# KOLMOGOROV-TYPE TESTS FOR EXPONENTIALITY WHEN THE SCALE PARAMETER IS UNKNOWN

BY

M. A. STEPHENS

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# KOLMOGOROV-TYPE TESTS FOR EXPONENTIALITY WHEN THE SCALE PARAMETER IS UNKNOWN

by

M. A. Stephens*
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Summary.

This paper shows how four statistics (Kolmogorov-Smirnov, Cramer-von Mises, and the Kuiper and Watson extensions) may be used to test whether a given sample comes from the exponential distribution with unknown parameter. Simple modifications of the basic definitions make it possible to use each statistic with only one line of percentage points: in turn, these may be reduced to chi-square points. The tests are powerful than the usual Pearson chi-square test, and are very well adapted for use with a computer.

## 1.1 Introduction.

Suppose a random sample consists of $n$ values $x_1, x_2, \ldots, x_n$. We wish to test the null hypothesis $H_o$: the sample comes from the exponential distribution, with distribution function and density:

$$F(x) = 1 - e^{-\theta x} \; ; \quad f(x) = \theta e^{-\theta x} \; ; \quad x > 0 . \tag{1}$$

*This work was supported also by the National Research Council of Canada.

The parameter $\theta$ is not known, and will be estimated by the maximum likelihood estimator $\hat{\theta} = 1/\bar{x}$ . The tests given will us Kolmogorov-type statistics, i.e. those based on a measure of the difference between the sample (or empirical) distribution function $F_n(x)$ and the hypothesised distribution function $F(x)$. We shall consider four of these statistics, usually known by $D$ (Kolmogorov-Smirnov), $W^2$ (Cramer-von Mises), $V$ (Kuiper) and $U^2$ (Watson); customarily, they are given a suffix $n$ to show the dependence of their distributions on sample size, but this will be omitted.

## 1.2 Null Distributions of Kolmogorov-type Statistics.

Kolmogorov-type statistics are used to test whether a random sample comes from a given distribution; let the distribution function be $G(x)$, to distinguish from the special $F(x)$ defined in (1). By null distribution is meant the distribution of the test statistic when the null hypothesis is true. It is well known that when $G(x)$ is completely specified, the null distributions of the four statistics above do not depend on $G(x)$, but only on sample size $n$; these distributions have all been tabulated, so that the goodness-of-fit test is available. Further, the statistics have recently been modified to remove the dependence on sample size (Stephens, 1970). When $G(x)$ contains one or more parameters which must be estimated from the sample, the null distributions are changed, and the standard percentage points of $D$, $W^2$, $V$ and $U^2$ do not apply. It has been shown that, for certain types of parameter, and for certain estimators, the null distribution will depend on the family of distributions specified by $G(x)$, but not on the specific true parameter values (Darling, 1955). This will be so for the situation treated in this paper, where $\theta$ in (1) is a scale parameter and $\hat{\theta}$ is the maximum likelihood estimator. Nevertheless, the exact null distributions of the test statistics are still difficult to find; this paper gives Monte Carlo results for the percentage points. Modifications of the test statistics are also given; the modified test statistics each require only one line of percentage points, independent of $n$. These in turn may be reduced to values in a chi-square table. Results for the statistic $D$ have been given also by Lilliefors (1969).

## 1.3 Practical Considerations.

It has been well known that Kolmogorov-type statistics possess good power properties compared with the Pearson chi-square statistic; difficulty of calculation, together with the fact that $G(x)$ had to be completely specified, has presumably inhibited their use until now. For the present application there are several merits to the statistics: (a) the difficulty of estimating the parameter has been removed; (b) the power properties will still be good (see section 2.9); (c) with a computer routine, the statistics are easy to calculate, and, with the modifications removing the need for long tables of percentage points, the tests become extremely easy to apply. Similar remarks apply to testing for the normal distribution when parameters are not known; recent work on this subject is in Lilliefors (1967) and Stephens (1969a).

In section 2 the test of $H_o$ is set out. The formulas given for $D$, $W^2$, $V$ and $U^2$ come from their definitions, with the estimate of $\theta$ used in $F(x)$. The modifications are then given, and the percentage points of the modified forms are in Table 1. These percentage points are the points for the asymptotic distributions of $\sqrt{n}\,D$, $W^2$, $\sqrt{n}\,V$, $U^2$, assuming $H_o$ true and the estimate of $\theta$ used. It is possible to get some theoretical results on the asymptotic distributions of $W^2$ and $U^2$ and these are used to give $\chi^2$ approximations to the percentage points; similar $\chi^2$ approximations are given also for $D$ and $V$. A short table of smoothed Monte Carlo points for the unmodified statistics is included; comparison may then be made with the results, for $D$, of Lilliefors (1969).

## 2. Kolmogorov-type Statistics: Modifications for Testing for The Exponential Distributions.

2.1. The test is of $H_o$: that a given random sample of size $n$ comes from $F(x) = 1-e^{-\theta x}$, $\theta$ unknown. For all the four statistics we first follow these steps.

(a) Assume the $x_i$, $i=1,2,\ldots,n$, are in ascending order.

(b) Calculate $\bar{x}$, the mean of the sample, and the values

$y_i = x_i/\bar{x}$, $i=1,2,\ldots,n$.

(c) Calculate $z_i = 1-\exp(-y_i)$, $i=1,2,\ldots,n$.

The four statistics are calculated from the $z$ values.

## 2.2 The Kolmogorov Statistic D.

(1) Calculate $D^+ = \max_i (i/n - z_i)$, $D^- = \max_i (z_i - (i-1)/n)$ and

$D = \max(D^+, D^-)$

(2) Modification. Calculate

$$D^* = (D - 0.2/n)(\sqrt{n} + 0.26 + 0.5/\sqrt{n})$$

(3) Test of $H_0$. Compare $D^*$ with its upper tail percentage points given in Table 1: if $D^*$ exceeds a given value, reject $H_0$ at the corresponding significance level.

## 2.3 The Cramer-von Mises Statistic $W^2$.

(1) Calculate $W^2 = \sum_i (z_i - (2i-1)/2n)^2 + 1/(12n)$.

(2) Modification. Calculate $W^* = W^2(1+0.16/n)$.

(3) Test of $H_0$. Compare $W^*$ with its upper tail percentage points, given in Table 1.

## 2.4 The Kuiper Statistic V.

(1) Calculate $D^+$, $D^-$ as in section 2.2, and $V = D^+ + D^-$.

(2) Modification. Calculate

$$V^* = (V - 0.2/n)(\sqrt{n} + 0.24 + 0.35/\sqrt{n})$$

(3) Test of $H_0$. Compare $V^*$ with its upper tail percentage points, given in Table 1.

2.5 The Watson Statistic $U^2$.

(1) Calculate $W^2$, as in section 2.3, and then $U^2 = W^2 - n(\bar{z} - \frac{1}{2})^2$, where $\bar{z}$ is the mean of $z_i$, i.e., $\bar{z} = \sum_i z_i/n$ .

(2) Modification. Calculate $U^* = U^2\ (1+0.16/n)$ .

(3) Test of $H_o$. Compare $U^*$ with its upper tail percentage points, in Table 1.

2.6 Table of Percentage Points.

The percentage points for each statistic, for values of $n = 6,8,10,16,20,40,50,60,80,100$, were found by drawing Monte Carlo samples from $f(x) = e^{-x}$, and then calculating the statistics. 10,000 samples were drawn for each n. The percentage points for $\sqrt{n}\ D$ were plotted against $1/n$, and extrapolated to $1/n = 0$ to give the asymptotic points for $\sqrt{n}\ D$; these are the same as those for $D^*$, quoted in Table 1. Similarly for the other statistics; the points in Table 1 are the asymptotic points for $W^2$, $\sqrt{n}\ V$, and $U^2$. The actual percentage points, at the 5% and 1% level, obtained from the smoothed graphs, are given in Table 2. Those for $\sqrt{n}\ D$ may be compared with those for D in Lilliefors (1969). They give excellent agreement for low values of n; for higher n, Lilliefors' asymptotic values are lower than those in Table 1, but are based on samples not larger than 35. In Table 3, we give a table of estimated moments of the distributions; for a statistic, say T, we give $m_1$ = (sum of 10,000 T-values)/10,000, and similarly $m_k = \Sigma\ T^k/10{,}000$, for k = 2, 3 and 4. These will be of interest if any attempt can be made on the exact distributions of the four statistics.

## 2.7 Modifications.

The modifications effectively give approximations for the percentage points of a statistic; for example, setting $D^* = 0.99$ and solving for $D$, for any $n$, finds an approximation to the 1% point for $D$ at that value of $n$. Table 4 compares the approximations with the smoothed Monte Carlo values. If $\alpha'$ is the true significance level attained by an approximate point calculated for level $\alpha$, the error $|\alpha'-\alpha|$ can be seen to be negligible.

## 2.8 Chi-square Approximations to True Asymptotic Points.

An excellent approximation to the percentage points for $D^*$, given in Table 1, is

$$(2) \qquad D^*(\alpha) = 0.017 + 0.0545\, \chi^2_{20}(\alpha) .$$

Where $D^*(\alpha)$ and $\chi^2_{20}(\alpha)$ are the upper tail percentage points, at level $\alpha$, of $D^*$ and of $\chi^2$ with 20 degrees of freedom. Such an approximation is useful for computer work; given a sample, $H_0$ is tested by calculating $D$, then $D^*$, and then $U = (D^*-0.017)/0.0545$; $U$ is then output and referred to the upper tail of the $\chi^2_{20}$ table. Chi-square approximations are also useful in combinations of tests.

For the approximation (2), the degrees of freedom of $\chi^2$ was chosen to give the curvature in the tail close to that of $D^*$. Strictly, $\chi^2_{19}$ is slightly better; but the $D^*$ is derived from Monte Carlo results.

and $\chi^2_{19}$ is often not tabulated, so $\chi^2_{20}$ was used. The constants 0.017 and 0.0343 were found by matching the 5% and 1% points. Table 5 contains the percentage points given by this approximation and those for V, $W^2$ and $U^2$ which follow. Comparison with the Monte Carlo points, from Table 1, shows that they are all very good. The V* approximation, obtained as for D*, is

$$(3) \qquad V^*(\alpha) = -0.336 + 0.0295\ \chi^2_{50}(\alpha) .$$

2.8 For the $W^2$ and $U^2$ statistics, further information is available on the asymptotic distributions; the mean $\mu$ and variance $\sigma^2$ may be found exactly by methods of Darling (1955). Darling gives, for the asymptotic distribution of $W^2$, $\mu = 0.09259$ and $\sigma^2 = 0.004357$; similar calculations for $U^2$ give $\mu = 0.07176$ and $\sigma^2 = 0.0019838$ (Stephens, 1969b). This information may be incorporated to give $a + b\chi^2_p$ approximations in several ways; for a full discussion, see Stephens (1969a), where the technique was applied in connection with tests for normality. We give here the approximations obtained by choosing p as before, and then matching the mean and the 5% point. They are:

$$(4) \qquad W^*(\alpha) = 0.0460 + 0.0466\ \chi^2_1(\alpha) ;$$

$$(5) \qquad U^*(\alpha) = 0.0265 + 0.0266\ \chi^2_2(\alpha) .$$

Percentage points given by these approximations are in Table 4, together with the means and variances. The latter compare excellently with the exact values quoted above.

2.9 Power of the Tests.

It has been mentioned that Kolmogorov-type statistics would be expected to be more powerful than the usual Pearson chi-square statistic in the situation considered here. Lilliefore (1969) has confirmed this, for the statistic $D$, and has also given some comparisons, for $D$, when the distribution of the sample is actually $\chi_1^2$ or lognormal.

We have supplemented Lilliefors' results by also taking Monte Carlo samples from these two distributions, so that the four statistics may be compared. Samples were also taken from the half-normal distribution; i.e., $x$ was chosen from a $N(0,1)$ population and the absolute value of $x$ used as the sample observation. Results are given in Table 6. $W^2$ seems a better statistic than $D$, and $U^2$ than $V$. Since $W^2$ and $U^2$ are essentially a measure of the "sum" of the discrepancies between $F_n(x)$ and $F(x)$ at every point, they might be expected to detect more subtle departures from the null hypothesis than $D$ or $V$; when $U^2$ is better than $W^2$ is itself an interesting question. There are, of course, many other ways of testing for exponentiality; other power comparisons are being made and will be published separately.

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TABLE 1

Upper tail percentage points of modified Kolmogorov-type statistics

| Statistic | % level 10 | 5 | 2.5 | 1 |
|---|---|---|---|---|
| D* | 0.990 | 1.094 | 1.190 | 1.308 |
| W* | 0.178 | 0.225 | 0.276 | 0.349 |
| V* | 1.527 | 1.655 | 1.774 | 1.910 |
| U* | 0.131 | 0.162 | 0.193 | 0.233 |

TABLE 2

5% and 1% Upper tail percentage points for $\sqrt{n}D$, $W^2$, $\sqrt{n}V$, and $U^2$ for use in testing for exponentiality when the scale parameter must be estimated.

| | $\sqrt{n}D$ | | $W^2$ | | $\sqrt{n}V$ | | $U^2$ | |
|---|---|---|---|---|---|---|---|---|
| n | 5% | 1% | 5% | 1% | 5% | 1% | 5% | 1% |
| 6 | 1.006 | 1.174 | 0.216 | 0.317 | | 1.733 | 0.158 | 0.224 |
| 8 | 1.017 | 1.197 | 0.219 | .325 | 1.537 | 1.757 | .159 | .226 |
| 10 | 1.025 | 1.212 | .220 | .330 | 1.551 | 1.776 | .159 | .227 |
| 12 | 1.033 | 1.223 | .221 | .334 | 1.562 | 1.790 | .160 | .228 |
| 15 | 1.042 | 1.235 | .222 | .337 | 1.574 | 1.808 | .160 | .229 |
| 20 | 1.052 | 1.248 | .223 | .340 | 1.587 | 1.828 | .160 | .230 |
| 25 | 1.058 | 1.258 | .224 | .342 | 1.597 | 1.840 | .161 | .231 |
| 30 | 1.064 | 1.264 | .224 | .344 | 1.604 | 1.838 | .161 | .231 |
| 40 | 1.070 | 1.274 | .224 | .345 | 1.614 | 1.861 | .161 | .231 |
| 50 | 1.074 | 1.278 | .225 | .346 | 1.621 | 1.868 | .161 | .232 |
| 100 | 1.083 | 1.291 | .225 | .348 | 1.638 | 1.889 | .162 | .233 |

**TABLE 3**

Estimated moments of null distributions: Monte Carlo results, not smoothed.

| n | Statistic / Moments | $\sqrt{n}D$ $m_1'$ | $\sqrt{n}D$ $m_2'$ | $\sqrt{n}D$ $m_3'$ | $\sqrt{n}D$ $m_4'$ | $W^2$ $m_1'$ | $W^2$ $m_2'$ | $W^2$ $m_3'$ | $W^2$ $m_4'$ | $\sqrt{n}V$ $m_1'$ | $\sqrt{n}V$ $m_2'$ | $\sqrt{n}V$ $m_3'$ | $\sqrt{n}V$ $m_4'$ | $U^2$ $m_1'$ | $U^2$ $m_2'$ | $U^2$ $m_3'$ | $U^2$ $m_4'$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | | 0.673 | 0.485 | 0.375 | 0.308 | 0.094 | 0.013 | 0.0024 | 0.0006 | 1.084 | 1.232 | 1.466 | 1.824 | 0.074 | 0.007 | 0.001 | 0.0002 |
| 10 | | .687 | .507 | .400 | .337 | .094 | .013 | .0026 | .0007 | 1.106 | 1.285 | 1.563 | 1.989 | .074 | .007 | .001 | .0002 |
| 20 | | .703 | .531 | .431 | .375 | .095 | .014 | .0029 | .0009 | 1.136 | 1.355 | 1.696 | 2.225 | .074 | .007 | .001 | .0002 |
| 50 | | .712 | .545 | .448 | .395 | .094 | .013 | .0027 | .0008 | 1.154 | 1.400 | 1.780 | 2.373 | .073 | .007 | .001 | .0002 |
| 100 | | .720 | .557 | .462 | .412 | .094 | .013 | .0027 | .0007 | 1.169 | 1.433 | 1.843 | 2.483 | .073 | .007 | .001 | .0002 |

TABLE 4

Comparison of Monte Carlo and approximate percentage points for four Statistics

The values given are for the 5% and 1% Upper tail percentage points

| n | Statistic % Level: | $\sqrt{n}D$ 5 | 1 | $W^2$ 5 | 1 | $\sqrt{n}V$ 5 | 1 | $U^2$ 5 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| 10 | M.C. | 1.025 | 1.212 | 0.220 | .330 | 1.551 | 1.776 | 0.159 | 0.227 |
| | Approx. | 1.030 | 1.219 | .221 | .343 | 1.553 | 1.783 | .159 | .229 |
| 20 | M.C. | 1.052 | 1.248 | .223 | .340 | 1.587 | 1.828 | .160 | .230 |
| | Approx. | 1.055 | 1.252 | .223 | .346 | 1.590 | 1.828 | .161 | .231 |
| 50 | M.C. | 1.074 | 1.278 | .225 | .346 | 1.621 | 1.868 | .161 | .232 |
| | Approx. | 1.07 | 1.28 | .224 | .348 | 1.62 | 1.87 | .161 | .232 |
| 100 | M.C. | 1.083 | 1.291 | .225 | .348 | 1.638 | 1.889 | .162 | .233 |
| | Approx. | 1.08 | 1.29 | .225 | .348 | 1.64 | 1.89 | .162 | .233 |

TABLE 5

Chi-square Approximations for Asymptotic Distributions

| Statistic | $\mu$ | $\sigma^2$ | Percentage Points 10 | 5 | 2.5 | 1. |
|---|---|---|---|---|---|---|
| $D^*$ | 0.70 | 0.047 | 0.991 | 1.094 | 1.190 | 1.308 |
| $W^*$ | 0.0926 | 0.00434 | 0.172 | 0.225 | 0.280 | 0.355 |
| $V^*$ | 1.14 | 0.087 | 1.528 | 1.655 | 1.771 | 1.910 |
| $U^*$ | 0.0718 | 0.00204 | 0.131 | 0.162 | 0.193 | 0.233 |

TABLE 6

Power Comparisons

The table gives the percentage of 1000 samples significant, when the test for exponentiality was applied at the 10% level, and the true distribution is as shown, n is the number in each sample.

| Sample Size n | Statistic : Distribution | D | W | V | U |
|---|---|---|---|---|---|
| 10 | $\chi_1^2$ | 316 | 349 | 291 | 302 |
| 20 | $\chi_1^2$ | 545 | 599 | 473 | 498 |
| 10 | lognormal | 170 | 171 | 155 | 173 |
| 20 | lognormal | 206 | 213 | 197 | 229 |
| 10 | halfnormal | 201 | 216 | 184 | 200 |
| 20 | halfnormal | 305 | 337 | 257 | 281 |

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