Dynamic Analysis of the Graze Module of the Hi-Performance Point Detonating Fuge (U)

Final Report

Dr. Joseph F. Shelley
July, 1971

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Dover, New Jersey 07801

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The equations of motion are presented for the inertia weight, firing pin and detent balls of the graze module of the high performance point detonating fuze. These equations are all in terms of the generalized graze forcing functions. The equations are also presented for the case where the graze forcing function-time plot is assumed to have a triangular shape. The criteria are established for the minimum values of forcing functions required to activate the graze module. All differential and constraint equations are presented in numerical form, but no numerical results are obtained. The numerical constants used are for the 105mm Howitzer Shell, M1.
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1. INTRODUCTION

The Fuze Research and Engineering Division, AED, at Picatinny Arsenal presently has an ongoing development program for the Hi-Performance fuze. One element of this program is the response of the graze module of the fuze to various forcing functions acting on the shell casing. The senior scientist of Edutronics Analysis, Inc., Dr. Joseph F. Shelley, had formerly been a consultant to the Fuze Research and Engineering Division, Picatinny Arsenal, through the Army Research Organization at Duke University. In this capacity he performed a preliminary analysis of the operation of the graze module. The results of this analysis identified the conditions which are required to obtain dynamic response of the graze module under the application of the grazing forces. This study indicated that a more detailed analysis of the motion of the several elements of the graze module, with various values of forcing functions, was required in order to define the dynamic force-motion history of this fuze mechanism.

The present Picatinny Arsenal design of the graze module of the Hi-Performance fuze is a purely mechanical device. Its essential elements are a weight which is concentric with the long axis of the shell, and which has an inclined face, eight steel balls which are equally spaced about the long axis of the shell, three springs which hold the weight in a rearward position against the balls, and a housing which contains the entire assembly.

In the case of normal impact of a shell on a terrain the firing function is straightforward, since the force of the decelerating shell is available to activate the central firing plunger in the nose of the shell. When the shell grazes the terrain, however, there will be some angle of graze below which the central firing plunger will not be activated. The graze module is designed to effect the firing function in this case.

When the shell grazes the terrain, the resulting lateral acceleration field acting on the graze module has the effect of forcing several of the steel balls against the inclined face of the weight. The weight then moves forward towards the nose of the shell. As this weight moves forward, two detent lock-

---

In order to evaluate the effectiveness of the graze module, both as an individual mechanism and as an element of the fuze system, the following information is required.

1. The minimum magnitudes of the external graze forcing functions on the shell casing which will cause the device to function.
2. The time for the weight to move forward sufficiently to release the two detent balls.
3. The time, after the release of the two detent balls for the internal firing pin to move to the closed position with sufficient energy to activate the initiator.

2. OPERATION OF THE GRAZE MODULE

Figures 1 and 2 show the elements of the graze module. The vertical center axis in Figure 1 is the shell longitudinal center axis. In the inactive position, the 8 steel balls, A, and the 4 detent locks, C, are in the position shown. The 3 creep springs, F, force the inertia weight, B, downward at all times. When the shell leaves the barrel, the spin velocity causes the detent locks, C, to assume maximum outboard positions, so that the inertia weight is free to move upward. When the shell grazes on the terrain, several of the steel balls, A, are forced radially inward and the inertia weight, B, is forced upward. When this inertia weight has moved upward by a sufficient amount, the two firing pin detent balls, D, move radially outward. After sufficient movement of these two balls, the firing pin, E, moves downward under the influence of the firing pin spring, G. When the firing pin assumes its maximum downward position, the firing process is initiated.

3. ASSUMPTIONS

The assumptions used in the present study are listed below.

a. The present analysis is for the case where the graze module is in a partially static condition. As the shell grazes the terrain, there will be a graze force acting on it for a short duration. Due to the spin of the shell, there will be a continually varying position of any graze ball with respect to the line of action of this force. It will be assumed in this first analysis, subject to confirmation, that the time for the total firing process to occur is less than the time for any ball to occupy a successive position. The effect of this assumption is to reduce the problem to the situation de-
Figure 1 - Graze Module - Side View
Figure 2 - Graze Module - Top View
picted in Figure 3. In this Figure, $a_T$ is the total lateral acceleration of the graze module housing. Balls 1 and 2 will be referred to as the primary and secondary balls, respectively. The assumption implied by Figure 3 is that the present study considers the case where the radial position vector of ball 1 is collinear with the direction of $a_T$, and that these directions are constant with respect to each other.

b. All friction forces are neglected except the friction forces between:
   1. the inertia weight and the firing pin housing of the graze module.
   2. the firing pin and the firing pin housing of the graze module.

c. The coefficient of friction between all sliding parts is constant.
d. The spin velocity of the shell is constant, and the other components of rotation of the shell are small.

e. During graze the shell moves in a plane which is normal to the graze terrain, and the shell remains at a constant angle with respect to this terrain.

f. The graze module activating balls translate only in a plane through the center axis of the graze module, and thus the tendency of the graze balls to move circumferentially out of their radial groove is neglected.

g. The direction of $a_m$ is collinear with the line joining the two detent balls. This orientation is also assumed to be constant throughout graze, for the reasons given in assumption a.

h. The dimension $C$, is constant, and the details of this conclusion are recorded in Appendix D.

i. The graze force $P$ acts at the tip of the ogive shell nose.

j. The mass moment of inertia of the firing pin, about an axis through its center of mass and normal to the longitudinal axis of the shell, is negligible.

k. The firing pin only moves forward with respect to the graze module housing.

l. The firing pin spring is a massless element with a constant spring rate.

m. The forcing function for one case is assumed to have a triangular shape, and for this case the equations of motion are expressed in numerical form.

n. The centripetal acceleration of the graze module, with respect to an axis through the center of mass of the shell and normal to the center axis of the shell, is negligible.
4. ACCELERATION FIELD OF THE GRAZE MODULE DUE TO THE SHELL FORCING FUNCTIONS

The typical shell is shown in Figure 4. The dimensions $d$ and $l$ are peculiar.

\[ a_T = a_N + (l - d - 4) \alpha \]  
\[ c_T = \frac{P_N}{m} + (l - d - 4) P_N \frac{(l - d)}{I_{CM}} \]

Figure 4 - Shell and Graze Force Relationship

to a given shell. $\alpha_A, \alpha_N$ and $\alpha$ are components of acceleration of the center of mass, CM, of the shell. $P_A$ and $P_N$ are components of the graze force $P(t)$ and $\alpha$ is the graze angle. The graze module is at a constant distance from the nose for all shells. The graze force $P$ is some complicated function of time, and it will be carried through the present derivation in symbolic form. The equations of motion of the shell are

\[ P_N = m \alpha_N \]  
\[ P_A = m \alpha_A \]  
\[ P_N (l - d) = I_{CM} \alpha \]
Using

\[ \overline{I_{CM}} = m \overline{r} \]

where \( I_{CM} \) is the radius of gyration,

\[ a_T = \frac{p_N}{m} \left[ 1 + \frac{(k_2 - k_1)(k_1 - k_2 - 4)}{2 k_1 k_2} \right] \]  

Equation (4-7) may be used directly to find \( a_T \), when \( p_N \) is known.

From Equation (4-7)

\[ a_T \sim p_N \]  

and from Equation (4-2),

\[ a_A \sim p_A \]  

\( p_A \) and \( p_N \) are components of the same force, so that

\[ p_A \sim p_N \]  

and therefore

\[ a_A \sim p_N \]  

Finally, from Equation (4-3)

\[ a \sim p_N \]  

Consideration of the above results now leads to the conclusion

\[ a_A, a \sim a_T \]  

or

\[ a_A = k_1 a_T \]  

\[ a = k_2 a_T \]  

where \( k_1 \) and \( k_2 \) are known constants. These results will subsequently be used to express the right hand sides of the equations of motion in terms of \( a_T \) only.

For the purpose of the present analysis, the graze module will be assumed to experience acceleration components which are oriented as in Figure 5.
5. ABSOLUTE ACCELERATION OF THE GRAZE MODULE ACTIVATING BALLS

To obtain the accelerations of the balls and of the inertia weight, a secondary coordinate system will be used. Figure 6 shows the general system.

\[ \ddot{r} = \ddot{r}_\omega + \ddot{r}_n + \frac{d\omega}{dt} \times \ddot{r}_n + 2 \omega \times \ddot{r}_n + \ddot{\omega} \times (\ddot{\omega} \times \ddot{r}_n) \]

Each of these terms will now be identified. \( \ddot{r} \) is the absolute acceleration of \( O' \), the origin of the secondary coordinate system.

*The senses of certain components, for convenience, are different from those shown in Figure 4.
\[ \ddot{\mathbf{r}}_n = \dot{\mathbf{c}} + \dot{\mathbf{j}} + \dot{\mathbf{k}} \cdot \dot{\mathbf{z}} \], where \( \mathbf{c}, \mathbf{j}, \mathbf{k} \) are unit vectors in the \( x, y, z \) coordinate system, is the relative acceleration, measured in the \( x, y, z \) frame of reference.

\[ \ddot{\mathbf{r}}_n = \dot{\mathbf{c}} + \dot{\mathbf{j}} + \dot{\mathbf{k}} \cdot \dot{\mathbf{z}} \] is the relative velocity, measured in the \( x, y, z \) frame of reference.

\[ \mathbf{r}_n = \mathbf{c} + \mathbf{j} + \mathbf{k} \cdot \mathbf{z} \] is the position vector, in the \( x, y, z \) system, of the point \( P \).

\( \frac{d\mathbf{\omega}}{dt} \) is the time rate of change of \( \mathbf{\omega} \)

\( \frac{d\mathbf{\omega}}{dt} \times \mathbf{r}_n \) is the tangential acceleration effect

\( 2\mathbf{\omega} \times \mathbf{r}_n \) is the coriolis acceleration

\( \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}) \) is the centripetal acceleration.

From Figure 3, the primary activating ball, Ball 1, lies on the line of action of the graze module lateral acceleration \( a_m \). The secondary balls, Balls 2, are located symmetrically with respect to Ball 1. Equation (5-1) will now be used to obtain the absolute accelerations of Balls 1 and 2.

The secondary \( x, y, z \) coordinate axes are attached to the graze module housing. The \( x \) axis passes through the center of the primary Ball 1 and the \( y \) axis is collinear with the axis of the inertia weight. The orientation is shown in Figure 7. These coordinates will rotate with the graze module and thus possesses the spin velocity of the shell. For this orientation of \( a_m \), the primary ball is forced radially inward and it tends to move the inertia weight forward.

The absolute acceleration \( \ddot{\mathbf{r}}_{0,1} \) of the primary ball will now be obtained. The rotation \( \mathbf{\omega} \) is

\[ \dot{\mathbf{\omega}} = \dot{\mathbf{j}} \cdot \omega_y \]  \hspace{1cm} (5-2)

since \( \dot{\mathbf{j}} \cdot \omega_y \) and \( \omega_x \cdot \omega_y \), the spin velocity, is assumed to be constant. The position vector \( \mathbf{r}_n \) of Ball 1 is

\[ \mathbf{r}_n = \mathbf{c} \cdot x \]  \hspace{1cm} (5-3)

so that \( x \) locates the center of Ball 1.
Figure 7 - Coordinate Axes Orientation on Graze Module

The several other terms now appear as

\[
\frac{\ddot{R}}{R} = \bar{a}_T + \bar{J}(-\dot{a}_A) \tag{5-4}
\]

\[
\frac{\dot{R}}{R} \times R = \bar{a} \times
\]

\[
\frac{\ddot{R}}{R} \times R = \bar{c} \times R = 0 \tag{5-7}
\]

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The final result for the absolute acceleration is then

\[
\ddot{\bar{a}}_{o,l} = \int \left[ \dot{\bar{X}} - \dot{\bar{x}}_w \bar{a}_T \right] + \int \left[ -\bar{a}_A \right] + \bar{K} \left[ -2\dot{\omega}_y \dot{X} \right] \tag{5-11}
\]

Figure 8 shows the primary ball 1 and the two secondary balls 2. The acceleration field \( \dot{a}_T \) was assumed to be uniform along a direction normal to the shell axis and have a constant direction with respect to the position of the three balls. Thus, if Ball 1 experiences \( \dot{a}_T \), then Balls 2 experience a corresponding acceleration \( \dot{a}_T \) and \( \bar{a}_0 \). The acceleration of Balls 2, \( \bar{a}_{o,l} \), would then be given by Equation (5-11) with \( \dot{a}_T \) replaced by \( 0.707 \dot{a}_T \).
Figure 8 - Acceleration Components Acting on Graze Balls

For this case, the secondary x axis would pass through Ball 2.

It may be seen from Equation (5-11) that there will be an inertia force on the balls in the z direction, of magnitude \( 2m_2 \omega^2 \), where \( m_2 \) is the mass of the activating ball. This force will tend to displace the balls in a circumferential direction. This effect is assumed to be small compared to the other forces on the ball and accounts, in retrospect, for assumption f.

6. DYNAMIC EQUILIBRIUM REQUIREMENTS OF THE GRAZE MODULE ACTIVATING BALLS

The free body diagram of the primary ball 1, together with the inertia forces, is shown in Figure 9. The ramp angle of the inertia weight is \( \Theta \).

\( F_{w,1} \) is the reaction force on Ball 1 by the floor of the graze module, and \( F_{n,1} \) is the normal reaction force between Ball 1 and the inertia weight. The friction forces induced by \( F_{w,1} \) and \( F_{n,1} \) are zero, in view of assumption b. These effects could be included in a more refined study.
The components of $F_{11}$ are related by

$$F_{1x_1} = F_{1y_1} \tan \theta \quad (6-1)$$

A necessary constraint condition is that

$$F_{11} \geq 0 \quad (6-2)$$
$$F_{ww} \geq 0 \quad (6-3)$$

since these forces are compressive contact forces.

Since Figure 9 is a concurrent force system, the equilibrium requirements are
The constraint equation is

\[ \dot{x} = x_0 \]

where \( x_0 \) is defined on page 88.

\[
F_{1y,1} = m_G c_A^L + F_{u,1}
\] (6-5)

\( F_{1x,1} \) is eliminated between Equations (6-1) and (6-4), with the result

\[
F_{1y,1} = m_G \cot \theta (\ddot{x} + a_T - x \dot{y}^2)
\] (6-6)

Equations (6-5) and (6-6) are now combined to find \( F_{u,1} \) and

\[
F_{u,1} = m_G \cot \theta (\ddot{x} + a_T - x \dot{y}^2) - m_G a A
\] (6-7)

The above equation is obviously true only for \( F_{u,1} > 0 \).

The opening effect of the ball on the inertia weight is indicated by the magnitude of \( F_{u,1} \). From Equation (6-6), as the ball moves inward \( x < 0 \) and \( x \) decreases and only the term \( a_T \) is always positive.

Since the desired final result of this study is an equation of motion of the inertia weight in terms of \( y \), all \( x \) quantities and their derivatives will be expressed in terms of \( y \). Figure 10 shows the kinematic relationships as the ball moves inward.

![Figure 10 - Kinematic Relationship Between Graze Ball and Inertia Weight](image-url)
\[
\tan \theta = \frac{\Delta y}{\Delta x} \quad (6-8)
\]
\[
\Delta y = -\Delta x \tan \theta \quad (6-9)
\]
\[
\frac{\Delta y}{\Delta t} = -\frac{\Delta x}{\Delta t} \tan \theta \quad (6-10)
\]
\[
\ddot{y} = -\ddot{x} \tan \theta \quad (6-11)
\]
\[
\ddot{x} = -\ddot{y} \cot \theta \quad (6-12)
\]

\(x\) and \(y\) displacements are related by an equation of the form

\[
x = C_\theta y + C_\varphi \quad (6-13)
\]

where \(C_\theta\) and \(C_\varphi\) are constants to be chosen later. Equation (6-6) now appears as

\[
F_{1y,1} = m_G \omega^2 \theta \left[ -\dot{y} \omega \theta + a_T - (c_\theta y + c_\varphi)w\right] \quad (6-14)
\]

The \(y\) components of the reaction forces on Balls 2, designated by \(F_{1y,2}\), are then

\[
F_{1y,2} = m_G \omega^2 \theta \left[ -\dot{y} \omega \theta + 0.707a_T - (c_\theta y + c_\varphi)w\right] \quad (6-15)
\]

Each of the remaining five balls will exert a direct axial force \(F_{1y,3}\), on the inertia weight of magnitude

\[
F_{1y,3} = m_G a_A \quad (6-16)
\]

and acting in the positive \(y\) sense.

The total force \(F_{y,\text{tot}}\) of the balls on the inertia weight, acting in the positive \(y\) sense, is then

\[
F_{y,\text{tot}} = F_{1y,1} + 2F_{1y,2} + 5F_{1y,3} \quad (6-17)
\]

\[
F_{y,\text{tot}} = m_G \omega^2 \theta \left[ -\dot{y} \omega \theta + a_T - (c_\theta y + c_\varphi)w\right]
\]

\[
+ m_G \omega^2 \theta \left[ -\dot{y} \omega \theta + 0.707a_T - (c_\theta y + c_\varphi)w\right]
\]

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The free body diagram of the secondary balls 2 is shown in Figure 11. The equilibrium equations for balls 2 are

\[ F_{y,\text{tot.}} = m_G \left\{ \cot \theta \left[ -s^2 \cot \theta + \varepsilon + \dot{\alpha}_T \right] \right\} + S a_A \] (6-18)

\[ + 5 m_G \ddot{a}_A \] (6-19)

The free body diagram of the secondary balls 2 is shown in Figure 11. The equilibrium equations for balls 2 are

\[ F_{1y,2} = F_{w,2} + m_G \ddot{a}_A \] (6-20)

\[ F_{1x,2} = F_{1y,2} \tan \theta = m_G \left[ 0.707 \alpha_T - \ddot{y} \cot \theta - (C_R y + C_Q) \dot{\omega}_y \right] \] (6-21)

Each of balls 2 exert less axial force on the inertia weight because of the term \( 0.707 \alpha_T \) in Equation (6-21) compared with the term \( a_T \) in Equation (6-14). A limiting condition for ball 2 is reached when

\[ F_{b,2} \to 0 \] (6-22)

At this condition, following Equation (6-20),

\[ F_{1y,2} \to m_G \ddot{a}_A \] (6-23)
Figure 11 - Free Body Diagram of Secondary Ball

The graze forcing functions are applied to the shell at $t = 0^+$. If $t_o$ is the time corresponding to Equation (6-23), then Equation (6-21) reduces to

\[
\begin{align*}
\begin{cases}
m_G a_A \tan \theta &= m_G \left[0.707 a_T - \ddot{y} \cot \theta \right] \\
&- (c_y y + c_y) w_1^2 \right] \bigg|_{t = t_o} 
\end{cases}
\end{align*}
\]

or

\[
\begin{align*}
\begin{cases}
\ddot{y} &= \tan \theta \left[0.707 a_T - (c_y y + c_y) w_1^2 - a_A \tan \theta \right] \bigg|_{t = t_o}
\end{cases}
\end{align*}
\]
With the numerical values in Appendix A, the above equation appears as

\[ \begin{cases} \dot{y} - 2.68 \times 10^6 y = 0.10 \times (u_T - 0.35) \omega_H - 1.1 \times 10^6 \bigg( t - t_c \bigg) \\ \omega_n \bigg( t - t_c \bigg) \end{cases} \]  (6-27)

After \( y = y(t) \) is obtained, the time \( t \) which satisfies Equation (6-27) may be obtained. The \( y \) component of force, \( F_y, u_T \), on the inertia weight for \( t \geq t_c \) is then

\[ F_{y, u_T} = F_{y1, t} + 7 F_{y3, t} \]  (6-28)

\[ F_{y, u_T} = m_G \omega_H \bigg[ - \dot{y} u_T + \alpha_T - (C_y y + C_q) \omega_H + 7 m_G \omega_n \bigg] \]  (6-29)

\[ F_{y, u_T} = m_G \bigg\{ \omega_n \bigg[ - \dot{y} u_T + \alpha_T - (C_y y + C_q) \omega_H \bigg] \bigg] + 7 \alpha_H \bigg\} \quad t \geq t_c \]  (6-30)

where \( t_c \) is the time which satisfies Equation (6-27).
The term \( T F_{1y,5} \) in Equation (6-28) reflects the direct axial effect of the balls which do not contribute wedging action on the inertia weight. The two possible regimes of motion will now be summarized.

**Case I** - For \( 0 \leq \psi \leq \psi_0 \), the primary and secondary balls contribute wedging action and the remaining five balls contribute direct axial effect. Equation (6-19) expresses the axial force acting on the inertia weight.

**Case II** - For \( \psi_0 < \psi \), the primary ball only contributes wedging action while the remaining seven balls contribute direct axial effect. For this case, Equation (6-30) expresses the axial force on the inertia weight.

A continuous test of whether Case I or Case II operation occurs may be handled by consideration of Equation (6-20). If \( F_{b,2} > 0 \), then the secondary balls contribute to the wedging effect of Balls 2 on the inertia weight. For this situation Equations (6-20) and (6-21) are valid. At times \( \psi \geq \psi_0 \), when \( F_{w,2} = 0 \), Equation (6-20) will still be valid, resulting in

\[
F_{1y,2} = m_6 \omega_A \tag{6-31}
\]

However, Equation (6-21) will no longer be true, since the kinematic relationship between Balls 2 and the inertia weight, given by Equation (6-13), is no longer true.

\( F_{1y,2} \) is eliminated between Equations (6-20) and (6-21), with the result

\[
F_{b,2} = m_6 \omega_A \left[ 0.707 \alpha_T - \gamma \cot \theta \right.
\]

\[
- \left( C_8 y + C_q \right) \omega_{y}^2 - \alpha_A \tan \theta \left. \right] \tag{6-32}
\]

In a similar fashion, from Equation (6-7), (6-12) and (6-13) appears as

\[
F_{w,1} = m_6 \omega_A \left[ \alpha_T - \gamma \cot \theta - (C_8 y)
\right.
\]

\[
+ C_q \right) \omega_{y}^2 - \alpha_A \tan \theta \left. \right] \tag{6-32a}
\]
The condition \( F_{\omega, z} > 0 \) is thus, following Equation (6-32), equivalent to

\[
\left[ 0.707 \, a_{\tau} - \dot{y} \, \cos \theta - (C_y + C_{\eta}) \, \omega_y - a_A \tan \theta \right]
\]

or

\[
[-1.73 \, \dot{y} + 4.65 \times 10^6 \, y + 0.107 \, a_{\tau} - a_A - 2.77 \times 10^6] > 0 \quad (6-34)
\]

Equation (6-34) may be used to determine whether Case I or Case II operation prevails, and the time \( \tau_c \) at which Case II operation commences is found from Equation (6-27).

7. ABSOLUTE ACCELERATION OF THE CENTER OF MASS OF THE INERTIA WEIGHT

With the secondary \( x, y, z \) coordinates positioned as described before, the absolute acceleration \( \ddot{\rho}_{CM} \) of the center of mass of the inertia weight will now be obtained. The several terms needed in Equation (5-1) are

\[
\ddot{\omega} = \ddot{y} \omega_y
\]

(7-1)

\[
\ddot{r} = \ddot{y} \, \dot{y}
\]

(7-2)

so that \( \dot{y} \) is the position of the CM of the inertia weight.

\[
\ddot{r} = \dot{z} (a_{\tau}) + \ddot{y} (-a_A)
\]

(7-3)

\[
\dddot{r}_{\rho} = \ddot{y} \, \dot{y}
\]

(7-4)

\[
\dddot{r}_{\rho} = \dddot{y}
\]

(7-5)

\[
\frac{d \dot{\omega}}{d \tau} \times \ddot{r} = \ddot{\omega} \times \dot{\omega} = 0
\]

(7-6)
\[ \mathbf{\ddot{\omega} \times \mathbf{r}_n} = \begin{bmatrix} \ddot{\omega} & \dot{\mathbf{r}}_n \\ \dot{\mathbf{r}}_n & \mathbf{r}_n \end{bmatrix} = \mathbf{\omega} \] (7-7)

\[ \mathbf{\ddot{\omega} \times \mathbf{r}} = \begin{bmatrix} \ddot{\omega} & \dot{\mathbf{r}} \\ \dot{\mathbf{r}} & \mathbf{r} \end{bmatrix} = \mathbf{\omega} \] (7-8)

\[ \mathbf{\ddot{\omega} \times (\ddot{\omega} \times \mathbf{r})} = \mathbf{0} \] (7-9)

The absolute acceleration \[ \mathbf{\ddot{\alpha}_{o, \tau}} \] is then

\[ \mathbf{\ddot{\alpha}_{o, \tau}} = \ddot{\omega}(\mathbf{a_T}) + \mathbf{J}(-\alpha_T + \dot{\gamma} ) \] (7-10)

8. DYNAMIC EQUILIBRIUM REQUIREMENTS OF THE INERTIA WEIGHT

Figure 12 shows the applied and inertia forces acting on the inertia weight. These forces are identified below.

- \( F_{l,1}, F_{l,2}, F_{l,y,1}, F_{l,y,2}, F_{l,z,1}, F_{l,z,2}, F_{l,y,2} \) - reaction forces and components of Balls 1 and 2, as previously defined, on the inertia weight.
- \( F_{2}, F_{5} \) - Normal reactions of center rod on inertia weight
- \( F_{5}, F_{4} \) - Friction forces induced by \( F_{2} \) and \( F_{5} \)
- \( F_{7} \) - Inertia force of inertia weight.
- \( F_{8} \) - Total force of three creep springs. \( \kappa \) is the combined spring constant of the three springs.
- \( F_{q} \) - Total initial force of 3 creep springs
- \( I_{o, \alpha} \) - Inertia moment due to angular acceleration of shell. \( I_{o} \) is the mass moment of inertia about the CM.

The sense of the inertia force \( \mathbf{\mathcal{M}}_{\mathbf{\mathcal{T}}}, \mathbf{\mathcal{L}}_{\mathbf{\mathcal{T}}} \) which acts through the CM is obtained from the \( \kappa \) component of Equation (7-10),
\[ F_B = F_g + K(y-y_0) \]
\[ F_I = m_I(\ddot{y} - a_A) \]

Note: \( A \) is the point of application of \( F_{1,2} \), \( F_{1x} \), and \( F_{1y} \) (see Fig. 13), and the forces \( F_{2x} \) and \( F_{2y} \) are not shown in the figure.

Figure 12 - Free Body Diagram of Inertia Weight
while the sense of the term $I_0 \zeta^\alpha$ is obtained from Figure 5.

The object of the subsequent analysis is to obtain the friction forces $F_3$ and $F_4$, so that a summation of forces in the $y$ direction may be made. The moments will be summed about the CM. Figure 13 shows the configuration of the components of ball force which contribute to moment about the CM depicted in Figure 12. There will be a slight additional contribution to the sum of moments about the CM caused by the forces $F_{1y} = m_0 \zeta^\alpha$, and these effects will be neglected.

![Figure 13 - Components of Force of Primary and Secondary Graze Balls](image-url)
The equilibrium requirements, from Figure 12, are

\[ \sum M_{CM} = 0 \quad (\text{clockwise moments are positive}) \]

\[ -I_{o} \alpha + F_{2} C_{6} + F_{3} C_{4} + F_{1} x_{1} (C_{5} - C_{3}) - F_{1} y_{1} C_{2} + F_{5} C_{5} - F_{4} C_{4} + 2 \left\{ 0.707 F_{1} x_{1} \right\} (C_{5} - C_{3}) \]

\[ - F_{1} y_{1} \left( 0.707 C_{2} \right) \right) = 0 \quad (8-1) \]

\[ \sum F_{X} = 0 \quad (\text{Forces positive in positive X sense}) \]

\[ F_{2} - m_{1} a_{T} - F_{1} x_{1} - F_{5} - 1.41 F_{1} x_{1,2} = 0 \quad (8-2) \]

Motion of the inertia weight is assumed to be impending, so that

\[ F_{2} = \mu F_{2} \quad (8-3) \]

\[ F_{4} = \mu F_{5} \quad (8-4) \]

With the results

\[ F_{1, j} = F_{1, y, j} \tan \Theta \quad j = 1, 2 \quad (8-5) \]

The normal reactions \( F_{2} \) and \( F_{5} \) are

\[ F_{2} = \frac{1}{C_{1}} \left\{ I_{o} \alpha + C_{2} (C_{5} - \mu C_{4}) m_{1} a_{T} \right\} \]

\[ + \left[ C_{2} + C_{3} - \mu C_{4} \right] \tan \Theta \left( F_{1, y_{1}} + 1.41 F_{1, y_{2}} \right) \quad (8-6) \]

\[ F_{5} = \frac{1}{C_{1}} \left\{ I_{o} \alpha - (C_{6} + \mu C_{4}) m_{1} a_{T} \right\} \]
\[ + \left[ C_2 - C_7 + \mu C_4 \right] \tan \Theta \right] \left( F_{iy,1} + 1.41 F_{iy,2} \right) \tag{8-7} \]

The total friction force \( F_{F_i, \text{TOT}} \) is

\[ F_{F_i, \text{TOT}} = F_3 + F_4 = \mu \left( F_L + F_{c_i} \right) \tag{8-8} \]

For equilibrium of the inertia weight in the \( y \) direction, from Figure 12, for

\[ y \geq y_o \]

\[ F_{iy, \text{TOT}} - F_{F_i, \text{TOT}} - m_i \left( \dot{y} - a_A \right) - \left[ F_q + k(y - y_o) \right] = 0 \tag{8-9} \]

For the regime of the motion where \( 0 \leq t \leq t_o \), the above equation appears as

\[ m_i \left\{ \cot \Theta \left[ -3 \dot{y} \cot \Theta + 2 \pi a_T + 3 (C_0 y + C_q) \omega_y^2 \right] + 5 a_A \right\} \]

\[ - \frac{\mu}{C_1} \left\{ I_0 a_T + \left( C_3 - \mu C_4 \right) m_i a_T \right\} \]

\[ + \left[ C_2 + C_7 - \mu C_4 \right] \tan \Theta \right] \left( F_{iy,1} + 1.41 F_{iy,2} \right) \]

\[ + \left[ C_2 + C_7 - \mu C_4 \right] \tan \Theta \right] \left( F_{iy,1} + 1.41 F_{iy,2} \right) \]

\[ - m_i \left( \dot{y} - a_A \right) - \left( F_q + k[y - y_o] \right) = 0 \quad 0 \leq t \leq t_o \tag{8-10} \]
where \( M_I \) is the mass of the inertia weight. Using \( F_{Iy, l} \) and \( F_{Iy, t} \) from Equations (6-14) and (6-15), the above equation finally appears as

\[
\begin{align*}
\left[ m_I + m_{e, \theta} \cot \theta \right]^2 y - 2.41 \frac{U_c}{C_1} y + \tan \theta \left( C_3 - C_7 - 2 \mu C_4 \right) \right] y = \left[ m_I + 5 m_6 \right] a_A \\
+ \left[ m_{e, \theta} \cot \theta \left\{ 2.41 - \frac{2 \mu}{C_1} \left[ 2 C_2 + \tan \theta \left( C_3 - C_7 - 2 \mu C_4 \right) \right] \right\} \right] \frac{U_c}{C_1} (C_5 - C_6 - 2 \mu C_4) \left[ m_I \right] a_T - \left[ C_7 w_y \right] m_{e, \theta} \cot \theta \left\{ 3 - 2.41 \frac{U_c}{C_1} \left[ 2 C_2 - 2 \mu C_4 \right] \right\} \right] y + \frac{2 \mu U_o \alpha}{C_1} + F_q - K_y \omega \left( 8-11 \right)
\end{align*}
\]

For Case II, where only the primary ball has a wedging effect on the inertia weight and \( t \geq t_c \), the equilibrium requirements of the inertia weight, from Figure 12, are

\[
\begin{align*}
-I_0 x + F_2 C_6 - F_3 C_4 + F_{1x} \left( C_5 - C_3 \right) - F_{1y} C_4 \\
+ F_5 \left( C_5 - F_4 C_4 \right) = 0
\end{align*}
\]

(8-12)
\[ F_2 - m_i a_T - F_{1x} - F_5 = 0 \] (8-13)

Motion is again assumed to be impending, so that

\[ F_3 = \mu F_2 \] (8-14)
\[ F_4 = \mu F_5 \] (8-15)

The forces \( F_3 \) and \( F_4 \) are then found to be

\[ F_3 = \frac{\mu}{C_1} \left\{ I \alpha + \left( \frac{C_5 - \mu C_4}{C_4} \right) m_T a_T + \left[ c_2 + \left( C_5 - \mu C_4 \right) \tan \theta \right] F_{1y} \right\} \] (8-16)
\[ F_4 = \frac{\mu}{C_1} \left\{ I \alpha - \left( C_6 + \mu C_4 \right) m_T a_T + \left[ c_2 - \left( C_7 + \mu C_4 \right) \tan \theta \right] F_{1y} \right\} \] (8-17)

The total friction force \( F_{F, \text{TOT}} \) is

\[ F_{F, \text{TOT}} = F_3 + F_4 \] (8-18)

so that

\[ F_{F, \text{TOT}} = \frac{\mu}{C_1} \left\{ 2 I \alpha + \left( \frac{C_5 - C_6 - 2 \mu C_4}{C_4} \right) m_T a_T \right. \]
\[ + \left[ 2 c_2 + C_5 - C_7 - 2 \mu C_4 \right] \tan \theta \left. \right\} F_{1y} \] (8-19)

For Case II operation, where only the primary ball acts on the inertia weight,

\[ F_{1y, \text{TOT}} = F_{1y, 1} \] (8-20)
The equilibrium requirement of the inertia weight in the \( \hat{y} \) direction, from Figure 12, is

\[
\begin{align*}
F_{\hat{y},I} - F_{\hat{y},RT} - m_T \left( \hat{y} - c_A \right) - \left[ F_{\hat{y}} + K \left( y - y_c \right) \right] = 0 \quad (8-21)
\end{align*}
\]

The above equation may be written as

\[
\begin{align*}
&\dot{\omega}_r \dot{\gamma} \cos \gamma + \ddot{\gamma} - \left( c_3 - c_5 - 2 u c_4 \right) \omega_r \omega_r + \ddot{\gamma} \cos \gamma \n
&+ m_{\text{eq}} \left[ 2 c_3 + \left( c_3 - c_7 - 2 u c_4 \right) \tan \theta \right] \omega_r \omega_r + \ddot{\gamma} \cos \gamma \\
&+ \left( c_3 - c_7 + c_4 \right) \omega_r e \right] \left[ -m_T \left( \hat{y} - c_A \right) - \left[ F_{\hat{y}} + K \left( y - y_c \right) \right] \right] = 0 \quad (8-22)
\end{align*}
\]

The term \( (7 m_{\text{eq}} a_A) \) in the above equation is added directly to account for the effect of the remaining seven balls. The terms in Equation (8-22) are combined, and the final result is

\[
\begin{align*}
\left[ m_T + m_{\text{eq}} \omega_r e \right] \ddot{\gamma} - \frac{M}{C_1} \left[ 2 c_3 + \left( c_3 - c_7 - 2 u c_4 \right) \omega_r e \right] \ddot{\gamma} = 0
\end{align*}
\]
The dimension $C_L$ in Figure 12 is a function of $y$. The effect of this is to make the equations of motion of the inertia weight, Equations (8-11) and (8-23), nonlinear. However, as is shown in Appendix B, the term $C_L$ exhibits a weak dependence on $y$. It will be assumed that the term $C_L$ is a constant, and the magnitude of this constant will be chosen (2) to yield conservative results for the motion.

(2) See Appendix B for the details of this computation.
of the inertia weight. If $C_L$ is a constant, the equations of motion of the inertia weight, Equations (8-11) and (8-23), are second order linear differential equations with constant coefficients, of the form

$$A_1 y'' + A_2 y' = A_3 c L + A_4 c L T + A_5 x + A_6$$

(8-24)

where the $A_j$ are constants.

The initial conditions are

$$t = 0, \quad y = y_c = c \text{ const}.$$  

(8-25)

$$t = 0, \quad y' = 0.$$  

(8-26)

where $y_c$ is the initial displacement of the center of mass of the inertia weight. The above conditions are chosen to correspond to the application of the graze forcing function at $t = 0$.

9. NUMERICAL FORMS OF THE EQUATIONS OF MOTION OF THE INERTIA WEIGHT

Appendix A contains a tabulation of the numerical constants of the present problem. The particular shell considered is the 105mm Howitzer Ml. Appendix B compares the forms of the equations of motion of the inertia weight for maximum and minimum values of the dimension $C_L$. It is seen in this appendix that the maximum percent differences between the coefficients in the equations of motion of the inertia weight, in the extreme cases, are less than 8%. It will thus be assumed, in order to obtain linear forms for the equations of motion of the inertia weight, that $C_L = \text{constant}$. In addition, the constant value of $C_L$ will be taken to be $C_L = \text{constant}$. So that conservative results will be obtained for the motion of the inertia weight. With the above assumptions, the equations of motion of the inertia weight for the two regimes $0 < t \leq t_0$ and $t_0 \leq t$ are, from Equations (8-11) and (8-23),

$$1.69 \times 10^{-4} y'' = 3.64 \times 10^{-5} a_T + 1.37 \times 10^{-4} a_H$$

(3) One of the purposes of this study is to determine whether the graze module functions under the application of minimum values of the forcing functions. Equations (8-6) and (8-7) reveal that the retarding friction forces, $F_3$ and $F_4$, acting on the inertia weight will be maximum for maximum values of $C_L$. 

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Case I - Operation, \( c_t < t \leq t_c \), Balls 1 and 2 both exert wedging action on inertia weight,
\[ C_2 = C_{2, \text{max}} = 0.47 \omega \] (Equation 8-11)

\[ 1.10 \times 10^{-4} \ddot{y} - 5\omega_s \dot{y} = 1.29 \times 10^{-5} \dot{\alpha}_T + 1.57 \times 10^{-4} \dot{\alpha}_H \]

\[ -3.20 \times 10^{-6} \dot{x} - 47.7 \] (9-2)

Case II - Operation, \( t = t_c \), Ball 1 only exerts wedging action on inertia weight, \( C_2 = C_{2, \text{max}} = 0.47 \omega \), Equation (8-23)

The constraint equation which determines whether Case I or Case II operation prevails, Equation (6-27), appears as

\[
\left[ \begin{array}{l}
\ddot{y} - 2.68 \times 10^6 y - 0.408 \dot{\alpha}_T + 0.333 \dot{\alpha}_H + 1.60 \times 10^6 \\
\end{array} \right] = 0 \] (9-3)

For \( 0 \leq t \leq t_c \), the motion of the inertia weight will be given by Equation (9-1). For \( t \geq t_c \), the motion is given by Equation (9-2). It may be observed that, for certain magnitudes of the forcing functions, there may be no positive \( t_c \) which satisfies Equation (9-3). For this case the equation of motion of the inertia weight would be given by Equation (9-2).

10. EQUATIONS OF MOTION OF THE INERTIA WEIGHT WHEN THE GRAZE FORCING FUNCTIONS ARE APPROXIMATED BY TRIANGULAR SHAPES.

The lateral acceleration field \( a_T \) of the graze module housing will now be assumed to have the shape shown in Figure 14. By properly choosing \( \alpha_c, \tau_1, \) and \( \tau_2 \), the best approximation to the \( \alpha_T(t) \) curve may be obtained. The three regimes of time are then
0 \leq t \leq t_1 \quad \phi_T = \frac{\alpha c t}{t_1} \quad (10-1)

t_1 \leq t \leq t_2 \quad \phi_T = \frac{\alpha c}{t_2-t_1} (t_2 - t) \quad (10-2)

t_2 \leq t \quad \phi_T = c \quad (10-3)

Figure 14 - Triangular Approximation of Graze Forcing Functions

where $t_1$ and $t_2$ are the times of peak and cessation, respectively, of the acceleration function.

Equation (9-1) is divided by $1.69 \times 10^{-4}$, with the result

$$\frac{\dot{y}}{0.0016} = 0.216 a_T + 0.810 a_A - 0.019 \times 8.32 \times 10^5 \quad (10-4)$$
Using Equations (4-14) and (4-15), the above equation appears as

\[ \ddot{y} - 1.2 \times 10^6 y = \left[ C_{21} L + C_{22} E \right] T + 0.0 \left[ \dot{K}_2 \right] T \]

\[- 8.32 \times 10^5 \]

Equation (10-5)

\( K_3 \) is defined as

\[ K_3 = \left[ C_{21} L + C_{22} E \right] K \]

\[- C_{22} \cdot \left[ \dot{K}_2 \right] \]

Equation (10-6)

and Equation (10-5) now appears as

\[ \ddot{y} - 1.2 \times 10^6 y = K_3 a_T - 8.32 \times 10^5 \]

Equation (10-7)

Equation (10-7) will now be solved for the three regimes of operation

\[ 0 \leq t \leq t_1 \]

\( a_T \) is given by Equation (10-1) as

\[ a_T = \frac{a_0 t}{t_1} \]

(10-8)

With

\[ K_4 = \frac{K_3 a_0}{t_1} \]

(10-9)

Equation (10-7) appears as

\[ \ddot{y} - 1.2 \times 10^6 y = K_4 t - 8.32 \times 10^5 \]

(10-10)
The complementary solution satisfies the equation

\[ y_c - 1.26 \times 10^6 y_c = 0 \]  
(10-11)

or

\[ y_c = A \sinh 1120 t + A_2 \cosh 1120 t \]  
(10-12)

The particular solution \( y_p \) is assumed in the form

\[ y_p = A t + B \]  
(10-13)

Equation (10-13) is substituted into Equation (10-16) with the result

\[ 0 - 1.26 \times 10^6 (A t + B) = K_4 t - 8.32 \times 10^5 \]  
(10-14)

\[ -1.26 \times 10^6 A = K_4 \]  
(10-15)

\[ -1.26 \times 10^6 B = -8.32 \times 10^5 \]  
(10-16)

\[ A = -7.94 \times 10^{-7} K_4 \]  
(10-17)

\[ B = 0.660 \]  
(10-18)

\( y_p \) is then

\[ y_p = -7.94 \times 10^{-7} K_4 t + 0.660 \]  
(10-19)
and the general solution is

\[ y = y_c + y_p \]  

(10-20)

or

\[ y = A_1 \sinh 1120t + A_2 \cosh 1120t. \]

(10-21)

The velocity \( \dot{y} \) is

\[ \dot{y} = 1120 A_1 \cosh 1120t + 1120 A_2 \sinh 1120t \]

(10-22)

At \( t = 0 \), \( \dot{y} = 0 \), so that

\[ 0 = 1120 A_1 - 7.94 \times 10^{-7} K_4 \]  

(10-23)

\[ A_1 = 7.06 \times 10^{-16} K_4 \]  

(10-24)

At \( t = 0 \), \( y = y_c = 0.273 \), so that

\[ 0.273 = A_2 + 0.660 \]  

(10-25)

\[ A_2 = -0.387 \]  

(10-26)
The complete solution for the interval \( 0 \leq t < t_1 \) is finally

\[
\dot{y} = 7.08 \times 10^{-16} K_4 \sinh 1120t - 0.387 \cosh 1120t
\]

\[-7.94 \times 10^{-7} K_4 t + C_{\text{CC}}\]  \hspace{1cm} (10-27)

\[
\dot{y} = 7.94 \times 10^{-7} K_4 \cosh 1120t - 434 \sinh 1120t
\]

\[-7.94 \times 10^{-7} K_4\]  \hspace{1cm} (10-28)

In the interval \( t_1 \leq t \leq t_2 \) the equation of motion is

\[
\ddot{y} - 1.26 \times 10^6 y = k_3 \left[ \frac{a_0}{t_2 - t_1} \right] - 8.32 \times 10^5 \] \hspace{1cm} (10-29)

\[
= \frac{k_3 a_0 t_2}{t_2 - t_1} - \frac{k_3 a_0 t}{t_2 - t_1} - 8.32 \times 10^5 \] \hspace{1cm} (10-30)

With the notations

\[
k_5 = - \frac{k_3 a_0}{t_2 - t_1} \] \hspace{1cm} (10-31)
\[ K_L = \frac{K_2 C \omega t_2}{t_2 - t_1} \quad (10-32) \]

Equation (10-29) appears as

\[ \ddot{y} - 1.26 \times 10^6 y = K_5 \omega^2 + K_L - 8.32 \times 10^5 \quad (10-33) \]

The complementary solution is

\[ y_c = A_3 \sin 1120t + A_4 \cos 1120t \quad (10-34) \]

The particular solution is assumed to be

\[ y = Ct + D \quad (10-35) \]

Equation (10-35) is substituted into Equation (10-33), with the result

\[ -1.26 \times 10^6 \left[ Ct + D \right] = K_5 \omega^2 + K_6 - 8.32 \times 10^5 \quad (10-36) \]

\[ -1.26 \times 10^6 C = K_5 \quad (10-37) \]

\[ C = -7.94 \times 10^{-7} K_5 \quad (10-38) \]

\[ -1.26 \times 10^6 D = K_6 - 8.32 \times 10^5 \quad (10-39) \]

\[ D = \frac{K_6 - 8.32 \times 10^5}{-1.26 \times 10^6} \quad (10-40) \]
\[ D = -7.94 \times 10^{-7} \kappa \omega + 0.660 \]  
\hspace{1cm} \text{(10-41)}

and

\[ y_p = -7.94 \times 10^{-7} K_5 \omega - 7.94 \times 10^{-7} \kappa \omega + 0.660 \]  
\hspace{1cm} \text{(10-42)}

The final solutions for \( y \) and \( y' \) are then

\[ y = A_3 \sin 1120 \omega t + A_4 \cos 1120 \omega t - 7.94 \times 10^{-7} K_5 \omega t \]
\[ - 7.94 \times 10^{-7} \kappa \omega + 0.660 \]  
\hspace{1cm} \text{(10-43)}

\[ y' = 1120 A_3 \cos 1120 \omega t + 1120 A_4 \sin 1120 \omega t - 7.94 \times 10^{-7} K_5 \omega t \]  
\hspace{1cm} \text{(10-44)}

The boundary conditions are that, at \( t = t_1 \), Equations (10-27) and (10-28) must be equal to Equations (10-43) and (10-44), or

\[
\begin{bmatrix}
\text{Equation (10-27)}
\end{bmatrix} = \begin{bmatrix}
\text{Equation (10-43)}
\end{bmatrix}
\quad t = t_1
\]
\hspace{1cm} \text{(10-45)}

\[
\begin{bmatrix}
\text{Equation (10-28)}
\end{bmatrix} = \begin{bmatrix}
\text{Equation (10-44)}
\end{bmatrix}
\quad t = t_1
\]
\hspace{1cm} \text{(10-46)}
Equations (10-45) and (10-46) are solved simultaneously for $A_3$ and $A_4$. The final result for the equation of motion in the interval $\tau_1 \leq t \leq \tau_2$ is then

$$y_3 = \begin{cases} 7.08 \times 10^{-10} K_4 \sinh 112 \cdot \tau_1 - 0.587 \tau_1 - \sinh \tau_1 \\ - 7.94 \times 10^{-7} K_4 \tau_1 + 7.94 \times 10^{-7} K_5 \tau_1 \\ + 7.94 \times 10^{-7} K_6 \cos 112 \cdot (t - \tau_1) \end{cases}$$

$$+ \frac{1}{1120} \left\{ 7.94 \times 10^{-7} K_4 \cos 112 \cdot \tau_1 - 484 \sin 112 \cdot \tau_1 \right\}$$

$$- 7.94 \times 10^{-7} K_4 + 7.94 \times 10^{-7} K_5 \right\} \sinh 112 \cdot (t - \tau_1)$$

$$- 7.94 \times 10^{-7} K_5 \tau_1 - 7.94 \times 10^{-7} K_6 + C_{66c} \ (10-47)$$

$$y_4 = 1120 \left\{ 7.08 \times 10^{-10} K_4 \sinh 112 \cdot \tau_1 - 0.587 \tau_1 - 7.94 \times 10^{-7} K_5 \tau_1 \right\}$$
\[-7.94 \times 10^{-7} K_4 t_1 + 7.94 \times 10^{-7} K_5 t_1\]

\[+ 7.94 \times 10^{-7} K_6 \ln 1120 (t-t_1)\]

\[+ \left\{ 7.94 \times 10^{-7} K_4 \ln 1120 t_1 - 434 \ln 1120 t_1 \right\} \]

\[-7.94 \times 10^{-7} K_4 + 7.94 \times 10^{-7} K_5 \ln 1120 (t-t_1)\]

\[-7.94 \times 10^{-7} K_5 \quad t_1 \leq t \leq t_2\]

(10-48)

For \(t_2 \leq t\), \(\alpha_T = 0\) and the equation of motion appears as

\[-1.26 \times 10^6 y = -8.32 \times 10^5\]

(10-49)
The solution to Equation (10-49) is

\[ y = A_0 \sin \omega t + A_1 \cos \omega t + A_2 \sin \omega t \quad (10-50) \]

and the velocity \( \dot{y} \) is

\[ \dot{y} = 1120 \sin \omega t + 1120 \cos \omega t \quad (10-51) \]

The above two functions must be equal to Equations (10-47) and (10-48), at \( t = t_\perp \). This condition is expressed as

\[
\begin{bmatrix}
\text{Equation (10-47)} \\
\end{bmatrix}
= 
\begin{bmatrix}
\text{Equation (10-50)} \\
\end{bmatrix}
\quad \text{at } t = t_\perp \quad (10-52)
\]

\[
\begin{bmatrix}
\text{Equation (10-48)} \\
\end{bmatrix}
= 
\begin{bmatrix}
\text{Equation (10-51)} \\
\end{bmatrix}
\quad \text{at } t = t_\perp \quad (10-53)
\]

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Equations (10-52) and (10-53) are solved simultaneously for $A_5$ and $A_6$. The final result for the equation of motion in the interval $t_2 \leq t$ is then

$$y = \left\{ \begin{array}{l}
7.08 \times 10^{-10} K_4 \sin \left( \frac{1120}{125} t_1 \right) - 0.367 \cosh \left( \frac{1120}{125} t_1 \right) \\
-7.94 \times 10^{-7} K_4 t_1 + 7.94 \times 10^{-7} K_5 t_1 \\
+ 7.94 \times 10^{-7} K_6 \right\} \cosh \left( \frac{1120}{125} (t - t_1) \right)
+ \frac{1}{1120} \left\{ 7.94 \times 10^{-7} K_4 \cosh \left( \frac{1120}{125} t_1 \right) - 434 \sinh \left( \frac{1120}{125} t_1 \right) \\
-7.94 \times 10^{-7} K_4 + 7.94 \times 10^{-7} K_5 \right\} \sinh \left( \frac{1120}{125} (t - t_1) \right)
-7.94 \times 10^{-7} K_5 \left[ t_2 \cosh \left( \frac{1120}{125} (t - t_2) \right) + \frac{1}{1120} \sinh \left( \frac{1120}{125} (t - t_2) \right) \right]
+ (-7.94 \times 10^{-7} K_6 + 0.660) \cosh \left( \frac{1120}{125} (t - t_2) \right)
- 0.660 \cosh \left( \frac{1120}{125} (t - t_2) \right) + 0.660 \\
t_2 \leq t \ (10-54)
\end{array} \right.$$
\[
\dot{y} = 1120 \left\{ 7.08 \times 10^{-10} K_4 \Delta N_{120} \Delta N_{120} \Delta N_{120} t_1 - 0.387 \Delta N_{120} \Delta N_{120} t_1 \\
- 7.94 \times 10^{-7} K_4 t_1 + 7.94 \times 10^{-7} K_5 t_1 \\
+ 7.94 \times 10^{-7} K_6 \Delta N_{120} \Delta N_{120} (t-t_1) \\
+ \left\{ 7.94 \times 10^{-7} K_4 \Delta N_{120} \Delta N_{120} t_1 - 434 \Delta N_{120} \Delta N_{120} t_1 \\
- 7.94 \times 10^{-7} K_4 + 7.94 \times 10^{-7} K_5 \right\} \Delta N_{120} \Delta N_{120} (t-t_1) \\
- 7.94 \times 10^{-7} K_5 \left[ 1120 t_2 \Delta N_{120} \Delta N_{120} (t-t_2) \\
+ \Delta N_{120} \Delta N_{120} (t-t_2) \right] \\
+ (-7.94 \times 10^{-7} K_6 + 0.660) \Delta N_{120} \Delta N_{120} (t-t_2) \\
- 0.660 \Delta N_{120} \Delta N_{120} (t-t_2) \quad t_2 \leq t \right\}
\]
The above equations are all for the case where both the Primary Ball, 1, and the Secondary Balls, 2, exert wedging action on the inertia weight. In all of these cases, \( 0 < \tau < \tau_0 \). The equations of motion for the case where Ball 1 alone exerts wedging action on the inertia weight, where \( \tau_0 < \tau \), can be derived in a similar manner.
11. COMBINED MOTION OF INERTIA WEIGHT, DETENT BALLS AND FIRING PIN

When the inertia weight moves forward sufficiently, the two detent locking balls will be released. The orientation of the line of centers of these two balls with respect to the line of action of \( a_T \), for the duration of the graze, is arbitrary. Figure 15 shows two extremes of this orientation. In Figure 15a both balls will have the same forces acting on them and the problem is symmetric. In Figure 15b the problem is antisymmetric, since one ball will have the \( a_T \) effect tending to release it while the other ball will have the \( a_u \) effect opposing it.

![Figure 15 Detent Ball Orientation](image)

Figure 15 shows the configuration of the firing pin, the detent ball and the inertia weight. \( y_i \) is an axial coordinate of the firing pin, positive downwards, and \( y_o \) is an arbitrary positive constant, so that the relative axial displacement of the firing pin is \( y_i - y_o \).

\( x_i \) is a radial coordinate of the ball and \( x_o \) is some positive constant, so that the displacement of the ball, positive outwards, is \( x_i - x_o \). The displacement
of the inertia weight is \( y - y_0 \), as derived earlier, and the radius of the ball is designated by \( r \cdot X_0 \), and \( y_0 \) will presently be defined.

As the inertia weight moves upward, the detent balls will move radially outward. The edge of the inertia weight will be in contact with the ball, Figure 16, for some part of the motion.

The Kinematic relationship between the motion of the firing pin and the ball is given in Figure 16. From this figure,

\[
\tan \beta = \frac{X_1 - X_{01}}{y_1 - y_{01}} \quad X_1 \geq X_{01} \quad (11-1)
\]

or

\[
X_1 = 0.577 \cdot y_1 - 0.037 \cdot \omega^2 \quad (11-1a)
\]
The Dimension A in Figure 16 is given by

\[ A = r - \left( x_i - x_{c_i} \right) \]  \hspace{1cm} (11-2)

The relationship between the motion of the inertia weight and the ball, while they are both in contact, is

\[ r^2 = (x_i - x_{c_i})^2 + A^2 \]  \hspace{1cm} (11-3)

or

\[ \rho^2 = (y_i - y_{c_i})^2 + \left[ r - \left( x_i - x_{c_i} \right) \right]^2 \]  \hspace{1cm} (11-4)

Equation (11-4) will be true only for the time that the ball is in contact with the inertia weight. This effect is very significant in the present derivation, and it will be subsequently discussed. From Figure 16,

\[ \tan \phi = \frac{y_i - y_{c_i}}{r - (x_i - x_{c_i})} \]  \hspace{1cm} (11-5)

The relationship between the velocities \( \dot{x}_i \) and \( \dot{y}_i \) is now found by taking the first time derivative of Equation (11-4). This result is

\[ 0 = 2 (y_i - y_{c_i}) \dot{y}_i + \dot{r} \left[ \rho - (x_i - x_{c_i}) \right] (\dot{x}_i - \dot{x}_{c_i}) \]  \hspace{1cm} (11-6)

\[ \frac{y_i - y_{c_i}}{\rho - (x_i - x_{c_i})} \dot{y}_i = \dot{x}_i \]  \hspace{1cm} (11-7)
With the aid of Equation (11-5), the above equation appears as

\[ x_1 = y \tan \phi \]  

(11-8)

Equation (11-8) is a kinematic relationship between the velocities of the detent ball and the inertia weight. As the ball moves radially outward, it will exert an upward component of force on the inertia weight. This force will be assumed to be small, and thus it will not influence the equation of motion of the inertia weight. Thus, the previously obtained equations of motion of the inertia weight will be valid.

As

\[ \phi \rightarrow \frac{\pi}{2} \]  

(11-9)

and, following Equation (11-8),

\[ \tan \phi \rightarrow \infty \]  

(11-10)

and

\[ \dot{x}_1 \rightarrow \infty \]  

(11-11)

The physical significance of Equation (11-11) is that at some angle, designated by \( \phi_0 \), the ball and the inertia weight are no longer in contact. For \( 0 \leq \phi \leq \phi_0 \), the motion of the detent ball bears a purely kinematic relationship to the motion of the inertia weight, in accordance with (11-8). For \( \phi_0 < \phi \), the motion of the ball is governed by its own equation of motion. This limiting value of \( \phi_0 \) will be identified in a subsequent section.

The antisymmetric case of Figure 15b will now be analyzed, with the previous assumption of quasi-static behavior. All friction forces on the detent balls will be neglected, but the friction force between the firing pin and its housing will be included. As the inertia weight moves forward, several regimes of operation are possible, depending on the magnitude of the forcing functions. These regimes are identified below.

Case A - Both Detent Balls in Contact with Inertia Weight and with Firing Pin

For this case the motion relationship is purely kinematic.
The '3' (inertia weight) motion is related to the \( X_1 \) (detent ball) motion through Equation (11-4). The \( X_1 \) motion is related to the \( y_1 \) (firing pin) motion by Equation (11-1). From Equation (11-4),

\[
y = y_0 + \sqrt{h^2 - \left[ h - (X_1 - X_{c1}) \right]^2} \quad (11-12)
\]

is the kinematic relationship between the inertia weight and the detent balls. Case A operation will exist until

\[
y > y_0 + \sqrt{h^2 - \left[ h - (X_1 - X_{c1}) \right]^2} \quad (11-13)
\]
or

\[
y > 0.273 + \sqrt{0.4 \left[ X_1 - X_{c1} - 3 \right]} \quad (11-14)
\]

This limiting condition for Case A operation will be discussed in a subsequent section of this report.

Case B - Both Detent Balls in Contact with Firing Pin, but Not in Contact with Inertia Weight

For this case, the free body diagram will be as shown in Figure 17. The forces \( P_{3,3} \) and \( P_{4,4} \) are normal contact forces on Balls 3 and 4, respectively. Both balls will have equal radial displacements, since they are assumed to be in contact with the firing pin. \( P_{3,3} \) and \( P_{4,4} \) are reaction forces, of the detent ball guides on the detent balls, acting in the axial direction. The coordinate \( X_\text{c1} \) is measured from the center axis of the shell. \( X_\text{c1} \) will be taken to be the coordinate of the balls in the locked position. Thus, \( X_1 - X_{c1} \) is the relative, outward, radial displacement of the balls.

Figure 18 shows the free body diagram of the firing pin, corresponding to the situation depicted in Figure 17. The coordinate \( y_\text{f} \) is measured from the ceiling of the grage module and \( y_{\text{df}} \) is a known constant.

If the free length of the spring is designated by \( a \), then the spring force \( F_\text{s} \) is

\[
F_\text{s} = \kappa_\text{s} (a - y_\text{f}) \quad (11-15)
\]

where \( \kappa_\text{s} \) is the spring constant. The initial spring force \( F_\text{io} \) is given by

\[
F_\text{io} = \kappa_\text{s} (a - y_{\text{df}}) \quad (11-16)
\]
Figure 17  Free Body Diagram of Detent Balls, Case B Operation

Figure 18  Free Body Diagram of Firing Pin
Equations (11-15) and (11-16) may be combined, with the result

\[ \begin{bmatrix} x_1 \\ \dot{x}_1 \\ y_1 \\ \dot{y}_1 \\ z_1 \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} x \cdot \mathbf{v} \\ \dot{x} \cdot \mathbf{v} \\ y \cdot \mathbf{v} \\ \dot{y} \cdot \mathbf{v} \\ z \cdot \mathbf{v} \\ \dot{z} \cdot \mathbf{v} \end{bmatrix} \] (11-17)

The friction force due to the normal force difference

\[ \begin{bmatrix} \nabla \cdot \mathbf{u} \\ \nabla \cdot \mathbf{u} \end{bmatrix} = \begin{bmatrix} \nabla \cdot \mathbf{u} \\ \nabla \cdot \mathbf{u} \end{bmatrix} \]

has been neglected. The equilibrium requirement of the firing pin is

\[ \mathbf{F}_0 \cdot \mathbf{F} = \begin{bmatrix} x \cdot \mathbf{v} \\ \dot{x} \cdot \mathbf{v} \\ y \cdot \mathbf{v} \\ \dot{y} \cdot \mathbf{v} \\ z \cdot \mathbf{v} \\ \dot{z} \cdot \mathbf{v} \end{bmatrix} \]

and that of the detent balls is given by

\[ \mathbf{P}_{1,4} \cos \beta - m_0 \mathbf{X}_1 = m_0 \mathbf{X}_1 \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{bmatrix} \]

(11-19)

\[ \mathbf{P}_{1,5} \cos \beta - m_0 \mathbf{X}_1 = m_0 \mathbf{X}_1 \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{bmatrix} \]

(11-20)

where \( m_0 \) and \( m_f \) are the masses of the balls and the firing pin, respectively. Equations (11-19) and (11-20) are solved for \( \mathbf{P}_{1,5} \) and \( \mathbf{P}_{1,4} \) with the result

\[ \mathbf{P}_{1,3} = \mathbf{P}_{1,3} = \frac{1}{\epsilon \mathbf{P}_{1,3}} \left[ m_0 \mathbf{X}_1 + m_0 \mathbf{X}_1 \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{bmatrix} \right] \]

(11-21)

\[ \mathbf{P}_{1,4} = \mathbf{P}_{1,4} = \frac{1}{\epsilon \mathbf{P}_{1,4}} \left[ m_0 \mathbf{X}_1 + m_0 \mathbf{X}_1 \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{bmatrix} \right] \]

(11-22)
From Equation (11-1),

\[ \dot{X}_1 = \int \tan \beta \, dt \]  \hspace{1cm} (11-23)

Equations (11-1), (11-14) and (11-21) through (11-24) are now combined and the result is

\[ \left[ m_F \omega \cot \beta + 2 m_0 \tan \beta \right] \ddot{X}_1 - \left[ a_{11} \bar{w}_y \tan \beta - K_0 \cot \beta \right] X_1 = \]

\[ = F_0 + K_0 X_{01} \left( \omega \cot \beta - \eta \right) F \left( a_A + \mu a_T \right) \]  \hspace{1cm} (11-24)

or

\[ 1.15 \times 10^{-5} \ddot{X}_1 + 8.80 X_1 = 5.35 \]

\[- 6.30 \times 10^{-6} \left( a_A + 0.1 a_T \right) \]  \hspace{1cm} (11-25)

Equation (11-24) is the desired result for the motion of the balls when both balls are in contact with the firing pin in case B operation. It may be observed that \( a_T \) appears in Equation (11-24) only in the form of a friction force effect. After Equation (11-24) is integrated, the \( X_1 \) and \( Y_1 \) motions are related by Equation (11-1). The necessary condition for case B operation is that

\[ P_{1,5} > 0 \]  \hspace{1cm} (11-26)

\[ P_{1,4} > 0 \]  \hspace{1cm} (11-27)

since these forces are compressive contact forces. These conditions, from Equations (11-21) and (11-22), are
\[ \ddot{x}_1 = (x_1 + \omega_1^2 + a_T^2) \]

or

\[ \ddot{x}_1 = (x_1 + \omega_1^2 + a_T^2) \]

Case C - One Ball Only in Contact with Inertia Weight

The two detent balls, from Figure 10, experience different outward forces due to the terms ±\(a_T\). Since Ball 3 has a greater outward force on it than Ball 4, there is a possibility that it might lose contact with the firing pin before Ball 4 does. The case will now be analyzed where Ball 4 is in contact with the firing pin and Ball 3 is not.

The free body diagram for this condition is shown in Figure 19.

![Free Body Diagram of Detent Balls, Case C Operation](image)
The displacement $x_{1,4}$ of Ball 4 will be related to the displace- 
ment $y$, of the firing pin by Equation (11-1). The displace- 
ment $x_{1,3}$ of Ball 3, if $x_{1,3} \neq x_{1,4}$, will be in- 
dependent of $y$. The free body diagram of the firing 
pin will be the same as that shown in Figure 19, if the 
force $P_{1,3}$ is removed.

The equilibrium requirement of the firing pin is

$$E_{0} = k_{0}(y_{1} - y_{01}) - m_{f} \alpha_{A} - m_{f} y_{j} - m_{w} \alpha_{T} \cdot \alpha_{1,4} \Delta \tan \beta = 0$$ (11-32)

and that of Ball 4 is

$$P_{1,4} \cos \beta - m_{D} x_{1,4}^{*} + m_{D} (x_{1,4} w_{y}^{*} - \alpha_{T}) = 0$$ (11-33)

$P_{1,4}$ is eliminated between the above two equations and, 
using Equation (11-1), the final result is

$$m_{f} \sin \beta + m_{D} \tan \beta) x_{1,4}^{*} = m_{D} w_{y}^{*} \tan \beta - k_{0} \cos \beta x_{1,4}^{*}$$

$$= F_{0} + k_{0} x_{01} \cos \beta - m_{f} (a_{A} + \mu \alpha_{T}) - m_{A} \tan \beta$$ (11-34)

or

$$1.12 \times 10^{-5} x_{1,4}^{*} + 9.6 x_{1,4} = 5.35 - 6.5 \times 10^{-6} a_{A}$$

$$- 9.68 \times 10^{-7} a_{T}$$ (11-35)

The equilibrium requirement for Ball 3 is

$$- m_{D} x_{1,3}^{*} + m_{D} (x_{1,3} w_{y}^{*} + \alpha_{T}) = 0$$ (11-36)

and the constraint equation is

$$(x_{1,3} - x_{01}) > (y_{1} - y_{01}) \tan \beta$$ (11-37)
The numerical form of Equation (11-36) is

\[ X_{1,5} = ... \]  

(11-38)

Equation (11-37) appears as

\[ X_{1,4} > \epsilon, \quad \forall i \neq 1, 2, 3, 4 \]

The constraint condition for Equation (11-34) is

\[ \beta_{1,4} > \epsilon \]

(11-39)

or, from Equation (11-29),

\[ X_{1,4} - (X_{1,4} \omega_1 - \alpha_T) > \epsilon \]  

(11-40)

or

\[ X_{1,4} - (Z \omega_2 \times 1 \omega_3 \times X_{1,4} - \alpha_T) > \epsilon \]

(11-41)

Case D - Both Balls Not in Contact With Firing Pin or "With Inertia Weight"

For another regime of motion, where the firing pin exerts no force on the balls and the balls do not contact the inertia weight, the free body diagram is as shown in Figure 20.

For this case the displacements of the two balls, \( X_{1,3} \) and \( X_{1,4} \), are different, due to the terms \( \pm \alpha_T \). The corresponding free body diagram of the firing pin will be the same as Figure 18, with the forces \( P_{1,3} \) and \( P_{1,4} \) omitted.

From Figure 18,

\[ F_c - k_i (\omega_i - \omega_c) - m_F \alpha_A - m_F \omega_i H - \omega_i m_F \alpha_T = 0 \]  

(11-42)
and, from Figure 20,

\[-m_D \ddot{x}_{1,4} + m_D (x_{1,4} \omega_4^L - a_T) = 0 \tag{11-43}\]

\[-m_D \ddot{x}_{1,3} + m_D (x_{1,3} \omega_4^L + a_T) = 0 \tag{11-44}\]

The conditions for operation in this regime are that

\[(x_{1,3} - x_{01})(x_{1,4} - x_{01}) > (y_1 - y_{c1}) \tan \beta\]

and that Equation (11-13) is satisfied.
Equations (11-42) through (11-44) are now rearranged to appear as

\[ m_F \ddot{y}_1 + k_0 y_1 = F_0 + k_0 y_{01} \cdot \eta F (\alpha H + \mu \cdot \xi T) \]  

(11-45)

\[ \ddot{x}_{1,4} - \omega y x_{1,4} = -\alpha_T \]  

(11-46)

\[ \ddot{x}_{1,3} - \omega y x_{1,3} = \alpha_T \]  

(11-47)

or

\[ 6.30 \times 10^6 \dot{y}_1 + 6.00 y_1 = 5.94 - 6.30 \times 10^6 (\alpha_H + \mu \cdot \xi T) \]  

(11-48)

\[ \ddot{x}_{1,4} - 2.65 \times 10^6 \dot{x}_{1,4} = -\alpha_T \]  

(11-49)

\[ \ddot{x}_{1,3} - 2.65 \times 10^6 \dot{x}_{1,3} = \alpha_T \]  

(11-50)

The coordinate \( y \) is a measure of the axial displacement of the inertia weight, while the coordinate \( y_1 \) measures the axial displacement of the firing pin. For computer solutions of the present problem, it may be more efficient to express all results in terms of \( y \). The transformation equations which relate \( y \) and \( y_1 \), and their derivatives, are

\[ y + \dot{y}_1 = \xi_H = 0.54 \]  

(11-51)

\[ \dot{y} = -\dot{y}_1 \]  

(11-52)

\[ \ddot{y} = -\ddot{y}_1 \]  

(11-53)
12. LIMITING CONDITIONS FOR INERTIA WEIGHT TO BE IN CONTACT WITH DETENT BALLS

The combined motion of the inertia weight, detent balls and firing pin is an effect which exhibits bilinear behavior. In addition, for a small part of the motion of the detent balls the motion may be nonlinear, and this situation shall be discussed in the following paragraph. Numerical values of the forcing functions are required before a definite identification can be made of the various regimes of combined motion of the detent balls and firing pin.

Figure 21 shows the detent ball at the last point at which it is tangent to the inclined surface of the firing pin. As the pin moves down further, and if it still contacts the ball, the magnitude and direction of the contact force will continually be changing. This will be a regime of nonlinear motion because of the changing direction of the force.

Figure 21 - Kinematic Relationship of Firing Pin and Detent Ball
Since this motion is a small part of the total motion, it will be treated as a linear motion. That is, when the pin goes down past the position shown in Figure 21, it will be assumed that there is no further contact between the pin and the detent ball. From Figure 22, when a detent ball has moved through a radial distance $C_{12}$, it will be assumed to no longer be in contact with the firing pin.

![Figure 22 - Limiting Position of Firing Pin and Detent Ball](image)

In the initial configuration, at time $t=0^-$, the inertia weight is down, Figure 1, and the detent balls are in the locked position. At $t=c$ the graze forcing functions are applied and at some later time, for sufficiently large values of the forcing functions, the inertia weight begins to rise. In this initial stage of the motion balls 1 and 2 are in contact with the inertia weight, in Case A Operation. A method of superposition will now be employed to determine the time and position at which this contact is lost.
For Case A operation, the displacements and velocities of the balls and the inertia weight are related by Equations (11-4) and (11-8). Also, at the outset of motion both dent balls are in positive contact with the firing pin.

The inertia weight will now be imagined to be removed and the equation of motion, Equation (11-24), for the situation of Figure 17, will be solved. This solution will be called $X_{i,a}(t)$. The kinematic relationships between the inertia weight and the balls are repeated below.

\[ \mathbf{r}_i^2 = (y_j - y_{c,i})^2 + \left[ r - (x_i - x_{d,i}) \right]^2 \]  
\[ X_i = y_j \tan \varphi_i \]  

The function $y$ is the motion of the inertia weight and at this stage of the analysis is a known function of time. The term $X_i$ in the above two equations will be designated by $X_{i,b}(t)$ and these two equations are rewritten as

\[ X_{i,b} = X_{d,i} + r - \sqrt{r^2 - (y_j - y_{c,i})^2} \]  
\[ X_{i,b} = y_j \tan \varphi_i \]

The inertia weight is now placed back in the problem. Contact between the balls and the inertia weight will then be maintained so long as

\[ X_{i,b} \geq X_{i,b} \]  

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This expression implies that the $X_a$ motion of the balls for this hypothetical case is "ahead" of the $X_b$ motion of the inertia weight. The limiting condition when contact is lost occurs when

$$X_{1,a} = X_{1,b}$$

Both $X_{1,a}$ and $X_{1,b}$ are known functions of time, after the forcing functions have been described. The time $t_4$ which satisfies Equation (12-6) may now be found, and this time $t_4$ is the time at which the inertia weight loses contact with the detent balls, at the end of Case A operation.

The relationship

$$X_{1,a} = X_{1,b}$$

may be used to find $\phi_c$, the limiting angle which was defined in the section following Equation (11-11).
13. SUMMARY OF EVENTS DURING THE TOTAL FUNCTIONING TIME
FROM GRAZE TO FIRING

In order for the graze module to be effective in performing its design function it must open sufficiently to release the detent balls which, in turn, release the firing pin. The test of whether the inertia weight moves forward with respect to the shell is as follows.

a. The solution for $\ddot{y}$ from Equation (8-11) and (9-1), or from Equation (8-23) and (9-2), must be positive. A positive value of $\ddot{y}$ would indicate that the inertia weight moves forward with respect to the graze module housing.

b. The velocity, $\dot{y}$, of the inertia weight must be positive. In view of the definition of the orientation of the secondary $x,y,t$ coordinates, a negative value of $\dot{y}$ would be incompatible with the physical constraints of the problem.

The graze forcing functions are assumed to be applied at time $t = 0^+$. At this outset of motion of the inertia weight two regimes of motion are possible, if the magnitude of the graze force is sufficiently large. In one of these regimes, both the primary and the secondary balls will exert wedging action on the inertia weight, and the equation of motion of this weight is expressed, in its most fundamental form, by Equation (8-11) or (9-1). This will be Case I Operation, as previously defined. For this regime of motion the inequality, Equation (6-33), must be satisfied. If Equation (6-33) is not satisfied, then only the primary ball will exert wedging action on the inertia weight. For this case, previously defined as Case II Operation, the equation of motion of the inertia weight is given by Equation (8-23) or (9-2).

If the magnitudes of the graze forcing functions are not sufficiently large, the inertia weight will not open. In the following discussion it will be assumed that the forcing functions which are applied at $t = 0^+$ result in

\[ y > 0 \quad t > 0 \quad (13-1) \]

\[ \dot{y} > 0 \quad t > 0 \quad (13-2) \]
This assumption may be tested when the numerical results are obtained.

The several possible regimes of motion which follow the opening of the inertia weight will now be considered. It must be emphasized that a positive identification of a particular regime of operation will be possible only after the numerical values of the forcing functions have been specified. Consideration will now be given to the combined motion of the inertia weight, detent balls and firing pin. There are several possible sequences of events which may occur after the application of the forcing functions. Two typical sequences will be presented below.

In the first of these sequences, designated as Circuit 1, the sequence will be

Case A → Case B → Case D

where the cases are defined in section 11.

When both detent balls have moved through a distance \( C_{\text{L}} \), Figure 22, the firing pin is free to close. In the second sequence, designated by Circuit 2, Case A Operation will be assumed to exist until both detent balls have moved radially through a distance \( C_{\text{L}} \). The equation of motion of the firing pin will then be as outlined in Case D. Other sequences may exist, depending on the numerical values. The computations below, however, outline the typical details of going through a particular circuit.

**Circuit 1**

At the outset of motion, at \( t = 0^+ \), the inertia weight is in the downward position displayed in Figure 1. In this position, which is Case A Operation, both detent balls are in positive contact with the inertia weight. If the forcing functions are of sufficient magnitude to cause the inertia weight to commence its opening process, then a limiting condition occurs when the inertia weight loses contact with the two detent balls. This condition will occur at a time designated by \( t_4 \), where \( t_4 \) is the time which satisfies Equation (12-6). The time \( t_4 \) is now compared with \( t_0 \), where \( t_0 \) is given by Equation (6-26). If \( t_4 < t_0 \), then the solution of Equation (9-1) is evaluated at \( t = t_4 \). This value will be designated by \( y_A \), where \( y_A \) is a constant. It will represent the upward travel of the inertia weight which corresponds to the time \( t_4 \).

If \( t_4 > t_0 \) the problem is inherently more difficult. A test is first made to ascertain whether \( t_0 \) is positive. If \( t_0 \) is not positive, then the two secondary balls do not exert any wedging effect on the inertia weight. For this case the solution of Equation (9-2) is evaluated at \( t = t_4 \).
This value will be designated by \( y_0 \), where \( y_0 \) is a constant, and it will represent the upward travel of the inertia weight which corresponds to \( t_4 \).

If \( t_4 > t_c \), and \( t_0 > t_c \), then for part of the travel of the inertia weight all three balls will exert wedging action, and for another part of the travel only the primary ball will exert this action. The solution for \( t < t_c \) will be designated by \( y_C \), and it may be found from Equation (9-1). The solution for \( t_c < t < t_4 \) will be designated by \( y_D \). The initial conditions for this problem are then

\[
\begin{align*}
\dot{y}_C &= 0 & y_C &= y_c & (13-4) \\
\dot{y}_C &= C & y_C &= y_c & (13-5) \\
\dot{y}_C &= y_D & y_C &= y_d & (13-6) \\
\dot{y}_C &= \dot{y}_D & y_C &= \dot{y}_D & (13-7)
\end{align*}
\]

The total displacement of the inertia weight at time \( t_4 \), designated by \( y_E \), is then

\[
y_E = y_D \bigg|_{t=t_4} (13-8)
\]
The initial conditions for the solutions which result in $\gamma_A$ and $\gamma_B$ are

$$t = 0 \quad \gamma = \gamma_c \quad (13-9)$$

$$t = 0 \quad \gamma = 0 \quad (13-10)$$

At this stage of the analysis there are three possible displacements of the inertia weight which may exist, and these are $\gamma_A$, $\gamma_B$, or $\gamma_C$. The several values of the displacement of the inertia weight, at the point where it loses contact with the detent balls, are summarized below.

$$t_0 > 0 \quad t_4 < t_0 \quad \gamma \bigg|_{t=t_4} = \gamma_A \quad (13-11)$$

$$t_0 < 0 \quad t_4 > t_0 \quad \gamma \bigg|_{t=t_4} = \gamma_B \quad (13-12)$$

$$t_0 > 0 \quad t_4 > t_0 \quad t < t_0 \quad \gamma = \gamma_C \quad (13-13)$$

$$t_0 > 0 \quad t_4 > t_0 \quad t_0 < t < t_4 \quad \gamma = \gamma_D \quad (13-14)$$

$$t_0 > 0 \quad t_4 > t_0 \quad \gamma \bigg|_{t=t_4} = \gamma_D = \gamma_E \quad (13-15)$$
Figure 23 shows a detail from Figure 16. The subscript $j$ may be $A$, $B$ or $E$. $\varphi_o$ is the limiting angle at which the inertia weight loses contact with the detent ball, defined by Equation (12-7), and $A_v$ corresponds to $\psi_o$. From Equation (11-2)

$$A = r_{ci} - (x_i - x_{ci}) \quad (13-16)$$

From the figure

$$A_o = (y_j - y_{ci}) \omega t \psi_o \quad (13-17)$$

$x_{i,j}$ will be the radial displacement of the detent balls, at the position $\varphi = \psi_o$, which corresponds to $y_j$, where $j = A, B$ or $E$. Equation (13-16) is substituted into Equation (13-17), with the result

$$(y_j - y_{ci}) \omega t \psi_o = r - (x_{i,j} - x_{ci}) \quad (13-18)$$

or

$$x_{i,j} = x_{ci} + r - (y_j - y_{ci}) \omega t \psi_o \quad (13-19)$$
Equation (13-19) gives the radial location of the detent balls at the time \( t_4 \) when the inertia weight loses contact with these balls. The corresponding velocities of the detent balls may be found with the aid of Equation (11-8) which appears as

\[
\left[ \dot{X}_i = \ddot{y}_j \tan \alpha \right]_{t = t_4} \tag{13-20}
\]

The functional forms of \( y_j \) have already been obtained for the several regimes of time which are defined by Equations (13-11) through (13-15), and the initial conditions. The values of \( \dot{y}_j \), where \( j = A, B \) or \( E \), may then be obtained for \( t = t_4 \). The detent ball velocity \( \dot{x}_{1,j} \) is then related to the inertia weight velocity \( \dot{y}_j \), following equation (13-20), by

\[
\dot{x}_{1,j} = \dot{y}_j \tan \alpha \dot{y}_0 \tag{13-21}
\]

where \( \dot{y}_0 \) is as previously defined and \( j = A, B \) or \( E \).

In Circuit 1 Operation, Case B Operation is assumed to follow Case A Operation. This implies that, from Figure 22,

\[
X_{1,j} - X_{0,1} < C_{12} = 0.05q \tag{13-22}
\]

\[
X_{1,j} < 0.17q \tag{13-23}
\]

The two detent balls are in contact with the firing pin, but no longer in contact with the inertia weight. The motion of the detent balls is given by Equation (11-24). This equation is integrated, and the solution is designated by \( \dot{x}_B \), where the superscript \( B \) signifies Case B Operation.

The initial conditions are

\[
t = t_4 \quad \dot{x}_B = \dot{x}_{1,j} \quad j = A, B \text{ or } E \tag{13-24}
\]

\[
t = t_4 \quad \ddot{x}_B = \ddot{x}_{1,j} \quad j = A, B \text{ or } E \tag{13-25}
\]
When

\[ X'_1 - X_{c1} = C_{12} \]  \hspace{1cm} (13-26)

or

\[ X'_1 = t^{1.7} \]  \hspace{1cm} (13-27)

the detent balls lose contact with the firing pin and Case B Operation ceases. The corresponding time is \( t_5 \).

For \( t > t_5 \), Case D Operation prevails and the equation of motion of the firing pin is given by Equation (11-45).

Following Equation (11-1),

\[(y_1 - y_{c1}) \tan \beta = x_1 - x_{c1} \]  \hspace{1cm} (13-28)

For the conditions of Equation (13-26), Equation (13-28) appears as

\[(y_1 - y_{c1}) = C_{12} \cdot e^{t \beta} \]  \hspace{1cm} (13-29)

and, from Equation (11-1),

\[ y'_1 = x'_1 \cdot e^{t \beta} \]  \hspace{1cm} (13-30)

Equation (11-45) is now integrated, and the solution is designated \( y^D_1 \). The initial conditions are

\[ t = t_5 \hspace{1cm} y^D_1 - y_{c1} = C_{12} \cdot e^{t_5 \beta} \]  \hspace{1cm} (13-31)

\[ t = t_5 \hspace{1cm} y^D_1 = x^B_1 \]  \hspace{1cm} (13-32)

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If the maximum travel of the firing pin is $\frac{\text{M}}{\text{VKR}}$, and the corresponding time at the end of this travel is $t$, then

$$\left( y_1^D - y_{z1} \right) t = \frac{\text{M}}{\text{VKR}} - y_{z1}$$

Equation (13-33) may be used to find $t$. The velocity of the firing pin on striking the detonator is

$$y_1^D \mid t = t$$

and the kinetic energy of the firing pin on impact is

$$KE = \frac{1}{2} m_r \left[ \dot{y}_1^D \right] t = t$$

Circuit 2

In Circuit 2 Operation, the inertia weight is assumed to remain in contact with both detent balls until these balls have moved through a radial distance $C_1$. The corresponding time will be designated by $t_7$. As in Circuit 1 Operation, a preliminary test is whether $t_7 > t_c$ or $t_7 < t_c$, and this analysis would follow the steps outlined earlier. The condition when the detent balls lose contact with the firing pin is

$$t = t_7, \quad X_1 - X_\omega$$(13-35)
The displacement of the inertia weight at \( t = t_7 \) is given by
\[
\left( y_{|t=t_7} - y_{01} \right) = + \sqrt{\mathfrak{c} - \left( \mathfrak{c} - \mathfrak{c}_{12} \right)^2}
\]  
(13-36)

Time \( t_7 \) may now be found from Equation (13-36).

The corresponding velocity of the detent balls is, from Equations (11-5) and (11-8),
\[
\dot{X}_1 \bigg|_{t=t_7} = \left[ y_{|t=t_7} - y_{01} \right] \frac{\omega t}{\mathfrak{c} - \mathfrak{c}_{12}}
\]  
(13-37)

The displacement and velocity of the firing pin at \( t = t_7 \), following the form of Equation (13-29), are
\[
\begin{align*}
t = t_7 & \quad \left( y_{|t=t_7} - y_{01} \right) = \mathfrak{c}_{12} \omega t \beta \\
\frac{\mathfrak{c}}{t = t_7} & \quad \left( y_{|t=t_7} - y_{01} \right) = \frac{\mathfrak{c}_{12}}{t = t_7} \omega t \beta
\end{align*}
\]  
(13-38)

The motion of the firing pin for \( t > t_7 \) is given by Equation (11-45). The integral of this equation is designated by \( y_{E_1} \). The initial conditions are, using Equations (13-36) through (13-39),
\[
\begin{align*}
t = t_7 & \quad \left( y_{E_1} - y_{01} \right) = \mathfrak{c}_{12} \omega t \beta
\end{align*}
\]  
(13-40)
When the final form of $\mathcal{V}$ is obtained, the solution proceeds as before, starting with Equation (13-33).

All of the numerical values of the constants in this section are contained in Appendix A.

14. DISCUSSION OF ASSUMPTIONS

The purpose of the present report is to investigate the effectiveness of the graze module of the Hi-Performance point detonating fuze. The assumptions which are the basis for this study will now be commented on. It should be emphasized that, since no numerical results are obtained in this study, certain conclusions must, of necessity, be general.

Assumption a, which states that the direction of $\mathcal{A}_T$ is constant with respect to the position of a particular graze ball, reduces the situation to a static analysis of a dynamical problem. If $t_e$ is the time for any graze ball to occupy a successive position, then

$$t_e = \pi / 4 \omega_y$$

where $\omega_y$ is the assumed constant spin velocity of the shell. If the total time from the initiation of graze to the firing event is much less than $t_e$, then assumption a can be justified. If this situation does not prevail, the analysis would have to be modified to show the time varying effect, due to shell rotation, of $\mathcal{A}_T$ on the graze balls. In the present analysis, $\mathcal{A}_T$ is assumed to have a constant direction with respect to the graze balls, as shown in Figure 3. In the more refined analysis, the effect of $\mathcal{A}_T$ along a radial axis through the graze balls would be of the form

$$\mathcal{A}_T \propto (\omega_y t + \omega_o)$$

(14-2)
where $\alpha_o$ is a conveniently chosen phase angle. The above conclusion also applies to assumption $\mathcal{G}$ (page 11), which assumed that the direction of $\alpha_T$ is constant with respect to the direction of the two detent balls.

Assumption $b$ states that friction forces exist only between the inertia weight and the center post, and between the firing pin and the center post. The friction forces which have been neglected are those between the graze balls and their mating surfaces. As the graze balls move radially inward, they will possess a combined rotating and translating motion. The normal contact forces on the graze and detent balls were defined earlier. It is thus possible to compute the tangential friction forces acting on the graze balls, after the motion has been established. A similar line of reasoning applies to the tangential friction forces acting on the detent balls. When these friction forces are obtained, they may be compared to the other forces acting on the balls, and the validity of the assumption may be tested.

Assumption $c$ states that the coefficient of friction between all sliding parts is constant. This standard assumption is usually true for the case of low contact pressures between the surfaces in contact. In the present problem, the impact nature of the graze forcing functions may cause considerable contact pressures between the graze balls and their contacting surfaces. As a result, the coefficients of static and kinetic friction may be considerably higher than the present estimates. In these cases, the coefficients are found to be dependent on the relative velocity, at a given pressure value, of the mating surfaces. The maximum value of the constant coefficient of friction in the present problem is assumed to be 0.1. If solutions are obtained for $\mu = 0.1$ and $\mu = 0$, the range of the effect of friction forces on the problem can be determined. In a more refined analysis of the problem, the variation of the coefficient of friction should be considered.

Assumptions $d$ and $e$ exclude all gyrodynamic effects of the moving shell. Assumption $d$ describes the shell as possessing only constant spin velocity. When the shell grazes the terrain, there will be a decrease in this spin velocity of unknown magnitude. The centrifugal forces on the graze balls, caused by the spin velocity, tend to retard the functioning of the graze module assembly. By assuming the spin velocity to be constant, a test is made of whether the graze module will function under the most adverse condition. If the numerical results indicate a borderline situation of operation, then a more refined analysis could include an assumed functional form for the decrease with respect to time of the spin velocity.
For this case the tangential (i.e. in a direction normal to the radial grooves) inertia forces on the graze balls would have to be included. The remaining two rotation components in the present problem, $\omega_x$ and $\omega_z$, of the shell absolute rotational velocity are assumed to be much smaller than $\omega_y$, and are thus neglected.

Assumption 6 states that the shell moves in a plane which is normal to the graze terrain, and remains at a constant angle with respect to this terrain. This assumption implies that the shell impacts the terrain with zero angle of attack, and that there is no spin decay. As the shell grazes the terrain, it is acted upon by some net external moment. Since there will be a change in all three components of the shell absolute rotational velocity, the shell must change its direction in such a way as to continually satisfy the moment-moment of momentum equation. The net result of this will be that the shell, at some time, will move out of its original direction. The magnitudes and durations of the graze forcing functions cover a wide range of values. In addition, these values are not known with great accuracy. It is the anticipation of the present design of the graze module that the total time from the initiation of graze to the firing event is small compared to other times in the problem. It thus appears that the above assumption, which states that the shell direction is constant with respect to the graze terrain, is reasonable. This assumption can more accurately be tested when the numerical results are obtained.

Assumption 4 states that the graze balls translate only in a plane through the center axis of the graze module. As the graze balls move radially in their grooves, there will be coriolis inertia forces which tend to displace the balls in a direction normal to these grooves. In addition, there will be components of the inertia force which tend to displace certain of the balls laterally. This latter effect is shown in Figure 24.

The two balls in the upper part of the figure, for the position shown, will have acting on them inertia forces, in a direction normal to the grooves, of magnitude $\omega_y Q_T \Delta z$. It is assumed in the present study that the effects of these forces, and of the coriolis inertia forces, are small compared to the other forces acting on the balls. These forces could be included in a more refined analysis of the graze module. The tangential inertia forces on the graze balls, which are absent because of assumption 6, are also neglected.
Assumption H assumes that the dimension $C_2$ is constant, so that the equations of motion will have linear forms. Appendix B contains the details of the computation of the range of values of $C_2$. A constant value of this term is chosen which will yield conservative results for the motion of the inertia weight. It is the feeling of the present author that the error introduced by assuming $C_2$ to be constant is less than the uncertainty in the magnitudes of the actual graze forcing functions.

Assumption I assumes, merely for convenience, that the graze force acts at the tip of the ogive shell nose.

Assumption J states that the mass moment of inertia of the firing pin, about an axis through its center of mass and normal to the longitudinal axis of the shell, may be neglected. This assumption is made since the firing pin is a slender element which, for a considerable part of its length, has the shape of a thin walled shell. It should be observed that this assumption was not made for the inertia weight, since this element has a shape which resembles a thick disc.

Assumption K states that the firing pin moves only forward with respect to the graze module housing. When the detent balls move outward sufficiently, the firing pin is free to
move forward under the influence of the firing pin spring. The axial friction force on the firing pin will always act to oppose the motion, so that this friction force on the firing pin will have a sense which is towards the nose of the shell. This effect is reflected in the sign of the terms $\mu N_f \alpha_T$ in Equations (11-18), (11-32) and (11-42). The criteria of whether the firing pin does indeed move forward are

$$j_f - j_{0f} > 0 \quad (14-3)$$

and

$$u_f > 0 \quad (14-4)$$

If this situation does not prevail, then the signs of the terms $\mu N_f \alpha_T$ in the above referenced equations are changed for the regime of motion where the firing pin moves towards the nose of the shell. The initial conditions of the firing pin would, of course, have to be modified accordingly.

Assumption 1 states that the firing pin spring is a massless element with a constant spring rate. This appears to be a valid assumption for a first approximation solution of the present problem. This spring tends to be compressed by the inertia forces induced by the direct effect of $\alpha_A$ on the mass particles of the spring. In addition, the firing pin spring has a mass which is not negligible compared with the mass of the firing pin. It is possible, then, that the spring force which is actually available to accelerate the firing pin may be much less than the design value which is used in this study.

Assumption 2 approximates the true graze forcing functions by a triangular shape. The general shape of the actual forcing function is shown in Figure 25. An improvement in the representation of the forcing functions may be obtained by a combination of the functions

$$e^\alpha t \quad \text{and} \quad e^{\nu t}$$
This new function for the acceleration \( \alpha_T \) would be of the form

\[
\alpha_T = \alpha \left( e^{\mu t} - e^{\nu \tau} \right)
\]  
(14.5)

where \( \alpha, \mu \), and \( \nu \) are constants to be chosen, and

\( \mu, \nu < \alpha \) and \( \mu > \nu \)

These constants may then be selected so that the best approximation to the actual curve is obtained.
15. CONCLUSIONS AND RECOMMENDATIONS

An analysis has been made of the major elements of the graze module of the Hi-Performance point detonating fuze, subject to the assumptions which are described in this report. The analysis shows a favorable form of solution for the equation of motion of the inertia weight, and this is an important factor in the sequential operation of the graze module. The analysis identifies two cases for the combined motion of the inertia weight and the graze balls, and four cases for the sequential motion of the inertia weight - detent balls - firing pin. Any combination of these latter cases may occur, depending on the values of the graze forcing functions. A numerical form of each equation was obtained, based on present design parameters for the 105 mm Howitzer. The forcing functions in one section of the report were also approximated by triangular shapes. No numerical values are obtained in this report. These values must be known before definite judgements may be made of the assumptions.

The following recommendations are offered for future study.

a. the dependence of the acceleration \( a \) on the shell rotation
b. the effect of spin decay on the equations of motion
c. the effect of the tangential inertia forces (i.e. in a direction normal to the radial grooves) on the graze balls
d. the dynamic effects of the firing pin spring, and of the inertia weight creep springs
e. the inclusion of a more accurate representation of the graze forcing functions
f. the inclusion in the impact configuration of the initial yaw angle of the shell. This analysis, together with the inclusion of spin decay, would provide more detailed results for the gyrodynamic motion of the shell during impact, and its instability behavior thereafter.
g. the effect of a variable coefficient of friction on the motion of the graze module components.
APPENDIX A - EVALUATION OF THE CONSTANTS OF THE PROBLEM

a. Location of The Centroid of The Inertia Weight

Figure A-1 shows the nominal dimensions of the inertia weight. In the calculation of the location of the centroid and the moment of inertia about the centroid, the following portions of the inertia weight will be neglected:

1. The groove, as shown in Figure A-1
2. The C.0436 locating hole
3. The C.2146 hole

As a preliminary calculation, the centroid of a frustrum of a right circular cone will be obtained. Figure A-2 shows the solid, and it is also piece "A" in Figure A-1.

---

Figure A-1 - Inertia Weight
For a right circular cone, the volume is \( \frac{1}{3} \pi r^2 h \) and the centroid is at \( \frac{h}{4} \).

The coordinate \( \bar{h} \) is

\[
\bar{h} = \frac{1}{4} (0.280) \left[ \frac{1}{3} \pi (0.487)^2 (0.240) \right] - \frac{1}{4} (0.122 + \frac{1}{4} (0.308)) \left[ \frac{1}{3} \pi (0.275)^2 (0.158) \right]
\]

\[
\frac{1}{3} \pi (0.487)^2 (0.240) - \frac{1}{3} \pi (0.275)^2 (0.158)
\]

\[\text{(A-1)}\]

\[
\bar{h} = \frac{0.60486 - 0.01202}{0.0955 - 0.0125}
\]

\[\text{(A-2)}\]

\[
\bar{h} = 0.498
\]

\[\text{(A-3)}\]
The computation of \( C_t \), the location of the centroid, is shown on the following page. The result is

\[
C_t = 0.121 \quad (A-4)
\]

\textbf{b. Moment of Inertia of The Inertia Weight About The Centroid}

Figure A-3 shows the elements which comprise the inertia weight. The frustrum of a cone, Piece A, is approximated as a circular disc. For a right circular cylinder, the mass moment of inertia about a diameter through the centroid is \( n \left( \frac{b}{2} \right)^2 + \frac{L^2}{12} \), where \( b \) is the diameter and \( L \) is the height. The moment of inertia about an axis parallel to the centroidal diameter is

\[
\mathbf{I}_o = \mathbf{I}_{\phi, 1} + \mathbf{I}_{\phi, 2} + \mathbf{I}_{\phi, 3} + \mathbf{I}_{\phi, 4} \quad (A-8)
\]

where \( R_o \) is the distance between the two axes. From Figure A-3, \( \mathbf{I}_o \) may be expressed as

\[
\mathbf{I}_o = \mathbf{I}_{\phi, 1} + \mathbf{I}_{\phi, 2} + \mathbf{I}_{\phi, 3} + \mathbf{I}_{\phi, 4} \quad (A-9)
\]

The mass \( \mathbf{m} \) may be written as \( \sqrt{V} \), where \( V \) is the mass density and \( V \) is the volume. \( \mathbf{I}_o \) now appears as

\[
\mathbf{I}_o = \frac{\pi(1.6)^2}{4} (0.315) \left[ \frac{1.125^2}{16} + \frac{0.315^2}{12} + 0.024 \frac{L^2}{12} \right]
\]

\[
+ \frac{\pi(0.762)^2}{4} (0.122) \left[ \frac{0.762^2}{16} + \frac{0.122^2}{12} + 0.193 \frac{L^2}{12} \right]
\]

\[
+ \frac{\pi(0.550)^2}{4} (0.194) \left[ \frac{0.550^2}{16} + \frac{0.194^2}{12} + 0.349 \frac{L^2}{12} \right]
\]

\[
- \frac{\pi(6.462)^2}{4} (0.625) \left[ \frac{6.462^2}{16} + \frac{0.625^2}{12} + 0.13 \frac{L^2}{12} \right] \quad (A-10)
\]
\[ C_6 = \frac{\pi (1.18)^2 (0.313)}{4} \left( \frac{0.513}{2} \right) + 0.0125 (0.313 + 0.0498) + \frac{\pi (0.550)^2 (0.190)}{4} \left( 0.313 + 0.125 + \frac{0.190}{2} \right) \]

\[ = \frac{\pi (1.18)^2 (0.313)}{4} (0.313) + 0.0125 + \frac{\pi (0.550)^2 (0.190)}{4} (0.190) \]

\[ - \frac{\pi (0.400)^2 (0.625)}{4} \left( \frac{0.625}{2} \right) - 3 \left\{ \frac{\pi (0.156)^2 (0.250)}{4} \left( \frac{0.250}{2} \right) \right\} \]

\[ = \frac{0.342 (0.157) + 0.0125 (0.363) + 0.045 (0.530) - 0.081 (0.313) - 0.145 (0.125)}{0.342 + 0.0125 + 0.045 - 0.081 - 0.0145} \]

\[ C_6 = \frac{0.0550}{0.304} = 0.181 \]
Using a specific weight for aluminum of $0.1 \text{ lb/ft}^3$, the inertia weight $I_o$ can be calculated as:

$$I_o = \frac{C_o \cdot \frac{\text{lb}}{\text{ft}}}{\text{in}^3} \left( \frac{C_o \cdot \text{lb} \cdot \text{in}}{\text{in}^3} \right)$$ (A-12)

$$I_o = 1.04 \times 10^{-5} \text{ lb \cdot in}$$ (A-13)

Figure A-3 - Subsections of Inertia Weight

C. Relationship of Inertia Weight Motion to Graze Ball Motion

Figure A-4 shows the kinematic relationships between the inertia weight and the ball. $X$ is the position of the ball and $X_o$ is the most outboard position. $Y$ is the position of the CM and $Y_o$ is the position which corresponds to $X_o$. From consideration of the triangle

$$\tan \theta = \frac{Y - Y_o}{X_o - X}$$ (A-14)
so that

\[ X = ( - \omega t e ) y + ( X_1 + X_2 e^{-t} ) \]  \hspace{1cm} (A-15)

Figure A-4  Kinematic Relationship of Graze Ball and Inertia Weight

From Equation (6-13),

\[ X = C_B y + C_q \]  \hspace{1cm} (A-16)

Comparison of Equations (A-16) and (A-17) results in

\[ C_B = - \omega e t \theta \]  \hspace{1cm} (A-17)

\[ C_q = X_0 + y_0 \omega e t \theta \]  \hspace{1cm} (A-18)
Figure A-5 shows the assembly of the inertia weight in the graz module housing. All of the dimensions shown are nominal. The ball diameter D is $5/16''$, and the radius is 0.156.

From Figure A-5,

$$x_0 = \frac{1.42}{2} - 0.156 = 0.554 \quad (A-19)$$

$$0.171 + y_0 + 0.181 = 0.625 \quad (A-20)$$

$$y_0 = 0.273 \quad (A-21)$$

$$c_3 = 0.171 + 0.156 \cdot \cos 30^\circ = 0.171 + 0.156(0.866) \quad (A-22)$$

$$c_3 = 0.306 \quad (A-23)$$
The dimension $C_{10}$ represents the maximum possible motion of the inertia weight with respect to the housing. From the figure,

$$C_{10} + 0.125 = 0.725 \quad (\Lambda-24)$$

$$C_{10} = 0.60 \quad (\Lambda-25)$$

The maximum displacement, $y_{\text{MAX}}$, of the CM of the inertia weight is then

$$y_{\text{MAX}} = y_0 + C_{10} \quad (\Lambda-26)$$

$$= 0.273 + 0.60 \quad (\Lambda-27)$$

$$y_{\text{MAX}} = 0.878 \quad (\Lambda-28)$$

The permissible range of $y$, the coordinate of the CM, is thus

$$y_0 \leq y \leq y_0 + C_{10} \quad (\Lambda-29)$$

$$0.273 \leq y \leq 0.878 \quad (\Lambda-30)$$

The dimension $C_2$ is a function of $X$. When $X = X_0$, $C_2 = C_2(X_0)$. For any other $X$, $C_2 = C_2(X)$. From the figure,

$$C_2 + \frac{D}{L} \tan \theta = X \quad (\Lambda-31)$$

$$C_2 + 0.156 \tan 30^\circ = X \quad (\Lambda-32)$$

$C_2$ will now be found in terms of $y$. From Equation (\Lambda-15)

$$X = -y \cot \theta + X_0 + y_0 \cot \theta \quad (\Lambda-33)$$

so that

$$C_2 = X_0 + y_0 \cot 30^\circ - 0.156 \tan 30^\circ - y \cot 30^\circ \quad (\Lambda-34)$$

In this maximum outboard position, $y = y_0 = 0.273$, so that

$$C_{2,\text{MAX}} = X_0 + y_0 \cot 30^\circ - 0.156 \tan 30^\circ - y_0 \cot 30^\circ \quad (\Lambda-35)$$

$$C_{2,\text{MAX}} = 0.554 - 0.156 (0.5) \quad (\Lambda-36)$$

$$C_{2,\text{MAX}} = 0.476 \quad (\Lambda-37)$$
When the inertia weight touches the ceiling of the graze module housing, the displacement is given by Equation (38) and $C_{z,MIN}$ is

$$C_{z,MIN} = 0.554 + 0.273(1.73) - C_{z,MAX}(0.5) - 1.73(0.362)$$ (39)

The possible range for $C_z$ in the present problem is then

$$0.313 \leq C_z \leq 0.476$$ (40)

e. Summary of Constants of Problem

The several constants of the problem are summarized below. The system of units is pound-seconds-inches.

$$m_b = 1.08 \times 10^{-5} \text{ lb. sec}^2/\text{in}.$$  

$$m_1 = 8.29 \times 10^{-5} \text{ lb. sec}^2/\text{in}.$$  

$$m_d = 5.18 \times 10^{-7} \text{ lb. sec}^2/\text{in}.$$  

$$m_f = 6.30 \times 10^{-6} \text{ lb. sec}^2/\text{in}.$$  

$C_1 = 0.625$  

$C_{z,MIN} = 0.313$  

$C_{z,MAX} = 0.476$  

$C_3 = 0.306$  

$C_A = 0.243$  

$C_5 = 0.444$  

$C_6 = 0.181$  

$C_7 = 0.319$  

$C_8 = -1.73$  

$C_9 = 1.03$  

$C_{10} = 0.095$  

$C_{11} = 0.549$  

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$C_{12} = 0.040$
\[\theta = 30^\circ\]
\[\beta = 30^\circ\]
\[\tan \theta = 0.577\]
\[\cot \theta = 1.73\]
\[I_0 = 1 \times 10^{-3} \text{lb.-sec}^2 \text{ in.}\]
\[F_q = 8.52 \text{ (Maximum)}\]
\[F_0 = 3.9 \text{ (Minimum)}\]
\[y_{\text{MAX.}} = 0.368\]
\[u = 0.1\]
\[x_0 = 0.554\]
\[y_0 = 0.273\]
\[x_{01} = 0.140\]
\[y_{01} = 0.340\]
\[K = 18 \text{ lb./in.}\]
\[K_0 = 6 \text{ lb./in.}\]
\[n = 0.063\]
\[D = 0.313\]

The spin velocity is assumed to be for the 105 mm. Howitzer, M1, so that

\[\omega y = 15,700 \text{ RPM} = 1,640 \text{ rad./sec.}\]
APPENDIX B - COMPARISON OF THE EQUATIONS OF MOTION OF THE INERTIA WEIGHT FOR $C_z = C_z, \text{MIN.}$ AND $C_z = C_z, \text{MAX.}$

The equations of motion of the inertia weight for the two regimes of operations, Equations (8-11) and (8-23), will now be evaluated for the two extreme values of $C_z$. It will be seen that the differences in these equations which result from this variation in the magnitude of $C_z$ are small.

a. Both Primary and Secondary Balls Exert Wedging Effect on The Inertia Weight, Case I Operation

Using the numerical values in Appendix A, the equation of motion of the inertia weight, Equation (8-11), appears as

\[
1.69 \times 10^{-4} \ddot{y} - 2.12 \dot{y} = 3.64 \times 10^{-5} \alpha_T + 1.37 \times 10^{-4} \alpha_A
\]

\[-3.2 \times 10^{-6} \alpha - 14\]

$C_z = C_z, \text{MAX.} = 0.476$  \hspace{1cm} (B-1)

\[
1.73 \times 10^{-4} \ddot{y} - 222 \dot{y} = 3.81 \times 10^{-5} \alpha_T + 1.37 \times 10^{-4} \alpha_A
\]

\[-3.2 \times 10^{-6} \alpha - 147\]

$C_z = C_z, \text{MIN.} = 0.313$  \hspace{1cm} (B-2)

The above two equations will be compared term by term using a percent difference, $\%D$, as defined below.

\[
\%D = \frac{\text{Coefficient of Equation (B-2)} - \text{Coefficient of Equation (B-1)}}{\text{Coefficient of Equation (B-1)}}
\]

The results are tabulated in Table B-1

<table>
<thead>
<tr>
<th>Term</th>
<th>$\ddot{y}$</th>
<th>$\dot{y}$</th>
<th>$\alpha_T$</th>
<th>$\alpha_A$</th>
<th>$\alpha$</th>
<th>Const.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$%D$</td>
<td>+2.4</td>
<td>+4.7</td>
<td>+4.7</td>
<td>0</td>
<td>0</td>
<td>+4.3</td>
</tr>
</tbody>
</table>

Table B-1

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b. Primary Ball Alone Exerts Wedging Effect on Inertia Weight, Case II Operation

The equations of motion of the inertia weight, Equation (8-23), for the two extreme values of $C$, now appear as

$$1.1 \times 10^{-4} \ddot{y} - 5C \dot{y} = 1.29 \times 10^{-5} \alpha_T + 1.3 \times 10^{-4} \alpha_H$$

$$-3.2 \times 10^{-4} \dot{x} - 4.1 \times 10^{-4} \dot{x} \dot{y} = 1.3 \times 10^{-5} \alpha_T + 1.3 \times 10^{-4} \alpha_H$$

The percent differences, $\%D$, in the above two equations are defined by Equation (B-6) as

$$\%D = \frac{\text{Coefficient of Equation (B-5)} - \text{Coefficient of Equation (B-4)}}{\text{Coefficient of Equation (B-4)}}$$

The results are tabulated in Table B-2

<table>
<thead>
<tr>
<th>Term</th>
<th>$y_j$</th>
<th>$y_j$</th>
<th>$\alpha_T$</th>
<th>$\alpha_T$</th>
<th>$\alpha$</th>
<th>$C$</th>
<th>Const.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>+1.8</td>
<td>+7.8</td>
<td>+7.8</td>
<td>$C$</td>
<td>$C$</td>
<td>+5.7</td>
<td></td>
</tr>
</tbody>
</table>

APPENDIX C - IMPACT OF INERTIA WEIGHT ON CEILING OF GRAZE MODULE

For very large magnitudes of the graze forcing functions, it is possible that the inertia weight might rebound off the ceiling of the graze with sufficient velocity to "pinch" a detent balls on their way out, and thus prevent the closing of the firing pin. This section establishes the conditions for the impact of the inertia weight on the ceiling of the graze module. The time $t_1$ is defined as the time when the inertia weight strikes the ceiling of the graze module housing. Since the mass of the inertia weight is much less than the mass of the shell, it will be assumed that the velocity of the shell is unaffected by this impact. Thus, the rebound velocity of the inertia weight will be governed by the definition of the coefficient of restitution.
For two spheres in direct central impact, the coefficient of restitution $e$ is defined to be

$$e = \frac{V_B' - V_A'}{V_A - V_B}$$  \hspace{1cm} (C-1)$$

where $A$ and $B$ signify the two spheres and the primes designate conditions after impact. The velocities in Equation (C-1) are absolute. The impact of the inertia weight will be considered to be comparable to the case of spheres impacting, and a representative value of $e$ will subsequently be chosen.

Figure C-1 shows the inertia weight and graze module ceiling, before impact. The velocity of the $\chi\eta\zeta$ coordinate system is $V_\zeta$ so that

$$V_A = V_B + i\jmath$$  \hspace{1cm} (C-2)$$

In view of the assumption,

$$V_\zeta' = V_B'$$  \hspace{1cm} (C-3)$$

and thus

$$V_A' = V_\zeta' + i\jmath' = V_B + i\jmath'$$  \hspace{1cm} (C-4)$$

where $i\jmath'$ is the relative velocity of the inertia weight after impact.
The results are substituted into Equation (C-1), with the result

\[ \varepsilon = \frac{\dot{V}_{11} - \dot{V}_{44}}{V_{44} - V_{11}} = \frac{V_{11} \left( V_{11} \varepsilon + \dot{\varepsilon} \right)}{(V_{11} \varepsilon + \dot{\varepsilon}) - V_{11}} \]  
(C-5)

\[ \varepsilon = -\frac{\dot{\varepsilon}}{\varepsilon} \]  
(C-6)

The final result is

\[ \dot{\varepsilon} = -\varepsilon \dot{\varepsilon} \]

The boundary conditions at \( t = t_3 \) are then

\[ t = t_3 \quad \dot{\varepsilon} = \dot{\varepsilon}_M \max \]  
(C-8)

\[ t = t_3 \quad \varepsilon \bigg|_{t_3} = -\varepsilon \bigg|_{t_3} \]  
(C-9)

where \( \varepsilon_M \max \) is the maximum displacement of the CM of the inertia weight.

**APPENDIX D - NOMENCLATURE**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition (Page on which first used)</th>
</tr>
</thead>
<tbody>
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<td>(\alpha)</td>
<td>Shell Dimension (12)</td>
</tr>
<tr>
<td>(\xi)</td>
<td>Shell Dimension (12)</td>
</tr>
<tr>
<td>(\alpha_h)</td>
<td>Component of shell acceleration (12)</td>
</tr>
<tr>
<td>(\alpha_k)</td>
<td>Component of shell acceleration (12)</td>
</tr>
<tr>
<td>(\alpha_{k-h})</td>
<td>Component of shell acceleration (12)</td>
</tr>
<tr>
<td>(\bar{F}_n)</td>
<td>Axial component of graze force (12)</td>
</tr>
<tr>
<td>(\bar{F}_F)</td>
<td>Normal component of graze force (12)</td>
</tr>
<tr>
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<tr>
<td>$3$</td>
<td>Graze angle (12)</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of shell (12)</td>
</tr>
<tr>
<td>$I_{cm}$</td>
<td>Moment of inertia of shell about an axis normal to the longitudinal axis, with respect to center of mass (12)</td>
</tr>
<tr>
<td>$a_T$</td>
<td>Lateral acceleration of graze module housing (12)</td>
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<tr>
<td>$K_{cm}$</td>
<td>Radius of gyration of shell, about an axis normal to the longitudinal axis, with respect to the center of mass (12)</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Constant of proportionality (13)</td>
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<tr>
<td>$K_2$</td>
<td>Constant of proportionality (13)</td>
</tr>
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<td>$x_0, y_0, z_0$</td>
<td>Inertial coordinates (14)</td>
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<td>$x, y, z$</td>
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<td>$\ddot{x}_c$</td>
<td>Absolute acceleration of an arbitrary point (14)</td>
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<tr>
<td>$\ddot{\ell}$</td>
<td>An arbitrary point (14)</td>
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<td>$\ddot{R}, \ddot{z}$</td>
<td>Terms in expression for absolute acceleration (15)</td>
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<td>$\ddot{\ell}, \ddot{j}, \ddot{k}$</td>
<td>Unit vectors in the $x, y, z$ coordinate system (15)</td>
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<tr>
<td>$\ddot{a}_{0,1}$</td>
<td>Absolute acceleration of primary graze ball (15)</td>
</tr>
<tr>
<td>$\omega_{y}$</td>
<td>Spin angular velocity of shell (15)</td>
</tr>
<tr>
<td>$\ddot{a}_{0,2}$</td>
<td>Absolute acceleration of secondary graze ball (17)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Ramp angle of inertia weight (18)</td>
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<tr>
<td>$\bar{F}_{w,1}$</td>
<td>Reaction force on primary graze ball 1, by floor of graze module (18)</td>
</tr>
<tr>
<td>$\bar{F}_{l,1}$</td>
<td>Normal contact force between inertia weight and primary graze ball (18)</td>
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<tr>
<td>$F_{ix,1}, F_{iy,1}$</td>
<td>$\bar{x}$ and $\bar{y}$ components of $\bar{F}_{l,1}$ (19)</td>
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<tr>
<td>$m_G$</td>
<td>Graze ball mass (20)</td>
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<td>$c_\gamma$</td>
<td>Constant (21)</td>
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<td>$c_q$</td>
<td>Constant (21)</td>
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<td>$\bar{F}<em>{L}$, $\bar{F}</em>{R}$</td>
<td>Normal reactions on inertia weight (27)</td>
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<tr>
<td>$F_x$, $F_y$</td>
<td>Friction forces induced by $F_L$ and $F_R$ (27)</td>
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<td>Constant coefficient of friction (30)</td>
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<td>Inertia force of inertia weight (27)</td>
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<tr>
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<td>Total force of three inertia weight creep springs (27)</td>
</tr>
<tr>
<td>$K$</td>
<td>Total spring constant of three inertia weight creep springs (27)</td>
</tr>
<tr>
<td>$F_i$</td>
<td>Total initial force of three inertia weight creep springs (27)</td>
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<td>$I_w$</td>
<td>Mass moment of inertia of inertia weight, about an axis through the center of mass and normal to the longitudinal axis (27)</td>
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<td>Inertia weight mass (27)</td>
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<td>$C_1$, $C_2$, $C_3$, $C_4$, $C_5$, $C_6$, $C_7$</td>
<td>Constants (28)</td>
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<td>$F_{TR}$</td>
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<tr>
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<td>$C_L$</td>
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<tr>
<td>$t_L$</td>
<td>Time which corresponds to $C_L$ (38)</td>
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<tr>
<td>$t_c$</td>
<td>Time of cessation of triangular approximation of graze module lateral acceleration (38)</td>
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<td>$K_C$</td>
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<td>Axial displacement of firing pin (51)</td>
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<td>$u_{\theta 1}$</td>
<td>Positive constant (51)</td>
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<tr>
<td>$\chi_1$</td>
<td>Radial displacement of detent ball (51)</td>
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<tr>
<td>$\chi_{\theta 1}$</td>
<td>Positive constant (51)</td>
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<tr>
<td>$r$</td>
<td>Radius of detent ball (52)</td>
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<tr>
<td>$\beta$</td>
<td>Ramp angle of firing pin (52)</td>
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<td>$\varphi$</td>
<td>Variable angle (52)</td>
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<td>Maximum value of $\varphi$ (54)</td>
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<tr>
<td>$P_{1,3}, P_{1,4}$</td>
<td>Normal contact forces between firing pin and detent balls (55)</td>
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<tr>
<td>$P_{1,2}, P_{1,4}$</td>
<td>Reaction forces, in the axial direction, on detent balls (55)</td>
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<td>$l_0$</td>
<td>Free length of firing pin spring (55)</td>
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<td>Spring constant of firing pin spring (55)</td>
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<td>$F_0$</td>
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<tr>
<td>$F_0$</td>
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<td>$m_D$</td>
<td>Mass of detent ball (57)</td>
</tr>
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<td>$m_F$</td>
<td>Mass of firing pin (57)</td>
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<tr>
<td>$C_{11}$</td>
<td>Constant (63)</td>
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<tr>
<td>$A, B, C, D$</td>
<td>Designation of cases of combined motion of inertia weight, detent balls and firing pin (54)</td>
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<td>$C_{12}$</td>
<td>Constant (65)</td>
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<tr>
<td>$t_A$</td>
<td>Time at which inertia weight loses contact with detent balls (67)</td>
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<tr>
<td>$t_0$</td>
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</tr>
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<td>( \gamma_A )</td>
<td>Constant value of ( e ) (69)</td>
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<tr>
<td>( \gamma_B )</td>
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<td>( \gamma_C, \gamma_D )</td>
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<tr>
<td>( j )</td>
<td>Dummy subscript to represent ( A, B ) or ( F ) (72)</td>
</tr>
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<td>( x_i )</td>
<td>( x_i ), motion for Case ( F ) Operation (73)</td>
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<tr>
<td>( \tau_s )</td>
<td>Time at which detent balls lose contact with firing pin (74)</td>
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<tr>
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<td>( y_i ), motion for Case ( F ) Operation (74)</td>
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<td>( y_i_{\text{MAX}} )</td>
<td>Maximum displacement of firing pin (75)</td>
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<td>( \tau_u )</td>
<td>Time at which firing pin achieves its maximum displacement (75)</td>
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<td>( \tau_7 )</td>
<td>Time at which detent balls have moved through the distance ( C_{11} ) (75)</td>
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<td>Motion of firing pin for ( t &gt; \tau_7 ) (76)</td>
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<tr>
<td>( X_{1, \text{A}}(t) )</td>
<td>Certain function of ( x_1 ) (66)</td>
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<tr>
<td>( X_{1, \text{B}}(t) )</td>
<td>Certain function of ( x_1 ) (66)</td>
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<td>( X_0 )</td>
<td>Maximum outward displacement of graze balls (88)</td>
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<td>( C_{10} )</td>
<td>Maximum relative displacement of inertia weight with respect to graze module housing (91)</td>
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<tr>
<td>( y_{\text{MAX}} )</td>
<td>Maximum displacement of ( C_{11} ) of inertia weight (91)</td>
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<tr>
<td>( D )</td>
<td>Diameter of graze balls (90)</td>
</tr>
<tr>
<td>( e )</td>
<td>Coefficient of restitution (96)</td>
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<tr>
<td>( V_{10} )</td>
<td>Absolute velocity, before impact, of the coordinate system (96)</td>
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<tr>
<td>( V_{1A} )</td>
<td>Absolute velocity, before impact, of the inertia weight (96)</td>
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<td>( V_{1'} )</td>
<td>Absolute velocity, after impact, of the coordinate system (96)</td>
</tr>
<tr>
<td>Term</td>
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<td>------</td>
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<td>$K_3$</td>
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<td>$y_c$</td>
<td>Complementary solution for $y$ (40)</td>
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<td>$y_p$</td>
<td>Particular solution for $y$ (40)</td>
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<tr>
<td>$V_{A'}$</td>
<td>Absolute velocity, after impact, of the inertia weight (96)</td>
</tr>
<tr>
<td>$t_{45}$</td>
<td>Time for the center of a graze ball to rotate 45' with respect to the shell axis. (77)</td>
</tr>
</tbody>
</table>

The reference for dimensions used in this report is drawing KD-84300, entitled "Graze Delay Assembly." All of the other subassembly drawing numbers are included on this drawing.