<td>CL_δ_a</td>
<td>Rolling moment coefficient due to aileron deflection</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$C_{r}$</td>
<td>rolling moment coefficient due to yaw rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{r_{\text{fin}}}$</td>
<td>fin contribution to rolling moment coefficient due to yaw rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{n_{p}}$</td>
<td>yawing moment coefficient due to roll rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{n_{\text{fin}}}$</td>
<td>fin contribution to yawing moment coefficient due to yaw rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{n_{r}}$</td>
<td>yawing moment coefficient due to yaw rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{n_{\text{fin}}}$</td>
<td>fin contribution to yawing moment coefficient due to yaw rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{n_{\beta}}$</td>
<td>yawing moment coefficient due to sideslip angle</td>
<td>-</td>
</tr>
<tr>
<td>$C_{n_{\beta_{r}}}$</td>
<td>yawing moment coefficient due to rudder deflection</td>
<td>-</td>
</tr>
<tr>
<td>$C_{y_{p}}$</td>
<td>side force coefficient due to roll rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{y_{r}}$</td>
<td>side force coefficient due to yaw rate</td>
<td>-</td>
</tr>
<tr>
<td>$C_{y_{\beta}}$</td>
<td>side force coefficient due to sideslip angle</td>
<td>-</td>
</tr>
<tr>
<td>$C_{T_{1}}$</td>
<td>empirical coefficient in thrust equation</td>
<td>$\text{fps}^{-1}$</td>
</tr>
<tr>
<td>$C_{T_{2}}$</td>
<td>empirical coefficient in thrust equation</td>
<td>$\text{fps}^{-2}$</td>
</tr>
<tr>
<td>$D$</td>
<td>aircraft drag</td>
<td>lbs</td>
</tr>
<tr>
<td>$e$</td>
<td>aircraft efficiency factor</td>
<td>-</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational constant = 32.2</td>
<td>$\text{ft/sec}^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>altitude = $-z_{L}$</td>
<td>ft</td>
</tr>
<tr>
<td>$h_{\text{ATM}}$</td>
<td>characteristic density altitude of atmosphere</td>
<td>ft</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>i,j,k</td>
<td>unit vectors along the X, Y, and Z, axes of the () coordinate frame, respectively</td>
<td></td>
</tr>
<tr>
<td>IAS</td>
<td>indicated airspeed</td>
<td>mph</td>
</tr>
<tr>
<td>I_x, I_y, I_z</td>
<td>aircraft rolling, pitching, and yawing moment of inertia, respectively</td>
<td>slug-ft²</td>
</tr>
<tr>
<td>J_{xz}</td>
<td>product of inertia = ( \int x z , dm )</td>
<td>slug-ft²</td>
</tr>
<tr>
<td>L</td>
<td>aircraft lift</td>
<td>lbs</td>
</tr>
<tr>
<td>L,M,N</td>
<td>scalar component of the applied external moment along the X_A, Y_A', and Z_A axis, respectively</td>
<td>ft-lbs</td>
</tr>
<tr>
<td>L_P</td>
<td>rolling moment due to roll rate</td>
<td>ft-lbs/\text{rad/sec}</td>
</tr>
<tr>
<td>L_R</td>
<td>rolling moment due to yaw rate</td>
<td>ft-lbs/\text{rad/sec}</td>
</tr>
<tr>
<td>L_V</td>
<td>rolling moment due to sideslip velocity</td>
<td>ft-lbs/fps</td>
</tr>
<tr>
<td>L_{\delta_a}</td>
<td>rolling moment due to aileron deflection</td>
<td>ft-lbs/\text{rad}</td>
</tr>
<tr>
<td>1/m</td>
<td>l/aircraft mass</td>
<td>slugs⁻¹</td>
</tr>
<tr>
<td>N_P</td>
<td>yawing moment due to roll rate</td>
<td>ft-lbs/\text{rad/sec}</td>
</tr>
<tr>
<td>N_R</td>
<td>yawing moment due to yaw rate</td>
<td>ft-lbs/\text{rad/sec}</td>
</tr>
<tr>
<td>N_V</td>
<td>yawing moment due to sideslip velocity</td>
<td>ft-lbs/fps</td>
</tr>
<tr>
<td>N_{\delta_r}</td>
<td>yawing moment due to rudder deflection</td>
<td>ft-lbs/\text{rad}</td>
</tr>
<tr>
<td>P,Q,R</td>
<td>scalar components of the angular rotation vector of the aircraft along the X_A, Y_A', and Z_A axis, respectively</td>
<td>rad/sec</td>
</tr>
<tr>
<td>q</td>
<td>dynamic pressure</td>
<td>lbs/ft²</td>
</tr>
<tr>
<td>S</td>
<td>wing area</td>
<td>ft²</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>$T$</td>
<td>THRUST aircraft thrust</td>
<td>lbs</td>
</tr>
<tr>
<td>$-T_{TMDLE}$</td>
<td>elevator input delay (See Section IV-B)</td>
<td>sec</td>
</tr>
<tr>
<td>$-T_{TMTHR}$</td>
<td>throttle input delay (See Section IV-B)</td>
<td>sec</td>
</tr>
<tr>
<td>$T_{static}$</td>
<td>aircraft thrust at zero airspeed</td>
<td>lbs</td>
</tr>
<tr>
<td>$U,V,W$</td>
<td>$U,V,W$ scalar component of aircraft velocity along the $X_A$, $Y_A$, and $Z_A$ axis, respectively</td>
<td>fps</td>
</tr>
<tr>
<td>$U_{w},V_{w},W_{w}$</td>
<td>scalar component of aircraft with respect to airmass along the $X_A$, $Y_A$, and $Z_A$ axis, respectively</td>
<td>fps</td>
</tr>
<tr>
<td>$V_R$</td>
<td>VR resultant velocity of aircraft with respect to airmass</td>
<td>fps</td>
</tr>
<tr>
<td>$W$</td>
<td>WEIGHT aircraft weight</td>
<td>lbs</td>
</tr>
<tr>
<td>$X,Y,Z$</td>
<td>$X,Y,Z$ scalar component of the applied external non-gravitational force along the $X_A$, $Y_A$, and $Z_A$ axis, respectively</td>
<td>lbs</td>
</tr>
<tr>
<td>$X(),Y(),Z()$</td>
<td>axes defining the () coordinate frame</td>
<td></td>
</tr>
<tr>
<td>$X_L,Y_L,Z_L$</td>
<td>displacements along the respective axes of the L coordinate frame</td>
<td>ft</td>
</tr>
<tr>
<td>$X_{DOT},Y_{DOT},Z_{DOT}$</td>
<td>velocities along the respective axes of the L coordinate frame</td>
<td>fps</td>
</tr>
<tr>
<td>$X_{SS},Y_{SS},Z_{SS}$</td>
<td>steady state airmass velocity along the $X_L$, $Y_L$, and $Z_L$ axes, respectively</td>
<td>fps</td>
</tr>
<tr>
<td>$Y_P$</td>
<td>YP side force due to roll rate</td>
<td>lbs/rad/ sec</td>
</tr>
<tr>
<td>$Y_R$</td>
<td>YR side force due to yaw rate</td>
<td>lbs/rad/ sec</td>
</tr>
<tr>
<td>$Y_V$</td>
<td>YV side force due to sideslip velocity</td>
<td>lbs/fps</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle from the remote wind vector $V_R$ to the $X_A$ axis</td>
<td>rad</td>
</tr>
<tr>
<td>$\dot{\alpha}$</td>
<td>$\frac{d\alpha}{dt}$</td>
<td>rad/sec</td>
</tr>
<tr>
<td>$\alpha_B$</td>
<td>angle from the remote wind vector $V_R$ to the $X_B$ axis</td>
<td>rad</td>
</tr>
<tr>
<td>$\alpha_{B0}$</td>
<td>angle between the body-fixed $X_A$ and $X_B$ axes</td>
<td>rad</td>
</tr>
<tr>
<td>$\alpha_{B0L}$</td>
<td>value of $\alpha_B$ for which no lift is developed by the aircraft</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta$</td>
<td>aircraft sideslip angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>angle from the horizontal reference line to the remote wind vector $V_R$: $\gamma = \theta - \alpha$</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_a$</td>
<td>aileron deflection</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>elevator deflection</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_r$</td>
<td>rudder deflection</td>
<td>rad</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>(See definition of Euler angles $\psi, \theta, \phi$ below)</td>
<td></td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>angle from the horizontal reference line to the $X_B$ axis</td>
<td>rad</td>
</tr>
<tr>
<td>$\xi$</td>
<td>pilot throttle input as fraction of maximum input</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>atmospheric air density</td>
<td>slugs/ft$^3$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>atmospheric air density at sea level, std day</td>
<td>slugs/ft$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\rho/\rho_0$</td>
<td></td>
</tr>
<tr>
<td>$\psi, \theta, \phi$</td>
<td>Euler angles relating $L, C,$ and $A$ coordinate frames (further defined in Figure 3)</td>
<td>rad</td>
</tr>
</tbody>
</table>
SUBSCRIPTS

A  aircraft body coordinate frame
B  body reference coordinate frame
C  Earth-aircraft control coordinate frame
L  Earth local-vertical coordinate frame
cr design economy cruise condition
 o  equilibrium or reference condition
 OL zero lift value
I. Introduction

Simulation models of two representative STOL aircraft have been generated. The models are documented in this report.

The computer simulation is to be used as a tool in the development of STOL terminal area guidance and navigation systems.

This intended use has determined the form of the simulation: The aircraft are described by means of non-linear equations that will accommodate gross changes in angle of attack, pitch angle, flight path angle, velocity, and power setting. Aircraft motions in response to control inputs and external disturbances are related to Earth-fixed coordinates. The equations are programmed to run in "real time" so that they can be used in conjunction with a manned cockpit simulator. Provisions are made for pilot control inputs to the simulation, and conventional panel display parameters are generated.

The aircraft which are modeled - the DHC "Twin Otter" and the DHC "Buffalo" - are described in Figures 1 and 2, respectively. They were selected as representative light and medium propeller-driven STOL transports. Their selection does not imply that there are not other STOL aircraft representative of these classes. Similarly, the material contained in this report should not be used as the basis for an evaluation of the flying qualities of the "Buffalo" or "Twin Otter" or of the suitability of these aircraft for any specific mission.
The aircraft are modeled only to the extent necessary to yield a representative vehicle model controllable by a guidance or navigation system. Certain simplifying assumptions - specified in the following sections - are made. These assumptions are justified for the present model application but may render the model unsuitable for other possible applications.

The simulation is described in detail in the following sections of this report. In Section II, all required equations are developed. Section III tabulates numerical values to be used in these equations for the "Buffalo" and "Twin Otter".

The simulation program is presented in Section IV. A listing of all computer statements is included. Finally, in Section V, representative simulation results are shown. These results demonstrate that the simulation is an adequate representation of the two STOL aircraft.
II Description of Mathematical Model

The mathematical model consists of all equations required to describe the motions of the aircraft in space resulting from external disturbances, control inputs, and the aircraft's aerodynamic characteristics. These equations are presented in this Section. First, however, it is necessary to define the reference coordinate frames to be used.

IIA Definition of Reference Coordinate Frames

Reference coordinate frames to be used in this analysis are defined in this section. Insofar as possible, axis systems have been defined so that senses of rotation and translation are similar for small rotations. Positive force, moment, and motion vector components are defined to be in the positive sense of the axis. To the largest extent possible, the symbols and conventions used are consistent with those in common usage in the guidance and control fields and with those used by NASA for aircraft stability and control work.

The Earth Local-Vertical Frame \( (L) \) is a local geographic frame. Its origin is fixed at a point on the Earth's surface with \( Z_L \) along the vertical defined by the local gravity vector (positive downward), \( X_L \) parallel to geographic North (positive to the North), and \( Y_L \) parallel to geographic East (positive to the East).

The Aircraft Body Coordinate Frame \( (A) \) is fixed to the aircraft and translates with the aircraft. Its origin is the center of mass of the aircraft. The \( X_A \) axis is chosen in a forward direction in the plane of symmetry that
is parallel to the initial or equilibrium direction of the remote wind. Thus the A-frame axes, by the commonly accepted definition, are "stability axes". Because the $X_A$ axis is initially aligned with the remote wind, the initial angle of attack $\alpha(0) = \alpha_0$ is zero. (In this report, $\theta$ and $\alpha$ when not subscripted to indicate reference frame, are assumed to be referenced to the A-frame. Further, since in the simulation documented in this report the aircraft is placed in equilibrium at $t = 0$, "equilibrium" and "initial" conditions are equivalent.)

The $Y_A$ axis is normal to the aircraft's plane of symmetry (positive to the right), and the $Z_A$ axis is in the plane of symmetry (positive downward) and orthogonal to the $X_A$ and $Y_A$ axes. The A-frame is related to the L-frame (and to the next-defined C-frame) in Figure 3.

The Earth-Aircraft Control Coordinate Frame (C) is also centered at the center of mass of the aircraft. The $Z_C$ axis is aligned with the local gravity vector (positive downward) and is therefore parallel to the $Z_L$ axis. The $X_C$ axis is the intersection of the horizontal plane with the vertical plane containing the $X_A$ axis. The $Y_C$ axis completes the orthogonal right-hand system. The C-frame is an intermediate frame needed to define the Euler angles describing the relationship between the Earth local-vertical (L) frame and the Aircraft body (A) frame. In their order of rotation (which must be preserved) the Euler angles are defined as:
1. **Heading (Ψ)**: angle of rotation about Z from X to \(X_C\);

2. **Pitch (Φ)**: angle of rotation about Y from X to \(X_A\);

3. **Roll (Θ)**: angle of rotation about X from Y to \(Y_A\).

These Euler angle rotations are shown in Figure 3.

The **Body Reference Coordinate Frame** \((B)\) is introduced and defined in this report primarily to clarify the definition of trim angle of attack. Like the A-frame, this frame is fixed to, and translates and rotates with, the aircraft and has, as its origin the center of mass of the aircraft. The \(X_B\) axis, however, is fixed in a forward direction in the plane of symmetry parallel to a fuselage waterline or datum line. The \(X_B\) axis is displaced from the \(X_A\) axis by the angle \(α_{BO}\). The \(Y_B\) axis coincides with the \(Y_A\) axis, and the \(Z_B\) axis (positive downward) forms an orthogonal set.

The angle \(α_{BO}\) is sometimes called \(α_{trim}\), the trimmed angle of attack. It is the angle between the initial (equilibrium) remote wind vector and the \(X_B\) axis. Unlike \(α_{O}\), it has a non-zero value. It is evident from Figure 4 that

\[
α_B = α + α_{BO} \quad Ω_B = Ω + α_{BO}.
\]

*Numbered equations are mechanized in the simulation. Other equations are introduced as necessary for purposes of clarification, but are not numbered.*
and, for equilibrium level flight, that

$$\theta_{B0} = \alpha_{B0}$$

The trim angle $\alpha_{B0}$ can be approximated in the following fashion in the absence of wind tunnel or flight test data:

Assuming a constant aircraft lift curve slope, $a$, sketch the aircraft's lift curve:

From the sketch it is apparent that

$$C_L = a (\alpha_B - \alpha_{BOL})$$

or, at equilibrium,

$$C_{LO} = a (\alpha_{BO} - \alpha_{BOL})$$

Next, assume that wing incidence has been chosen by the aircraft manufacturer to produce a level fuselage attitude ($\alpha_{B0} = 0$) when the aircraft is in flight at "Economy Cruise Speed" at 10000 ft and at an arbitrarily chosen average gross weight. Using the relation $W_{cr} = C_{Lcr} q_{cr} S$, calculate the lift coefficient at the flight condition. The angle of attack for zero lift can then be calculated from the above equation as

$$\alpha_{B0} = - \frac{C_{Lcr}}{a} \alpha_{BOL}$$
The same equation can be manipulated to give an expression for the trim angle \( \alpha_{B_O} \) at any other trim lift coefficient:

\[
\alpha_{B_O} = \frac{C_{L_O}}{a} + \alpha_{B_{OL}} \tag{2}
\]

(In Appendix B of Reference 1, \( C_{L_{CR}} \) was estimated to be .44 for the "Buffalo" and .48 for the "Twin Otter". For both aircraft, \( a = 5.2/\text{rad} \), so

\[
\begin{align*}
\alpha_{B_{OL}} & = -0.085 = -4.8^\circ \text{ (Buffalo)} \\
& = -0.092 = -5.3^\circ \text{ (Twin Otter)}
\end{align*}
\]

These values are used in this report.)

IIB Velocity Resolutions

Use must be made of the above-defined Euler angles to relate a vector quantity in the A-frame to its components in the L-frame and vice versa. In general, a vector \( \vec{R} \) can be resolved into its A-frame or L-frame components:

\[
\vec{R} = R_X A \hat{i}_A + R_Y A \hat{j}_A + R_Z A \hat{k}_A
\]

\[
= R_X L \hat{i}_L + R_Y L \hat{j}_L + R_Z L \hat{k}_L
\]

where \( \hat{i}, \hat{j}, \) and \( \hat{k} \) are unit vectors in the indicated frames.

L-frame components of \( \vec{R} \) can be expressed in terms of A-frame components of \( \vec{R} \) and the Euler angles:
where \( B_{11} = \cos \psi \cos \theta \) \hfill (3)

\[ B_{12} = \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi \] \hfill (4)

\[ B_{13} = \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \] \hfill (5)

\[ B_{21} = \sin \psi \cos \theta \] \hfill (6)

\[ B_{22} = \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi \] \hfill (7)

\[ B_{23} = \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \] \hfill (8)

\[ B_{31} = -\sin \theta \] \hfill (9)

\[ B_{32} = \cos \theta \sin \phi \] \hfill (10)

\[ B_{33} = \cos \theta \cos \phi \] \hfill (11)

Conversely, A-frame components of any vector \( \mathbf{R} \) can be expressed in terms of L-frame components:

\[
\begin{bmatrix}
R_{x_A} \\
R_{y_A} \\
R_{z_A}
\end{bmatrix} = \begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33}
\end{bmatrix} \begin{bmatrix}
R_{x_L} \\
R_{y_L} \\
R_{z_L}
\end{bmatrix}
\]
Thus, in the simulation, the A-frame components of aircraft velocity \( \hat{x}_A = U, \hat{y}_A = V, \) and \( \hat{z}_A = W \) are computed and used to obtain velocity components with respect to the ground:

\[
\begin{align*}
\dot{X}_L &= B_{11} U + B_{12} V + B_{13} W \quad \text{(fps)} \\
\dot{Y}_L &= B_{21} U + B_{22} V + B_{23} W \quad \text{(fps)} \\
\dot{Z}_L &= -B_{31} U - B_{32} V - B_{33} W \quad \text{(fps)}
\end{align*}
\]

**IIC Provisions for Atmospheric Disturbances (Winds)**

Winds are input into the simulation in the L-frame. Components are \( \hat{x}_w \) (positive North), \( \hat{y}_w \) (positive East), and \( \hat{z}_w \) (positive downward). The winds are resolved into A-frame components in equations 15-17 in order to compute airspeed components:

\[
\begin{align*}
U_w &= U - [B_{11} \dot{x}_w_L + B_{21} \dot{y}_w_L + B_{31} \dot{z}_w_L] \quad \text{(fps)} \\
V_w &= V - [B_{12} \dot{x}_w_L + B_{22} \dot{y}_w_L + B_{32} \dot{z}_w_L] \quad \text{(fps)} \\
W_w &= W - [B_{13} \dot{x}_w_L + B_{23} \dot{y}_w_L + B_{33} \dot{z}_w_L] \quad \text{(fps)}
\end{align*}
\]

Material contained in this report is sufficient to allow introduction of steady state wind components. The desired winds are simply input as \( \dot{x}_w_L, \dot{y}_w_L, \) and \( \dot{z}_w_L \). The report does not document wind gust or wind shear models. However, these models, when developed, can be readily incorporated into the simulation with only minor modifications to the program being required.
In Reference 1, general 6 degree of freedom airframe equations of motion were developed as

\[
\begin{align*}
\dot{m} [\dot{U} + QW - RV + g \sin \theta] &= X \quad \text{(longitudinal force)} \\
\dot{m} [\dot{V} + RU - PW - g \cos \theta \sin \phi] &= Y \quad \text{(side force)} \\
\dot{m} [\dot{W} + PV - QU - g \cos \theta \cos \phi] &= Z \quad \text{(normal force)} \\
I_x \dot{P} + (I_x - I_z) Q R - J_{xz} (\dot{R} + PQ) &= L \quad \text{(rolling moment)} \\
I_y \dot{Q} + (I_y - I_z) R P - J_{xz} (R^2 - P^2) &= M \quad \text{(pitching moment)} \\
I_z \dot{R} + (I_y - I_x) P Q - J_{xz} (\dot{P} - QR) &= N \quad \text{(yawing moment)}
\end{align*}
\]

where the body-axis angular rates \( P, Q, \) and \( R \), can be used to obtain Euler angle rates according to the equations

\[
\begin{align*}
\dot{\psi} &= Q \frac{\sin \phi}{\cos \theta} + R \frac{\cos \phi}{\cos \theta} \quad \text{(rad/sec)} \\
\dot{\theta} &= Q \cos \phi - R \sin \phi \quad \text{(rad/sec)} \\
\dot{\phi} &= P + \dot{\psi} \sin \theta \quad \text{(rad/sec)}
\end{align*}
\]

These nine equations, together with equations 12-14, provide an almost exact description of the motions of an aircraft operating near the Earth's surface. They involve, as shown in Reference 1, only four assumptions:

1. Aircraft mass is constant
2. The Earth can be considered an inertial frame
3. The aircraft is a rigid body
4. The aircraft is symmetrical about its \( x - z \) plane.
For purposes of this simulation, the above 6 rigid body airframe equations have been approximated as

\[
\begin{align*}
\dot{U} &= RV - QW - g \sin \theta + X/m \\ (\text{ft/sec}^2) \\
\dot{V} &= PW - RU + g \cos \theta \sin \phi + Y/m \\ (\text{ft/sec}^2) \\
\dot{W} &= QU - PV + g \cos \theta \cos \phi + Z/m \\ (\text{ft/sec}^2) \\
\dot{P} &= L/I_x \\ (\text{rad/sec}^2) \\
\dot{Q} &= M/I_y \\ (\text{rad/sec}^2) \\
\dot{R} &= N/I_z \\ (\text{rad/sec}^2)
\end{align*}
\]

The omitted terms in the moment equations involve either products of angular velocities (e.g. QR) felt to be small compared with other equation terms, or terms containing \( J_{xz} \) which will be neglected. Experience has shown that, for purposes of this simulation, these terms can be omitted with negligible effect on results.

The terms \( X, Y, Z, L, M, \) and \( N \) of equations 21 - 26 represent the aerodynamic forces and moments acting on the aircraft. The lateral terms \( (Y, L, N) \) will be expressed in a quasi-linear form (as in Reference 1), but the longitudinal forces and moment \( (X,Z,M) \) must be non-linear in order to permit large excursions in forward velocity.

The longitudinal aerodynamic force terms are, from the sketch,

\[
\begin{align*}
X &= T - D \cos \alpha + L \sin \alpha \\ (\text{lbs}) \\
Z &= - (L \cos \alpha + D \sin \alpha) \\ (\text{lbs})
\end{align*}
\]
The terms \( X_q', Z_q', Z_w', \) and \( Z_{\delta_e} \) have been neglected in this analysis because of their small contribution to the overall forces.

It is also assumed that all thrust forces act along the \( X_A \) axis. Thus moment effects of thrust changes are neglected, as are forces and moments produced by special lift devices operating within or outside of the propeller slipstream. These effects are neglected because the airframe data required to model them are not available.

Equations 27 and 28 are solved (as are the other simulation equations) once every computer iteration cycle. Thrust, drag, and lift force components are summed to produce resultant \( X \) and \( Z \) forces acting on the aircraft.

Expressions for the total thrust, lift, and drag forces are next developed.

Thrust is computed from an empirically-derived expression (developed in the appendix) which accounts for the effects of altitude \( h \), airspeed \( V_R' \), and throttle setting \( \xi \):
\[ T = \frac{\sigma T_{\text{static}} \cdot \xi}{1 + C_{T_1} V_R + C_{T_2} V_R^2} \quad (\text{lbs}) \quad (29) \]

where \( 0 \leq \xi \leq 1.0, \)
\[ \sigma = e^{-h/h_{\text{atm}}} \quad (-) \quad (30) \]

and
\[ V_R = \left[ U_w^2 + V_w^2 + W_w^2 \right]^{1/2} \quad (\text{fps}) \quad (31) \]

Lift and drag are calculated from the standard relationships:
\[ L = C_L qS \quad (\text{lbs}) \quad (32) \]
\[ D = C_D qS \quad (\text{lbs}) \quad (33) \]

where
\[ C_L = C_{L_0} + \alpha \alpha \quad (-) \quad (34) \]
\[ C_D = C_{D_0} + C_L^2/\pi eAR \quad (-) \quad (35) \]
\[ q = \frac{1}{2} \rho V_R^2 \quad (\text{lbs/ft}^2) \quad (36) \]
\[ \rho = \sigma \rho_0 \quad (\text{sl/ft}^3) \quad (37) \]

and
\[ \alpha = \tan^{-1} \frac{W_w}{U_w} \quad \text{(rad)} \quad (38) \]

The expression for pitching moment used in the simulation is
\[ M = qSc[C_{m_t} + C_m \alpha + \frac{c}{2V_R} (C_{m_\alpha} \alpha + C_{m_q} \dot{q}) + C_{m_\delta_e} \delta_e] \quad (\text{ft/lbs}) \quad (39) \]

where the coefficients of the variables are constants. The term \( C_{m_t} \) is zero in this report, but is included to facilitate
later shaping of the trimmed $\delta_e$ vs $V_R$ curve. To do this, $C_{m_t}$
would be made a function of $V_R$.

Rate of change with time of angle of attack is obtained by
differentiating equation 38:

$$\dot{\alpha} = \frac{d}{dt} \left( \tan^{-1} \frac{W}{U} \right)$$

$$= \frac{U_W \dot{W}_W - W_W \dot{U}_W}{U_W^2 + W_W^2}$$

If the approximation is made that $U = \dot{U}$ and $W = \dot{W}$, the above
expression can be manipulated to produce

$$\dot{\alpha} = (\dot{W} - \frac{U}{U_W} \dot{U}) \frac{\cos^2 \alpha}{U_W}$$

(rad/sec) (40)

which is the expression used in the simulation.

The lateral force ($Y$) and moments ($L$ and $N$) are
developed in conventional linearized form (as in Reference 1)
except that total variables are used rather than perturbation
values, and that coefficients of the lateral variables are made
functions of lift and drag coefficient, airspeed, and dynamic
pressure, all of which are determined by solution of the
longitudinal equations.

The lateral force and moment expressions used in the
simulation are:

$$Y = Y_v V_W + Y_r R + Y_P P$$ (lbs) (41)

$$L = L_v V_W + L_r R + L_P P + L_{\delta_a} \delta_a$$ (ft-lbs) (42)

$$N = N_v V_W + N_r R + N_P P + N_{\delta_r} \delta_r$$ (ft-lbs) (43)
The terms $Y_\delta$, $L_\delta$, and $N_\delta$, sometimes included in the lateral equations, have been omitted in the present analysis because of their negligible effects.

The coefficients of these equations are

$$Y_v = \frac{1}{2} \rho V_R S C_{\gamma\beta} \quad \text{(lbs/fps)}$$

(44)

$$Y_r = \frac{1}{4} \rho V_R Sb C_{\gamma_r} \quad \text{(lbs/rad/sec)}$$

(45)

$$Y_p = \frac{1}{4} \rho V_R Sb C_{\gamma_p} \quad \text{(lbs/rad/sec)}$$

(46)

$$L_v = \frac{1}{2} \rho V_R Sb C_{\kappa\beta} \quad \text{(ft-lbs/fps)}$$

(47)

$$L_r = \frac{1}{4} \rho V_R Sb^2 C_{\kappa_r} \quad \text{(ft-lbs/rad/sec)}$$

(48)

$$C_{\kappa_r} = C_{\kappa_{FIN}} + C_L/4 \quad \text{(-)}$$

(49)

$$L_p = \frac{1}{4} \rho V_R Sb^2 C_{\kappa_p} \quad \text{(ft-lbs/rad/sec)}$$

(50)

$$L_{\delta a} = q Sb C_{\kappa_{\delta a}} \quad \text{(ft-lbs/rad)}$$

(51)

$$N_v = \frac{1}{2} \rho V_R Sb C_n \quad \text{(ft-lbs/fps)}$$

(52)

$$N_r = \frac{1}{4} \rho V_R Sb^2 C_{n_r} \quad \text{(ft-lbs/rad/sec)}$$

(53)

$$C_{n_r} = C_{n_{FIN}} - C_{d_{wing}}/4 \quad \text{(-)}$$

(54)

$$N_p = \frac{1}{4} \rho V_R Sb^2 C_{n_p} \quad \text{(ft-lb/rad/sec)}$$

(55)

$$C_{n_p} = C_{n_{FIN}} - \frac{C_L}{4} \left(1 - \frac{a}{\pi AR}\right) \quad \text{(-)}$$

(56)

$$N_{\delta_r} = q Sb C_{n_{\delta_r}} \quad \text{(ft-lbs/rad)}$$

(57)
The equation for sideslip angle is

\[ \beta = \tan^{-1} \frac{V}{U_W} \] (rad) \hspace{1cm} (58)

Linear and angular rates are integrated to produce the required linear and angular displacements. Initial values of displacements are provided for where necessary:

\[ U = U(0) + \int_0^t U \, dt \] (fps) \hspace{1cm} (59)

\[ V = V(0) + \int_0^t V \, dt \] (fps) \hspace{1cm} (60)

\[ W = W(0) + \int_0^t W \, dt \] (fps) \hspace{1cm} (61)

\[ P = \int_0^t P \, dt \] (rad/sec) \hspace{1cm} (62)

\[ Q = \int_0^t Q \, dt \] (rad/sec) \hspace{1cm} (63)

\[ R = \int_0^t R \, dt \] (rad/sec) \hspace{1cm} (64)

\[ \psi = \int_0^t \psi \, dt \] (rad) \hspace{1cm} (65)

\[ \theta = \int_0^t \theta \, dt \] (rad) \hspace{1cm} (66)

\[ \phi = \int_0^t \phi \, dt \] (rad) \hspace{1cm} (67)

\[ x_L = \int_0^t x_L \, dt \] (ft) \hspace{1cm} (68)

\[ y_L = \int_0^t y_L \, dt \] \hspace{1cm} (69)

\[ h = - \int_0^t h \, dt \] \hspace{1cm} (70)
**Definition of Required Display Quantities**

Provisions are made in the simulation for displaying parameters that are commonly available on a cockpit instrument panel. These parameters are tabulated here (and are defined if they have not been previously defined):

- **Indicated Airspeed IAS**
  \[
  \text{IAS} = \sqrt{\frac{\sigma}{1.46}} V_R \quad \text{(mph)}
  \]

- **Altimeter Output \( h \)**
  \( \text{(ft)} \)

- **Directional Gyro Output**
  \( 57.3 \psi \) \( \text{(deg)} \)

- **Pitch Attitude Gyro Output**
  \( 57.3 \Theta_B \) \( \text{(deg)} \)

- **Roll Attitude Gyro Output**
  \( 57.3 \phi \) \( \text{(deg)} \)

- **Rate of Climb Indicator Output**
  \( \frac{h}{60} \) \( \text{(fpm)} \)

- **Turn Rate Indicator Output**
  \( 57.3 R \) \( \text{(deg/sec)} \)

- **Slip Indicator Output**
  \[
  \frac{g \cos \Theta \sin \phi - V - RU + PW}{g \cos \Theta \cos \phi - \dot{W} - PV + QU} \quad \text{(rad)}
  \]
III Tabulation of Numerical Data for "Buffalo" and "Twin Otter"

Numerical data for the two aircraft to be modeled are tabulated in this section. Unless otherwise indicated, the values have been taken from Reference 1. It should be recognized that stability derivative values tabulated here are not based on wind tunnel or flight test results, but have been generated using analytical expressions presented in Reference 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Buffalo</th>
<th>Twin Otter</th>
</tr>
</thead>
<tbody>
<tr>
<td>a, rad⁻¹</td>
<td>5.2</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>AR</td>
<td>9.75</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>b, ft</td>
<td>96</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>c, ft</td>
<td>10.1</td>
<td>6.5</td>
<td></td>
</tr>
<tr>
<td>C_Df</td>
<td>.032</td>
<td>.039</td>
<td></td>
</tr>
<tr>
<td>ΔC_D</td>
<td>.030</td>
<td>.035</td>
<td></td>
</tr>
<tr>
<td>C_m_t</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C_m_q</td>
<td>-35.6</td>
<td>-24.6</td>
<td></td>
</tr>
<tr>
<td>C_m_α</td>
<td>-.78</td>
<td>-.78</td>
<td></td>
</tr>
<tr>
<td>C_m_ā</td>
<td>-6.05</td>
<td>-6.15</td>
<td></td>
</tr>
<tr>
<td>C_m_δ_e</td>
<td>2.12</td>
<td>1.73</td>
<td></td>
</tr>
<tr>
<td>C_α_p</td>
<td>-.53</td>
<td>-.53</td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>-------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Buffalo</td>
<td>Twin Otter</td>
<td></td>
</tr>
<tr>
<td>$C_{L \beta}$</td>
<td>-.125</td>
<td>-.103</td>
<td></td>
</tr>
<tr>
<td>$C_{L \delta a}$</td>
<td>.20</td>
<td>.38</td>
<td></td>
</tr>
<tr>
<td>$C_{L \gamma \text{r fin}}$</td>
<td>.038</td>
<td>.033</td>
<td></td>
</tr>
<tr>
<td>$C_{n \text{p fin}}$</td>
<td>.025</td>
<td>.033</td>
<td></td>
</tr>
<tr>
<td>$C_{n \text{r fin}}$</td>
<td>-.169</td>
<td>-.168</td>
<td></td>
</tr>
<tr>
<td>$C_{n \beta}$</td>
<td>.101</td>
<td>.121</td>
<td></td>
</tr>
<tr>
<td>$C_{n \delta r}$</td>
<td>.107</td>
<td>.107</td>
<td></td>
</tr>
<tr>
<td>$C_{y p}$</td>
<td>-.055</td>
<td>-.085</td>
<td></td>
</tr>
<tr>
<td>$C_{y r}$</td>
<td>.368</td>
<td>.429</td>
<td></td>
</tr>
<tr>
<td>$C_{y \beta}$</td>
<td>-.362</td>
<td>-.492</td>
<td></td>
</tr>
<tr>
<td>$C_{T_1, fps^{-1}}$</td>
<td>.00370</td>
<td>.00378</td>
<td></td>
</tr>
<tr>
<td>$C_{T_2, fps^{-2}}$</td>
<td>6.51x10^{-6}</td>
<td>9.07x10^{-6}</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>.75</td>
<td>.75</td>
<td></td>
</tr>
<tr>
<td>$h_{ATM, ft}$</td>
<td>32500</td>
<td>32500</td>
<td></td>
</tr>
<tr>
<td>$I_x, slug-ft^2$</td>
<td>273000</td>
<td>243000</td>
<td></td>
</tr>
<tr>
<td>$I_y, slug-ft^2$</td>
<td>215000</td>
<td>22000</td>
<td></td>
</tr>
<tr>
<td>$I_z, slug-ft^2$</td>
<td>447000</td>
<td>41000</td>
<td></td>
</tr>
<tr>
<td>$J_{xz, slug-ft^2}$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td>Buffalo</td>
<td>Twin Otter</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------</td>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>$S, \text{ft}^2$</td>
<td></td>
<td>945</td>
<td>420</td>
</tr>
<tr>
<td>$T_{\text{static}}, \text{lbs} \ (1)$</td>
<td></td>
<td>22400</td>
<td>5750</td>
</tr>
<tr>
<td>$W, \text{lbs} \ (3)$</td>
<td></td>
<td>40000</td>
<td>12000</td>
</tr>
<tr>
<td>$\alpha_{\text{OL}}, \text{rad} \ (3)$</td>
<td></td>
<td>-.085</td>
<td>-.092</td>
</tr>
<tr>
<td>$\rho_0, \text{slugs/ft}^3$</td>
<td></td>
<td>.002378</td>
<td>.002378</td>
</tr>
</tbody>
</table>

Notes:
1. From Appendix, this report.
2. Atmospheric density ratio calculated as $\sigma = e^{-h/32500}$ compares with standard atmosphere data as follows:

<table>
<thead>
<tr>
<th>$h$</th>
<th>standard $\sigma$</th>
<th>calculated $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5000</td>
<td>.862</td>
<td>.858</td>
</tr>
<tr>
<td>10000</td>
<td>.738</td>
<td>.735</td>
</tr>
<tr>
<td>15000</td>
<td>.629</td>
<td>.630</td>
</tr>
<tr>
<td>20000</td>
<td>.533</td>
<td>.540</td>
</tr>
</tbody>
</table>

3. From Section IIA, this report
IV Simulation Program

The equations of Section II have been programmed for real-time solution on an XDS9300 digital computer at the TSC Simulation Facility.

Because the simulation is a simple one, a flow chart is not presented. The program listing, together with the discussion presented here, should be sufficient to completely describe the simulation. The listing is included in this report as Table I.

IV-A Interface with GAT-1 Cockpit

Provisions are made to drive the simulation manually using a GAT-1 fixed-base cockpit modified for the purpose. Commands from the cockpit are:

- Elevator trim (ELTRM)
- Longitudinal stick displacement (DLE)
- Lateral stick displacement (DLA)
- Rudder pedal displacement (DLR)
- Throttle setting (THROT)

The scaling voltages used are given in Table I.

Similarly, the display quantities presented at the GAT-1 panel (listed in Section II-E) are scaled as shown in Table I.

IV-B Definition of Initial Values of Variables

It is convenient to be able to begin a simulation run with the aircraft trimmed at a level flight condition. Accordingly, provisions are made in the simulation for inputting desired initial conditions, and then for calculating required initial values of other parameters to produce a trimmed flight condition.
Non-zero initial values are normally input for altitude h(0) and airspeed V_R(0). In addition, non-zero steady state wind values can also be specified. Zero initial values are set in the first computer iteration for these parameters:

\[ U, V, W, P, Q, R, \psi, \phi, P, Q, R, \]

\[ \dot{\psi}, \dot{\vartheta}, \phi, V_w, W, x_L, y_L, \alpha, \dot{\alpha}, \beta \]

An initial computation is made to calculate initial values of other parameters, using the following equations:

\[ \sigma = e^{-\frac{h}{h_{ATM}}} \]
\[ \rho = \sigma \rho_0 \]
\[ q = \frac{1}{2} \rho \ V_R^2 \]
\[ U_w = V_R \]
\[ \dot{x}_L = U = V_R + x_{w_L} \]
\[ \dot{y}_L = V = y_{w_L} \]
\[ \dot{z}_L = W = z_{w_L} \]
\[ C_L = C_{L_0} = \frac{W}{qS} \]
\[ C_D = C_{D_f} + C_L^2 / \pi eAR \]
\[ D = C_D qS \]
\[ \theta_B = \alpha_B = C_{L_0} / a + \alpha_{OL} \]
\[ \delta_e = 0 \]
\[ \xi = D (1 + C_{T_1} V_R + C_{T_2} V_R^2) / \sigma \ T_{static} \]
The last two equations define required pilot inputs for initial trim. In the simulation, provision is made for inputting these trim values for a specified length of time, after which the actual control signal from the cockpit is used. The magnitude of the delays are TMTHR seconds for throttle setting $\xi$, and TMDLE seconds for elevator input $\delta_e$. This scheme permits setting up an initial trimmed condition without the need for cockpit control manipulation. It is useful when, for example, step response runs are to be made.
V  Simulation Results

Simulation results are presented in this section. These results are in the form of time responses to various step control inputs.

The time responses are presented in a manner that permits direct comparison with the linearized results generated in Appendix D of Reference 1. In general, agreement between the two sets of responses is very close.

It should be noted, however, that Reference 1 and this report utilize the same analytically-derived data. Therefore agreement between these two reports does not in itself prove the validity of either set of results. This proof can only be obtained by comparing the present results with data obtained from some other independent source. Unfortunately, however, specific data on "Buffalo" and "Twin Otter" responses from other sources are not currently available.

Accordingly, it is possible to say at this time only that this report is consistent with Reference 1 and that both sets of results are "reasonable". The time constants, frequencies, and damping ratios of the various modes presented in Appendix D of Reference 1 agree with results presented in this report. The values of these parameters are in the expected ranges, and show the normal variation with airspeed for each aircraft. Similarly, control power values appear to be within the expected ranges and in proper proportions.

Responses shown in this report are for the Cruise Flight Condition. For the "Buffalo" this is level flight at 400 fps and
10,000 ft altitude with a gross weight of 40,000 lbs. For the "Twin Otter", cruise is defined as level flight at 278 fps and 10,000 feet with a gross weight of 12,000 lbs.

Figure 5 shows the response in pitch rate $Q$, pitch angle $\theta$, angle of attack $\alpha$, altitude rate $\dot{h}$, and forward speed $U$ resulting from a $1^\circ$ step elevator input $\delta_e$ for the "Buffalo". Lateral degrees of freedom were suppressed during this run. This figure compares with Figure D1 of Reference 1.

Figure 6 shows the same information for the "Twin Otter". This figure corresponds to Figure D13 of Reference 1.

Figures 7 and 8 present lateral responses for the "Buffalo". Here, longitudinal modes are suppressed. Figure 7 shows the response in sideslip angle $\beta$, roll rate $\dot{P}$, roll angle $\phi$, yaw rate $R$, and yaw angle $\psi$ resulting from a $1^\circ$ step aileron input $\delta_a$. Figure 7 compares with Figure D7 of Reference 1.

Figure 8 shows the response in the same parameters resulting from a $1^\circ$ step rudder input $\delta_r$. This figure corresponds to Figure D8 of Reference 1.

Figures 9 and 10 present lateral responses for the "Twin Otter" for $1^\circ$ aileron and rudder inputs, respectively. These figures correspond to Figures D19 and D20 of Reference 1.
References


TABLE I
SIMULATION PROGRAM LISTING

```
C****REVISED DATA FOR BUFFALOTER MG 6/16/71
REAL IX, IY, IZ,
COMMON/CONST/WEIGHT, RHOBSEA, HATM, A, B, C, S, SEPIAR1, CT1, CT2, CMT,
1 CMAFZ, CMALF, CMQ, CMODEL, CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD,
2 CNPFIN, CLB, CLRFIN, CLP, CLDLA, STAT, IX, IY, IZ, 0.00, 0.00, 0.00, 0.00,
END

C****REVISED DATA FOR TWIN OTTER MG 6/16/71
COMMON/CONST/WEIGHT, RHOBSEA, HATM, A, B, C, S, SEPIAR1, CT1, CT2, CMT,
1 CMAFZ, CMALF, CMQ, CMODEL, CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD,
2 CNPFIN, CLB, CLRFIN, CLP, CLDLA, STAT, IX, IY, IZ, 0.00, 0.00, 0.00, 0.00,
END

MAIN PROGRAM
DIMENSION DERIV(12), VINT(12)
REAL LIFT, LV, LP, LDLA, NV, NR, NP, NDLR, IX, IY, IZ
COMMON/3IPRN/BLPFQG, ITPB, IBLIPRN, ITPQ, IPRN
COMMON/CONST/WEIGHT, RHOBSEA, HATM, A, B, C, S, SEPIAR1, CT1, CT2, CMT,
1 CMAFZ, CMALF, CMQ, CMODEL, CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD,
2 CNPFIN, CLB, CLRFIN, CLP, CLDLA, STAT, IX, IY, IZ, 0.00, 0.00, 0.00, 0.00,
END

COMMON/BLFLB/
DATA NVHE/21/
DATA WEIGHT/RHOBSEA, HATM, ALFBOL/A0000, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
DATA A, B, C, S, SEPIAR1/5.2, 96.0, 1.0, 1.945, 0.0, 0.035,
DATA CT1, CT2, CMAFZ, CMALF, CMQ, CMODEL/0.00, 78.0, 0.00, 35.6, 2.12,
DATA CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD/A0000, 0.00, 0.00, 0.00, 0.00,
DATA CNRFIN, CLRFIN, CLP, CLDLA/7.169, 0.035, 0.038, 0.033,
DATA CT1, CT2, DELCD, STAT/0.037, 0.034, 0.032, 0.031,
DATA IX, IY, IZ/273000.0, 215000.0, 447000.0,
END

C MAIN PROGRAM
DIMENSION DERIV(12), VINT(12)
REAL LIFT, LV, LP, LDLA, NV, NR, NP, NDLR, IX, IY, IZ
COMMON/3IPRN/BLPFQG, ITPB, IBLIPRN, ITPQ, IPRN
COMMON/CONST/WEIGHT, RHOBSEA, HATM, A, B, C, S, SEPIAR1, CT1, CT2, CMT,
1 CMAFZ, CMALF, CMQ, CMODEL, CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD,
2 CNPFIN, CLB, CLRFIN, CLP, CLDLA, STAT, IX, IY, IZ, 0.00, 0.00, 0.00, 0.00,
END

COMMON/BLFLB/
DATA NVHE/21/
DATA WEIGHT/RHOBSEA, HATM, ALFBOL/A0000, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
DATA A, B, C, S, SEPIAR1/5.2, 96.0, 1.0, 1.945, 0.0, 0.035,
DATA CT1, CT2, CMAFZ, CMALF, CMQ, CMODEL/0.00, 78.0, 0.00, 35.6, 2.12,
DATA CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD/A0000, 0.00, 0.00, 0.00, 0.00,
DATA CNRFIN, CLRFIN, CLP, CLDLA/7.169, 0.035, 0.038, 0.033,
DATA CT1, CT2, DELCD, STAT/0.037, 0.034, 0.032, 0.031,
DATA IX, IY, IZ/273000.0, 215000.0, 447000.0,
END

C MAIN PROGRAM
DIMENSION DERIV(12), VINT(12)
REAL LIFT, LV, LP, LDLA, NV, NR, NP, NDLR, IX, IY, IZ
COMMON/3IPRN/BLPFQG, ITPB, IBLIPRN, ITPQ, IPRN
COMMON/CONST/WEIGHT, RHOBSEA, HATM, A, B, C, S, SEPIAR1, CT1, CT2, CMT,
1 CMAFZ, CMALF, CMQ, CMODEL, CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD,
2 CNPFIN, CLB, CLRFIN, CLP, CLDLA, STAT, IX, IY, IZ, 0.00, 0.00, 0.00, 0.00,
END

COMMON/BLFLB/
DATA NVHE/21/
DATA WEIGHT/RHOBSEA, HATM, ALFBOL/A0000, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
DATA A, B, C, S, SEPIAR1/5.2, 96.0, 1.0, 1.945, 0.0, 0.035,
DATA CT1, CT2, CMAFZ, CMALF, CMQ, CMODEL/0.00, 78.0, 0.00, 35.6, 2.12,
DATA CYB, CYR, CYP, CNB, CNDLR, CNRFIN, DELCD/A0000, 0.00, 0.00, 0.00, 0.00,
DATA CNRFIN, CLRFIN, CLP, CLDLA/7.169, 0.035, 0.038, 0.033,
DATA CT1, CT2, DELCD, STAT/0.037, 0.034, 0.032, 0.031,
DATA IX, IY, IZ/273000.0, 215000.0, 447000.0,
END
```

---

-27-
TABLE I (Cont)

X  NAMELIST BLFPRQ,PRNFQ
   NAMELIST GDLQ,GDLK,TRANPRE,SLPBPR,SELTRM
   NAMELIST XSS,YSS,ZSS,XSD,YSD,ZSD,XTAU,YTAU,ZTAU,JK,JY,JZ

CALL SETPST(32,3000,2000,0000,2000)
CALL SETPST(132,3000,2000,0000,2000)

12 NTYPE=0
CALL STANDBY
S: JK 031010
C:*** 3 DEG/SEC/POINTER WIDTH FOR RATE OF TURN
   SLPB=5.73
C:*** 4 DEG/SEC/POINTER WIDTH FOR PITCH
   PTCPR=1.6325
C:*** 2 DEG/VOLT
   GELTRM=GDLQ=0.349666
C:*** 2 3 DEG/VOLT
   GDLR=716356
C:*** 4 3 DEG/VOLT
   GDLA=-0132712
C
C SET INITIAL CONDITIONS, IDLE L9P
C
H=6000.
VR=2000.
Y*PSI*PHI=P-Q-R
TMH=TMODE-ZSS=0

X BLFPRQ=10.

X PRNFQ=*1
   XK=JY=JZ=*1
   XSS=1*
   YSS=1*
   XSD=2.3
   YSD=1.6
   ZSD=1
   XTAU=YTAU=ZTAU=1.5
   DEL=05
   CALL IF INITIA
   INPUT(105)

C:*** CALL COMPUTE
S
E 0H 031013
X
TEMP=ALFPRG/DEL
X
ITTH=EMP
X
TEMP=PHYFPRG/DFL
X
ITTH=EMP
X
BLIP=IPREF=1
OM=32.7/HEIGHT
DB 1 1=1
1
DEJIV(1)=0.
SIG*EXP(-H/HATM)
RHO-SIG*RHOSEA
CLO2=2.7/WEIGHT/(RHO*S*VR*VR)
ALFRO=CLO2/ALFROL
TMETA=ALFW=0.
IF(SENSERSWITCHS)20,21

20 CONTINUE
   UV+VSS
   V=VSS
   W=WSS

-28-
TABLE I (Cont)

21 CONTINUE
VW = 0.
UVR
22 CONTINUE
VH = 0.
UVR

CUMF = CLC*EPHAI
DYN = RBN*VR
DRAG = CD*MS +
DRAGI = DRAG
ALFI = ALF
VR = VR
SRT = SRT
T = DEL
RNS = DEL/VT
RNy = DEL/VA
RZA1 = DEL/ZTAU
RMEM = XOR*SORT(2*DE/VA)
RNS = ZSO*SORT(12*DE/VA)
SRT = SORT(XS*SS*YS*Y)
CALM ARM(0)
CALM ENINT
CONNECT(40, AERB)

C**** IDLE COMP + TEST-CASE CALLS OF AER
10 CONTINUE
X 1 CALL AERB
C**** IDLE LOOP + FAKE INT 00 USING F/F T/L S/L S IF INT. SYSTEM DOWN

S SKS 030004
S BRU s-1
S ED0 030008
S ED0 030020
IF SENSES SWITCH(1), 12, 10
END

SUBROUTINE AER
DIMENSION BTE(3,3)
DIMENSION DERIV((17), VINT(12))

REAL LV, LR, LP, LDLAY, NV, NR, NP, NDLR, IY, IZ,

CUMN = SLIP/PLFR/IBLER/ITTP/IPRN
COMM = CON/HEIGHT, RSHSHA, CMAT, AX, CC, S, P, T1, C1, C2, CMR,

CUMN = LAMDA, CM, CYL, CYR, S, CYN, C1, C2, CYG

COMM = NRC, NP, NDLR, CM, DEL, T
CUMN = VTHR, XDLA, DVL, ZOLT, GDLA, GDLR, GEI

1 TNPVT, SLPSR, PTDOD
CUMN = ALF, VR, LIFT, DRAG, DALF, THRUST, UDOT, WDOT, C0, VDOT, PDOT,

1 RDOT, THEODT, PSI0, THEODT, XDOT, YDOT, HDOU, UDL, VDL, HTH, PSI1
2 PHI, Y, HRETA

CUMON CLO/ALFBO

EQUIVALENCE (DERIV(1), UNBT), (DERIV(2), WDOT), (DERIV(3), QDOT),
1 (DERIV(4), VDOT), (DERIV(5), PDOT), (DERIV(6), RDOT), (DERIV(7), THEODT)
2 (DERIV(8), PSI0), (DERIV(9), PHODT), (DERIV(10), XDOT), (DERIV(11),
3 THEODT, THEODT, PSI0), (VINT(1), U),(VINT(2), X), (VINT(3), Z), (VINT(4)
4 Y), (VINT(5), P), (VINT(6), R), (VINT(7), THEA), (VINT(8), PSI0), (VINT(9)
5 PSI1), (VINT(10), X), (VINT(11), Y), (VINT(12), A)
TABLE I (Cont)

<table>
<thead>
<tr>
<th>C</th>
<th><strong>TIME SIGNAL</strong></th>
<th><strong>SET FP</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>E04 030000</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>RECTANGULAR INTEGRATION</td>
<td></td>
</tr>
<tr>
<td>T=T+DEL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DO 10 I=1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. VINT(I)*VINT(I)+DERIV(I)*DEL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*NOTE HERE*

C | **DLE FROM** | **=10 V, DOWN TO +15 V, UP** |
|---|---|---|
C | **DLA FROM** | **=15 V, RIGHT TO +15 V, LEFT** |
C | **DLE FROM** | **=30 V, RIGHT TO +30 V, LEFT** |
C | **ELTRM FROM** | **=15 V, DOWN TO +15 V, UP** |
C | **THROT FROM** | **=32 V, IDLE TO 0 V, FULL** |

*CALCULATE COEFFICIENTS*

SIG=EXP(-H/HATM)
RHO=RHOSE#SIG
IF(T/E=THTR/ID)GB TO A1
IF(VE=EG+P)SIG=1
THROT=SIG*(1+CT1+VT+CT2+VT+VT)/(SIGI+1)

81 | IF(T/E=THTR/ID)GB TO A2
DLE=G.

82 | **CONTINUE**

C | **CONV AIR IS SUPERCHARGED** | **USE SIG=1 FOR THRUST CHP** |
|---|---|---|
| IF(VE=EG+P)SIG=1
THROT=SIG*$SIGI+1/(1+CT1+VT+CT2+VT+VT)$ |

C | **DYNAMIC PRESSURE** | **DYN*51+H0#VRSG** |

C | **SPHI=SIGI(SPHI)** |
| **SPSI=SIGI(PSI)** |
| **STH=SIGI(THEETA)** |
| **CPHI=SIGI(PI)** |
| **CPPS=SIGI(PSI)** |
| **CTH=SIGI(THEETA)** |

C | **BODY TO EARTH TRANS MATRIX** |
| **STCS**=**CT**=**CPHI** |
| **SSCP**=**PSI**=**CPHI** |
| **SSSP**=**PSI**=**SPHI** |
| **BTE(1,1)**=**CT**=**CPHI** |
| **BTE(2,1)**=**CT**=**SPHI** |
| **BTE(3,1)**=**CT**=**SPHI** |
| **BTE(4,1)**=**CT**=**SPHI** |
| **BTE(5,1)**=**CT**=**SPHI** |
| **BTE(6,1)**=**CT**=**SPHI** |
| **BTE(7,1)**=**CT**=**SPHI** |
| **BTE(8,1)**=**CT**=**SPHI** |
| **BTE(9,1)**=**CT**=**SPHI** |

C | **VEN 11-19 M/D** |
C | **F S 21 12 23 DOWN FOR JUST IN X Y Z RESP.** |
C | **S S S 5 SET FOR STEADY STATE** |
S | **SKS 230006** |

-30-
<table>
<thead>
<tr>
<th>S</th>
<th>BRU 11S</th>
<th>CALL GUST(xGUS, RNAX, RNB, JX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>xGUS = 0</td>
<td></td>
</tr>
<tr>
<td>S12</td>
<td>SKS 030007</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>BRU 13S</td>
<td>CALL GUST(yGUS, RNY, RNB, JY)</td>
</tr>
<tr>
<td>13</td>
<td>yGUS = 0</td>
<td></td>
</tr>
<tr>
<td>S14</td>
<td>SKS 030010</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>BRU 15S</td>
<td>CALL GUST(zGUS, RNZ, RNB, JZ)</td>
</tr>
<tr>
<td>15</td>
<td>zGUS = 0</td>
<td></td>
</tr>
</tbody>
</table>

CONTINUE

**TABLE I (Cont)**

---

C**ADD XY AXIS VELOCITIES INCLUDING WINDS**

\[
\begin{align*}
\text{UW} & = 1 \times (\text{XX} \times \text{BTE}(1,1) + \text{YY} \times \text{BTE}(P,1) - \text{ZZ} \times \text{BTE}(3,1)) \\
\text{VM} & = 1 \times (\text{XX} \times \text{BTE}(1,2) + \text{YY} \times \text{BTE}(2,2) - \text{ZZ} \times \text{BTE}(3,2)) \\
\text{VW} & = 1 \times (\text{XX} \times \text{BTE}(1,3) + \text{YY} \times \text{BTE}(2,3) - \text{ZZ} \times \text{BTE}(3,3)) \\
\end{align*}
\]

**C** ANGLE OF ATTACK, LIFT, DRAG

\[
\begin{align*}
\text{CL} & = \text{ATAN2(WW, UW)} \\
\text{CD} & = \text{ATAN2(CL, CL)} \\
\text{CU} & = \text{ATAN2(CL, CL)} \\
\text{CL} & = \text{ATAN2(CL, CL)} \\
\text{CL} & = \text{ATAN2(CL, CL)} \\
\end{align*}
\]

**C** SIDESLIP

\[
\begin{align*}
\text{CL} & = \text{ATAN2(VW, UK)} \\
\text{CL} & = \text{ATAN2(CL, CL)} \\
\text{CL} & = \text{ATAN2(CL, CL)} \\
\text{CL} & = \text{ATAN2(CL, CL)} \\
\text{CL} & = \text{ATAN2(CL, CL)} \\
\end{align*}
\]

**C** CALCULATE AERODYNAMIC FORCES
TABLE I (Cont)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CALF = \cos(\alpha)</td>
<td>Longitudinal Equations</td>
</tr>
<tr>
<td>SALF = \sin(\alpha)</td>
<td></td>
</tr>
</tbody>
</table>

**Longitudinal Equations**

- If \( \text{NTYPE} = 2 \) fly only lateral
- \( \text{UDOT} = \text{HDOT} = \text{QDOT} = 0 \)
- **GO TO 31**

**31 X-Force**

- \( \text{UDOT} = 0 \)
- \( \text{THrust} = (\text{CALF} \times \text{LIFT} + \text{SALF} \times \text{WEIGHT} + \text{STH}) = \text{QW} + \text{REV} \)

**32 Z-Force**

- \( \text{VDOT} = 0 \)
- \( \text{W} \times \text{REV} + \text{W} \times \text{REV} + \text{REV} \times \text{REV} + \text{REV} \times \text{REV} \)

**C**

**33 Euler Angle Transformation**

- \( \text{THETDOT} = 0 \)
- \( \text{R} \times \text{SPH1} \)

**34 Continue**

**35** Derivatives in Earth-Fixed Coordinates

- **GO TO 34**
- \( \text{HDOT} = \text{THETDOT} = 0 \)
- \( \text{PSIODT} = \text{R} \)
- \( \text{PHIDOT} = \text{R} \)

**36 BLIPs for Time on Strip Chart Recorder**

- **GO TO 63**
- \( \text{D8} = 25 \)
- \( \text{D8} = 25 \)
- \( \text{D9} = 25 \)
- \( \text{D11} = 143 \times 25 \)
- \( \text{D12} = 143 \times 25 \)

---
TABLE I (Cont)

C

C

C

IF (NTYPE.1) GO TO 66
D5=PHI*57.3

C

IF (NTYPE=1) GO TO 66
D5=PHI*57.3
D7=DLA*114.6

C

IF (HDIT=.GT.0) R6CF = +002*HDIT
IF (HDIT=.LE.0) R6CF = -008*HDIT

C

40 CONTINUE
CALL DALC2ODI,022P50*D5#(j,0)M(72)
CONTINUE
TURNO,TINPNT=7.5*PS1DBT
SLIPF=15*SLPBL*BETA

C

-6.67 750 VOLTS/1000 CEIMB, +8.33 VOLTS/1000 DESCEND

C

IF (VKT5=.LT.175) 200, 210, 211
IF (VKT5=125) .01, 209, 208
IF (VKT5=44) .02, 208, 207
AIRSPD=.
GO TO 211

C

DIRCPRN.=PSI*15@916676
DT=GYR*DYR*1100.0
IF (ABS(ROLL) .GT. 29.9) ROLL=SIGN(29.9,ROLL)
TH0=THETE+ALFA
RPUP3+TH0=ROT027*
CALL DAL(28#

C

33
TABLE I (Cont)

```
X 4 F11+4*/2X*THRT**F11+2*/LIFT*F11+2*/DRAG**F11+4**10X
X 5 DLE**F11+2*12X*DLR**F11+2**4X*DLR**F11+2*11X**BETA**
X 6 F11+4//
X 62 CONTINUE
C
C**** TIMING SIGNAL, RESET F/F
S EOM 030001
END
```

-34-
**DH-4 TWIN OTTER**

Announced in August 1944, the Twin Otter is a STOL transport powered by two Pratt & Whitney PT6A-20 turboprop engines. Design work began in January 1945. Construction of an initial batch of Twin Otters was started in November of the same year and the first of these flew on May 20, 1946.

At the beginning of 1947, a total of 25 Twin Otters had been delivered or were on order, with options for 11 more. They included eight for the Chilean Air Force, two for Trans-Australian Airlines, one for the Canadian Department of Lands and Forests, four for Alenia of Italy, one for Northern Consolidated Airlines, and others for Viking Airlines and Air Wisconsin, USA. Production was scheduled to be at the rate of six a month through 1947.

Under development for delivery in 1948 is a version of the Twin Otter with more powerful (840 shp) Pratt & Whitney PT6A-27 turboprop engines, longer nose to provide more baggage space, and AEW of 12,500 lb (5,670 kg). The following data refer to the current production model.

**Type:** Twin-turboprop STOL transport.

**Wings:** Braced high-wing monoplane, with a single streamlined-section bracing strut on each side. Wing section NACA 8A series mean line; NACA 6015 (modified) thickness distribution. Aspect ratio 19. Constant chord of a 6 ft 6 in (1.98 m). Induced 2°; Incidence 2° 30'. No sweepback. All-metal skin-life structure. All-metal ailerons which also droop for use as flaps; double-slotted all-metal full-span trailing-edge flaps. No spoilers. Trim-tabs in ailerons. Pneumatic-deicing equipment optional.

**Fuselage:** Conventional all-metal structure with monocoque skin-life structure.

**Tail Unit:** Cantilever all-metal structure of high-strength aluminium alloys. Pin integral with fuselage. Fixed-unidrome tailplane. Trim-tab in rudder and port elevator, latter interconnected with flaps. Pneumatic de-icing boots on tailplane leading edge optional.

**Landing Gear:** Non-retractable tricycle type, with steerable nose-wheel. Rubber shock absorption on main units. Hydropneumatic nose-wheel shock-absorber. Goodyear main wheel tyres size 11 x 14, pressure 22.8 kgf/m (2.23 kN/m). Goodyear nose-wheel type size 6 x 8 00 (1250 kgf/m). Goodyear hydraulic brake; provision for alternative foot and ski gear.

**Power Plant:** Two 840 shp Pratt & Whitney (CMA) PT6A-20 turboprop engines, each driving a Garrett three-blade reversible-pitch, fully-feathering metal propeller, diameter 8 ft 0 in (2.44 m). Fuel in two tanks (stills) under cabin floor; total capacity 919 Imp gallons (4178 litres). Two refuelling points on port side of fuselage: 11 sq ft capacity 2 Imp gallon (9.9 litres) per engine. Electric de-icing system for propellers and airscrews optional.

---

<table>
<thead>
<tr>
<th>Accommodation: Two seats side-by-side on flight deck. Seating for 12-16 passengers in main cabin. Cabin divided by bulkhead into main passenger or freight compartment and baggage or toilet compartment. Door on each side of main cabin, at rear. Baggage compartments in nose and aft of cabin, each with upwinding door on port side.</th>
<th>Width: 2 ft 6 in (0.76 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall: 45 ft 0 in (13.72 m)</td>
<td>Height to all: 5 ft 10 in (1.77 m)</td>
</tr>
<tr>
<td>Height overall: 10 ft 6 in (3.20 m)</td>
<td>Baggage compartment door (nose): Height to all: 6 ft 10 in (2.07 m)</td>
</tr>
<tr>
<td>Tailplane span: 51 ft 0 in (15.54 m)</td>
<td>Baggage compartment door (port, rear): Height: 4 ft 0 in (1.22 m)</td>
</tr>
<tr>
<td>Wheel track: 12 ft 0 in (3.65 m)</td>
<td>Width: 4 ft 0 in (1.22 m)</td>
</tr>
<tr>
<td>Wheelbase: 14 ft 0 in (4.26 m)</td>
<td>Height to all: 5 ft 10 in (1.77 m)</td>
</tr>
</tbody>
</table>
| Passenger door (port side): Height: 4 ft 2 in (1.27 m) | Dimensions, External: |}

---

**Dimensions, Internal:**

Cabin, recording flight deck, galley and baggage or toilet compartments.

Length: 10 ft 0 in (3.05 m)

Max width: 5 ft 0 in (1.52 m)

Max height: 3 ft 11 in (1.2 m)

Volume: 484 cu ft (13.91 m³)

Baggage compartment (nose) volume: 22 cu ft (0.62 m³)

Baggage compartment (port) volume: 32 cu ft (0.91 m³)

**Aerodynamics:**

Wings: gross 440 sq ft (13.000 ft²)

Aeroplane (total): 440 sq ft (13.000 ft²)

Trailing-edge flaps (total): 112 sq ft (3.39 m²)

Fin: 40 sq ft (1.19 m²)

Rudder, including tab: 24 sq ft (0.71 m²)

---

**Source:** Reference 3

**FIGURE 1**

---

**-35-**
Source: Reference 3  
Figure 2

-36-
L: EARTH LOCAL VERTICAL COORDINATE FRAME
C: EARTH-AIRCRAFT CONTROL COORDINATE FRAME
A: AIRCRAFT BODY COORDINATE FRAME

EULER ANGLES

\[ \begin{align*}
\psi &= \text{ROTATION ABOUT } Z_L \text{ AXIS} \\
\theta &= \text{ROTATION ABOUT } Y_C \text{ AXIS} \\
\phi &= \text{ROTATION ABOUT } X_A \text{ AXIS}
\end{align*} \]

Figure 3: Reference Coordinate Frames
Figure 4: Sketches showing Relationship of A and B Frames
FIG. 5 Response to 1° Step Elevator Input (Buffalo, Cruise)
FIG. 6 Response to 1° Step Elevator Input (Twin Otter, Cruise)
FIG. 7 Response to 1° Step Aileron Input (Buffalo, Cruise)
FIG. 8 Response to 1° Step Rudder Input (Buffalo, Cruise)
FIG. 9 Response to 1° Step Aileron Input (Twin Otter, Cruise)
FIG. 10 Response to 1° Step Rudder Input (Twin Otter, Cruise)
Appendix: Development of Expression for Thrust

The longitudinal force equation of Section II includes the total thrust force $T$. Since directly applicable data on the propulsive system installation of the "Buffalo" and "Twin Otter" are not available, an expression for $T$ is developed here for use in the simulation. Although the expression is adequate for the simulation documented in this report, it must be considered an approximate one.

Thrust developed by a propeller is

$$ T = \eta_p \frac{P}{V} $$

where $P$ is the power supplied to the propeller, $V$ is the velocity of the propeller with respect to the air, and $\eta_p$ is the propeller efficiency. Power supplied to the propeller is expressed in this report as

$$ P = \sigma P_o \xi $$

where $\sigma$ is the atmospheric density ratio, $P_o$ is the rated power output of the engine at sea level, and $\xi$ is the pilot's throttle deflection, expressed as a fraction of the deflection for rated power.

Propeller efficiency, $\eta_p$, is obtained from Figure 3-17 of Reference 2 (reproduced here)

![Figure 3-17. Propeller efficiency (measured and theoretical)](image)

- A-1 -
as a function of advance ratio $J$ and power coefficient $C_p$. By definition,
\[
J = \frac{60V}{ND}
\]
and
\[
C_p = \frac{.5P/1000}{\sigma(N/1000)^3(D/10)^5}
\]
where $V$ is in ft/sec, $N$ is propeller speed in rpm, $D$ is propeller diameter in feet, and $P$ is power in horsepower units.

For the "Buffalo" (Figure 2) with its two T64-GE-10 engines, $N = 1160$ rpm, $D = 14.7$ ft, and $P_o = 2850$ ESHP/engine, so, at sea level,
\[
C_p = .137
\]
or
\[
C_p^{1/3} = .515
\]
Entering Figure 3-17 at $J/C_p^{1/3} = 2.0$ gives $n_p = .79$. This value of $J/C_p^{1/3}$ corresponds to $J = 1.03$ or $V = 293$ fps. Therefore
\[
T = \frac{.79 (2850)(550)}{293} = 4220 \text{ lbs/engine}
\]
or, for two engines, 8440 lbs. Repeating this calculation for other values of $J/C_p^{1/3}$ produces the required thrust vs speed relationship.

This thrust - speed curve can be represented by an equation of the form
\[
T_{\text{rated power, sea level}} = \frac{T_{\text{static, sea level}}}{1 + C_{T_1} V_R + C_{T_2} V_R^2}
\]
By curve-fitting techniques, it can be established that, for the "Buffalo", 

- A-2 -
\[ T_{\text{static}} = 22400 \text{ lbs} \]
\[ C_{T_1} = 0.00370 \text{ fps}^{-1} \]
\[ C_{T_2} = 6.51 \times 10^{-6} \text{ fps}^{-2} \]

The process is repeated for the "Twin Otter" (Figure 1). For this aircraft (with two PT6A-20 engines), \( N = 2200 \text{ rpm}, D = 8.5 \text{ ft}, \) and \( P_0 = 652 \text{ ESHP/engine}. \) The required constants are established as:

\[ T_{\text{static}} = 5750 \text{ lbs} \]
\[ C_{T_1} = 0.00378 \text{ fps}^{-1} \]
\[ C_{T_2} = 9.07 \times 10^{-6} \text{ fps}^{-2} \]

These values are tabulated in Section II where simulation input quantities are listed.