Interior Ballistic Effects
of
Gun Erosion

by

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INTERIOR BALLISTIC EFFECTS OF GUN FRICTION

Abstract

The primary cause of loss of velocity in worn artillery tubes is the decrease of engraving resistance in the early stages of projectile motion. Certain results of this decrease of engraving resistance, which is not accounted for in the present interior ballistic tables, are of interest in connection with problems encountered in ammunition calibration, rifling design and related interior ballistic problems.

The equations which serve as the basis of the present interior ballistic tables for multi-perforated powder are modified herein to account for arbitrary engraving resistance "patterns". From empirical data, estimates are made of the engraving resistance patterns for a particular type of weapon. Using these patterns and the modified equations, interior ballistic trajectories are computed by numerical integration under conditions designed to investigate certain effects of the engraving resistance and its interaction with other interior ballistic variables.

The engraving resistance encountered in short distances from the start of projectile travel is investigated with respect to its effect on muzzle velocity. Information relative to the effect of seating of rotating bands, at various positions of the band, is deduced from the above investigations. Changes of calibration of ammunition as a result of differences of powder quickness, cartridge
Case crimp and shell weight are computed for various engraving resistance patterns which are intended to represent tubes at various stages of wear. In particular, the linearity of these changes of calibration relative to a measure of tube erosion is investigated since this is a problem of critical importance in designing ammunition calibration programs.

Modifications of rifling design indicated by the computations are suggested.
INTRODUCTION TO EFFECTS OF GUN EROSION

Introduction

Statement of Problem

The primary problem of the field of interior ballistics is the prediction of the interior ballistic trajectory, i.e., the velocity and position of the projectile and the magnitude of the pressure of the powder gas as a function of time during the time that the projectile is in the gun. The interior ballistic trajectory has obvious uses in the design of weapons, ammunition, propellants and in the testing of ammunition.

Many difficulties in the theoretical determination of the interior ballistic trajectory arise from the extremes of temperature, pressure and acceleration encountered in gun firings. Little or no reliable data is available, for example, on such items as the heat loss during firing, the frictional force encountered by the projectile, or the forces encountered by the projectile as it begins to move and encounters the lands. Although the knowledge of many other important factors is now far more extensive than ever before, it can hardly be regarded as entirely satisfactory.

The complete knowledge of these factors, however, if it were attainable, would certainly not solve the problems of the interior ballisticsian. The calculation of interior ballistic trajectories accounting for the multitude of variables that exist would be a complicated task. The differential equations obtained can be solved only by numerical integration. The solution of these
differential equations for all combinations of the parameters which might be expected to occur would be an impracticable task.

These difficulties are handled, in practice, by appropriate changes of variables designed to eliminate certain parameters and, in the case of some rather poorly known parameters, by the assignment of universal, though arbitrary, values. By these devices, the number of parameters is sufficiently reduced to permit tabulation of a relatively small number of interior ballistic trajectories which are representative of all firing conditions that may be expected. This tabulation is then available for use in design or experimental work as mentioned previously.

This thesis is a consideration of one of the parameters to which it is customary to assign an arbitrary value. This parameter is the resistance encountered by the projectile in the first few inches of travel. During this initial period of travel, the rotating band of the projectile encounters the lands of the rifling and is engraved by them. Large forces will, therefore, oppose motion of the projectile and, as will be seen later, these forces seriously affect the combustion of the powder. The variations of these forces, which occur as the result of erosion of the lands by firing, will be shown to be a major factor in the variation of velocity levels between different guns of the same type or in a given gun under different conditions of erosion.

This engraving resistance is accounted for in the current interior ballistic tables by the use of an initial resistance which must be overcome before motion of the projectile begins. These tables have been developed using
one chosen fixed value of the initial resistance, which is assumed to exist only before, but not during, the projectile travel. In using the tables, no variation of the initial resistance is permitted, either to account for magnitude or region of application. The reasons for this are sound and obvious. First, practically nothing was known about the actual magnitude and region of action of these forces. Second, the addition of two parameters (i.e., magnitude and region of application of engraving resistance) would cause such an increase in the volume of the tables as to make their preparation and use very difficult.

The question as to just what effects the introduction of this engraving resistance might have, however, is an enticing one. Many effects have been observed in actual firings which appear to find a ready explanation in the vicissitudes of the engraving resistance. These effects will be discussed in detail later but it is worth noting at this time that among the phenomena which are attributable largely to changes of engraving resistance are (1) the loss of velocity in a worn gun (2) the decrease of velocity dispersion in a worn gun (3) the change of relative velocity levels of ammunition between firings in new and worn guns (4) the absence of certain "tube conditioning" effects on velocity in worn guns.

The purpose of this thesis is to investigate the theoretical effects of engraving resistance in a selected weapon for which considerable experimental data is available. An attempt will be made to estimate the magnitude and region of action of the engraving resistance from physical considerations and from the observed effects which are believed to be largely attributed to the resistance. Using the values so estimated, other factors, more difficult to determine experimentally, may then be computed theoretically. It is anticipated that, if
successful, these results may be of value in the design of
the milling of a weapon and in the specification of such
values as the limits of quickness of powders for use in a
weapon.

The method employed in adding the factor of engraving
of resistance to calculations of trajectories will be the
simple and straightforward one of subtracting the force
of engraving from the accelerating force in the numerical
integration of the equation of motion. This scheme is
employed without claim of its being either novel or clever.
It has unquestionably been given consideration by many per-
sons concerned with the field of interior ballistics but
it has, to the limits of knowledge of the author, been
seldom employed. In part, at least, this may be explained
by the fact that until the recent war many of the serious
effects of engraving resistance were not recognized, or were
of less consequence, because of the less stringent require-
ments placed on ammunition. This method shows promise of
providing an insight to certain interior ballistic phenomena
which, at present, are of interest in conjunction with the
calibration of ammunition.

Ballistic Terminology

The purpose of this section is to introduce the reader
unfamiliar with the field of interior ballistics to certain
terms which are peculiar to this field.

Figure 1 (a) gives a cutaway view of a typical complete
round of ammunition. The rotating band is made of copper
or killing metal and serves the dual purpose of minimizing
the escape of gas during firing and imparting rotational
velocity to the projectile. An important aspect of its
behavior in connection with this study is the engraving
resistance which exists when the rotating band engages the rifling of the tube. The setting of the rotating band or clearance between its inner face and the outer face of the band seat of the projectile may be seen to have a pronounced effect on the magnitude of the engraving resistance.

The cartridge case is fastened to the projectile to facilitate handling of the round prior to firing by means of crimping it at the position of the crimping grooves of the projectile.

The powder charge is contained in the cartridge case. The space contained in the cartridge case with the projectile in place is referred to as the powder chamber. The same term is applied to the portion of the gun which houses the cartridge case, however, the term volume of the powder chamber will always refer specifically to the volume in the cartridge case in this discussion. An end view of a multi-perforated powder grain is shown in Figure 1 (b). The least distance between the perforations of the grain specifies its web thickness. This type of grain is known as a progressive burning grain because the surface of the grain increases as the burning proceeds until the point of slivering is reached. The term quickness is defined specifically in certain interior ballistic tables, but is frequently used, rather loosely, to denote the relative rate at which powders differing in dimensions or chemical composition would release energy under identical conditions. The burning rate of a powder is the linear rate of burning of a surface of the powder under standard pressure.

Figure 1 (c) shows a typical cross section of a gun or gun tube showing the rifling. The recesses are called grooves and the projections lands. The lands spiral around the bore or inner face of the gun to impart rotation.
the protective. The rate of this spiralling is denoted by the twist or pitch of rifling, which may be specified in either calibers per revolution (where caliber is the diameter of the bore of the gun as shown in Figure 1 (a)), or in degrees referring to the slope of a plane development of the lands relative to the gun axis. The rifling may be either of uniform or progressive twist. The latter, rather uncommon in American artillery, serves to reduce and shift the peak of rotational acceleration which in the case of uniform twist coincides with the peak of translational acceleration.

The chamber of the gun is, of course, smooth, since this portion houses the cartridge case during firing. The chamber tapers down to a region immediately forward of the position of the rotating band when the round is inserted in firing position. At this region, known as the origin of rifling, the rifling begins. The lands do not assume full depth at the origin of rifling but gradually reach full depth by virtue of a taper of generally of about 9 degrees to the axis of the tube. Erosion of the gun, that is the removal of metal from the bore by the friction and hot gases during firing, is greatest at the origin of rifling.

The term ammunition lot is applied to a group of items either complete rounds, i.e. the assembled round ready for firing, or its components which were designated with the same lot number by their manufacturer. Requirements imposed on the production of lots of complete rounds give reasonable assurance that the lots are homogeneous in their interior and exterior ballistic behavior.
The interior ballistic equations of Pennett

The interior ballistic tables for use with multi-perforated powders which have been adopted by the Ordnance Department are those contained in Ordnance Department Document No. 2039(1). The equations which serve as the basis of these tables will be discussed in the following sections. The fundamental equations required for solving the motion of the projectile are those of (1) translation, (2) burning rate and (3) energy.

Granulation Function

The granulation function used in Pennett's Tables (Ordnance Department Document No. 2039) expresses the fraction of the charge burned as a function of the fraction of the web burned. This function was determined from geometrical considerations of the standard multi-perforate grain.

This method of determination of the granulation function necessitates the assumption that layers of equal depth are burned from all surfaces in equal time intervals and that all grains are of the same shape. The latter assumption is sound inasmuch as little variation in dimensions is found between grains of a charge. The former apparently is not too well met because of the tendency for more rapid burning on the inner surfaces of the perforations than on the outer grain surface. This phenomenon, which has been observed on partially burned powder grains, is attributed to either the higher pressures existing in the perforations during burning, or the erosion of the grain by gas escaping from the perforations during burning or both. The manner in which the table are used,
However, normally minimizes the effects of the failure of the assumptions of equal burning by assuming to the charge an effective quickness empirically determined under firing conditions approximating those desired.

**Burning Rate Equation**

The rate of burning of the powder is assumed to be given by the equation

$$\frac{dz}{dt} = \frac{z}{R} F^k$$

where $z =$ fraction of web burned

$k =$ burning rate constant

$F =$ pressure of powder gas

$k =$ burning rate exponent

$h =$ web thickness

This form of the burning rate equation, which is common to practically all interior ballistic systems, is attributed to Vieille (1893) by Crans (2) and is in agreement with experimental data. The only value in this expression of interest at the moment is that of the burning rate exponent. This value was, at the time of the preparation of Bennett's Tables, most generally accepted as $2/3$ although, as mentioned by Bennett, the evidence was far from conclusive. More recent experimental data (3) indicates that a somewhat higher value, about .85, is more representative of present powders.

**Energy Equation**

This expression, which equates the energy released by the combustion to that taken up by various processes
in the sun, is the source of most, though not all, improvements required in interior ballistics.

Cronz(2) tabulates and considers ten processes which absorb the energy of combustion as follows: (1) kinetic energy of translation of the projectile (2) kinetic energy of rotation of the projectile (3) energy of motion of the powder charge and powder cases (4) energy of motion of the recoiling parts of the gun (5) work of overcoming exterior atmospheric pressure (6) energy of motion of air ahead of the projectile (7) work of friction between the projectile and the gun (8) work of engraving the rotating band (9) heat loss to projectile and gun and energy loss by escape of powder gas past the projectile (10) internal energy of the powder gas.

Item (1) and (10) are of the greatest importance in the disposition of energy, and most investigations of interior ballistics are based on equations expressing all ten terms relative to these two as nearly as possible. Thus, Bennett's equations assume adiabatic behavior of the powder gas to account for item (10), and introduce an effective projectile mass to account for all other items. This system accounts well for all processes except item (5), (6) and (8), which are small and item (9), which is not necessarily small. At present, the heat loss is considered to be roughly accounted for in Bennett's equation by the use of a ratio of specific heats somewhat larger than the actual value for powder gas. This method of handling the heat loss is justifiable from the viewpoint of expediency because too little is known about the actual heat loss to justify the use of any other form.

It should be apparent at this point, that in the use of interior ballistic tables, the normal procedure is not
Based on purely theoretical considerations, but on an empirical foundation approximating, as nearly as possible, the desired conditions. That is, for example, to estimate the velocity expected from a weapon under certain proposed conditions, one would first attempt to find the results of actual firings under conditions approximating those proposed. With these empirical results, he may then evaluate the parameters which, in the theoretical attack, would have given the observed result. Using these parameters, modified so as to account for the change from the actual to the proposed conditions, he then may compute the expected result, having accounted for the vagaries of such items as heat loss in a semi-empirical manner.

Proceeding, then, with the development of Bennett's energy equation under the assumption of adiabatic behavior of the powder gas, the energy remaining in the gas, $E_0$, may be shown to be

$$E_0 = \frac{P\Omega}{k-1} \quad (1)$$

where $P$ is the gas pressure.

$\Omega$ is the free volume of the powder gas.

$k$ is the ratio of the specific heats of the powder gas.

Expressing the free volume as the total volume of the powder chamber and the portion of the bore through which the projectile has travelled, minus the volume occupied by the unburned powder and the co-volume of the powder gas, and taking the co-volume of the powder gas as $3/2$ the volume of the powder burned, the equation for $\Omega$ becomes.
\[ \Omega = S C' - \frac{C}{W} \left[1 - G(z)\right] - \frac{3}{2} \frac{C}{W^2} G(z) \]

\[ = S C' - \frac{C}{W} \left[1 + \frac{G(z)}{2}\right] \]

where \( C' \) is the chamber volume
\( S \) is a travel parameter such that the total volume behind the projectile is \( S C' \); thus \( S=1 \) at the start of travel,
\( c \) is the charge weight
\( \delta \) is the specific gravity of the solid powder
\( W \) is the density of water
\( G(z) \) is the granulation function

Defining \( \Delta = \frac{C}{c \cdot W} \)

and substituting for \( \Omega \), (1) becomes

\[ E_g = \frac{P' C'}{K-1} \left[ S - \frac{\Delta}{\delta} \left(1 + \frac{G(z)}{2}\right)\right] \]  \( \text{(2)} \)

Bennett then introduces the reduced projectile weight, \( P' \) defined as the weight of the projectile plus one-third the weight of the powder charge. It is readily shown that this reduced weight will account for items (1) and (3) of the energy absorption if the powder gas and unburned powder are assumed to be uniformly distributed through the volume behind the projectile at all times. Using this reduced weight, and introducing the factor \( \frac{1}{r_0^2} \) as the constant of proportionality required to take account of all other energy losses, the effective kinetic energy, \( E_g \), then becomes.
Then denoting the energy liberated by combustion per unit weight of charge by \( n' \), or the total energy liberated at any instant by \( n'c \, G(z) \), the energy equation may be written

\[
E_e = \frac{1}{r_o} \frac{P' \, v^2}{2g}
\]  

Then, denoting the energy liberated by combustion per unit weight of charge by \( n' \), or the total energy liberated at any instant by \( n'c \, G(z) \), the energy equation may be written

\[
n'c \, G(z) = E_g + E_e
\]  

\[
= \frac{Pc'}{k-1} \left[ \frac{\theta}{6} (1 + \frac{G(z)}{2}) \right] + \frac{1}{r_o} \frac{P' \, v^2}{2g}
\]  

At this point, the pressure is written as

\[
P = \frac{1}{r_o^2} \frac{L}{\delta t} \frac{P' \, \delta v}{\delta t}
\]

where \( L \) is the length such that \( \frac{G'}{L} \) equals the area of the bore, and \( r_o^2 \) is assumed to be the same as the corresponding value in equation (3). It should be noted that the definitions of \( S \) and \( L \) imply the relation,

\[
S = 1 + \frac{u}{L}
\]

where \( u \) is the displacement of the projectile from the rest position.

Now, to reduce the number of parameters, a new independent variable will be introduced. This is done by first defining
\[ d = \frac{F'}{\rho_0 v} \]

and

\[ r = \frac{r_0}{d^{1/2}} \frac{n'}{n_o} \]

(6)

where \( n_o' \) is a standard value of specific energy of the powder. The variable \( \phi \), called the reduced time, is now defined, using \( t \) to denote time:

\[ \phi = \frac{rt}{r'} \]

From (5) and (6)

\[ v = \frac{du}{dt} = \frac{Lds}{dt} = r \frac{ds}{d\phi} \]

(7)

and, since

\[ \frac{dv}{dt} = \frac{r^2}{r'} \frac{d^2s}{d\phi^2} \]

the new expression for the pressure, called reduced pressure, becomes

\[ P = \frac{n'}{n_o} \frac{d^2s}{d\phi^2} \]

(8)

Substituting (6) and (7) in the energy equation (4) gives
\[
n'(z) = \frac{n'}{n_0} \left( \frac{C'}{k-1} \right) \frac{d^2s}{d\phi^2} \left[ s - \frac{\delta}{\delta} (1 + q(z)) \right] + \frac{n'}{n_0} \frac{2}{C} \left( \frac{d\phi}{d\phi} \right)^2 \]

or
\[
n'(k-1)W(z) = \frac{d^2s}{d\phi^2} \left[ s - \frac{\delta}{\delta} (1 + q(z)) \right] + \frac{k-1}{2} \left( \frac{d\phi}{d\phi} \right)^2 \quad (9)
\]

Bennett assigns the value 1.3 to \( k \) and \( 1.25 \times 10^6 \) ft. lbs./lb. to \( n_0 \) and, replacing \( \delta \) and \( W \) by their known values, obtains the energy equation
\[
162,400 \Delta G(z) = \frac{d^2s}{d\phi^2} \left[ s - 0.632\Delta - 0.316\Delta G(z) \right] + 0.15 \left( \frac{d\phi}{d\phi} \right)^2 \quad (10)
\]

The burning rate equation of the previous section, expressed in terms of the new variable \( \phi \), becomes, using equations (6) and (8)
\[
\frac{dz}{d\phi} = \frac{b r L}{R} \frac{d^{1/2}}{n' \left( \frac{n_0}{n_0} \right) \left( \frac{d\phi}{d\phi} \right)^{2/3}}
\]

This is normally written as
\[
\frac{dz}{d\phi} = q \left( \frac{d\phi}{d\phi} \right)^{2/3}
\]

in which \( q \) is the quickness as used in Bennett's Tables.

**Tabulation**

The energy equation above, the burning rate equation, and the granulation function form a set of equations.
expressing s, \( \frac{ds}{d\phi} \) and \( \frac{d^2s}{d\phi^2} \) in terms of the parameters \( \xi \) and \( \Delta \) and the independent variable \( \phi \). Thus, by numerical integration of the three equations, the displacement, velocity and pressure may be determined as functions of time. These values, in the reduced units, are tabulated in O.D.T. No. 2030 for representative values of the parameters.

These integrations were performed using an initial value of \( \frac{d^2s}{d\phi^2} \) corresponding to about 2500 lb./sq.in. This "shot start" pressure was considered to account, in part at least, for the engraving resistance of the rotating band which cannot be reasonably accounted for by the factor \( \frac{1}{r_0^2} \) used for so many of the other effects.

Modified Interior Ballistic Equations

The distinction between engraving resistance and frictional bore resistance, as used herein, should be recalled at this point. The latter seems to be reasonably well taken into account, in the previous theory, by the multiplication of the projectile weight by a factor slightly larger than one. This procedure implies that the frictional resistance is proportional to the acceleration which, in a weapon having uniform rifling twist, is roughly equivalent to assuming a coefficient of friction independent of velocity. Aside from any consideration of the adequacy of a constant coefficient of friction, this method permits an empirical evaluation of the frictional resistance, since the factor \( r_0 \) is so evaluated, and the frictional energy losses are both small and reasonably constant in any individual tube. The magnitude of the
frictional resistance is large only in regions in which it has no appreciable effect on the combustion of the powder.

On the other hand, engraving resistance always occurs early in the projectile travel and, in this region, can have large effects on the combustion of the powder. Whereas the energy expended in overcoming engraving forces is trivial relative to the muzzle energy of the projectile, the delay caused by engraving may so affect the combustion of the powder as to change the energy delivered to the projectile by more than ten percent. Since both the magnitude and region of action of the engraving forces are seriously affected by gun erosion, these forces are, obviously, not well accounted for in the preceding theory, although the frictional resistance does appear to be adequately handled.

The energy and burning rate equations of Bennett will now be modified so as to permit the introduction of an engraving resistance of arbitrary magnitude operating over an arbitrary region. This may be readily done by first rewriting equation (4) as

\[ n'cG(z) = \left( \frac{P + P_f}{k-1} \right) C' \left[ s - \frac{A}{b} \left( 1 + \frac{C(s)}{z} \right) \right] + \frac{1}{r_0^2} \frac{P^2v^2}{2} + A\cdot F(P_f, u) \]

In terms of

\[ A = \text{Area of bore} \]
\[ P_f = \text{pressure equivalent to engraving resistance} \]

where the new values introduced are

\[ A\cdot F(P_f, u) = A \int_0^1 P_f \, du = \text{work of engraving} \]
Using the same change of variables as before (11)

becomes

\[ n'G(z) = \frac{n'}{n_0}(A) \left( \frac{d^2s}{dp^2} + \frac{n'P_f}{n} \right) \left[ s - \frac{A}{6} (1 + \frac{G(z)}{2}) \right] + \]

\[ \frac{n'C}{n_0} \left( \frac{ds}{dp} \right)^2 + A \cdot F(P_f, u) \]

or, since,

\[ A \int_0^1 P_f \, du = AL \int_1^s P_f \, ds = AL \cdot F(P_f, s) = C \cdot F(P_f, s) \]

\[ n'(k-1)W\Delta G(z) = \left( \frac{d^2s}{dp^2} + \frac{n'P_f}{n'} \right) \left[ s - \frac{A}{6} (1 + \frac{G(z)}{2}) \right] + \]

\[ \frac{k}{2} \left[ \left( \frac{ds}{dp} \right)^2 + \frac{n'}{n'} \right] F(P_f, s) \]

and, for brevity, defining

\[ P_f' = \frac{n'P_f}{n'} \]

\[ \frac{d^2s}{dp^2} = \frac{d^2s}{dp^2} + P_f' \]

then
\[ r_0'(k-1) \Delta G(z) = \frac{d^2s'}{d\phi^2} \left[ s - \frac{A}{b} \left( 1 + \frac{r(z)}{2} \right) \right] + \frac{k-1}{2} \]

\[ \left[ \left( \frac{ds}{d\phi} \right)^2 + 2P(P_f', s) \right] \]

Equations (12) and (13), together with the granulation function, provide a means of accounting for the engraving resistance in computation of the interior ballistic trajectory, assuming that \( P_f \) is known as a function of \( u \), the projectile travel. A discussion of the evaluation of \( P_f \) is contained in the following section.

Other factors than change of engraving resistance accompany tube erosion. The increase of bore diameter permits the escape of a greater quantity of powder gas and, in addition, increases the volume of the bore. Both of these effects tend to lower the velocity. A comparison of the relative magnitudes of these effects by the British (4) indicates that the combined effects of gas escape and volume increase account for about one-third of the total velocity loss in their 3.7" gun, the remaining two-thirds being
caused by loss of engraving resistance. The internal contours of the guns on which this is based show greater enlargement by a factor of nearly two than do representative U. S. 90mm guns (for example, see the star case curve of 90mm M1, tube No. 156 in reference 5). This indicates strongly that the effect of engraving resistance is the predominate cause of velocity loss in the U. S. 90mm gun. In the following sections, computations of the effect of engraving resistance are made considering it as the only cause of velocity loss. Although this is obviously not true it appears to be a good enough approximation to indicate the major trends and effects which will be sought.

In the computations reported in the following sections the process of numerical integration was carried out in steps of .0005 units of the reduced time throughout the trajectories. For convenience in computation trapezoidal integration was used rather than one of the quadrature formulae which account for higher order differences. With the small steps employed, the trapezoidal method was considered sufficiently accurate. As a test of this, a comparison was made of the velocity obtained by the trapezoidal method with that obtained using Simpson's One-Third Rule which accounts for second order differences. The greatest difference between the two methods was less than one foot per second throughout the trajectory in the computation which led to the result tabulated as A-12 in the Summary of Computations.
Application

The modified interior ballistic equations may be employed to determine the effect of resistance at various positions along the bore and the interaction of this effect with changes of powder quickness, cartridge case crimp and shell banding. A problem of this nature of considerable importance is the estimation of the effect on velocity of variations in cartridge case crimp under various conditions of tube erosion (i.e. under various conditions of engraving resistance.)

This particular problem is of interest in the calibration of ammunition for the 90mm gun. In current calibration firings at Aberdeen Proving Ground, frequent observation has been made of the fact that the calibration of complete round lots, relative to a reference lot (a control series) vary by large amounts, depending upon the condition of the tube in which calibration is performed. These variations of calibration are the result of interactions between the engraving resistance, which varies with the tube condition, and certain properties of the ammunition being calibrated such as cartridge case crimp, powder quickness and shell banding. Because of this, the calibration of these lots must be tabulated as a function of the tube condition. Actually, the tube condition is most conveniently indicated for ballistic purposes by the muzzle velocity of a reference complete round lot, and it is relative to this that the calibration is tabulated.

The calibration of a lot of ammunition in tubes of all conditions is, of course, impractical. An economical limit to the number of tube conditions which can be employed per lot in an extensive calibration program is two,
i.e. a reasonably new and reasonably worn tube. To go beyond this would, first, entail considerable expense and, second, require the destruction of an excessively large number of tubes and ammunition in testing. Thus, it is important to investigate methods for predicting the calibration of an ammunition lot, under any desired condition, from observation of the calibration at only two reference velocity levels. Essentially, this becomes a problem of determining the linearity of the calibration as a function of the reference velocity level or, since the calibration is, in effect, the sum of the effects of powder quickness, crimp and banding differences between the reference and the test lots, the problem is the linearity of these effects.

In order to compute the effects of such differences in tubes of various conditions, some knowledge of the manner in which the engraving resistance varies from a new to a worn tube is needed. The following sections summarize the available data concerning the magnitude of the engraving resistance in new tubes.

The engraving resistance in very worn tubes is known from bore measurements to be negligible until after a projectile travel of several inches. Subsequent calculations show that if the engraving resistance occurs after a travel of several inches it has very little effect on velocity. The pattern of engraving resistance in tubes of intermediate wear will be discussed at a later point in this paper.

Static Engraving Studies

Tube stresses caused by forcing projectiles through the bore mechanically have been studied recently by Watertown Arsenal and by the Catholic University of America. (6, 7, 8).
These studies, although conducted primarily to provide information for use in tube and shell design, have also given data regarding the magnitude of the static engraving pressure. Tests have been made in 37mm, 75mm, and 105mm tubes.

All but a few of the tests made at Watertown were conducted by pushing shell through the bore with a push rod. In a few of the tests, the shell was forced through the tube by simulated gas pressure, but this was done only after the engraving process had been completed. Thus all the static tests of engraving thrust were made without experimental simulation of the effect of gas pressure.

The results obtained by the Catholic University in measurement of axial thrust are summarized in NDRG Rept. No. A 442, pg. 47, as follows,

"The axial load behaves very irregularly, depending on the tube. Usually a maximum, which is 110-200 percent of the value found after engraving, occurs near the end of engraving."

The pattern of thrust against travel during engraving in new tubes showed, in general, a rapid and reasonably linear increase during the first one-third of the band width and a more gradual increase thereafter extending nearly to the end of engraving.

In eroded tubes the axial thrust increased more slowly with tube travel and did not attain values as high as those in the new tubes. The region of engraving in eroded tubes appeared to be equal to two or three band widths. The maximum thrusts during engraving were little, if any, higher than those attained after engraving.
The magnitudes of the maximum thrusts during erev-
ing are of interest, since they provide a rough estimate of
the pressure required to overcome forcing resistance dur-
ing firing in a new tube. These thrusts ranged from 25,000
to 35,000 lbs. for the 37mm tubes, 50,000 to 75,000 lbs.
for the 75mm tube and 100,000 to 150,000 lbs. for the 105mm
tube. Corresponding pressures (i.e. chamber pressures
equivalent to these thrusts) are 15,000 to 20,000 p.s.i.
for the 37mm tube and 7500 to 15,000 p.s.i. for the 75mm
and 105mm tubes.

Whereas the prediction of the shape and magnitude of
the engraving resistance-travel curve for the 90mm tube
from the above information would be a rather uncertain
procedure, the above information provides the means of
verifying the validity of an approximation of this curve
obtained from other sources.

Dynamic Engraving Studies

The difficulties inherent in any attempt to measure
dynamic engraving resistance are obvious. The projectile
is in the bore for only a few milli-seconds after motion
begins. During this time, one must obtain a continuous
record of velocity and of pressure on the base of the pro-
jectile, or at least discrete measurements of these values
at frequent intervals during initial travel, in order to
have information from which the engraving resistance could
be deduced. Unusual precision is necessary in these
measurements in order to give reasonably accurate estimates
of the forcing resistance, inasmuch as it is determined
from the difference of functions of the measured values.
Despite these difficulties, two recent reports contain the results of dynamic engraving force measurements. One of these, a German report by Rossmann,\textsuperscript{(9)} gives curves of resistance against travel from the start of motion to a considerable distance down the bore. Although this report does not indicate the weapon for which the curves were made, it appears from the magnitude of certain measurements to be the German 88mm gun. The curves indicate the engraving resistance to reach a maximum of about 400 kg/cm (5700 psi). The projectile for the German 88mm gun has a rotating band of considerably less cross-sectional area than that of the American 90mm gun. Although this result is of interest, it must be accepted rather conditionally because the report fails to indicate the means employed in determining the pressure at the base of the projectile and, in fact, gives only a slight description of the instrumentation employed in other measurements.

The other report, prepared by the Bureau of Standards,\textsuperscript{(10)} contains an excellent account of methods employed in instrumentation and reduction of data. This report covers firings of the M62A1 projectile from the 90mm M1 gun. The M62A1 projectile has a rotating band identical with that of the M71 projectile; however, for purposes of instrumentation, the taper of the leading edge of the band was machined down to present a square shoulder. Even with this modification, the projectile employed in this test should give values of engraving resistance quite similar to those encountered with the M71 projectile.

The abstract of the Bureau of Standards report, in summarizing the results of measurement of bore resistance
"The bore friction, after engraving is complete, is small (less than 10,000 lb). The engraving friction with a maximum value of about 100,000 lbs., is variable from round to round. The starting pressure is about 3,000 lb/in²."

Several rounds which were forced through the bore mechanically in this program gave resistance-travel patterns similar to those obtained by Watertown Arsenal and the Catholic University of America, in that the resistance after engraving was about one-half as large as that during engraving. The maximum resistances during engraving in the static tests corresponded in magnitude to those derived from the ballistic tests.

**Engraving Resistance Patterns**

Figure 2 shows the position of the rotating band of the 90mm M71 projectile relative to the rifling when in firing position. If one assumed the engraving resistance met by this rotating band to be proportional to the volume of metal displaced per unit travel, then the pattern of engraving resistance would increase sharply to a maximum after a travel of about 0.1" and would maintain this maximum (neglecting the effect of the cannelures) until the band was almost completely engraved. Such a pattern is, at least, not inconsistent with the empirical data previously cited. For convenience in computation, the engraving resistance pattern used was rectangular in shape and equal in length to the width of the rotating band (about 1.2").

Using this type of engraving resistance pattern, an
attempt was then made to determine the magnitude of the engraving resistance which, in computation, would produce results equivalent to those observed in actual firing, in regard to velocity drop from new to worn tubes. This was done by selecting a representative firing in a worn tube (11) and determining from Bennett's Tables the quickness and velocity parameters required to give the observed velocity and pressure. Using these parameters, computations were then performed with engraving resistances corresponding to tabular pressures of 2500, 5000 and 7500 psi. These results are included in the Summary of Computations-Part A-lines A-3, A-7, A-12 and appear in Fig. 3 as Curve II.

The highest value, 7500 psi, gives a velocity increase of 123 f/s over the value obtained with no engraving pressure. This velocity difference is near, though perhaps slightly under, the value normally observed as the drop in velocity between new and worn 90mm tubes. Thus, from the standpoint of loss of velocity, a satisfactory analogue of the new and worn tube engraving patterns was obtained by using a 7500 psi engraving pressure for one band width to simulate the new tube and no engraving pressure to simulate the old tube. The value of the engraving pressure in the new tube is in good agreement with that previously cited from the Bureau of Standards firings (10).

* All pressures mentioned in connection with interior ballistic trajectories will, for convenience, be given in the tabular unit (see Equation 8) used in computation which is 0.85 times the actual pressure. Thus the pressure of 7500 units above is actually 6375 p.s.i.
Muzzle velocity as a function of starting pressure, (a pressure which must be exceeded before motion starts, but which offers no resistance thereafter), was next computed (lines A-1, A-2, A-6 & A-9 in the Summary of Computations) and is shown as Curve III of Figure 3. Starting pressure only, as is used in most interior ballistic tables, cannot serve to simulate the change of engraving resistance from new to worn tubes, because unreasonably high pressures would be required to provide the observed velocity differential. This may be seen by comparing curves II and III of Fig. 3, from which it appears that a starting pressure of 7500 p.s.i. is required to produce the same effect on muzzle velocity as an engraving pressure of only 2000 p.s.i.

A check on the simulated new and old tube resistance patterns was possible by using these to compute the relative velocity levels of powders of different quickness in new and old tubes, and comparing the computed values with values observed in actual firings. Such a firing test was conducted at Southwestern Proving Ground in which a ballistically "slow" powder (lot RAI 15665) was compared with a ballistically "fast" powder (lot SUN 14653) in both new and worn tubes. Using the data of this firing record, the quickness and velocity parameters (q and r) were obtained from Bennett's Tables for both the fast and the slow powders. Integrations were then performed for both powders using the new and the worn tube resistance patterns previously adopted. The results of these computations (Summary of Computations Cl, C4, C5 & C6) show a change of 18 f/s in the relative velocity levels of the two powders. That is, the slow powder, which was 15 f/s higher in velocity than the fast powder in the new tube,
was 3 f/s lower than the fast powder in the worn tube. In the actual firing, this change of relative velocity levels was 23 f/s. The computed difference varies from the observed difference by an amount which is within limits of the experimental error of the firing. The new and worn tube patterns of engraving resistance thus appear to provide a satisfactory analogue from the standpoint of interaction with powders of varying quickness.

Length of Engraving Region

One method of inspection of artillery shells for defective (loose) rotating band seating involves the estimation of clearance between the band and the band seat by measurements of external indentations made on the rotating band. In connection with this type of inspection, knowledge of the relative effects on muzzle velocity of clearance under the band at various positions longitudinally along the band is valuable.

A thorough investigation of this effect would involve the integration of a large number of trajectories with various combinations of high and low engraving resistance at various positions. Rather than undertake this sizable task, it was felt that a sufficient insight into these effects could be obtained from determination of the effect of various lengths of engraving. The effect of extending the engraving region an additional band length was found to be negligible (see computations designated A-5 and A-8 and their relation to A-3 and A-7 respectively). Therefore, trajectories were integrated with 7500 p.s.i. engraving pressures beginning at the start of projectile travel and acting over regions up to one band width in length. These computations (A-9 through A-12) are the
The engraving resistance encountered over a surprisingly small travel exerts a large influence on the muzzle velocity. Figure 4 shows that a pressure of 7500 p.s.i. active over only 0.3" of travel causes an increase in velocity of over 100 f/s above that obtained with no engraving resistance. The engraving resistance encountered in the remaining 0.9" of travel while engraving is taking place accounts for only an additional 20 f/s. One might reason properly, as is shown by subsequent computations, that the effect of the latter 0.9" of engraving would be greater if the first 0.3" of engraving had not already taken place, however, the fact that the front portion of the rotating band plays a predominant part in determining the muzzle velocity can hardly be disputed. This is, in fact, in qualitative agreement with a firing program (13) in which metal was removed from the front, middle, and rear portions of the band successively. These two sources of information provide convincing evidence that the measurements of rotating band clearance should be made on the forward part of the band. Another logical inference from this data is that the process of shell banding might be both simplified and improved by designing banding machines to apply greater pressure to the forward regions than to the rearward regions of the rotating band.

Cartridge Case Crimp

In order to assess the effects of cartridge case crimp on velocity, a pattern of crimp resistance as a function of projectile travel was required. The length of effective crimp action was estimated from observation of
static debulleting operations in which 90mm shell were
mechanically pulled out of cartridge cases. In this
operation the shell are pulled at the rate of 1/4" per
minute and the force required to maintain this rate of
pull is recorded. From observation of a large number of
debulletings performed in conjunction with current tests
of 90mm of ammunition, it was found that the length of
effective action "as practically independent of the magni-
tude of the debulleting force and averaged about 1/8".
Using this length of action and a constant pressure of
1500 psi (tabular), trajectories were computed for new
and worn tubes (computations B-1 and B-4). Results of
recent firings are available (14) in which 90mm rounds, with
the standard cartridge case crimp, which gives a debullet-
ing force of about 9000 lbs., were compared with uncrimped
rounds in both new and worn tubes. The average differ-
ence of 29 f/s found in the actual firing in the worn
tube compares favorably (within experimental error) with
the computed of 25 f/s for the worn tube. The tabular
pressure corresponding to the debulleting force of 9000
lbs. is only 1100 p.s.i., which is considerably lower than
the 1500 p.s.i. necessary in computation to give reason-
able results. The discrepancy here may be accountable to
the existence of higher debulleting forces under firing
conditions than under the relatively static conditions
imposed by the debulleting machine.

The comparison between the empirical and the computed
values in the new tube seems, at first sight, to present
a discrepancy. The firing data (14) indicates no effect
of crimp in a new tube or, at least, so small an amount
as to be insignificant. The computations indicate the
crimped rounds to fire higher by 12 f/s, an amount which
should be detectable in ordinary firings in this weapon. A plausible explanation of this difference will develop in the further considerations of the interaction of crimp with tube erosion and thermal expansion.

For reasons previously stated, the behavior of the crimp effect under various conditions of tube erosion was next investigated. This was done using the crimp resistance pattern which was found above to agree with experimental data in the old tube, and imposing this resistance on the engraving resistance, which was varied to simulate different stages of wear.

The manner in which the engraving resistance changes with tube erosion is undoubtedly quite variable from tube to tube, and also must be affected by the thermal expansion of the tube during firing. For example, if one were to cause erosion by repeatedly forcing rounds down the bore mechanically, thereby eliminating the effects of gas erosion, surface melting and thermal expansion, he might reasonably expect the bore resistance pattern to maintain a fairly rectangular shape, decreasing in magnitude and increasing in length as erosion progressed. Peaks of resistance would be leveled if the bore metal were homogeneous because these would cause peaks of abrasion. The effect of surface melting, on the other hand, is greater near the origin of rifling than further down the bore, because of the greater time of exposure to the combustion temperature. This effect would tend to decrease the magnitude of the engraving resistance near the beginning of travel than later. Thermal expansion of a new tube, by itself, may be seen by reference to Fig. 2 above a short time was followed by an engraving pattern similar to that of an unlathed new
tube. Thermal expansion of 0.01° diametrically, which often occurs during the firing of 100 rounds in the 30mm tube, provides a free run of 0.04°. Free runs of even this small distance will be shown later to have remarkably large effects on muzzle velocity in new tubes.

Two distinct patterns of engraving resistance were investigated with respect to their interaction with crimp. The first (Assumption 1) was that suggested by pure mechanical erosion. This pattern, being rectangular and acting over a region of one band length, simulated various stages of erosion by various magnitudes of engraving resistance. No account was taken of the elongation of the pattern with advancing erosion because the effect of this, as previously mentioned, was insignificant. The muzzle velocity using this assumption of engraving resistance is shown in Fig. 3; both for crimped rounds (Curve I) and uncrimped rounds (Curve II).

The other pattern of engraving resistance investigated (Assumption 2) was that formed by delaying the start of engraving varying amounts, and keeping the shape and magnitude of the engraving resistance identical with that used for the new tube. This type of engraving pattern would result, for example, if the origin of rifling was unchanged in shape by erosion but progressively advanced down the bore. Under this assumption computations were performed with (3-4 through 3-9) and without (A-12 through A-17) crimping resistance at various stages of 'crimping'. The results of these computations are shown in Figs. 5 and 6.

The effect of crimp as a function of the velocity of the uncrimped rounds is plotted in Fig. 7 for the two
assumptions of engraving resistance patterns. This plot indicates that the effect of crimp may be largely influenced by the manner in which tube erosion actually takes place in a moderately worn tube, but should be fairly predictable in worn tubes. The rapid reversal of direction of the crimp effect under Assumption 2 in a new tube provides a strong suggestion that thermal expansion may account for the failure to observe any significant crimp effect in actual firings in new tubes. The fact that the crimp can, under certain conditions, cause a substantial decrease in velocity, which had previously been considered unlikely, provides a means of explaining the occurrence of the apparent anomaly in recent firings at Aberdeen.\(15\)

These results indicate that the linearity of crimp effect with respect to muzzle velocity, which is so desirable in calibration, very likely does not exist. The effect of crimp in a moderately worn tube, using any particular reference velocity level as an indication of tube erosion, may have various values, depending on the tube temperature. The fact that the crimp effect is more predictable in a worn tube will be mentioned again in the section on rifling design.

Powder Quickness

The linearity of the effect of differences in powder quickness was next investigated using the two types of engraving resistance patterns described in the preceding section. This was done using the quickness and velocity parameters for 'fast' and 'slow' powders previously mentioned in the section entitled Engraving Resistance Patterns. Since the new and worn tube velocities for rounds having
these powders had been computed in connection with the section on Engraving Resistance Patterns, computations for tubes of intermediate wear only were required. The two assumed engraving resistance patterns differ only for tubes of intermediate wear, therefore, the new and worn tube computations were usable under either assumption.

The results of all computations made for this purpose are included in Part C of the Summary of computations, and are shown graphically in Figure 8. Briefly, these computations consisted of determining the velocity level of rounds with the fast and slow powders under each of three conditions corresponding to tubes of intermediate wear in addition to the new and worn tube computations previously performed. Two of the intermediate conditions were of the type of Assumption 1, preceding section, and one was of the type of Assumption 2.

Figure 8 indicates the effect of differences of powder quickness to be reasonably linear under either assumption. These assumptions are believed to represent two opposing extremes of the possible patterns of engraving resistance. Thus, although proof of the linearity is impossible in the absence of more exact knowledge of the engraving resistance, this evidence gives credence to the hypothesis of linearity of the powder quickness effect which is desired in ammunition calibration.

**Projectile Weight**

The estimation of the change of muzzle velocity which results from the change of projectile weight is frequently desired in analysis of ballistic data and in
as shown above, decrease only about 10 percent. The use
of a constant differential coefficient, regardless of tube
condition, appears to be entirely reasonable, inasmuch as
this value is only used for small deviations from standard
conditions. Also, the value deduced by Hitchcock's method
is intermediate between the new and worn tube values, which
is ideal if a constant value is to be used.

The shell weight effect, further, may be seen to be
linear relative to the change of velocity produced by tube
wear, if the effect of quickness is linear in this manner.
The reduced equation contains the projectile weight only
in the burning rate equation (page 16), in which it enters
as one of the factors of the quickness term. A change in
shell weight is thus related to a change in quickness and
is linear in its effect if the quickness effect is linear.

Rifling Design

Several aspects of the computations may already have
suggested to the reader that the engraving resistance
in the early stages of projectile travel accounts for many
of the variations of velocity level in ammunition. All
these velocity variations contribute to the inaccuracy of
artillery fire and are, therefore, most undesirable in
service. The primary goal of much research in the design,
production and usage of ammunition is the elimination of
these velocity variations and their effects. A summary
of the velocity variations stemming from engraving resist-
ance shows the following:

First, there is the large drop of velocity from a new
to a worn tube condition, which is indicated to be at least
ballistic design. These differentials are ordinarily determined using a method devised by Hitchcock (16) which is based on Bennett's Tables. This method gives a differential coefficient which expresses, for example, the percentage change in velocity corresponding to one percent change in projectile weight. Thus, although the original tables do not make any allowance for the change of the effect of weight with erosion, if the same differential coefficient is used for a new and a worn gun, as is customarily done in practice, the procedure tacitly implies that the differential effect is proportional to the velocity level.

The use of the new and worn tube engraving resistance patterns permits the testing of the proportionality of the weight effect to the velocity level. For this purpose, computations were made with the shell weight increased by 10 percent and decreased by 10 percent, with both new and worn tube engraving resistance patterns (Part II—Summary of Computations). A comparison of differential coefficients obtained in this manner with the one determined by Hitchcock's method follows:

<table>
<thead>
<tr>
<th>Method of Determination</th>
<th>Differential Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Tube Engraving Pattern</td>
<td>-.55</td>
</tr>
<tr>
<td>Worn Tube Engraving Pattern</td>
<td>-.51</td>
</tr>
<tr>
<td>Hitchcock</td>
<td>-.33</td>
</tr>
</tbody>
</table>

The computed differentials decrease from 96 f/s in a new tube to 79 f/s in a worn tube, a change in the differential of about 20 percent. Because of the corresponding decrease of muzzle velocity, the differential coefficients,
two-thirds attributable to loss of engraving resistance. Thus, if a tube could be designed so that the engraving resistance remained constant, at least near the start of projectile travel, the primary cause of velocity loss would be eliminated.

Next, there is the variation of velocity caused by differences of crimp which, if the engraving resistance were constant, would continue to exist but would then be predictable and constant. This would eliminate the fluctuations of the crimp effect which appear to render ammunition calibration uncertain. Similarly, differences of velocity which result from differences of powder quickness would be constant, as would differences accountable to variations of rotating band seating. These constitute all the factors which cause the calibration of an ammunition lot to vary; hence a single calibration should be sufficient to specify the velocity behavior of a lot of ammunition, if the engraving resistance could be held constant. Even more desirable, ammunition lots could be produced so that all lots would have the same velocity in any one weapon. At present, the differences of velocity between lots are largely affected by the weapon used and its condition of erosion, temperature, etc.

This naturally leads to the question of the feasibility of producing tubes in which the engraving resistance remains constant in the critical region near the start of travel. The obvious way to achieve this is to make the engraving resistance zero or nearly so during the first few inches of travel. A proposal of essentially this nature has been previously made by the author, (5) based on empirical data. The only alternative method of
maintaining constant engraving resistance near the origin of rifling in artillery tubes requires the use of tube metal which is neither subject to erosion during firing nor to thermal expansion. The former method is the more reasonable.

A rifling design which would cause the engraving to take place after a few inches of projectile travel, as proposed by the reference cited above, is quite similar to the rifling which exists in a worn tube. The engraving resistance is concerned. Two immediate objections to such a design are apparent. First, shearing of the rotating bands and, second, low muzzle velocity occur in worn tubes. It is these things which eventually render a worn tube unserviceable. These objections have been countered in the proposed design: the first by the use of a progressive twist rifling and the second by demonstrating that, by adding powder, full service velocity may be attained with somewhat less chamber pressure than results with the present rifling. One might also reasonably expect to attain service velocity without any increase in powder charge, if this were desirable, by the use of a powder of smaller web.

Two other results of the proposed rifling design may be anticipated from its analogy to the worn tube. The dispersion of velocity within an ammunition lot is known to be smaller in a worn 90mm tube than in a new tube. This probably happens because of the elimination of the effect on velocity of certain variables, such as the rotating band seating, with the decrease of engraving resistance. In current calibration firings in the 90mm 52, the standard deviation of velocity within ammunition lots is only 4 1/2/o in worn tubes as compared with 7 1/2/o in new tubes. Another undesirable peculiarity of the present new
tubes, the conditioning effect of powder of one chemical composition on subsequent rounds of another chemical composition, disappears in worn tubes. This conditioning effect causes velocity trends of as much as 30 f/s with consequent increase of velocity dispersion. Both of these effects, which are discussed and documented in the report mentioned above, should, in a tube having the engraving delayed for a few inches, behave in much the same manner as they do in a worn tube. These are decided advantages to be expected from delaying the engraving.

The computations undertaken herein provide data from which the proposed rifling design might be modified. For example, the original proposal called for a smooth bore travel (no engraving resistance) for a distance of five inches followed by a region five inches in length in which the lands gradually assumed full height, but did not spiral. The progressive twist began after an additional straight travel of five inches. The results shown in Fig. 5 indicate that a free run of less than that originally proposed, perhaps one and one-half inches instead of five inches, should be adequate to move the region of engraving to a position at which its effect on the muzzle velocity is small. If this change is adopted, the twist of the rifling could be modified to reduce the maximum rotational acceleration imparted to the shell. This would extend the usable life of the tube, in so far as failure caused by the shearing of rotating bands is concerned.

A preliminary investigation of the behavior of such a rifling design could be made by modifying a 90mm tube of the present type so as to approximate the proposed design. The existing tubes have rifling of uniform twist;
however, by machining one of these tubes so as to move the origin of rifling forward to provide the desired free run, and then tapering the width of the lands to give the effect of a progressive twist for a short distance, the proposed design would be approximated. Arrangements are now being made to modify a tube in this manner at Aberdeen Proving Ground, and to conduct firing tests to investigate the velocity behavior of various types of ammunition in the modified tube.
Conclusions

The magnitude of the engraving resistance in a new 90mm, M1, tube appears, from interior ballistic computations, to correspond to 7500 psi. This value is consistent with the limited amount of experimental data available. Resistance encountered after a fairly short travel, about 1-1/2" or one rotating band length, has only a small effect on powder combustion and, hence, on velocity.

The resistance produced by the engraving of the forward half of the rotating band has much more effect on velocity than that produced by the rear half. For this reason, design, production and inspection of rotating bands for artillery shells should give primary consideration to minimization of dimensional variations on the forward half of the band.

The variation of the velocity effects of differences in powder quickness relative to tube erosion, as indicated by the velocity of a reference complete round, are reasonably linear. The same is true of variations of projectile weight. This actually was only shown to hold for two widely differing types of engraving resistance patterns but, since these appear to represent opposing extremes, the conclusion seems valid. On the other hand, the effect of cartridge case crimp was shown to be remarkably affected by the engraving pattern existing in the tube. Under one assumption as to the form of the engraving pattern, the effect of a difference of crimp is distinctly curvilinear. Engraving resistance patterns of the type assumed, in the above instance, may result from changes of tube temperature. Therefore, the effect of cartridge case crimp does not
appear to be linear relative to the tube erosion, as indicated by a reference round velocity level. This effect is not indicated to be predictable, even in algebraic sign, from the reference velocity level except in very worn tubes.

Many benefits may be achieved by elimination of engraving resistance, or by causing it to occur after the projectile has travelled a short distance. Among these are (1) reduction of rate of loss of velocity caused by tube erosion (2) reduction of dispersion of velocity within ammunition lots (3) elimination of differences of velocity accountable to differences of rotating band seating between shell lots (4) elimination of tube conditioning effects on velocity and (5) elimination of the change in relative velocity level which accompanies tube erosion in firing rounds containing powder of differing quickness. The computations made in this report have provided the basis for modifications to be made to a 90mm, M1 tube in an attempt to realize the above benefits.

Grateful acknowledgment is made of the advice and suggestions of Dr. V. E. Parker and Dr. Harold Peeny of the University of Delaware and of Dr. J. P. Vinti of the Ballistic Research Laboratories. Valuable assistance was given by Mr. V. E. Hon, Ballistic Research Laboratories, in the computation of the interior ballistic trajectories.
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Summary of Computations

A - Engraving Resistance

<table>
<thead>
<tr>
<th>Designation</th>
<th>Engraving Pressure Magnitude</th>
<th>Region of Action</th>
<th>Crimp Resistance</th>
<th>Projectile Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
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<td>No</td>
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</tr>
<tr>
<td>A-2</td>
<td>2500</td>
<td>Start only</td>
<td>No</td>
<td>2596</td>
</tr>
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<td>A-3</td>
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<td>0 to 1</td>
<td>No</td>
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</tr>
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<td>A-5</td>
<td>&quot;</td>
<td>0 to 2</td>
<td>No</td>
<td>2639</td>
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<td>A-6</td>
<td>5000</td>
<td>Start only</td>
<td>No</td>
<td>2613</td>
</tr>
<tr>
<td>A-7</td>
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<td>0 to 1</td>
<td>No</td>
<td>2577</td>
</tr>
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<td>A-8</td>
<td>&quot;</td>
<td>0 to 2</td>
<td>No</td>
<td>2581</td>
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<td>Start only</td>
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<td>No</td>
<td>2690</td>
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<td>A-11</td>
<td>&quot;</td>
<td>0 to 1/2</td>
<td>No</td>
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</tr>
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<td>A-12</td>
<td>&quot;</td>
<td>0 to 1</td>
<td>No</td>
<td>2710</td>
</tr>
<tr>
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<td>No</td>
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<td>No</td>
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</table>

(a) Region of action is shown in units of rotating band widths from start of projectile travel. Thus 1 to 2 indicates no engraving pressure until after one band width of travel and engraving pressure active in the region between one and two band widths of travel only.
### Summary of Computations (cont'd.)

**B - Crimp**

<table>
<thead>
<tr>
<th>Designation</th>
<th>Engraving Pressure Magnitude p.s.i.</th>
<th>Region of Action (a)</th>
<th>Crimp Resistance (b)</th>
<th>Muzzle Velocity f/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>0</td>
<td></td>
<td>Yes</td>
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</tr>
<tr>
<td>B-2</td>
<td>2500</td>
<td>0 to 1</td>
<td>Yes</td>
<td>2652</td>
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<td>B-3</td>
<td>5000</td>
<td>0 to 1</td>
<td>Yes</td>
<td>2652</td>
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<tr>
<td>B-4</td>
<td>7500</td>
<td>0 to 1</td>
<td>Yes</td>
<td>2722</td>
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<tr>
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<tr>
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</table>

(b) Crimp resistance was simulated by a pressure of 1500 p.s.i. (tabular) acting over a region of approximately 1/8" from the start of projectile travel.
### Summary of Computations (cont'd.)

#### C - Powder Quickness

<table>
<thead>
<tr>
<th>Designation</th>
<th>Engraving Pressure Magnitude</th>
<th>Region of Action</th>
<th>Relative Quickness</th>
<th>Muscle Velocity</th>
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<tbody>
<tr>
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<td></td>
<td>1.091</td>
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<td>C-2</td>
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<td>0 to 1</td>
<td>1.091</td>
<td>2653</td>
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<tr>
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<td>5000</td>
<td>0 to 1</td>
<td>1.091</td>
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<td>7500</td>
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<td>1.091</td>
<td>2900</td>
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<td>C-5</td>
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<td>0.939</td>
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<td>C-8</td>
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<td>0.939</td>
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#### D - Shell Weight

<table>
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<th>Engraving Pressure Magnitude</th>
<th>Region of Action</th>
<th>Relative Shell Weight</th>
<th>Muscle Velocity</th>
</tr>
</thead>
<tbody>
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<td>D-2</td>
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<td>7500</td>
<td>0 to 1</td>
<td>0.90</td>
<td>2703</td>
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</tbody>
</table>
(a) Sectional View of Complete Round

(b) Multiperforated Powder Grain - end view

(c) Sectional View of Gun Cylinder
Muzzle velocity as a function of engraving pressure plus crimp force

Engraving pressure maintained for one band width plus crimp force of 1500 lbs maintained for 1/16 band width

Muzzle velocity as a function of engraving pressure

Engraving pressure maintained constant for excess of one band width

Muzzle velocity as a function of starting pressure only
Fig. 4
Muscle Velocity
vs
Length of Engraving Region

Engraving Pressure 7500 psi - beginning at start
of projectile travel

Length of Engraving Region

Length in Units of Band Width
Fig. 8

Interaction of Powder Quickness with Tube Erosion

Points 1, 2, 4 and 5 computed using Assumption 1 (see fig. 7) for engraving resistance pattern. Points 1, 3 and 5 computed using Assumption 2.