FINAL REPORT

"Fundamental Investigations in Operations Research"

U. S. Army Research Office - Durham
ARO-D Project Number 968-M
Contract DA-31-124-ARO-D-209
M.I.T. DSR 75217

Contract Period: July 1, 1964 - September 30, 1970

Operations Research Center
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Cambridge, Massachusetts 02139

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This final report covers the contract period, July 1, 1964 through September 30, 1970. Professor Philip M. Morse, Director of the Operations Research Center, was Principal Investigator from July 1, 1964 through June 30, 1967. Joining him as Co-Principal Investigator from July 1, 1967 through September 30, 1970 was Professor Alvin W. Drake, Associate Director of the Operations Research Center. Professor Morse retired June 30, 1969, but continues at the Center part time. Professor John D. C. Little was appointed Director July 1, 1969; Professor Drake continues as Associate Director.

STATEMENT OF PROBLEM STUDIED

This project has been concerned with extending the fundamental theories underlying operations research. In particular it has focused on certain areas in mathematical programming, adaptive control, decision theory, and queuing theory. The research in mathematical programming has centered on integer and mixed integer problems. The adaptive control work has dealt with special classes of stochastic decision problems. The multidimensional utility function problem has been studied in decision theory. In queuing theory, which has long been descriptive in its approach, the emphasis has been on normative models, especially Markovian decision processes.

PUBLICATIONS supported entirely or in part by ARO-D during the contract period:

**Journal Articles and Books**


Publications (cont’d)


Publications (cont'd)


Obregon, I., "Linear Programming," (in Spanish), DELTA, 1, No. 4, 3-20 (December, 1969).


In Process


Eisenberg, M., "Two Queues with Changeover Times," to be published in Operations Research.


Publications (cont'd)


Technical Reports and Working Papers


Publications (cont'd)


Publications (cont'd)


Scientific Personnel supported entirely or in part by ARO-D during the contract period:

Prof. Philip M. Morse
July 1, 1964 - August 31, 1964
Sept. 16, 1964 - June 15, 1965
July 11, 1965 - August 31, 1965
Sept. 1-10 & 16-30, 1965 - June 15, 1966
Sept. 16, 1966 - June 15, 1967
Sept. 16, 1968 - June 15, 1970

Prof. E. Farnsworth Bisbee
Oct. 16, 1964 - June 15, 1965

Dr. George M. Murray
Jan. 1, 1965 - July 31, 1965

Prof. Jeremy F. Shapiro
Sept. 16, 1968 - Sept. 30, 1969

Prof. Alvin W. Drake
Feb. 1, 1969 - June 15, 1969

Prof. John D. C. Little
Sept. 16, 1969 - June 15, 1970
Scientific Personnel (cont'd)

Prof. Roy E. Welsch
July 1, 1970 - July 31, 1970

Harold D. Cluck
E.E. E. E. February 1965

E. Gerald Hurst, Jr.
S.M. Ph. D. June 1966

Romulo H. Gonzalez
Sept. 16, 1964 - June 15, 1965
Ph. D. June 1965

Terence E. Daniel
Sept. 16, 1965 - Oct. 31, 1965
S.M. June 1967

Ivan Obregon
Sept. 16, 1965 - Sept. 15, 1966
Ph. D. September 1966

Avinaash Dixit
Sept. 16, 1965 - June 15, 1966
Sept. 16, 1966 - June 15, 1967
Ph. D. June 1968

Hans A. Herriger
S.M. February 1967

Martin Eisenberg
June 16, 1967 - Sept. 15, 1967
Ph. D. September 1967

John D. Steinbruner
Ph. D. February 1968

Donald E. Lewin
(Ph. D. expected Feb. 1971)

Joel Shwimer
(Ph. D. expected Feb. 1972)

Laurence A. Wolsey
Sept. 16, 1968 - June 15, 1969
Ph. D. June 1969

Norman W. G. Wilde
Feb. 1, 1969 - June 15, 1969
Sept. 10, 1969 - June 15, 1970
(Ph. D. expected Feb. 1971)
Scientific Personnel (cont'd)

Thomas K. Zaslavsky

Hubert M. Tavernier

Michael H. Wagner

DEGREES AWARDED during contract period, supported (in part) by ARO-D:

June 1964


June 1965


September 1965


September 1966

Degrees Awarded (cont'd)

February 1967


June 1967


September 1967


February 1968


Degrees Awarded (cont'd)

June 1968


February 1969


June 1969


September 1969


Degrees Awarded (cont'd)

February 1970


June 1970


September 1970


SUMMARY OF RESEARCH FINDINGS NOT PREVIOUSLY REPORTED

Mathematical Programming


A class of resource constrained network scheduling problems is defined in which a set of start times for tasks must be determined which minimize some function of the task completion times. Included in this class is the job shop scheduling problem. Three major ideas for the optimal solution of these problems are presented.

First, the special case in which resources are available and used in unit amounts is formulated as a 0-1 IP problem in which all coefficients are 0 or 1 and all right-hand sides are 1. It is shown that every edge of the
Summary of Research Findings (cont'd)

integer polyhedron for this problem is also an edge of the continuous polyhedron and the implications for applying the simplex method are developed.

In the second major idea, we generalized the 0-1 IP formulation previously developed to include the entire class of scheduling problems which have been defined. The formulation is used to derive a family of lower bounds on the optimal objective value for a particular scheduling problem. A particularly strong bound is incorporated in a tree search algorithm for solving the general scheduling problem.

The third idea is that a scheduling problem may be modified by dividing all task process times by a constant $\lambda > 1$ and rounding down to the nearest integer to obtain a new problem. The optimal objective value of the new problem, multiplied by $\lambda$, is a lower bound on the optimal objective value for the original problem.


The optimization of a stationary staircase problem is studied with an approach that mixes concepts of mathematical programming and optimal control. Some results are first derived about feasibility and controllability. Then optimization is approached by dynamic programming to show that the problem can be reformulated as a decoupled optimization problem. Suggestions are made to take advantage of this fact in designing practical algorithms.


A certain class of integer programming problems, called asymptotic or steady-state, has been shown by Gomory to be cost equivalent to a group-optimization problem. This paper extends the algebraic characterization by demonstrating that there are cost-equivalent group problems for all integer programming problems. Finally, the result is interpreted from the viewpoint of dynamic programming, and this provides the turnpike theorem.
Summary of Research Findings (cont'd)


Group theory is used to integrate a wide variety of integer programming methods into a common computational process. Included are group optimization algorithms, Lagrangian methods, the cutting plane method, and the method of surrogate constraints. These methods are controlled by a supervisor which performs four main functions: set-up, directed search, subproblem analysis, and prognosis.

Some computational experience is given. An appendix contains an algorithm for dynamically solving unconstrained group problems. A second appendix gives an algorithm for solving zero-one group problems.


In this paper, the theory of generalized Lagrange multipliers is combined with a reformulation of the integer programming problem due to group theory. The use of multipliers enhances the algorithmic efficiency of group theory in a variety of ways. One particular application is the approximation of generalized Lagrange multipliers by generalized linear programming as suggested by Brooks and Geoffrion. This procedure is shown to be closely related to the cutting plane method of Gomory.


The usefulness of group theoretic methods in solving integer programming (IP) problems is extended by procedures for controlling the size of the groups. The main procedure shows how an optimal linear programming basis can be altered to reduce the magnitude of its determinant, thereby reducing the size of the group induced by the basis. An adaptation of Bender's mixed IP algorithm is given which uses these methods. Some limited computational experience is given.
Summary of Research Findings (cont'd)


A previous paper contains a branch and bound integer programming (IP) algorithm which exploits a group optimization problem identified by Gomory. This note presents an improved branching rule which (1) eliminates certain computation that is useless, and (2) serves to intensify the search for an optimal solution in an area of promising solutions.


The Wilson, or classical, economic lot size model is generalized to multi-stage assembly systems in which lot sizes need to be determined for constituent parts and subassemblies as well as for final products. Under the usual assumption of constant demand over an infinite horizon, a characterization of the form of the optimal solution is obtained. Dynamic programming is then used to find optimal solutions, and comparisons are made with results obtained with various heuristics.


Many linear, quadratic, or convex programming problems should be viewed in a dynamic environment. This is particularly true for planning problems. The present research is designed to apply some of the concepts of a system theory and optimal control to the programming of discrete time, state constrained, dynamic systems of the staircase type.


This study analyzes the numerical properties of dynamic programming algorithms, emphasizing the effects of approximation procedures on the accuracy of the result. Error propagation formulas are developed and sufficient conditions are provided for the convergence of approximate dynamic programs to the true solution. A "state reduction" dynamic programming algorithm for the deterministic case is proposed which offers the possibility of perfect convergence over continuous state spaces using only a finite amount of computer memory.
Summary of Research Findings (cont'd)


Lot size determination for multi-stage assembly systems is considered for both a finite horizon model with varying, but known, demands, and for infinite horizon models with constant demands. Characterizations of the form of optimal solutions are developed under the assumption of concave, time-invariant, production and inventory cost functions. Heuristics and a dynamic programming algorithm for the infinite horizon cases are described and computational results reported. Solution of the finite-horizon case by dynamic programming and by Bender's method for mixed integer programming is discussed.

Adaptive Control


A manager tries to put together the various resources under his control into an activity that achieves his objectives. A model of his operation can assist him but probably will not unless it meets certain requirements. A model that is to be used by a manager should be simple, robust, easy to control, adaptive, as complete as possible, and easy to communicate with. By simple is meant easy to understand; by robust, hard to get absurd answers from; by easy to control, that the user knows what input data would be required to produce desired output answers; adaptive means that the model can be adjusted as new information is acquired; completeness implies that important phenomena will be included even if they require judgmental estimates of their effect; and, finally, easy to communicate with means that the manager can quickly and easily change inputs and obtain and understand the outputs.

Such a model consists of a set of numerical procedures for processing data and judgments to assist managerial decision making and so will be called a decision calculus. An example is described.

Markov Models


A model is developed for the flow of automobiles in one direction along a two-lane country road. The model takes into account the fact that cars differ in speed, that slower cars accumulate queues behind them, and that the rate of
Summary of Research Findings (cont'd)

escape from such a queue by passing the lead car depends on the speed of the lead car, on the nature of the road, and on the density of traffic going in the opposite direction. Equations for the stochastic steady state of the system reduce to an integral equation for the mean queue length, as a function of lead-car speed and of a queue-delay parameter. Solutions are obtained, with tables and graphs, for two different assumptions regarding the dependence of passing delay on lead-car speed. In both cases the model exhibits a sudden change from sparse-traffic conditions (where queuing is rare and delay of the faster cars is minimal) to the dense-traffic conditions (where nearly all cars find themselves trapped in a slow queue as the passing-delay parameter is increased beyond its transitional value). Such a sudden "phase change" in traffic character is typical of actual traffic under the specified conditions. Measured dependence of queue length on lead-car speed checks nicely with the model, for sparse-traffic conditions.

Queuing Theory and Markov Models


A mathematical analysis is given of a round-robin computer time sharing system that is characterized by homogeneous Poisson arrivals, exponentially distributed service times and an ordered priority queue. Each new arrival buys a position in this queue by offering a non-negative payment to the manager of the computer facility. The system is modeled as a continuous time stochastic process and analyzed by an imbedded Markov chain. Relevant queuing statistics are identified and derived in terms of system parameters. An optimization problem in revenue maximization is formulated and solved with two system parameters as decision variables.


The basic concepts of Markov decision processes are employed to measure and compare the effectiveness of a queuing system consisting of a facility that accepts units of two different types, each characterized by a different pair of probability density functions for the arrival and service times. Five alternative schemes of servicing units that belong to two different classes have been analyzed.
Summary of Research Findings (cont'd)

Order Statistics


This paper characterizes the possible limit laws for a sequence of normalized extreme order statistics (maximum, second maximum, etc.) from a stationary strong-mixing sequence of random variables. It extends the work of Loynes who considered only the maximum process. The maximum process leads to limit laws that are the same three types that occur when the underlying process is a sequence of independent random variables. The results presented here show that the possible limit laws for the k-th maximum process \((k > 1)\) from a strong-mixing sequence form a larger class than can occur in the independent case.

ANNUAL REPORT

The Center publishes an Annual Report which presents an overview of all its research activities, educational activities, publications, seminars, symposia, special programs, and funding sources. Distribution is made to all those on the distribution lists of contract and grant agencies that support the Center, to other centers, and to interested individuals.

By indicating how the staff and students interact and contribute to each other's work, the Report gives a unified picture of the Center's activities to every contractor and shows how each contractor contributes to the development of operations research as a whole, not just to the support of particular staff personnel and graduate assistants.

The reports relevant to the reporting period covered herein are:

This project has been concerned with extending the fundamental theories underlying operations research. In particular it has focused on certain areas in mathematical programming, adaptive control, decision theory, and queuing theory. The research in mathematical programming has centered on integer and mixed integer problems. The adaptive control work has dealt with special classes of stochastic decision problems. The multidimensional utility function problem has been studied in decision theory. In queuing theory, which has long been descriptive in its approach, the emphasis has been on normative models, especially Markovian decision processes.