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Introduction

This final report describes research completed under Contract AF49(638)-1243, during the period 1 July 1963 to 31 December, 1969. The primary objectives of the project were as follows:

(1) The development of theoretical methods for treating the amplitude and phase characteristics of body waves in the earth.

(2) The development of parallel numerical methods for computing the amplitudes and phases of body waves in a radially heterogeneous earth.

(3) The application of these results to a more complete understanding of observed long- and short-period signals from explosions and earthquakes.

Major results:

The primary outcome of the research may be summarized:

(1) Further understanding of the effect of the crust on long-period body waves. In Phinney (1964) and Phinney (1965), the distortion effect of the crust on teleseismic P waves, and the theory of the body wave portion of the crustal long-period seismogram were treated.

(2) A full treatment of the effects produced by the core boundary and the lower mantle on body waves is now available, from a series of papers beginning in 1966. The core diffraction proved to be especially important for
testing these ideas because it is about the highest-quality, least-cluttered signal on existing long-period records.

(3) A full theory for the interaction of body waves with regions of high velocity gradients and with negative gradients is now available (Phinney, 1970 and Chapman, 1969). The kinematic behavior of waves in a shadow region may be described.

(4) The development of a scheme for the exact numerical representation of a long-period body wave signal. This scheme has been implemented in an existing computer program, and is being applied in the synthesis of signals from a realistic earth model.

(5) The discovery of the utility of a complex frequency variable as a means of smoothing data, with maximum weight given to early-arriving signals.

Most of the published theoretical and numerical results, in addition to a great deal more in the form of notes, represents the source material for a monograph now in preparation by the Principal Investigator. The publication of this material will probably take another 18 months.

Resources generated:

In addition to the principal published results, the following resources for data analysis and theoretical work were generated:

(1) Computer programs for (a) calculation of the response of a layered crust to body waves; (b) cal-
calculation of the signal generated by a point source in a layered medium; (c) calculation of the signal generated by a point source in a spherical earth; (d) reduction and analysis of long-period core diffractions; (e) computation of spectral ratios from long-period body wave components.

(2) A data archive of 70 mm film chips from the WWSSN for selected large earthquakes and blasts.

(3) A data archive of digitized long-period P waves (3-component) from selected stations of the WWSSN.

(4) A data archive of digitized long-period diffracted core waves.

The project objective to acquire and analyze a data library of digital long-period body wave signals from the LRSM and LASA networks was never realized, owing to the impracticability of large-scale access to these collections.

Graduate student research:

L. M. Cathles was supported by the contract for one year. He collaborated as a co-investigator on the detection of the core shadow (Phinney & Cathles, 1969).
Research by visiting scientists:

The following visiting researchers were supported by the contract, and contributed to the project output.

Shelton S. Alexander: Six months as visiting fellow in 1965, while on leave from the Air Force. Contributed to papers on diffraction by the core-mantle boundary.

Christopher Chapman: Six months as visiting student in 1968-69. Carried out research on body wave calculations, leading to a Ph. D. thesis at Cambridge in September, 1969.

### Appendix

**Bibliography with Abstracts**

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PUBLISHED PAPERS:


ABSTRACT

Long-period P waves from distant earthquakes have been analyzed from seismograms recorded at Albuquerque and Bermuda in light of Haskell's theory of the spectral response of a layered crust. By using the ratio of the vertical spectrum to the horizontal component spectrum, we obtain a function which depends on structure beneath the station. Because of the poorly understood nature of the signal which follows the first P wave motion, the methods of power spectrum analysis are applied and a lag window selected to discriminate against long time correlations within the signal. Corrections for the differing responses of the three components are made by using the power spectral matrix of calibration signals. A range of crustal models has been found which agrees with the data. We are restricted primarily in choosing models for the lower half of the crust; variations of structure in the upper half do not noticeably affect the theoretical curves in the period range considered (0.02 to 0.20 cps). At Albuquerque the crust is about 40 km thick and the lower crust has velocities in the range 6.6 to 7.0 km/sec. The Mohorovicic (M) discontinuity under Bermuda is 12 km below sea level, and the structure appears to be a normal oceanic crust depressed elastically by the weight of the volcanics which compose the island.

**ABSTRACT**

A practical method for the calculation of the spectral parameters of first-arriving signals in seismology has been the object of much theoretical work in recent years. The difficulty has been that the first arrival usually behaves as an imperfectly trapped mode. Mathematically, it arises from the contributions of branch line integrals and complex poles. Attempts to transform the solution into a generalization of the normal modes have been a mathematical success only. Because of the complexity of this solution, a different, less elegant approach is demanded. A practical technique is proposed. By a change of variable, the twice transformed solution is separated into a product of the form \( f(u)e^{-iuR} \). This can be integrated with respect to the phase variable \( u \), using standard quadrature methods, the real part of \( u \) changing most rapidly along the integration path. By making the frequency complex, it is possible to displace any singularities away from the vicinity of the contour. This gives the spectrum of the signal as viewed through an exponentially decaying time window, making it possible to work with the first arrival by itself.
ABSTRACT

Many of the applications of body waves to detection and identification of explosions have depended on the understanding of the behavior of body wave amplitudes in a radially heterogeneous earth. It is possible, by means of theory now available, to compute wave-theoretical body wave spectral amplitudes which take account of focusing and diffraction phenomena not described by ordinary ray theory. The existence of these phenomena demands that amplitudes of body waves be determined only by careful study of the spectral properties of the signals and after corrections have been made for known distortions.

ABSTRACT

Ratios of the spectrums of diffracted P waves can be used to isolate effects of propagation along the core-mantle boundary, and the rate of attenuation with distance as a function of frequency is a useful quantitative measure of core-mantle properties. Spectra of the attenuation coefficient for a number of patches on the core-mantle boundary were measured in the frequency range 0.01 to 0.12 cps, using data from large earthquakes in different locations. Although the general trend of the observations follows the theoretically expected $f^{1/3}$ frequency dependence, significant lateral variations occur both in level and shape of the attenuation spectrum, implying lateral variations in core-mantle properties. In particular, resonance absorption peaks are observed at certain frequencies which may be a consequence of layering in the deepest part of the mantle. The results are correlated with geographic location in an effort to commence to map the lateral variations in core-mantle properties. In addition, the observed phase spectrums indicate that the diffracted P waves are dispersed little or none at all for core-mantle boundary propagation.
PUBLISHED PAPERS (cont):


Abstract

The theory for diffraction of elastic waves in a spherical geometry is reviewed and modified to apply to a multilayer geometry. We have made numerical calculations of the attenuation function for P waves in the shadow of the core for certain basic geometries and for models which have a single layer at the core-mantle boundary. Resonance phenomena due to the presence of the layer are discussed, and an attempt is made to explain observed peaks in the attenuation spectrum for various regions of the core boundary. A region between 30 and 160 km thick, with a reduced shear velocity and an increased density, appears to be the most likely explanation of the data.
ABSTRACT

Theoretical expressions for the amplitude of P waves in the neighborhood of the core shadow boundary have been formulated and evaluated by computer. Comparison with 2.5-sec data of Sacks confirms his determination of the shadow position at 96.5°. A square-root frequency scaling law is found for the high-frequency diffraction pattern and permits the use of the Fresnel integral function at short periods. Long-period amplitude data for three paths have been spectrun-analyzed and compared against the theoretical amplitude dropoff going into the shadow. The agreement is about as good as the data will permit and no indication is found at these periods of a low-amplitude defocused region at 90° to 96°, after diffraction has been accounted for.
ABSTRACT

Asymptotic expressions for the P-wave solution near a turning point may be combined with the boundary conditions prevailing at the mantle-core boundary to obtain an asymptotic expression for the reflection coefficient at this boundary that preserves the diffraction effects of curvature and velocity gradient. Increasing the velocity gradient is shown to increase the decay and coefficient for the diffracted wave in the shadow. Consideration of recent published data and some new short-period data suggests that regional differences in the decay of this signal may be due to difference in the gradient at the base of the mantle.
PUBLISHED PAPERS (cont):


ABSTRACT

Reflection-transmission problems for plane waves can be analytically solved in a restricted number of special cases for which asymptotic connection formulas exist. The technique is demonstrated for the case of a parabolic barrier. A method due to Epstein, which makes use of the properties of hypergeometric functions, may be used to obtain solutions for a variety of transitional structures. This method is taken over from the field of ionospheric radio wave propagation and adapted to the acoustic problem without neglect of the density gradient. The solution for a monotonic transitional zone is obtained, and the behavior of the subcritical reflection discussed, with reference to the inner core reflection as an example.
MANUSCRIPTS AND UNPUBLISHED REPORTS:


**ABSTRACT**

The problem of body wave propagation is considered for a radially heterogeneous earth. The approach is based on an expansion of the formal solution into integrals representing "generalized rays". The various ray integrals for a homogeneous mantle/homogeneous core model have been studied for both high frequencies (asymptotic ray theory) and exactly, by numerical integration. The numerical integration method involves complex integration in the neighborhood of the saddle point representing the geometric ray. For the above model, the signals P, PcP, diffracted P, PKP,diffracted PKP, S, ScS, SKS, SnKS, were treated.

Extension of these methods was made to the case of a general earth model. The various analytic approximations leading to ordinary ray theory and their extensions were discussed. Numerical contour integration of the ray integrals was done as above. The computation of the response function for the general earth models is done by a combination of Runge-Kutta integration of the equations of motion and asymptotic expressions, for regions where these are valid. The following phases were computed: P, PcP, diffracted P, S, ScS, diffracted S, PKP, PKIKP.

This thesis is being divided into two or three distinct papers, which are being submitted for publication. The first has already been submitted to the Geophysical Journal.
PAPERS AT MEETINGS: (Not published elsewhere).


*Diffraction Theory for PKP.* Annual meeting of the American Geophysical Union.

**ABSTRACT**

I have computed numerical results for the wave-theoretical PKP signal as a function of frequency for an elastic/fluid model of the mantle-core interface. This shows the kinematic behavior of the pulse in the neighborhood of the caustic and puts limits on the frequencies for which ray theory is adequate. The predicted amplitude decay into the shadow region takes place over several degrees, even at periods of 1-2 sec. Some preliminary analyses of long-period data are available for the vicinity of the PKP caustic at 141°.
PAPERS AT MEETINGS: (not published elsewhere)


ABSTRACT

The increasing resolution and precision of modern seismic systems has made it necessary to more fully understand the behavior of body wave amplitudes in the neighborhood of cusps and endpoints of the travel-time curve. These arise when the turning point of a ray is sufficiently close to a region where the density and velocities do not satisfy the smoothness criteria of JWKB (ray) theory. Most situations of interest may be modeled by regions where the wave solutions may be shown to be connected with the interface waves traveling along the structures of interest; Analysis of a wide variety of first and second order discontinuities is possible. An example is given showing the role of the velocity gradient in determining the decay of the diffracted P phase in the core shadow.
RELATED WORK:


ABSTRACT

Longitudinal seismic velocities have been measured in 24 cores from 8 sites in the Stillwater igneous complex, Montana, and in 3 cores from a sample of the Tinaquillo peridotite, Venezuela. The velocity anisotropy, $\Delta V/V$, for the Stillwater sites ranges from 2% to 5%. Correlation with petrofabric diagrams shows that bronzitites have minimum velocities normal to the layering in the Stillwater due to preferred orientation of the $b$ axes. Anorthosites show a velocity maximum normal to the layering with $c$ crystallographic axes concentrated in the layering plane. The major characteristics of the preferred orientation of both bronzite and plagioclase are consistent over distances of 14 miles and support a gravitational settling mechanism for the layering. Although the Tinaquillo peridotite samples appear to have experienced intensive shearing in their history, a lack of preferred orientation in the olivine is inferred from the low seismic velocity anisotropy (2%). This is interpreted as being due to a degradation of shear-induced preferred orientations by continued shearing.
RELATED WORK:


ABSTRACT

The problem of determining the refractivity profile of a planetary atmosphere from optical or radio occultation data is identical in principle to the problem of determining the variation of seismic velocities in the earth from the observed travel times of seismic body waves. In either case, a complete set of data can be inverted uniquely, the only constraints being those fundamental to geometric optics. Expressions are given for converting observed Doppler shifts to the index of refraction as a function of depth in the atmosphere. The effect of various approximations on the analysis is discussed; it is found that a "thin atmosphere" approximation simplifies the mathematics and preserves the singularity at the critical ray curvature.
Research in Seismology

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c. The application of these results to a more complete understanding of observed long and short period signals from explosions and earthquakes.

Key words:
Long Period
Seismogram
P waves
Radially heterogeneous earth.