ERRATA SHEET


Page 7, table should be:

<table>
<thead>
<tr>
<th>Distance Units</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>u₁, user type 1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>u₂, user type 2</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Page 21, should read:

Case B - Assume a completely centralized library where the average trip is 1-1/2 units.

Page 22, first paragraph, should read:

Substituting a centralized library (Case B) for the high crossover situation (Case A) costs not $238,000 but ($238,000 - $193,000) $45,000 more. Economies of scale, however, may offset the added transportation costs, though it is our opinion that real economies of scale are probably not very large. If the low crossover situation (Case C) actually prevails at the outset, substituting a centralized library costs $67,000 more.

Page 22, Table 5:

Cases should be A, B, and C.
One of the questions that seems to perplex many university and special librarians is whether to move in the direction of centralizing or decentralizing the library's collections and facilities. The Association of Research Libraries, in a report entitled Problems in University Library Management, has stated the problem this way:

Librarians are caught between conflicting pressures for centralization and decentralization of collections and, consequently, facilities. University administrators desire to hold duplication of collections and dispersal of services to a minimum. Faculty and graduate students press for decentralized departmental libraries.

The issue of centralized versus decentralized facilities poses major management problems for university librarians. In planning new construction and considering changes in existing space utilization, the librarian must decide whether it is more efficient and effective to decentralize or to centralize operations. Librarians indicate that little data are available to assist them in making such decisions.

Locating university libraries near the classrooms, offices, or dormitories of those who most frequently use them requires a larger budget.

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expenditure than combining these decentralized, smaller libraries into a large, single facility. Yet many have argued that there is a cost to the university community which is not shown in the university budget—a cost in time, energy, and decreased use resulting from locating the library a longer distance from users. Location theory allows the analyst to examine economies of scale and the cost of overcoming distance simultaneously to determine the optimal location and size of university libraries for a given level of services.²

Location theory considers the overcoming of distance as an input of the production process. Naturally there is a cost incurred in doing this. The library problem is not entirely analogous. University libraries are not producing commodities but rather providing services. Similarly, the objective of the library presumably is not to maximize profits. Defining the library’s objective for this paper in terms of minimizing the cost of maintaining a specific level of benefits simplifies the problem by not allowing the level of services (or "production output") to vary. It is assumed in our model that the level of output is determined by university policy (which itself may be a function of the costs and benefits associated with different levels of library activities).

Presented below is a theoretical approach to the library centralization-decentralization question and several specific applications for M.I.T., though applications to special libraries should be readily apparent.

The Market Orientation of Libraries

Libraries are "market" oriented, that is, their location is sensitive to the location of users, because (1) the average cost of transporting books from the central processing unit is less than transporting books between the library and its users, and (2) the number of books entering the library from the central processing unit is less than the number transported to and from users. At M.I.T., for example, while about 60,000 items must be processed annually (catalogued, stamped, and so forth), these items are delivered only once on weekdays at a regular time by an unskilled messenger to the decentralized library collections. Given the large cost of processing (over $500,000 annually) and the small cost of transporting books to libraries (under $10,000) the economies of scale associated with book processing lead to the maintaining of a single book processing unit. Leaving (and then returning) to the libraries are about 250,000 books per year, transported by highly paid and highly skilled people (faculty and students) up to 14 hours daily at irregular intervals.³

We can model the information production process in the following way: the processing station is a source of inputs, that is, books; the library branches are sites of circulation production using the books received from the processing station; finally, the places where consumers of books and material originate, for example, offices, dormitory rooms, can be considered the market sites. Transportation costs are incurred at each stage of the process.

³ This paper was written after the publication of Systematic Analysis of University Libraries in response to a request for an analysis of the centralization-decentralization question for the M.I.T. collection as a whole and not just the reserve collection which was discussed in the book.
These costs can be lowered in general by locating the circulation production sites, that is, branch libraries, near the markets, if other costs (wages, rents, and so forth) do not vary by location. The analyses below concentrate on the location of libraries vis-a-vis the location of users.

The Location Analysis Approach

East location may be expressed in terms of its map coordinates \((a,b)\). Figure 1, for example, shows a 5 by 11 unit grid, where each unit equals approximately one-tenth of a mile, superimposed over a map of the M.I.T. campus. Thus Building E52 is located at \((5,11)\). Of course, the units could be made finer or coarser in order to adjust to the desired precision. Similarly, the grid could be extended to include off-campus areas such as Harvard Square and Boston.

The distance between any two points \((a_1, b_1)\) and \((a_2, b_2)\) is determined by the formula:

\[
d' = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}
\]

Thus \((5,3)\) is 5 units from \((1,6)\). To reduce the calculations necessary in the quantitative analyses below, an approximation was used. Distance was measured by counting the number of adjacent or diagonally adjacent squares along the shortest route between two points, so that \((5,3)\) would be 4 units from \((1,6)\). For "coarse" grids this distance, \(d\), could also be approximated by the formula \(d = \max(\{|a_2 - a_1|, |b_2 - b_1|\})\) where \(|x|\) is the absolute value of \(x\), that is, the sign of \(x\) is ignored.

\(^{4}\)Let us adopt the convention that \((a,b)\) means row \(a\) and column \(b\). This is convenient since it is consistent with the convention of identifying matrix elements by a row denoter followed by a column denoter.
It is convenient to summarize the distance of a specific location from all other locations. For example, if we know that all students have classes in the "main building," how many distance units away is each possible location of a library? The distance matrix below is a convenient way of summarizing this information:

\[
\begin{bmatrix}
    d_{11} & \cdots & d_{1m} \\
    \vdots & \ddots & \vdots \\
    d_{n1} & \cdots & d_{nm}
\end{bmatrix}
\]

From the location \((a, b)\), the distance matrix tells us that location \((i, j)\) is \(d_{ij}\) units away, where \(i = 1, \ldots, n\) and \(j = 1, \ldots, m\). For example, let us assume a grid which is three units by four units, then

\[
D(2, 2) = \begin{bmatrix}
    1 & 1 & 1 & 2 \\
    1 & 0 & 1 & 2 \\
    1 & 1 & 1 & 2
\end{bmatrix}
\]

The cost of traveling from one location to another is not necessarily proportional to the distance from one location to another. For example, traveling less than one unit may not require the user to leave a building while traveling four units requires a trip outside, travel fatigue, and general inconvenience. The latter trip is usually seen as more than four times as costly as the first. A method of transforming distance into the cost of "overcoming" this distance is required. This transformation must depend on the individual (in general, the type of user), and the method of transportation (walking, automobile).

The transformation factors for each type of user can be expressed in a table. For example, Table 1 below shows the cost of overcoming distance for two types of users.
Table 1
COST OF OVERCOMING DISTANCE
(units arbitrary)

<table>
<thead>
<tr>
<th>Distance Units</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$, user type 1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>$u_2$, user type 2</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

The distance matrices can now be transformed into cost matrices.

Using the rate structures of Table 1,

$$D(2,2) = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

becomes

$$C_{u_1} (2,2) = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$ and

$$C_{u_2} (2,2) = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

The cost matrix $C_{u(a,b)}$ thus tells us the cost of traveling from location $(a,b)$ to each point on campus for each user type $u$. 
Given the cost of one trip to a location, what is the number of trips a set of users will make in a given time period? For this paper we have chosen a time period of one year. To allow for the analysis of decentralized collections, the total library collection is divided into a set of \( J \) nonoverlapping subcollections (each book belongs to one and only one subcollection). Let \( L_j \) denote the \( j \)th subcollection. Clearly it is possible for all subcollections to be located at one point, that is, a centralized library, or for several subcollections to have a common location, or for each subcollection to be housed separately. The whole collection is just the sum \( L = L_1 + L_2 + \ldots + L_J \). Just as the library can be divided into subcollections by location, each user type can be divided by his location or "market." A market is the origin or starting point of users going to the various library subcollections. Thus, for most faculty members their offices are considered as their origin or market. Let there be \( I \) markets; each market can then be denoted \( M_i \), where \( i \) is an integer number from 1 to \( I \). Given these subdivisions of users and collections, the number of trips made by user type \( u \) to library \( j \) from market \( i \) is expressed as \( t_{ij}^u \). The number of trips can be conveniently expressed in a "trip" matrix:

\[
T^u = \begin{bmatrix}
t_{11} & \cdots & \cdots & t_{1J} \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
t_{I1} & \cdots & \cdots & t_{IJ}
\end{bmatrix}
\]
This trip matrix is nothing more than a convenient and economical way of displaying a large amount of information. The same information could be displayed in tables such as Table 2.

Table 2

NUMBER OF TRIPS BY USER TYPE \( u_1 \)

<table>
<thead>
<tr>
<th>Subcollection 1</th>
<th>Subcollection 2</th>
<th>Subcollection 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market or point of origin 1</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>Market of point of origin 2</td>
<td>500</td>
<td>0</td>
</tr>
</tbody>
</table>

NUMBER OF TRIPS BY USER TYPE \( u_2 \)

<table>
<thead>
<tr>
<th>Subcollection 1</th>
<th>Subcollection 2</th>
<th>Subcollection 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market or point of origin 1</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>Market of point of origin 2</td>
<td>250</td>
<td>1000</td>
</tr>
</tbody>
</table>

In this example, the trip matrices would be respectively

\[
T_1 = \begin{bmatrix}
500 & 250 & 1000 \\
500 & 0 & 250
\end{bmatrix}
\]

\[
T_2 = \begin{bmatrix}
1000 & 0 & 500 \\
250 & 1000 & 250
\end{bmatrix}
\]

The total numbers of trips made by all users from all markets to all subcollections is just the sum of all the entries of the trip matrices. In the above example, it would be 5500.
The total cost to a particular user type of traveling from a given market to each possible location of a given library subcollection is found by multiplying the number of trips made by the user from that market to each possible location by the cost of each such trip.

Following the example begin on page 6 and using the data from Tables 1 and 2, the total cost of user type 1 traveling from market 1 located at, say, (2,2) to subcollection 3 is

\[
\begin{bmatrix}
1000 & 1000 & 1000 & 3000 \\
1000 & 0 & 1000 & 3000 \\
1000 & 1000 & 1000 & 3000
\end{bmatrix}
\]

This total cost matrix was obtained by multiplying each entry of \( C_{2,2} \) by \( 1000 ( = t_{13}^1 ) \) from Table 2. Clearly then if library subcollection 3 (\( L_3 \)) were located at (2,2), the same location as user type 1's market 1, the cost would be zero.

It should be evident that the number of trips made by users to a library depend on the distance which the library is from these users. Thus the trip matrix in general is a function of location and is not constant. Practically speaking, determining a trip matrix for every set of locations would involve immense difficulties in data collection and calculations. For this reason we have assumed that the trip matrix is constant. Thus whether \( L_1 \) is close or far, we assume a group of users at \( M_1 \) would make, for example, 1000 trips annually to \( L_1 \). However, it is also assumed that the library user is rational and substitutes other ways of obtaining what the library provides if the traveling costs are too high. In particular, we assume that the library user refuses to incur costs in excess of what the information is worth.
ANALYSIS OF THE MODEL

Locating a Centralized Library

If a campus is to have only one centralized library, it should be located where, for a given level of benefits, transportation costs are minimized. This location can be determined by adding the cost matrices for each user type at each market. Subcollections cannot be divided so there is but one total cost matrix for each user type at each market.

We shall continue our example using data from Tables 1 and 2. Type 1 users make a total of 1750 (500 + 250 + 1000) trips to the library, that is, all subcollections taken together, from M₁ (market 1) and 750(500 + 0 + 250) trips to the library from M₂ (market 2). Similarly, type 2 users make 1500 trips to the library from M₁ and 1500 trips from M₂. The total cost matrix is obtained by multiplying each element of the cost matrix by the total number of trips, as shown in Table 3.

Table 3

TOTAL TRANSPORTATION COST FOR CENTRALIZED FACILITY BY MARKET AND USER TYPE

<table>
<thead>
<tr>
<th>User Type</th>
<th>Number of Trips</th>
<th>Cost Matrix</th>
<th>Total Cost Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1750</td>
<td>[1 1 1]</td>
<td>1750 1750 1750 5250</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1 0 1 3]</td>
<td>1750 0 1750 5250</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1 1 1 3]</td>
<td>1750 1750 1750 5250</td>
</tr>
<tr>
<td>2</td>
<td>1500</td>
<td>[1 1 1 2]</td>
<td>1500 1500 1500 3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1 0 1 2]</td>
<td>1500 0 1500 3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1 1 1 2]</td>
<td>1500 1500 1500 3000</td>
</tr>
</tbody>
</table>

From M₂; assume M₂ at (3,4)

<table>
<thead>
<tr>
<th>User Type</th>
<th>Number of Trips</th>
<th>Cost Matrix</th>
<th>Total Cost Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>750</td>
<td>[5 3 3 3]</td>
<td>3750 2250 2250 2250</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[5 3 1 1]</td>
<td>3750 2250 750 750</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[5 3 1 0]</td>
<td>3750 2250 750 0</td>
</tr>
</tbody>
</table>
Table 3, continued.

<table>
<thead>
<tr>
<th>User Type</th>
<th>Number of Trips</th>
<th>Cost Matrix</th>
<th>Total Cost Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1500</td>
<td>3 2 2 2</td>
<td>1500 3000 3000 3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 2 1 1</td>
<td>4500 3000 1500 1500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 2 1 0</td>
<td>4500 3000 1500 0</td>
</tr>
</tbody>
</table>

The total cost for all users is then

\[
\begin{bmatrix}
11,500 & 8,500 & 8,500 & 13,500 \\
11,500 & 5,250 & 5,500 & 10,500 \\
11,500 & 8,500 & 5,500 & 8,250 \\
\end{bmatrix}
\]

In this example, one location minimizes transportation costs. That one location is, of course, (2,2).

When the transportation rate structure is linear with distance, the center of gravity minimizes transportation costs; when the marginal cost of traveling increases with distance, the markets become more attractive as library locations.

The model allows an analysis of the effects of building new libraries. If, in the example immediately above, two libraries could have been built with identical collections, they would have been located at each market in order to minimize transportation costs. Yet this might be uneconomical where the number of users of each library differs greatly. One library's collection would be overused, the other's underused, and perhaps one library would be overcrowded while the other one was empty. There is a force toward dividing the trips and users (these may not be directly related) equally between the two. When there are more than two markets, it is possible to estimate which users will use each collection by assuming users will travel
so as to minimize their transportation costs until the libraries are used about equally.

For example, assume that besides the total cost matrices for \( M_1 \) at (2,2) and \( M_2 \) at (3,4) the example included markets \( M_3 \) at (1,1) and \( M_4 \) at (1,3). The appropriate total cost matrices would be:

<table>
<thead>
<tr>
<th>Total cost for all users</th>
<th>( M_1 ) at (2,2)</th>
<th>( M_2 ) at (3,4)</th>
<th>( M_3 ) at (1,1)</th>
<th>( M_4 ) at (1,3)</th>
</tr>
</thead>
</table>
|                          | \[
|\begin{array}{cccc}
3500 & 3500 & 3500 & 9000 \\
3500 & 0   & 3500 & 9000 \\
3500 & 3500 & 3500 & 9000 \\
\end{array}
\] | \[
|\begin{array}{cccc}
9500 & 6000 & 6000 & 6000 \\
9500 & 6000 & 2500 & 2500 \\
9500 & 6000 & 2500 & 0 \\
\end{array}
\] | \[
|\begin{array}{cccc}
0 & 5000 & 10,000 & 15,000 \\
5000 & 5000 & 10,000 & 15,000 \\
5000 & 5000 & 10,000 & 15,000 \\
\end{array}
\] | \[
|\begin{array}{cccc}
5000 & 1000 & 0 & 1000 \\
5000 & 1000 & 1000 & 1000 \\
5000 & 1000 & 1000 & 1000 \\
\end{array}
\] |

Those at \( M_1 \) will use a library at (1,4), (2,4), or (3,4) only as a last resort, and will prefer the adjacent library which is least crowded. If two identical libraries are to be built, each set of feasible locations must be examined. Thus set I might include \( L \) at (3,3) and \( L' \) at (2,2).

5These total cost matrices have not been derived from any previously given data. They were assumed in order to illustrate our point.
Those at $M_1$ would prefer (2,2), those at $M_2$, (3,3), those at $M_3$, (2,2), and those at $M_4$ would be indifferent as far as transportation considerations. Set II might be $L$ at (1,1) and $L'$ at (2,3). For each possible set of library locations for $L$ and $L'$, the users of each library and the total cost of transportation to users can be determined, as shown in Table 4.

Table 4

SET I, L AT (3,3) AND L AT (2,2)

<table>
<thead>
<tr>
<th>Users at:</th>
<th>Library Used</th>
<th>Cost to Users</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>$L'$</td>
<td>0</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$L$</td>
<td>2500</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$L'$</td>
<td>5000</td>
</tr>
<tr>
<td>$M_4$</td>
<td>$L$</td>
<td>1000</td>
</tr>
</tbody>
</table>

Total Transportation Cost 8500

SET II, L AT (1,1) and L AT (2,3)

<table>
<thead>
<tr>
<th>Users at:</th>
<th>Library Used</th>
<th>Cost to Users</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>$L$</td>
<td>3500</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$L'$</td>
<td>2500</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$L$</td>
<td>0</td>
</tr>
<tr>
<td>$M_4$</td>
<td>$L'$</td>
<td>1000</td>
</tr>
</tbody>
</table>

Total Transportation Cost 7000

Comparing these two possible sets of locations, set II would be preferred by the library planners trying to minimize transportation costs.
The above analysis could also be applied to the case of adding an identical library to determine who would use it and the resulting savings in transportation costs. Thus if only L at (1,1) was operative, users from all four markets would use it at a total transportation cost of 18,000. Building L' at (2,3) reduced these costs to 7,000, a saving of 11,000 cost units. Thus the benefits of building new libraries extend beyond the reduction of overcrowding and may be measured quantitatively.

In most cases, however, decentralizing the library means subdividing the collection and not building identical collections. Assume

\[
\begin{bmatrix}
1000 & 0 & 0 \\
0 & 5000 & 1000 \\
0 & 5000 & 1000
\end{bmatrix}
\]

In \( T_1^u \) only those from \( M_1 \) use \( L_1 \), that is, subcollection 1, and only those from \( M_2 \) use \( L_2 \) and \( L_3 \). Assuming no economies of scale, \( L_1 \) should be located at \( M_1 \), and both \( L_2 \) and \( L_3 \) at \( M_2 \) to minimize costs. Note that it makes no difference how far those at \( M_1 \) will be from \( L_2 \) and \( L_3 \) since they never use these libraries. Thus in the case of no crossover and no economies of scale decentralization is optimal.

Now assume:

\[
\begin{bmatrix}
1500 & 500 & 1500 \\
1000 & 1000 & 500
\end{bmatrix}
\]

The case for decentralization is not easily made. Since crossover exists, for example, those at \( M_1 \) sometimes must travel to \( L_2 \) or \( L_3 \), the cost of these excursions and thus the distance between \( M_1 \) and \( L_2 \) and \( L_3 \) becomes a factor. This is handled adequately by the model discussed above.
For each market, the cost of traveling to each possible location of the three libraries is found as before by multiplying the number of trips to each library by the cost matrix. For each library, then, the total cost matrices are summed to determine the total transportation cost for each possible location of that library.

**An Example**

Given the trip matrix for user type u

\[
T_u = \begin{bmatrix}
1500 & 500 & 1500 \\
1000 & 1000 & 500
\end{bmatrix}
\]

and given the fact that market 1 is located at (2,2) while market 2 is located at (3,4), then cost matrices are respectively:

\[
C_u^{(2,2)} = \begin{bmatrix}
1 & 1 & 1 & 3 \\
1 & 0 & 1 & 3 \\
1 & 1 & 1 & 3
\end{bmatrix}
\]

\[
C_u^{(3,4)} = \begin{bmatrix}
5 & 3 & 3 & 3 \\
5 & 3 & 1 & 1 \\
5 & 3 & 1 & 0
\end{bmatrix}
\]

From the above data:

- From M_1 trips to L_1 = 1500, Cost for L_1 =
  \[
  1500 \quad 1500 \quad 1500 \quad 4500 \\
  1500 \quad 0 \quad 1500 \quad 4500 \\
  1500 \quad 1500 \quad 1500 \quad 4500
  \]

- L_2 = 500, Cost for L_2 =
  \[
  500 \quad 500 \quad 500 \quad 1500 \\
  500 \quad 0 \quad 500 \quad 1500 \\
  500 \quad 500 \quad 500 \quad 1500
  \]

- L_3 = 1500, Cost for L_3 =
  \[
  1500 \quad 1500 \quad 1500 \quad 4500 \\
  1500 \quad 0 \quad 1500 \quad 4500 \\
  1500 \quad 1500 \quad 1500 \quad 4500
  \]
From \( M_2 \), trips to \( L_1 \) = 1000, Cost for \( L_1 \) =
\[
\begin{bmatrix}
5000 & 3000 & 3000 & 3000 \\
5000 & 3000 & 1000 & 1000 \\
5000 & 3000 & 1000 & 0 \\
\end{bmatrix}
\]

\( L_2 \) = 1000, Cost for \( L_2 \) =
\[
\begin{bmatrix}
5000 & 3000 & 3000 & 3000 \\
5000 & 3000 & 1000 & 1000 \\
5000 & 3000 & 1000 & 0 \\
\end{bmatrix}
\]

\( L_3 \) = 500, Cost for \( L_3 \) =
\[
\begin{bmatrix}
2500 & 1500 & 1500 & 1500 \\
2500 & 1500 & 500 & 500 \\
2500 & 1500 & 500 & 0 \\
\end{bmatrix}
\]

Summing the two matrices for each \( L_i \):

**Total cost for \( L_1 \): Sum** =
\[
\begin{bmatrix}
6500 & 4500 & 4500 & 7500 \\
6500 & 3000 & 2500 & 5500 \\
6500 & 4500 & 2500 & 4500 \\
\end{bmatrix}
\]

**Total cost for \( L_2 \): Sum** =
\[
\begin{bmatrix}
5500 & 3500 & 3500 & 4500 \\
5500 & 3000 & 1500 & 2500 \\
5500 & 3500 & 1500 & 1500 \\
\end{bmatrix}
\]

**Total cost for \( L_3 \): Sum** =
\[
\begin{bmatrix}
4000 & 3000 & 3000 & 6000 \\
4000 & 1500 & 2000 & 5000 \\
4000 & 3000 & 2000 & 4500 \\
\end{bmatrix}
\]

If there were no economies of scale, \( L_1 \) should be located at \((2,3)\) or \((3,3)\), \( L_2 \) at \((3,4), (2,3), \) or \((3,3)\), and \( L_3 \) at \((2,2)\) to minimize transportation costs.
Economies of scale, however, may make minimizing transportation costs a less than optimal solution. For example, if \( L_1 \) were built at (2,3) \( L_2 \) at (3,4) and \( L_3 \) at (2,2) the total cost of transportation would equal \((5500 + 1500 + 1500)\) or 5500 cost units. If \( L_1 \) were built at (2,2), thus combined with \( L_3 \), the total transportation cost would rise 500 units to 6000 cost units. Would combining \( L_1 \) and \( L_3 \) be an optimal solution? Without economies of scale it would not, but with such economies, that is, cost savings in building and operating due to increased size, the question becomes one of the magnitude of such savings.

A Further Application: The Complete M.I.T. Library System

As will be noted below, the data necessary for a complete location analysis relevant to current M.I.T. library planning is not now available. This discussion is thus limited to specifying the information that is required and further illustrating the value of location analysis.

Figure 2 is the trip matrix which cannot now be filled with the necessary data. The libraries are divided into three sections, study hall, reserve and required reading, and research. This division allows for the separate consideration of the centralization of each part of the library. This is suggested because of the (1) apparent differences in the economies of scale among the three divisions (moderate economies for study halls, small, if any, for reserve and research), and (2) the probable differences in user transportation behavior among the three sections (given the cyclical nature of research and required reading and the use of study halls between classes). For example, it is hypothesized that research work is specific to a given library collection and allows for little substitution by purchasing books at the Coop.\(^6\)

\(^6\)M.I.T.'s campus bookstore.
Figure 2

TRIP MATRIX FOR M.I.T. LIBRARY SYSTEM

<table>
<thead>
<tr>
<th>Research</th>
<th>Study Hall</th>
<th>Reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1L_2L_3L_4L_5L_6L_7$</td>
<td>$L_8L_9L_{10}L_{11}L_{12}L_{13}L_{14}L_{15}$</td>
<td>$L_{16}L_{17}L_{18}L_{19}L_{20}L_{21}L_{22}$</td>
</tr>
</tbody>
</table>

- **Origins**
  - M_1
  - M_2
  - M_3
  - M_4
  - M_5

- **Departments, Offices**
  - M_6
  - M_7
  - M_8
  - M_9
  - M_{10}
  - M_{11}
  - M_{12}
  - M_{13}

- **Arrival Points on Campus**
  - M_{14}
  - M_{15}

Note: The table represents the movement matrix for different locations in the MIT library system.
or consolidating trips, while the use of a specific study hall would be very sensitive to transportation costs. If this were true, then for the former it would be possible to assume an "inelastic" trip matrix, while the latter implies an "elastic" trip matrix. The above hypothesis could be partially tested by determining if a given library branch tended to be used as a study hall by those with the closest origins.

To separate these three functions which the library serves, users could be asked to indicate their purpose for each trip to the library. Where more than one purpose is indicated, the purposes may be placed in some hierarchical order. Research in a specific library usual does not have a substitute, required reading may be postponed or curtailed, and studying may be done at many convenient locations on campus. Thus, where motives are reported as mixed, the more restrictive motive could be assumed. (There are alternative ways of analyzing these responses, but a priori this appears the most fruitful.)

The remainder of the information would include:

1. Origin of the trip to library (campus building or its approximate locations, home, or transit stop).

2. Background information—user's status, department.

3. Library information—specific library, day, time that individual entered.

4. Next destination, if known (is library a convenient or inconvenient stopping off point?).

This type of survey has been done (in less specific terms) by Bush, Gallanter, and Morse in 1956 for the Science Library at M.I.T.¹ It is feasible

and could be easily attached to a general user survey, which might include questions about user behavior while in the library. The number of times a user is not satisfied and must go to another library could also be determined. Given the requirement for data about each of the libraries and for relatively infrequent users, samples would have to be taken periodically, at different hours on different days, for each library.

Rather than assume a trip matrix and develop an analysis as above, a simplified example illustrates the great impact of the factor termed crossover, the degree to which those at one market or point of origin use libraries at more than one location. Assume students make 1,000,000 trips to the library annually and faculty 100,000; students walk at a rate of 4 miles per hour and faculty at 3 m.p.h.; and one distance unit equals one-tenth of a mile. The cases can now be analyzed as in Table 5.

Case A - Assume high crossover where 25 percent of the student and 10 percent of the faculty trips average 2 units in length, the remainder, 1 unit.

Case B - Assume a completely centralized library where the average trip is 3 units.

Case C - Assume low crossover where 10 percent of the students and none of the faculty trips average 2 units, the remainder, 1 unit.

8In reality, the averages should all be weighted, probably by a rate structure reflecting increasing marginal cost as distance traveled increases.
Table 5
ANALYSIS OF THE EFFECT OF "CROSSOVER"

<table>
<thead>
<tr>
<th>Case</th>
<th>User Type</th>
<th>Units Traversed</th>
<th>Miles per Unit</th>
<th>Miles Walked per Hour</th>
<th>$/hr.</th>
<th>Approximate Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Students</td>
<td>1,250,000</td>
<td>1/10</td>
<td>4</td>
<td>$5</td>
<td>$156,000</td>
<td>$193,000</td>
</tr>
<tr>
<td></td>
<td>Faculty</td>
<td>110,000</td>
<td>1/10</td>
<td>3</td>
<td>10</td>
<td>$37,000</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Students</td>
<td>1,500,000</td>
<td>1/10</td>
<td>4</td>
<td>$5</td>
<td>$188,000</td>
<td>238,000</td>
</tr>
<tr>
<td></td>
<td>Faculty</td>
<td>150,000</td>
<td>1/10</td>
<td>3</td>
<td>10</td>
<td>$50,000</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Students</td>
<td>1,100,000</td>
<td>1/10</td>
<td>4</td>
<td>$5</td>
<td>$138,000</td>
<td>171,000</td>
</tr>
<tr>
<td></td>
<td>Faculty</td>
<td>100,000</td>
<td>1/10</td>
<td>3</td>
<td>10</td>
<td>$33,000</td>
<td></td>
</tr>
</tbody>
</table>

In this illustration three special cases have been examined; in a complete analysis all possible locations are carried through the analysis. Substituting a centralized library (Case B) for the high crossover (Case A) costs not $238,000 but ($238,000 - $173,000) $45,000 more. Economies of scale, however, may offset the added transportation costs, though it is our opinion that real economies of scale are probably not very large.

The decision to centralize thus depends upon the amount of crossover in alternative systems and economies of scale. Location analysis provides a framework to combine these elements. The assumptions which must be made for such an analysis, pertaining to the linearity and absolute level of transportation rate structures, may be partially tested. At worse, such an analysis could change the problem from one of coping with general impressions to one of making some quantitative determinations.