A MECHANICS APPROACH TO PROJECTILE PENETRATION

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SCIENTIFIC REPORT NO 6

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A Mechanics Approach to Projectile Penetration* 

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Abstract

The subject of the penetration and perforation of different materials has been studied for many years. Due to the intricacy of this subject, involving many parameters, the main body of knowledge is based on experimental work. Such theoretical approaches as there are, are simplified by some basic assumptions. Nevertheless, there appears to be room for further investigation of the mechanism of penetration. In the program reported in this paper, a mathematical model was developed which describes the mechanism of the normal penetration of metallic targets. The model considers all the forces acting on the projectile during penetration, bearing in mind that it is deformed during the penetration and that its effective mass increases during penetration due to the concomitant motion of part of the target mass. With the aid of the mathematical expressions the projectile's velocity after perforation can be calculated by substituting the information on the cavity diameter obtained experi-

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Another part of this program consisted of a series of penetration experiments. For the sake of convenience and from practical considerations, the experimental studies were carried out with 0.22" caliber lead bullets. The experimental results for the velocity drop due to normal perforation of metallic plates were compared with the mathematical model, and excellent agreement was established. In the frame of this research an explanation for the dependence of the velocity drop on the angle of impact was attempted, and ballistic tests were performed with different materials.

**Notation**

- $A$ - the area of the projectile nose projected on the target
- $b$ - width of the shearing surface
- $d$ - diameter of the hole
- $F$ - resisting force
- $F_f$ - resisting inertial force
- $F_c$ - resisting compressive force
- $F_s$ - resisting shearing force
- $h$ - plate thickness
- $K$ - numerical constant
- $L$ - length of projectile nose
- $m_o$ - mass of the projectile
- $m$ - instantaneous mass of the projectile
- $R$ - radius of projectile body
t - time

V - instantaneous velocity

V_i - impact velocity

V_f - final velocity

V_b - velocity of projectile at end of first stage of penetration

x - depth of penetration

α - angle of impact

ρ - density of the target material

σ - stress

σ_Y - yield stress

σ_ult - ultimate stress

τ - maximum shearing stress

**INTRODUCTION**

The subject of penetration and perforation has been investigated for many years, the main interest being in the military field. There has been a long-standing mutual escalation between improving projectiles on the one hand and increasing target resistance on the other, but since the mechanism of penetration and perforation and the effects of the various parameters influencing the perforation process are little understood, most of the relevant advances have been based upon the empirical knowledge derived from long series of ballistic experiments. In addition, a great deal of uncertainty surrounds the partition of the projectile's initial kinetic energy into heat, plastic deformation of
target and projectile, elastic-plastic waves, kinetic energy of the fragments, etc. The problem is complicated by the fact that the partition depends on many parameters, e.g., the mechanical and constitutive properties of the projectile and target materials, their densities, the geometrical data of the projectile's nose, and its impact velocity. Only since the Second World War have investigators begun to tackle the subject. Among the many are Bethe (n. d.), Taylor (1948), Goldsmith (1960), Said and Paul (1957, 1958, 1959), Thomson (1955), Nishiwaki (1951), Kucher (1967), Goldsmith et al. (1965), and Masket (1949). All the theories suggested by the different investigators were, it is true, simplified by some basic assumptions, yet they succeeded in contributing to the progress of research on the subject.

The process of the impact and penetration of projectiles is attended by high deformation and a high strain rate. Under such high strain rates ($10^2 - 10^5 \text{sec}^{-1}$) the mechanical properties of materials differ from those under static loading. The large amounts of heat generated by the plastic deformation and friction also affect the mechanical properties of the material. In the last two decades, a vast amount of knowledge on the effects of high strain rates and temperatures on various material properties has accumulated.

In addition, analytical and experimental research on the problem of elastic-plastic wave propagation in bars and plates has in the past few years been undertaken by many workers. The fields of dynamic plas-
ticity and of the theory of dislocation dynamics have also been extensively explored.

It is therefore generally accepted that the theoretical approach to the composite subject of impact, penetration, and perforation should be renewed and that the above general theories be applied to this problem.

**THEORETICAL BACKGROUND**

In response to the impact of a projectile, the target may fail by a variety of mechanisms, such as petal formation (or dishing), hole enlargement (ductile type of penetration), plug formation, and the fragmentation (scabbing) of the target material. Goldsmith (1960) and Rinehart and Pearson (1965) schematically show the possible mechanisms for plate perforation.

Many investigators have suggested various theories explaining the resistance of an armour plate to projectile penetration. They separate the discussion of the perforation of thin plates from that of plug formation and consider the different shapes of the projectile's nose. In all the theories mentioned only undeformed projectiles are taken into account. Two main approaches were applied in tackling this problem: the first was the aspect of the energy balance, and the other was based on the law of the conservation of momentum.

Thomson (1955), using a quasi-dynamic approach, derives equations for the energy dissipation due to plastic deformation, heating and
inertial resistance of the target material. The following assumptions were made by that author in common with some other investigators: (1) the circumferential stress, \( \sigma_\theta \), is the important stress component in crater development; (2) the yield condition is: \( \sigma_\theta = \sigma_y \); (3) the materials of both projectile and target are incompressible; (4) the plate thickness is much smaller than the diameter of the hole; (5) there is no plastic deformation outside the region of the perforated hole. From these basic assumptions the author derives the total work required for perforation of thin plates.

Bethe and Taylor (1948) analysed the stress distribution in a plate due to the enlargement of a circular hole by a conical-head projectile, and derived the total work required for plastic deformation. Zaid and Paul (1957; 1958) attacked the problem of the perforation of thin plates, and based their approach on the law of conservation of momentum which requires that the terminal shape of the perforated plate be specified. Nishiwaky (1951) proposed a theory for the penetration of plates, which is based upon data derived from static tests. Finally, there are some papers studying the case of plug formation, such as Recht and Ipson (1963) and others who dealt with the cases of high-velocity impact.

Most of the theories enumerated discuss the case of a high impact velocity, using it to justify a number of their assumptions: constant velocity during the perforation of thin plates, absence of plastic
deformation beyond the immediate zone surrounding the hole (perforation velocity greater than the plastic stress wave velocity of the target material), negligibility of the effects of the material strength of the target. They also restrict their consideration to the case in which the projectile is not deformed during perforation. Moreover, each of the different investigators discussed only one of the mechanisms of failure mentioned above. Therefore, none of these approaches appear to be applicable to the case of projectiles impinging at ordnance velocity. The residual velocity derived from any of those theories is invariably higher than that shown by the experimental results.

ANALYTICAL CONSIDERATIONS

In the course of this experimental study, a long series of ballistic experiments were carried out. Observations of the different specimens, the shape of the hole, and a large amount of ballistic photographs have led to some conclusions which permit the construction of a mathematical model representing the problem of normal perforation of metallic plates by 0.22" caliber lead bullets.

The analysis considers all the forces acting on the projectile during its perforating action. According to the second law of Newton:

$$\frac{d}{dt}(mV) = -F$$  \hspace{1cm} (1)

From observations of the perforated specimen it is concluded that
the force $F$ is the resultant of three main components: the resisting inertial force of the target material $- F_i$; the compressive force $- F_c$; and the shearing force $- F_s$. Substituting these forces in Eq. (1) yields

$$\frac{d}{dt}(mV) = -(F_i + F_c + F_s)$$

(2)

The frictional forces are neglected. The time interval for which contact between the projectile's nose and the target lasts is very short. It is about 10-30 μsec (for a 0.22" caliber lead bullet moving at a velocity of 400 m/sec). Therefore the target material cannot dissipate the large amount of heat generated and a thin film of fluid ($10^{-2}$ mm thick)* is produced, which separates the two solid materials (of the projectile and of the target) from each other and reduces the resisting frictional forces. Thomson (1955), Zener and Peterson (1943), and others have shown that the frictional forces dissipate only a small part of the impact energy in the form of heat, Kraft (1955) giving the proportion as about 3%.

The inertial resistance force $F_i$ has been discussed by several investigators. Zener and Peterson (1943), for example, assume that "as long as the speed of the projectile is small compared with the speed of sound in the plate material...the pressure that the projectile must sustain due only to the plate inertia is given approximately by the

* See Zener and Peterson (1943), Thomson (1955).
equation for the force per unit area due to air resistance at velocities considerably below the velocity of sound in air; \( F = K ρ V^2 \)." The authors give some calculated values of \( K \) (a numerical constant depending on the geometrical data of the projectile's nose) for various projectiles. This relationship is also quoted by other authors.

Equating the work done on the target material by the reaction of the inertial force to the change in the kinetic energy of a mass element of the target material yields:

\[
F_1 = \frac{1}{2} pXAV^2
\]  \( (3) \)

For a cylindrical nose, \( K = 1 \); for an ogival nose

\[
K = 1 + \frac{16L^2}{\pi^2D^2} \ln \frac{16L^2}{\pi^2D^2 + 16L^2}
\]

The resisting compressive force is taken to be uniformly distributed over the projectile's nose, and may be written down in the form \( F_c = \sigma A \). A is the area of the projectile's nose as projected onto the target, and the question is asked what the value of \( \sigma \) is that must be substituted in the equation, taking account of the strain rate effects. Manjoine (1944) found that at high strain rates the stress-strain curves for mild steel flatten out above a certain very low strain and that the ratio \( \sigma_f/\sigma_u \) increases with the strain rate until it approaches 1.0 at high
strain rates (10^3 sec^{-1}). Another effect studied by many investigators is the increase in yield stress with increasing rate of deformation. It was found to reach 2-3 times its value in static conditions. It follows that for targets made of mild steel σ = 80,000-100,000 psi is reasonable. Similar investigations carried out with aluminum alloys showed that these are relatively insensitive to strain rate effects (at least up to 10^3 sec^{-1}). It is, therefore, justifiable to substitute σ ≈ σ_{ult}. Generally speaking, for every material its particular sensitivity to strain rate effects must be taken into account.

The resisting shearing force is F_s = τπDb, where b is the width of the punched plug, which may be equal either to zero (no punching, hence this is the ductile type of perforation), or to h_o (plug formation), or it may be smaller than h_o. The ratio b/h_o, of course, depends on many parameters.

The mass of the projectile is assumed to be "not constant." A projectile moving at the instantaneous velocity V, penetrating a resisting medium, accelerates a certain mass element of the penetrated medium. At the same time, the neighbouring mass element remains immobile. The velocity of the projectile is reduced to (V-ΔV), while the mass element acquires the same velocity and begins to move together with the projectile. To the next mass element, therefore, a momentum is imparted by a mass m_o + Δm, a "change" of mass which must be taken into account. Part of the kinetic energy imparted to Δm by the pro-
Jectile is stored in the "composite projectile", the remainder is converted into plastic deformation and heat.

When b is smaller than h (the last of the three possibilities mentioned before) the penetration process may be divided into two stages (see Fig. 1). In the first, only the inertial and compressive resistive forces are acting, and the projectile's mass is enlarged:

$$\frac{d}{dt}(mV) = -\frac{1}{2}KpAV^2 - \sigma A = \frac{dm}{dt}V + \frac{dv}{dt}m \tag{4}$$

where \(\frac{dm}{dt}\) is the rate of increase of the projectile's mass.

Substituting the relations:

\[
m = m_0 + \Delta m = m_0 + \rhoAx
\]

\[
\frac{dm}{dt} = \rhoA \frac{dx}{dt} = \rhoAV
\]

\[
\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}
\]

in Eq. (4) yields:

$$\rhoAV^2 + (m_0 + \rhoAx)\frac{dv}{dx} = -\frac{1}{2} KpAV^2 - \sigma A \tag{5}$$

For 0.22" caliber lead bullets, photography of the bullet while perforating metallic plates showed that the projectile's nose deforms and takes on a cylindrical shape immediately after impact. K may therefore be assumed to be about 1 (see Fig. 2). Solving the differential Eq. (5) yields:
This is the final velocity of the projectile when there is no plug formation. It is also seen that varying any one of the parameters in Eq. (6) changes $V_f$ in a manner predictable by either intuition or experience. When there is plug formation, $h_o - b$ will be substituted instead of $h_o$ in equation (6), yielding $V_b$, the velocity at the end of the first stage of penetration:

$$V_f = \left[\left(v_i^2 + \frac{\alpha}{2D}\left(\frac{m_o/\rho A}{h_o + m_o/\rho A}\right)^3 - \frac{\alpha}{3D}\right)^{1/2}\right]$$ (6)

$$V_b = \left[\left(v_i^2 + \frac{\alpha}{2D}\left(\frac{m_o/\rho A}{h_o - b + m_o/\rho A}\right)^3 - \frac{\alpha}{3D}\right)^{1/2}\right]$$ (7)

In the second stage of the penetration process, only the shearing force needs to be taken into account, there being no "change" in the projectile's mass. The impulse-momentum equation for this stage is:

$$[m_o + \rho A(h_o - b)]V_b - [m_o + \rho Ab + \rho A(h_o - b)]V_f = \pi \rho Db(\Delta t)$$ (8)

where: $m_o + \rho A(h_o - b)$ - the "composite projectile mass" at the end of the first stage

$\rho Ab$ - the mass of the plug

$V_b$ - the velocity of the projectile at the end of the first stage of penetration

Equations (7) and (8) are two equations with two unknowns, namely $V_f$. 

$$V_f = \left[\left(v_i^2 + \frac{\alpha}{2D}\left(\frac{m_o/\rho A}{h_o + m_o/\rho A}\right)^3 - \frac{\alpha}{3D}\right)^{1/2}\right]$$
and $V_b$. In order to find $\Delta t$, the velocity drop during penetration is assumed to be as shown in Fig. 3 (an assumption, the appropriateness of which will be proved later on), viz.:

$$\Delta t = \frac{b}{V_b + V_f}$$  \hspace{1cm} (9)

Substituting $\Delta t$ in Eq. (8) and solving (7) and (8) for $V_f$ produces:

$$V_f = \frac{-AB + \sqrt{(AB)^2 + 4C(EB^2 - F)}}{2C}$$  \hspace{1cm} (10)

where

$$A = \rho Ab$$

$$B = \left[ (V_1^2 + \frac{\sigma}{3D}) \left( \frac{m_0/\rho A}{h_o - b + m_0/\rho A} \right)^{-1} - \frac{\sigma}{3D} \right]^{1/2}$$

$$C = m_o + (\rho Ah_o)$$

$$E = m_o + \rho A(h_o - b)$$

$$F = 2\pi Db^2\tau$$

Substituting $b = 0$ in Eq. (10) leads to Eq. (6). In the case of the penetration of thin plates (1.0 mm. thick) made of any soft material such as, for example, commercially pure aluminum, in which the projectile's nose is not deformed, $K = 0.225$, since $\frac{1}{D} \approx 1$ for 0.22" caliber lead bullets. Equation (6) becomes, in this case,
**Equation (10) contains two empirical values, namely b and d. These values were measured on the perforated specimen and substituted in Eq. (10). Since these values are only empirical, the effects of the various parameters influencing the mechanism of perforation, on the geometrical data of the hole should be more closely studied. However, after performing some ballistic tests in order to determine the character of d and b as a function of the thickness of the target, it is possible to calculate from Eq. (10) the thickness that will stop the projectile. This statement was verified by some ballistic experiments actually carried out.**

Figure 4 shows the curves \( \Delta V = f(h_o) \) for commercially pure aluminum and for aluminum 2024-T3. It will be seen that, for thin plates, these two curves merge. With thick plates the differences in mechanical properties between these two materials becomes apparent. This is in agreement with the well-known fact that on penetration the mechanical properties of the target material bring their influence to bear only after the projectile has penetrated to a certain depth. In the first
stage, the process of penetration is of a hydrodynamic nature and the parameters influencing it being the densities of the target and projectile materials, the impact velocity, the mass of the projectile, and the shape of its head. In that stage the data are the same for both kinds of aluminum (1100-H14 and 2024-T3), hence the velocity drop must be one and the same for all thin aluminum plates. Figure 5 shows the curve $\Delta V = f(h_o)$ for mild steel and, incidentally, demonstrates the good agreement between the experimental results and theory.

One of this author's assumptions was that the velocity drop is as described in Fig. 3. Figure 6, showing the dependence of $V_b$ and $V_f$ on the target thickness, as calculated from Eqs. (7) and (8), justifies this assumption.

OBLIQUE PERFORATION

In order to explain the influence of the angle of impact on the velocity drop, a series of ballistic tests were undertaken. The target materials were two kinds of aluminum, 1100-H14 and 2024-T3, and mild steel.

In an investigation of this nature, two different stages must be considered: The first stage is that of the immediate impact, and the second that of the progress of penetration. In the first stage, a force $F$ arises as a resultant of the compressive (normal to the target) and the frictional forces. This resultant force must not necessarily
and, in fact, rarely does pass through the center of mass of the projectile. If it does not, a moment results which turns the projectile in a direction that will depend on the nature of the target surface. In the second stage, the balance of the forces acting on the projectile is strongly affected by the type of the target's failure. If plug formation occurs before the target has deformed by an appreciable amount, the projectile's angle of obliquity tends to decrease (see Fig. 7). If penetration is of the ductile type, the angle of obliquity initially increases. But once the projectile has penetrated to a certain depth in a ductile manner, the resultant force will act so as to reduce the angle of penetration (see Fig. 8). In the first as well as in the second stage the resultant force acting on the projectile has a shearing component which causes the fragmentation of the projectile's tail. The partition of the momentum in oblique perforation will therefore be as shown in Fig. 9, and this was borne out by the author's tests (see Fig. 10): The projectile continues in its initial direction, the plug (or fragments) moves in a direction normal to the target plane, and the fragments of the projectile's tail move in the direction indicated. Furthermore, enlargement of the hole due to the rotation of the projectile is to be expected.

In the case of oblique perforation, it might be expected that only the thickness \( h' = \frac{h}{\cos \alpha} \) of the medium, or only the normal component of the striking velocity \( (V_1 \cos \alpha) \), need be taken into
account  If such were the case, the curve $\Delta V = f(h_o)$ for normal perforation would merge with the curve $\Delta V = f(h')$ for oblique perforation.

Figure 11 shows the two curves for perforation in commercially pure aluminum 3.0 mm thickness (The curved lines show the trend of the experimental results), and it can be seen that such a merger occurs only for small angles. Therefore, bearing in mind the above discussion of the change in $\alpha$ due to perforation, the curve $\Delta V = f(h'')\left(h'' = \frac{h_o}{\cos(\alpha + \Delta \alpha)}\right)$ would appear to be more suitable. ($\Delta \alpha$ is measured on the perforated specimen). But when this is done - Fig. 11 - it is seen that the curve $\Delta V = f(h'')$ does not merge with the curve $\Delta V = f(h_o)$. This leads to the conclusion that the velocity drop at oblique angles of impact is not only a function of the thickness of the medium penetrated.

From Fig. 12 it can be seen that in the case of a 3.0 mm thick target there is almost no change in the velocity drop dependent on $\alpha$ up to values of $\alpha = 40^\circ$. Figure 8 shows that at this particular angle of impact considerable plastic deformation occurs as well as a considerable increase in the diameter of the hole. It can safely be concluded, therefore, at least for those cases in which the projectile's caliber is of the same order as the target thickness, that it is the plastic deformation of the target material which absorbs the kinetic energy of the projectile and that the resistance to penetration increases with the increase in the size of the hole. Similar results were obtained for other target dimensions and different impact angles, as well as for the case of plug formation.
CONCLUSIONS

1) A theoretical equation was developed which permits the calculation of the projectile velocity after perforating a metallic plate as a function of the mechanical and physical properties and the thickness of the target and of the projectile's geometrical data, mass, and striking velocity. This requires the measurement of two empirical geometrical values which must be substituted in the equation.

2) The mathematical expressions derived in this work enable the residual velocity to be calculated for the cases of petal formation, plug formation, and ductile enlargement of the hole or for any combination of the latter two.

3) The theory suggested takes into account the constitutive properties of the material and also applies to the case of the ordnance impact velocity. In that case the effect of the mechanical properties of the material, such as its strength, cannot be neglected. It is possible to predict the velocity of a given projectile after perforation of a metallic plate by measuring the geometrical data of the hole.

4) In oblique impact the dependence of the velocity drop on the angle of impact is influenced not only by the fact that the medium penetrated is thicker than in normal impact, but also and in fact principally by
the amount of plastic deformation accompanying the process of perforation and by the enlargement of the hole, the latter being due to the projectile's turning during penetration.

ACKNOWLEDGEMENT

I am greatly indebted to Professor S.R. Bodner - who as my M.Sc. Supervisor proposed the subject of this research - for his interest, guidance and valuable advice. I wish to express my appreciation to Mr. Y. Wachtel and Mr. N. Bareket for constructing the test apparatus and for their devoted aid in the experimental work.
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List of Captions

Fig. 1 : Schematic cross section of the penetration hole.

Fig. 2 : 0.22" caliber lead bullets and fragments of target after perforation of commercially pure aluminum plates 1 0-5.0 mm thick.

Fig. 3 : Assumed velocity drop versus penetration depth at the end of each penetration stage.

Fig. 4 : Velocity drop of 0.22" caliber lead bullet perforating aluminum plates versus thickness.

Fig. 5 : Velocity drop of 0.22" caliber lead bullet perforating mild steel versus thickness.

Fig. 6 : Velocity of the projectile perforating aluminum 2024-T3 plates at the end of each penetration stage.

Fig. 7 : Cross section of aluminum 2024-T3 plates 5.0 mm thick after perforation by 0.22" caliber lead bullets at different oblique angles.

Fig. 8 : Cross section of commercially pure aluminum plates 3.0 mm thick after perforation by 0.22" caliber lead bullets at different oblique angles.

Fig. 9 : Momentum partition in oblique perforation.

Fig. 10 : Photographs of 0.22" caliber lead bullet and fragments of commercially pure aluminum plate after perforation at an oblique angle of 31°.
Fig 11: Comparison of velocity drop in normal perforation through commercially pure aluminum plate (10-60 mm) and in oblique perforation through 30 mm plate.

Fig 12: Velocity drop of 0.22" caliber lead bullet perforating commercially pure aluminum plates of different thicknesses versus angle of impact.
FIG 2

NOT REPRODUCIBLE
FIG 4
FIG 5

EXPERIMENTAL RESULTS
THEORY

VELOCITY DROP $\Delta v$ [m/sec]

THICKNESS $h_0$ [mm]
FIG 6
\[ O\Delta V = f(h_0) \]
\[ \bullet \Delta V = f(h') \]
\[ \times \Delta V = f(h'') \]

\[ h' = h_0 / \cos \alpha \]
\[ h'' = h_0 / \cos(\alpha + \Delta \alpha) \]

**FIG 11**
FIG 12

VELOCITY DROP $\Delta v$ [m/sec.]

IMPACT ANGLE $\alpha$

1.0 mm
2.0 mm
3.0 mm
4.0 mm
5.0 mm
6.0 mm

10° 20° 30° 40° 50° 60° 70° 80° 90°
### Abstract

The subject of the penetration and perforation of different materials has been studied for many years. Due to the intricacy of this subject, involving many parameters, the main body of knowledge is based on experimental work. Such theoretical approaches as there are, are simplified by some basic assumptions. Nevertheless, there appears to be room for further investigation of the mechanism of penetration. In the program reported in this paper, a mathematical model was developed which describes the mechanism of the normal penetration of metallic targets. The model considers all the forces acting on the projectile during penetration, bearing in mind that it is deformed during the penetration and that its effective mass increases during penetration due to the concomitant motion of part of the target mass. With the aid of the mathematical expressions the projectile's velocity after perforation can be calculated by substituting the information on the cavity diameter obtained experimentally. Another part of this program consisted of a series of penetration experiments. For the sake of convenience and from practical considerations, the experimental studies were carried out with 0.22" caliber bullets. The experimental results for the velocity drop due to normal perforation of metallic plates were compared with the mathematical model, and excellent agreement was established. In the frame of this research an explanation for the dependence of the velocity drop on the angle of impact was attempted, and ballistic tests were performed with different materials.
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14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical content. The assignment of limits, rules, and weights is optional.