THE SIMPLE THEORY OF PUBLIC LIBRARY SERVICES

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In this note a simple normative theory applicable to public library services is presented. The theory was developed as a tool to aid libraries in answering the question: which books should be bought by the library? Although developed for normative purposes, the theory generates testable predictions. Further, the theory is applicable to a number of public services such as neighborhood health centers where the individual has the option of buying a similar service on the market. In particular, the theory is relevant to measuring benefits from services which are provided publicly because when "uncrowded" they have zero marginal cost (library books, swimming pools) or because the market may be monopolistic (library books, health services). The theory will be developed in the context of the library.

If the aggregate demand curve for library books could be measured, it would be a relatively simple task to determine which books the library should buy. Ignoring distributional considerations, it should buy those books whose appropriately discounted benefit stream over the
life of the book is greatest, where benefit in any period can be approximately measured by the area under the income-compensated demand curve. That is, suppose \( q_L \) is the quantity of a particular library book demanded in a given period. It is a function of the price of using the library for this book \( p_L \), the price of the book \( p_B \), and \( x \), a vector of other variables,

\[
q_L = f(p_L, p_B, x).
\]

There are certain complications such as possible non-price rationing of the library book, but ignoring those complications we could, if we knew the demand curve, measure benefit in the usual fashion, as

\[
\bar{q}_L \int_0^{\bar{p}_L} dq_L,
\]

where \( \bar{q}_L \) is the quantity demanded at the existing price \( \bar{p}_L \).

Unfortunately such a demand curve is very hard to measure because preferences are never revealed in the marketplace. This, of course, is the usual problem of publicly supplied goods. The point of this paper is that the benefit from publicly provided goods which are close substitutes for goods that can be purchased on the market can be approximately measured by using the demand curve for the private good, a demand curve which not only is measurable but frequently is not difficult to measure. Further, in the case of the library it will turn out that the theory can be implemented without actually measuring any demand curve.
Suppose we write the demand curve to purchase a book as:

\[ q_B = g(p_L, p_B, y) \]

where:
- \( q_B \) = quantity of book demanded
- \( p_L \) = price of using book at library
- \( p_B \) = price of book
- \( y \) = vector of other variables.

Two such demand curves are shown in Fig. 1. Of is the price of the book.

The outer demand curve CD is the demand curve for the book if the library does not add a copy of the book; AB is the demand curve if it does. Thus, if the library buys an additional copy, the availability of the book at the library will increase, \( p_L \) will fall, and AC fewer
books will be bought. Call those who would have bought the book had
the library not purchased it Type A borrowers. Type B borrowers are
those excluded by the price of the book. They are willing to borrow
the book from the library because they will pay more than \( p_L \) to read
the book, but they are unwilling to buy the book. The most Type A
borrowers will pay to borrow the book is the price of the book;
Type B borrowers will pay at most something less. Hence, an upper
bound on the benefit that could be provided the community by having
the book available at the library is the shaded area ACDBE. Note
that, cet. par., the shaded area is larger: 1) the more elastic the
demand curve below \( C \); 2) the greater the price of the book; 3) the
greater the shift of the demand curve when the library buys a copy.
The area is not increased by a simple shift outward in both demand
curves.

Before setting out a decision rule for libraries, we must take
account of some qualifications. First, trips to the library cost
something in time and money and individuals have different values for
time. Likewise, the marginal cost of obtaining a second book from
the library is small if one is making a trip for another book. Neither
of these considerations should much affect the theory, however. So

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*Edmund Brunner has pointed out to me that a Type C borrower may
exist: one who will read a book in the library and then buy it. This
reader is receiving a service whose value we are not measuring, but it
should not much affect the theory. See below.

**It would be theoretically better to compare the library with a
more similar service, the book rental market. However, data from book
rentals are not as easily available to the library as data on book
prices. Data from rental libraries will be useful to our framework,
however. See below.
long as the tastes of individuals in books are not correlated with
the cost to them of using the library or with propensities to check
out more than one book, the effect of such transaction costs is like
a lump-sum tax on the total benefit the library is providing the
community.* It should not affect the relative desirability of various
books and periodicals.** We ignore costs of using the library in what
follows.

Second, not all those who demand the book can use it at the same
time. Some users' reading must be deferred. Hence a discount factor
should be applied to the benefits supplied to those individuals whose
reading is deferred. There is more to the problem than that, however.
Time will also affect what an individual will pay to have the services
of the book. Some books have a fad value; the value one places on
reading last year's best seller may be much lower this year. Refer-
ence works are different; even if one has already used a reference
book in the past, it may continue to be useful in the future. In short,
although some books depreciate very rapidly, others do so much more
slowly. Thus, an individual's demand for the services of a book in
future time periods will depend upon: a) whether or not he has read
the book; b) what type of book it is.

One of the services which the library provides its users is in-
formation about the book market, since the library provides a convenient
means of finding out which books are available, what a particular

*The amount of the tax might be reduced by a different location policy.
**If those with a high value for time like, say, art books, and
those with a low value for time like, say, mysteries and westerns,
benefit would be higher, cet. par., for mysteries and westerns.
author's style is like, etc. This service -- which we do not value -- increases the benefit which the library is providing the community. Still, the theory of book selection should not be much affected unless some classes of books provide more information about the book market than others.

On the assumption that the borrower of the book gets some fraction $b_i$ of the benefits of ownership, we can approximately measure the benefit which the $i$th library book provides the community in time period $t$ as:

$$B_{it} = a_{it} b_i p_{it} q_{it}$$

where $a_{it}$ is a fraction between 1/2 and one which reflects the proportion of Type A borrowers to Type B borrowers ($a_{it}$ is one if all borrowers are Type A borrowers, 1/2 if all borrowers are Type B borrowers), $b_i$ is as defined above, $p_{it}$ is the price of the book, and $q_{it}$ is circulation of the book. If $a_{it}$ and $b_i$ were constant across books, the library's decision rule is particularly simple: maximize discounted weighted circulation (using price weights) for any given book budget subject to distributional constraints. (The distributional constraints exist because of the following problem: If circulation does not fall as holdings of one type of book increase, the entire book budget might be spent upon one type of book (e.g., mysteries);

*This is the product the Type C borrower, who buys the book after he has read it, receives.

**Since a larger collection of books may well provide more information than a smaller collection, the criterion developed below (which ignores this point) undervalues cheaper books. In the real world the bias seems sufficiently small to ignore, however.
in the absence of compensation, those who didn't read that type of book would be worse off. Hence, some constraints to insure that the library spends a certain amount in each area do not appear unreasonable.)

Unfortunately, \( a_{it} \) and \( b_i \) are likely to differ among books, which means estimates of their values are necessary to implement this decision rule. Consider first \( a_{it} \). For very high priced reference-type books (e.g., *Dictionary of New Hampshire Butterflies*) relatively few readers are likely to purchase the book even if the library does not; consequently, \( a_{it} \) is likely to be near \( 1/2 \). For a popular novel which is available in paperback \( a_{it} \) may be near one. Likewise, \( b_i \) will probably be smaller for the expensive reference book. For the popular novel, the owner is likely to read it once, put it down, and seldom, if ever, pick it up again. The additional benefits he receives relative to the borrower would consist of little more than pride of ownership. An expensive reference-type book, however, may be consulted several times; hence, if there are transactions costs in using the library, the book which is used several times is likely to provide a smaller fraction of the purchase price in benefits in any one usage than will books which will only be used once. (That is, if the book is expected to be used several times and transactions costs are significant, the reader will buy. Hence, those who don't buy a book they use several times are likely to place a lower value relative to the purchase price for each usage than those who borrow a book only once.)

Our measure also assumes that Type B borrowers would pay on the average \( 1/2 \ b_i \ p_i \) to borrow the book. This does not seem unreasonable.
We also assume order of borrowing is random. However, those to whom the book is worth more may have a stronger incentive to borrow the book from the library. Hence, initial borrowers would be disproportionately concentrated in the left part of the figure. If so, because the benefits of later borrowers are more heavily discounted, this would mean that the above benefit measure is slightly downward biased.

To make this criterion operational, we redefine \( i \) to be an index of book types and then have the library choose \( B_i \) to maximize

\[
\sum_{t} \sum_{i} a_{it} b_i p_{it} q_{it} / (1 + r)^t
\]

subject to

\[
q_{it} = c_{it} B_i, \sum_{i} p_i B_i \leq B, B_i \geq 0, Q \geq 0,
\]

where all symbols are as defined above, \( r \) is the discount rate, \( B_i \) are the number of books of type \( i \) bought with the current book budget, \( c_{it} \) is the fraction of books of type \( i \) which circulate in time period \( t \), \( B \) is the current book budget, and \( Q \) is the vector of minimum additions to holdings which is based upon distributional considerations. Note that this formulation assumes additional books of type \( i \) will circulate at the same frequency as existing books of type \( i \).

The parameters \( a_{it} \) and \( b_i \) are to be estimated. The variable \( q_{it} \) can be estimated from circulation data for classes of books of

*Data from rental libraries might be helpful in establishing the magnitude of the b's.
type i. (We assume that within a class the library can predict relatively well the books which are most in demand; its problem occurs in deciding how much of its budget to allocate to each type of book.)

The variable $p_{\text{it}}$ presents some difficulty. When the book is first released, the measure is straightforward; problems occur if a paperback edition is released or if the book goes out of print. Since buying a paperback probably provides more services than borrowing a hardback edition from the library, it seems reasonable to treat the $p_{\text{it}}$ variable as the price of the paperback, if it is issued. (In this case, an upward adjustment in the parameter $b$ would seem warranted.) A more difficult problem is caused if the book is out of print, because the subscriber does not have the option of purchasing the book on the market. Hence, the subscriber could be willing to pay much more than the old market price for the privilege of borrowing the book. If the library does not own an out-of-print book, the potential reader's alternatives are inter-library loan, rental libraries, other libraries, or conceivably advertising for a copy of the book. We have arbitrarily assumed the cost of doing so is again at most the price of the book. The error introduced by this approximation will depend upon the proportion of circulation attributable to out of print books.

The effect of this decision rule is that the library will tend to buy books which are expensive and which circulate frequently (circulation should be a function of price elasticity). To be precise about "expensive," in general as price rises, the size of the area CAEBD will increase rapidly. For linear demand curves, ceteris paribus the increase will always be greater than the increase in $p$ up to the
point where the demand curve intersects the price axis (the demand curve would have to be reasonably concave for this not to be true); thus, \textit{cet. par.}, net benefit will be greater for higher period books. In practical terms, this means the library should not necessarily buy very popular books since an equal shift in both demand curve will not increase the area CAEBD (it should, however, buy books which circulate frequently); also it should tend not to buy books which will later be published in paperback (when the paperback edition comes out, the size of the area will diminish).

Some other prescriptions that may be drawn from this model are:

1) Ignoring distributional considerations, the library can increase welfare by charging a fee to reserve a book. By charging such a fee, the library will increase the probability that those who place the greatest value on the book will read it first; thus, the larger discount factors will be applied to the lesser benefits.\footnote{Since the value of time differs among people, those who place the greatest value on the book will not necessarily read it first.} Also, the possibility of reservation by paying a fee reduces the probability of socially inefficient trips or calls to the library to obtain or inquire about a book that is not in; 2) the library should not simply accumulate books, but should sell off some older books whose circulation has dropped off; by using the proceeds from the sale to buy new books, it may be able to increase the community's welfare;\footnote{Likewise it may be able to profitably buy second-hand books.} 3) except for the above mentioned reserve charges, prices to use the library should be low or zero; like a collective good with a zero marginal
cost, a positive price which excludes individuals creates a deadweight loss; insofar as there are positive marginal administrative costs in issuing cards and loaning books, however, a fee is justified; 4) fines for not returning books on time should reflect demand for the book; in particular, theoretically no fine should be imposed for a book which is not demanded. Two qualifications of this rule are in order: 1) it is difficult to know that a book is not demanded; thus some small fine for lateness may be justified by the presumption that any book on the shelf might be demanded; 2) keeping a book longer may raise the probability of loss. Fines should, however, be higher on books which are known to be demanded.

Before setting out positive implications of the theory, we should take note of a probable relationship between price, quantity, and demand elasticity.

Publishers of a book have a legal monopoly to publish the book; therefore, pricing of books becomes a straightforward exercise in the theory of monopoly pricing. For simple monopoly, that theory, as is well known, predicts that \( \hat{p} \), the profit maximizing price, will equal \( \frac{MC}{1 - 1/e} \), where \( MC \) is marginal cost and \( e \) is the price elasticity of demand. For constant marginal cost (probably a reasonable approximation) \( d\hat{p}/de \) equals minus \( MC \left( e^{-2}\right)/(1 - e^{-1})^2 \), which is negative. Hence, the lower the price elasticity of demand, the higher \( \hat{p} \). Thus,

*It is interesting to note that this may be one reason for the existence of a library; that is, its existence helps reduce the deadweight loss from a price above marginal cost. Our measure does not take account of differential amounts of deadweight loss which might be elimin.
two of the factors in a library's decision to purchase a book -- price and elasticity -- can often be predicted to work at cross-purposes.

There is also the possibility that the book will be brought out in paperback. The possibility of paperback publication means that the theory of discriminating monopoly rather than the theory of simple monopoly is applicable. In this case, cross-elasticities between paperback and hardback editions becomes important in pricing. However, it can be shown that as price responsiveness (3 quantity demanded/
2 own price) increases, cet. par., the profit-maximizing price for that product will usually fall.* (The profit-maximizing price for the other product, if price responsiveness for the first product increases, may rise or fall.) Hence, a more realistic picture of the market than simple monopoly does not change the conclusion of the previous paragraph that price and elasticity will usually be negatively related.

These theories generate testable predictions. First, it would be interesting to know whether publishers do price as monopolists. Some survey data on demand functions for books may be available. If so, the hypothesis that as elasticity rises, price falls can be tested. Also, decision rules for bringing out a paperback can be explored, so that a library can predict which books are likely to appear in paperback.

Second, the theory predicts that an individual's demand for a library book relative to purchasing the book will be higher, cet. par.,

* It will fall provided \( \frac{\partial f}{\partial p_L} \cdot \frac{\partial g}{\partial p_N} - \frac{\partial f}{\partial p_N} \cdot \frac{\partial g}{\partial p_L} > 0 \),

where \( f \) is the demand function for product \( L \), \( g \) is the demand function for product \( N \), \( p_L \) and \( p_N \) are the price of \( L \) and \( N \) respectively. Otherwise, it will rise. In other words, it will fall if the product of the partials with respect to own price exceeds the product of the partials with respect to cross price.
the higher the price of the book. Hence, aggregate demand for a library book will be higher, *ceteris paribus*, the higher the price of the book. This could be tested by forming homogeneous classes of books (such as novels that have not appeared in paperback) and comparing the ratios of library circulation to book sales for various priced books. It is important to note, however, that if higher price is associated with lower price elasticity of demand, this hypothesis may be rejected. Third, the theory predicts that demand for a library book will fall when a paperback edition is issued. Although the *ceteris paribus* conditions are again difficult to fulfill, this could be tested by computing the ratio of circulation after issuance of paperback to circulation before issuance for a number of books of one type (e.g., novels). One needs a reference group; this might be derived from a sample of books of the particular type which have not been issued as paperbacks; summing their circulation in each period (e.g., month) after acquisition and dividing by the number of books in the sample yields a time profile of circulation for books which did not appear in paperback. This could be used to derive a ratio to compare with the previously defined ratio for books which appear in paperback.

In this note we have presented both the normative and positive aspects of a theory of book purchase for libraries. Although developed in the context of library services, the theory is formally applicable to a number of publicly-provided services which are substitutes or near-substitutes for privately marketed goods. In effect, it uses part of the area under the demand curve for the private good.
to approximate the area under the demand curve for the public good.

Although formally applicable, the theory is not always helpful. Because the non-excludability condition does not hold (otherwise the good would not be produced on the market), user charges may be imposed. Such charges can be manipulated to estimate a demand curve for the public service. For a service such as a swimming pool this appears to be a more promising kind of analysis, since the price of a swimming pool vastly exceeds what most of its users would pay. (They are almost all Type B users.) Thus, the shape of the demand curve in the region of low prices is important, but the private market provides little relevant information. A somewhat similar type of analysis has, however, been applied to hospital outpatient operations with some success.*