INVESTIGATION OF THE NONLINEAR MECHANICS OF UNSTABLE PLASMAS

The research supported by this grant has been devoted to exploring the possibility of applying theoretical and experimental techniques that are well-known in lumped-parameter systems such as vacuum-tube oscillators, microwave devices, etc. to the study of distributed-parameter systems such as plasmas. One long-term objective of the work is the investigation of dynamic-stabilization methods for various instabilities in typical distributed-parameter systems; particular interest attaches to plasma instabilities and turbulence phenomena that affect high-speed plasma flow and plasma confinement. This objective, however, means that the relevant theoretical and experimental aspects of nonlinear behavior in plasmas must be understood and this has been the primary objective of the program. This report contains a broad statement of the problem area and a description of the theoretical and experimental approaches taken, a summary of the major results obtained, and a list of published articles and reports where the complete details of the work are available,
INTRODUCTION

The research supported by this grant has been devoted to exploring the possibility of applying theoretical and experimental techniques that are well-known in lumped-parameter systems such as vacuum-tube oscillators, microwave devices, etc. to the study of distributed-parameter systems such as plasmas. The results are also felt to be relevant to other distributed-parameter systems such as lasers, fluids and gases. One long-term objective of the work is the investigation of dynamic-stabilization methods for various instabilities in typical distributed-parameter systems; particular interest attaches to plasma instabilities and turbulence phenomena that affect high-speed plasma flow and plasma confinement. This objective, however, means that the relevant theoretical and experimental aspects of nonlinear behavior in plasmas must be understood and this has been the primary objective of the program.

The work carried out under the program for the past five years has proceeded along two main channels. These are: 1) the theoretical program to investigate the applications of concepts well-known in lumped-parameter systems in the analysis of nonlinear phenomena in distributed-parameter systems, and 2) verification of the theoretical concepts by means of laboratory experiments in a low-temperature plasma device known as a Q-machine, which has been built for this purpose.

It is felt that the general objectives of the program have been realized. The theoretical work, based on the use of the Van der Pol oscillator as a phenomenological model to describe plasma instabilities, a concept started in the present program, appears to be gaining wider acceptance, and it is felt
that a number of nonlinear effects that have been studied both theoretically and experimentally in the program are now becoming established in the literature. These include the use of the Van der Pol model in multi-mode systems; mode competition between unstable modes of the same plasma instability; mode-locking and entrainment of one mode by another; a new phenomenon called periodic pulling, fi. reported in this program, which is of particular interest in weak plasma turbulence in bounded systems; parametric excitation of ion-acoustic waves; the nonlinear interaction between waves and instabilities, for example, the nonlinear interaction between ion-acoustic waves and drift waves. This phenomenological approach has provided a useful tool for the description of nonlinear plasma phenomena and also provides a basis for the study of dynamic-stabilization methods.

The general interest in this work has resulted in the solicitation of a monograph on the subject by the Principal Investigator. The monograph will be devoted to providing the general background required for extending the analysis of lumped-parameter systems to the distributed-parameter systems, primarily the mathematical analysis of nonlinear phenomena. It is felt that this book will be useful in providing plasma physicists and other scientists with the background needed to carry out such analyses. Much, if not all, of the material in this volume is the result of work carried out with the support of the program.

Work in this program has resulted in certain technological spin-off developments. These include a new oscilloscope display system, for which a patent has recently been received, and a new method of making microwave measurements based on so-called "active" systems. Interest in this new microwave technique has resulted in funding of a modest project to explore the technique by the Division of Engineering of the National Science Foundation. A new development in microwave diagnostics based on the use of so-called Lecher wires, which provide high spatial resolution in plasma diagnostics, has also come out of the program. This technique is finding application in other work in plasma physics, in particular,
a project concerned with laser-generated plasmas.

Finally, a Q-machine (low-temperature plasma device) has been constructed and has been in operation for some five years. It is felt that this system represents a state-of-the-art development in Q-machines. There are about ten others in the United States at the present time.

The material in this final report is divided into three sections which follow immediately. The first, Section II, is a review of the results of the program. Section III describes the technological spin-off developments, and Section IV contains a list of publications, awards, etc.

II REVIEW

1 - Nonlinear Phenomena

The primary interest in plasma instabilities is the effect the instabilities have on plasma confinement, especially in connection with attempts to achieve thermonuclear fusion. One class of plasma instabilities of particular interest is the class known as universal or drift instabilities, which arise as a result of transverse gradients of density, temperature or magnetic field in a plasma confined by a magnetic field. These instabilities, in particular, the nonlinear behavior of these instabilities, is of great interest since this behavior determines the ultimate level of the turbulence that is set up in the plasma and the rate at which plasma is lost. The analysis of the plasma instabilities is based on the *Van der Pol equation*, which has been used for many years in the description of vacuum tube oscillators. In this model the unstable modes of an instability are represented by an ensemble of weakly coupled Van der Pol oscillators, since this picture provides a phenomenological description of many nonlinear effects that have been observed experimentally.

The use of the Van der Pol model has made it possible, for example, to derive an instability criterion for the simultaneous existence of more than two modes in a bounded system. This work is of significance in lasers and in bounded plasmas.
The conditions required for simultaneous existence of two modes in a bounded system, in which each mode competes for the available energy, had been derived earlier in connection with the helium-neon laser and in connection with earlier plasma work.

The new criterion extends this concept to the case of N modes. In the new method, the coefficients associated with the growth rates, damping rates, and coupling terms for the various modes are assembled in a particular way to form a matrix. The characteristic equation for the matrix then determines the stability of the system. In turn, the stability of the matrix can be determined by various graphical techniques, so that the stability problem does not have to be computed exactly. In particular, the new approach makes use of the Nyquist diagram.

Another result obtained in this program pertains to a new phenomenon which has been discovered, analyzed theoretically, and verified experimentally. This effect, which has been called "periodic pulling" is characteristic of any distributed parameter system that can be described by the Van der Pol equation and, for this reason, should be observed in other kinds of oscillators. This was indeed found to be the case and the effect has been reported in work on phase-locked magnetrons used in microwave radar systems, carried out at Air Force Cambridge Research Laboratories. The periodic-pulling phenomenon was also observed in work on Tonks-Dattner resonances in mercury vapor plasmas in work carried out in France. It has also been learned that periodic pulling has been observed in work on gas discharges being carried out at Wright-Patterson Field in connection with the development of quiet helium-neon lasers.

The phenomenon we call periodic pulling is closely related to a well-known phenomenon known as entrainment. Periodic pulling is essentially the incomplete entrainment of an oscillator or an unstable plasma mode by an injected perturbation whose frequency and amplitude are such that the parameters are just beyond the range at which the oscillator would be "locked" to the injected perturbation. Under these conditions, the mode tries periodically to become locked and, in so doing, exhibits a periodic frequency variation which is essentially a frequency
modulation. Since the amplitude and frequency of a nonlinear oscillator are always interdependent, this frequency modulation also produces an amplitude modulation. The net result is to produce a multiplicity of sidebands at frequencies that are not related to the initial mode frequencies. Periodic pulling introduces incommensurate frequencies into a spectrum of discrete modes associated with a plasma instability in a bounded system. These incommensurate frequencies then mix and give rise to turbulence effects. These turbulence effects lead to the production of low-frequency electric fields which are believed to play an important role in the diffusion of the plasma across a magnetic field.

**Parametric Excitation**

The parametric excitation of ionacoustic waves is of interest in connection with the general program designed to show that concepts well-known in lumped-parameter systems can be used to analyze the behavior of a plasma in a magnetic field. The effect follows directly from the application of the Mathieu equation. The electron temperature in the plasma is controlled by the application of rf power so that the electron temperature can be varied periodically by amplitude modulation of the rf power. In these experiments the temperature is modulated at a frequency equal to twice the frequency of the ion-acoustic wave. Ion-acoustic waves have been examined in detail in other parts of the program. The usefulness of the method lies in the fact that it provides an additional means for excitation of ion-acoustic waves in a plasma. In turn, the ion-acoustic waves can be used to study anomalous plasma conductivity in the presence of a known wave spectrum.

The work on ion-acoustic waves was originally carried out with the intention of using these waves as a means of carrying out diagnostics to measure plasma-parameters such as the plasma streaming velocity and the effect of collisions on ion waves. These parameters are of interest in the evaluation of the collisional
environment under which drift instabilities are excited in the Q-machine. Subsequently, however, the interest in ion-acoustic waves has been intensified as a consequence of recent Soviet work with the Tokomak, a device which is of interest in connection with thermonuclear fusion. It is believed that ion-acoustic waves are in some way responsible for the high electron temperatures that are observed in this machine. These waves are also of interest in connection with current-driven instabilities that arise in collisionless shock waves that are of importance in high-speed plasma flow.

In the work on plasma stabilization carried out in this program primary interest has been given to a comparison of so-called open-loop methods and closed-loop methods. In the closed-loop case the correction forces are derived from the instability; in the open-loop case, the correction forces are independent of the instability. The Van der Pol model, developed in the earlier parts of the program, was found to be quite useful in this connection. The Van der Pol analysis has shown that feedback (closed-loop) systems have three important shortcomings. 1) The feedback system must function as a linear amplifier.

2) The feedback system is a narrow band system which must provide the required phase-shift at a given instability frequency. To suppress an instability in a time-varying plasma the parameters of a feedback system must change in time.

3) A single feedback system cannot stabilize several modes which are unstable simultaneously. Quenching of one mode by feedback causes the growth of others because of an effect known as mode-competition, which has been investigated in earlier work in the program. All of the unstable modes must be quenched simultaneously. For these reasons, in this program interest has been concentrated on so-called open-loop stabilization methods. These can be broken down into two classes. A prototype problem for the first class is the inverted pendulum. The advantage of this method is the fact that high power is required to achieve stabilization inasmuch as the parameters of the energy storage component
of the system must be varied. For this reason we have investigated a method known as asynchronous quenching, or dithering, which is well-known in lumped-parameter systems. Analog computer experiments were carried out to simulate the effect of dithering on the Van der Pol oscillator, and it is found that stabilization can indeed be achieved. In the course of this work it was found possible to resolve certain questions in the literature that derive from the confusion between the effect known as asynchronous quenching and an earlier effect known as entrainment. The analog computer results obtained in this program did in fact demonstrate that these two are the same phenomenon. It is felt that this work has contributed to clearing up a long-standing misconception in the literature.

In connection with our general work on nonlinear analysis a model was developed to describe the well-known Bénard instability in terms of an equivalent electrical circuit which is found to be the circuit for the so-called dynatron oscillator, which is well-known in electronics. The viscosity and thermal conductivity of the fluid play the role of the resistance in the electronic system; the gravitational potential energy plays the role of electrostatic energy stored in the capacity, and so on. Although the Bénard instability, which is the instability of a fluid heated from below, has been already investigated extensively, it is of interest in that it represents a prototype for various kinds of convective instabilities which frequently appear in plasmas.

A further development in this connection has been the derivation of a new equation which describes the nonlinear saturation of the Bénard instability. This model equation, which is an analog of the Van der Pol equation in certain respects, is useful in providing a phenomenological description of experimentally observed effects in convective instabilities.

As indicated above, it is felt that the Q-machine constructed at the University of Maryland in connection with this program represents a state-of-the-art development. Two guiding principles have been observed in the design
of the University of Maryland Q-machine (UMQ). In order to exploit sophisticated data-acquisition and data-processing methods, such as synchronous detection, correlation, and continuous signal averaging, it is necessary to provide adequate control, stability, and reproducibility. In accordance with this objective, UMQ has been designed to provide control of all experimental parameters to 0.1 percent or better. The basic mechanical design of UMQ differs from that of other Q-machines. All the operating elements used in an experiment are mounted on a single platform which can be readily removed from the vacuum chamber and replaced with another. This platform is provided with two moving stages driven by lead screws by means of which various experimental components can be moved axially in the course of an experiment. In addition, provision is made for alignment of the platform and the experiment elements in the magnetic field while the machine is in operation. Another desirable feature of this design is the fact that several experiments can be carried out almost simultaneously, each one being provided with its own platform and elements. The air-core solenoid used to provide the magnetic field provides fields up to 10 kilogauss with a stability and reproducibility of one part in $10^5$ over a working volume 10 inches in diameter and some 40 inches in length. The system is stabilized to one part in $10^4$ for periods of several hours. The water cooling system is provided with a deionization chamber and an oxygen scavenging arrangement in order to prevent chemical deterioration of the coils. Another novel feature in UMQ is the use of a sealed-off electron bombardment diode. This design isolates the alkali metal vapor in the main chamber from the gun. It is found to be possible to heat the plate, which is 4 cm in diameter, to a temperature of the order of 2200° K with somewhat less than 1.5 kW. The functions of heat removal and trapping of spent alkali vapor are accomplished by means of a water-cooled shroud which is readily removed from the machine for cleaning. This feature has been found to be extremely useful in that the walls of the
Q-machine remain essentially free of any trapped alkali metal vapor after many hours of operation. As indicated in the introduction, this machine has been in use for some five years and has provided trouble-free, reliable operation.

III TECHNOLOGICAL SPINOFF

Oscilloscope Display

A two-dimensional oscilloscope display has been developed in connection with plasma physics research carried out in this program. The device is used in two general applications:

1) Signal-to-noise enhancement by continuous signal averaging.

2) Data presentation in the form of an X-Y plot. In this application a conventional oscilloscope that presents a functional dependence of the form \( y+y(t) \) is effectively converted into an X-Y plotter that presents a functional dependence of the form \( y+y(x) \).

This display is also found to be highly convenient for the presentation of spectral data in compact form and also functions as an optical signal averager. In this application it is found to be extremely useful in plasma turbulence experiments and in the detection of signals whose frequencies are not known a priori. In the detection of drifting signals the present system seems to have certain advantages over conventional electronic signal averagers. A patent application was filed for this device by the Research Corporation, acting for the University of Maryland, and the device is now covered by U. S. Patent 3,609,580. It is expected that this device will provide a useful accessory for use in connection with oscilloscopes in various applications in science and engineering.

Active Microwave Systems

In the course of our plasma research a technique has been developed in connection with microwave diagnostics being used with the Q-machine. Although
originally intended for plasma applications, the new technique was found to exhibit features that may be of value in a number of problems of current scientific and technological interest. Conventional systems for the determination of the dielectric constant at microwave frequencies are "passive" in the sense that the quantities of interest are determined from a shift in phase or frequency in a passive element such as a microwave interferometer bridge or cavity resonator, the microwave energy required for the measurement being derived from an external source.

In contrast, the new approach makes use of an "active" system in which a positive feedback loop containing a microwave amplifier is used to provide gain. The entire configuration, including the measurement element, then comprises a microwave oscillator, the frequency and amplitude of the oscillator signal containing information on the quantities being measured. Experimental results obtained in plasma experiments indicate that the technique is attractive from the point of view of sensitivity, dynamic range, time-resolution and convenience of readout. It appears that the system can measure the real and imaginary parts of the dielectric constant of solids, liquids and gases at microwave frequencies with the capability of convenient transient measurements in the submicrosecond range as well as measurements in the presence of high losses which would be prohibitive for passive systems. This system would appear to be useful in process control in cases in which it is desired to monitor the dielectric constant or loss tangent, at microwave frequencies of any solid, gas, or fluid coming off a production line. The material being tested can move through the system on a continuous basis. The fact that the data can be read out on a high-speed basis means that the information can be used to provide feedback to control various production processes.

There also appears to some application for a microwave refractometer capable of high time resolution in the study of clear air turbulence. Certain current experiments in clear air turbulence use an airborne microwave refractometer to
sample localized fluctuations in the refractive index of the atmosphere. The aircraft carrying the microwave refractometer is simultaneously tracked by a high-resolution ground-based radar. The radar returns are then correlated with the localized readings of the fluctuations in refractive index of the atmosphere as obtained with the airborne refractometer. Conversations with people in the field have indicated that an active system might be capable of improving the available time resolution by an order of magnitude or more. A proposal was recently submitted to the National Science Foundation to support the development of this method at a modest level for nonplasma applications. This proposal has been funded by the Engineering Systems Division of the National Science Foundation.

Microwave Plasma Diagnostics with Lecher-Wire Systems

Conventional systems used for plasma microwave diagnostics employ bounded resonant structures, such as cavity resonators, or non-resonant radiating structures, such as focused microwave horns. In either case, the spatial resolution that can be achieved in the measurement of plasma density is of the order of a wavelength of the microwave probing signal. On the other hand, in many cases it is necessary to achieve high spatial resolution even though the electron density is so low that adequate sensitivity cannot be obtained with microwaves of short wavelength and high frequency. Moreover, when radiating structures are used inside the metal chambers characteristic of plasma laboratory experiments the measurements can be confused by spurious reflections from the metal walls.

It is possible to circumvent these problems by using wire conductors to guide the microwave probing radiation through plasma. In this case the microwave energy is carried by surface waves that propagate along the wires and the radiation field is confined to the immediate vicinity of the conductors so that high spatial resolution and freedom from spurious wall reflections can be realized.
Coupling of a wire conductor configuration to a standard microwave system that employs waveguides or coaxial cables requires special measures. The coupling section must minimize frequency-dependent reflections and spurious radiation, at the same time providing a smooth impedance match between the wire conductor configuration and the waveguide or coaxial system.

Lecher-wire systems have been used for microwave diagnostics and are reported in the literature. A standard nonresonant microwave interferometer is generally used. The coupling is achieved by so-called finline sections. In the present work we have made use of a so-called single wire transmission line in conjunction with an active Fabry-Perot resonator, making use of the active microwave concept described above. The single-wire line has a number of interesting features as compared with the usual Lecher-wire systems. There is much more flexibility in the choice of the wire diameter so that the diameter can be dictated by considerations of mechanical strength. In the Lecher-wire system the values of the diameter that satisfy the electrical requirements generally limit the mechanical strength. Moreover, it has been found possible to use a so-called flared-horn launcher with the single wire line. This type of transition section has a much higher efficiency than the tapered waveguide or finline and is also free from spurious radiation and spurious resonances. The results of this work have culminated in the development of a diagnostic system which will be installed in the Q-machine. Also, this work on Lecher-wire systems has stimulated a great deal of development resulting in the installation of a Lecher-wire system in a laser-plasma experiment being carried out at the University of Maryland.

IV PUBLICATIONS, AWARDS, ETC.

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**Awards etc.**

Dr. E. J. Yadlowsky, a graduate research assistant supported by this grant, was awarded a National Research Council Postdoctoral Associateship at the ESSA Laboratories in Boulder, Colorado (1970-72).

U. S. Patent 3,609,540 "Raster Display Method and Apparatus", September 1971, has been awarded for a device developed in this program.

Excitation of Ion-Acoustic Waves by Parametric Heating and Strong High-Frequency Electric Fields, 3rd International Conference on Quiescent Plasmas, Riso, Denmark, 1971 (T. Ohe, R. H. Abrams, Jr. and H. Lashinsky)


Periodic Pulling of the van der Pol Oscillator, Proceedings of the Fifth International Conference on Nonlinear Oscillations, Kiev, U.S.S.R., August 1969 (H. Lashinsky)


Asynchronous Quenching of the van der Pol Oscillator, Institute of Electrical and Electronics Engineers, Transactions of the Professional Group on Auto-
LOW-FREQUENCY CONDUCTIVITY OF A COLLISIONLESS TURBULENT PLASMA, BULL. AM. PHYS. SOC., 16, 1276 (1971) (T. Ohe, R. H. Abrams, Jr. and H. Lashinsky)


