ANALYTIC MODEL OF BORDER CONTROL

G. F. Schilling

prepared for
ADVANCED RESEARCH PROJECTS AGENCY
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The effectiveness of a counter-infiltration program to inhibit the movement of hostile forces across defined boundaries depends on military, technical, geopolitical, socio-economic and other factors. The interrelations and mutual interactions of these factors are complex, but an examination of problems of border security requires their explicit consideration. This Memorandum describes an analytic model of border control that structures and clarifies some of the problems involved. It makes it possible to perform quantitative sensitivity analyses to assist in comparative evaluations of candidate border security systems.

The model was developed as part of a study of infiltration and invasion control for the Advanced Research Projects Agency (Project AJILE). An expansion and application of some of the basic model concepts to the 1969 situation in Vietnam can be found in *A Model Relating Infiltration Retractation System and Force Levels (U)*, RM-6021-1-ARPA (Conf./4), by M. B. Schaffer.

Descriptions of several computerized versions of the analytic model will be published as a separate Memorandum. These computer programs will be made available in a JOSS* library file and will permit on-line use.

The Memorandum should be of interest also to other agencies concerned with counter-insurgency research, or the development of contingency plans and programs for various areas of the world.

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SUMMARY

The situation of a country subjected to guerrilla activity is modeled in terms of mathematical parameters that relate both functionally and quantitatively the principal problems of infiltration, invasion, and insurgency. The basic model reflects geopolitical and economic as well as military and technical aspects, and provides some insight into their complex interrelationships. It treats specifically the situation where not only guerrillas and their opponents are active in an area, but where also infiltration or exfiltration occurs along stretches of national borders or other lines of defense. Computerized versions of the analytic model permit the ready investigation of specific situations, the rapid testing of new concepts and ideas with regard to their probable effects under various contingencies, and the conduct of quantitative sensitivity analyses of candidate border security systems and programs.

The model shows conclusively that a border security system is a must for any attempts to deal successfully with insurgent conflicts supported from outside. It illustrates why there is no obvious military way to end a conflict as long as there is actual infiltration or the opportunity for relatively unopposed infiltration. This result is in implicit agreement with other studies that have indicated that force ratios alone do not determine the outcome of guerrilla/counter-guerrilla warfare.

For the situation in Vietnam, the model implies that even a low-efficiency border security system will deny the enemy his freedom of infiltrating and exfiltrating men and supplies at will to a usable degree. Trial solutions suggest that, with a border security system, it would require far less internal combat activity (or a considerably lower guerrilla attrition rate than now) to prevent excessive enemy accumulations.

However, the model makes clear that the quantitative interactions between infiltration, interdiction, recruitment, and attrition are complex, and that it could be very misleading to generalize. Each specific situation and combination of circumstances represents
a case by itself that must be individually investigated with regard to optimum system mixes for different contingencies. The greatest value of the model is its ability to permit doing such analyses rapidly and efficiently.
SYMBOLS

A = size of area where guerrilla activity occurs
A' = size of area where guerrilla recruitment occurs
E = interdiction efficiency of border security system
K = number of guerrillas attrited in the area
L = length of area border or boundary subject to infiltration attempts
M = number of defenders in the area
N = number of guerrillas in the area
P(D) = probability of detection
P(I|D) = conditional probability of interdiction, if detection occurs
P(I|ND) = conditional probability of interdiction if no detection occurs
P(L,W,At) = probability of penetration of an interdiction zone of length L and width W during a time interval At
p = probability of successful penetration
R = number of guerrillas newly recruited in the area
S = number of successful infiltrators
T = number of attempted infiltrations
t = time
W = width of border interdiction zone
α = constant rate of attempted infiltrations (dT/dt)
γ = attrition efficiency of internal area security program
Γ = constant guerrilla attrition rate (dK/dt)
λ = constant rate of change of guerrilla force level (dN/dt)
ν = attrition efficiency of individual defender
ν̅ = Lanchester coefficient of proportionality
ρ = efficiency of guerrilla recruitment
P = constant guerrilla recruitment rate (dR/dt)
σ = constant rate of successful infiltrations (dS/dt)
Δτ = time interval of evaluation
Subscripts

i = type of guerrilla
j = type of newly recruited guerrilla
o = initial value
t = value at time t
τ = value after time interval Δt
∞ = after infinite time

Time Rates of Change

\( \frac{dK}{dt} \) = guerrilla attrition rate (number of guerrillas attrited per unit of time)

\( \frac{dN}{dt} \) = guerrilla survival rate (change in the number of guerrillas per unit of time = variation of guerrilla force level)

\( \frac{dR}{dt} \) = guerrilla recruitment rate (number of new guerrillas recruited per unit of time)

\( \frac{dS}{dt} \) = infiltration rate (number of successful infiltrators per unit of time)

\( \frac{dT}{dt} \) = rate of infiltration attempts (number of guerrillas attempting to infiltrate per unit of time)
CONTENTS

PREFACE ....................................................... iii
SUMMARY ........................................................... v
SYMBOLS ......................................................... vii
Section
I. INTRODUCTION ............................................. 1
II. BORDER CONTROL MODEL ..................................... 2
   Basic Theory .................................................. 2
   Reality ....................................................... 5
   Implications of Trivial Solutions ......................... 6
III. MODEL PARAMETERS ......................................... 9
   Infiltration .................................................. 9
   Recruitment ................................................ 14
   Attrition .................................................. 16
IV. MODEL SOLUTIONS AND ENEMY STRATEGIES ................. 20
   Model Objectives ........................................... 20
   General Solutions .......................................... 21
   Enemy Strategy X ........................................... 24
   Enemy Strategy Y ........................................... 26
   Special Cases .............................................. 29
V. SENSITIVITY ANALYSES ...................................... 31
VI. EXAMPLE OF JOSS VERSION OF MODEL ......................... 43
   Synopsis of JOSS Example ................................ 43
   Border Control Model ...................................... 46
VII. RESUME AND CONCLUSIONS .................................. 55
   Model Characteristics ..................................... 55
   Model Capabilities ......................................... 57
   Model Uses ................................................ 57
   Conclusions ............................................... 58
REFERENCES ..................................................... 60
I. INTRODUCTION

In this Memorandum, an analytic model of border control is described that interrelates both quantitatively and functionally a number of the principal factors in the problems of infiltration, invasion, and insurgency. It permits the consideration of parameters that reflect geopolitical and socio-economic, as well as military and technical, aspects and provides insight into their complex interrelationships. Computerized versions of the basic model make it possible to perform numerical sensitivity analyses to assist in comparative evaluations of candidate border security systems.

While models of military conflict can never properly reflect the true complexity of all factors possibly associated with insurgent conflicts, the approach described here is indicative of the power of mathematical analysis in structuring and clarifying the essential problems.

As will be shown, the nature of the functional interdependence of the various factors is such that intuitive expectations alone will seldom point in the direction of the correct solutions. In this respect, computerized JOSS versions of the analytic model are especially helpful through their capability of readily testing new concepts and ideas by showing the probable consequences or outcome. Detailed descriptions of these on-line computer programs will be published separately, and the programs made available in a JOSS library file.

An example of the use of one such JOSS version, usable for studying infiltration problems of any country, is given in Section VI of this Memorandum.
II. BORDER CONTROL MODEL

BASIC THEORY

The model treats a situation where a country, or any part of it, is subjected to guerrilla activity, and where counter-insurgent measures are planned or in progress. Specifically, the model is concerned with situations where not only guerrilla activity and counter-activity are taking place in a given area, but where also additional infiltration -- or exfiltration -- of guerrillas occurs along stretches of the national border or other lines of defense.

In its simplest concept, the model situation can be viewed as sketched in Fig. 1. At any instant of time, the number of guerrillas (N_t) in the area will be equal to the initial number (N_0) in the area, plus the number of guerrillas (S_t) that have successfully infiltrated into the area and the number of new guerrillas (R_t) that have been recruited in the area, less the number of guerrillas lost by attrition or that have otherwise disappeared from the area (K_t). Further, the number of successful infiltrators will be the number of guerrillas that have attempted to infiltrate (T_t) less those that were prevented from infiltrating at the border zone and never reached the area of interest.

The basis for consideration is then the following differential equation which governs activity in the area of interest:

\[
\frac{dN}{dt} = \frac{dS}{dt} + \frac{dR}{dt} - \frac{dK}{dt}
\]

(1)

where \( \frac{dN}{dt} \) is the survival rate of guerrillas in the area, i.e., the increase or decrease in the number of guerrillas per unit of time;

\( \frac{dS}{dt} \) is the infiltration rate, i.e., the number of infiltrators that successfully penetrate into the area per unit of time;

\( \frac{dR}{dt} \) is the guerrilla recruitment rate, i.e., the number of new guerrillas recruited per unit of time by the guerrillas already in the area; and
Fig. 1 -- Schematic representation of the situation investigated by the border control model. The size of the area of interest may vary from that of a whole country exposed to Guerrilla activity, to any small part of it.
\( \frac{dk}{dt} \) is the guerrilla attrition rate, i.e., the number of guerrillas that are killed, captured, pacified, or otherwise neutralized in the area per unit of time.

In the next section, each of these parameters will be quantitatively related to the appropriate factors that influence its magnitude. The functional relationships can be expressed generally as follows:

\[
\frac{dS}{dt} = (1 - E) \frac{dT}{dt}
\]

Equation (2) states that the infiltration rate is related to the rate of attempted infiltrations \( \frac{dT}{dt} \), i.e., the number of guerrillas that attempt to infiltrate the area per unit of time, and to \( E \), the efficiency of a border security system in preventing such attempted infiltrations. Equation (3) assumes that the guerrilla recruitment rate is proportional to the number of guerrillas in the area:

\[
\frac{dR}{dt} = \rho N_t
\]

Equation (4) assumes that the guerrilla attrition rate is also proportional in some way to the number of guerrillas in the area of interest:

\[
\frac{dK}{dt} = \gamma N_t
\]

The basic differential equation, (1), now assumes the general form

\[
\frac{dN}{dt} = (1 - E) \frac{dT}{dt} + \rho N_t - \gamma N_t
\]

It can be integrated and solved under various conditions, depending primarily on assumptions concerning the strategy of the enemy. Numerical solutions are then easily obtained for different values of the
coefficients $E$, $p$, and $\gamma$, that express the efficiency of the border security system, of guerrilla recruitment, and of internal security measures, respectively. This is discussed in detail in Sections III and IV.

REALITY

Before dealing with the individual parameters and coefficients, the following comments will clarify the applicability of this theoretical formulation to the real world of insurgent conflict.

For certain situations, it may be of interest to consider explicitly different kinds of guerrillas. For this purpose the above equations can be subscripted where, for example, $S_i$ can refer to a specific type of infiltrator, and $E_i$ would reflect the efficiency of the border security system to deal with this type of infiltration. Different subscripts can stand for members of a combat unit, members of a civilian cadre, saboteurs, unarmed smugglers, and others. Analogously, subscripts can also be introduced to refer to equipment instead of human beings, differentiating perhaps between ammunition, weapons, trucks, food supplies, and so on.

For some evaluations, it may be important to identify guerrillas of different origin and/or tactical history. Additional subscripts can be used for this purpose to distinguish, for example, indigenous guerrillas, guerrillas that have infiltrated from outside, or guerrillas recruited by other guerrillas.

For all these applications, the basic equations can be replaced by a series of subscripted equations and the appropriate solutions are obtained by summations. But unless a differentiation is made specifically, the term "guerrillas" will be used in the text to denote all enemies present in the area of interest, regardless of their individual origin or type.

An important application involves the likely situation where separate areas within a country are subject to various guerrilla conditions, and where the manner of infiltration may differ in separate stretches of border with varying geopolitical characteristics. Again
the basic equations are replaced by a series of similar equations and the solutions are obtained by appropriate summations.

**IMPLICATIONS OF TRIVIAL SOLUTIONS**

It is instructive even at this stage of development to illustrate the insight that can be gained by the model, and to check its results for two extreme cases that lend themselves to intuitive verification.

**Schematic Case No. 1**

Let us assume that the border security system or interdiction zone can be penetrated by infiltrators (i.e.: \( \frac{dS}{dt} > 0 \)), that there is only negligible recruitment of new guerrillas (i.e.: \( \frac{dR}{dt} \approx 0 \)), but that there is no efficient internal security or counter-insurgency activity in the area, that is, \( \gamma = 0 \). The governing equations (1) and (2) then reduce to:

\[
\frac{dN}{dt} = \frac{dS}{dt} = (1 - \varepsilon) \frac{dT}{dt}
\]

Taking a constant infiltration rate \( \frac{dS}{dt} = \sigma \) of any number (other than zero) of infiltrators succeeding in penetrating per day, month, or whatever unit of time, the mathematical solution for this case is:

\[
N_t = N_0 + \sigma t \quad \text{as} \quad t \to \infty
\]

where \( N_0 \) is the initial number of guerrillas in the area, and \( N_t \) is the number of guerrillas in the area after time \( t \). The model solution states that when \( t \) is large, \( N_t \) becomes very large.

In words: If there is no effective internal security activity in the area, it is only a matter of time until the guerrillas in this area can reach tremendous numbers. In this case, and only in this case, neither the degree of efficiency -- barring the concept of an impenetrable border barrier -- of a border security system, nor the efficiency of any guerrilla recruitment would affect the eventual
outcome. It could only affect the rate at which the guerrilla force increases. Accordingly, since in the real world there is no completely impenetrable barrier, the best border interdiction system available will not completely solve infiltration problems without internal security activity in the area invaded.

**Schematic Case No. 2**

Let us look at the opposite case and assume an interdiction zone or barrier that is indeed impenetrable (i.e.: \( E = 1 \), and that there is, therefore, no successful infiltration (i.e.: \( dS/dt = 0 \)). To further simplify, we shall also assume that there is no recruitment of new guerrillas (i.e.: \( dR/dt = 0 \)). The governing equations (1) and (4) then reduce to:

\[
\frac{dN}{dt} = -\frac{dK}{dt} = -\gamma N \tag{7}
\]

Taking an attrition efficiency \( \gamma \) of any value other than zero, the mathematical solution for this case is:

\[
N_t = N_0 e^{-\gamma t} \quad \therefore N \to 0 \quad t \to \infty
\]

The model solution states that when \( t \) is large, \( N_t \) becomes very small.

In words, if there were an impenetrable barrier surrounding an area, it would be a matter only of time until the number of guerrillas in this area reached zero, provided there were an internal attrition process, with some (albeit low) efficiency, which nevertheless exceeds that of guerrilla recruitment.

**Inferences**

The significance of these two trivial cases lies in the indications they give for the direction which model solutions will take in realistic cases. They also reflect that infiltration control is a
dynamic problem throughout, and that attempts to control guerrilla activity in an area either by border security measures alone, or by internal security measures alone, cannot be successful. Hence, all practical solutions will require combinations of border security programs and internal security programs.

A priori, one might be inclined to expect, for example, that an internal security program with a high enough attrition efficiency should be able to overcome both guerrilla infiltration and guerrilla recruitment. But the analytic formulation indicates that this is not the case. Later, complete model solutions will explain why it is not. Essentially, the best possible outcome for this situation -- an internal security program but no effective border interdiction -- is a precarious equilibrium where the defending forces just manage to keep the number of guerrillas in an area from deviating from a certain balance level. This will be shown explicitly in Sections IV and V and will be discussed in Section VII.

One objective of our study becomes an investigation of the overall distributions of resources that result in the combinations most effective in border control. As will be shown, however, potential tradeoffs between the two principal components of counter-guerrilla activity, border interdiction and internal attrition, are nonlinear in nature, and numerical solutions of the model are needed to determine the probable consequences of specific system or program mixes. Of special interest will be results that can be obtained by supplementing an on-going internal area security program with a border security system.
III. MODEL PARAMETERS

INfiltration

Equation (2) related the rate of successful infiltration of guerrillas \( \frac{dS}{dt} \) to the rate of attempted infiltration \( \frac{dT}{dt} \) and the efficiency \( E \) of a barrier or border security system. As sketched in Fig. 2, we shall use the term "interdiction zone" as indicating a zone of a certain width \( W \) over which a border security system is active.

The term "interdiction zone" will cover every phase of operation of all components of a border security system. In some potential systems, it may include not only physical barriers and technical detection and monitoring devices, but also weapon systems and the verification or reaction forces charged with the prevention of infiltration along this stretch of the interdiction zone. Hence, in terms of width, the zone may range from yards to many miles.

Penetration Probabilities

The probability of penetration of an interdiction zone which has uniform properties over a length \( L \) and width \( W \) during a time interval \( \Delta t \), can be simply defined as:

\[
P(L,W,\Delta t) = \frac{\text{Number of Successful Infiltrators}}{\text{Number of Attempted Infiltrations}} = \frac{\Delta S}{\Delta T} \quad (8)
\]

It might be kept in mind that this penetration probability is defined more precisely as referring to unit length and unit time, i.e.:

\[
p = \frac{\Delta S}{\Delta L \Delta t} = \frac{\Delta (\Delta S/\Delta t)}{\Delta L} = \frac{\Delta (\Delta S/\Delta L)}{\Delta t} \quad (9)
\]

But for an interdiction zone with uniform properties over a stretch of length \( L \), during a time interval \( \Delta t \), this equation reduces for most
Fig. 2 -- Schematic representation of the interdiction zone of a border security system. The zone under consideration may be of any length L, and the width W may vary widely for different systems.
practical purposes (i.e.: \( dp = 0 \)) to:

\[
p_i = \frac{\partial s_i}{\partial T_i}
\]  

(10)

where the subscripts \( i \) can refer to specific kinds of infiltrators, as discussed earlier.

For many systems, it is important to distinguish between two functional parameters, namely detection and interdiction. *The probability of successful penetration of an interdiction zone, of specific length \( L \) and depth \( W \) during a time interval \( \Delta t \), by infiltrators of the type \( i \), then becomes

\[
p_i = P(D) \times [1 - P(I|D)] + [1 - P(D)] \times [1 - P(I|ND)]
\]  

(11)

where \( P(D) \) is the probability of detection

\( P(I|D) \) is the conditional probability of interdiction, if detection occurs, and

\( P(I|ND) \) is the conditional probability of interdiction, if no detection occurs.

**System Efficiency**

In general, the probability of penetrating an interdiction zone will be a function of the technical and operational properties of the border security system, including the attritive actions of mechanical devices and reaction forces.

The efficiency of the border security system is then expressed as

\[
E_i = 1 - p_i
\]  

(12)

*Interdiction is used in the broad sense of any activity which prevents the infiltrator from penetrating the border security zone. It will be accomplished through attrition devices such as mined barriers, H and I fire, air strikes, patrol or counter-force actions, and other means.*
that is, in terms of the probability of nonpenetration. It is this quantity which is most readily related to resource costs, terrain features, and other appropriate factors.

From Eqs. (10, 12)

\[ E_i = \frac{3T - 3S}{3T} \]  

(13)

we note that the numerical value of the efficiency can also represent the percentage of attempted infiltrations that are interdicted or stopped, i.e., the percentage of would-be infiltrators stopped in the border zone, or the probability that at most the fraction p of the attempts is successful.

Combining Eqs. (11) and (12), we obtain an expression that has practical utility for the evaluation of the efficiency of candidate border security systems, viz.:

\[ E_i = P(I|ND) + P(D) \times [P(I|D) - P(I|ND)] \]  

(14)

In the context of the model, Eqs. (2) and (14) explain quantitatively that in terms of infiltration control, the ability of a border security system to interdict attempted infiltrations through attrition of guerrilla forces attempting infiltration in the border zone predominates in determining its efficiency. If a system only detects or monitors, but can not interdict (i.e.: \( P(I|D) \) and \( P(I|ND) \) are both zero), whether through technical devices, reaction forces, or at least some form of coercion or deterrence, Eq. (14) correctly shows that its efficiency, \( E \), equals zero. If, on the other hand, there is no difference between interdiction capabilities with or without detection [i.e.: \( P(I|D) = P(I|ND) \)], a way of saying that detection capability does not matter or does not exist, Eq. (14) shows that the efficiency becomes equal to the interdiction probability alone.

Note that numerical values of the efficiency of a border security system for Eq. (2) can be supplied in two ways: through design specifications of the system (principally Eqs. (11) or (14) or specific
equivalents]; or through empirical experience with an existing system, either in actual operation or from field tests [principally through Eqs. (8) or (9)].

This is especially important because, to a higher order of approximation, the efficiency of a border security system may also be a function of time and space gradients. In other words, the efficiency may be related to the number of infiltration attempts during a given time interval, and to the density of these attempts [see Eq. (9)]. For example, the system efficiency may decrease if more than a certain number of guerrillas attempt to penetrate within a short time. It may also decrease rapidly if more than a certain number of guerrillas attempt to penetrate over a short stretch.

In addition, there may be a learning curve for certain systems, leading to counter-countermeasures and causing a time variation of the efficiency. Finally, above a definable limit of penetration attempts ($\frac{\partial^2 T}{\partial L^2}$), infiltration could be considered as changing to invasion. In other words, it may be necessary to express penetration probability as a time-varying function (i.e.: $\frac{dp}{dt} \neq 0$), and substitute for Eq. (10) a more rigorous expression.

**Penetration Rates**

To test candidate border defense systems, or to use the model for the analysis of empirical data, different forms of Eq. (2) can be used:

$$\frac{dS}{dt} = p \frac{dT}{dt} = (1 - E) \frac{dT}{dt} = \frac{2S}{\partial T} \frac{dT}{dt} \quad (2a)$$

for $dp = 0$ during $\Delta t$;

and

$$E = 1 - p = 1 - \frac{2S}{\partial T} = P(I|ND) + P(D) \times [P(I|D) - P(I|ND)] \quad (14a)$$

for $\frac{2S}{\partial T} = \text{constant during } \Delta t$. 
But note that, for example, doubling the interdiction efficiency $E$ does not simply halve the penetration probability $p$ or the rate of successful penetration $dS/dt$. This will be discussed further in Section V in connection with sensitivity analyses.

Table 1 summarizes expressions that correspond to frequently available input information. Depending on what combinations of data are available for a specific case, Eq. (2) can provide estimates of the other ones. (All relations in Table 1 refer to an interdiction zone with uniform properties over a length $L$ during a time interval $\Delta t$; subscripts denoting types of infiltrators have been omitted.)

**RECRUITMENT**

Equation (3) related the rate of recruitment of new guerrillas $(dR/dt)$ to the number of guerrillas $(N)$ in the area, viz.:

$$\frac{dR}{dt} = \rho N_t$$  \hspace{2cm} (3)

This formulation expresses the general concept that the more guerrillas there are in the area, the more new guerrillas are likely to be recruited by them.

The efficiency of the recruitment process is expressed by the proportionality factor $\rho$, where

$$\rho = \frac{1}{N} \frac{dR}{dt}$$  \hspace{2cm} (15)

and its unit of measure is the percent increase in the guerrilla force per unit of time due to new recruits. It is also numerically equal to the number of new guerrillas recruited by each guerrilla in the area per unit of time. Thus, each group of guerrillas of size $1/\rho$ recruits one new guerrilla every such period.

For any suitable time interval, this coefficient $\rho$ can be taken as constant. But in practice, the recruitment efficiency will depend on a variety of circumstances, including the general enemy strategy.
Table 1

INfiltrATION TERMS FOR QUANTITATIVE ANALYSIS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relation</th>
<th>Typical Unit of Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of attempted infiltrations</td>
<td>$\frac{dT}{dt}$</td>
<td>Number of infiltration attempts per month</td>
</tr>
<tr>
<td>Rate of attempted penetrations</td>
<td>$\frac{1}{L} \frac{dT}{dt}$</td>
<td>Number of infiltration attempts per mile per month</td>
</tr>
<tr>
<td>Rate of successful infiltrations</td>
<td>$\frac{dS}{dt}$</td>
<td>Number of infiltrators per month</td>
</tr>
<tr>
<td>Rate of successful penetrations</td>
<td>$\frac{1}{L} \frac{dS}{dt}$</td>
<td>Number of infiltrators per mile per month</td>
</tr>
<tr>
<td>Density of penetration</td>
<td>$\frac{3S}{3L}$</td>
<td>Number of infiltrators per mile</td>
</tr>
<tr>
<td>Penetration probability</td>
<td>$\frac{3S}{3T}$</td>
<td>Percentage of infiltrators successful</td>
</tr>
<tr>
<td>Efficiency of border security system</td>
<td>$\frac{\frac{dT}{dt} - \frac{3S}{3T}}{dt}$</td>
<td>Percentage of infiltrators unsuccessful (interdicted)</td>
</tr>
<tr>
<td>Probability of detection</td>
<td>$P(D)$</td>
<td>Percentage of infiltration attempts detected</td>
</tr>
<tr>
<td>Probability of interdiction, if detected</td>
<td>$P(I</td>
<td>D)$</td>
</tr>
<tr>
<td>Probability of interdiction, without detection</td>
<td>$P(I</td>
<td>ND)$</td>
</tr>
</tbody>
</table>
Other circumstances which can be considered by the model are the
likelihood that the recruiting may be carried on primarily by special
types of guerrillas. Where this constitutes a factor for considera-
tion, Eq. (3) is replaced by

$$\frac{dR_i}{dt} = \rho_{i,j} N_i$$

(16)

where the subscript $i$ denotes the guerrillas doing the recruiting, and
the subscript $j$ identifies the type of new guerrilla being recruited.

Another consideration involves situations where only specific
portions of the area of interest are suitable for guerrilla recruit-
ment. For this purpose, Eq. (3) can be expanded to an expression

$$\frac{dR}{dt} = \rho N \frac{A'}{A}$$

(17)

where $A'$ is the size of that portion of the general area $A$, where guer-
rilla recruitment is occurring.

The actual form of Eq. (3) used in the model depends on the type
of input data available or being tested, and on the objectives of any
specific model run. Forms other than the variations mentioned briefly
here can also be used.

**ATTRITION**

Equation (4) related the rate of attrition of guerrillas ($dK/dt$)
to the number of guerrillas ($N$) in the area; viz.:

$$\frac{dK}{dt} = \gamma N_t$$

(4)

This formulation expresses the concept that the more guerrillas
there are in an area, the more of them are likely to be eliminated.
The numerical value of $dK/dt$, i.e., the actual number of guerrillas
eliminated in a given time interval, depends then both on the number
of guerrillas present in the area and on the efficiency of the attrition process of the internal-area security program.

In this general form, the efficiency of the attrition process is expressed by the proportionality factor $\gamma$, where

$$\gamma = \frac{1}{N} \frac{dK}{dt}$$

(4a)

and its unit of measure is the percentage of the guerrilla force that is attrited per unit of time. This attrition process represents the results of all internal security measures and may consist of kills, captures, defection, pacification, or any other activity that reduces the number of guerrillas present in the area. If desired, the numerical value of $\gamma$ can be given as an integral measure of the efficiency of this process, or it can be structured to reflect these activities separately.

In general, the attrition efficiency $\gamma$ will depend on the strategies and tactics adopted by both guerrillas and defenders.

If, for example, the efficiency of the attrition process is assumed to vary with the strength of the defending forces, then

$$\gamma = vM$$

(18)

where $M$ is the number of defenders in the area.

From Eqs. (4) and (18), we then obtain

$$\frac{dK}{dt} = vNM$$

(19)

where the measure of the coefficient $v$ is the percentage of the guerrilla force attrited per unit of time per individual defender. In this formulation of the guerrilla attrition rate, Eq. (19) corresponds to one of the well known Lanchester equations of combat. (1)
The other Lanchester equation would express the attrition rate as proportional only to the number of defenders, i.e.:

\[ \frac{dK}{dt} = -\alpha M \]  

(20)

This formulation would not be valid for the situation modeled here. It would neglect the nontrivial condition, implicit in Eq. (4), that there is a practical upper limit to the attrition rate \( \frac{dK}{dt} \), even for an attrition efficiency of 100 percent per unit of time. In other words, Eq. (20) would numerically permit more of the guerrilla force to be attrited in an area during a given time interval than is there.

The general applicability of the Lanchester equations to guerrilla warfare was shown by Deitchman, and they were applied to different stages of insurgency engagements by Schaffer. But as was pointed out by Deitchman, guerrilla warfare does not usually represent symmetrical firing cases, and therefore, neither the "square law" nor the "linear law" for equality of fighting strength gives the condition under which neither side wins. As will be shown in Section IV, the border-control model implicitly confirms the resulting conclusion that force ratios alone do not determine the outcome of guerrilla/counter-guerrilla warfare.

It should be noted here that the basic differential equation of the model [Eq. (1)] reflects that the number of defenders per se is not a dominating factor on an area-wide basis, although it may be very important for limited combat engagements occurring over small areas. The influence of defending strength comes indirectly in terms of resource-allocation costs. It is introduced through Eqs. (4) and (18), where it may affect the efficiency of guerrilla attrition by means of internal security measures, and through Eqs. (2) and (11), where it may affect the infiltration rate by altering the interdiction efficiency of border security measures. In other words, the model implies that the defender has the ability and the resource capacity to alter his strategy, including his force strength, when the progress of activities reveals a tendency towards a direction unfavorable for him.
For a suitably chosen time interval, the coefficients $\gamma$ and $\nu$ can then be considered as invariant. Table 2 summarizes the expressions that correspond to frequently available input information. Depending on what combinations of data are available for a specific case, Eqs. (4) and (18), or expanded versions, can provide estimates of the other ones.

Table 2

ATTRITION TERMS FOR QUANTITATIVE ANALYSIS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relation</th>
<th>Typical Unit of Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of attrition</td>
<td>$\frac{dK}{dt}$</td>
<td>Number of guerrillas eliminated per month</td>
</tr>
<tr>
<td>Attrition efficiency</td>
<td>$\gamma = \frac{1}{N} \frac{dK}{dt}$</td>
<td>Percentage of guerrilla force attrited per month</td>
</tr>
<tr>
<td>Relative efficiency of defender</td>
<td>$\nu = \frac{1}{MN} \frac{dK}{dt}$</td>
<td>Percentage of guerrilla force attrited per month per defender</td>
</tr>
<tr>
<td>Relative rate of attrition</td>
<td>$\nu N = \frac{1}{M} \frac{dK}{dt}$</td>
<td>Number of guerrillas eliminated per month per defender</td>
</tr>
<tr>
<td>Force requirement</td>
<td>$\frac{N}{dK/dt} = \frac{1}{\nu N}$</td>
<td>Number of defenders required to eliminate one guerrilla per month</td>
</tr>
<tr>
<td>guerrilla force requirement</td>
<td>$\frac{N}{dK/\gamma} = \frac{1}{\gamma}$</td>
<td>Size of guerrilla group which loses one guerrilla per month</td>
</tr>
</tbody>
</table>
IV. MODEL SOLUTIONS AND ENEMY STRATEGIES

MODEL OBJECTIVES

The basic differential equation [Eq. (1)] of the border control model, in its general form [Eq. (5)], expresses quantitatively the interrelationships and interactions between the various parameters and coefficients discussed in the previous section. Before investigating the solutions, let us look at its functional significance:

\[
\frac{dN_i}{dt} = (1 - E_i) \frac{dT_i}{dt} - (\gamma - \rho) N_i
\]  \hspace{1cm} (5)

Through numerical solutions, the model can serve three principal purposes:

a. By using such empirical data as are available, it is possible to determine the values of individual parameters and coefficients for guerrilla activities taking place, or having taken place, in specific areas. Of special interest, in this application, is the knowledge that can be gained about the relative importance of different parameters in affecting activities.

b. By using conditional input data for candidate border security systems, it is possible to investigate the overall efficiency of system mixes and variations, and to test the applicability and usefulness of planned or actual security systems and programs under different contingencies or scenarios.

c. By using different functional solutions that correspond to different enemy strategies, it is possible to assess the probable consequences of system implementations in terms of likely enemy response and resulting requirements for system changes.
Essentially, all of these applications constitute sensitivity analyses where computerized versions of the model allow ready evaluations. Of special practical use have been several JOSS computer versions that permit the user to obtain meaningful results rapidly for a variety of input data and theater conditions (see also Section VI).

GENERAL SOLUTIONS

As was discussed in Section III, the model is capable of accepting a variety of combinations of input data, and can provide results from a minimum of assumptions. For the general case where no information is available about enemy strategy, the evaluation of Eq. (5) considers that the parameters and coefficients listed in Table 3 do not vary during the time interval of evaluation, \( \Delta t \). The solutions for this general case are summarized in Table 4.

In addition to the listed quantities, the model provides, if desired, a variety of supplemental information such as the density of guerrillas in the area of interest at any time, or their rate of accumulation (compare with the reproduction of JOSS computer print-out in Section VI).

Table 3
TIME INVARIANT PARAMETERS AND COEFFICIENTS FOR GENERAL SOLUTIONS (CONSTANT DURING TIME INTERVAL \( \Delta t \)).

<table>
<thead>
<tr>
<th>Parameter/Coefficient</th>
<th>Symbol/Relation</th>
<th>Equation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Number of guerrillas in area</td>
<td>( N_0 )</td>
<td>Input</td>
</tr>
<tr>
<td>Interdiction efficiency of border security system</td>
<td>( E )</td>
<td>12, 13, 14</td>
</tr>
<tr>
<td>Efficiency of guerrilla recruitment</td>
<td>( \mu )</td>
<td>15, 16</td>
</tr>
<tr>
<td>Attrition efficiency of internal area security program</td>
<td>( \gamma )</td>
<td>4a, 18</td>
</tr>
<tr>
<td>Rate of attempted infiltrations</td>
<td>( \frac{dT}{dt} = \alpha )</td>
<td>See Table 1</td>
</tr>
<tr>
<td>Rate of successful infiltrations</td>
<td>( \frac{dS}{dt} = \sigma = (1 - E)\alpha )</td>
<td>See Table 1</td>
</tr>
</tbody>
</table>
The solutions illustrate the ability of the model to compare quantitatively as well as conceptually the interactions between various border-control parameters. Note that the eventual outcome — if no changes are made by either side in the quantities listed in Table 3 — is determined by balance relations between them.

The equations reveal, for example, that the best possible outcome, even with a 100 percent attrition efficiency and a minimal recruitment efficiency, is only an equilibrium state as long as there is any successful infiltration at all. This equilibrium state, expressed in Table 4 as the final number of guerrillas in the area, corresponds somewhat to an acceptable level of violence. Together with the balance rate which influences how fast this equilibrium is being reached, these terms can be used to characterize the over-all effectiveness of a border-control system.

For realistic situations of guerrilla warfare, it must be assumed that the strategy of the enemy, as well as that of the defender, might be adjustable depending on the progress as well as the projected outcome of the conflict. The model can reflect this by permitting changes in the basic quantities given in Table 3 at any suitable time \( t \), and by continuing with the changed values for subsequent time intervals \( \Delta t \).

In addition, it will be shown in the following how the model can be adapted to a priori assumptions about a specific enemy strategy. Conversely, border control systems can be tested as to their ability to deal with different enemy strategies.
### Table 4

**GENERAL SOLUTIONS OF BORDER CONTROL MODEL**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Equation/Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of guerrillas in area at time $t$</td>
<td>$N_t = \frac{\sigma}{\gamma - \rho} - \left[\frac{\sigma}{\gamma - \rho} - N_o\right] e^{-(\gamma - \rho)t}$</td>
</tr>
<tr>
<td>Number of successful infiltrators up to time $t$</td>
<td>$S_t = \sigma t$</td>
</tr>
<tr>
<td>Number of infiltration attempts up to time $t$</td>
<td>$T_t = \frac{S_t}{1 - e}$</td>
</tr>
<tr>
<td>Number of infiltration attempts interdicted up to time $t$</td>
<td>$T_t = S_t$</td>
</tr>
<tr>
<td>Number of guerrillas recruited in area up to time $t$</td>
<td>$R_t = \frac{\rho}{\gamma - \rho} \left[N_o + \sigma t - \frac{\sigma}{\gamma - \rho} + \left(\frac{\sigma}{\gamma - \rho} - N_o\right) e^{-(\gamma - \rho)t}\right]$</td>
</tr>
<tr>
<td>Number of guerrillas eliminated in area up to time $t$</td>
<td>$K_t = \frac{\gamma}{\gamma - \rho} \left[N_o + \sigma t - \frac{\sigma}{\gamma - \rho} + \left(\frac{\sigma}{\gamma - \rho} - N_o\right) e^{-(\gamma - \rho)t}\right]$</td>
</tr>
</tbody>
</table>

**Guerrilla recruitment rate**

$$\frac{dR}{dt} = \frac{\sigma}{\gamma - \rho} - \left[\frac{\sigma}{\gamma - \rho} - \rho N_o\right] e^{-(\gamma - \rho)t}$$

**Guerrilla attrition rate**

$$\frac{dK}{dt} = \frac{\gamma}{\gamma - \rho} - \left[\frac{\gamma}{\gamma - \rho} - \gamma N_o\right] e^{-(\gamma - \rho)t}$$

**Rate of change of guerrilla force in area**

$$\frac{dN}{dt} = \left[\sigma - (\gamma - \rho)N_o\right] e^{-(\gamma - \rho)t}$$

**Final number of guerrillas in area (at $t = \infty$)**

$$\frac{\sigma}{\gamma - \rho} \quad [= \text{if } \gamma \leq \rho]$$

**Final recruitment rate (at $t = \infty$)**

$$\frac{\rho}{\gamma - \rho} \quad \sigma \quad [= \text{if } \gamma \leq \rho]$$

**Final attrition rate (at $t = \infty$)**

$$\frac{\gamma}{\gamma - \rho} \quad \sigma \quad [= \text{if } \gamma \leq \rho]$$

**Balance rate (at any time $t$):**

- $\sigma > (\gamma - \rho)N_o$ : Guerrilla force increases
- $\sigma < (\gamma - \rho)N_o$ : Guerrilla force decreases

**Time to reach 99% of final number of guerrillas in area**

$$t = \frac{1}{\gamma - \rho} \ln \frac{\sigma - (\gamma - \rho)N_o}{0.91 \sigma}$$

*The time to reach 99 percent of the final number is chosen because, in some situations, the final number is reached asymptotically, i.e., only after infinite time.*
ENEMY STRATEGY X

Principal Enemy Objective: Maintain strength of guerrilla forces in the area at the initial level of \( N_0 \).

Equation (5) for this case reduces to:

\[
(1 - E) \frac{dT}{dt} + pN_0 - \gamma N_0 = 0
\]

and the appropriate model formulations become:

\[
\frac{dN}{dt} = 0 \quad N_t = N_0
\]

\[
\frac{dS}{dt} = (\gamma - p)N_0 \equiv \sigma
\]

\[
\frac{dT}{dt} = \frac{\gamma - p}{1 - E} N_0 \equiv \alpha
\]

It follows that both the rate of attempted infiltrations \( (dT/dt) \) and the rate of successful infiltrations \( (dS/dt) \) remain constant over the time interval \( \Delta t \) during which no changes are made in the efficiencies of recruitment \( (p) \), attrition \( (\gamma) \), and border security \( (E) \).

Thus, the enemy can achieve his principal objective by trying to maintain his rate of attempted infiltrations \( (\alpha) \) at an approximately constant level, dictated by the relative efficiencies of operations that prevail, i.e., his rate of attempts must be:

\[
\alpha = \frac{\gamma - p}{1 - E} N_0
\]

Alternatively, the enemy can try to adjust his efficiency of recruiting new guerrillas in the area \( (\rho) \) taking into account his attrition losses and his rate of successful infiltrations \( (\sigma) \), so that:

\[
\rho = \gamma - \frac{\sigma}{N_0}
\]

For both cases, the model will readily provide quantitative answers. The model solutions are simple and are summarized in Table 5. Of practical interest is the model's capability of testing the overall efficiency of different counter-measure systems for this situation.
**Table 5**

**SOLUTIONS OF BORDER CONTROL MODEL FOR ENEMY STRATEGY X**

**MAINTAINING CONSTANT LEVEL OF GUERRILLA FORCE**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Equation/Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of successful infiltrations</td>
<td>[ \frac{dS}{dt} = \sigma = (\gamma - \rho)N_0 ] = constant</td>
</tr>
<tr>
<td>Required rate of attempted infiltrations</td>
<td>[ \frac{dT}{dt} = \alpha = \frac{\gamma - \rho}{1 - E} N_0 ] = constant</td>
</tr>
<tr>
<td>Number of guerrillas in area</td>
<td>[ N_0 = \text{input constant} ]</td>
</tr>
<tr>
<td>Number of successful infiltrators up to time t</td>
<td>[ S_t = \sigma t = (\gamma - \rho)N_0 t ]</td>
</tr>
<tr>
<td>Number of infiltration attempts up to time t</td>
<td>[ T_t = \frac{S_t}{1 - E} = \alpha t = \frac{\gamma - \rho}{1 - E} tN_0 ]</td>
</tr>
<tr>
<td>Number of infiltration attempts interdicted up to time t</td>
<td>[ T_t - S_t = (\alpha - \sigma)t ]</td>
</tr>
<tr>
<td>Number of guerrillas recruited in area up to time t</td>
<td>[ R_t = \rho N_0 t ]</td>
</tr>
<tr>
<td>Number of guerrillas attrited in area up to time t</td>
<td>[ K_t = \gamma N_0 t ]</td>
</tr>
<tr>
<td>Guerrilla recruitment rate</td>
<td>[ \frac{dR}{dt} = \rho N_0 ]</td>
</tr>
<tr>
<td>Guerrilla attrition rate</td>
<td>[ \frac{dK}{dt} = \gamma N_0 ]</td>
</tr>
<tr>
<td>Rate of change of guerrilla force in area</td>
<td>[ \frac{dN}{dt} = 0 ]</td>
</tr>
</tbody>
</table>
| Balance rate (at any time t):                  | \[ (1 - E)\alpha = \sigma = (\gamma - \rho)N_0 \]  
  Guerrilla force constant
ENEMY STRATEGY Y

Principal Enemy Objective: Increase (or decrease) the strength of guerrilla forces in the area from an initial level $N_0$ to a level $N_T$ over a period of $\Delta t$ months.

Variation Y-1: Enemy wishes to implement this strategy with an approximately constant rate of increase (or decrease) in his force level ($\lambda$).

Variation Y-2: Enemy wishes to implement this strategy with an approximately constant rate of infiltration (or exfiltration) attempts ($\alpha$).

For Strategy Y-1, Eq. (5) becomes:

$$\lambda = (1 - E) \frac{dT}{dt} + \rho N_T - \gamma N_T = \frac{N_T - N_0}{\Delta t}$$

where, for given input quantities $N_0$, $N_T$, and $\Delta t$, the value of the input constant $\lambda$ will be positive or negative, depending on whether an increase or a decrease of his force level is the enemy's objective.

For Strategy Y-2, Eq. (5) remains:

$$\frac{dN}{dt} = (1 - E) \alpha + \rho N_T - \gamma N_T$$

where, for the same input quantities, the value of $\alpha$ is a positive or negative input constant, depending on whether infiltration or exfiltration is required by the enemy to achieve his objective.

The model solutions for Strategies Y-1 and Y-2 are summarized in Tables 6 and 7, respectively.
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Equation/Relation</th>
</tr>
</thead>
</table>
| Rate of successful infiltrations (exfiltrations)                        | \[
\frac{dS}{dt} = (\gamma - \rho)(N_0 + \lambda t) + \lambda
\] |
| REQUIRED rate of attempted infiltrations (exfiltrations)                | \[
\frac{dT}{dt} = \frac{\gamma - \rho}{1 - E} (N_0 + \lambda t) + \frac{\lambda}{1 - E}
\] |
| Number of guerrillas in area at time t                                  | \[ N_t = N_0 + \lambda t \] |
| Number of successful infiltrators (exfiltrators)                        | \[ S_t = \lambda t + (\gamma - \rho)N_0 + \frac{\gamma - \rho}{2} \lambda t^2 \] |
| Number of infiltration (exfiltration) attempts up to time t             | \[ T_t = \frac{S_t}{1 - E} \] |
| Number of infiltration (exfiltration) attempts interdicted up to time t | \[ T_t - S_t \] |
| Number of guerrillas recruited in area up to time t                     | \[ R_t = \rho N_0 t + \frac{\rho}{2} \lambda t^2 \] |
| Number of guerrillas attrited in area up to time t                      | \[ K_t = \gamma N_0 t + \frac{\gamma}{2} \lambda t^2 \] |
| Guerrilla recruitment rate                                              | \[ \frac{dR}{dt} = \rho N_0 + \rho \lambda t \] |
| Guerrilla attrition rate                                                | \[ \frac{dK}{dt} = \gamma N_0 + \gamma \lambda t \] |
| Rate of change of guerrilla force in area                               | \[ \frac{dT}{dt} = \lambda \equiv \text{input constant} \] |
| Final number of guerrillas in area (at \( t = \Delta t \))             | \[ N_t = N_0 + \lambda \Delta t = \text{input value} \] |
Table 7

SOLUTIONS OF BORDER CONTROL MODEL FOR ENEMY STRATEGY Y-2

(CHANGE IN FORCE LEVEL WITH CONSTANT RATE OF INFILTRATION ATTEMPTS)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Equation/Relation</th>
</tr>
</thead>
</table>
| Rate of successful infiltration (exfiltrations) | \[
\frac{dS}{dt} = \sigma = (\gamma - \rho)N \frac{e^{(\gamma - \rho)T} N_0}{N_T} = \text{constant}\] |
| REQUIRED RATE OF ATTEMPTED infiltrations      | \[
\frac{dT}{dt} = \alpha = \frac{(\gamma - \rho)N_0}{1 - e^{(\gamma - \rho)T}} = \text{constant}\] |
| Number of guerrillas in area at time t        | Same as Table 4                                                                  |
| Number of successful infiltrators (exfiltrators) up to time t | Same as Table 4                                                                |
| Number of infiltration (exfiltration) attempts up to time t | Same as Table 4                                                                |
| Number of infiltration (exfiltration) attempts interdicted up to time t | Same as Table 4                                                                |
| Number of guerrillas recruited in area up to time t | Same as Table 4                                                                |
| Number of guerrillas attritioned in area up to time t | Same as Table 4                                                                |
| Guerrilla recruitment rate                    | Same as Table 4                                                                  |
| Guerrilla attrition rate                      | Same as Table 4                                                                  |
| Rate of change of guerrilla force in area     | \[
\frac{dN}{dt} = (\gamma - \rho)N \frac{1 - \frac{N_0}{N_T} e^{(\gamma - \rho)(T-t)}}{e^{(\gamma - \rho)T - 1}}
\] |
| Final number of guerrillas in area (at t = \Delta t) | \[N_T = \text{input value}\]                                                    |
SPECIAL CASES

In addition to the enemy strategies shown in detail, the model can easily deal with a variety of special cases that represent simplified solutions of the general case given in Table 4. Of interest in connection with the availability of empirical data may be the following application.

If the guerrilla attrition rate remains constant over time interval $\Delta t$, Eq. (5) reduces to:

$$\frac{dN}{dt} = (1 - E) \frac{dT}{dt} + \rho N - \Gamma$$

where $\Gamma = \frac{dK}{dt} = \text{constant}.$

If the guerrilla recruitment rate remains constant over time interval $\Delta t$, Eq. (5) reduces to:

$$\frac{dN}{dt} = (1 - E) \frac{dT}{dt} + P - \gamma N$$

where $P = \frac{dR}{dt} = \text{constant}.$

The cases where either the guerrilla attrition rate ($dK/dt$) or the guerrilla recruitment rate ($dR/dt$) are zero, correspond to setting the respective efficiency ($\gamma$ or $\rho$) equal to zero. The solutions of Table 4 apply. Note, however, that if the guerrilla recruitment efficiency is higher than the guerrilla attrition efficiency (i.e.: $\rho > \gamma$), the number of guerrillas in the area will, of course, continue to increase with or without successful infiltration.

But if the attrition efficiency is equal to the recruitment efficiency (i.e.: $\gamma = \rho$), the solutions given in Table 8 must be substituted for the relevant quantities of Table 4. The eventual outcome of this situation is related to the trivial solution that was discussed in Section II, Subsection "Implications of Trivial Solutions," as case No. 1.
Table 8
SOLUTIONS OF BORDER CONTROL MODEL FOR SPECIAL CASE OF ATTRITION EFFICIENCY EQUAL TO RECRUITMENT EFFICIENCY

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Equation/Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of guerrillas in area at time t</td>
<td>( N_t = N_0 + \sigma t )</td>
</tr>
<tr>
<td>Number of guerrillas recruited in area up to time t</td>
<td>( R_t = \rho N_0 t + \frac{\rho}{2} \sigma t^2 )</td>
</tr>
<tr>
<td>Number of guerrillas eliminated in area up to time t</td>
<td>( K_t = \gamma N_0 t + \frac{\gamma}{2} \sigma t^2 )</td>
</tr>
<tr>
<td>Guerrilla recruitment rate</td>
<td>( \frac{dR}{dt} = \rho N_0 + \rho \sigma t )</td>
</tr>
<tr>
<td>Guerrilla attrition rate</td>
<td>( \frac{dK}{dt} = \gamma N_0 + \gamma \sigma t )</td>
</tr>
<tr>
<td>Rate of change of guerrilla force in area</td>
<td>( \frac{dN}{dt} = \sigma = (1 - E) \alpha )</td>
</tr>
<tr>
<td>Final number of guerrillas in area (at ( t = \infty ))</td>
<td>( N_\infty = \text{infinite} )</td>
</tr>
</tbody>
</table>
V. SENSITIVITY ANALYSES

The general solutions of the model, given in the previous section, show why and how, for most situations, the outcome of insurgent warfare is not determined by the force ratios of the opponents. They also show that it is not simple to define victory or defeat for either side. In fact, the solutions imply that the final result under most circumstances is an equilibrium where the defending forces just manage to keep the number of guerrillas from deviating from a certain balance level.

Whether this final equilibrium level is a balance of terror, or a perhaps acceptable, rather low level of violence, is determined primarily by interactions between rates of infiltration, guerrilla recruitment, and attrition. The numerical magnitude of the balance level, that is, the eventual equilibrium number of guerrillas in an area, is dependent on the efficiencies of these operations.

It will therefore be of interest to study quantitatively what effects are produced by different changes in the individual parameters. In the following, such sensitivity analyses are illustrated as deviations from a simple, schematic base case. In order to clearly show the effects, only one parameter at a time was varied, and all others were kept constant for each specific example. In each example, the static base case divides the situations that lead to increases or decreases in the number of guerrillas in the area.

Table 9 lists the adopted values of the model parameters for the base case. The subsequent Tables 10 through 14, and the companion Figs. 3 through 6, are examples of the major effects resulting from various changes in the modes of operation of the opposing forces.

It should be recalled that the interactions between the various model parameters are quite complex in nature. The schematic examples shown here illustrate what can be expected to happen in the situation depicted by the base case. It would be misleading, however, to generalize and to expect to be able to predict by analogy what should happen in different, even though similar situations. Unfortunately, each specific situation must be dealt with specifically, and may show quite
different effects. For this reason, JOSS computer versions have been
developed that allow the user to make quantitative sensitivity analy-
ses for any desired input data without the necessity of delving into
the mathematical complexities of the border control model. They pro-
vide considerably more information than is shown in Tables 10 through
14, and are discussed in Section VI.
## Table 9
VALUES OF PARAMETERS FOR THE SIMPLE BASE CASE

<table>
<thead>
<tr>
<th>Basic Input:</th>
<th>Resultant Values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Number of Guerrillas in Area ( N_0 ) = 100,000</td>
<td>( \frac{dS}{dt} = 8,000 ) per month</td>
</tr>
<tr>
<td>Rate of Attempted Infiltrations ( \frac{dT}{dt} ) = 10,000 per month</td>
<td></td>
</tr>
<tr>
<td>Border System Interdiction Efficiency ( E ) = 20% of attempts</td>
<td></td>
</tr>
<tr>
<td>Guerrilla Recruitment Efficiency ( \rho ) = 2% increase in force per month (^a)</td>
<td></td>
</tr>
<tr>
<td>Internal Attrition Efficiency ( \gamma ) = 10% of force in area attrited per month (^b)</td>
<td></td>
</tr>
</tbody>
</table>

This means that every group of 100 guerrillas in the area recruits 2 new guerrillas each month.

This means that every group of 100 guerrillas in the area suffers a loss of 10 guerrillas each month due to all kinds of attrition.

Input values have been chosen to reflect a static situation.

Note that the rates of recruitment and attrition, i.e., the actual numbers recruited or lost each month, are variables that depend not only on the efficiencies of these operations, but also on the number of guerrillas present in the area at the time.
Table 10

EFFECTS OF VARYING THE INTERDICTON EFFICIENCY E
OF A BORDER SECURITY SYSTEM

<table>
<thead>
<tr>
<th>Initial Number of Guerrillas in Area</th>
<th>$N_o = 100,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Attempted Infiltrations</td>
<td>$\frac{dT}{dt} = 10,000$ per month</td>
</tr>
<tr>
<td>Guerrilla Recruitment Efficiency</td>
<td>$\rho = 2%$ increase per month</td>
</tr>
<tr>
<td>Internal Area Attrition Efficiency</td>
<td>$\gamma = 10%$ attrited per month</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Border Interdiction Efficiency (E)</th>
<th>Rate of Successful Infiltrations ($\frac{dS}{dt}$)</th>
<th>Final Number of Guerrillas in Area ($N_\infty$)</th>
<th>Time to reach 99% of final number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 %</td>
<td>10,000 per month</td>
<td>125,000</td>
<td>37 months</td>
</tr>
<tr>
<td>10 %</td>
<td>9,000 per month</td>
<td>112,500</td>
<td>30 months</td>
</tr>
<tr>
<td>20 % (BASE)</td>
<td>8,000 per month</td>
<td>100,000</td>
<td>0 months</td>
</tr>
<tr>
<td>50 %</td>
<td>5,000 per month</td>
<td>62,500</td>
<td>51 months</td>
</tr>
<tr>
<td>60 %</td>
<td>4,000 per month</td>
<td>50,000</td>
<td>58 months</td>
</tr>
<tr>
<td>80 %</td>
<td>2,000 per month</td>
<td>25,000</td>
<td>71 months</td>
</tr>
<tr>
<td>90 %</td>
<td>1,000 per month</td>
<td>12,500</td>
<td>82 months</td>
</tr>
<tr>
<td>100 %</td>
<td>0 per month</td>
<td>0</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

This set of cases is illustrated in Fig. 3. Note that under these conditions the curves labelled $E = 0$ (i.e.: no border security system) and $E = 100\%$ represent the limiting boundaries for all possible developments.
Fig. 3 -- Effects of varying the interdiction efficiency $E$ of a border security system from no system ($E = 0$) to an ideal system of 100% efficiency (see also Table 10).
Table 11  
EFFECTS OF VARYING THE ATTRITION EFFICIENCY $\gamma$  
OF AN INTERNAL AREA SECURITY PROGRAM  

<table>
<thead>
<tr>
<th>Initial Number of Guerrillas in Area</th>
<th>$N_0 = 100,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Attempted Infiltrations</td>
<td>$dT/dt = 10,000$ per month</td>
</tr>
<tr>
<td>Border Interdiction Efficiency</td>
<td>$E = 20%$ of attempts</td>
</tr>
<tr>
<td>Rate of Successful Infiltrations</td>
<td>$dS/dt = 8,000$ per month</td>
</tr>
<tr>
<td>Guerrilla Recruitment Efficiency</td>
<td>$\rho = 2%$ increase per month</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Internal Area Attrition Efficiency ($\gamma$)</th>
<th>Final Number of Guerrillas in Area ($N_e$)</th>
<th>Time to reach 99% of final Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>infinite</td>
<td>$\infty$</td>
</tr>
<tr>
<td>2</td>
<td>infinite</td>
<td>$\infty$</td>
</tr>
<tr>
<td>7.3 $%$ per month</td>
<td>150,000</td>
<td>66 months</td>
</tr>
<tr>
<td>8.4 $%$ per month</td>
<td>125,000</td>
<td>47 months</td>
</tr>
<tr>
<td>10 $%$ per month (BASE)</td>
<td>100,000</td>
<td>0 months</td>
</tr>
<tr>
<td>14.8 $%$ per month</td>
<td>62,500</td>
<td>32 months</td>
</tr>
<tr>
<td>34 $%$ per month</td>
<td>25,000</td>
<td>18 months</td>
</tr>
<tr>
<td>100 $%$ per month</td>
<td>8,100</td>
<td>7 months</td>
</tr>
</tbody>
</table>

This set of cases is illustrated in Fig. 4. Note that for $\gamma \leq 2\%$ per month, the situation is explosive and developments do not lead to a balance solution. Conversely, even a $\gamma$ of 100% does not decrease the final number of guerrillas in the area below the balance value of 8,100.
Fig. 4 -- Effects of varying the attrition efficiency \( \gamma \) of an internal area security program from no program (\( \gamma = 0 \)) to a program with an efficiency of 100% per month (see also Table 11).
Table 12
EFFECTS OF VARIATIONS IN THE RATE OF
ATTEMPTED INFILTRATIONS $\frac{dT}{dt}$

<table>
<thead>
<tr>
<th>Initial Number of Guerrillas in Area</th>
<th>$N_0 = 100,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Border Interdiction Efficiency</td>
<td>$E = 20%$ of attempts</td>
</tr>
<tr>
<td>Guerrilla Recruitment Efficiency</td>
<td>$\rho = 2%$ increase per month</td>
</tr>
<tr>
<td>Internal Area Attrition Efficiency</td>
<td>$\gamma = 10%$ attrited per month</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rate of Attempted Infiltrations ($dT/dt$)</th>
<th>Rate of Successful Infiltrations ($dS/dt$)</th>
<th>Final Number of Guerrillas in Area ($N_0$)</th>
<th>Time to Reach 99% of Final Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>15,000 per month</td>
<td>12,000 per month</td>
<td>150,000</td>
<td>44 months</td>
</tr>
<tr>
<td>12,500 per month</td>
<td>10,000 per month</td>
<td>125,000</td>
<td>37 months</td>
</tr>
<tr>
<td>10,000 per month</td>
<td>8,000 per month</td>
<td>100,000</td>
<td>0 months</td>
</tr>
<tr>
<td>5,000 per month</td>
<td>4,000 per month</td>
<td>50,000</td>
<td>58 months</td>
</tr>
<tr>
<td>2,500 per month</td>
<td>2,000 per month</td>
<td>25,000</td>
<td>71 months</td>
</tr>
<tr>
<td>0 per month</td>
<td>0 per month</td>
<td>0</td>
<td>$\infty$ months</td>
</tr>
</tbody>
</table>

This set of cases is illustrated in Fig. 5. Note that the base case ($dl/dt = 10,000$ per month and $E = 20\%$) divides the situations that lead to increases or decreases in the number of guerrillas in the area.
Fig. 5 -- Effects of variations in the rate of attempted infiltrations from none (0.month) to 15,000/month (see also Table 12).
Table 13

EFFECTS OF VARIATIONS IN THE INITIAL NUMBER OF GUERRILLAS $N_o$ IN THE AREAb

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Attempted Infiltrations</td>
<td>$dT/dt = 10,000$ per month</td>
</tr>
<tr>
<td>Border Interdiction Efficiency</td>
<td>$E = 20%$ of attempts</td>
</tr>
<tr>
<td>Rate of Successful Infiltrations</td>
<td>$dS/dt = 8,000$ per month</td>
</tr>
<tr>
<td>Guerrilla Recruitment Efficiency</td>
<td>$\rho = 2%$ increase per month</td>
</tr>
<tr>
<td>Internal Area Attrition Efficiency</td>
<td>$\gamma = 10%$ attrited per month</td>
</tr>
<tr>
<td>Final Number of Guerrillas in Area</td>
<td>$N_\infty = 100,000$ after infinite time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Number of Guerrillas ($N_o$)</th>
<th>Number of Guerrillas in Area after 24 Months</th>
<th>Time to Reach 99$%$ of Final Number of 100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>200,000</td>
<td>114,661</td>
<td>58 months</td>
</tr>
<tr>
<td>150,000</td>
<td>107,330</td>
<td>49 months</td>
</tr>
<tr>
<td>100,000 (BASE)</td>
<td>100,000</td>
<td>0 months</td>
</tr>
<tr>
<td>50,000</td>
<td>92,670</td>
<td>49 months</td>
</tr>
<tr>
<td>0</td>
<td>85,339</td>
<td>58 months</td>
</tr>
</tbody>
</table>

bThis set of cases is illustrated in Fig. 6. Note that the final results in terms of the eventual number of guerrillas in the area ($N_\infty = 100,000$) are the same, independent of the initial number of guerrillas in the area. The other parameters, however, influence how fast this final stage is reached, and whether it is favorable or unfavorable (i.e., whether the guerrilla force strength decreases or increases from the initial value).
Fig. 6 -- Effects of variations in the initial number of guerrillas in the area from none \((N_0 = 0)\) to 200,000 (see also Table 13).
## Table 14

**ILLUSTRATIVE COMPARISON OF DIFFERENT METHODS OF REDUCING NUMBER OF GUERRILLAS IN AREA TO ONE-HALF OF INITIAL NUMBER**

<table>
<thead>
<tr>
<th>Description</th>
<th>Method I</th>
<th>Method II</th>
<th>Method III</th>
<th>Method IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Number of Guerrillas</td>
<td>$N_0 = 100,000$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of Attempted Infiltrations</td>
<td>$\frac{dT}{dt} = 10,000$ per month</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Border Interdiction Efficiency</td>
<td>$E = 20%$ of attempts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Internal Area Attrition Efficiency</td>
<td>$\gamma = 10%$ attrited per month</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Guerrilla Recruitment Efficiency</td>
<td>$\rho = 2%$ increase per month</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DESIRED Final Number of Guerrillas in Area</td>
<td>$N_\infty = 50,000$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Method I: Increase only of border interdiction efficiency from 20\% to 60\%.

Method II: Increase only of internal area attrition efficiency from 10\% to 18\% per month.

Method III: Increase of internal area attrition efficiency from 10\% to 16\% per month, and reduction of Guerrilla recruitment efficiency from 2\% per month to 0.

Method IV: Rate of attempted infiltrations decreases from 10,060 to 5,000 attempts per month.

<table>
<thead>
<tr>
<th>STATUS after 24 months:</th>
<th>Method</th>
<th>Method</th>
<th>Method</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Infiltration Attempts</td>
<td>240,000</td>
<td>240,000</td>
<td>240,000</td>
<td>120,000</td>
</tr>
<tr>
<td>Total Number interdicted at Border</td>
<td>144,000</td>
<td>48,000</td>
<td>48,000</td>
<td>24,000</td>
</tr>
<tr>
<td>Total Number of successful Infiltrators</td>
<td>96,000</td>
<td>192,000</td>
<td>192,000</td>
<td>96,000</td>
</tr>
<tr>
<td>Total Number recruited in area</td>
<td>34,667</td>
<td>30,116</td>
<td>0</td>
<td>34,667</td>
</tr>
<tr>
<td>Total Number eliminated in area</td>
<td>173,337</td>
<td>271,041</td>
<td>240,925</td>
<td>173,337</td>
</tr>
<tr>
<td>NUMBER OF GUERRILLAS IN AREA</td>
<td>57,330</td>
<td>51,075</td>
<td>51,075</td>
<td>57,330</td>
</tr>
<tr>
<td>ADDITIONAL TIME needed to reach 99% of desired final number of 50,000 Guerrillas in area:</td>
<td>34 months</td>
<td>5 months</td>
<td>5 months</td>
<td>34 months</td>
</tr>
</tbody>
</table>


VI. EXAMPLE OF JOSS VERSION OF MODEL

The problems of infiltration control in guerrilla warfare are not simple ones, and it cannot be expected that a simple model would illuminate them. The border control model described here has the capability of treating a variety of situations, but the mathematical formulations are by necessity somewhat complex.

However, computerized versions of the basic model have been developed and programmed for JOSS, and these permit the investigation of many realistic situations without the need of following the mathematical manipulations. One such JOSS version uses language exclusively rather than mathematical symbolism, and is readily usable -- on-line -- without external instructions. The JOSS user need have no knowledge of the analytic process described here, and is free to concentrate on manipulating the strategic and tactical situations of his own choosing.

The utility and capability of this JOSS border control model are best shown by an example. Pages 46 through 52 are a copy of a model run that investigated and analyzed the fictitious situation outlined below. Figure 7 shows one aspect of the results, and Table 15 translates the language input for this example into the mathematical symbols of the basic model.

The example will suffice to illustrate the ease and rapidity with which different situations or modifications of a situation can be investigated with this JOSS version. Detailed descriptions of the various computerized programs will be published in a separate Memorandum, and the programs will be available in a JOSS library file for on-line use.

SYNOPSIS OF JOSS EXAMPLE (Fictitious Situation)

Starting Situation: A country with an area of 66,000 square miles is exposed to hostile infiltration along a 1000-mile stretch of its border. At the start of the analysis, there is a force of 100,000 guerrillas dispersed over the area.
Additions to the guerrilla force from the outside are occurring at a level of 10,000 attempted infiltrations per month over the 1000 miles of open border. In the area, the guerrillas are able to recruit new guerrillas with an efficiency of one percent per month. (In other words, every group of 100 guerrillas recruits, on the average, one new guerrilla for its group each month.)

The defenders have a force level of 500,000 and conduct counter-insurgent activities through an area security program that operates at an average attrition efficiency of 4 percent per month. This corresponds to an initial attrition -- when there are 100,000 guerrillas in the area -- of 4000 guerrillas per month. (In other words, it takes initially 125 defenders to eliminate one guerrilla per month.)

**Situation after 8 months:** This situation prevails for eight months and, as shown by the model results (p. 48), the guerrilla force strength in the area has increased to about 150,000. The defenders decide therefore at this time (p. 49) to double the efficiency of the area security program to 8 percent per month. This leads immediately (p. 50) to a relatively high initial guerrilla attrition rate of approximately 12,000 per month. (This simple example does not specify the resource costs for doubling the attrition efficiency, and retains the number of defenders as constant.)

**Situation after 16 months:** As the model shows (p. 50), the guerrilla force strength has remained at about the level of 150,000. Even with the high attrition efficiency, the area security measures are not able to decrease the number of guerrillas in the area noticeably below this level. The defenders decide to install at this time (p. 51) a border security system -- along the 1000-mile stretch of open border -- which has an efficiency of 75 percent. (In other words, 75 percent of the infiltration attempts are interdicted or deterred.)

**Situation after 24 months:** The installation of a border security system, together with the continued program of area security, has shown immediate results (p. 52). The guerrilla force strength has dropped back down to slightly less than 100,000 and, importantly, continues to decrease. This is taking place, although the monthly guerrilla attrition rate is going down too. (In other words, no unrealistically high
demands are made on the performance of the area security program.)

The model projects that the eventual outcome -- if no further changes are made by either side -- would be an equilibrium level of about 35,000 guerrillas in the area, suffering losses of 7500 per month through border interdiction, and about 2900 per month through area security measures.

JOSS ROUTINE

Input: JOSS automatically raises a series of questions that set the general framework. The sequential demand for answers translates the situation under investigation easily and efficiently into the appropriate model parameters (pp. 46 and 47).

Output: After a brief recapitulation of the input data and their implications (p. 47), the principal output is given for the dates originally specified (e.g., at months 0, 4, and 8). The guerrilla situation on these dates is reflected in historical numbers (e.g.: 40,284 guerrillas eliminated from the area by month 8).

When the originally specified date for a re-evaluation has been reached, additional output is provided in the form of a time projection, i.e., the eventual outcome is predicted for the continuation of the general situation without any changes. At this date, JOSS is ready to accept changed input values as a result of the user's assessment of desirable alterations, and to continue evaluations with these new characteristics.
Do part 1.

BORDER CONTROL MODEL

Version A: Sensitivity Analysis

Would you like a brief program description? Answer yes (1) or no (0) = 1

This program evaluates the performance of counter-infiltration programs in a country subjected to guerrilla activity. It permits the investigation of a variety of situations and presents the results in terms of situation projections and eventual outcome. Changes in the efficiencies of border control and area security systems can be made at any desired time to evaluate the probable consequences. The program is based on the basic model of border control, described in this Memorandum.

There are four different modes of operation available, depending on the type of input information given.

Mode A: Infiltration or Penetration Rate only.

Mode B: Infiltration or Penetration Rate, AND Barrier Efficiency.

Mode C: Infiltration or Penetration Rate, AND Threat Characteristics.

Mode D: Barrier Efficiency, AND Threat Characteristics.

If there is no input to a question, please answer = -1.

The program will automatically select the appropriate mode.

GEOGRAPHICAL DATA
Area of Interest [square miles] = 66000
Length of Border [miles] = 1000

CALENDAR
Starting date is Month Zero.
What date of re-evaluation is wanted?
(New input data after how many months) = 8
Size of time intervals of output:
(Results every how many months?) = 4

STARTING SITUATION
Initial Number of Guerrillas in Area at Month Zero = 100000
Initial Number of Defenders in Area at Month Zero = 500000

STAND BY
INTERNAL SECURITY

Efficiency of (Internal) Area Attrition Measures
[percent of Guerrilla Force attrited per month; e.g.: 15] = 4

Efficiency of Guerrilla Recruitment
[percent increase of Guerrilla Force per month due to new recruitments; e.g.: 4] = 1

INPUT DATA

Rate of Successful Infiltration [per month] = -1
Rate of Successful Penetration [per mile per month] = -1

Barrier Interdiction Efficiency
[give as Probability of Non-Penetration in percent] = 0

THREAT CHARACTERISTICS

Rate of Attempted Infiltrations [per month] = 10000

YOUR INPUT DATA AND IMPLICATIONS:

Length of Border: 1000 miles
Infiltration Area: 66000 square miles
Border Parameter: .0152 miles per square mile

Initial Number of Guerrillas in Area: 100000
Initial Number of Defenders in Area: 500000

Rate of Attempted Infiltrations: 10000 per month
Rate of Attempted Penetrations: 10.00 per mile per month

Barrier Penetration Probability: 100 percent
Barrier Interdiction Efficiency: 0 percent of infiltration attempts stopped.

Rate of Successful Infiltrations: 10000 per month
Rate of Successful Penetrations: 10.00 per mile per month

Internal Security Efficiency: 4.00 percent attrited per month
8.0-06 percent attrited per month per defender

Guerrilla Recruitment Efficiency: 1.0 percent increase per month
(This means that each group of about 1000 guerrillas recruits ten new guerrillas each month in the area.)

According to the input given, the program is operating in MODE D.
## RESULTS

<table>
<thead>
<tr>
<th>DATE Month</th>
<th>CUMULATIVE NUMBERS</th>
<th>DENSITY</th>
<th>TIME RATE OF CHANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Attempted Infiltrations: 0</td>
<td>0 per mile</td>
<td>10000 per month</td>
</tr>
<tr>
<td></td>
<td>Attempted infiltrations interdicted at Barrier: 0</td>
<td>0 per mile</td>
<td>0 per month</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border: 0</td>
<td>0 per mile</td>
<td>10000 per month</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area: 0</td>
<td>.0 per mile*2</td>
<td>1000 per month</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area: 0</td>
<td>.0 per mile*2</td>
<td>4000 per month</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA: 100000</td>
<td>1.5 per mile*2</td>
<td>7000 per month</td>
</tr>
<tr>
<td>4</td>
<td>Attempted Infiltrations: 40000</td>
<td>40 per mile</td>
<td>10000 per month</td>
</tr>
<tr>
<td></td>
<td>Attempted infiltrations interdicted at Barrier: 0</td>
<td>0 per mile</td>
<td>0 per month</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border: 40000</td>
<td>40 per mile</td>
<td>10000 per month</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area: 4538</td>
<td>.1 per mile*2</td>
<td>1264 per month</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area: 18153</td>
<td>.3 per mile*2</td>
<td>0.05 per month</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA: 126385</td>
<td>1.9 per mile*2</td>
<td>6208 per month</td>
</tr>
<tr>
<td>8</td>
<td>Attempted Infiltrations: 80000</td>
<td>80 per mile</td>
<td>10000 per month</td>
</tr>
<tr>
<td></td>
<td>Attempted infiltrations interdicted at Barrier: 0</td>
<td>0 per mile</td>
<td>0 per month</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border: 80000</td>
<td>80 per mile</td>
<td>10000 per month</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area: 10071</td>
<td>.2 per mile*2</td>
<td>1498 per month</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area: 40284</td>
<td>.6 per mile*2</td>
<td>5991 per month</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA: 149787</td>
<td>2.3 per mile*2</td>
<td>5506 per month</td>
</tr>
</tbody>
</table>

### PROJECTED OUTCOME

( eventual balance situation )

The Guerrilla force strength has been increasing from its initial value.

There will be eventually (after infinite time) 333333 Guerrillas in the area.

If conditions remain unchanged, 99 o/o of this number will be reached at calendar date: Month 142

The final attrition rate will be 13333 per month;
The final recruitment rate will be 3333 per month.

Ready for different assumptions and/or data.
CHANGE OF SITUATION:

INTERNAL SECURITY
Efficiency of (Internal) Area Attrition Measures
[percent of Guerrilla Force attrited per month; e.g.: 15] = 8

Efficiency of Guerrilla Recruitment
[percent increase of Guerrilla Force per month due to
new recruitments; e.g.: 4] = 1

INPUT DATA
Rate of Successful Infiltration [per month] = -1
Rate of Successful Penetration [per mile per month] = -1

Barrier Interdiction Efficiency
[give as Probability of Non-Penetration in percent] = 0

THREAT CHARACTERISTICS
Rate of Attempted Infiltrations [per month] = 10000

YOUR INPUT DATA AND IMPLICATIONS:

Length of Border: 1000 miles
Infiltration Area: 66000 square miles
Border Parameter: .0152 miles per square mile

Initial Number of Guerrillas in Area: 149787
Initial Number of Defenders in Area: 500000

Rate of Attempted Infiltrations: 10000 per month
Rate of Attempted Penetrations: 10.00 per mile per month

Barrier Penetration Probability: 100 percent
Barrier Interdiction Efficiency: 0 percent of infiltration attempts stopped.

Rate of Successful Infiltrations: 10000 per month
Rate of Successful Penetrations: 10.00 per mile per month

Internal Security Efficiency: 8.00 percent attrited per month
1.6-05 percent attrited per month per defender

Guerrilla Recruitment Efficiency: 1.0 percent increase per month
(This means that each group of about 1000 guerrillas recruit
ten new guerrillas each month in the area.)

According to the input given, the program is operating
in MODE D.
RESULTS

<table>
<thead>
<tr>
<th>DATE</th>
<th>CUMULATIVE NUMBERS</th>
<th>DENSITY</th>
<th>TIME RATE OF CHANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month</td>
<td>Attempted Infiltrations:</td>
<td>80000</td>
<td>80 per mile</td>
</tr>
<tr>
<td></td>
<td>Attempted infiltrations interdicted at Barrier:</td>
<td>0</td>
<td>0 per mile</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border:</td>
<td>80000</td>
<td>80 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area:</td>
<td>10071</td>
<td>.2 per mile^2</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area:</td>
<td>40284</td>
<td>.6 per mile^2</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA:</td>
<td>149787</td>
<td>2.3 per mile^2</td>
</tr>
<tr>
<td>8</td>
<td>Attempted Infiltrations:</td>
<td>120000</td>
<td>120 per mile</td>
</tr>
<tr>
<td></td>
<td>Attempted infiltrations interdicted at Barrier:</td>
<td>0</td>
<td>0 per mile</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border:</td>
<td>120000</td>
<td>120 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area:</td>
<td>16027</td>
<td>.2 per mile^2</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area:</td>
<td>87933</td>
<td>1.3 per mile^2</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA:</td>
<td>148094</td>
<td>2.2 per mile^2</td>
</tr>
<tr>
<td>12</td>
<td>Attempted Infiltrations:</td>
<td>160000</td>
<td>160 per mile</td>
</tr>
<tr>
<td></td>
<td>Attempted infiltrations interdicted at Barrier:</td>
<td>0</td>
<td>0 per mile</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border:</td>
<td>160000</td>
<td>160 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area:</td>
<td>21924</td>
<td>.3 per mile^2</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area:</td>
<td>135109</td>
<td>2.0 per mile^2</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA:</td>
<td>146815</td>
<td>2.2 per mile^2</td>
</tr>
</tbody>
</table>

PROJECTED OUTCOME
(Eventual Balance Situation)

The Guerrilla force strength has been decreasing from its initial value. There will be eventually (after infinite time) 142857 Guerrillas in the area. If conditions remain unchanged, 99 o/o of this number will be reached at calendar date: Month 31

The final attrition rate will be 11429 per month; The final recruitment rate will be 1429 per month.

Ready for different assumptions and/or data.
CHANGE OF SITUATION:

INTERNAL SECURITY
Efficiency of (Internal) Area Attrition Measures
[percent of Guerrilla Force attrited per month; e.g.: 15] = 8

Efficiency of Guerrilla Recruitment
[percent increase of Guerrilla Force per month due to new recruitments; e.g.: 4] = 1

INPUT DATA

Rate of Successful Infiltration [per month] = -1
Rate of Successful Penetration [per mile per month] = -1

Barrier Interdiction Efficiency
[give as Probability of Non-Penetration in percent] = 75

THREAT CHARACTERISTICS
Rate of Attempted Infiltrations [per month] = 10000

YOUR INPUT DATA AND IMPLICATIONS:

Length of Border: 1000 miles
Infiltration Area: 66000 square miles
Border Parameter: .0152 miles per square mile

Initial Number of Guerrillas in Area: 146815
Initial Number of Defenders in Area: 500000

Rate of Attempted Infiltrations: 10000 per month
Rate of Attempted Penetrations: 10.00 per mile per month

Barrier Penetration Probability: 25 percent
Barrier Interdiction Efficiency: 75 percent of infiltration attempts stopped.

Rate of Successful Infiltrations: 2500 per month
Rate of Successful Penetrations: 2.50 per mile per month

Internal Security Efficiency: 8.00 percent attrited per month
                               1.6-05 percent attrited per month per defender

Guerrilla Recruitment Efficiency: 1.0 percent increase per month
(This means that each group of about 1000 guerrillas recruits ten new guerrillas each month in the area.)

According to the input given, the program is operating in MODE D.
### RESULTS

<table>
<thead>
<tr>
<th>Month</th>
<th>Attempted Infiltrations:</th>
<th>CUMULATIVE NUMBERS</th>
<th>TIME RATE OF CHANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Attempted Infiltrations</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>interdicted at Barrier:</td>
<td>0</td>
<td>0 per mile</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 per mile</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border:</td>
<td>160000</td>
<td>160 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area:</td>
<td>21924</td>
<td>160 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area:</td>
<td>135109</td>
<td>2.0 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA:</td>
<td>146815</td>
<td>2.2 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-7777 per month</td>
</tr>
<tr>
<td>20</td>
<td>Attempted Infiltrations:</td>
<td>200000</td>
<td>200 per mile</td>
</tr>
<tr>
<td></td>
<td>interdicted at Barrier:</td>
<td>30000</td>
<td>30 per mile</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>30 per mile</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border:</td>
<td>170000</td>
<td>170 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area:</td>
<td>27229</td>
<td>.4 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area:</td>
<td>177546</td>
<td>2.7 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA:</td>
<td>119683</td>
<td>1.8 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-5878 per month</td>
</tr>
<tr>
<td>24</td>
<td>Attempted Infiltrations:</td>
<td>240000</td>
<td>240 per mile</td>
</tr>
<tr>
<td></td>
<td>interdicted at Barrier:</td>
<td>60000</td>
<td>60 per mile</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>60 per mile</td>
</tr>
<tr>
<td></td>
<td>Infiltrators across Border:</td>
<td>180000</td>
<td>180 per mile</td>
</tr>
<tr>
<td></td>
<td>Guerillas recruited in Area:</td>
<td>31587</td>
<td>.5 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td>Guerillas attrited in Area:</td>
<td>212411</td>
<td>3.2 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td>GUERRILLAS in AREA:</td>
<td>99176</td>
<td>1.5 per mile$^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-4442 per month</td>
</tr>
</tbody>
</table>

**PROJECTED OUTCOME**

(Eventual Balance Situation)

The Guerrilla force strength has been decreasing from its initial value. There will be eventually (after infinite time) 35714 Guerrillas in the area. If conditions remain unchanged, 99 o/o of this number will be reached at calendar date: Month 98.

The final attrition rate will be 2857 per month; The final recruitment rate will be 357 per month.

Ready for different assumptions and/or data.
Fig. 7 -- Illustration of the effects of increasing the attrition efficiency of an internal area security system, and of installing a border security system.
<table>
<thead>
<tr>
<th>Model Symbol</th>
<th>Parameter</th>
<th>Time in Months after Start</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t = 0</td>
</tr>
<tr>
<td>A</td>
<td>Size of Area</td>
<td>66,000 miles (^2)</td>
</tr>
<tr>
<td>L</td>
<td>Length of Exposed Border</td>
<td>1,000 miles</td>
</tr>
<tr>
<td>N(_0)</td>
<td>Initial Number of Guerrillas in this area</td>
<td>100,000</td>
</tr>
<tr>
<td>M</td>
<td>Number of Defenders in the area</td>
<td>500,000</td>
</tr>
<tr>
<td>Y</td>
<td>Attrition Efficiency of Area Security Program per month</td>
<td>4%</td>
</tr>
<tr>
<td>p</td>
<td>Efficiency of Guerrilla Recruitment in Area per month</td>
<td>1%</td>
</tr>
<tr>
<td>E</td>
<td>Interdiction Efficiency of Border Security System</td>
<td>0</td>
</tr>
<tr>
<td>dT/dt</td>
<td>Rate of Attempted Infiltrations per month</td>
<td>10,000</td>
</tr>
</tbody>
</table>
VII. RESUMÉ AND CONCLUSIONS

MODEL CHARACTERISTICS

The border-control model is essentially a tool to assist in the analysis of actual or potential insurgent conflicts. It provides a new capability for investigating the specific problems of counter-guerrilla activities in a country or area where infiltration can or does occur along stretches of national border or other lines of demarcation. The basic analytic formulation of the model allows investigations a wide latitude, ranging from critical studies and evaluations of past or current conflict situations to contingency plans for various areas of the world.

The problems of guerrilla warfare are not simple ones, and it could not be expected that a simple model would illuminate them. In fact, no mathematical model of military conflict can properly reflect the true complexity of all associated factors. But the border model succeeds in structuring and clarifying the essential problems of infiltration by emphasizing the real world rather than mathematical abstraction. Its computerized JOSS versions use plain language instead of symbolism, allow the user to manipulate strategic and tactical situations according to his own choosing, and provide him with quantitative answers with respect to the projected outcome of postulated situations.

The nature of the border control problem, as depicted by the model, is illustrated by a differential equation that says:

The rate of change of guerrillas in an area is equal to
the rate of border infiltration plus
the rate of guerrilla recruitment minus
the rate of attrition in this area.
The three input rates in this equation can assume different values primarily as follows:

<table>
<thead>
<tr>
<th>The rate of border infiltration</th>
</tr>
</thead>
<tbody>
<tr>
<td>depends on: Characteristics of a border security system, and</td>
</tr>
<tr>
<td>the rate of infiltration attempts.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The rate of guerrilla recruitment</th>
</tr>
</thead>
<tbody>
<tr>
<td>depends on: Number of guerrillas in the area, and</td>
</tr>
<tr>
<td>the efficiency of the recruiting process.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The area attrition rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>depends on: Number of guerrillas in the area at any time, and</td>
</tr>
<tr>
<td>the efficiency of area-wide attrition measures.</td>
</tr>
</tbody>
</table>

Integration of this differential equation solves the mutual interactions among its various component parts, and can take into account the enemy strategy, if that is known. The general solution is not easily paraphrased in words, but it says essentially that:

<table>
<thead>
<tr>
<th>The NUMBER OF GUERRILLAS IN AN AREA at any specific time</th>
</tr>
</thead>
<tbody>
<tr>
<td>depends, in a complex way,</td>
</tr>
<tr>
<td>on the relative efficiencies of the processes of infiltration,</td>
</tr>
<tr>
<td>recruitment, and attrition,</td>
</tr>
<tr>
<td>rather than on the force-strength ratio of guerrillas and defenders,</td>
</tr>
<tr>
<td>and varies exponentially with time.</td>
</tr>
</tbody>
</table>
A glance at Tables 4 through 8 will illustrate why assessment and analysis of the implications and probable outcome of any specific insurgent situation is better left to numerical evaluation rather than intuitive prediction. In this case, the projected outcome can be reflected by the actual number of guerrillas present in the area at any time of interest in the future, and the numbers of guerrillas that were interdicted at the border, that successfully infiltrated, that were newly recruited, or that were eliminated in the area, respectively. A judgment whether these results constitute a desirable or undesirable status can then be made rationally on this basis.

MODEL CAPABILITIES

The principal functional capabilities of the model are as follows:

a. It considers the relations between infiltration, invasion, border security, internal guerrilla activity, and area-wide security;

b. investigates the relative efficiencies of such operations as infiltration detection and interdiction, attempted and successful penetrations, guerrilla recruitment and attrition;

c. relates variations in the guerrilla force strength to enemy strategies and the efficiency of counter-guerrilla operations;

d. is able to take into account a variety of factors; for example, distinctions can be made between types of guerrillas and the relative efficiency of a border security system to deal with members of a combat unit or civilian cadre, saboteurs or unarmed smugglers.

MODEL USES

The principal uses of numerical solutions of the model are as follows:

a. Through the use of historical data such as enemy orders of battle and casualty figures, the model can reproduce the actual course of events and determine the relative importance of different factors as they influenced past activities;
b. through the use of conditional input data, the model can investigate the applicability and probable usefulness of candidate border security systems and programs for different contingencies and scenarios;
c. through the use of different model solutions that correspond to different enemy strategies, it is possible to assess the probable consequences of system implementation in terms of likely enemy response and resulting requirements for system changes;
d. through time projections, different mixes of border security systems and internal area security programs can be tested with respect to optimal resource allocations.

CONCLUSIONS

During the course of the study of infiltration and invasion control, the model has been used primarily to support the on-going analyses. Emphasis to date has therefore been on the situation in Vietnam, but a number of the lessons learned have wider applicability. Throughout the text of this Memorandum, a number of conclusions have been discussed under the appropriate topic or heading. In the following, only the principal ones are reiterated:

It is clear that a counter-infiltration system cannot be considered a separate entity, because the efficiency of a border security program is completely interwoven with and tied to the efficiency of any area-wide counter-guerrilla program. However, the model makes clear that the interactions among the parameters are complex in nature, and it could be dangerous to generalize and to expect to be able to predict by analogy or by intuition what should happen in different, even though similar situations. Each specific realistic situation must be dealt with specifically, and may show quite different effects. This the model is able to do rapidly and efficiently.

As long as there is infiltration or the opportunity for infiltration, the best possible military outcome of a guerrilla conflict can only be a dynamic balance, an equilibrium situation where the defending forces just manage to keep the number of guerrillas from deviating
markedly from this level. Whether this final equilibrium level represents full combat activity, sabotage, or perhaps only an acceptable, rather low level of violence similar to crime, is determined by interactions among infiltration, guerrilla recruitment, and attrition, and the relative efficiencies of these operations.

In the absence of a border security system that at least hinders or deters the enemy from determining freely his desired infiltration rates, no model solution leads to conflict termination. That is true even for minimal guerrilla recruitment and the highest possible efficiency of area attrition measures. The conflict continues at a level of activity determined by the enemy, rather than by the defender. Changes in force ratio influence this level of activity, but even markedly different resource potentials of enemy and defender do not result in a clear conflict termination. This conclusion can best be paraphrased as stating that even the largest and most efficient defending force cannot eliminate more guerrillas in an area during a given time interval than there are guerrillas in the area — and new ones are relatively free to come in at any time of their own choosing.

As illuminated by the model in Section IV, it is a peculiar characteristic of insurgent conflict that the enemy is relatively free to adopt a strategy favorable to him, in response to almost any realistic defending strategy. However, similar to the defender's situation, the best possible outcome for the enemy is also only a balance situation, which can go on forever — at least analytically — without leading to any military victory or decisive military advantage.

In conclusion, it would appear that an efficient border security system is a necessity for any attempt to deal successfully with insurgent conflicts. Even a low-efficiency system will deny the enemy some of his freedom to bring men and supplies in and out at will. Information about desirable resource distributions among border security and internal area security can then be provided by the model for various candidate programs or systems to deal with various contingencies.
REFERENCES


## Abstract

Description of an analytic model of border control that interrelates a number of the principal factors in problems of infiltration, invasion, and insurgency. The basic model reflects geopolitical and economic as well as military and technical aspects, and provides some insight into their complex relationships. In particular, it treats the situation in which not only guerrillas and their opponents are active in an area, but in addition, infiltration or exfiltration occurs along stretches of national borders or other lines of defense. The model says, in essence: At any instant in time, the rate of change of guerrillas in an area is equal to the rate of border infiltration, plus the rate of guerrilla recruitment, minus the rate of attrition in the area. Computerized versions of the model permit the ready investigation of specific situations, the rapid testing of new concepts with respect to their probable effects under various contingencies, and the development of quantitative sensitivity analysis of candidate border security systems and programs.