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FREQUENCY-DEPENDENT AMPLITUDE-DISTANCE CURVE
FOR P-WAVES FROM 87° TO 110°

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<td>Seismological Institute</td>
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<td>Uppsala, Sweden</td>
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<td>Contract Termination Date</td>
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<tr>
<td>Project Scientist</td>
<td>Professor Markus Båth</td>
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<td></td>
<td>Tel. 130256</td>
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<td>Short Title of Work</td>
<td>Seismic Body-Wave Research</td>
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Summary

This work is an attempt to clarify the nature of the amplitude-distance curve for P-waves between 87° and 110°, using the spectral amplitudes of earthquakes in the Indonesian region recorded at the Swedish and Finnish seismograph stations. At the present time the results are inconclusive, because even after allowing for station and source terms there is a large unexpected scatter.

Introduction

The amplitude decrease of P-waves which are incident at the surface beyond 95° and which have passed through the deeper part of the mantle has been observed for a long time. As the quality and distribution of seismographs have increased, it has been possible to make more detailed studies. Gutenberg (1960) with limited data demonstrated the frequency dependence of the amplitude decrease, while Sacks (1966) with better data illustrated the effect very clearly. These results, however, are more qualitative than quantitative and cannot be used to test hypotheses on the nature of the core-mantle boundary region which produces the shadowing effect. Also these studies use a few earthquakes and do not consider the effects of station geology on the results. More recently, Alexander and Phinney (1966) have worked with long-period waves in the shadow region, but their data has large scatter, they do not consider station effects and they do not combine data from different earthquakes.

Recently, Carpenter, Marshall and Douglas (1965); and Cleary (1967) have worked on the amplitude-distance curve between 30° and 102° and have
used a joint analysis method described by Carpenter et al. This method allows the combination of different earthquakes, finds corrections for the station effect and produces an amplitude-distance curve independent of earthquakes and stations. These amplitude-distance curves are valid for short-period vertical-component records of about 1 sec period, but there is some disagreement between them at distances beyond 90° probably because of the different methods of measuring amplitudes. Both authors have little data over 95° and neither investigate the effect of frequency on the amplitude-distance curve. The present work has been done to attempt to clarify the frequency dependence of the amplitude-distance curve using the joint analysis technique to combine data from many earthquakes and many stations.

The observational material used in the present investigation consists of short-period vertical-component records of P-waves from the network of Swedish and Finnish stations for a number of earthquakes in the Indonesian archipelago.

Analytical method

At teleseismic distances we can express the frequency-dependent amplitude $A$ of the body waves in the form

$$ A = B R S $$

where $B$ is the frequency-dependent source function which includes the effect of the crust and upper mantle at the source, $R$ is the transmission coefficient for passage through the mantle which includes the effects of reflection and diffraction by the core-mantle boundary region (if the wave concerned is affected by this region), the effect of transmission at any boundaries and the effect of geometrical spreading of the waves, and
finally $S$ is the receive function which includes the effects of the station seismographic response curve and the crust and the upper mantle below the station. Each of $B$, $R$ and $S$ also includes the effect of the anelastic dissipation and scattering by inhomogeneities in the regions concerned. $B$ and $S$ vary with azimuth and also with angle of incidence on the surface. To the extent that the lower mantle is inhomogeneous $R$ is dependent on the particular path through the mantle.

In our problem then, we have selected the stations and earthquakes such that we make the assumption that $B$ and $S$ vary little over the small variation of azimuthal angles and small variation of angles of incidence involved (this assumption may not be valid!). $R$ will apply to the mantle between Indonesia and Fennoscandia and to the core-mantle boundary region under Central Asia. All of $A$, $B$, $R$ and $S$ are frequency-dependent and complex.

If we use the base ten logarithms of these quantities then we have

$$a = b + r + s$$  \hspace{1cm} (2a)

where $a = \log_{10}|A|$, $b = \log_{10}|B|$ etc. ($|A|$ is the amplitude of the complex $A$) and

$$\text{phase} (A) = \text{phase} (B) + \text{phase} (R) + \text{phase} (S)$$  \hspace{1cm} (2b)

For any particular measurement of $a$ we have

$$a = b + r + s + \epsilon$$  \hspace{1cm} (3)

where $\epsilon$ is an error term which includes the inaccuracies of measurement of $a$ and the effect of inadequacies of the model we have set up. This formulation is the same as derived by Carpenter et al. (1967) except that our $a$ is the log of the spectral amplitude and not $\log_{10} \left( \frac{A}{T} \right)$ and in our model the azimuths and angles of incidence are very limited in range.
Following Carpenter et al., we find that if we make a number of observations of $a$ at a number of stations for a number of earthquakes we can obtain estimates of $b$, $r$ and $s$. If we denote by subscript $i$ the particular earthquake considered then $b_i$ is the source term of the $i$th earthquake. Similarly, if we denote by subscript $j$ the particular station considered then $s_j$ is the station (crustal + seismograph) function for the $j$th station. Finally, if we divide the distance range into intervals over which the amplitude-distance curve is assumed constant and we denote by $k$ the $k$th such interval, then $r_k$ is the mantle transfer function for this distance range. If some estimate $r_e$ of the amplitude-distance curve is available, then we can subtract $r_e$ from both sides of equation (3) and $r_k$ is considered as the actual difference of $r$ and $r_e$. When $r_e$ is a reasonable approximation to $r$, the constancy of $r_k$ over a distance interval is a less imposing condition and yet we retain the flexibility of the histogram representation.

Thus if earthquake $i$ is observed at station $j$ and the separation of the two is in distance range $k$, the observed amplitude $a_{ijk}$ may be expressed in the form

$$a_{ijk} = b_i + r_k + s_j + \varepsilon_{ijk}$$

(4)

For $N_re$ observations of $a_{ijk}$ from $N_ep$ epicentres at some or all of $N_st$ stations using $N_pa$ distance ranges, we have a set of $N_re$ linear equations for $a_{ijk}$. We have $N_ep$ unknowns $b_i$, $N_pa$ unknowns $r_k$, $N_st$ unknowns $s_j$ and $N_re$ unknowns $\varepsilon_{ijk}$. If we remove from $b_i$, $r_k$ and $s_j$ their respective averages so that

$$c = \bar{b}_i + \bar{r}_k + \bar{s}_j$$

$$b_i - \bar{b}_i = b_i'$$

$$r_k - \bar{r}_k = r_k'$$

$$s_j - \bar{s}_j = s_j'$$

(5)
then the new $b_i'$, $r_k'$ and $s_j'$ averaged over $i$, $k$ and $j$ respectively are zero. Hence we have $N_{re} + 3$ equations:

$N_{re}$ equations

$$a_{ijk} = C + b_i' + r_k' + s_j' + \epsilon_{ijk}$$

and 3 equations

$$\sum_i b_i' = 0, \quad \sum_k r_k' = 0 \quad \text{and} \quad \sum_j s_j' = 0$$

We can henceforth drop the primes on $b_i'$, $r_k'$ and $s_j'$.

We can represent these equations in the matrix formulation

$$P = QX + E$$

where $P$ is the row vector of $a_{ijk}$ in some order, $E$ is the error row vector of $\epsilon_{ijk}$ in the same order as $a_{ijk}$, $X$ is the column vector $(C, b_1, b_2, \ldots, b_{N_{pe}}, r_1, r_2, \ldots, r_{N_{pa}}, s_1, s_2, \ldots, s_{N_{st}})$ and $Q$ is the matrix of indicator variables such that if the $n$th element of $P$ is $a_{ijk}$ then the $n$th row of $Q$ multiplied by $X$ gives $C + b_i' + r_k' + s_j'$ and the last three rows of $Q$ when multiplied by $X$ give equations (6).

It is possible to solve this matrix equation by the least squares method to minimise $|E|$ and get an estimate of $X$ and hence of $C$ and of $b_i'$, $r_k'$ and $s_j'$.

The least squares estimate for $X$ is given by

$$X = (Q^T Q)^{-1} (Q^T P)$$

where $Q^T$ is the transpose of $Q$ and $(Q^T Q)^{-1}$ is the inverse of $Q^T Q$, i.e. $(Q^T Q)^{-1} (Q^T Q) = I$, the identity matrix. Problems may arise with calculation of $(Q^T Q)^{-1} (Q^T P)$ and these problems are discussed by Anderssen (1969). In the present work straightforward matrix inversion
was used to form \((q^T q)^{-1}\) and the difference \((q^T q)^{-1}(q^T q) - I\) was used as a guide to the accuracy of the inversion of \(q^T q\). Since \(q\) is composed of integer indicator variables, \(q^T q\) can be calculated exactly and is not affected by computational rounding errors. If there are a sufficient number \(N_i\) of linearly independent equations (4), then a solution \(X\) of equation (7) can be found. \(N_i + 3\) must be greater than \(1 + N_{ep} + N_{pa} + N_{st}\), and the greater \(N_i\) the better the statistical estimate of \(X\).

**Observational material**

The stations used are those of the high quality Swedish and Finnish networks situated on the relatively homogeneous Fennoscandian shield (table 1). The earthquakes used occurred in the Indonesian area between the beginning of 1963 and the end of 1968 (table 2). See also figure 1. The particular earthquakes selected were such that the signal-to-noise ratio was generally good, the amplitude of the signal was sufficient to make further analysis worthwhile and the energy of the signal was concentrated near the onset. Any selection of the data will affect the final result as the criteria used are subjective. If, for example, a record is rejected because of low signal-to-noise ratio - then it may be that the noise level is high or that the amplitude level is low. However, some selection must be made and the criteria used seem reasonable.

The stations and earthquakes are related so that for any one earthquake the stations cover an azimuthal range of less than \(10^\circ\) and that for one station the earthquakes cover a back azimuthal range of less than about \(20^\circ\) (cf figure 2). For the range \(90^\circ-110^\circ\) epicentral distance, the angle of approach of the seismic P-wave changes little. So for each earthquake the station net covers a small solid angle and for each station the
earthquake epicentres cover a small solid angle. As we shall see later, these conditions should make the joint analysis method suitable for analysing the data.

The eleven Swedish and Finnish stations originally chosen are given in table 1. Of these SOD was later rejected, because of the nonstability of its amplification curve, and UDD, which because of its later construction recorded only four of the earthquakes (two on the earlier Grenet instrument and two on the later installed Benioff).

Sixteen earthquakes were initially selected, listed in table 2. Eleven of these earthquakes lie in a narrow back azimuthal range from Scandinavia and the other five are outside this band. The latter five are treated as suspect, as the station terms may vary too much with large changes in back azimuth. Of the original 176 possible records, 109 were selected and digitised. 14 records were not available, 20 were at too short epicentral distances and 33 were rejected because the signal-to-noise ratio was too low or the record amplitude was not large enough to make Fourier analysis worthwhile. Figure 3 shows a typical record.

For each earthquake the epicentral distance, azimuth and back azimuth to the stations of the net were calculated. The epicentral distances were corrected for depth of focus using the results of Buchbinder (1968). These corrections are such that all the earthquakes can be considered as surface focus events with regard to the amplitude-distance curve.

**Experimental method**

For the records from each earthquake a suitable record length was chosen, either 20, 30 or 40 sec, and this length was such that the main
part of the energy was in the earlier portion of the record and at the end of the record the amplitude was much smaller or reduced to near noise level. This selection of record lengths should minimise the effects of truncation of the record. The start of the record was taken just before the apparent onset of the arriving P-wave.

The records were photographically enlarged four or five times. Then the top and bottom of the trace were digitised on a DMac pen follower and the data were converted to cards. They were then interpolated to the desired interpolation intervals: \( \frac{20}{256} \) sec for 20 sec records, \( \frac{30}{512} \) sec for 30 sec records and \( \frac{40}{512} \) sec for 40 sec records. The average of the two traces was taken and the Fourier transform of the average computed in the form of amplitude and phase spectra. The theory of the spectral analysis of digitised seismic data is well covered by Huang (1966). If the seismic trace is the time function \( f(t) \), then the computed Fourier spectrum is given by \( F(v_n) \) where

\[
F(v_n) = \frac{1}{m} \sum_{k=0}^{m-1} f(k\Delta t) e^{- \frac{2\pi i n k \Delta t}{T}}
\]  

(9)

\( T \) is the length of the record, \( v_n \) is the \( n^{th} \) frequency in cycles per sec and \( v_n = \frac{n}{T} \) where \( n \) runs from 0 to \( m \), \( m \) is the number of digitised points, \( \Delta t \) is the digitising interval and \( m\Delta t = T \), and finally for most efficient computation \( m \) is a power of 2 (in our case \( m = 256 \) or 512). We avoid aliasing by using a digitising interval sufficiently small so that the amplitude spectra are negligible for frequencies above the folding frequency \( \frac{1}{2\Delta t} \).

Various methods of windowing were considered but none was applied...
as none seemed suitable. Using longer record lengths with low cut-off amplitudes should minimise the effect of truncating the records.

One record of average quality was photographically enlarged and digitised separately three times. The agreement between the three amplitude spectra is very good as shown in figure 4. The maximum variation throughout most of the frequency range was 0.25 units compared with the maximum amplitude of 4 units. For the larger amplitude components the difference is less than 8%. For better quality records the agreement should be better and the opposite for poorer quality records.

The amplitude spectra have been smoothed using a 3-point smoothing with \( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \) weighting for the 20 sec records, a 3-point smoothing with \( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \) weighting for the 30 sec records and a 5-point smoothing with \( \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \) weighting for 40 sec records. This smoothing should improve the consistency of the results and also make the comparison of the spectra from different length records more meaningful. This smoothing is not windowing but an attempt to smooth the insignificant fluctuations in the amplitude spectra.

All the spectra obtained are divided by the instrument magnification factor at 1 sec, and so the amplification curves are normalised at 1 sec. Also the inverse of the magnification factors for the photo enlargement is applied such that the amplitude is in units of 0.1 microns and after we have taken the \( \log_{10} \) of the amplitude spectra, we add one to the results, i.e. the \( \log \) (amplitude) of the spectra is such that the amplitude is measured in 0.01 microns. PnP is always included in the pulse, and if the earthquake is shallow, pP is included in the record. If
the earthquake is deeper, pP either does not affect the record or is small and appears near the end of the record.

Computations and results

In the experimental work we find estimates of $a_{ijk}$ for various earthquakes and stations. Then we apply the analytical method of joint analysis to estimate the amplitude-distance curve and the station terms. A computer program to solve equation (7) and find $X$ in the form of equation (8) has been developed. The program calculates the station and source terms $s_j$ and $b_i$ and also the amplitude-distance curve using a histogram of $2^\circ$ intervals. See Appendix.

The results are very poor. In figure 5 we show a plot of the raw data from which is subtracted the appropriate source and station terms and the constant introduced in equation (5). Even though the station and source terms are allowed for, the scatter is very high and certainly no amplitude decrease is seen beyond $90^\circ$ – as would be expected. This behaviour seems to come from the data and not from the inversion program. So far no explanation has been found for the anomalous behaviour of the results. The data has been divided into smaller groups of earthquakes with narrower azimuthal and distance ranges but there is no substantial improvement in the results. It is possible that the model we have set up is based on invalid assumptions on the nature of the source and station functions.

As an example we present the amplitude data for 1 sec period for all stations which recorded the earthquake on the 29 July, 1968. We list the stations in order of azimuth (epicenter to station) and epicentral distance. It is obvious that no clear pattern emerges.
Again for the earthquake on the 15 July, 1965, we have the following results at 1 cycle per sec frequency. (Obviously the data is not accurate to four decimal places!)

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<th>Station</th>
<th>Azimuth</th>
<th>( \log_{10}(\text{Amp}) ) at 1 sec</th>
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<tr>
<td>KLS</td>
<td>329.1</td>
<td>-0.1664</td>
</tr>
<tr>
<td>GOT</td>
<td>331.5</td>
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<tr>
<td>NUR</td>
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<tr>
<td>UPP</td>
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<tr>
<td>KJN</td>
<td>334.8</td>
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<tr>
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<tr>
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<tr>
<td>KIR</td>
<td>339.3</td>
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<table>
<thead>
<tr>
<th>Station</th>
<th>Distance (reduced to zero focus)</th>
<th>( \log_{10}(\text{Amp}) ) at 1 sec</th>
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<tr>
<td>Station</td>
<td>Distance</td>
<td>$\log_{10}(\text{Amp})$ at 1 sec</td>
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In the first example the back azimuths vary from $61^\circ$ to $75^\circ$ and in the second example from $67^\circ$ to $74^\circ$. The change in back azimuths between the two examples is about $5.5^\circ$ for each station.

The Fourier spectral program from seismogram to amplitude spectrum has been checked against an independent program. The spectral estimates seem therefore to be valid.

The problem remains - which of the assumptions we have made is not valid? Possible it is that the source function can vary very rapidly over very small azimuthal angles. In both the above examples the smallest and the largest amplitudes are next to each other in the distribution of azimuth. (This effect has not been checked on other data sets). If the source spectrum does vary so much with such small angles, then spectral analysis of short-period P-waves from earthquakes could only be done on a statistical basis. Explosions provide much more symmetrical sources, but their limited distribution prohibits their application to our present problem.
References


Acknowledgements

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The CDC 3600 computer of the Uppsala University Computer Centre was used for the calculations.
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<td>UME</td>
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<td>Skalstugan</td>
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Table 2

Earthquakes used (see also figure 1)

Data from USCGS

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<th>Epicentre Longitude</th>
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<td>101.5</td>
<td>173</td>
<td>7.0</td>
</tr>
<tr>
<td>26.8.1967</td>
<td>00.36.42.1</td>
<td>12.2</td>
<td>140.7</td>
<td>33</td>
<td>6.6</td>
</tr>
<tr>
<td>24.5.1968</td>
<td>15.43.54.2</td>
<td>-6.8</td>
<td>118.9</td>
<td>605</td>
<td>6.3</td>
</tr>
<tr>
<td>29.7.1968</td>
<td>23.52.15.0</td>
<td>-0.2</td>
<td>133.4</td>
<td>12</td>
<td>6.7</td>
</tr>
<tr>
<td>27.9.1968</td>
<td>03.58.55.1</td>
<td>-6.8</td>
<td>129.1</td>
<td>127</td>
<td>6.9</td>
</tr>
</tbody>
</table>
**Figure captions**

Fig. 1. Mercator projection of area of interest (showing Fennoscandian stations and Indonesian epicentres used).

Fig. 2. Cross-section of the earth showing diagrammatically the ray paths from Indonesia to Fennoscandia (the diagram is merely suggestive and not to scale).

Fig. 3. Short-period vertical-component P-wave recorded at Umeå from the Banda Sea earthquake of 21 March, 1964.

Fig. 4. The effect on the spectral amplitude of treating the record at Kiruna from the earthquake of 21 March, 1964, three times as a separate unit.

Fig. 5. Plot of all raw data for 1 cps with station and epicentre terms removed (shows failure of method to give the expected result and also shows large scatter).
FIG. 2

Fennoscandia

Indonesia

crust

mantle

core
Appendix

Program SOLVE is used to invert the system of linear equations described in the section "Analytical method". The program is not particularly efficient but is effective. Subroutine DIST 1 puts the earthquake-station pairs into their distance ranges. Subroutine SHUFFLE is used to vary the input without changing the coding of the original seismograms - earthquake 3 at station 6 has code 1316. Subroutine SELECT selects the frequencies required from the 20 frequencies of the input. Subroutine DIST 2 is here only formal but may be used to remove any prior estimate of station, distance or source terms.
PROGRAM SOLVE
CALL WORK
CALL EXIT
END

SUBROUTINE WORK

C THIS PROGRAM COULD BE MORE EFFICIENT
C MAXIMUM DIMENSIONS 110 RECORDS, 15 FREQUENCIES AND
C NST STATIONS, NPA PARAMETERS AND NLF EPICENTERS WHERE
C NST+NPA+NPF IS LESS THAN OR EQUAL TO 49
DIMENSION A(113,50),B(113,15),DISTANCE(110),FNQ(20),ANUM(60)
DIMENSION X(15,15),AS(50,50),AT(125,20)
DIMENSION TPR(113),ATCOL(113),TITLE(6)
EQUIVALENCE(AS,AT)
COMMON/10/AA(50,50)
READ 91,NST,NEP,NPA,NRE,NFR
901 FORMAT(5I5)
PRINT 992,NST,NEP,NPA,NRE,NFR
92 FORMAT(1X,15,* STATIONS, *,15,* EPICENTRES, *,15,* PARAMETERS,1
      *,15,* RECORDS, *,15,* FREQUENCIES, *)
      NCOL=NST+NEP+NPA+1! NKROW=NRE+3
READ 1, (EPQ(I,J)=I,NFR)
1 FORMAT(15F5.2)
      KST=1+NST+KPA=1+NST+4PA;JST=25JPA=2+NST+JEP=2+NST+NPA
      NEST=NCOL
      TITLE(I)=RHFQ.
DO 555 I=1,6
      TITLE(I)=RH
555 CONTINUE
C PUT A(I,J)=0.0
DO 35 J=1,NROW
DO 35 I=1,NCOL
A(I,J)=0.0
3 CONTINUE
2 CONTINUE
C PUT IN VALUES OF FIRST COLUMN OF A(I,J) AND BOTTOM THREE ROWS OF
C A AND B
DO 45 J=1,NRE
4 J=1
NRT=NR+1
DO 55 J=1,KST
55 A(NRT,J)=1.0
NRT=NRT+2
DO 65 J=1,JEP+NCOL
65 A(NRT,J)=1.0
NRT=NRT-1
DO 75 J=1,JPA+KPA
75 A(NRT,J)=1.0
NRT=NRT+1
DO 7 J=1,NFR
C  ANUM(I) GIVES THE NUMBER OF NON-ZERO ELEMENTS IN EACH COLUMN OF A
C  PRINT 12
12 FORMAT(1X,10X,INPUT DATA)
PRINT 10
10 FORMAT(*
PRINT 15,(FREQ(I),I=1,NFR)
15 FORMAT(9X,FREQUENCY *,15F7.2)
PRINT 10
16 FORMAT(1X,* FREQUENCY)
DO 9,1=2,NCOL
9 ANUM(I)=0
ANUM(I)=FLOOR(NRF)

C READ IN DATA
DO 100, I=1,NRF
READ 10,MPECIC,MSTAT,DISTA
10 FORMAT(1X,2I2,F6,1)
DISTANCE(I)=DISTA
MPECIC=MPECIC-10 SMSTAT=MSTAT-10
ANUM=MSTAT
CALL SHUFFLE(MPECIC,NFP,MSTAT,NST)
M=MSTAT+1 ANUM=MPECIC+KPA
A(I,M)=1.0
A(I,M)=1.0
ANUM(1)=ANUM(1)+1.0
ANUM(N)=ANUM(N)+1.0
CALL DIST1(DISTA,NPA,KDIST)
KA=KDIST+14+NST
IR(I)=KA
A(I,KA)=1.0
ANUM(KA)=ANUM(KA)+1.0
READ 29,(FREQ(K),K=1,20)
29 FORMAT(5F12.6)
CALL SELECT(FREQ,NFR)
C FREQ IS HERE THE INPUT AMPLITUDE DATA
PRINT 24,MPECIC,MSTAT,DISTA,(FREQ(K),K=1,NFR)
24 FORMAT(1X,2I2,F4.11F10.6)
Z=DISTANCE(I)
C SUBTRACT FITTED CURVE, SPECIFIED IN DIST2, AND INST EFFECT
CALL DIST2(FREQ,Z,NFR,MSTAT,NST)
DO 9881, K=1,NFR
9881 B(I,K)=FREQ(K)
100 CONTINUE
C HAVE NOW READ DATA INTO B AND PARAMETERS INTO A, AND SUBTRACTED
C FITTED CURVE AND INST EFFECTS FROMB
C

IF NOW FORM ATA AND ATB DATA IS INAA AND ATB IS IN X
DO 10 10=1,NCOL
DO 107 J=1,L
AA(I+J)=0.0
DO 16 16=1,NCOL
16 AA(I+J)=AA(I,J)+A(K,I)*A(K,J)
AS(I+J)=AS(I,J)
106 CONTINUE

L=1-L
DO 107 J=1,L
AA(I,J)=AA(I,J)
107 AS(I,J)=AS(I,J)
104 CONTINUE
DO 112 K=1,NCOL
DO 111 J=1,NEQ
X(I,J)=0.0
DO 112 K=1,NCOL
112 X(I,J)=X(I,J)*A(K,I)*B(K,J)
111 CONTINUE
110 CONTINUE
PRINT 1010
1010 FORMAT(* MATRIX ATA*)

DO 1011 I=1,NCOL
PRINT 1002(AA(I,J),J=1,NCOL)
1011 CONTINUE
DETERM=0.0
NMAX=0
CALL :MATINV(NCOL,X,NEQ,DETERM,NMAX)
PRINT 3992,DETERM
1002 FORMAT(1X,DETERM=**E12.4)
CGF=9.0
DO 2001 I=1,NCOL
DO 2002 J=1,NCOL
VAP=9.0
NOT REPRODUCIBLE
DO 20 J=1,NFP
20 VAR=VAR+AA(I,J)*AS(K,J)
   IF(I.EQ.J) VAR=VAR-1.0
   VAR=VAR*VAR
2002 IF(VAR.GT.GRE) GRE=VAR
2001 CONTINUE
   PRINT 14
   GRE=SORTF(GRE)
   PRINT 2004,GRE
   DO 205 I=1,NFP
   DO 205 J=1,NFR
   DUM=0.0
   DO 122 K=1,NCOL
122 DUM=DUM+AT(I,K)*X(K,J)
   AT(I,J)=DUM-B(I,J)
   B(I,J)=AT(I,J)*AT(I,J)
   AT(I,J)=X(IDR(I),J)-AT(I,J)
   121 CONTINUE
   120 CONTINUE
   DO 500 J=1,NFR
   DO 500 I=1,NFP
   502 ATCOL(I)=AT(I,J)
   CALL :NCODE(ITITLE)
   CALL FMTS(R)
   CALL FMTI(J,1)
   CALL GRAPH1(DISTANCE,ATCOL,-NRE,3H7X8,4HAUTO,ITITLE,10HDISTANCE,15HAMP,526060606060606060606)
   CONTINUE
   C X CONTAINS THE SOLUTIONS AND B CONTAINS THE ERROR SQUARED
   C WE FIND NOW THE STANDARD DEVIATIONS
   SOAN1=SORTF(ANUM(1)-1.0)
   SOAN2=SORTF(ANUM(1)-NPA+1.0-1.0)
   ANNN=ANUM(1)-1.0*(NPA+NST+NEP+1)
   ANNN=MAX1F(ANNN+1.0)
   SOAN3=SOPTF(ANNN)
   DO 150 I=1,NFR
   AA(I,1)=0.0
   DO 150 J=1,NFR
150 AA(I,J)=AA(I,1)+B(J,1)
   DUM=SOPTF(AA(I,1))
   AA(I,1)=DUM/SOAN1
   AA(I,2)=DUM/SOAN2
   AA(I,3)=DUM/SOAN3
   150 CONTINUE
   C EXCEPT FOR FIRST COL ANUM(1) IS NO IN COL MINUS ONE
   DO 160 K=J+3
160 L=J+1
   DO 160 I=1,NFR
AA(I,K)=0.0
DO 160 I=1,N=1
160 AA(I,K)=AA(I,K)+B(M,I)*A(M,L)
1A0 AA(I,K)=SORTF(AA(I,K)/ANUM(L))

CONTINUE
PRINT 14
PRINT 200
20 FORMAT(* RESULTS*)
PRINT 201
201 FORMAT(* CONSTANT*)
202 FORMAT(1X,16FR*,4)
DO 204 J=1,3
PRINT 202,(X(1,J),AA(I,J),I=1,NFR)
204 CONTINUE
PRINT 14
PRINT 205
205 FORMAT(1X,*STATIONS*)
207 FORMAT(1X,*ID)!
DO 206 I=2,KST
J=1-1 SIL=I+2
PRINT 207*,J
PRINT 202,(X(I,K),AA(K,L),K=1,NFR)
206 CONTINUE
PRINT 14
PRINT 208
208 FORMAT(1X,*DATAFTERS*)
209 FORMAT(1X,3X,$DA=KDA)$
DO 200 ,I=JOA,KDA$
J=1-1-NST < L=I+2$
PRINT 207*,J
PRINT 202,(X(I,K),AA(K,L),K=1,NFR)
200 CONTINUE
PRINT 14
PRINT 211
211 FORMAT(1X,*EPICENTRES*)
212 FORMAT(1X,3X,$EP=KEP)$
DO 212 ,I=JFP,KEP$
J=1-1-NST-NPA SIL=I+2$
PRINT 207*,J
PRINT 202,(X(I,K),AA(K,L),K=1,NFR)
212 CONTINUE
OPTION
END

SUBROUTINE DIST1(DISTA,NPA,KDIST)
X IS DISTANCE TO RIGHT OF FIRST INTERVAL,N IS NUMBER OF FIRST LONG
1 INTERVAL, Y IS THE DISTANCE BETWEEN THE TOP OF FIRST AND BOTTOM
1 OF FIRST LONG INTERVAL, W IS WIDTH OF SHORT INTERVAL, LONG IS 5.
X=1.0J, S=4 SY=4.0 SW=2.0
Z=DIISTRA-X
IF(Z<LT.0.0)Z=0.125
IF(Z,GT,Y) GO TO 1
  7=7/W
  KDIST=INTF(7)+1
  RETURN
1 Z=(Z-Y)/5.0
  KDIST=INTF(7)+N
  RETURN
END
SUBROUTINE DIST?(FREQ,Z,NFR,MSTAT,NST)
  DIMENSION FRO(1)
  GO TO 2
2 CONTINUE
  RETURN
END
SUBROUTINE SHUFFLE(MFEPIC,NEP,MSTAT,NST)
  DIMENSION NUTT(16),NURT(1)
  SORTS EPICENTRES AND STATIONS INTO NUMBERED ORDER
  DATA((NUTT(1),I=1,16)=7,0,0,0,1,0,0,0,2,3,0,5,0,4),((NURT(I),I=1,11)
1,11)=123456789010)
  MFEPIC=NUTT(MFEPIC)
  MSTAT=NURT(MSTAT)
  RETURN
END
SUBROUTINE SELECT(FREQ,NFR)

  DIMENSION FRO(1),FL(20)
  SELECTS THE FREQUENCIES REQUIRED FROM THE 20 READ
  DATA((NUM(I),I=1,10)=1,2,3,4,5,6,7,8,9,10)
  DO 1 I=1,20
1 F(1)=FRO(I)
  DO 2 I=1,NFR
2 FRO(I)=FL(NUM(I))
  RETURN
END
*LOAD,
( 1, 12(1))
This work is an attempt to clarify the nature of the amplitude-distance curve for P-waves between 87° and 110°, using the spectral amplitudes of earthquakes in the Indonesian region recorded at the Swedish and Finnish seismograph stations. At the present time the results are inconclusive, because even after allowing for station and source terms there is a large unexpected scatter.

14. Key Words

Indonesian earthquakes
Fennoscandian stations
Diffracted P-waves
Spectra, spectral analysis
Amplitude-distance curve
Computer program SOLVE