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The Theory Of The Motion Of A Bullet About Its Center Of Gravity In Dense Media, With Applications To Bullet Design

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A B E R D E E N   P R O V I N G   G R O U N D ,   M A R Y L A N D

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Aberdeen Proving Ground, Md.
January 14, 1930

THE THEORY OF THE MOTION OF A BULLET ABOUT ITS CENTER OF GRAVITY IN DENSE MEDIA, WITH APPLICATIONS TO BULLET DESIGN

ABSTRACT

An investigation is made of the motion of the bullet about its center of gravity in a dense medium. It is found that approximately the yaw of the bullet in such a medium is an exponential function of the time, that is to say, that it increases according to e^{kt}. It is shown that the value of the quantity k is much more significant for the motion than even the size of the yaw upon impact. Methods of determining k are given. It is shown that the size of the yaw in the medium is approximately independent of the striking velocity and therefore for a given bullet, the amount of energy absorbed will be proportional to the square of the striking velocity. It is shown that the twist of rifling has no appreciable effect upon the size of the yaw in a dense medium except in so far as it affects the size of the yaw on impact. It is pointed out that a large value of k may be obtained by the use of bullets having light noses and it is indicated that for a given muzzle energy there will be greater energy absorbed from light bullets than from heavy bullets. The theory is applied to the effect of the caliber on the amount of energy absorbed in the medium. It is discovered that at short ranges, the amount of energy absorbed will tend to increase as the caliber is reduced. Certain other aspects of the influence of caliber upon the general effectiveness of infantry weapons are also discussed.

Experiments have shown that there is a fair degree of correlation between the effects of bullets fired into animal tissues and into water cans, in other words, that the bullets which have been found to produce very destructive effects on water cans have also been found to produce destructive effects on animal tissues. It is therefore believed that the theory of the motion of the bullet in a dense medium may throw considerable light on the matter of the wounding effects of bullets on animal tissues.

Consider a projectile fired in air and allowed to impinge on the boundary of a dense medium such as water.

We proceed to discuss the motion of the projectile when it has entered the dense medium. A second order differential equation for the motion of the projectile may be deduced by Lagrange's method as follows:

(1) $B\ddot{\delta} - B\cos\delta\sin\delta\,\dot{\phi}^2 + AN\dot{\phi}\sin\delta = \mu\sin\delta.$*

In this equation, B is the transverse moment of inertia, A is the axial moment of inertia, N is the spin in radians per second, δ is the angle of yaw, ϕ is the angle of orientation of the yaw, that is to say, the angle between a plane including the trajectory and the axis of the projectile and a standard reference plane through the trajectory, and μ is the couple factor and is defined by the relation

$$\text{Overturning Couple} = \mu\sin\delta.^{**}$$

It is assumed that μ is constant for angles of yaw less than 20°.

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*The kinetic energy is $\frac{1}{2}\left[AN^2 + B(\dot{\delta}^2 + \dot{\phi}^2\sin^2\delta)\right]$ and the potential energy is $\mu\cos\delta$. (See Jeans "Theoretical Mechanics," p. 311). (See also pp. 329-332 for Lagrange's equation.)

**For a projectile of given shape and caliber fired at a given velocity, μ is proportional to the distance from the center of pressure to the center of gravity. The nearer the center of gravity is to the base, the greater will be the value of μ.

The dots are used to indicate the differentiation with respect to time, that is to say,

$$\dot{\phi} = \frac{d\phi}{dt}$$

and

$$\ddot{\delta} = \frac{d^2\delta}{dt^2} \quad .$$

It is desired to reduce this equation to a simple form by means of which the increase in δ, the angle of yaw, may be readily calculated.

From (1), we obtain

$$(2) \quad \ddot{\delta} = \frac{\mu}{B} \sin \delta \left(1 + \frac{B}{\mu} \cos \delta \, \dot{\phi}^2 - \frac{AN\dot{\phi}}{\mu}\right).$$

From the condition that the component of the moment of momentum of the projectile about the trajectory is constant, we obtain the relation

$$(3) \; B \sin^2\delta \, \dot{\phi} + AN \cos \delta = \text{Const.}^*$$

By calculating the value of ϕ upon impact, it may be shown that for the motion in the medium, the constant on the right hand side of equation (3) is AN to a close approximation and we find for $\dot{\phi}$

$$\dot{\phi} = \frac{AN}{B} \frac{(1 - \cos\delta)}{\sin^2\delta} = \frac{AN}{B(1 + \cos\delta)} \quad .$$

By virtue of this, (2) may be changed to

$$\ddot{\delta} = \frac{\mu}{B}\sin \delta \left(1 - \frac{4s}{(1 + \cos\delta)^2}\right),$$

where s is used to represent $\frac{A^2N^2}{4B\mu}$.

If δ is small, we have

$$(4) \quad \ddot{\delta} = \frac{\mu}{B} \delta(1 - s).$$

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* See Crabtree "The Elementary Treatment of the Theory of the Spinning Top and Gyroscopic Motion."

"s" is known as the stability factor of the projectile. In air, near the muzzle, its value is 2 for the Cal. .30 M1 bullet, but in water, near the muzzle, its value will be only 1/400.* Thus, so far as our computations are concerned, it may be neglected.

Before proceeding with the solution of this equation, we shall consider the consequences of the results just obtained.

It may be seen from equation (4) that unless the stability factor is as high as 0.1, the stability factor and hence the spin may be considered to have a practically negligible influence upon the motion of the bullet in the medium.

For the M1 bullet in water to have a stability factor as great as .1, would require that the linear velocity be reduced to value of about 1/7 of the muzzle velocity or 400 f/s, the spin being assumed constant. Such a low velocity is not attained until the projectile has reached a range of about 4000 yards, and thus for all ranges of practical importance, it may be seen that the twist of rifling will not have any appreciable effect on the motion of the bullet in a medium like water except in so far as the initial yaw of the bullet is dependent upon the stability factor in air.

We proceed to the solution of equation (4) on page (6). If s is neglected, we obtain

$$(5) \quad \ddot{\delta} = \frac{\mu}{B}\,\delta \,.^{**}$$

The general solution of the equation is therefore seen to be

$$\delta = C\,e^{\sqrt{\frac{\mu}{B}}\,t} + D\,e^{-\sqrt{\frac{\mu}{B}}\,t} \quad \text{(assuming } \mu \text{ constant)},$$

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* There is a certain justification for this since Bauer ("Anne̲len der Physik," Volume 80, 1928, page 232) has shown that the ratio of the resistance of the sphere in water to that of a sphere in air for a given velocity is approximately proportional to the ratio of the densities.

** In the "Abstract" k is used to denote $\sqrt{\mu/B}$.

as may be proved by differentiating δ and inserting in (5).

We will assume that at the instant the bullet has penetrated the medium, the value of the yaw is δ_0, or that

$$\delta_{t=0} = \delta_0,$$

and that the initial value of the angular velocity of yaw $\dot{\delta}$ is $\dot{\delta}_0$, or

$$\dot{\delta}_{t=0} = \dot{\delta}_0.$$

From these conditions, it may easily be shown that

$$C = \frac{1}{2} \left(\delta_0 + \frac{\dot{\delta}_0}{\sqrt{\frac{\mu}{B}}}\right),$$

and

$$D = \frac{1}{2} \left(\delta_0 - \frac{\dot{\delta}_0}{\sqrt{\frac{\mu}{B}}}\right).$$

We have then as the solution corresponding to initial conditions:

$$\delta = \frac{1}{2} \left(\delta_0 + \frac{\dot{\delta}_0}{\sqrt{\frac{\mu}{B}}}\right) e^{\sqrt{\frac{\mu}{B}}\, t} + \frac{1}{2} \left(\delta_0 - \frac{\dot{\delta}_0}{\sqrt{\frac{\mu}{B}}}\right) e^{-\sqrt{\frac{\mu}{B}}\, t} .$$

Calculations on the motion of a bullet which is projected under ordinary conditions indicate that $\dfrac{\dot{\delta}_0}{2\sqrt{\frac{\mu}{B}}}$ is negligible compared with $\delta_0/2$.*

It is also evident that the terms with the negative exponent of e will be very soon damped out. Thus for most practical purposes, we may

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* In the theory no account has been taken of the motion while entering the medium. The entrance lasts such a short time that it probably does not appreciably affect the value of δ. The experiments suggested later should be capable of giving information on this point.

write the solution of the motion as follows:

$$(6) \quad \delta = \frac{1}{2} \; \delta_0 \, e^{kt},$$

where k is used instead of $\sqrt{\mu/B}$.

We shall apply this result to the penetration of the M1 bullet. The characteristics of this are as follows:

$$A = 1.7 \times 10^{-6} \text{ lbs. ft.}^2,$$

and $$B = 16.7 \times 10^{-6} \text{ lbs. ft.}^2.$$

When this bullet is fired in a rifle with a twist of 14" and a velocity of 2700 f/s, its stability factor is found to be about 1. Now

$$s = \frac{A^2 N^2}{4 \, B\mu} \, ,$$

and from this, we find that for air

$$\mu = 9.15 \text{ lbs. ft.}^2/\text{sec.}^2.$$

The density of water is approximately 800 times that of air, and thus the value of μ for water is

$$\mu = 7 \times 10^3 \text{ lbs. ft.}^2/\text{sec.}^2.$$

Thus, we finally find

$$\sqrt{\mu/B} = \sqrt{\frac{7 \times 10^3}{17 \times 10^6}} = \sqrt{4 \times 10^8} \approx 2 \times 10^4.$$

Thus, if our assumptions are correct, the motion of the M1 bullet in water is given by the formula

$$\delta = \frac{1}{2} \quad \delta_0 \, e^{20,000t}.$$

Let us also consider the motion of two other fictitious bullets, one of these bullets, which we shall designate as the heavy bullet, is so long that the value of μ/B is 1/2 as great as it is for the M1 bullet and the other bullet, we shall suppose to have an aluminum nose, which will tend to reduce its moment of inertia and increase the couple coefficient to such an extent that μ/B for this will be twice as great as it is for the M1 Bullet.

With these assumptions, we shall have for the motion of the heavy bullet

$$\delta = (\delta_0/2)\ e^{1.4 \times 10^4 t},$$

and for the yaw of the light bullet, we will have

$$\delta = (\delta_0/2)\ e^{2.8 \times 10^4 t}.$$

In the following tables are given yaws for the three bullets as calculated, first on the assumption that the initial yaw is 1° and second on the assumption that the initial yaw is $1/4^\circ$.

TABLE OF COMPUTED YAWS (1° Initial Yaw).

Penetration Distance in.	Time Sec.	M1 Bullet Yaw in Deg.	Heavy Bullet Yaw in Deg.	Light Bullet Yaw in Deg.
0.0	.00000	1	1.	1.
1.6	.00005	1.5	1.25	2.1
3.2	.00010	3.7	2.1	8.
4.8	.00015	10.	4.1	33.
6.4	.00020	27.	8.	-
8.0	.00025	-	17.	-
9.6	.00030	-	33.	-

TABLE OF COMPUTED YAWS ($1/4^\circ$ Initial Yaw).

Penetration Distance in.	Time Sec.	M1 Bullet Yaw in Deg.	Heavy Bullet Yaw in Deg.	Light Bullet Yaw in Deg.
0.0	.00000	.25	.25	.25
1.6	.00005	.4	.30	.5
3.2	.00010	.9	.5	2.0
4.8	.00015	2.5	1.0	8.0
6.4	.00020	7.0	2.0	32.0
8.0	.00025	18	4.	-
9.6	.00030	50*	8.	-

* The values calculated from the formula are given although it is rigorously applicable only to yaws less than 20°.

It is evident from these tables how great an effect the value of μ/B has on the size of the yaw. For example, the light bullet with an initial yaw of $1/4^{\circ}$ when it has travelled 6.4" in the medium has a yaw which is very much greater than that of the heavy bullet with an initial yaw four times as great when it has travelled 8".

It follows that, although the initial angle of yaw, δ_0, is of great importance in connection with the yaws of the bullet in the medium, yet the most important factor is the value of μ/B.

Now for small angles of yaw, the resistance offered to the motion of the projectile increases as the square of the yaw, in such a manner that approximately a yaw of 13° doubles the resistance of the projectile. It follows from this that the light bullet with the aluminum nose, by virtue of its large angle of yaw, will transmit a great deal more energy to the medium than will the heavier bullets.

In view of the overwhelming importance of the quantities μ and B as affecting motion of the projectile in a dense medium, it would seem desirable that measurements of these quantities be obtained. The facilities for obtaining measurements of B are already available at the Proving Ground, and there remains the problem of developing the method for measuring the value of μ for motion in water. The theory which has been developed in the foregoing suggests an easy method of obtaining approximate values of this quantity.

Let a bullet be fired in a gun having such a twist that reasonably large initial angles of yaw are produced, and let this yaw be measured immediately before the bullet enters the water by means of a paper screen. Let the front side of the water tank consist of waxed paper so as not to interfere seriously with the motion of the bullet, and let the backside of the tank be made of tin of such thickness that a fairly good imprint of the projectile is obtained from which the angle of yaw of the bullet upon emergence may be determined. Let the initial angle of yaw be δ_0, the angle of yaw upon emergence be δ_1, and the time required for the passage through the water tank be t_1. Then we have from the foregoing theory that

$$\delta_1 = (\delta_0/2)\, e^{\sqrt{\frac{\mu}{B}}\; t_1} \quad .$$

From this relation, it follows that $\sqrt{\mu/B}\; t_1 = \log_e (2\, \delta_1/\delta_0)$, where $\log_e$ is used to denote the logarithm to the base e. From the thickness of the tank and the velocity of the bullet upon impact, the value of t_1 may be approximately determined; and B having already been measured, it will be easily possible to determine an approximate value of μ.*

Let us next consider the resistance of the bullet as dependent upon shape in various media. At a velocity of 2700 f/s, a bullet when moving in air has a velocity greater than that of sound, and experiment has shown that at velocities less than that of sound, a blunt-nosed projectile has less resistance than a sharp-pointed projectile. Now, the velocity of sound in water is about 4800 f/s and thus it appears that the bullet we have considered having a muzzle velocity of 2700 f/s when immersed in water will have a velocity less than that of sound. One would expect under these considerations that so far as the resistance in water is concerned, a blunt-nosed projectile would have a smaller resistance than a sharp-pointed bullet. This reduced resistance of the blunt-nosed bullet is probably one of the reasons why a long blunt-nosed bullet would have a greater penetrating ability than a sharp-pointed one. Of course, the most important factor in the penetration of the long blunt-nosed bullet is its smaller value of $\frac{\mu}{B}$, but it is probable that the bluntness of its nose also contributes to its penetrating power.

In the foregoing analysis, the velocity of the bullet has been taken to be 2700 f/s, and the effect of changes in velocity on the sizes of yaw and on the resistance offered by the medium have not been discussed. The value of μ, the couple factor, is proportional to the

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* It may be found necessary to modify these formulae somewhat to take account of the angular velocity $\dot{\delta}$ generated by the entrance of the bullet into the medium.

square of the velocity, and if we can neglect the changes in the velocity in the medium for the first few inches, we may replace the time "t" by s/v where s is the penetration distance and v is the velocity.

If "v" is considered a constant, then we have

$$\ddot{\delta} = v^2 \frac{d^2\delta}{d\,s^2},$$

and if we replace μ by the expression $C_\mu \rho v^2 d^3$, where C_μ is a coefficient which is independent of the dimensions and varies only slightly and ρ is the ratio of the density to that of normal air, we finally obtain from (5)

$$v^2 \frac{d^2\delta}{ds^2} = \frac{C_\mu \rho d^3 v^2}{B} \delta.$$

If we divide both sides of the equation by v^2, we obtain an equation which does not to a first approximation involve the velocity and which indicates that the yaw in the medium as a function of the distance of penetration is independent of the velocity of the bullet. Thus approximately, it follows that the same sized yaws will be produced in the medium no matter what the velocity of the bullet.

Now, the resistance to the motion is proportional to the square of the velocity, and thus it follows that as the velocity is changed, the energy absorbed by the medium will be proportional to the square of the velocity.

We shall make another application of the theory to the design of bullets in discussing the problem of the influence of the caliber of a bullet on the energy absorbed by the medium at short ranges.

The resistance of the projectile, R, is given by the expression:

$$R = \rho v^2 d^2 C_R(1 + \frac{\delta^2}{169}),$$

where

ρ is the ratio of the density to that of normal air,

v is the velocity with respect to medium,

d is the diameter of the projectile,

C_R is a coefficient depending upon the shape and also to a certain extent on the relation of the velocity to certain characteristic qualities of the medium, e.g., the velocity of sound in the medium,

and δ is the angle of yaw.

The expression $\frac{\delta^2}{169}$ is used to express the proportional increase in resistance due to yaw.

The energy absorbed in a trajectory of length s in the medium is

$$\int_0^s R\, ds = \rho d^2 \int_0^s v^2\, C_R \left(1 + \frac{\delta^2}{169}\right) ds.$$

From equation (6) on page (9), we obtain

$$(7) \quad \delta^2 = \frac{\delta_0{}^2}{4}\, e^{\frac{\mu}{B} t^2}$$

It may be shown that for a series of similar projectiles μ may be expressed by

$$\mu = \rho v^2 d^3 C_\mu,$$

where C_μ is the same constant for all the projectiles. Similarly for a group of homologous projectiles, we may write

$$B = C_B d^5.$$

If we confine our attention to a short trajectory a few inches long, we may replace t by $\frac{s}{v}$.

We then obtain from (7)

$$\delta^2 = \frac{\delta_0{}^2}{4}\, e^{\frac{\rho v^2 d^3 C_\mu t^2}{C_B\, d^5}} = \frac{\delta_0{}^2}{4}\, e^{\frac{\rho C_\mu}{C_B}\left(\frac{s}{d}\right)^2}$$

From this it follows that for like values of $\frac{s}{d}$, equal values of the yaw will result. In other words, a .25" projectile will yaw as much after a motion of 1 inch in water, as a homologous .50" projectile will in two inches.

We obtain for the absorbed energy, E_{ab},

$$(8) \quad E_{ab} = \rho d^2 \int_0^s v^2 C_R \left[1 + \frac{\delta_0^{\,2}}{4 \times 169} e^{\frac{\rho C_\mu}{C_B}\left(\frac{s}{d}\right)^2} \right] ds.$$

As the caliber of the projectile is changed, let us consider what the changes in velocity will be. Let us assume that the average pressure on the base of the projectile is unchanged and that the length of travel is unchanged.

Then the muzzle energy of the bullet will vary as d^2. The mass is proportional to d^3, and thus we find for the muzzle velocity, v,

$$d^2 v^2 = kd^*$$

and v will vary as $1/\sqrt{d}$. If $\sqrt{\frac{k}{d}}$ is substituted for v in (8), assuming that the velocity is practically constant for the first few inches, we obtain for the absorbed energy:

$$E_{ab} = \rho k d \int_0^s C_R \left[1 + \frac{\delta_0^{\,2}}{4 \times 169} e^{\frac{\rho C_\mu}{C_B}\left(\frac{s}{d}\right)^2} \right] ds.$$

We proceed to apply this result to the calculation of the absorbed energy of a series of bullets homologous to the cal. .30 M1.

The values of $d \left[1 + \frac{\delta_0^{\,2}}{4 \times 169} e^{\frac{\rho C_\mu}{C_B}\left(\frac{s}{d}\right)^2} \right]$ or $d(1 + \frac{\delta^2}{169})$,

assuming that the initial yaw δ_0 is 1°, are plotted as a function of the range, s, in inches for projectiles of calibers of .30, .25, and .20 in Figure 1. In Figure 2 are shown plots of the absorbed energy E_{ab} as a function of the range s for the three different calibers .30, .25, and .20. From this it may be seen that for ranges of more than four inches in water, the greatest energy is absorbed from the smallest bullet. If

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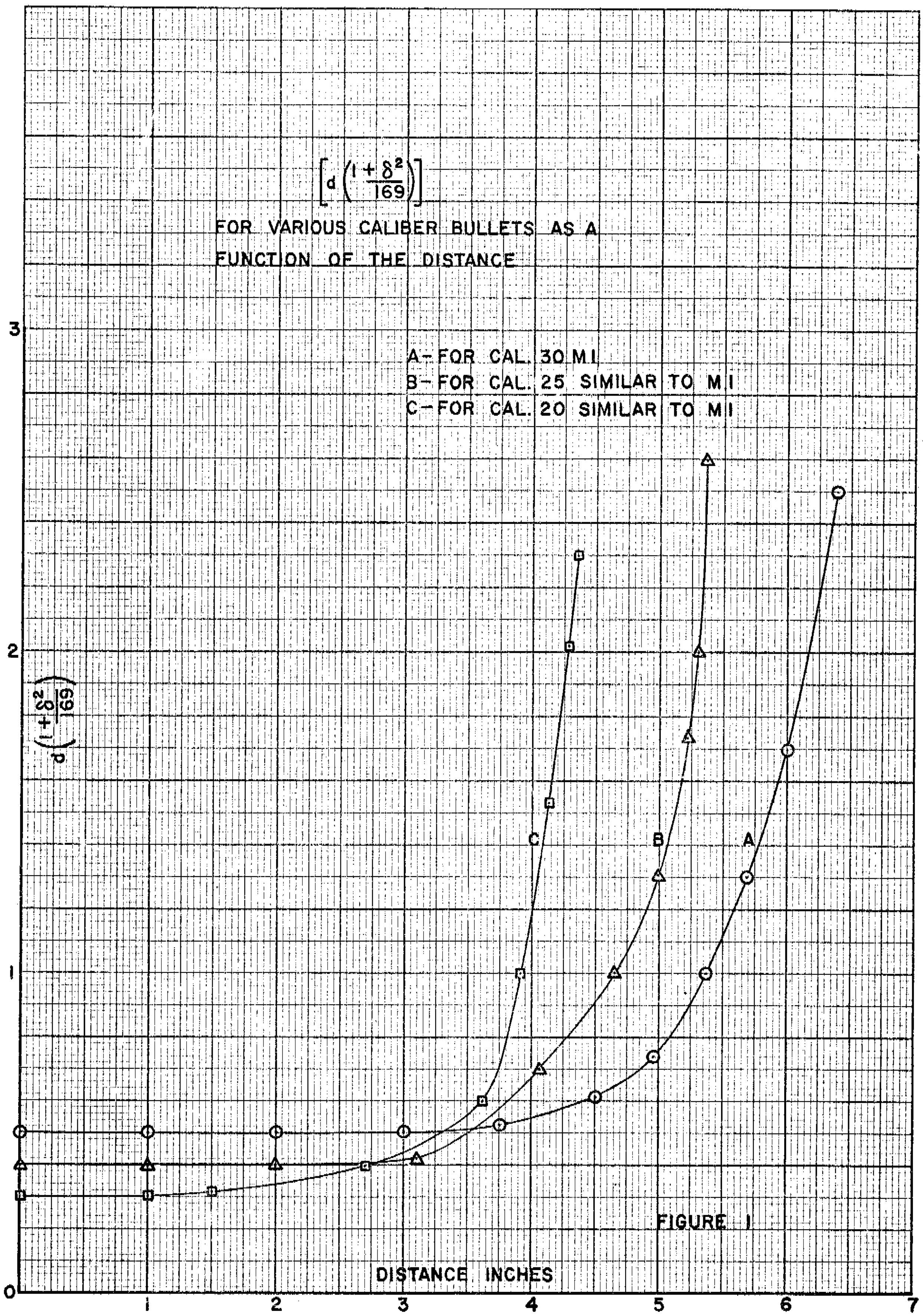
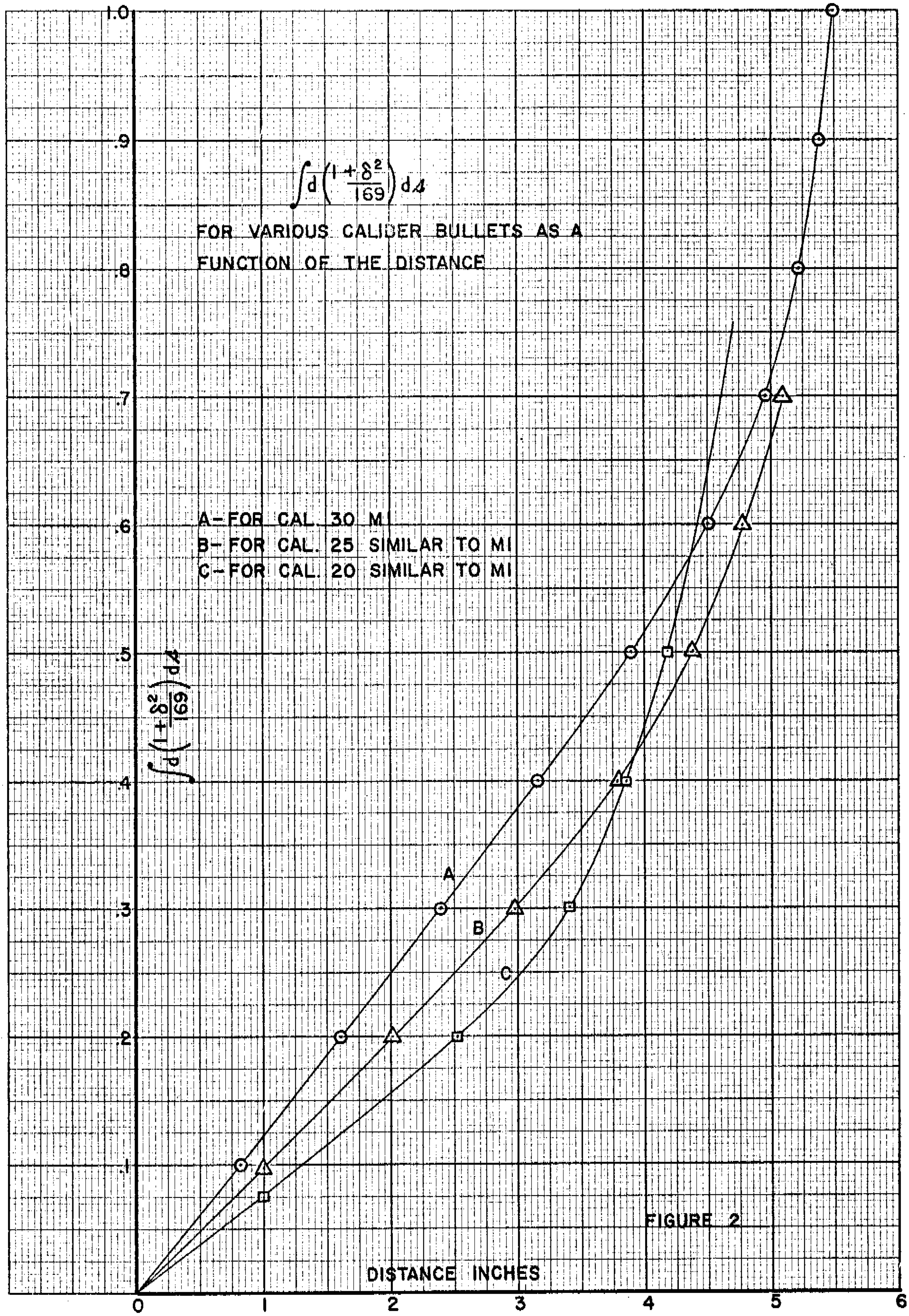
* k is a constant for the different calibers.


$\left[d\left(1+\frac{\delta^2}{169}\right)\right]$
FOR VARIOUS CALIBER BULLETS AS A
FUNCTION OF THE DISTANCE
A- FOR CAL. 30 M1
B- FOR CAL. 25 SIMILAR TO M1
C- FOR CAL. 20 SIMILAR TO M1
$d\left(1+\frac{\delta^2}{169}\right)$
C
B
A
FIGURE 1
DISTANCE INCHES
0
1
2
3
4
5
6
7



$\int d\left(1+\frac{\delta^2}{169}\right) d\mathit{\Delta}$
FOR VARIOUS CALIDER BULLETS AS A
FUNCTION OF THE DISTANCE
A- FOR CAL. 30 MI
B- FOR CAL. 25 SIMILAR TO MI
C- FOR CAL. 20 SIMILAR TO MI
A
B
C
DISTANCE INCHES
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1
2
3
4
5
6

FIGURE 2

the bullet were to hit an object like a bone, the smallest bullet would show a still greater superiority as far as the amount of energy absorbed is concerned.

From the preceding discussion, it may be seen that if the caliber of the infantry rifle is reduced that no reduction in effectiveness at short ranges will be obtained and that in fact at these ranges the stopping and shocking power will probably increase. At long ranges, the smaller bullets will have lost more velocity than the larger bullets and will thus have a smaller energy absorption at long ranges. This characteristic of the smaller bullet should prove advantageous since, at such ranges, it is probably desirable that the bullet wounds shall not be lethal.

Another characteristic of great importance in a projectile is the flatness of its trajectory. As the caliber of the bullet is reduced and its velocity increased, the flatness of the trajectory for short ranges will be increased and decreased for long ranges. The greater flatness of trajectory of the smaller bullets at short ranges may more than compensate for the reduced flatness at long ranges.

Data relating to the remaining velocities at various ranges of the bullets are given in the following table:

Caliber	1/C(J)	V_o	V(500 yds)	V(1000 yds)
.30	3.70	2700	1940	1274
.25	4.44	3000	2054	1249
.20	5.55	3300	2083	1106

From the results given, it appears practically certain that the Cal. .25 bullet will have the flattest trajectory of all the bullets for the 1000 yard range and that the caliber .20 bullet will have the flattest trajectory for the 500 yard range.

A great advantage of the smaller bullet will be that the recoil velocity of the rifle will be considerably reduced and thus also the fatigue of firing.

The reduced recoil velocity and bore diameter as the caliber is reduced will probably make it practicable to employ muzzle velocities still higher than those considered in the foregoing, and thus the flatness of the trajectory and effectiveness of the small caliber bullet may still be further augmented. In short, it appears that considerable advantages may result from the selection of a caliber considerably less than .30. A rational choice of caliber could only be made after an exhaustive study of all the factors involved.

RÉSUMÉ

It has been pointed out that, at points near the muzzle, the ratio of the spin to the linear velocity of the projectile is so low that the spin has practically no effect so far as the size of the yaw in the medium is concerned; that is to say, that, at points near the muzzle, the gyroscopic stability of the bullet is so small that it is not able to interfere seriously with the tendency of the medium to overturn the bullet. For the bullet to have a sufficient gyroscopic stability to be effective in preventing the tumbling, such great ranges would be required that the stability factor of the bullet in air would have an approximate value of 800.

For such a high value to be reached, if the stability factor of the bullet in air at the muzzle were 2, the remaining velocity would have to be as low as 150 f/s and thus, for the ordinary bullets, the spin would not have any appreciable effect in tending to prevent yaw in the medium except at ranges over 2000 yards.

It has been shown in the foregoing that the angle of yaw at any given point in the dense medium depends upon the initial angle of yaw δ_0 and upon the value of μ/B. It has also been pointed out that the resistance of the motion of the bullet increases as the square of the yaw and that it depends upon the shape of the projectile. It is thus apparent that the resistance of the bullet at a given velocity depends upon the following factors:

(1) The initial angle of yaw,

(2) The couple factor, μ,

(3) The transverse moment of inertia, B,

(4) The shape of the bullet.

A method is already available for the determination of B and a method has been outlined for the determination of μ.

It has been shown that the angle of yaw of the bullet in the medium is approximately independent of the velocity, and since the resistance is proportional to the square of the velocity, it follows that if the velocity of the bullet is changed, the energy absorbed will vary as the square of the velocity.

The application of the foregoing theory to the design of bullets is obvious. We have seen that the lighter the bullet, the greater the absorbed energy and also the higher the velocity of the bullet, the greater the absorbed energy. If the muzzle energy of the bullet is kept constant, it follows therefore that greater absorbed energy will be obtained by the use of lighter bullets since the lighter bullet will result in increased velocity and also in increased yaws in the medium. Another result to be obtained from the use of the lighter bullet if the muzzle energy is constant is that the recoil velocity will be less and therefore the effect upon the firer of the discharge of the piece will be reduced.

On the other hand, if it is desired to design a bullet which will make a small hole, it can be done by augmenting the transverse moment of inertia, B, and by making the nose heavy and the base light.

The influence of the caliber upon the amount of energy absorbed in the medium has been treated. It has been found that if the caliber of a series of homologous projectiles is reduced larger yaws will be obtained in the medium, and thus if the muzzle velocity is augmented as the caliber is diminished, it will result in a greater absorption of energy.

It has been pointed out that if the caliber is reduced the flatness of the trajectory at short ranges will be augmented, although it will be reduced for longer ranges; and it has been noted that the reaction due to recoil on the firer will be diminished as the caliber is reduced.

These considerations indicate that, probably, considerable improvement in the effectiveness of the infantry weapon may be obtained by a reduction of the caliber below that which now exists, but a rational choice of the exact caliber would have to be based on a very extensive investigation.

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