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THE BUCKLING OF STIFFENED AND UNSTIFFENED SHELL STRUCTURES

JOSEF SINGER

Technion — Israel Institute of Technology

Department of Aeronautical Engineering

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13 ABSTRACT

Theoretical and experimental research on the buckling of stiffened and unstiffened cylindrical and conical shells, and arches and rings carried out over a period of 2 years in the Department of Aeronautical Engineering is summarized. The topics of earlier work are otulined and the more recent topics are summarized. These include: the influence of in-plane boundary conditions for stringer and ring-stiffened cylindrical shells; extensive tests on integrally stringer-stiffened and ring-stiffene cylindrical shells under axial compression; thermal buckling of cylindrical shells; a collocation method for buckling analysis of elastically restrained conical shells; buckling of cylindrical panels under non-uniform axial compression; and instability of closely ring-stiffened conical shells.

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JOSEF SINGER
Department of Aeronautical Engineering
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SUMMARY

Theoretical and experimental research on the buckling of stiffened and unstiffened cylindrical and conical shells, and arches and rings carried out over a period of 2 years in the Department of Aeronautical Engineering is summarized. The topics of earlier work are outlined and the more recent topics are summarized. These include: the influence of in-plane boundary conditions for stringer and ring-stiffened cylindrical shells; extensive tests on integrally stringer-stiffened and ring-stiffened cylindrical shells under axial compression; thermal buckling of cylindrical shells; a collocation method for buckling analysis of elastically restrained conical shells; buckling of cylindrical panels under non-uniform axial compression; and instability of closely ring-stiffened conical shells.

INTRODUCTION

The studies briefly described in this report are part of a continuing investigation of the stability of stiffened and unstiffened shells under different loads and load combinations as well as heating. The purpose of the investigation is not only a better understanding of the phenomenon of buckling but also better methods of analysis and improved structural efficiency of aerospace vehicles. The work reported was performed as the Department of Aeronautical Engineering of the Technion, Israel Institute of Technology, under Contract F61052-69-C-0040 during the period September 16, 1969 to September 15, 1971. Previous Studies were summarized in the Final Report of Contract AF 61(052)-905, TAE Report No. 102 " The Buckling of Stiffened and Unstiffened Conical and Cylindrical Shells, October, 1969.

The earlier phases of the present work were reported in the following Scientific Reports and Publications:

1) SR - 1 (TAE Report 93) - Singer,J., Baruch,M., and Reichenthal,J., "Influence of In-Plane Boundary Conditions on the Buckling of Clamped Conical Shells", September 1970. Slightly condensed version published in Proceedings XIII Israel Ann. Conf. on Aviation and Astronautics, Israel J. of Technology, Vol.9, No.1, March 1971, pp.127-139.
2) SR - 2 (TAE Report 133) - Dym, C.L., "On the Buckling and Postbuckling of Circular Arches and Rings", August 1971.
3) SR - 11 of Contract AF 61(052)-905 (TAE Report 100) - Weller, T., Singer, J. and Nachmani, S., "Recent Experimental Studies on the Buckling of Intergrally-Stiffened Cylindrical Shells", April 1970. (Publication of this report was delayed to include also some results of work carried our under the present contract).

4) Weller, T., Baruch, M., and Singer, J., " Influence of In-Plane Boundary Conditions on Buckling Under Axial Compression of Ring-Stiffened Shells," Israel Journal of Technology, Vol. 9, No. 4, pp. 397-410, July 1971. (Condensed version of SR 10 of Contract AF 61(052)-905 with some extensions).

The earlier phases are summarized in Section 1 of the present report. The other phases of the work are outlined in Sections 2-7. More details are given in the following Scientific Reports that are in preparation:

SR - 2 (TAE Report 135) - Weller T., and Singer, J., "Experimental Studies on the Buckling of 7075 Aluminium Alloy Integrally Stringer-Stiffened Shells", 1971.

SR - 3 (TAE Report 129) - Weller, T., " Further Studies on the Effect of In-Plane Boundary Conditions on the Buckling of Stiffened Cylindrical Shells", 1971.

SR - 4 (TAE Report 136) - Durban, D. and Singer, J., "Buckling of Cylindrical Panels Under Non-Uniform Axial Compression", 1971.

SR - 6 (TAE Report 138) - Weller, T. and Singer, J., "Further Experimental Studies of Integrally Ring-Stiffened Cylindrical Shells under Axial Compression," 1971.

SR - 7 (TAE Report 137) - Shalev, A., Baruch, M. and Nissim, E. " A Collocation Method for Buckling Analysis of Elastically Constrained Conical Shells under Hydrostatic Pressure", 1971.

SR - 8 (TAE Report 140) - Haftka, R. and Singer, J., "Instability of Closely Ring-Stiffened Conical Shells", 1971.

The experimental study of thermal buckling of cylindrical shells outlined in Section 5 will be reported in detail later in a Scientific Report that will also cover work on the same topic to be carried out under Contract F 44620-71-C-0116.

The writer would like to take this opportunity to thank the authors whose names appear on the list of Scientific Reports and publications for their invaluable contributions to the work performed. Also he would like to acknowledge the continuous assistance and encouragement given by the Air Force Office of Scientific Research and its European Office of Aerospace Research.

1. BUCKLING OF STIFFENED AND UNSTIFFENED CONCIAL AND CYLINDRICAL SHELLS AND ARCHES AND RINGS

The work reported in SR-1 and SR-2 and paper No. 4 deals with the following topics:

(a) Influence of Boundary Conditions on Buckling of Conical Shells.

The influence of the different in-plane boundary conditions on the buckling loads and deflection shapes of clamped conical shells under external pressure was studied with the displacement method developed earlier (Refs. 1 and 2). The effects on the in-plane boundary conditions for clamped conical shells were found to be in general similar to those observed in simple supported shells, however, the stiffening of $v = 0$ and $u = 0$ is slightly less here.

(b) Influence of Boundary Conditions on Buckling of Ring-Stiffened Cylindrical Shells.

An analysis for buckling of ring-stiffened cylindrical shells, by the displacement method, developed earlier for isotropic conical shells (Refs. 1 and 2), was derived and employed for an extensive parametric study of the influence of in-plane boundary conditions on the buckling of ring-stiffened cylinders. Similar in-plane boundary effects to those observed in unstiffened cylindrical shells - buckling loads of about half the classical (SS3) ones for cases SS1 and SS2, and identical loads for SS3 and SS4 were found.

(c) Experimental Studies of Stringer-Stiffened Cylindrical Shells.

The test program on the buckling of closely stringer-stiffened cylindrical shells, using accurate integrally machined steel specimens was continued and extended to provide additional data. Effects of material non-linear behavior were introduced in the correlation of tests with theory. A further test program with aluminum alloy specimens is discussed in Section 3.

(d) Buckling and Postbuckling of Arches and Rings.

In connection with imperfection sensitivity studies initiated for shells, a study of the buckling and initial postbuckling of circular arches and rings under constant directional pressure was carried out. The assumption of inextensibility was shown to be reasonable for buckling of steep arches and rings and to have no effect on the stable postbuckling behavior.

2. INFLUENCE OF IN-PLANE BOUNDARY CONDITIONS ON THE BUCKLING OF STIFFENED CYLINDRICAL SHELLS.

The influence of the in-plane boundary conditions on the buckling loads of isotropic and orthotropic cylindrical shells has been the subject of many studies in recent years (see for example the bibliography of Ref. 3, Ref. 4 and Ref. 5)and has been found to be significant. For stiffened shells, however, these studies (Refs. 6 and 7) were not complete. Hence an analysis of stiffened cylindrical shell by the displacement method was initiated (see Section 1) and employed for a parametric study of ring-stiffened cylinders. The method was now extended to stringer stiffened shells. The analysis starts with the Donnell type stability equations for stiffened shells from Ref. 8 and the same basic non-dimensional displacements . In order to satisfy the in-plane boundary conditions the displacements are assumed to have the following form:

$$u_n = C_n \sin t\phi[a_n \cos(n\beta y) + u_{on}(x)]$$

$$v_n = C_n \cos t\phi[b_n \sin(n\beta y) + v_{on}(x)]$$

$$w_n = C_n \sin t\phi[\sin(n\beta y) + w_{on}(x)] \qquad (1)$$

w_{on} is chosen to be for internal stringers, $\underline{\chi_1 > 0}$,

$$w_{on}(x) = \overline{A_{1n}} + \overline{A_{2n}}x + \overline{A_{3n}} \cos(\mu x) + \overline{A_{4n}} \sin(\mu x) \qquad (2a)$$

where

$$\mu^2 = (\nu/\chi_1)$$

and for external stiffening, $\underline{\chi_1 < 0}$

$$w_{on}(x) = \overline{A_{1n}} + \overline{A_{2n}}x + \overline{A_{3n}}ch(\mu x) + \overline{A_{4n}}sh(\mu x) \qquad (2b)$$

where $\mu^2 = -(\nu/\chi_1)$

Substitution of Eqs. (2a) and (2b) into the stability equations of Ref. 8 yields for the additional displacements $u_{on}(x)$ and $v_{on}(x)$ the following solutions for internal stringers, $\underline{\chi_1 > 0}$,

$$u_{on}(x) = \overline{\gamma_1}\overline{A_{2n}} + \overline{\gamma_2}\overline{A_{3n}}\sin(\mu x) + \overline{\gamma_2}\overline{A_{4n}}\cos(\mu x) + A_{1n}sh(\alpha_1 x) +$$

$$+ A_{2n}sh(\alpha_3 x) + A_{3n}ch(\alpha_1 x) + A_{4n}ch(\alpha_3 x)$$

$$v_{on}(x) = -\overline{\gamma_3}\overline{A_{1n}} - \overline{\gamma_3}\overline{A_{2n}}x + \overline{\gamma_4}\overline{A_{4n}}\sin(\mu x) + \overline{\gamma_4}A_{3n}\cos(\mu x) +$$

$$+ \theta_1 A_{1n}ch(\alpha_1 x) + \theta_3 A_{2n}ch(\alpha_3 x) + \theta_1 A_{3n}sh(\alpha_1 x) +$$

$$+ \theta_3 A_{4n}sh(\alpha_3 x) \qquad (3)$$

and slightly different expressions of similar form for external stringers, $\underline{\chi_1 < 0}$

The constants of integration $\overline{A_{jn}}$ and A_{jn} $(j = 1 \ldots 4)$ are determined by enforcement of the in-and out-of plane boundary conditions.

The α_j's are defined as follows

$$\alpha_{1,3} = t\{[(T-\nu) \pm \sqrt{(T-1)(T-\nu)}\,]/[(1-\nu)(1+\mu_1)]\}^{1/2} \qquad (4)$$

where $T = (1 + \mu_1)(1 + \mu_2)$

θ_j is given by

$$\theta_j = \frac{2(1 + \mu_1)\alpha_j^2 - (1 - \nu)t^2}{\alpha_j t(1 + \nu)} \tag{5}$$

and the $\bar{\gamma}$'s are determined by

$$\bar{\gamma}_1 = [(1 + \nu)\bar{\gamma}_3 - (2\nu/t)]/(1 - \nu)t$$

$$\bar{\gamma}_3 = \{(1/t) - [\chi_2 t/(1 + \mu_2)]\}$$

$$\bar{\gamma}_2 = \frac{ut^2\ (\frac{1+\nu}{2})[(1 + \mu_2) - \chi_2 t^2]}{[\mu t(\frac{1+\nu}{2})]^2 - [(1+\mu_2)t^2 + (\frac{1-\nu}{2})\mu^2][(1 + \mu_1)\mu^2 + (\frac{1-\nu}{2})t^2]}$$

$$\underline{\text{for } \chi_1 > 0}$$

$$\bar{\gamma}_4 = \bar{\gamma}_2[(1 + \mu_1)\mu^2 + (\frac{1 - \nu}{2})t^2]/[ut(\frac{1 + \nu}{2})]$$

$$(6)$$

and expression of similar form for external stringers.

a_n and b_n are obtained as in Ref. 8 by substituting of the basic displacements in the first two stability equations, and since the origin is taken in the middle of the shell of length 2ℓ ,

$$y = x + \frac{\ell}{R}$$

and $$n\beta y = n\beta x + \frac{n\pi}{2} \tag{7}$$

where $$\beta = \frac{\pi R}{2\ell}$$

After enforcement of the in and out-of plane boundary conditions, the third stability equation is solved by the Galerkin method.

Axisymmetric buckling is considered separately, since the above formulation does not permit a direct application to the case of t = 0.

The numerical work covers a wide range of shell and stiffener geometries. Results indicate that the influence of in-plane boundary conditions is affected by stiffener geometry and location. For very light stiffening the influence is similar to that for unstiffened shells, but less sensitive, whereas for heavy stiffening, the effect of the in-plane boundary conditions is completely different. Here, as in the case of buckling under external pressure, the buckling loads are found to be sensitive to the axial boundary restraints, $u = 0$ (SS2 and SS4), or $N_x = 0$ (SS1 and SS3). The SS2 boundary conditions yield critical loads almost the same as the SS4 ones, whereas the SS1 boundary conditions yield identical loads to the SS3 ones. The SS2 and SS4 critical loads are higher than those of the SS1 and SS3 ones and this difference is very noticeable for internally stringer-stiffened shells.

For shells stiffened by rings and stringers the sensitivity of the rings to the "weak" circumferential boundary conditions $N_{x\phi} = 0$ (SS1 and SS2) dominates and the effect of in-plane boundary conditions is very similar to that for unstiffened or ring-stiffened shells. Shells stiffened with external rings and stringers are found to yield the highest critical loads.

More details are given in SR-3 (TAE Report 120).

3. EXPERIMENTAL STUDIES ON THE BUCKLING OF INTEGRALLY STIFFENED CYLINDRICAL SHELLS.

The test programs on integrally stiffened cylindrical shells under axial compression described in Refs. 9 and 10 were continued. Additional tests on integrally ring and stringer-stiffened cylindrical shells were carried out to verify and extend the earlier conclusions. The main effort was devoted, however, to extensive tests of aluminum alloy stringer-stiffened cylindrical shells.

The earlier experimental studies by other investigators on buckling of stringer stiffened cylindrical shells (for example Refs. 11, 12 and 13) did not discern the influence of various geometrical parameters on the applicability of linear theory. The aim of Refs. 9, 10 and the present study was therefore to determine the effect of the stiffener parameters on the "linearity" $\rho = (P_{exp}/P_{th})$, which defined the applicability of classical theory. The stiffener parameters studied were the cross-sectional area, A_1, or in non-dimensional form (A_1/bh), the eccentricity e_1, or non-dimensionally (e_1/h), the non-dimensional moment of inertia (I_{11}/bh^3), and the Koiter panel parameter θ (Ref. 14) for local buckling behavior of the panels.

In the program 37 integrally stringer-stiffened cylindrical shells were tested. Most of the specimens were designed to fail in general instability and hence the stringers were closely spaced. Some of them, however, were intentionally designed for unstable panel postbuckling behavior. The values of Koiter's panel parameter θ for the shells tested vary between $\theta = 0.2$ to $\theta = 0.75$ (the limiting value for a simply supported panel is $\theta = 0.64$). The specimens were manufactured of 7075-T6 Aluminum

Alloy, which has a fairly high yield stress and a very low $(E/\sigma_{y.p.})$ ratio to ensure elastic buckling and avoid the effects of non-linear material behavior. The (R/h) of the specimens was 400 to 800. The stringer area parameter was varied from $(A_1/bh) \simeq 0.15$ to $(A_1/bh \simeq 0.8$. By variation of the stringer spacing (b/h), (A_1/bh) and θ could easibly be changed. The variation of the eccentricity parameter (e_1/h) was somewhat limited by the rectangular cross-section of the stringer, dictated by manufacturing difficulties. Hence also (I_{11}/bh^3) was not independent.

The stringer-stiffened shells were designed, as were the ring-stiffened shells in Ref. 9, to ensure loading along the middle surface of the shells, and hence eliminate effects of load-eccentricity effects.

The present specimens were machined on a "heated" mandrel. Placement of blank on the mandrel before machining and releasing of the machined shells was made within a controlled heated oil reservoir. Stringers are machined on a milling machine, with a division head, by special curved form cutters (which have the shape of the shell between the stringers). There are more accumulating inaccuracies in this milling process than in the turning process of the ring-stiffened shells, and therefore the stringer-stiffened shells are slightly less precise. The deviations in shell thickness, in the worst shells were here less than 10% of the lowest thickness, though usually considerably less.

The specimens were again "covered" by many strain gages to assist in the detection of incipient buckling. Southwell plots were obtained from most strain gage readings, using the procedure of Ref. 15. The buckling loads predicted from the Southwell plots were between the experimental ones and the predicted values for a perfect shell.

If the "linearity" ρ obtained in the present tests and the previous tests is plotted versus the stringer area parameter (A_1/bh) a trend can be observed that would suggest $(A_1/bh) > 0.50$ for reasonable applicability of linear theory. If ρ is plotted versus Z (Batdorf parameter) of the shells it is observed that the "linearity" is also dependent upon the value of Z. There is a clear trend towards $\rho = 1$ with increasing values of Z. Hence, (A_1/bh) and Z are the main parameters which determine the adequacy of linear theory. It should be noticed, however, that scatter of results is quite considerable for both kinds of plots, but if nonelastic effects are taken into account as well as panel unstable postbuckling behavior this scatter is noticeably reduced.

The theoretical buckling loads with which the experimental ones were compared, were obtained with the linear "smeared" - stiffener theory of Refs. 16 and 17 for SS3 and SS4 boundary conditions. The effect of discreteness of stringers was checked and found to be negligible.

More details are given in SR - 3 (TAE Report 132).

The test series of ring-stiffened shells included 29 specimens. Most of the shells were in the ring parameter range $A_2/ah < 0.2$, which was not covered by either Ref. 9 or other tests (Ref. 11 and 12). For correlation with other investigations, some specimens were more heavily stiffened to cover the range of (A_2/ah) up to 1.0.

Like in Ref. 9 it was found that the primary stiffener parameter for rings is (A_2/ah) and that for $A_2ah > 0.2$ linear theory is reasonably applicable. Scatter of results is only noticeable in the low range of A_2/ah $(A_2/ah < 0.2)$ where the ring-stiffened shells are nearly as sensitive to imperfections as unstiffened ones. Beyond this value of (A_2/ah) scatter of results is less pronounced and there is a trend towards $\rho = 1$. The "linearity" obtained in the present test program is slightly higher than that obtained in most other tests, probably due to the very careful and precise fabrication of the specimens.

For "heavily" stiffened shells, axisymmetric initial buckling modes were observed. The patterns changed, however, rapidly into diamond-shapes at collapse. More details are given in SR-6 (TAE Report 138).

A special test program was initiated to study the effect of eccentricity of loading on the buckling loads of stringer stiffened shells, which has been predicted to be very significant (see Refs. 18 to 21 and 7). To date, 9 shells, made of 7075-T6 aluminum alloy were tested. To obtain eccentric loading, the shells were loaded through the tips of their stringers, which were designed to protrude. The experimental results were compared with those obtained for the "twin" specimens of the above shells in which the load was introduced through their mid-skins. It was observed that for long shells the influence of eccentricity of loading is indeed very significant. For the geometries tested, the load was reduced up to almost a half of that corresponding to a shell loaded through its mid-skin. For medium length shells the effect still exists, but is very small. It was also observed that the buckling is much less violent for the eccentrically loaded shells and resembles the behavior of a beam column. After unloading, the shell was found to be practically undamaged and could be reloaded to approximately the initial critical load.

This test program is being continued under contract F44620-71-C-0116 and details will appear in a Scientific Report of this contract.

4. COLLOCATION METHOD FOR BUCKLING ANALYSIS OF ELASTICALLY RESTRAINED CONICAL SHELLS.

The collocation method has been studied and employed as an alternative approach to the buckling analysis of shells. Two methods are proposed to deal with the main difficulty inherent in the method, the choice of the collocation points.

The first is based on collocating at the point of maximum deviation from the computed eigenvalue. The deviation, $\delta\lambda$, is mathematically defined as

$$\delta\lambda = - \frac{E(x)}{L_2(y)} \tag{8}$$

where $E(x)$ is the error in the solution of the differential equation

$$L_1(y) + \lambda L_2(y) = 0 \tag{9}$$

The second approach is based on collocating at optimum points for numerical quadrature which also fulfil the integral equation yielded by the minimum potential principle

$$\int_{x_1}^{x_2} [L_1(y) + \lambda L_2(y)\} \delta y \, dx = 0 \tag{10}$$

Another difficulty of the collocation method is the finding of the smallest eigenvalue of the generalized stability matrix. This is treated by matrix transformation and by application of the known Stodola procedure.

The collocation method was first employed to problems with known solutions and there yielded good results. The problems solved included buckling of columns subject to axial compression with different boundary conditions; buckling of columns of linearly varying cross section (EIαx); buckling of conical shells under hydrostatic pressure with different types of simple supports and various geometries.

Then the method was also applied to as yet unsolved problems of the buckling of conical shells under hydrostatic pressure with elastic supports. The results are related to the four major types of in-plane and out-of-plane springs and checked in the limits (spring equal zero or infinity) with known results. More details are given in SR - 7 (TAE Report 137).

5. EXPERIMENTAL STUDY OF THE THERMAL BUCKLING OF CYLINDRICAL SHELLS

As part of the general study of the effects of the combination of mechanical loads and rapid heating on buckling of aerospace structures, an experimental study of thermal buckling of cylindrical shells heated along lines was carried out. The first phase of the program was a series of tests with slow heating along a line reported in Ref. 22. Based on the results of this test series, the multipurpose test rig (Ref. 22) was modified and improved test techniques were initiated. In order to eliminate bending effects, two opposite line heaters were installed and the heating network was rebuilt to facilitate rapid heating and achieve better temperature uniformity.

In the present program the cylindrical shells (made of 2024-T3 Alclad) are submitted to a combination of mechanical axial load and rapid heating. 18 shells were tested. An initial axial load is applied to the shell and then the line heaters are switched on instantly. The cylinders are clamped (approaching conditions $w = w_{,x} = v = u = 0$) at their ends. The compliance with the condition $u = 0$ is achieved by a "manual feed back" - an operator who gradually activates a hydraulic jack during the heating time. In the tests u was found to be about 0.01 mm or less at the moment of buckling.

Each thermal buckling tests is followed by a regular buckling test of the same specimen in the same rig under uniform axial compression. This is possible since the thermal buckling is of a more local nature and less violent. Hence the shell is usually undamaged and can be retested after cooling. The test data is recorded simultaneously on the Beckman data acquisition system and on X-Y recorders. The parallel data on the X-Y recorders serves as visual control during the test and for general verification of the taped results.

Instrumentation was further improved as the tests progressed. Thermocouples, LVDT's, high temperature strain gages and audio signal devices were employed for measurements. For data reduction, a set of special computer programs were developed.

The test program is being continued under contract F 44620-71-C-0116 and details will be reported in a Scientific Report of this contract.

6. BUCKLING OF CYLINDRICAL PANELS UNDER NON-UNIFORM AXIAL COMPRESSION

The earlier investigations on the buckling of cylindrical panels under axial compression were all limited to the case of uniform distribution of the prebuckling stress. The actual case, met in practice, where the axial compression is non-uniformly distributed in the circumferential direction, has however not been studied. Hence the present study was initiated.

The study deals with the buckling of a circular cylindrical panel under axial compression varying in the circumferential direction and constant in the axial direction. Various distributions of the load are investigated including bending and concentrated load. The boundary conditions at the curved edges are those of classical simply supports (SS3). At the straight edges three types of boundary conditions are investigated: SS3, SS4 and a free boundary.

The solution is based on the Donnell equations. A displacement field is chosen so as to satisfy the boundary conditions and two of the equilibrium equations. On the third equation, in the radial direction, the Galerkin method is then applied, yielding for buckling the symmetrical determinant equation:

$$\left\| I^{*}_{mn} + \Theta_n \delta_{mn} - 2\rho\, \sigma_{mn} \right\| = 0 \qquad m,n=1,2,\ldots \qquad (11)$$

The minimum eigenvalue ρ is minimized with respect to the number of axial half waves ($k\beta$). Equation (11) is suitable for investigation of various combinations of load distribution and boundary conditions. The latter influences only I^{*}_{mn}, while the load distribution influences only σ_{mn}.

The final results for ρ_{min} are plotted versus a single parameter representing the geometry of the panel:

$$k^* = 4\sqrt{3(1 - \nu^2)}\,(\frac{\phi_o}{\pi})^2(\frac{R}{h}) \qquad (12)$$

Results show that, unless k^* is very small or the load is very concentrated, the panel will buckle when the maximum stress is slightly higher than the uniform buckling stress for the same boundary conditions. The buckling stress for the SS4 boundary condition is slightly higher than that for SS3, while for the free boundary the buckling stress is much smaller and presents a lower bound for design.

More details are given in SR - 4 (TAE 136).

7. INSTABILITY OF CLOSELY RING STIFFENED CONICAL SHELLS

Earlier analyses of the instability of closely ring stiffened conical shells with "smeared" stiffener theory (Refs. 23 and 24) did not satisfy the in-plane boundary conditions. Hence an improved analysis was derived for stiffened shells based on the displacement method developed for unstiffened conical shells (Refs. 1 and 2). Similar displacements are employed, but whereas the assumed displacements solve the first two stability equations there rigorously, they do not in the case of the stiffened conical shell. Hence a different approach was employed in discretely stiffened conical shells (Ref. 25), and the same approach is applied here with "smeared" stiffener theory. The coefficients in the assumed displacements are adjusted to satisfy the in-plane boundary conditions and the three stability equations are then solved with these "adjusted" displacements by the Galerkin method.

The detailed analysis for buckling of closely ring stiffened conical shells is carried out at the most rigid in plane-boundary conditions SS4 $(u = v = 0)$. More details are given in SR - 8 (TAE 140).

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