REVIEW OF NEW GEOGRAPHIC METHODS AND TECHNIQUES

ROBERT J. TATA

DEPARTMENT OF GEOGRAPHY

FLORIDA ATLANTIC UNIVERSITY

FINAL TECHNICAL REPORT

DECEMBER 1969

CONTRACT DAAK02-68-C-0219

RESEARCH INSTITUTE

U.S. ARMY ENGINEER TOPOGRAPHIC LABORATORIES

Reproduced by the CLEARINGHOUSE for Federal Scientific & Technical Information Springfield Va. 22151
REVIEW OF NEW GEOGRAPHIC METHODS AND TECHNIQUES

ROBERT J. TATA
DEPARTMENT OF GEOGRAPHY
FLORIDA ATLANTIC UNIVERSITY
# TABLE OF CONTENTS

## INTRODUCTION

### Chapter

<table>
<thead>
<tr>
<th>I. Development of Geographic Thought</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>II. New Techniques in Geography</td>
<td>19</td>
</tr>
</tbody>
</table>

### I. Location

A. Bachi's Standard Distance Statistic, Nearest Neighbor | 26 |
B. Central Tendency and Dispersion | 30 |
C. Chi Square | 35 |
D. Computers, Use of | 36 |
E. Contiguity Ratio C | 38 |
F. Correlation - Regression Analysis | 39 |
G. Factor Analysis - Principal Components Analysis | 41 |
H. Fourier (Harmonic) Analysis | 43 |
I. Graph Theory | 44 |
J. Linkage Analysis | 46 |
K. Location Patterns | 47 |
L. Poisson Distribution | 49 |
M. Probability Maps | 50 |
N. Quadrant Sampling | 51 |
O. Sampling, Geographic | 51 |
P. Taxonomy Based on Distance | 52 |
Q. Testing Geographic Hypotheses | 53 |
R. Urban and Settlement Models | 54 |
S. Variance, Analysis of | 56 |

### II. Extent

A. Classification and Regionalization | 57 |
B. Correspondence of Isarithmic Maps | 58 |
C. General Field Theory of Spatial Behavior | 59 |
D. Geographic Data Matrix | 60 |
E. Geometric Interpretation | 61 |
F. New Mathematics | 63 |
G. Isotropic Plain | 64 |
H. Map Transformations | 65 |
I. Quantitative Definition of Regions | 68 |
J. Regions as Models | 69 |
K. Residuals from Regression, Maps of | 71 |
TABLE OF CONTENTS - CONTINUED

L. Robinson's Solution ........................................ 72
M. Statistical Inference ........................................ 74
N. Trend Surface Analysis ...................................... 75
O. Venn Diagrams .................................................. 76

III - \( \Sigma \)
A. Analog Models .................................................. 78
B. Gravity and Space Potential Models ....................... 81
C. Spatial Efficiency ............................................. 83

IV - \( \Sigma \)
A. Harmonic Analysis .............................................. 85
B. Markov-Chain Models .......................................... 86
C. Monte-Carlo Simulation ........................................ 88

V - \( \Sigma \)
A. Extended Point Set Theory .................................... 90
B. Game Theory .................................................... 91
C. Spatial Systems Analysis ...................................... 92

III. MOST PROMISING NEW TECHNIQUES .......................... 97

ANNOTATED BIBLIOGRAPHY ......................................... 109

APPENDIX A, Gravity Model Use to Analyze a Transportation Network .... 131

APPENDIX B, Computer Mapping ..................................... 146
INTRODUCTION

In the purchase description that the Research Institute of the U.S. Army Topographic Laboratories, U.S. Army Topographic Command published on 9 June 1967 for the initiation of this project, the following statement appears: "The discipline of geography has shown change and growth in its long history, and in recent years has made interesting advances in mathematical and statistical techniques applied to regional and systematic methodologies." "... but, before that can be done (incorporation of new techniques into problems of applied geography), a review of the literature must be made to identify the various current theories." This final technical report summarizes the results of an intensive effort to search geographic and related literature and to elicit information about new research methods from professional geographers. The scope of such a project is enormous, and obviously much selection and paring had to be done on the mass of data accumulated, in order to produce a report of manageable size and coherence. In this process, it is hoped that the "essence" of change in geographic methodology is reflected in the report, because certainly, comprehensiveness was impossible to achieve in the light of the rapidity and volume of new work development in geography.
I would like to thank the many people who helped directly or indirectly in the preparation of this report. Dr. Roger A. Leestma and several of his associates at the Geographic Sciences Division extended aid in many ways throughout the research effort. Richard W. McBride was the principal student assistant on the project, and his research efforts contributed significantly to the accomplishment of the laborious data gathering phase of the study. Joyce Polsac, Clara Field, and Thomas Kramer also provided input which was valuable in preparing the final report. The many geographers who took the time to reply to a questionnaire survey also had an impact on the ideas that emerged in the report, and I would especially like to thank them for their cooperation. Mrs. A. Becker and Sandra Harris, the indispensable typists, performed cheerfully and well. The author, of course, must accept responsibility for the final form and interpretation of the information presented. I hope that readers will find the report useful.
CHAPTER I

DEVELOPMENT OF GEOGRAPHIC THOUGHT
DEVELOPMENT OF GEOGRAPHIC THOUGHT

It can be said that geographic methods have changed very little since the time of the ancient geographers, while the techniques geographers use have changed tremendously. The basis of this statement is the distinction which Russell Ackoff draws between "Techniques" and "Methods." He defines techniques as the behavior and instruments used in performing research operations, such as making observations, recording data, analyzing data, and so forth; while methods refer to the behavior and instruments used in selecting and constructing techniques. He concludes that methods are therefore more general than techniques and thus the former are the foundations on which the latter are built. Applying these ideas to the development of the science of geography, it is apparent that "an analysis of earth space" has been the focal method for geographic studies from the days of Strabo, Ptolemy, and Erathosthenes to the present time. The technology and scientific attainment of the society in which a scholar works largely determines what data he can gather, the techniques he can use to analyze the data, and thus even the questions that can be answered by the data and techniques available.

1 Ackoff, Russell L. - The Design of Social Research
DEVELOPMENT OF GEOGRAPHIC IDEAS

The ancient geographers were concerned mostly with "filling in the outlines of the earth and its inhabitants." Prehistoric man likely had a keen sense of place and space, since his major goal of survival depended on accurate knowledge of watering places, game trails, the probable location of enemies, location of edible plants, and the location of cave home sites. In this situation, man was forced to be mobile over space, because the predominant factors of the environment on which his existence depended also were relatively mobile. When plants and animals were domesticated and surplus farm production permitted the growth of cities, man's spatial perspective changed somewhat. Because his locus of everyday activities was now more fixed in place, he had a need to define field lines, lay out city streets, determine which market center could buy farm surpluses, and where communication routes should go. Plane geometry was a technique used to solve some of these practical problems, probably as far back as 3500 B.C., by the Egyptians and Babylonians. Power accumulated in the cities and again modified man's perception of his spatial domain. Boundary lines around the inhabitants subject to an administrative - religious - economic center became a practical concern; information on the location, character, accessibility, and resources of neighboring communities was needed to

1 For a discussion of these and other geographic tasks, see: Ackerman, E. A. - Geography as a Fundamental Research Discipline
effect trade and military operations. Man's geographic horizons had thus expanded from a basic interest in the needs of the individual household to a "world-wide" concern with neighboring and distant people and places.

The drive to accumulate encyclopedic data was spurred by the necessities of developing civilization, whereas theoretical work was neglected because it was not immediately useful. The Phoenicians compiled data on far away places to aid their commerce with these lands. Alexander employed pacers and mappers in his armies to catalog the lands he conquered. And the Romans described travel routes in road books which crudely mapped roads, paths, distances, hills, rivers, and rest stations. The Romans Strabo and Ptolemy were the premier geographers of their day, the former amassing place descriptions of his Greek predecessors, while the latter wrote about map making and place determination. Questions that dealt with concepts and abstract reasoning were most often answered by mythical tales, except for the work that Greek geographers did concerning the size, shape, and generalizations about the content of earth space. The place of geography in the academic world was a puzzle to the Greeks, disciples of Plato claiming it was a branch of physics, while followers of Aristotle said it should be regarded as a part of applied mathematics. After the fall of Rome and concurrent with the rise of Christianity, a period of some 300 years elapsed during which learning was stagnant.

1 Thompson, J. O. History of Ancient Geography, pp. 124-130.
2 Brook, Jan - Geography: Its Scope and Spirit, p. 11
3 Kimble, George H. T. - Geography in the Middle Ages.
MEDIEVAL PERIOD

In the early part of the Medieval Period, learning in the Western World was dominated by church scholars, while the latter part saw the "resecularization" of learning. Geographic study in Europe at this time, like all study, was devoted to an explanation of Bible stories. With much geographic fact taken from pre-Ptolemaic work, the discipline considered such problems as Biblical site locations and the mapping of local habitations. Topics of the description of the physical environment and the peopling of the earth were considered as suitable vehicles for spreading Church teachings. Arabic geographers, drawing on the knowledge of the Greeks and Persians, added the most significant contributions of the times. Many Arabs were avid travelers, again in response to international trade and international military expeditions. Men like Ibn Batuta, Avicenna, and Edrisi described lands and peoples they had seen and even began to attempt simple scientific generalizations on climatic and erosional processes. One result of their intellectual endeavors was the establishment of some of Spain's finest universities. Instruments were invented to sharpen the perception of the eye and ear. Compasses, cross-staffs, and astrolabes aided sea and land navigation and permitted a more accurate determination of location. Reading maps, collections of written directions compiled by travelers, were used by sailors as an aid to navigation in unfamiliar territory.

1 Kimble, George H. T. - Geography in The Middle Ages
So it was by the end of this period, in the 15th Century, technology and intellectual curiosity were attuned to the rapid developments that lay on the immediate horizon.

AGE OF EXPLORATION

An explosion of knowledge occurred in the late 15th Century as a result of the trans-oceanic explorations, which in turn were made possible in part by the new spirit of inquiry and the accumulation of learning from previous centuries. Again it was an economic motive which justified the expense of the explorations. Trade routes to India and the Spice Islands were vigorously sought by competing maritime European nations. Initial successes in rounding Africa and in the discovery of the New World generated more explorations and again increased the rate of data accumulation. Subjects such as geodesy, oceanography, climatology, meteorology, geomorphology, hydrology, and of course cartography benefited greatly from the flow of knowledge. The job of cataloging information on the size and shape of land bodies, water bodies, and the sea routes among land areas was an urgent need. This descriptive task occupied most of the geographers of this era, and little was done to systematize the data for the sake of general explanations. Accomplishments of the three centuries beginning in 1500 included more precise surveying, the determination of time, size and shape of the earth, outlines of the continents, a grid system of longitude and latitude, the invention of the map projection, and a start in the filling in of the content of earth space with physical and biotic information was made. This period of data accumulation and mapping technique development was instrumental in setting the stage for the process-oriented geography of the nineteenth century.
NINETEENTH CENTURY

The effect of the scientific intellectual bent of the nineteenth century on geography was the great impetus in the discipline given to studies on physical geography and the relative neglect of human geography on the grounds that it could not be studied scientifically. With a large backlog of geographic data in hand, scientists of the times could concentrate on categorization and differentiation of mapped phenomena. Based on the philosophy of science of Immanual Kant, geography came to be known as a chorological science, one that deals with the association of diverse phenomena in segments of earth space. Two intellectual giants arose at this time in Germany, Alexander von Humboldt and Carl Ritter, both of whom molded the development of geography by the example of their work. Von Humboldt devoted much of his efforts to the systematic accumulation of carefully measured observations from far-ranging field work. Ritter's major contribution was the treatment of regions on the earth's surface as units which gained their character from the interplay of all sorts of human and physical factors. The deaths of these two men in 1859 left a void that proved difficult to fill, and the resulting lack of leadership precluded any strong development of the science of geography for several decades. By the close of the century, Friedrich Ratzel in Germany, William Morris Davis, John W. Powell, George P. Marsh, and Ellen C. Semple in the United States had established a strong physical base for geographic study, carrying it so far as to claiming that the physical environment was a major determinant of human behavior. So strongly was this latter idea presented in the United States, termed environmental determinism, its impetus influenced geographic thought even into the twentieth century.
THE MODERN ERA

The modern era in the development of geographic thought, measured by a fundamental reorientation of the methodology of the discipline, did not occur until the twentieth century was about half over. Perhaps the strongest component of the field of geography at the beginning of the century centered on the study of landforms. Process and the development of erosional surfaces occupied the best minds in geography and various descriptive models of landform development were hypothesized. Much of the work, and its carry-over into the underdeveloped human side of geography, was almost exclusively theoretical in nature, with virtually no thought given to the testing of hypotheses with empirical data. One of the most quoted apologies for this lack of a scientific approach to geography was the complaint that geographic variables were far too complex to permit valid scientific analysis. This line of thinking was reinforced in the 1920s when geography made a swing to the human aspects of the field, since it was thought that human behavior could not be subjected to rational analysis. Another important bit of philosophy in methodology characterized the thought of prominent geographers of the day, verbalized in Carl Sauer's classical article, "The Morphology of Landscape," reasoned that geographers should concentrate their efforts exclusively on the spatial structure of the phenomena they studied rather than to delve into the functional relationships among the phenomena. This idea was instrumental in retarding the scientific development of the field, because it encouraged geographers to think that "mere description" was the legitimate goal of their discipline, thereby seducing would-be scientists from the more difficult concern with process and functional relationships.
A generation of geographers, from about the 1920s through 1950, produced a literature which was long on fact and short on theory. Many studies were made, often based on data collected in the field, which described the geographic content of some, usually small, segment of earth space. This inventory stage, however, was rarely followed by an analytical stage which was designed to seek generalization from the specific data and relate the findings to some broader understanding. The superficiality and lack of scientific merit in this work began to nag on the conscience of the profession in the early 1950s. Fred K. Schaefer's 1953 article, "Exceptionalism in Geography: A Methodological Examination," sounded the call to revolution. In this article, Schaefer stated that the nature of geography did not make it analytically unique, as the previous generation claimed, thereby dispatching the notion that the scientific method could not be followed in geography. He called for theoretical constructs which could explain and predict the spatial dimensions of phenomena, and in this way escape the sterile description which was leading geography to intellectual naivety. A period of rapid development of a scientific approach to geographic problems characterizes the most recent period in the evolution of geographic science.

For the past sixteen years, a major portion of the new generation of geographers has been concerned with the development of scientific geographic methodology. For too long, the older generation of geographers have proclaimed that the scholarly goal of geography was to understand "the total integration of factors that give varying character to places on the earth's surface." The new generation believes that his ambitious goal is impossible
to attain at the present time. A first step towards the goal, however, can and must be made by an intensive investigation of the functional characteristics of component parts, or sub-systems, of the great man-environment complex. Furthermore, in conformance with the successes and trends in the other social sciences, an understanding of man's spatial behavior must be sought through the use of scientific methods which are firmly rooted in quantitative techniques. Armed with this mandate, the younger geographers have produced many statistical and mathematical models to describe, explain, and predict the spatial dimensions of human activities. Ian Burton claims that the scientific, quantitative revolution in geography is now over, 1 his implication being that the new approach to geographic study has proved its superiority to older approaches and the task remaining is to articulate the major concepts 2 that the new orientation of geography has laid open.

THE QUANTITATIVE ERA

Machine technology and quantitative analysis have given geography much more powerful tools with which to examine the man-environment system. As a result of the increased scientific capabilities provided by the computer age, geographers have joined with other scientists in developing problem-solving mathematical models that would have been too laborious or complex to solve without machine aid. Much of this work in model-building has been theoretical,


2 See: Kuhn, Thomas S. - The Structure of Scientific Revolutions - p. 24
without empirical verification, Peter Gould has often stated that our analytical abilities in geography today have outstripped the capabilities to collect pertinent data to test our ideas against. Thus today there are many statistical and mathematical models in the field which seem to hold much promise for adding depth and breadth to our geographical knowledge, but the concepts and theory to support these models have not been thoroughly tested. The specific ability of these models to explain real-world data and to sharpen our insights about the man-environment complex remains to be proved. Evaluation of the new techniques is therefore a worthwhile project for anyone who makes practical use of geographic methodology. Identification of the most promising new tools can be made based on the rationality of the analysis, and the trends in hardware development, even though hard data may not be available to verify the techniques.

TWO PREDOMINANT PHILOSOPHIES

The two predominant philosophies that exist in geography today might be termed "the scientific school," and for want of a better term, "the non-scientific school." Geographers who approach their subject matter scientifically are concerned with the "generation of knowledge without end." That is to say, they seek universals in the data they study, hoping to find generalizations about the real world that effectively explain reality. The discovery of such a system permits one to understand the interconnections between the factor studied and the other functional parts of the system to

---

1 Personal Communications with Peter Gould, 1960-1963

2 Ackoff, Russell - The Design of Social Research
which it belongs. Thus, not only is the phenomenon immediately under scrutiny understood, but a knowledge of the conditions of the environment (both physical and cultural) that in any way affect the phenomenon permits valid inferences about similar phenomena which have not been observed. One of the characteristics of the scientific study, then, is that its conclusions have wider applicability than the immediate bounds of the problem at hand. Hence much of the literature from this school concerns itself with techniques of analysis, ways to interpret a set of data whereby more information is acquired.

Typically, but not necessarily, the scientific school utilizes statistical and mathematical techniques in pursuing its studies. Mathematics is a system of logic. Mathematical definition of terms is precise, concise, and readily understandable to other trained scientists in the field. Furthermore, quantified variables can be manipulated by machine processes which are more accurate, faster, and far more complex than would be feasible by hand calculation. Vast quantities of data can be stored and recalled in various combinations and in various formats, including machine-printed maps. Results of statistical-mathematical studies can be checked and modified in the light of new data and thus possess the invaluable scientific property of a building-block nature. The scientific geographic approach uses the scientific method discussed above to expand geographic knowledge by a community effort, one project advancing the bounds of another project, the cumulative effect being the generation of knowledge without end.

The goals of the non-scientific school are the same in substance, but their methodology hinders rather than furthers these goals. This group seeks
inspiration and definition of their legitimate areas of scholarly
interest in the works of great geographers of the past.\(^1\) This is
patently a logical error, because it denies the value of the scientific
evolution of the discipline. A scholarly task to the nineteenth century
German geographers today could be a naive study, unworthy of attention
by professionals. Cataloging space data once was an essential job
for geographers, whereas today, profound work requires lucid analysis
and meaningful interpretation of the data. The attitude of the "Traditional
school" is that the complexity of geographic variables precludes the
application of scientific methods. This outlook produces studies which
have been called "descriptions of unique entities." Research in this vein
typically involves a careful empirical description of the structure or
functions of phenomena or events. Any generalizations derived from the study are
usually only descriptive summaries of the data at hand. Definitions of
terms are not often made, hypotheses concerning the relationships of vari-
ables in both the observed system and broader universe to which the system
belongs are not posed, and the results of the studies therefore rarely
generate information beyond the bounds of the original problem statement.

The non-scientific geographers are usually the regionalists of the field.
Regional geographers attempt to understand the overall effect of the inter-
relationships of all sorts of phenomena that are associated in various
places. In trying to comprehend the totality of factors in an area, these

---

\(^1\) For exposition of these ideas, see: Hartshorne, R. - *The Nature of
Geography*; James, P. E. and Jones, C. F. - *American Geography:
Inventory and Prospect*; Murphoy, Rhodes - *The Scope of Geography*;
and Brook, J. - *Geography: Its Scope and Spirit*
geographers most often condemn themselves to only a superficial understanding of these factors. Their real forte is an almost innate understanding of the cultural environment of their place of interest, which is inculcated in their thinking from years of study and/or residence in the area. All too often, however, this understanding is a nebulous, mystique-like part of their knowledge and it therefore is never communicated effectively to fellow workers in the discipline. Perhaps much of this "feel for the area" cannot be communicated, but in order to have scientific value, as many variables as possible must be put into a standardized form that permits other scholars to understand and utilize these factors. Because the traditional geographic tools and the results that they produced lack scientific merit, many geographers, spurred on by the rapid advances in all of science, have recently attempted to regenerate geography into a modern-scientific discipline.

APPROACH OF THIS REPORT

In light of the rapid development of geographic methodology, the purpose of this study is to describe some new techniques being utilized in geography and to evaluate their possible usefulness for various military applications. One major source of data for the study is published material, both books and articles from professional journals. Since a plethora of methodological works is being published these days, it is fortunate that various pairs of geographers have put together
anthologies of studies that the editors selected as particularly valuable contributions to the literature. The existence of these books, listed in the annotated bibliography, not only aids in the collection task, but it also provides a grouping of articles from which research trends can be deduced. From the outset of this project, it was felt that an attempt must be made to include some of the unpublished studies which represent current thinking of members of the profession. Each member of the Association of American Geographers as listed in the 1967 Directory of the organization who indicated a professional specialty in methodological topics was contacted by questionnaire and asked to report any unpublished ideas concerning research techniques. (The summary report of responses to this questionnaire survey is included in this report as Appendix C).

Personal contact was another method used to gather data. Discussions with geographers at the University of Kansas, Harvard University Center for Computer Graphics, the annual meetings of the Association of American Geographers in Washington, D. C., August, 1968, the annual meeting of the Southeast Division of the Association of American Geographers in Gainesville, Florida, November, 1967, the 21st International Geographical Congress in New Delhi, India, December, 1963, and the Statistical Techniques Seminar in Nysor, India, December, 1968, all contributed insights about new developments in geography.

In addition to the personal contacts, a search of geographic and related literature was accomplished. All the articles published in the Annals of the Association of American Geographers back to 1950 were
abstracted and analyzed; a similar procedure was followed for *Economic Geography*, with the idea that a more specialized journal such as this might reveal some differences in research trends from the more generalized journal. In order to discern changes in methodology and common problem-type geographers have been interested in, a study was done of the historical development of geographic thought. The major results of this study were summarized in the first section of this report, and some of the ideas are used in the discussion on specific techniques which follows. Several sources on general scientific methodology and on social science methodology were used to compare developments in geography with developments in the overall scientific community. This source also revealed many of the developments in machine technology which should be understood to evaluate the potentialities of development trends in geography’s techniques. Bibliographic accumulation from articles, books, questionnaires, and colleagues helped to identify the major groups of statistical models with which geographers are working today. They are analyzed and evaluated in the following sections according to the types of problems they are designed to solve.

Two geographers wrote reports on new developments in their fields for the project. Dr. Richard Witmer, Assistant Director of the Association of American Geographers Commission on Geographic Applications of Remote Sensing of the environment compiled a lengthy bibliography on various remote sensing systems and briefly commented on new developments in this area of methodology. Professor Cecil E. Palmer did a study of water resources techniques currently used in geographic analyses, with a major focus on the role of the geographer in water resources planning and management. Both reports are included as appendices to this report, published under separate cover.
CHAPTER II

NEW TECHNIQUES IN GEOGRAPHY
NEW TECHNIQUES IN GEOGRAPHY

The new scientific geography is developing techniques and procedures which are problem-solving in nature, a systematic methodology which asks and answers significant questions about spatial relationships in the real world. Basic to all scientific endeavors is the assumption of rationality of the universe in which science operates. This assumption specifies that real world phenomena and events are not random occurrences, but rather are tied to other phenomena and events in cause-effect systems. Scientific geography is organized around the formulation and testing of hypotheses that try to understand various spatial aspects of these cause-effect systems. The goals of this approach to the discipline are not only to describe the structure of spatial systems, but also to explain their functioning and to predict the replication of similar spatial distributions and events.

Modern geography starts with the notion that there are "Spatial Dimensions" of objects and events which fundamentally influence the character of the objects and the execution of the events. Location, spatial extent, flows over space, and succession of spatial patterns (addition of the time factor) are attributes or properties of objects and events that are the subject matter of geographic analysis. Space or distance is a cost-incurring, time-consuming, and energy-absorbing hindrance to the free geographic mobility

---

1 This idea is consistent with P. E. James' discussion of "Processes" in Chapter I, James, P. E. and Jones, C. F., American Geography: Inventory and Prospect, pp. 5-6.

2 Many of the ideas in this section are derived from the following sources: Chorley, R. and Haggett, P. (Eds), Models in Geography; Berry, Brian and Marble, D. (Eds.), Spatial Analysis: A Reader in Statistical Geography; NAS/NRC Ad Hoc Committee on Geography, The Science of Geography; Cole, J. P. and King, C. A. M., Quantitative Geography.
of people, commodities and ideas. Thus it can be argued that the "friction of distance" is a basic attribute of real-world human and physical systems which must be understood if there is to be a full comprehension of these systems. For the geographer, it also means that other aspects of the nature of phenomena and events must be understood if he is to interpret the significance of the spatial dimensions. Since many central problems within each of the spatial dimensions have been defined and identified as significant topics for examining real-world systems, analytical techniques are under development to probe these topics.

CENTRAL PROBLEMS OF SPATIAL DIMENSIONS

Some geographers have contended that the description and explanation of location is the central problem of all of geographic study. Certainly this spatial dimension has been a focus for geographers' attention since the earliest scholarly studies, because the "where" of data is an inherently geographic theme. The interpretation of the location problem has been sharpened and broadened by modern geographers. The concept of accessibility (the ability to move from one place to another) is an aspect of location that has direct application to many practical situations. The size and spacing of nodes (such as in a system of urban central places) can be studied to learn about the structure and functional relationships of nodal systems. Related to this latter idea is the concept of a hierarchy of nodes and their location within the system. Density and intensity of various types of point patterns in an area

1 See: Bunge, William, Theoretical Geography
is yet another important locational question. The functional connections among locations (as between manufacturing plant sites and source locations of the factors of production) emphasizes the dynamic qualities of a spatial system. The concept of a spatial equilibrium connotes the idea of efficiency of locations in a connected nodal system. These and other aspects of location form a set of problems which can be claimed as distinctive objects of geographic study.

SPATIAL EXTENT

Spatial extent of phenomena or events is almost universally recognized by geographers as a core problem in their discipline, usually comprising the idea of the homogeneous region. Regions can be defined and investigated on the basis of a single factor or on a composite of many factors. Traditional geographers have developed the concept of the "compage," which they define as the totality of factors that give character to places. Modern geographers have rejected the legitimacy of the compage as a current research goal on the theoretical grounds that the task is impossible, given the present capabilities of our analytical tools. They argue that the character and meaning of the spatial extent of individual factors must be thoroughly understood before a reasonable attempt can be made to understand the interplay of many factors.

Major efforts have been made by the new school of geography to develop concepts and techniques for analyzing single factor regions. The problem of studying the character of places based on one or several related factors

---

1 See: James, P. E. and Jones, C. F., American Geography: Inventory and Prospect; and James, P. E., "The Regional Concept and the Regional Method," p. 21.
attracts the attention of many geographers. Important in this effort is the definition of terms that are widely accepted and therefore usable by co-workers in geography and related disciplines. The terminology and concepts of systems analysis are currently being promoted as the means toward this end. Much work is needed in this area because many of the ideas in systems analysis as applied to spatial problems are nebulous and lack clear reference to real-world situations. The size, shape, and related functional characteristics of hinterlands (tributary areas) for economic, cultural, and physical regions are being defined and studied with new techniques of analysis. The statistical surface of variations in intensity of an explanatory factor, including the gradient and direction of change in the factor, is an important concept that is being operationalized. Related to this type of analysis is the problem of measurement of the covariation of two or more areas of spatial extent. A series of techniques is being explored to solve this problem. Another interesting experimental area concerns individual's perception of space. This idea indicates that some aspects of human behavior and some aspects of the "cultural environment" can be discerned by people's "space preferences."

SPATIAL FLOWS

Edward Ullman has pioneered the development of a new focus of geographic research, the movement of people, commodities, and ideas over space.¹ This is a topic of "hard core" geography because the significance of the distance

variable is central to the problem of spatial flows. Again the concept of accessibility looms significant, in terms of cost, time, and various expressions of energy absorption. Communications networks, linkages among places, centrality, constraints to flows, the direction and volumes of traffic, and complementarity among functionally related places all are aspects of the spatial flow problem to which new geographic techniques are being applied. Movement over space is a particularly important subject because process and functional relationships are stressed, thus implying a further significant dimension of geographic analysis, the historical evolution of earth-space content.

SPATIAL SUCCESSION

Changes in space over time is another major area of geographic concern, recognized by the "main stream" of geographers of the early 1950's, but infused and enhanced with new insights by the new scientific approach to geography. Topics such as the growth or decline of population, income, industry, trade patterns, or cities engaged geographers since at least the time of the 19th century regionalists, who attempted to explain the apparent growth advantages of one place at one time with the advantages of another place at another time. Torsten Hägerstrand's research opened up fruitful new lines of geographic inquiry with his work on the spatial diffusion of innovation waves. Spatial diffusion examines both time and space variables.

1 See: James, P. E. and Jones, C. F. (Eds.) - Op Cit.

2 See: Hägerstrand, Torsten - Innovation Diffusion as a Spatial Process.
to trace the quantitative growth of a factor or adoption of a new idea. This general formulation has many potential applications in geography. An older version of the basic ideas of change through time was the study of sequent occupancy of an area. A new topic along this line is the concept of zones of cultural contact, or frontiers, or zones of ideological conflict. Again the main theme concerns the spread of differing ideas, this time towards a place where assimilation or synthesis of the ideas occurs. Spatial succession is really a topic that cuts across all previously discussed lines of geographic research. It contains the normative implication of planning for the accomplishment of a desirable spatial allocation of resources.

METHODOLOGY

Methodology is a topic that is not a spatial dimension, but rather it is the means for studying and gaining knowledge from the spatial dimensions. Since geography today is undergoing a scientific revolution, much effort is being expended in developing new techniques to do the old jobs of analysis and synthesis better. In the course of these methodological explorations, new lines of inquiry become feasible as research technology advances and the new lines of inquiry are suggested as new questions are raised from better answers to older research problems which are now possible. In the following paragraphs, an analysis and evaluation of some of the new techniques currently under development in geography and closely related disciplines will be discussed. The discussion of techniques will be organized according to the four basic geographic problem areas outlined above: Location, Spatial
Extent, Spatial Flows, and Spatial Succession. Techniques sometimes overlap several or all problem areas, but the problem-oriented organization is here considered significant for emphasis of the ongoing nature of methodological development, comprehension of which should also permit useful predictions about future development. Specific topics to be discussed in the following analysis and evaluation of each technique are: basic premise and methodology; criteria for data measurement; educational level required for easy understanding and use; techniques for graphic presentation; and mathematical and/or statistical techniques employed; potential applications to military geography; adaptability to an ADP environment; and comparisons of contrasting schools of thought. While some techniques obviously fit more than one major line of geographic analysis, each technique is discussed only under its most appropriate application. At the end of the discussion on a technique that can be used for other purposes, the other purposes will be suggested. Techniques which apply to all problem areas will be treated under the heading of methodology.

SIGNIFICANT NEW GEOGRAPHIC TECHNIQUES

I - LOCATION

A. Bachi's Standard Distance Statistic, Nearest Neighbor - The objective of the nearest neighbor statistic is to describe a pattern of points in a given area, according to whether the points are evenly, regularly, or randomly spaced. The first step is to plot the point distribution on a true-scale map.

1 For additional reference, see: Haggett, P., Locational Analysis in Human Geography, pp. 231 - 236.
Measurement is then made directly from the plot in any convenient linear measure from each point to its nearest neighbor. Values are processed through the formula: $R = \frac{\bar{r}_A}{\bar{r}_E}$ (where $R$ is the measure of the distribution, randomness assumed), $\bar{r}_A$ (mean actual distance to nearest neighbor), $\bar{r}_E$ (mean expected distance to nearest neighbor if a random distribution exists). $\bar{r}_A$ is derived by dividing the total distances of all points to their nearest neighbor ($\frac{\sum d}{n}$) by the number of points being examined ($n$); $\bar{r}_E$ is derived by dividing unity (1) by two times the square root of the density of points in the region ($2 \sqrt{\frac{\rho}{\pi}}$), where $\rho$ is the overall density of points in the region. The value of $R$, the nearest neighbor statistic, can vary from 0 (points are agglomerated at one place), to 1 (signifying a random point distribution), to a maximum of 2.149 (which signifies maximum spread of points, thus even spacing).

The principal value of the nearest neighbor statistic is that it is a systematic procedure for describing a point pattern accurately and intelligibly for communication with interested parties. In dispelling ambiguity, the technique like all other scientific procedures permits comparability of point patterns at different time periods and at different places. Argument arises over the interpretation of intermediate values for the statistic. In describing a given pattern, definitions of 0, 1, and 2.149 values are clear, but the question about category dividing points (i.e. at what specific value does a distribution cease to be agglomerated and take on the property of randomness) is debatable. Another problem of interpretation arises
when point patterns are clustered in small groups which in turn are scattered throughout the entire area. Problems such as this of extreme spatial skewness can be handled by modifications to the basic program. One particularly valuable analysis that can be accomplished with the nearest neighbor technique is the comparison of the same area's point pattern at different times. Changes in the pattern over time might reveal whole new orientations of central place systems, or the rationalization of previously randomly located factories, or some other such significant change.

The mechanics of calculating the nearest neighbor statistic are simple enough to be handled by a technician with only a basic knowledge of college mathematics, but the interpretation of results requires graduate level training in modern geography. The measurements of distances between points is a common task among laymen who read road maps. This task will probably be completely automated in the near future with various kinds of scanning devices feeding information into a computer. Interpretation of the relative agglomeration - randomness - even spacing characteristics, however, requires a firm understanding of the various systems to which the points in the pattern belong. Central places, factories, administrative centers, command posts, and communications centers have some general functional likeness, but their specific functions and therefore the meaning of their spatial distributions and interconnections within their systems depend on their specialized purposes. Persons must be intimately familiar with these purposes for accurate and full interpretation of their spatial dimensions.
Potentially there are both tactical and strategic military applications of the nearest neighbor statistic. Many types of human (to include military) activities in earth space take the form of point patterns; they can thus be graphically presented and analyzed in terms of the geometry of the point pattern. On the strategic level, the nearest neighbor analysis could be used to explore such diverse spatial distributions as cities, industrial complexes for target analysis, watering points in an arid region, location of mineral deposits, or the growth of strong points in national defenses. Since the success of tactical level uses of nearest neighbor analysis would depend upon the speed of the analysis and sensitivity to relatively rapid changes in point patterns, use of the technique at this level would appear to hinge on the utilization of computers in tactical operations. Once information was gathered on a point pattern and fed to a digitizer, then periodic checks on the pattern and machine analysis might be used for such intelligence purposes as troop movements, target acquisition, planning a logistics system, shelling strike reports for artillery, and many other uses where spatial point patterns are significant. This application, along with most others based on new geographic techniques of analysis, will attain its maximum usefulness when tied into a central data bank which will permit recall and recombination of many divergent bits of information on an area to develop intelligence. These comments in general apply to most of the other techniques discussed below.
B. Central Tendency and Dispersion\(^1\) - geography's adoption of these standard statistical parameters indicates the long overdue attempt of the discipline to make its methodology more scientific. The "average" or "mean" of geographic data has been used for many years, all too often obscuring rather than illuminating relationships that the data contained. The recent application of other descriptive statistics to geographic problems has thus been employed to elicit more meaningful interpretations from geographic information.

Geographers are faced with a peculiar problem in the data with which they deal. Bunge in *Theoretical Geography* makes the point that a "single factor region" actually contains two factors, one of which is location. This idea points to the fact that by definition, a geographic factor is an object, a property of an object, or an event at a particular place. For the most part, geographers have tended to ignore this central issue, i.e. the effect of location on the variables which are commonly thought of as belonging to other disciplines. They have used statistical techniques which are designed to measure characteristics which are uni-dimensional, whereas their data are both topical and locational. The adaptation of standard statistical techniques to portray geographic relationships adequately is a topic that geographers must concern themselves with more. This subject will be explored more fully in the last paragraph of this section below.\(^2\)

---


The commonly used measures of central tendency and dispersion are the mean, mode, median, variance, and standard deviation. The value of these measures is that they accurately and concisely describe the numerical characteristics of a set of quantitative data. They comprise a shorthand representation of the original data that can then be used to analyze and synthesize data parsimoniously, to compare cumbersome sets of data, and to communicate results of data manipulation intelligibly and briefly. Central tendency and dispersion refer to the distribution of values in a set of data along the scale on which the data is measured.

An arithmetic mean is derived by dividing the sum total of all values in a set of data by the number of observations in the set. If the data values are normally distributed (the determination of normalcy will be discussed below), then the "mean value" is a valid representative of each of the individual values of the set. In a skewed distribution, one having some values much higher or lower than the majority of the values, the value of the mean is either inflated or deflated by the few deviants. This of course renders the mean value nonrepresentative of the individual observations. For a symmetrical (normal) distribution, the mean, mode, and median all have the same value. Because there are well-developed mathematical techniques for further analysis of the mean, it is the most useful measure of central tendency for normal distributions.

The mode and median have their greatest value in describing statistical distributions which are skewed. The mode is simply the value of a set of

---

1 Most standard statistical textbooks describe these measures.
data which occurs the most frequently. If there are two values that occur an equal number of times, the distribution is said to be bi-modal. The mode has only limited uses in the behavioral sciences. A median is the value in a statistical series which divides the number of observations into an equal number of values less than the median value and an equal number of values more than the median value. In a highly skewed distribution, therefore, the median value is usually more representative of the individual values in the distribution than the mean would be.

Dispersion of the values in a distribution around the mean or median is measured most commonly by the variance and the standard deviation.¹

The variance is the summation of the squares of each value in the distribution subtracted from the mean of the distribution, the whole of the summation then divided by one less than the number of observations in the distribution. In symbols: \[
S^2 \text{(variance)} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{(N-1)}.
\]
The resulting figure measures the dispersion of all observations from the arithmetic mean of the statistical distribution. The square root of the variance yields the standard deviation.

In the analysis of a normal distribution, the standard deviation is an exceedingly useful parameter. Constant percentages of the total observations in any normal distribution are defined by the mean and standard deviation of the distribution when used together. Sixty-eight percent of all observations are always found within the range of plus one standard deviation above the mean to minus one standard deviation below the mean in normal distributions. Ninety-five percent of all observations are found

within the range of plus two standard deviations above the mean to
minus two standard deviations below the mean; while ninety-nine per-
cent of all observations lie within the range of three standard devia-
tions above and below the mean value.

Standard deviation and mean have an outstanding use in geography
for rationally determining the categories of statistical data which are
to be mapped. When a choropleth map is to be drawn from a collection of
statistical data, the categories to be portrayed are often chosen rather
arbitrarily. Breaking the data into quintiles is a popular method, be-
cause this division gives an average category, high and low categories,
and very high and very low categories. Any greater or lesser number of
categories also may be used in this classification system, the result
being an approximately equal number of observations in each category.
In normally distributed statistical data that are to be mapped, the stand-
ard deviation can be used to determine the value range of data that de-
fines map categories rationally. A middle or "average" category of values
can be delimit by one-half a standard deviation above and one-half a
standard deviation below the mean value of the distribution. The next
categories would be defined as one and one-half standard deviations above
and one and one-half standard deviations below the mean. The last cate-
gories could be two and one-half standard deviations above and below the
mean. This method is considered a "rational" method for category defini-
tion, because it derives categories by following standardized statistical
procedures and because it defines categories according to understandable
parameters that accurately and meaningfully describe the statistical data
from which the map is to be drawn. To illustrate the latter point, in
normal distributions, 68% of all values fall between plus one and minus one standard deviation from the mean; while 95% of all values fall between plus two and minus two standard deviations from the mean; and 99% of all values fall between plus and minus three standard deviations from the mean. This imbues the qualitative category designations of "very high," "high," "average," "low," and "very low" with quantitative accuracy and objectivity. In this sense, then, it can be said that the categories into which data are divided are rationally derived.

The Kolmogorov-Smirnov test is one that can be applied to a statistical array of data to ascertain whether or not the data are normally distributed. Briefly, this technique compares the values of an actual statistical distribution and their cumulative percentages against a curve of values and cumulative percentages that would appear in a perfect normal distribution. The objective of the analysis is to determine how much deviation from a normal distribution exists in the actual data. A "Z" score value results from the calculation which then can be found in a table of "goodness of fit." This value reveals the probability that the actual distribution can be normal with its given amount of deviation from the ideal normal curve.

Techniques of central tendency and dispersion help geographers to describe the locations of their data more precisely, within the limitations of the "enumeration district problem." For much work in statistical analysis, geographers must accept data gathered by census units which may obscure real-world locational relationships. An example, the population of many cities


2 For example, see: Robinson, A. H., "The Necessity of Weighting Values in Correlation of Areal Data."
between 1940 and 1950 appears to have been static or to have grown only slightly. In reality, the cities often have doubled their populations, but most of the increase has occurred in the suburbs, outside the political city enumeration district. The functional city expresses the real-world city, whereas the political city becomes a fiction of the census taker. The ideal data unit for the geographer would be small hexagons (size depending on scale of problem) which would give total area coverage and at the same time reveal the actual spatial variation of the data. Geographers who can specify their own areal units, such as those gathering raw data directly from aerial photos or topographic maps, can insure that they gather data which reflects the actual situation, which in turn will insure more meaningful use of statistical techniques used to analyze the data. Perhaps the data problem is akin to the need for "normalcy" in statistical distributions, where geographers must add "areal normalcy" to the need for statistical normalcy.

C. Chi Square\(^1\) - this is a measure to ascertain whether or not there is a significant difference between two samples of statistical data. It is often used to compare an actual distribution to an hypothesized distribution. The formula for the statistic is:

\[
X^2 \text{ (chi square)} = \sum_{j=1}^{k} \frac{(f_j - F_j)^2}{F_j}
\]

where \(f_j\) is the observed frequency of an actual class (ith of k classes)

---

1 For more details, see: Gregory, S. *Statistical Methods and The Geographer*, pp. 163-179.

2 King, L. *Op. Cit.* p. 69
of objects, properties, or events, and \( F_j \) is a theoretical or expected frequency. Thus the statistic is used to test a definite hypothesis about the frequency distribution of values in a statistical series.

In order to utilize Chi Square to advantage, enough must be known about the situation under investigation to permit the researcher to advance a plausible estimate of the statistical distribution that could be expected under specified conditions. Chi Square in this instance would be used to validate or invalidate his hypothesis. If the deviation between actual and expected is too great (a high Chi Square value), the researcher must conclude that the two samples must be from different populations. While this latter result would deny the original hypothesized statistical distribution, it could conversely be used to differentiate categories in a group of statistical series. Thus Chi Square might be used to separate data groups into categories for mapping, or it may be used to demonstrate a relationship between classes of data. An example of the latter use might be a comparison of observed traffic flow along various categories of road types, based on the assumption that these road types have differing "carrying capacities," with an hypothesized distribution of flow that one might expect, given the different road qualities.

D. Computers, Uses of

Computer storage, analysis and retrieval of information promises not only greater accessibility to information and more facile handling of large amounts of data, but also the truly geographic interpretation of census gathered point and area data. Data stored at the

---

location it occurs is a word description of a map. Geographers have emphasized this aspect of the functions of a map, claiming that the map is thus a geographic tool par excellence because it shows "spatial relationships."

This is true to only a limited extent. Putting information "in place" means that some aspects of the Euclidean geometry of the data can be interpreted with the eye, or for more precision, with the aid of measuring instruments.

Geographers now recognize, however, that there is much more to "spatial relationships" than the traditional geometry would imply. Hence the familiar geographic concepts of orientation, centrality, nearness, friction of distance, connectivity, regional boundary, extent, covariation, and others are being rethought and reformulated to encompass new perspectives on real-world and abstract space relationships. Innovation diffusion, space perception, demographic potential, gradient of desire lines, spatial efficiency, spatial equilibrium, and others are new concepts under development. Part of the drive for new concepts has been stimulated and made feasible by the increased analytical capabilities provided by developments in computer technology.

Computer uses range from bookkeeping and computation to analysis of data, map compilation and printing, and new processes of learning. "It is by restoring the immediacy of sensory experience and by sharpening intuition that the computer is reshaping experimental analysis."\(^1\) Increasingly complex geographic data matrices of the type Berry suggests can be stored and manipulated as computer memories are expanded.\(^2\) New ideas for interpretation of data can be programmed and then experimentally run against stored data for a testing of the methodology or for the elicitation of new information from the old data. Tablet type input also promises much for geographic use as sketches, maps, or graphs can be fed directly to the computer by means of a special stylus, thereby permitting almost instantaneous interpretation.

\(^1\) Scientific American, Op. Cit., p. 114

\(^2\) Berry, B. J. L. and Marble, D. F., Spatial Analysis, pp. 24-34.
E. Contiguity Ratio C

This statistical technique performs a task for a choropleth map that nearest neighbor performs for a map of point distribution. The technique, a generalization of a one-dimensional analysis often used in time series problems, can be used for any number of dimensions. If contiguous areal units have similar statistical values, then the ratio tends to total less than unity; while a more random spatial distribution of units will yield a ratio of unity. The formula is:

\[
C = \frac{(n - 1) \sum_{i \neq t'} \left( z_t - z'_t \right)^2}{2 K_1 \sum_t (z_t - \bar{z})^2}
\]

where \( C \) is the contiguity ratio, \( n \) the number of counties, \( z_t \) the measure of the \( t \)th county, \( k_1 \) is the sum over all counties, and \( \sum_{t'} \) is the sum over contiguous counties.

One specific use for the contiguity ratio in geography would be the cartographic analysis of residuals from regression. Geographers often are interested in the establishment of functional relationships between a dependent and one or more independent variables. When there is some exploratory nature of the problem under investigation, that is when some important independent variables are unknown, the researcher will seek clues about the factors that can explain the remainder of the variance in the dependent variable which has not been explained by the known independent variables. Note here the definition of "explain": statistically, to explain means to discover a functional relationship between variables; causation is

\[\text{See: Geary, R. C., "The Contiguity Ratio and Statistical Mapping," in Berry and Marble, ibid, pp. 461-478.}\]
not claimed.) He can map the residuals in any convenient way for the purpose of further spatial analysis. Any method of analysis, the contiguity ratio for example, would be useful for seeking spatial regularity in the residuals, thereby suggesting additional independent variables.¹

F. Correlation - Regression Analysis - These standard statistical techniques have been adopted by geographers in order to define the degree of covariation of features in earth space in a more rigorous and universally understood way. King points out that there is a technical distinction between the two techniques, regression analysis specifying a dependent and one or more independent variables, while correlation analysis merely seeks functional relationships among variables without specifying dependent-independent status.² In practice this distinction is often irrelevant. Correlation and regression techniques help to shed light on one of the central problems of geographic analysis, the description and explanation of the location of phenomena.³

Correlation and regression analysis permit the geographer to describe and to predict the spatial association of data based on the average relationship between/among variables, expressed as a mathematical function. Linear regression, for instance, starts out with the plotting of the data on a cartesian coordinate graph called a scatter diagram. The resulting plot of points on the graph suggests whether or not a relationship exists between the variables, whether it is a strong or weak relationship, and whether the relationship is positive or negative by the trend of the points. The next step is the construction of the mathematical function curve which is fitted in to best approximate

³ Bunge claims that this is one of the two central problems in Geography Bunge, W., Theoretical Geography, pp. 195-196.
the average loci of the points displayed on the graph. The formula for
this best-fitting line is: \( Y = a + bX \), where \( a \) is the intercept of the
line on the Y axis, and \( b \) is the slope of the line. The parameters \( a \) and \( b \)
are derived by "least squares" methods, which are described in most standard
statistical textbooks. The coefficient of determination \( (r^2) \) is a parameter
of regression analysis that indicates the proportion of the variance in the
dependent variable which is explained by its relationship to the independent
variable. If a significant relationship is demonstrated between variables,
then the equation for the line can be used to make predictions of the value
of the dependent variable based on the known value of the independent variable.
The relative accuracy of the prediction, dependent on the sizes of the \( r^2 \)
and mean deviation, is known from the statistics that result from processing
the original data before predictions are made.

The residuals from regression are data derived from regression techniques
which help geographers to analyze the preliminary results of a problem further.
Residuals are given in regression analyses for each individual value of the
dependent variable. A residual is defined as the variance in the dependent
variable that remains unexplained, after the influence of the independent
variable has been considered. On the scatter diagram, the residual is
described as the distance between the locus of the data reading and the best-
fitting line, measured vertically from a line that represents the mean
value of the dependent variable. Residuals can be categorized by their
magnitude, and then mapped. By this procedure, the geographer can perform
a spatial analysis of the residuals, seeking meaning from the patterns
portrayed. The spatial patterns of residuals often suggest additional
independent variables that might have been overlooked in the initial stages
of the research.
This brief sketch of linear regression analysis was intended as an illustration of the general procedures and understandings that correlation-regression analyses provide. Geographers are no longer content merely to portray the spatial distribution of a phenomenon, but they now see an important scientific need to explain variations in locations and to predict the occurrence of phenomena at particular places. Correlation and regression analyses have provided the tools for the accomplishment of these tasks.

With the proper modification of variables, and definition of terms, these statistical techniques can be used for all types of geographic problems, analysis of point or area patterns, flows over space, or spatial succession.

G. Factor Analysis - Principal Components Analysis - One assumption of multiple regression analysis (not often fulfilled in practice) is that there is no interrelationships among the independent variables. Factor analysis and principal components analysis are multivariate techniques which start with a set of data which is assumed to consist of interrelated variables. The purpose of both models is to reduce the original set of variables to a simplified set of "basic dimensions." While both regression and factor-principal components analyses attempt to explain the variance of variables, the latter techniques provide a more secure estimate of the relative "weight" of each variable in describing the dependent variable. L. King states that there are mathematical and conceptual differences between principal components and factor analysis, but that often the two are treated as identical.

Because of their ability to draw the fundamental attributes out of masses of data, factor-principal components analyses are useful at the initial stages of an investigation when all of the dimensions of the problem are not well known. The variables with the highest factor loadings (derived from product-moment correlation and subsequent rotations of the factor axes) can be readily identified as those that most characterize the object, event, or attribute being studied. The principal components or factors that emerge from the original data manipulation must be interpreted to gain understanding from the analysis. Herein lies a danger, because either too much or too little read into the problem may drastically reduce the utility and power of the analysis. As in all other forms of statistical analysis, intimate knowledge of the subject will permit the analyst to make profound observations whereas ridiculous conclusions can result from interpretations by analysts with only superficial knowledge of their subject. For the geographer, factor-principal components analysis can define categories for the logical regionalization of an area. Once data has been characterized by the techniques and mapped, the geographic work of spatial analysis and interpretation can then be accomplished. Point and extent data are handled by these techniques. One caution about multivariate techniques is in order. The more complex a statistical analysis becomes, the more difficult it is to interpret the meaning of the results. Even in multiple regression analysis, the significance given to any one of the independent variables must be advanced with restraint. It is believed, for instance, that the weights of the same variables would be changed if they were entered into the problem in different orders. Whatever the drawbacks, techniques such as factor-principal components analyses are
important because most problems that concern the human occupation of earth space are multi-dimensional.

H. Fourier (Harmonic) Analysis - "Fourier series provide a convenient means for interpolation, and limited extrapolation of data that are oscillatory."¹ Data that vary cyclically along a time or distance continuum can be analyzed by the attempt to split the complex curve into its harmonic components. The technique has been used to analyze meteorological and geological data, but human geographers have not often used it. This is probably because of the need to assure periodicity and continuousness of the data.

The basic operation of the technique is to determine whether or not variations in a dependent variable can accurately be defined in terms of an oscillatory function. The Fourier series is composed of a mathematical set of terms containing sines and cosines. Amplitudes, phase angles, and frequencies are all derivable from the solution of the data, and these characteristics, may then be analyzed and interpolations may be attempted.² If the Fourier curve fits the data to an acceptable level, then the data can be described as periodic, and thus independent variables may be sought with the guideline that they too must be periodic. A single Fourier series may be used to analyze a curve, whereas a double Fourier series can be utilized to examine a surface feature. More and more statistical cultural surfaces are being explored in geographic studies and the latter technique might be appropriate to use in the conceptualization of this type of problem.


² See: King, L. op. cit., pp. 223-226, For Further Discussion.
I. Graph Theory

The technique of network geometry has been adopted by geographers and other scientists concerned with spatial problems to describe the connections and shapes of various kinds of linear networks. The relatively simple "dimensional" notions of length and area served by a transportation network have been topics that geographers have studied for some time. This conceptual base has recently been expanded to encompass such spatially significant ideas as "centrality," "connectivity," "flow capacity," "critical paths," "network efficiency," and others. Many of these new ideas have resulted from the application of graph theory to the study of the topological properties of networks. Further development of these concepts promise their extension to many other social systems whose operational characteristics can be conceptually related to those of a transport network.

Graphs of networks are composed of vertices (nodes) and edges (connecting lines among nodes). The analysis of graphs is thus a technique for examining the general geographic problem of a nodal region. Problems such as the hierarchal arrangement of central places, flows of people, commodities, and ideas among places, and classification of a major network into its sub-regions are now being studied through the use of graphs. Various indices have been proposed to describe the graph characteristics. The Beta index, defined as $e/v$ where $e$ is the number of edges and $v$ is the number of vertices, measures the connectivity of the vertices. The increasing value of the Beta index indicates that the vertices have an increasing amount of interconnectedness. Another index, sometimes called a König number, measures the centrality of nodes. This measure simply counts the edges on the shortest path between one

---

1 For a discussion of terms and basic concepts, see: Farary, F. and Norman, R., "Graph Theory as a Mathematical Model in Social Science."

2 Haggett, P., _Locational Analysis In Human Geography_, p. 238
nodr and all the others in the system. The smaller the value of the König number, the more central the vertex in the network. Shape of the network may be measured by an index designated as $\Pi$. In the expression $\Pi = C/d$, $C$ is the total mileage of the transportation network and $d$ is the total mileage of the network's diameter. The diameter of a network is defined as the number of edges in the shortest path between the most distant vertices. There is some evidence that the $\Pi$ index is both a good measure of the relative economic development status of the unit which the network serves as well as a good indicator of the economic state of the transport network. Matrix display and manipulation of data provide other useful means for analyzing networks in graph form.

While the application of graph theory to the analysis of networks seems somewhat mechanistic and stiff, more flexible ideas can be worked into the models to seek real-world interpretations on a variety of topics. Accessibility in a network might be measured in units that are more meaningful than mileage for various situations; spread of communications in a social network could be studied with only a slight modification of terms; flow barriers that occur when people, goods, or ideas must pass through nodes (bottlenecks of various sorts) would also be amenable to graph theory analysis. Graph Theory analysis of networks seems destined to be an even more significant tool of geographic inquiry, because a network is one form of "a system" and the formulation of problems in systems analysis is one of the current orientations of many young geographers. Other new techniques such as path geometry, circuit geometry, point set theory, simulation techniques, electric analogues, and more can be used with the basic ideas of graph theory to examine many types of network problems.

J. Linkage Analysis - The purpose of linkage analysis is to achieve rational taxonomy. Given a set of data about the characteristics of a population, linkage analysis can be applied to "collapse" the original set of data into smaller groups of data whose characteristics within each group are more similar to each other than they are to data in other groups. In other words, linkage analysis seeks a logical differentiation of data. One crucial influence on the success or failure of linkage analysis is the effective measurement of the attributes of the population under scrutiny.

Measurement scales should be identical for using linkage analysis when several variables are considered and the measures themselves should be clearly descriptive of a significant characteristic of the data. The original data which has one or more characteristics attributed to each observation unit can be thought of as representing complete detail in description. Linkage analysis is the vehicle to group observations according to likeness of characteristics values, all the while increasing generality while decreasing the number of observations. Linkage analysis provides a logical technique for accomplishing this task with a known amount of loss in detail as each step towards generality is taken. When many variables are involved for each observation and a technique such as factor analysis is used to produce composite factors, then the complexity of the analysis tends to obscure the meaning of the generalized categories that result from linkage analysis. It is true for the application of all statistical techniques, especially one such as linkage analysis, that judicious use of data by researchers who know the systems they are studying yields most satisfactory results. Computer processing of data, programmed for linkage analysis, supplies the researcher with a valuable tool for
regionalizing an area according to a logical procedure.

K. Location Patterns - One of the fundamental problems in geography is to describe and explain the locations of various types of phenomena. The description phase of much geographic work involves the construction of distribution maps. Basic types of distribution maps are dot maps, volume maps, choropleth maps, and isoline maps. A dot map assigns a value of the phenomenon to be mapped to one dot, and then the dots are placed on the map where the data exist on the actual earth's surface. The groupings and dispersions of the dots thus portray the spatial distribution of the phenomenon. Volume maps are similar to dot maps, except that symbols for the value of the mapped phenomenon vary in size to represent its different quantities or volumes according to an established scale. One of the most common examples of this type of map is that in which the area of a circle is varied proportional to the value for the phenomenon at a particular place. The distribution of symbols and their size portray the areal patterns in this case. Choropleth maps sum data by a set of areal units, the units are then shaded according to the categories into which the data are divided. A problem arises in this type of mapping, since it must be assumed (contrary to common knowledge) that the value for the data is spread evenly throughout the areal unit utilized. The problem theoretically reduces to one of scale, however, because the areal units can be shrunk in size (increasing the number of units) until a sufficient number of units is achieved to adequately portray the spatial distribution of data being studied. Practically, this procedure is seldom feasible. An isoline map also shows variations in the "intensity" of a factor as it varies over space. Each isoline is drawn through all the points on the map that have the same value.

1 King states that the old-fashioned linkage-cluster analysis has been replaced by principal components-factor analysis. King, L., op. cit., p. 165.
The patterns of lines on an isoline map indicate the locations and values of phenomena shown. An isarithmic map (special type of isoline) further shows the rate of change in the phenomenon as it varies from place to place.

Analysis of the patterns displayed on all four types of distribution maps can be done directly on the maps, or by machine or manual techniques from data extracted from the maps. Overlay study of covariant features, densitometer scanning of dot maps, profile construction on isoline maps are examples of the former; while many types of areal statistics might be extracted from the maps for other forms of analysis of the latter kind.

The above-mentioned uses of distribution maps are more or less traditional lines of geographic inquiry, but new techniques to pursue the same goals are emerging. One of the more interesting new techniques is computer mapping, suggested in section C. Data stored in appropriate computer format may be retrieved and analyzed in numerous ways, the results issuing forth in the form of a computer print-out map. Another new technique which shows great promise of eliciting extended types of understandings involves using non-Euclidean geometries and measures for the "friction of distance" other than simple physical distance. Map transformations, that is inversion, projection, topologic stretching, rotation, and others, are further manipulations of mapped phenomena that can provide new perspectives in geographic analysis.

Location theory has been developed principally by economic geographers and economists to help in the scientific description and explanation for locations. The report, "The Science of Geography," of the Earth Sciences Division N.A.S. - N.R.C. proclaims a hierarchy of Location Theory, based on a fusion of ideas from economic, urban, and transportation geography.

2 An IOC Committee on Geography, The Science of Geography, pp. 45-46.
Spatial structure, functional organization of spatial systems, temporal dynamics of spatial structure and functional spatial systems, and normative models are the major categories of this location study hierarchy. Spatial structure comprises the study of location patterns on the earth's surface, described and analyzed by various branches of mathematics and statistics. Functional systems in geography concerns linkages and flows, with themes such as accessibility, connectivity, dominance, and hierarchy. Temporal dynamics of course introduces the time elements. Comparative statics, process, equilibrium (borrowed from economics) are here imputed with spatial dimensions. Normative models try to derive efficiency solutions to spatial problems. They seek to discover the optimum (or minimum) spatial organization in the light of various constraints.

L. Poisson Distribution - The purpose of the poisson distribution is to analyze a pattern of points in two-dimensional space. One assumption of the model is that the process which generates the point pattern is random. Thus the criticism arises that the assumed "independence" of points in a system is not realistic. The technique can be utilized, however, where the underlying purpose of the analysis is merely to compare an actual distribution to a standard, in this case the standard is something other than a common real-world distribution. The Poisson distribution might be considered a probability model, because in its use, the researcher often asks the question: "what is the possibility that an 'event' occurs at a particular location?" A grid is selected to cover the mapped distribution, and the size of the grid squares and configuration of the grid itself become significant research operational problems that must be solved. Once the grid is selected,

1 For more detail, See: King, L., op.cit., pp. 41-45.
the procedure is to tabulate the occurrence of points in each grid cell and then compare this actual distribution to a theoretical distribution generated by the Poisson function. Perhaps the most serious criticism of this technique is that the model does not in fact describe the point pattern (as for instance the nearest neighbor statistic does), but rather it merely describes the possibility of finding points in any given quadrant. Obviously the success of the analysis provided by Poisson distribution depends heavily on the sector of reality being examined.

M. Probability Maps – One type of probability map has been suggested by Choynowski in an article entitled, "Maps Based On Probabilities." Rather than map the absolute incidence of some phenomenon over space, the technique applied in this study compares the probability against the actual occurrence of the phenomenon in order to gain further insight into the geography of variation of the phenomenon. Such a procedure eliminates non-significant spatial variation in the phenomenon that can easily be explained by differences in obviously important other variables. Taking the observed average occurrence of the phenomenon as a base and then comparing these data with a Poisson distribution, categories of deviations between actual and probable occurrence of the phenomenon helps to pinpoint areal variation which are not easily explained by change, poor sampling, or major influence from another dominant variable. Thus the probability map refines areal variation in a phenomenon into "truly significant" variation.

centers on the fact that in addition to the mathematical requirements that must be satisfied, there are also spatial requirements which must be satisfied, to insure valid geographic sampling. The usual types of samples, random, systematic, and stratified, may also be done with spatial data. For a random system, samples may be taken from the map by generating map coordinates from some pre-determined series of random numbers. In effect, this method utilizes whatever grid system is specified by the coordinate system. A systematic geographic sample starts from some randomly selected starting location. All subsequent points from which data is taken are determined by some fixed rules or repetitive procedures. A stratified geographic sample first subdivides the study area into zones or strata. Then the sample data is selected from the zones either randomly, systematically, or in some aligned fashion. Choice of the sampling system utilized of course depends upon the problem that is under consideration.

One slight modification of the above examples is the traverse. In a traverse, a sample of data is extracted from an area along some pre-determined line or traverse. One must be careful in this type to avoid biases such as taking the data sample along a road through an area. Land use, for instance, might be significantly different along the road from what it is in other parts of the region. The traverse lines across an area likewise might be random, systematic, and possibly even stratified. These types all refer to the methods in which traverse lines are selected.

P. Taxonomy Based on Distance - When a group of observations of areal data is at hand with no apparent classification scheme presented, then the distances among observations may be used to group the data.¹ This system

¹ See: King, L., op. cit., pp. 198-204.
is similar in procedure to linkage analysis, since the ultimate step in classification groupings is to have only one group composed of all observations, called a "complete linkage tree." A distance matrix is first constructed, in which observations are placed into the cells of the matrix according to their relative locations in their actual area occurrence. In a step-wise procedure, the two nearest observations can be grouped. When this is done, the distance matrix must be changed to reflect the group-of-two observations, all new distances reckoned from the center of this grouping. The question as to how many groupings shall constitute an adequate classification is a moot point. Probably the best that can be said on the subject is that the researcher with a "feel" for his data must make this decision as it arises with a view toward his objectives clearly in mind.

Q. Testing Geographic Hypotheses

Clearly, geographic hypotheses are concerned with explaining the location of phenomena or in interpreting the significance of distance for some specific situation. King states that these tests often relate to parameter values or the differences between parameter values. The means of two sets of data might be compared, for instance, to ascertain whether or not the observed mean fit a preconceived statistical pattern. A level of acceptance for the verification of the hypothesis is set up so that the researcher will have a yardstick for accepting the sample mean as close enough to the theoretical mean for an idealized distribution, or rejection of the hypothesis if too much deviation between actual and theoretical obtains.

1 For a brief discussion of this topic, See: King, L., Ibid., pp. 72-82.
Another general type of hypothesis testing concerns the supposed relationship between/among variables. The discovery of functional relationships among variables is a fundamental procedure in science which aims at the statistical explanation of a dependent variable in terms of one or more independent variables. Many new techniques have been adopted to the task of testing geographic hypotheses, including regression analysis, factor analysis, chi square, and others. Successful establishment of functional relationships after rigorous statistical assumptions have been fulfilled permits the valid extrapolation of results from the case study examined to other similar cases. If these assumptions are not fulfilled, the techniques are still useful for describing the parameters of the distribution that has been observed.

Various techniques to describe the "goodness of fit" of data also qualify as hypothesis-testing techniques. Essentially, these techniques compare different statistical models with the observed data. The Kolmogorov-Smirnov test first generates an idealized representation of some form of statistical distribution. Cumulative percentage of the actual distribution is compared to the idealized distribution to ascertain the degree of correspondence between actual and ideal. The underlying idea of this and such tests as Chi Square is the examination of the null hypothesis, that is that there is no difference between the observed sample and what might be expected from a given distribution. Many new hypothesis-testing techniques are being developed in all kinds of geographic studies.

R. Urban and Settlement Models - Urban places are of great interest in modern geography because they represent a specialized human organization of space. Recent trends in urban geography have been directed at the
formalization of concepts of urban geographic systems. Garner lists six premises that are important in the geographic study of urban systems: 1) the spatial distribution of human activity reflects an ordered adjustment to the friction of distance, 2) locational decisions are made to minimize the friction of distance, 3) some locations have better accessibility than others, 4) human activities tend to agglomerate to take advantage of economies of scale, 5) the organization of human activities in space is hierarchal, and 6) human occupancy is focal in character. "In this way, movement-minimization, accessibility, agglomerations and hierarchies are linked together to form a system of human organization in space."¹

Central place theory as innovated by Christaller and expanded upon by Berry and associates has been the core concept around which much of the new work in urban systems has centered. Trade areas, arrangement of central places, the hierarchy among central places, ranges of goods and services, and interaction among centers are all topics whose main outlines are now well known but whose functional interpretations demand much more work. The internal characteristics of cities can also be studied as geographic phenomena, by use of the full range of spatial tools and concepts. This aspect of urban systems has not received the treatment it deserves in comparison with the external characteristics of cities. A new behaviorist approach to the study of internal city spatial organization is now emerging. This idea pleads for a reversal in the traditional interpretations of cities towards interpretations that are derived from the urban dweller's perception of his spatial environment. The traditional approach propounded interpretations of city phenomena in terms

of economic-social-political geographic concepts which are far removed from the individual human city dweller's realm of existence. Because cities serve one major function as a habitation for man, study and planning for city development should be aware of this perspective on urban problems.

s. Variance, Analysis Of - This is a procedure that seeks to discover whether or not two or more statistical samples could have come from the same statistical population. The test examines the variation in the two samples to see if the variation within the sample is as great as the variation between the samples. Comparison of within and between variation yields a ratio which provides a value of "F." Given the appropriate degrees of freedom and a level of significance a "table of F" may be consulted to determine whether the "F" value obtained is high enough to support or reject the hypothesis that the samples are derived from the same population. This parametric test is obviously one of wide application. Any set of sampled statistics that are normally distributed can be examined rigorously w.r.t. the analysis of variance. One special application in geography might be the formulation of statistical regionalization categories.

1 For more information, See: Cole and King, op.cit., pp.126-128.
II - EXTENT

A. Classification and Regionalization - The formation and mapping of regions has been basic geographic methodology for many years. Field observation and mapping, choropleth mapping from published statistical sources, and mapping by tracing overlays from other maps have been traditional means for constructing regions. New techniques have sought rational systematized procedures for formulating regions. Particular emphasis has been placed on the capability to group single factor data into a logical taxonomy and to combine diverse factors into generalized locational characteristics of areas.

Discriminatory analysis aims at the allocation of data in a group to one or another class. In analysis of variance, for instance, two groups of data can be tested to ascertain whether or not the variance within the sample group is greater or lesser than the variance between the samples. If the latter is true, then the groups are considered to have internal cohesion, thus logical classification. Regions of the grouped data may then be plotted on a map if the actual place occurrences of the data are known. There are many different types of discriminant procedures, basically differentiated by the type of transformation used to separate data, the rules for determining class representative points, and the distance function used for measurement of the relative location of the points. King refers to work done by Casatti in describing the use of discriminatory analysis iterations, thusly: they are useful to evaluate the quality of a classification, they serve to identify the cores of classifications and regionalizations, and they can be used to

1 See: King, I., op. cit., pp. 204-215.
test the validity of classifications compared to classifications of the same
type of data, separated from the original set of data by space or time. 1

Use of the theory of sets in classification and regionalization problems
is discussed below under the heading of "Venn Diagrams."

B. Correspondence Of Isarithmic Maps - The purpose of this technique
is to make a comparison between the undulations of statistical surfaces on
two or more isoline maps. 2 Not only is the question of area investigated,
but also the problem of the area variation in intensity of two or more
factors (a three-dimensional analysis) is investigated by the technique.
Robinson relates the following decisions that must be made when this type
analysis is attempted: "(1) the manner by which the requisite paired data
are to be obtained from the maps; (2) the determination of the sizes of
the unit areas for which to obtain the paired data; (3) the selection of
the number of pairs to be used in calculating the coefficients of correlation,
and (4) the spatial organization of the areas to be represented by the #
pairs selected." 3 The technique thus uses linear regression analysis (when
two maps are being compared), selecting the observations in a regular way
so that the total area of both maps would be represented in the sample.

Robinson in a study of the correspondence between population density
and rainfall on the Great Plains plots the correlation coefficients on an
overlay of the area to determine the area variation in correspondence between

1 King L., Ibid. p. 214
2 See: Robinson, Arthur H., "Mapping the Correspondence of Isarithmic Maps," in
Berry and Marble, op. cit., pp. 301-312.
3 Robinson, Ibid., p. 305.
the two factors. A grid of forty offset large squares in adjacent columns, each consisting of sixteen 1,064-square mile units, was used as a base to gather and calculate data. Correlation coefficients were calculated by comparing the data of the sixteen small units and then placing that $r$ value in the center of the larger square. The forty control point $r$ values were used to construct an isoline map of correspondence by interpolating between control point values. Correspondence maps for three different dates were drawn and offered for analysis. The conclusion was that the degree of correspondence displayed on the maps could not have occurred by chance. Thus Robinson believes that the combination of cartographic and linear regression analyses offer great potential for comparing isarithmic maps in many kinds of geographic problems.

C. General Field Theory Of Spatial Behavior – "The field theory involves a spatial system that comprises places, the attributes of these places, and the interactions among them." 1 "Structure" and "Behavior" matrices are conceived as the bases for this system, the former composed of a reduction of the near-infinite number of place characteristics to a manageable number of meaningful concepts that describe places, while the latter is composed of the interactions between pairs of places. Since both types would represent characteristics for one time period, process or change with time could be included by constructing the same matrices for additional time periods.

Berry describes the basic postulates of the field theory in the following way: "1.(a) Dyadic spatial behavior is a function of the ways in which the fundamental spatial patterns characterize places. (b) changes in spatial behavior result from changes in the character of places as spatial processes

---

run their course. 2. (a) The characteristics of any place are largely dependent upon its relationships with other places. (b) Changes in spatial interactions give rise to changes in the character of places." 1 Berry then continues in the referenced article to describe the mathematics of such an analysis.

The field theory technique as Berry envisions it seems particularly sensitive to the collection of relevant data. Not only are large quantities of hard data needed for many places, but also the concepts that give meaning to the data must be unusually perceptive about the particular place being studied if significant results are to emerge. Given the quantity and quality of data, the field theory technique still appears to offer only a mechanistic, unrealistic analysis of real-world complex spatial systems. The term "spatial behavior" seems especially inappropriate in referring to the interaction between places because this idea connotes "choices," whereas the proposed model only considers determined attributes of the places in question.

D. Geographic Data Matrix - This analytical technique is proposed by Berry as an attempt to systematize and synthesize the major geographic approaches, regional and topical.2 The idea is to array data into columns and rows, the columns representing places, and the rows representing characteristics of those places. A cell, the intersection of a row and a column, is a geographic fact because it is a bit of information assigned to a specific place. A Column is a regional study because it describes many characteristics about one place; a row is a topical study because it describes the variation in one

1 Berry, Ibid., p. 420.

2 Berry, Brian J. L., "Approaches to Regional Analysis: A Synthesis," in Berry and Marble, Ibid., pp. 24-34.
factor as it differs from place to place. Several matrices can be constructed for different historical periods to add the time dimension. All sorts of combinations of rows, columns, and time periods can be permuted to study problems at different scales and at different notches along the regional-topical continuum.

The Geographic Data Matrix becomes an intriguing idea when associated with computer technology. One can imagine a huge mass of data about an entire country stored in the memory of an electronic computer. A researcher need only ask the machine for a given topic, area, or combination analysis and virtually instantly receive a printed map of the results. Or possibly the stored data could be recalled onto a cathode ray tube for manipulation by introducing changes into the matrix that has been in spatial equilibrium. All that these possibilities must await are computer storage facilities large enough to handle this size load, and collection capabilities to gather the large scale amount of information.

E. Geometric Interpretations 1 - The "Washington School" of geographers believes that geography can be methodologically described as applied geometry. This thesis surely emanates from the desire to make geographic analysis more scientific, to provide it with underpinnings for a conceptually significant discipline. The truth in this view of the field comes from the fact that most geographic problems concern the portrayal and interpretation of the spatial dimensions of phenomena. Taken literally, however, the applied geometry approach serves only to describe what might be called the structure of spatial systems. It is evident that this type of analysis must include

concepts and understandings beyond geometric form in order to attain realistic interpretation of real-world problems. Likewise, it is clear that geometries that are infused with valid real-world analog-type definitions can be exceedingly important in geographic analysis.

Because most of us have been taught to think only in terms of Euclidean plane geometry, the idea of different conceptions of space seems foreign. A corollary of this for geographers is that we have become accustomed to map orientation in which north upward is the "normal" position for viewing a map. The New Mathematics has now come along to show that there are other valid mathematical concepts that can be applied to scientific endeavors to obtain new perspectives on old spatial problems. Affine and projective geometries are of only marginal interest to geographers, although the latter has some importance in remote sensing imagery and in map projection work. Topology is a field of mathematics that concerns surficial relationships and thus has direct relevance for geographic studies.

Topology differs from Euclidean geometry in that it treats the basic problems of order and contiguity, rather than such concepts as distance, orientation, and straightness. On a map of cities served by an airline, the sequence of cities along the route is more important to show than the exact mileage among them. For a study of the spread of a disease, contiguity of observation units would perhaps be more significant than the size, distance, or orientation of the units. In both of these cases, topology might be used productively in place of Euclidean geometry; for problems in which scale and shape concepts are essential, then Euclidean geometry provides the

1 Cole and King, Ibid., p. 85.
appropriate tools. Another property of topology that proves useful is its "stretchability." This means that in preserving order, the shapes of a region could be deformed without loss of sequence of cities, or any other point information. Deformation of the two-dimensional figure is known as a transformation.

The terminology of topology also lends itself to geographic application, even though it is not entirely standardized among writers on topology. Nodes (vertices), arcs (edges), and regions (faces) have their counterparts in Euclidean geometry as points, lines, and surfaces. Whatever the specific terminology, the connotation of zero, one, and two-dimensional figures in a two-dimensional space is clear. These figures supply concepts for the analysis of networks, nodal and homogeneous regions, and sets which combine nodes, arcs, and regions. There also exists a three-dimensional topology with its own terminology and concepts.

F. New Mathematics - The new mathematics has created a body of ideas that permits us to manipulate and interpret data in novel ways. Two of the new concepts are Boolean algebra and set theory. Boolean algebra has only two constants, 1 and 0 and three basic operations, inversions, and, or. Inversion is performed when one constant becomes equal to another constant. For example, \( \overline{1} \) (not one) = 0. The and operation connects two or more constants or variables; the answer is 1 only if both constants are 1. Or also connects two or more constants or variables and the results equal 1 if either or both constants are 1. Boolean algebra has direct application to the operations of electrical circuits and therefore to digital computer

1 Cole and King, ibid., p. 87.
2 For a discussion of new mathematical concepts useful in Geography See: Cole and King, ibid., pp. 25-97.
Set theory concerns the manipulations of groupings of data. Sets are determined by the presence or absence of some significant characteristic(s) of the phenomenon under study. Terminology used is as follows: a universal set is one composed of all the observed units with the characteristic in common; an element is one observation unit with the appropriate characteristic, and this defines membership in a set; a null set is one with no membership; equal sets are those in which the elements are equal; a subset is a set composed of elements with similar characteristics to each other but different from the characteristics of other elements in the universal set, but at the same time, all subsets must have some characteristic in common to be grouped together as a universal set. Operational characteristics of sets are: complement of a set is all elements of a universal set that are not included in a given subset; union of sets is a combination of all members of two or more sets; intersection of sets is the common membership between two or more sets; disjoint sets are those which have no membership in common; a one-one correspondence is a correspondence in which each member of a set has an exact correspondence with a member in another set.

It is fairly obvious that set theory formalizes many ideas used in geography for many years in the construction of regions. The benefit to the discipline is thus not one of innovation, but rather it is one of more rigorous thinking about the same problems we have wrestled with before. More about set theory's visual representations will be discussed in the section on Venn diagrams.

G. Isotropic Plain - An isotropic plain is "...elementary, abstract, geographical space that has no difference from place to place or in one
direction to another; that is, not only are places the same, but movement
effort is the same in all directions from every place." ¹ This abstraction
is made so that attention can focus on the interrelationships of the
significant variables being examined. In other words, the isotropic plain
is the attempt to hold all non-essential variables equal. Nystuenn provides
the example of a teacher and students in a building with no furniture, the
flat polished tile floor representing an isotropic plain. The functions
of teacher, students, and the variables involved with the transmission and
reception of words determine how the students will arrange themselves
around the teacher. Chairs, pillars, glare from the floor and other
factors would all tend to distort this idealized teacher-students spatial
pattern. However, these factors are only of secondary importance, and the
isotropic plain assumption simplifies the analysis and isolates the
significant variables of voice and hearing ranges. The isotropic plain
concept is useful for a great many problems in geography, especially those
which involve movement over space.

H. Map Transformations ² - Map transformations change the size,
shape, or orientation of a mapped phenomenon in order to emphasize a feature,
or to show a "true" representation of the spatial dimension of the phenomenon.
The transformation is done strictly according to the rules of the trans-
formation system selected, and thus some map properties may be intentionally
distorted systematically, in order to create properties that demonstrate
a particular interpretation. This idea has recently gained significant

¹ Nystuenn, J., "Identification of Some Fundamental Spatial Concepts," in
Berry and Marble, op.cit., p. 37.
² See: Cole and King, op.cit., pp. 75-85.
status in geography because modern research has shown, that valid geographic interpretations involve not only concepts about physical space, but also concepts about abstract space dimensions. For instance, most geographers are familiar with William Warntz's work on the population space potential of the United States and other areas. His maps attempt to show the area variation of the statistical surface of aggregate accessibility to population. The idea he propounds in this work is that a common map of population density (based on Euclidean concepts) does not truly convey the cultural-economic effects of large agglomerations of population. His technique for generating population space potential surfaces thus transforms the Euclidean point density surface into a surface that considers aggregation. The "market view" of the United States is a well-known map that distorts the actual physical size (again according to Euclidean concepts) of the states into sizes proportional to the economic buying power of the inhabitants of the states. The result is to shrivel up the south, Great Plains, and mountain states and to expand the size of midwestern and northeastern states. The visual impression of such a map shows the states according to their "true" economic spatial dimension.

Numerous other types of map transformations can be made, ranging from a simple rotation of the map so that a direction other than north is to the top of the map to the utilization of unusual mathematical systems. Rotation and reflexion (showing the mirror image) are simple procedures for presenting the map of a place in a way that is different from the usual orientation of the map. The basic idea of the transformation is that a new perspective on the area might suggest relationships that are not clearly
Or readily seen from the usual orientation. Topological and some other geometric transformations have been briefly discussed above in section E. Again, the purpose of these transformations is the portrayal of a special interpretation of a map which might be obscured by its Euclidean representation. The possibility of novel map interpretations by either new mathematical techniques or new definitions of familiar concepts and variables bodes a whole field of geographic methodology that has only recently been recognized.

Maps have often been called geographers' models of space.\(^1\) Maps used as analytical tools rather than used as mere graphical displays have the character of an analog experiment through the device of cartographic symbols. Spatial variables, sometimes in combination with temporal variables, can be measured, manipulated, and the results presented on maps with the aid of computer or traditional methodology. A spatial diffusion problem could be presented on a series of maps; the growth of population in a state traced through a cartographic series; or projections about the possible spatial distribution of phenomena given various sets of constraints might be all analyzed by cartographic-computer systems. A "real-time", or condensed/expanded time spatial model of a variable such as traffic flows can be analyzed or be used as a teaching instrument by an electric analog system which combines cartographic and computer techniques. A series of maps based on a step-wise regression analysis plus the mapped residuals from regression is an example of an iterative solution to a spatial problem. This latter technique seems to hold much promise as a statistical-cartographic procedure that effectively utilizes both analytical approaches. Abstract

---

\(^1\) For a full discussion of the topic, see: Board, C., "Maps as Models," in Chorley and Haggett, *op. cit.*, pp. 671-725.
problems of space interpretations of course can also be handled through this type analysis.

I. Quantitative Definition Of Regions - David Grigg in his study entitled, "Regions, Models and Classes," delimits the following necessities for good regional classifications: 1 first that the objective of the regionalization scheme be as simple as possible. A single factor region is conceptually and methodologically easier to work with and to interpret than a multifactor region. His second point indicates that differences in Kind should be relegated to different classification systems. This rule has some important implications that might be glossed over because the main argument is so obvious. The significant idea here is that subtle but generic differences in what might appear to be a homogeneous category of things must be considered if the classification system is to be meaningful. A third point is that classifications, like all scientific knowledge, should be periodically subjected to critical scrutiny anew when additional information about the subject of the classification becomes available.

Fourth, regional classifications should be differentiated by the differences in properties of the objects that fit into the classification. The purpose of this rule is to insure that some inherent and concrete characteristic be differentiated rather than some peripheral and nebulous one. Fifth, that classes should be exhaustive and mutually exclusive. To eliminate any pertinent data or to create overlapping categories is to render the classification system scientifically unsound. Sixth, the criteria or

J. Regions As Models - Regions are tools of spatial analysis that geographers have utilized for many years. Regions are of two types, formal (nodal) and homogeneous, although the latter is the more general type and the former can almost be thought of as a special case of the latter. The common definition of a region states that a region is an area of any size, homogeneous in terms of specified criteria. The formulation and mapping of regions thus becomes a sort of a geographic classification of space. In one sense, regions are geographers' models of a type of spatial system. As an abstraction of reality (specified criteria), regions are in effect simplified descriptions of the real world. If the variables in a regional classification enable the researcher to understand the operations of the spatial system under study, then the regional scheme can be called a model of a segment of the real world.

See Discussion In: Chorley and Haggett, Ibid., pp. 494-500.
In a similar way, regions that are based on one or several related variables symbolically isolate these variables from others which may be associated with the original set in place but not functionally related. The region, an isomorphism of a segment of reality, specifies functional interrelationships of its parts, and therefore its growth and expansion may be traced from the past, through the present, and into the future. This procedure introduces the possible dynamic analysis of regional systems. In the past, this dynamic analysis was only accomplished through a system that might best be described as comparative statics, because regional change was studied by recreating various static cross-sections of time in the regional framework. New techniques permit a truly dynamic regional analysis because the variables can be treated as ongoing factors. The device of utilizing isomorphisms to study the real world is common practice in physical science, but it has only recently been widely accepted for use in social sciences. There is greater difficulty in the social sciences, because the one to one relationship between the real world variables and their analog counterparts is difficult to establish. Actually, this general type analog study has been fairly common practice in geography as often the researcher would study one area and extrapolate results to other similar areas. However, much of this type work ignored the formal requirements of isomorphism techniques and hence sometimes naïve conclusions resulted.

A slightly different conception of the region as a model might be its use under controlled situations. In this use, the region again specifies certain variables as critical to the understanding of the actual system under study and specifies that other variables in the actual system be held constant.
This procedure is important, because in the simplification of the experiment, the researcher has better opportunity of comprehending the basic operations of even complex systems. When the simplest regional systems are understood, then the controlled constants may be introduced at will to clearly ascertain their effects on the workings of the regions.

Another basic method of science, analysis, is also a task for which regionalization is well suited. Perhaps many interrelated variables in a system are quite well known. In order to better understand this complex real spatial system, the system might be more easily studied by breaking the component parts out into single factor regions. These single factor regions can lead to insights that might be missed from trying to see the entire complex picture at once. Conversely, some insights are better displayed through examination of whole systems. The important idea here is that the regional methodology of model building is flexible enough to permit all sorts of approaches for the study of spatial systems. This is the strongest rationale for espousing the use of regionalization as a model-oriented methodology in geography.

K. Residuals From Regression, Maps

The residuals from regression analysis provide excellent data for analytical mapping and interpretation. Regression analysis statistically explains the areal variation in one phenomenon in terms of the areal variation in one or more other factors. As described in section I-F above, the residuals from regression analysis

---

1 Sec: Haggert, P., Locational Analysis in Human Geography, pp. 278-280, 298-299.
represent the variation in the dependent variable which remains unexplained after account has been taken of the included independent variables. The well-trained geographer is often aware of major factors which might influence the spatial variation of a dependent variable he is trying to explain. Regression analysis can be employed with the suspected independent variables in order to ascertain the explanatory strength of these variables. Once this has been accomplished, assuming the usual case that less than 100% of the variation has been explained, then the focus of further investigation can center on the residuals.

The residuals are useful for the discovery of independent variables that may not have been considered at the outset of the research project. Analysis of the residuals is facilitated by the regression analysis because the results of regression specify magnitude and sign (whether a positive or negative residual) for each individual observation. For the geographer, this information can then be categorized and mapped for spatial analysis. The patterns of residuals on a map sometimes suggest the underlying causative factors that have influenced such a pattern. Hence the mapped residuals help the geographer probe deeper into the independent variables which influence the spatial variation of his dependent variable.

L. Robinson's Solution - Robinson's procedure was developed to provide geographers with a technique so that areal data could be used logically in such statistical techniques as regression analysis. Areal enumeration districts for census type material are often counties or some comparable unit

1 See Discussion In: Thomas, Edwin and Anderson, David, "Additional Comments on Weighting Values in Correlation Analysis of Areal Data," in Berry and Marble, op.cit., pp. 431-446.
that might vary widely according to size. Thus the magnitude of any geographic observation unit (county for instance) can be affected by the size of the unit. One way to handle the problem is to include size of the areal units as one of the independent variables in a multiple regression problem. Robinson's solution to the unequal areal unit problem is to include area size into the formulas for computing the regression parameters, \( a, b, \) and \( r. \) Whereas a sample of the exact same spatial distribution in three areas that had different size subdivision all yielded different regression parameters when unmodified areal data were plugged into the regression formula, the same parameters resulted from the three cases when areal weighting was employed.

Thomas and Anderson claim that the Robinson solution is special in nature and not a general solution to the problem of weighting areal data. They state that the only two cases in which Robinson's solution is adequate are: 1) the case in which the changed size areal units (aggregated) have exactly equal density point functions, and 2) the case in which the ratios of changed size areal units' density functions are constant. For their solution, Thomas and Anderson distinguish among sample (small collection from a larger group), population (a larger body of data from which samples are taken and whose parameters we try to determine in statistical inference), and universe (which is a population plus all the same type of events which might have happened). Given this data level differentiation, then areal data may be regarded as random samples, to which inferential tests of significance can be applied to determine whether or not the samples have indeed come from the same theoretical universe. Comparison of regression parameters of the different
size enumeration districts solutions then can be used to determine whether or not areal size does affect the correlation coefficients obtained.

The Thomas and Anderson solution is cumbersome, inconclusive, and only points to the need for further study of the subject. Perhaps the easiest solution was advanced above, that is to include size of enumeration district as one of the independent variables. In many studies up to the present time, the variation in size of the districts has not made a significant difference in statistical results of regression analysis.

M. Statistical Inference - Statistical inference concerns the estimation of population parameters from experiments on samples of the data and the testing of a whole "family" of a type of statistical hypothesis. Many subjects of geographic research interest are comprised of so much data that it is not economically feasible, or methodologically necessary to collect all of the data in order to understand the problem that the data describes. If a scientific sample of data is taken, then the sample is considered to be "representative" of the larger population of which it is a part. When this situation is satisfied, many types of interpolation and extrapolation of observed data may be made with a variety of statistical procedures. Thus more information is derived from the data then would be the case if purely descriptive techniques were used to analyze a set of data. Hypothesis testing usually takes the form of examining a given data sample by comparing it to some model distribution. Chi Square, the "t" Distribution, and the "F" Distribution are all standards for testing various properties of sample distributions. Future uses of inferential statistics in geography might include such approaches.

1 See: King L. op.cit., pp. 60-86.
as nonparametric techniques of statistical inference, rank order statistics, and statistics based on subjective probability.

N. Trend Surface Analysis - Sometimes referred to as Regional and Local Components Analysis, Trend Surface technique seeks to separate the "signal" (true process-response) from the "noise," (chance occurrence) factors in mapped information. Chorley and Haggett state that Trend Surface Analysis is a logical extension of Regression Analysis to two dimensions and is derived from information theory. A map of some type of geographical surface portrays information which must be learned through interpretation of the data shown. Typically, it is point data that generates the information for the mapping of a continuous surface, which is often represented by a pattern of isarithms. "... and the most obvious manner to treat them (isarithmic surface maps) is to attempt to disentangle the smooth, broader regional patterns of variation from the non-systematic, local, and chance variations, and then to ascribe mechanisms or causes to the different components."

Techniques to derive a trend surface from an isarithmic map vary from grid generalization to the use of regression analysis. Grid generalization is mostly a matter of scale, because it can be accomplished by averaging a series of control points to derive a more general set for drawing the isarithms. The trend surface in this case depends heavily upon the number of original control points that are grouped; because of this more or less subjective component, this approach could be termed a selective method. More objective in nature is the use of regression analysis to permit the original control point data to contribute its effect on the whole surface.


2 Chorley and Haggett, ibid., p. 196.
rather than to affect only its immediate locale. The results of both methods is a breakdown of the surface into two parts, the regional trends and the local residuals. There are many possible techniques in both the "selective" and "objective" methods of deriving trend surface data.

Trend Surface Analysis yields both descriptive and interpretative information. In the descriptive line, Trend Surface Analysis points to the morphology of the surface being studied, so that underlying causative factors that have perhaps been unsuspected might now emerge as significant factors in explaining the variation in the original surface. The analysis can also add input on the accuracy of the original data. As for interpretation, the further designation of the regional trend into other considerations such as the major trend, discrete trend, or interlocking trend. These interpretations have not been vigorously pursued in geography. An item of interpretation that has received considerable attention is the analysis of the residuals from Trend Surface Analysis. These of course are comparable in meaning and utilization to the residuals from regression analysis.

O. Venn Diagrams - For geographic applications, Venn Diagrams can be considered as diagrams of spatial sets. As such, they are merely a new way to look at the traditional geographic tool, the region.¹ Again, the concepts of set theory and Venn Diagrams are mostly a formalization of older concepts in the utilization of the regional method, which, unfortunately, has been pursued with only a haphazard application and adherence to fundamental concepts. In this formalization trend, the transformation of geography into a scientific

¹ Cole and King, op. cit., pp. 30-43.
discipline is served because it requires researchers to think and act with more precision.

Venn Diagrams put the spatial dimension into set theory. They show the area extent of such concepts as the universal set, subset, complement, union, intersection, and disjoint sets, all of which have their counterparts in the conceptualization of the regional method (that is the construction, analysis, and interpretation of regions). The universal set is analogous to the entire area to be regionalized; a subset to a particular homogeneous or nodal region within the larger region; the complement to all areas of the larger region outside of the specified regional system; union to the covariation of regional systems; intersection to the combination of single factor regions into a new system in which the covariation of single factor regions becomes a multiple factor region; disjoint set to the complete independence of two or more regional systems. It is fairly obvious from the above that wide avenues are opened up to the geographer who wishes to make more explicit analogies between set theory and the regional method. Doubtless, many fruitful new perspectives can be gained from a wedding of the largely unorganized techniques of the regional method and the more formalized procedures that set theory and Venn Diagrams offer.
Flows of people, commodities, or ideas over space is a topic that has emerged recently as a focus for much research work in geography, as a response to new techniques which permit an adequate investigation of this topic. Previously the common means for this type of map portrayal was either an arrow indicating direction and size of a flow, or the comparative static technique whereby the same factor was shown in its spatial distribution for two or more time periods. New quantitative methods are now available that put more realism and more analytical power into the analysis of spatial flows. Some of the more important techniques are discussed below.

A. Analog Models - Analog models can be applied to many different types of problems in geography. The basis of discussion of this topic here is that some significant new developments in flow studies have made use of analogs. The analog technique involves the construction of a model in which the real world variables are transformed into analogous and more familiar, simpler, or easier to manipulate terms. In essence, the technique tries to examine a real situation by comparing it to a situation which is more readily understood and which has essential characteristics that are similar to the real situation. Herein lies the crux of the adequacy of analog models: namely, the degree to which real world characteristics can be imputed to an "artificial" representation of reality.

Chorley lists four basic types of analog models, Abstraction, Mathematical models, Experimental models, and Natural models. Newton's model of gravitation

---

is an example of Abstraction. This type requires a simplification of real
world variables to the point where all minor variables and huge quantities
of real world data must be discarded in order to arrive at the underlying
fundamental relationships of the system being studied. Incisive insight
by the investigator is a prime requisite for Abstraction analog model building;
this fact has obviously limited successful attempts in this direction.
Mathematical models start with the language transformation of words in a
simplified model into mathematical symbols. Concepts are then built into
the model to produce a mathematical system of one or more equations which
specify the relationships of variables and constants within the system. The
mathematical model can be deterministic or stochastic. The former is based
on the mathematical notion of direct cause-effect relationships, while the
latter are based on probabilities rather than on mathematical certainty.
Again the success of these models depends directly on the relevance and
comprehensiveness of the mathematical symbols in replicating the real world.
In this regard, it should be kept clearly in mind that mathematics is
merely a system of logic; meaning and interpretation of mathematical procedures
and results must come from the scientist.

Experimental models may be divided into scale models and analog models.
Scale models in some way try to imitate the real world situation to which they
refer as closely as possible. A wind tunnel, or a stream table are examples
of this type. The wind tunnel recreates the parameters of atmospheric
conditions on a greatly reduced scale so that the affect of certain changes
in these variables on certain objects may be observed. The stream table
uses the same basic materials as its real world counterpart, running water
and earth mantle, in order to recreate in condensed time the actions of the causative variables and the effects (miniaturized landforms) that result from the interaction of these variables. The attempt to define variables of human behavior by analogs of physical world systems has recently received much attention. One of the more interesting analogs for spatial flow problems is the attempt to study transportation flows by the use of electrical circuits and the flow of electricity throughout the circuit. In this analog, electrical voltage is likened to complementarity, electrical wires to routes, length of wires to the friction of distance, resistors to bottlenecks, and so forth. Given a certain introduction of voltage into the system, the flow of electricity proceeds around the circuit until an equilibrium in space is reached between the factors which generate the flow and the factors that inhibit flow. New sources of current can then be introduced to ascertain its effect on the overall flow of the whole network. The one to one analog with the real world transportation system made, the electrical model can alert a researcher-planner about the possible effects of change in the variables on the overall flow equilibrium of the transportation network.

Natural models are used when a system under observation is thought to have an easier explored historical approach in that the process at work in the observed system repeats itself at other times or at other places, or both. For instance, a researcher might think that he can understand the growth of central places in the midwest by direct reference to the growth of central places that has occurred in a similar environment in upstate New York at a previous time period. Much "random noise" is generated in this type of analogous reasoning and it should only be used with utmost caution. Another
form of model is to try to study a human system by reference to a physical system. An example of this type is the study of human migration by analysis of the mixing of two liquids of different densities. A slight variation of this latter analog model type might be the study of the historical evolution of urban manufacturing region characteristics by use of an analogy to biologic growth of youth, maturity, and old age. The most infamous use of the biologic analogy was the Nazi German geopolitik interpretation of international relations.

B. Gravity and Space Potential Models - The Gravity and Space Potential models measure the possible interaction of complementary masses over a cost-incurring, time-consuming, and energy-absorbing distance. Thus many of the core concepts of geography are included in this family of models. Distance in the basic model is treated as a major hindrance to the movement of people, commodities, and ideas. Since cost, defined in a broad sense to include the expenditure of resources of all kinds, is cumulatively incurred for each unit of distance over which things are transported, there is a general and regular decrease in the movement of people, commodities, or ideas as distance from their source increases. Thus relationships of decreasing intensity of human activity with increases in distance has been observed in traffic flows, telephone calls, news items in newspapers, prices of agricultural commodities, the spread of innovations, pairing of marriage partners, population migrations, shopping center drawing power, the geographical distribution of the residences of students attending "national universities," rural rents, and in many other types of human relationships.

The gravity-space potential concepts were formalized in mathematical terms in several works by John Q. Stewart and George Zipf. In keeping with the physical analogy from which the idea of space potential was derived, concepts of "Force," "Energy," and "Potential" of human interactance were reduced to mathematical symbols. Force of interaction, interpreted as the attraction exerted by two masses for each other, is written as:

\[ F = K \frac{P_i P_j}{d_{ij}^2} \]

where \( K \) is a constant corresponding to the gravitational constant; \( P_i \) and \( P_j \) are two "social masses;" and \( d_{ij}^2 \) is the square of the distance between the two masses. The force of attraction sets up an energy for interaction between the masses, expressed as:

\[ E = K \frac{P_i P_j}{d_{ij}} \]

where \( P_i \) and \( P_j \) again represent two social masses with a mutual attraction; \( K \) is another constant factor; and \( d_{ij} \) is the distance between the masses (note that is simple distance, raised only to the first power.) This latter formulation of the gravity concept has been tested by a number of scholars in different disciplines to examine actual relationships of social interaction. A summation of the energy among all the masses in a given region yields the population space potential, interpreted as the total possibility for interaction at any point, and written:

\[ \Delta V_j = K \sum P_i \left( \frac{1}{d_{ij}} \right) \]

the symbols have similar meanings as the previous formulations. The major difference in this model is that the concept of population potential specifies that the total possibility for interaction at a point is determined

---

by the interrelationship of all such masses in a region.

The flow concept inherent in space potential analysis can be utilized to study problems in transportation and other types of movement over space. An example of what can be done with the model appears in a brief study in Appendix A to this report. Especially significant for geographers is the map presentation and analysis that the technique permits. The model is general in nature and applies to many types of human interaction. Because it comprises the ideas of complementarity of masses, location and density of the masses, and the effect of distance on movement of the masses, the gravity-space potential model is "hard core" geography.

C. Spatial Efficiency - Notions about the minimum distance path in a network of nodes and transportation lines is a general problem involving movement over constrained space. Described variously as "The Transportation Problem," or "The Traveling Salesman Problem," or others, the concept of spatial efficiency has attracted the attention of many scholars. \(^1\) The problem has many important implications in the attempts of man to organize his spatial activities in an "efficient" manner. Efficiency is a concept that must be defined by a society in terms of its principles and goals. Once the definition of efficiency is defined, then the model can be selected which minimizes distance, specifies order for travel through a system of nodes, maximizes number of nodes visited, or whatever other objective is desired.

One aspect of spatial order that has not been well examined is the idea of a "spatial equilibrium." If a region can be said to have achieved

---

\(^1\) Haggett, P., *Locational Analysis In Human Geography*, pp. 66-73.
spatial efficiency, then, according to the condition of the variables, the region at that time, it can be said to be in spatial equilibrium. As in economics where the concept is commonly used, the state of equilibrium can be regarded as a theoretical state. The purpose of hypothesizing such a condition is to focus on the values of the variables (and locations in geography) that must be attained to achieve equilibrium, the effect on the system of disequilibrating factors, higher level equilibria, and related concepts. All of these lines of inquiry appear to be significant for the study of spatial systems analysis.

Another type of spatial equilibrium analysis uses linear programming techniques. The problem in this analysis consist of minimizing some weight/distance function in a system where several sources of supply and markets are combined into an interrelated pattern of commodity flows. Efficiency in this context means that all demand must be satisfied from the sources of supply that can satisfy demand with the least cost. All sorts of variables might be included in the analysis, such as differing transportation rates, differing qualities of materials, time factors within which demand must be met, the effects of price competition and many others. While the technique has been known for some time, not many of the theoretical aspects of the analysis have been developed. This linear programming-spatial efficiency would seem to have many applications to problems of the underdeveloped countries where the economic system needs to be developed at the same time that scarce resources must be conserved.

IV. SUCCESSION

The idea of spatial succession adds the time dimension to geographic analysis. In traditional methodology, the time variable was most often
portrayed by the use of historic cross-section analysis. That is, the status of some spatial system would be carefully displayed for a particular date. Successive analyses of the same spatial system for different dates thus demonstrated the growth, decline, stagnation, or other spatial change in the system that might have occurred during the time span. As mentioned above in section III, this approach can be considered a comparative statics analysis. Some new techniques permit a dynamic approach to the study of spatial systems. A few illustrative examples will be described below.

A. Harmonic Analysis - Harmonic analysis is a technique for objectively describing and mapping a function which includes both space and time dimensions. In the referenced study by Sabbagh and Bryson, the authors attempt to explain the variance in month to month fluctuation of precipitation from place to place in Canada. The technique used, harmonic analysis, involves the comparison of the total variance of precipitation as measured by the month to month fluctuation, to the several variance components. A variance component is an independently calculated statistic that is specified by a sine curve (harmonic), "so that the annual variance component or first harmonic is a simple, smooth sine curve with one maximum and one minimum occurring six months apart, while the semiannual variance component, or second harmonic, is a sine curve with two maxima and two minima spaced three months apart." Each sine curve thus derived is fitted to the precipitation variation curve for each station studied by least squares methods.

2 Sabbagh and Bryson, Ibid., p. 250.
to determine the degree of correspondence between the harmonic and precipitation curve. The authors found that while six harmonics can describe the precipitation curve based on twelve monthly values, only the first and second harmonics were needed to account for a large portion of the annual precipitation variation. The fit between harmonics are measured in two parameters, the "amplitude," which is a measure of one-half the difference between the maximum and minimum of the sine curve, and the "phase angle," which measures the time of the year when maximum and minimum occur.

After the fit has been determined, the correspondence between harmonics and annual variance can be mapped. The resulting pattern for the first harmonic demonstrates the places where one maximum and one minimum precipitation regime obtains in the actual data of Canada. Likewise, the areas which do not fit this pattern well are also pointed out. The second harmonic, tantamount to a different type of precipitation regime, can then be mapped to ascertain the total amount of annual precipitation variance that is explained statistically by the first two "type precipitation regimes." Thus it is that harmonic analysis can wed the time and space dimensions to analyze a problem where the periodicity of the data is a vital factor. Since this type of problem is restricted by the nature of variables required, the technique of harmonic analysis is a specialized rather than a general one.

B. Markov-Chain Models - Models based on the Markov-Chain technique try to predict the outcome of some event. The underlying assumption is that the outcome of an event is directly dependent upon the outcome of some immediately preceding event, a sort of domino hypothesis of cause-effect relationships. Probabilities are generated in the model to predict the

1 Harvey, D., "Models of the Evolution of Spatial Patterns in Human Geography," in Chorley and Haggett, op. cit., pp. 577-582.
behavior of a unit of observation in the particular system under study. Thus
the full process of change is related to (1) the status of some immediately
preceeding event, and (2) a component of change expressed as a probability
function.

Utilization of the model requires much information and careful
formulation of the variables in the system. The outcome of any particular
event is termed the "state" of that event, which must be linked to the state
of a preceeding event by a probability value which is termed the "transition
probability." If states can be defined operationally, enough about the
system is known to permit the accurate hypotheses about transition probabilities,
and the initial state of the system is clearly known, then the behavior of the
system is amenable to calculation by the Markov-Chain Model. Only if the
transition probabilities can be assumed to be stable over time can we use the
model to predict the outcome of events in the future.

"There are, of course, difficulties. We assume that there are distinctive
states of the system in the models we have so far presented, and that change
occurs in a series of discrete time intervals. But space and time are
continuous variables. Similarly, the transition probabilities, which we
assume to be constant with time, may vary over time. Technically it is
possible to develop models which overcome all of these difficulties . . . " "But
such models are extremely complicated to operate and it will undoubtedly be
some time before such models can be satisfactorily applied to social and
economic development." ¹

¹ Harvey, Ibid., p. 582.
C. Monte-Carlo Simulation - This model is based on a stochastic process whereby some real world process is theoretically operationalized and then sampling techniques attempt to obtain approximate solutions to the problem. There are three basic operations in using the model to examine some real situation: (1) the probabilities of a particular event occurring must be determined, (2) a set of random numbers are obtained to allow the selection of samples from the probability distributions according to a specified method, and (3) the operation must be repeated many times to get an approximation to the model.

The example of migrations of people among country, suburban, and urban areas that Harvey offers helps to illustrate the use of Markov-Chain analysis. It is first determined that there is a 40% probability that people will move from a country to urban location; 10% probability that people will move from the country to a suburban location; and a 50% probability that people will remain in the country. Each probability set is translated into digits: 0, 1, 2, 3 for the 40% set, 4 for the 10% set, and 5, 6, 7, 8, 9 for the 50% set. Then, from a table of random numbers, a digit is selected for each of 10 units of observation, and the indicated result - stay in country, move from country to suburb, move from country to city - is charted for each unit for the "first stage" (first solution for the group.) At this first stage, the results can be tabulated to ascertain the percentage of people that remained in the country; likewise the other possibilities could be tabulated. The same process is repeated for the "second stage," and again for ten stages or trial runs of the model. While the percentage that remains in the country can vary widely between any two

1 Harvey, Ibid., pp. 582-588.
stages, there is a tendency to converge towards an average solution after a number of trials. This latter is also true in spite of the fact that there is great variation in total moves in the ten stages for any one observation unit, 8 in one case and 2 in another case.

This tendency towards an average solution that simulation procedures produce is further illustrated by Harvey when he cites a study of a water resource system in which a reasonable approximation to an optimal design was found after about 200 trial simulations. The total possible number of solutions was $10^{23}$.

V METHODOLOGY

There are a few new techniques which do not readily fit into the categories of the first four headings; these techniques are discussed below under methodology. Methodology has become an important topic for social scientists who are concerned with finding new perspectives on well-known issues. Methodology, having been spurred on by new technological capabilities, is closely related to the philosophy of science. Inquiry about the tools scientists use is an important task, because the information we derive from studies is directly dependent upon the methods we apply to the data. It is even possible that some analytical tools are constructed on a false philosophical base. For instance, many "optimizer" models have been developed wherein human behavior is thought to result in an optimization of alternative courses of action. Scientists have recently come to believe that even though men attempt to act rationally, they lack perfect knowledge about all situations and therefore could never hope to attain optimum solutions. The newer

1 Harvey, Ibid., p. 583.
philosophy states that models should be based on a "satisficer" principle, that is, offer solutions that satisfy a certain level of minimum acceptable conditions. Men with imperfect knowledge can then satisfy basic desires without having the impossible task of seeking optimum conclusions.

A. Extended Point Set Theory - John P. Cole has a system for classifying geographic problems which he calls "Extended Point Set Theory." To him this is a true "dimensional" classification, based on geometric form. The object which geography studies under this classification are zero-order (points and cities), first-order (lines and networks), second-order (areas and states), and third-order (surfaces and terrain). In addition to an established vocabulary, there are also standardized procedures and concepts for analyzing spatial problems when they are cast in the format of this extended point set theory.

The major fault with this type of classification is that in emphasizing the form of the objects studied in geography, the functional characteristics have been largely ignored. Often the analysis of a real world problem depends on a functional interpretation of variables and not only a structural interpretation. Classification should comprise a system whereby an essential quality of the objects, events, or characteristics being classified is differentiated. This essential quality should enable the observer to better understand the system of which the individual category is a part. From this point of view, the extended point set theory classification of geographic problems is sterile. Perhaps the classification would prove appropriate to one segment of geographic concern that has homogeneous functional characteristics.

---

and differs only in form. In any analysis of this type, it does seem the case
that function must be introduced into the study at some stage.

B. Game Theory - This technique is a segment of decision theory and
has as its objective, the drawing of rational decisions from uncertain evidence.1
The latter is accomplished by selecting strategies in such a way to outwit an
opponent, either human or a force of nature. In order to employ game theory to
a practical problem, many simplifying assumptions must be made. In the example
that Gould presents, the problem is for a farmer in Africa to decide ahead of
time the crop combination he should choose among his alternatives in order to
maximize the food value return in the light of uncertainty whether the weather
will be wet or dry. This is the first assumption, that there are only two
significant weather possibilities. Next, one must be able to predict the
yield of various crops under the wet or dry conditions hypothesized, all other
factors held constant. Once this has been accomplished, the analysis proceeds
with a "two-person-non-zero-sum game" to ascertain which crop combination to
grow and in what proportions to maximize food value yields.

Many assumptions made in this example could not be made in a real
situation. This means that the model would have to be made more complex
because more variables would need to be added. The simplified problem could
quite readily be handled with pad and pencil, whereas Gould describes the
expanded problem as "computationally miserable." Machine technology seems to
be the answer to this problem. The technique has proved to be a good
approximator of reality in one situation, when an anthropologist tried to
figure the proportions of fish traps that a Jamaican fisherman should set
close in to shore compared to those placed farther out to sea. The

1 Gould, P., "Man Against His Environment: A Game Theoretic Framework," in
Berry and Marble, op. cit., pp. 295-297.
problem here is that the farther out the traps are set, the better chance
to catch good fish, but the more risk is incurred for losing the traps.
The result of this game theoretic solution was found to closely approximate
the actual proportions of close-in as compared to far-out traps that the
fisherman arrived at through trial and error.

C. Spatial Systems Analysis - Peter Haggett has outlined a framework
for different types of models in the location of human activities which he
calls Spatial Systems Analysis.¹ He believes that this outline is consistent
with the new paradigm of geography, the scientific examination of spatial
aspects of the man-environment complex which was discussed in the first
part of this report. The outline is tied together in a systems theory
approach that a region needs a constant flow of people, goods, and ideas
to sustain itself, an excess of inward transactions means growth, while
an excess of outward transactions means decline. Adjustments in the
system are made in the direction of maintaining an efficiency among the
variables of the system, the regional flows and regional structure. Many
regularities of structure and flows can exist and persist over space and
time. Comparison of regional systems shows that differing causes can lead
to the same results. All of these concepts are cast in systems analysis
terms, and the topics which form the major groupings of model types are
all components of these regional systems of one sort or another.

The first major topic which Haggett lists is movement, with sub-topics
of movement and morphology, interaction, concepts of "field" and "territory,"
and diffusions. Movement of course is one of the prime areas of interest of

modern geography. Space among functionally related places means that time and physical resources must be expended to overcome the spatial separation in accomplishing the functioning of the system. Movement and morphology would encompass the types, sizes, and directions of flows within the region. Interaction combines movement and distance in the problem of satisfying complementarity among spatially separated places. Concepts of field and territory introduce the idea of a spatial sphere of influence, something like a field of magnetism would be in physical science. Diffusions concern movement and time as Haggett notes, but they also include the idea of area. Diffusion of human activities entails the historical expansion of the area of occupancy of these activities. All these avenues of inquiry have been truly opened for scholarly study only recently by new techniques of analysis and new technology which made the techniques a feasible tool.

Networks are the lines of communication over which regional flows must move, and their capacity and alignment are major influences on the volumes and directions of flows. Location of routes, density patterns of route networks, and models of network change are significant topics here. Location of routes comprises the notion of structure or form of the communication system in the region. All types of geometry, including graph theory, tree geometry, path geometry, topology, and others can be used to describe and analyze the network patterns. Density of routes is also a structural concept, and it includes such facets as connectivity of the system, flow capacity, centrality, flow efficiency, and accessibility. Models of network change examine the affect of "short circuiting" the system by link removal, growth in accessibility, or in effect, change in any one of the foregoing properties of the network.
Nodes are central places, or focii of human settlement and the underlying principle to justify this view is that human activity of all types tends to agglomerate in a few favored locations. Morphology of settlement patterns, population clusters and the size continuum, and size and spacing of clusters are particular concerns in this group. Settlements are man-made objects that reflect the culture and technological levels of their inhabitants, past, present, and even their attitudes towards the future. Physical manifestations of many cultural elements are revealed in the morphology of settlements. Size of settlements vary from a few habitations to huge metropolises. Size affects functions, living conditions, variety of goods and services available in a settlement, to name only a few factors. The size and spacing of settlements are related topics which include the functional idea of areas served by central places, and the interrelated dependence between node and area served.

Hierarchical ordering of human activities is a consequence of the need for people to contact one another, the frictions of the space component of their environment, the specialization of occupations and trading of surpluses, the economies gained through cooperative efforts, and the variety of settlement advantages in different locations. Haggett sees the following as major sub-topics: functional hierarchies of settlements, specialized centers in the hierarchy, distortion of patterns due to agglomeration, and the distortion of patterns due to uneven resource location.

There is a close relationship between settlement cluster size and the variety and scale of functions the settlement cluster performs for its hinterland. This problem concerns the seminal idea around which much geographic work is concentrated, the human organization of space. Specialized
functions often result from initial comparative advantage and, again initially, seem to distort the regularity of settlement type patterns. As the settlement system evolves, however, the specialized function might remain, but the overall settlement forces create a "leveling-out" effect which incorporates the specialized function advantage into a larger system of settlement advantages. Some types of human activity, especially with modern transportation and communications, thrive on the close areal association (agglomeration) of large numbers of people. This observation is based on economies of scale, and the agglomeration will proceed until diseconomies of scale work to disperse settlement agglomeration. Resource localization distorts a regular settlement pattern because this uneven distribution of resources in quantity or quality permits various savings in transportation expenses around the functional system.

In treating of surfaces, Haggett invokes more abstraction into his analysis and states that the previous topics concerned the skeletons of the regional systems, but that many understandings come from the generalizations of nodes, networks, hierarchies, and movement into density surface functions. Surface configuration and gradient, minimum movement, and distortions of regular gradients are major sub-headings in the study of surfaces. Throughout regional systems, there are various types of influence exerting forces whose intensity varies from place to place. These forces might vary from "income attraction" to "political intimidation and regimentation." Conceptually, any factor such as this can be considered a continuous variable, only varying in intensity from place to place; hence the surface analogy. Gradient and shape of this "social surface" can be examined to determine the nature of variation, nature of changes in intensity,
and so forth. Movement in relation to surfaces has the same implications about cost savings as in other types of analysis. Distortion of surfaces likewise can be studied to find which "other things are not equal" in a system that envisions a smoothly undulating surface where "all other things are equal."

Haggett's system described above is a convenient summary of many of the new lines of geographic inquiry in human geography. It provides some fundamental ideas for geographers to use in devising operational models for solving specific problems. While this is a notable contribution, the dynamics of the models, the theory of which gives essential meaning to this outline of models remains to be articulated. As a guidepost to aid researchers towards promising lines of inquiry, the system is a valuable tool. It is not a comprehensive categorization for modern geographic methodology.

Many of the new techniques described above will yield a rich harvest for the accumulation of geographic knowledge; some will prove to be dead ends. More detailed study and attempts at applications of these new techniques will separate the useful from non-useful. The concluding chapter of this report will suggest some of the avenues that appear to be especially promising to this author.
CHAPTER III

MOST PROMISING NEW TECHNIQUES
MOST PROMISING NEW TECHNIQUES

From the preceding sections of this report, it is plain to see that geographers have made a massive and rapid breakaway from purely descriptive work to a methodological status in which there is a multitude of explanatory techniques that promise fresh and productive perspectives on many geographic problems. Based on these new techniques, which are often adopted from other scientific disciplines, many hypotheses about spatial behavior have been advanced and many more will be formulated at an increasing rate. The great frustration of the immediate new generation of geographers is and will be a lack of adequate data for empirical verification of their hypotheses and models. If geography is to remain an important discipline for practical problem solving, it must make this step with the new tools of research. This chapter will suggest which of the previously discussed techniques seem to offer immediate practical applications, which offer practical applications but require longer-term development, and finally which techniques deserve intensive study to make them available for practical work as soon as possible.

CURRENTLY FEASIBLE TECHNIQUES

Techniques of statistical description and statistical inference can be used at the present time to elicit more information from collections of data. One of the prime requisites to implement these utilizations effectively is to set up a system of adequate and abundant data for future analysis. "The whole field of data storage, data retrieval, data analysis, and data display is expanding rapidly and we may expect that geography, as a special type of regional data - storage system, will reap considerable
benefits from this field of electronic technology." Adequate and reliable data are the sine qua non of statistical analysis. The term GIGO in computer jargon means "garbage in, garbage out," which translates as "the conclusions of statistical analysis are only as good as the data is that is subjected to the statistical analysis." Another important aspect of data concerns the meaningful definition and quantification of variables. It must always be remembered that statistical and mathematical methods are nothing more than systems of logic - meaning is put into the analysis by the definitions of variables, the interpretations of results, and the soundness of the theory upon which the previous two are based. Arthur Ross states the case in this way: "But suppose you administer an intelligence test to a group of children. The test is one thing, a mechanical instrument; intelligence is something else altogether, an abstraction devised by psychologists. While the scores of a test can be calculated, that which is measured remains an abstraction." 

Given adequate data and sound theory, all of the various parameter estimating techniques and descriptive - explanatory techniques can be utilized to make geographic inquiry more pertinent and more scientifically valid. Mean, mode, median, variance, standard deviation, tests for statistical distribution investigation, sampling, and many other data measures can aid geographers not only in the statistical description of their data but also in its spatial description. The statistical parameters can often be directly translated into map categories and subjected to cartographic

1 Chorley and Haggett, op. cit., p. 32.
analysis. Thus the "new approach" of statistical techniques use is combined with the "older approach" of map analysis to form more powerful tools of geographic analysis that yield more incisive insights into old and new problems. If more rigorous data standards are met, the statistical analysis of sampled data help geographers to make inferences about larger, still unobserved, systems they wish to study.

The idea of map transformation is particularly attractive for immediate exploitation. Even some of the simplest Euclidean Transformations described earlier in the report (such as rotation and reflection) can lead us to new insights in geographic data. More complex transformations can fundamentally alter our Euclidean distance concept of relative and absolute location and again point the way to more profound understandings of the cultural environment that exists, as well as the relatively more stable physical environment. Some of these latter developments must, however, await the valid reinterpretation of human relationships through sound theorizing.

Linear Regression analysis and various multivariate statistical techniques can—and should be used more often in geographic research. The major contribution of these and most of the other new techniques is their systematic, bias controlled, and rigorous analysis of data and their standardized, understandable presentation of results. Factor analysis and Principal Components analysis aim at valid, parsimonious description of large quantities of data. These two techniques are especially useful when the researcher at the beginning of a study is confronted by a mass of data on a subject which must be surveyed to ascertain the dimensions of the problem at hand. Also,
the interrelationships among many variables may be quickly learned and non-significant data can be discarded. Factor and Principal Components analyses thus prepare the data for further evaluation and investigation. Regression analysis, either linear or multiple, is another statistical technique for comparing sets of data to determine whether or not functional relationships exist between/among the variables. A most useful property of the analysis is the residuals from regression. These data can be mapped and then analyzed for the discovery of other causative variables which have perhaps been unsuspected. All of these techniques are useful for the testing of geographic hypotheses, a step in scientific geography that needs much more attention.

Graph theory, gravity-potential models, linear programming, and path geometry are a group of related techniques that offer an important "integrated" approach towards the interpretation of nodal regions. Geographers have found that there are many aspects of the human organization of space that are arranged in nodal region form. This is true of actual systems (such as a road network) and of abstract systems (such as a social communications network). The combination of these techniques will permit a structural and functional examination of nodal systems. Graph analysis is fairly rigid and somewhat mechanistic in its description. However, it does help to point out such factors as centrality, connectivity, and relative locations. Linear programming, gravity - space potential analyses, and path geometry all can add functional interpretation to the structural conclusions of Graph Theory study. Because nodes, lines of communication, and spatial flows are all considered in this group of techniques, and because these are all prime topics in geographic systems, they deserve con-
continued efforts to develop the theory which is needed for wider real-world application of the techniques.

Trend Surface analysis provides a technique for a more profound understanding of all sorts of statistical surfaces in geography. The prerequisite for successful Trend Surface studies is, of course, an accurate and meaningful isarithmic map of a physical or human phenomenon. New concepts about mapping abstract properties with isarithms will further expand the types of problems to which Trend Surface analysis may be addressed. Used with regression analysis, Trend Surface analysis tries to separate the true process-response factors which create the undulation in the surface from the chance occurrence factors which are unimportant in surface variations. Generalization of the significant causative factors can therefore produce a shape of the surface that is unlike the surface that contains "noise" factors. Hence a better understanding of the system which the surface seeks to describe.

POTENTIALLY USEFUL TECHNIQUES WHICH REQUIRE MORE DEVELOPMENT

Computers and various computer systems are useful for some types of geographic analysis at the present time, but their use in the future promises to be both more profound and more comprehensive as to various stages of research and to more varieties of geographic problems. Their usefulness of course will increase according to the development of new machine capabilities and to the adaptations geographers can make in conceptualizing their problems for machine analysis. New methods of input into computers such as the Rand tablet will mean that geographers can sketch spatial data and obtain
instantaneous analyses of the problem as data are added or subtracted. Computers will aid geographers in the study of dynamic spatial systems because storage facilities and additional inputs can portray growth or decline of regional systems. Furthermore, computers hold a great deal of promise as means to carry out various simulation problems. In a regional system of a human activity in which change is rapid, perhaps some type of sensing scanner could feed data directly to the computer for "real time" analysis. The development of different types of analog models is another promising use of computers. Electric circuit analogs or merely the programming of various systems and their characteristics into the computer can help to solve exceedingly complex problems based on analog techniques. Map printing by computers will improve in quality and in the types of distributions that can be made. This will aid in all kinds of geographic uses of computers. An obvious use of computers for geography is that of a spatial data storage bank. Storage of data "in place" can be handled now by digitizers, but more sophisticated storage facilities will permit more flexible use of this real-world model type.

Effective widespread use of Monte Carlo simulation techniques must await the development of sounder theory in more geographic systems. The technique's ability to produce a convergence towards an average solution to exceedingly complex problems would be a boon for the scientific study of many human spatial systems. Many human interrelationships are only poorly known and many others are unknown. Techniques to unravel some of these building blocks of the "total man-environment complex" will probably enjoy
the strenuous efforts of many of the good young minds of the coming generation of geographers. Monte Carlo simulation requires that enough be known about the particular system under study that the probabilities of particular events occurring will be known. The remainder of the technique is conceptually simple to apply.

Markov-Chain models also require much more knowledge about spatial systems before they can be effectively used. In the insistence of the model that the possibility for a given event depends entirely on the outcome of an immediately preceding event, the model seems unrealistic. In closer inspection, it becomes apparent that the terms of the model specify some exacting data requirements, thus qualifying the operational characteristics and rendering them more realistic. The model tries to predict the outcome of events based on a set of probabilities that will describe the outcome of the event in terms of some previous "state of the system," plus a component of change. The key to success here is accurate description of the previous state of the system, intimate knowledge about the processes whereby the system changes, and the effect on the system of various types of change plus the likelihood of their occurrence. It is fairly obvious that the "state of the geographic art" is not close to this type of understanding on any spatial system. The model will require long-term development, but the effort promises to be worthwhile.

TECHNIQUES WHICH MERIT IMMEDIATE THRUST

Two groups of new techniques seem to offer the greatest immediate rewards in practical applications and thereby deserve rapid development efforts.
The two groups are: (1) the Map Transformation - Trend Surface Analysis group, and (2) the Graph Theory - Gravity Model - Linear Programming group. While these have been singled out as most promising, practical-oriented techniques, it should be kept in mind that "basic" research and the development of longer-range techniques also deserve research encouragement. Haggett describes a whole new direction in which the quest for improved new techniques for studying geographic problems can be oriented. He says: "Simon . . . has drawn attention to two alternative models of individual behaviour, the optimizer and satisficer models. The optimizer concept has been tacitly introduced into human geography through the assumption of models like those of von Thünen, Weber, Christaller, and Lösch that individuals or groups would arrange themselves spatially so as to optimize the given set of resources and demands. Simon has argued, and Wolpert (1964) has demonstrated . . . , that the optimizer model is rather unsatisfactory. Optimization requires information and decision processes at the highest capacity of the individual or group, and as individuals and as groups there is plentiful evidence that we simply do not operate nor indeed can we operate (because of time uncertainties) at that level. Simon would replace this with a satisficer model which postulates that (1) we rank all the alternative courses of action of which we are aware along a preference scale and (ii) select from this set the course that will satisfy a set of needs. Clearly our choice is often sub-optimal, since ' . . . to optimize requires processes several orders of magnitude more complex than those required to satisfy.'"

---

1 Haggett, P., Location Analysis in Human Geography, p. 26.
Haggett continues the argument in favor of satisficer models as better real-world counterparts, thusly: "This is of course the world which as individuals we know: a world which is neither wholly rational nor wholly chaotic, but a probabilistic amalgam of choice, calculation, and chance." 1

Thomas Kuhn's view on the progress of scientific endeavors is pertinent to the current and future development of geographic methodology. 2 He differentiates between two major processes whereby scientific ideas are advanced, and he calls these procedures "paradigm" research, and "normal science" research. A paradigm according to Kuhn is a scientific achievement that reorients scientific procedures in a discipline and spawns a set of coherent traditions of research behavior. To qualify as a paradigm, the achievement must be such that (1) it is so unprecedented that it attracts an enduring group of adherents away from some previous orientation of scientific activity, and (2) it is sufficiently open-ended to leave all sorts of significant problems left open to solve for its practitioners. In regard to normal science, Kuhn says: "Instead, normal-scientific research is directed to the articulation of those phenomena and theories that the paradigm already supplies." 3

There is little question that geographic scholarship has indeed taken a substantial reorientation in the past two decades. Whether or not the new

1 Haggett, P., Ibid., p. 27
2 Kuhn, Thomas, The Structure of Scientific Revolutions.
3 Kuhn, Thomas, Ibid., p. 24.
scientific-quantitative approach can be called geography's paradigm is debatable, and not particularly important, because it is the results of scientific research which is the important measure of the progress of geography. For the moment assuming that the new geographic approach is a paradigm, it is interesting to note what Kuhn envisions to be the tasks for "filling-in" the new horizons that paradigms open up. He sees three focii for factual scientific investigations after a new paradigm has been discovered. The first step is the solution of problems to determine significant facts (the nature of facts can even change in a discipline under new paradigm metamorphosis!), to include factual precision, scope, and reliability. The second focus is empirical verification of the paradigm's spawned models. The third involves empirical work to articulate paradigm theory, the formulation of quantitative "laws." Paradigm or no, it does appear that the status of geographic research suggests a new expansion of lines of research with many new questions that need to be answered for the better interpretation of spatial systems.

The Map Transformation and Trend Surface analysis group of models should be studied intensively because the concept of fields of influence as they affect human activity is profound, far-reaching, and eminently spatial in orientation. In short, it represents a set of problems in hard-core scientific geography. Data needs to be gathered, astute observations need to be made, incisive intuitive hypotheses need to be advanced, and rigorous empirical verification needs to be performed. The same things can be said about the Graph Theory - Gravity Model - Linear Programming group. Are optimized or satisficer models most appropriate? Can we achieve
acceptable definitions of variables? Can we develop realistic theory?
Do the proposed models actually help us to understand and if necessary modify the real world system to which they apply? These and other questions need to be examined. The work ahead is laborious and many obstacles need to be overcome. But, if new geography is to have some significance for solving practical problems, the burden of this work must be shouldered.

SOFTWARE AND HARDWARE REQUIREMENTS

Any scientific task as large as the one outlined above obviously needs substantial means for its accomplishments. Probably the first need is for more highly trained scientist-geographers to lead the way in unraveling some of the knotty conceptual problems. Once this has been accomplished, the scientists should be able to communicate their findings to intelligent, educated laymen for the use of the techniques. Workers in any practical application of science need some training in the conceptual basis of the discipline from which the scientific ideas issue forth. Proper interpretation of maps in common use today need a "feel" for the nature of geography and geographic reality; this obligation will remain, even though geographic training today requires more rigorous thinking. We still need a volume entitled "Principles of Spatial Analysis," focusing on concepts that are not obscured to many people by excessive mathematical jargon, to include symbolism. All the techniques discussed in this report are either presently amenable to machine analysis, or certainly are potentially suited to this work saving grace. Many geographers should perhaps devote their efforts to the translation of models that interest them into machine capable format.
ANNOTATED BIBLIOGRAPHY
ANNOTATED BIBLIOGRAPHY

   This article discusses the sociological general marital pattern. It states that the trend has been to more marriages between people of close residence than in previous times.

   This is a theoretical and methodological discussion of modern geography's most important frontier, fundamental research. It is the base upon which the analysis and synthesis of earth-space content is the final step toward comprehending geographical reality.

   Presidential address to AAG speaks for a reorientation of the discipline to a "spatial systems approach."

   This is a methodological, sociological article which tests Zipf's and Stouffer's hypotheses on intervening opportunities and which presents an alternative, although similar, general, hypothetical model.

   This is a methodological paper using four independent measures to explain variation in certain migrations between metropolises in northeastern and northcentral regions of the U.S. between 1935-40.

   This is a methodological discussion of a geographical problem. He states that there is some difficulty in representing distance in terms of cost because of variability of cost.
   The population potential model does not identify forces in the sense of externally operating phenomena as much as it attempts to describe the internal relationships of the groups of men (societies).

Texts of presentations by a group of geographers on introductory geography courses to an NSF sponsored summer institute. Discusses the concept approach for introductory geography courses.


Describes method of deriving uniform regions of U.S. census data, using direct factor analysis and distance scaling of similarities.


Proposes geographic data matrix as means for organizing regional data and as a synthesis of ideas current in geography.


This is a practical attempt to clarify statistics' function in relation to scientific method and geography. In the inductive stage of scientific method (including geography's use of it) statistics can be effectively used. Mapping and summarizing can use statistics as can the process of acquiring knowledge of associations and relationships in the observed data.


This is a useful compendium of articles by several "new-school" geographers. Topics range from approaches to regional analysis to trend surface analysis. Discussions of spatial systems analysis and trend surface analysis are especially helpful in considering new developments in geography.


A basic treatment about the use of the concept of "system" in studying problems in social science.

Bogue, Donald J. and Harris, Dorothy L., Comparative population and Urban Research Via Multiple Regression and Covariance Analysis, Oxford, Ohio, Scripps Foundation, 1954.

Study of urban characteristics utilizing multiple regression and other statistical analyses. Good case study of technical application.

A very general application of systems analysis to geography is made here. Systems theory makes it possible to view geographic problems in terms of the variability of systems instead of the static view possible without this concept.


This is a book concerning the theory, methodology, and techniques of geography.

Bright, Margaret and Thomas, Dorothy S., "Interstate Migration and Intervening Opportunities," American Sociological Review, Vol. 6 (1941), pp. 773-783.

Stouffer's sociological theory of intervening opportunities is supported by the example stated here. The pattern of opportunities were followed closely by interstate migration. It is suggested that allowances be made for major disturbances in the pattern due to quantitative differences and certain directional factors.


One of social science seminar series, aimed at explaining geography to non-geographers.


Within the article development and underdevelopment are defined. The flow of output is said to depend on the factor supply, factor use, and socio-cultural environment. There is a discussion of how to solve the problems of underdevelopment.


This is an historical survey of geography. This volume deals with the very earliest development in the field.


This is a theoretical paper that discusses scientific theory and relates geographical investigation to it. Geographical research should not emphasize the unique.

This is Bunge's Ph.D. dissertation. He discusses geography and its place among the sciences; concludes that "Geography is the science of locations, regional geography classifies locations and theoretical geography predicts them." (p. 195) He sees two general topics: movement theory, and central place theory.


The various papers in this collection are mostly abstract discussions and add little to the understanding of possible or probable map uses.


Quantitative geography is here to stay; it promises to breed more scientific theoretical work.


This is a methodological article showing how to describe and find the magnitude of a city's influence on its surrounding hinterland. Quantitative methods are used.


This is a methodological discussion of metropolitan planning through study of the circulating system. Future population and land use, if known, could give future travel requirements and the utility network could be designed to meet those requirements.


This is an historical discussion of a sociological-geographic topic. The gravity and potential concepts of human interaction were formerly considered in terms of Newtonian physics of matter without considering the individual's ability to make decisions concerning his action.


This is a methodological paper which uses income potential to predict population in given regions through gravity models.
This book traces the work done by Indian geographers, according to major segments of the field, for the past 50 years.

This is a theoretical paper on using closed or open systems in geomorphology and then the advantages of the open system framework are specifically discussed.

This is a methodological paper concerned with model building in geography. It presents a model for models and also goes into the various kinds of models: mathematical, experimental, etc.

The concern of this chapter is the methodological problem of shrinking reality to make it comprehensible which is what geography as a macroscopic science must do.

This is a theoretical chapter on applying quantitative methods to geomorphology; why it has been so slow in developing within the field and some description of how it can be applied.

This is a collection of articles by various geographers that focus on new techniques of research. Discussions of game theory, Markov chain models, and simulation through Monte-Carlo methods are illuminating. The chapters are useful for suggesting needed research lines.

Classical work on the development of node-hinterland relationships. Many original concepts on central place systems which are today being articulated.

Committee on Geography, Advisory to the Office of Naval Research Division of Earth Sciences. Report of a conference on the Use of Orbiting Spacecraft in Geographic Research, Washington, D.C., National Academy of Sciences National Research Council, 1966. This is a report of the committee of what is being done with orbiting spacecraft in geographic research with such tools as side-looking radar.

Davie, M.R. and Reeves, R.J., "Propinquity of Residence Before Marriage." American Journal of Sociology, Vol. 44, 1939, pp. 510-517. This sociological discussion of marriage patterns in relation to distance between contracting partners suggests that part of the explanation for the close relationship between proximity and marriage may be due to 'natural' groupings of the same types of people and that like tend to marry like.

Davis, W.M., "An Inductive Study of the Content of Geography." Geographical Essays, New York, Dover Publications, 1909, pp. 3-22. The content of the subject of geography is discussed. It is important when determining whether a statement is geographical or not to know how far it constitutes or enters into causal relations. A geographical quality is gained when organic and inorganic elements are related in a cause and effect situation.

Dodd, Stuart C., "The Interactance Hypotheses: A Gravity Model Fitting Physical Measures and Human Groups," American Sociological Review, Vol. 15, 1950, pp. 245-256. This is a theoretical and methodological article relating dimension of physical quantities to social ones. Stouffer's intervening opportunities is a related index. Probability conditions are the base of the gravity models so larger masses are more reliably predicted.


Dohrs, Fred E. and Simmons, Laurence N. (eds.), Introduction to Geography: Selected Readings. New York, Thomas Y. Crowell Company, 1967. This is a good collection of articles on geographical topics including Carl Sauer's articles on the morphology of landscape.

This book deals with theory and method in dealing with the metropolis and region. Population potential is not emphasized in the physical science sense but as a convenient indicator of generalized accessibility.


This is a study by sociologists on their conception of problems in geography. This is advanced level in jargon, but basic and often naïve in geographical concept.


Uses space potential model for analysis of industrial location.


This is a statement by a prominent group of geographers about the trend in geographic science. It tries to fit old and new lines of research into a scientific mold. While a great improvement over the older statements on the nature of the field, the results of the study appear to be typical of committee work: too many compromises to be truly useful.


History of geographic knowledge from dawn of civilization to present. Lacks philosophy of geography today. Describes use of maps by Alexander the Great and Roman and Aztec armies.


Suggest lines of geographic inquiry amenable to quantitative analysis.


This is a theoretical, methodological paper that basically states that interaction concepts are essentially probability concepts.

This is a theoretical article concerned with the interrelations between the concepts of location and interaction.


Surveys of new approach to human and physical geography


Relationships between geographic facts can be discovered by "Laws;" cross-section, equilibrium, historical, developmental and statistical.


This is a methodological book for geographers presenting the statistical methods useful to the discipline.


A method for studying the spatial diffusion of ideas.


Discusses model of spatial diffusion as a geographic event.


This is a methodological paper describing some problems of scale standardization in geography. A proposal for standard geographical measurement is made.


Basic description of models now being used in locational analysis.


Discusses trend-surface analysis as a "Logical extension of regression analysis into two dimensions."

This is a methodological discussion of a topic dealing with interaction and distance. "The mechanism by which spatial distance is related to the frequency of human interactions is very involved and cannot be explained by some simple sociologic or economic interpretation."

Harary, Frank and Norman, Robert Z., *Graph Theory as a Model in Social Science*, University of Michigan Institute for Social Research, Ann Arbor, 1953.

Terminology and basic concepts for utilizing graph analysis in social science.


This is a methodological, geographical article discussing 'market potential' as a meaningful index and how it can be used as an indication for an area of the intensity of possible contact with markets.


This is a "classical" statement by an old-time geographer on the nature of the field. It is full of conceptual error such as: geography today should be defined in terms of what geographers of a century ago thought. This is basically wrong because he gives no consideration about the evolution of science. What was profound science to 19th century German geographers can be naive work to modern scholars because we have more powerful analytical tools today, hardware and software.


This paper concerns the history of the concept of geography as a chorological science rather than a systematic or historical science. In the conclusion four differences between geography and other sciences are noted.


This is a clearer, somewhat updated statement on earlier work. He sees a role for new methods in geography, but persists in stating the almost meaningless and impossible task for geography is to study phenomena in terms of the "areal variation of integrated phenomena."

Surveys literature and concludes that dynamic models, especially those based on stochastic processes, can be useful in agricultural geography.


This is a book that describes the history and present use of computing machines, and future possibilities.


This is a methodological paper that tries to put dynamic analysis in place of usual static by taking job opportunities and climate to predict net in migration to specific regions.


This is a methodological book for regional science. Gravity, potential, and spatial interaction models and the forces involved are discussed at length as are other techniques.


The theory of intervening opportunities seems to be verified with Swedish data in spite of circle measurements and doubtful suitability of data used as criteria for opportunities.


Papers varying from topics on glacier ice in Greenland to "kitchen middens" in Ghana. These mostly are case studies, using well-developed techniques.


Distinguishes between avoidable error and error in inadequate hypotheses. Former shows poor scholarship, latter is honest mistake and can help progress of science.

The theory and the method of study of the regional concept as the core of geography are discussed. The processes that form characteristic patterns for particular places on earth must be studied as dynamic features and the relations of regional systems to the underlying processes must be made as well as mapping regional systems.


This is a general paper describing modern geographic methodology. Today geography applies specific process information from the systematic sciences to its field. Two basic conclusions on geographic method: 1) regions with too many different processes involved make correct interpretation difficult, 2) the method cannot be solely contemporary.


This is an historical overview of the changes in the American geographical approach. The shifts from the early regional approach to the present two-way method of regional and systematic approach with the modern mathematical techniques involved are discussed.


Study of historical development of geography from ancients to modern practitioners.


Says the six most universal geographic regularities in physical processes are: 1. wholeness; 2. circulation systems for materials and energy; 3. rhythmic phenomena; 4. zonation; 5. azonal phenomena; 6. continuity of evolution engendered by exogenic-endogenic process struggle.


An analysis of possible use of new hardware for spatial data manipulation.


This is a theoretical book describing mathematical models used in the social sciences.

This book states that geography came to stand for the bits and pieces of knowledge unaccommodated by the seven liberal arts of the Middle Ages. Aristotle believed that geography ought to be regarded as a branch of applied mathematics, like geometry or astronomy; Plato felt it should be a branch of physics.


An introduction to statistical geography on first year graduate level.


This book traces the development of ideas and knowledge through scientific endeavors. It states that "normal science" is the common work that comprises most scientific endeavors to fill in the areas outlined by the theoretical "paradigms" or truly revolutionary ideas in science.


In the introduction to this article it states that the trend in social research today is toward singling out the basic variables from which all specific concepts and interrelationships can be derived. This is different from earlier times.


Kingsey Davis' article in this series is a detailed study of research steps. The frame of reference, deductive propositions, empirical propositions, and empirical generalizations are discussed in their content and functions.


This is a methodological paper which introduces the principle that "the most Probable condition exists when energy in a river system is as uniformly distributed as may be permitted by physical constraints. From these general considerations equations for the longitudinal profiles of rivers are derived that are mathematically comparable to those observed in the field.


A classical book on the fundamental economic concepts involved in locational analysis.

Says ancients were interested in location as description of place - "The analysis of process, cultural and natural, was the objective of description, and genesis was its mode of classification." Does not truly demonstrate this.


This is a methodological paper which consists of criticisms of gravity model assumptions. Potential models predict tendencies toward spatial equilibrium and only in this sense are they deterministic.


Studies college enrollment in Ohio by use of gravity model. He finds good correspondence between the model and reality.


Brief introduction into capabilities of computers as aids in geographic research.


Useful for bringing together many new ideas in economic geography into one source.

Marble, Duane F., *Some Computer Programs for Geographic Research*. Department of Geography, Northwestern University, Special Publication, No. 1, August 1967.

Gives purpose, and description of each program, as well as the format of input into the computer.


A short paperback book on the meaning that can be derived from the use of specific statistical techniques; shows how to use the techniques.


Within the article the distinctions between industrial location theory and central place theory are discussed.
Morril, R.I., "Quantitative Methods in Geography." Unpublished discussion paper, University of Washington, Department of Geography.

This is a theoretical paper discussing scientific approach in geography. The ideographic and nomothetic approaches are discussed and the qualifications of scientific endeavor are listed.


Lays foundation for development of algorithms that will facilitate the digital computer solution of problems involving contour line data.


First eleven chapters of "An Introduction to Geography." Fails to clearly define geography or geographic task.


Detailed examination of political problems, economic realities, problems of health, education, literacy, population size planning, labor utilization.


This work concerns a general theme in geography: movement over space.


He praises the work as a useful compendium of already published works but states that "applied geometry" approach to geography is the bias of "The Washington Group."


Discusses categories of industries according to the predominant length of haul of the differing industries.


Shows that high value-added industries are still located in the American manufacturing belt.
Seven studies by Hungarian geographers that describe various problems in cartography. Topics vary from an equal area world projection to the use of radar maps in river navigation.

Seeks regularity for migrations of people within England, Scotland, and Ireland.

Uses the gravity model to derive retail trade regions.

This is an article discussing and listing what, by 1956, had been published on statistical methods in geographic research.

Discusses the "distribution problem" for area data.

Compares population size in cities with number of services available - maps residuals - seeks regional vs. local effects with trend surface mapping. Linear 1, Quadratic 2 + 1, Cubic 3 + 2 + 1. Shows some regional effects.

This article cautions readers about the fundamental errors that can be caused by the uncritical use of data. Oversimplification of real-world problems and the drive for "hard data" to support policy decisions are the underlying causes of erroneous conclusions.

Within this article the mechanistic outlook is described.

A preliminary work in development of the idea of diffusion over space.

The concept, exceptionalism, that geography covers all phenomena in terms of space while history covers all in terms of time is here refuted. Although geography's core is regional, there must be a nomothetic approach in the field or it will stagnate as merely descriptive.


This is a theoretical (and methodological) criticism of the complete validity of the gravity models. He states that the gravity model is not and does not try to be explanatory and this is its failure.


This reprint of an issue of the Scientific American discusses the evolution of computer capabilities as information systems. It discusses input-output innovations, time sharing and potentialities of computers for various jobs.


This is a treatment about the scientific method in research. It is a "how to" study to teach students the scientific approach to learning. It is a good summary statement about such things as hypotheses, theory, cause-effect relationships, scales of measurement, analysis, and interpretation.


Discusses technique for testing empirical statistical distributions against model distributions.


Uses "basic cost" and "location cost" concepts as a foundation for a system of transportation explained location.


This is a theoretical discussion of the semantic problems of what is a scale of measurement which was discussed at a British Association for the Advancement of Science Meeting.


This is a sociological discussion of the directly proportional relationship between the distribution of residence and the total population of an area and inversely proportional relationship to the distance from the center. Undergraduate and alumni of a given college were samples. An isopotential map of population was used.

This is a methodological article on the proportional relationship between representation from each state to four "national" colleges to each state's white male population divided by the distance in miles to the campus.


This discusses the parameters of the gravity model in its use for describing population distribution.


This is a theoretical, sociological article discussing the quantitative study of society, social physics. Concepts from mathematics, economics, and physics are used when appropriate.


This is a socio-geographic discussion of the interrelation between population potential and other social-economic data which present a picture of the U.S. sociological structure different from that of the census bureau in 1948, who apparently did not see the intricate interaction of the factors involved.


This is a methodological discussion of the value of interdisciplinary work to solve a sociological problem, here in social physics. Social scientists in general are referred to as having the need for interdisciplinary techniques.


This shows how population space potential can be used as a market measure for locational problems.


The sociological field of social physics is here discussed and the form of analysis using concepts from other disciplines are here described (such as social temperature).

This is a theoretical article discussing the need for more abstraction and generally, basic theory in geography. It emphasizes integration in the social science.


This report describes on-the-average regularities which have been proved to exist in the distribution of people within cities and in rural areas and across countries as a whole.


A sociological theory of intervening (cumulative) opportunities is presented which may be used as a basic organizing principle to show patterns of population movement toward opportunities even if numerical data is inadequate.


This is a sociological discussion of the theory (Stouffer's) of intervening opportunities. He states that in different situations the ratio of opportunities to intervening opportunities need not be constant in the best fitting formula.


This methodological comment states that the gravity and potential models are not representative in certain cases and that perhaps the approximations made with present models will be improved upon by later methods.


Shows that large cities tend to draw migrants and concludes that it is not distance, but intervening opportunities which really affect numbers of people migrating from one state to another.


Small section using factor analysis to describe regionalizing New York State based on various indicators of "Economic Health."

This is an historical survey of ancient geography.


This article describes the data-processing system for map making and also automation in data gathering is discussed.


This seminal work discusses fundamental concepts in transportation, including the use of flow maps.


This volume contains a description and an interpretation, through text and maps, of rail and water traffic flows in American domestic and foreign trade.


Development occurs in concentrated areas in various countries. An area might have an initial advantage for development, and then the advantages of agglomeration compound the initial advantage, creating further concentration of development.


Use of gravity model in studying traffic flows.


This is a methodological paper which analysis the voting patterns of Newfoundland for confederation with Canada (isolated lower populations potential areas) or for responsible government (high population potential areas) in a geographic sense.


This is a methodological paper that suggests that space and time are more than cost incurring external frictions but rather, dimensions of the economic system which can be treated isomorphically in the rigid pattern of mathematical physics. The geographical distribution of farm prices of certain agricultural commodities are investigated.

The purpose of this study is to develop the theory of space potential and demonstrate its usefulness as a type of methodology in economic geography.


This is a methodological paper that deals with gravity models and their use to indicate average relations of people existing in a space continuum. For studying a geographical process as spatial flows in economic and social systems, properly weighted populations are substituted for mass.


Tries to develop concepts for aggregative analysis of human behavior.


Use of gravity model to the advance of a "front" of growing incomes and its impact on affected economy.


This work gives insight into the process by which a major scientific discovery was made. It depicts the continuous interchange between theoretical and empirical approaches. It demonstrates how intuition and guesses by highly trained people lead to clues about scientific fact that then can be checked out.


Classical book on the factors that tend to localize industries. Labor, materials, market, transportation costs are specified as major location factors.


Theory and method of regional research are discussed. Compages, a hierarchy of compages, and a ranking of regions are proposed as organizing factors of regionalism.

This is a very basic introduction to the systems analysis concept. Both systems for analysis and analysis of systems are discussed. Spatial systems and the defining of the role of functionally important spatial variables are appropriate to geography. Systems for analysis is not used much presently in geography and a geographic analysis of systems is as yet unclear.


Classical geography exemplified by Ratzel and Ritter; regional by Vidal de La Blache; modern by quantifiers. Geography has evolved to answer new questions being put to it.


Clear explanation of basic statistical techniques with practical examples of actual economic situations.


The hypothesis $P_1P_2/D$, which refers to all movement including persons was shown here to closely follow highway data concerning railway express shipments, telephone calls, and even airway shipments in 1933 although not as closely as the other two.


This is a methodological, sociological discussion of the minimum equation, used here as the assumption that work is always minimized in human behavior. This hypothesis is related to city growth, production and generally, economic development.


This is a sociological application of the economic principle of least effort (human behavior is taken into consideration). The factors that determine the location of the point that minimizes the work of transportation of persons to materials and materials to persons is discussed with equations.
APPENDIX A

GRAVITY MODEL USE TO ANALYZE A

TRANSPORTATION NETWORK
GRAVITY MODEL USE TO ANALYZE A TRANSPORTATION NETWORK

Data are presented in map and table form in the following pages to illustrate some lines of inquiry which can be opened up by the application of a particular statistical technique to a practical problem. This example is offered not as a paragon of either gravity model capabilities or specific real-world information that is generated by the model, but rather as an insight into the types of questions that may be answered through rational statistical analyses. The flexibility of statistical models in eliciting specialized information requirements and in the format of information presentation is also shown.

This brief study uses the flow concept inherent in space potential analysis to study transportation efficiency in an underdeveloped economy. The factor values generated by the space potential model (written as:

\[ V_i = P_i + \frac{P_j}{d_{ij}} \]

where \( V_i \) is the space potential at point \( i \), \( P_i \) is the population at point \( i \), \( P_j \) is the population at point \( j \), \( d_{ij} \) is the distance between \( i \) and \( j \), and \( j=1 \) means that the value of \( j \) varies from 1...n.) represent the "aggregate accessibility" of all masses in a region to a given point. Implicit in the idea of aggregate accessibility, and explicit in the mathematical formulation of the model, is the notion of a movement of masses over a cost-incurring, time-consuming, and energy-absorbing space. Because space potential suggests relative transportation efficiency, the analysis appears to be valid for the study

---

1 The full text of this paper will appear in the Proceedings of the Statistical Techniques Symposium of the 21st International Geographical Congress, New Delhi, India, 1968.
of the functional characteristics of a transportation network in a system of economic production.

Northeast Mexico is the locale for the space potential study of transportation. A "sample factor" of production, population of the 80 most populous municipios, must flow over the transportation networks of the region to accomplish the manufacturing operations of the system. The ability of the transportation network to move populations among the nodes in the system comprises the efficiency of the network, which in turn can be thought of as influencing the manufacturing productivity of the area.

The present transportation system, location of the cities in the network, and the population sizes of the cities are the major factors in determining the efficiency of the system. A comparison of two space potential solutions, one using straight line distance as the friction of distance factor and the other using actual route distance, is the procedure for measuring the degree of efficiency with which each city in the system is served by transportation. Indirect transportation links, rough terrain, variations in qualities of the transportation links, configuration of the transportation network, and transportation bottlenecks are some of the reasons why the actual transportation system might vary from the service which could be provided by a perfect transportation network. Efficiency of the various nodes can thus be defined as the difference between the population space potential value that could be attained with perfect transportation and that value that is attained, given the actual transportation routes. The percentage
difference between ideal and actual is the measure of relative efficiency.

**MAP AND TABLE ANALYSIS**

The information available through this analysis is presented on the following five maps and five tables. Map 1 shows the location of nodes (municipios) in northeast Mexico; maps 2 and 4 show the population space potential isoline distributions; map 3 shows the actual transportation system; and map 5 presents the conclusions of the study, showing the relative efficiency of transportation at each node. Tables 1, 2, and 3 compare the raw data from the analysis and array and categorize the results; table 4 enumerates and compares efficiency categories with other characteristics of the transport network; and table 5 reveals the centrality of each node, comparing this with other measures.

The transportation efficiency of each municipio in northeast Mexico can be evaluated both according to its ability to assemble the population of the region and to its connectivity to other centers in the system. Map 5 and tables 1-4 provide the data for the former problem while Map 3 and Table 5 summarize the data for the latter problem. Galeana (20), for example, ranks "average" in transportation efficiency on Map 3, based on the fact that its population space potential could be raised 14.60% if it had straight line access to all other municipios. Table 3, however, ranks Galeana 18th in inefficiency for assembling absolute numbers of population. Galeana could have its accessibility improved almost 15% with better connections to nearby major population centers.

Map 3 indicates poor location for Galeana in the transportation networks since it is the terminus of a major paved road extension.
NORTHEAST MEXICO
TRANSPORTATION MAP 3
with connections to the rest of the net to the south, but no direct connections to the major centers of the north. Physical centrality or connectivity among all nodes in the transportation system of northeast Mexico is listed on table 5. The basic data for the table, columns 2 and 3, are summations of the distance matrix columns for each municipio, which represents the total distance, actual and straight line, between the first and all other municipios. The table conveys the idea that the more total distance necessary to connect a municipio to all others, the poorer the centrality or connectivity. A "one" ranking in quintile position (last column) is a poor ranking, a "five" ranking is good. Galeana ranks number 70 on this table, the municipio ranking in the best categories of both centrality listings. Space potential and centrality/connectivity measures are different, but both highly significant dimensions of transportation efficiency. Space potential indicates that Galeana does not rank well in accumulating flows of population, while the centrality/connectivity measures demonstrate favorable position in the region's transportation net. A comprehensive transportation advantage evaluation of Galeana should include both conclusions, excellent location for possible high accessibility, but needing connections to major nearby centers.

Rational statistical analysis thus provides information; sound professional interpretation supplies the meaning of this information.
NORTHEAST MEXICO
POPULATION POTENTIAL
Actual Road Distance
MAP 4
NORTHEAST MEXICO TRANSPORTATION EFFICIENCY MAP 5
### Table 1. Comparison of Space Potential Analyses by Municípios

<table>
<thead>
<tr>
<th>Município</th>
<th>Space Potential</th>
<th>Acre</th>
<th>% Local</th>
<th>#</th>
<th>Line</th>
<th>Dist</th>
<th>Diff</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Monterey</td>
<td>424,643</td>
<td>618,675</td>
<td>5,948</td>
<td>9.55</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Torredó</td>
<td>405,310</td>
<td>609,005</td>
<td>6,784</td>
<td>10.70</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 San Luis Potosí</td>
<td>380,610</td>
<td>594,230</td>
<td>4,201</td>
<td>2.05</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Zacatecas</td>
<td>356,390</td>
<td>535,245</td>
<td>1,552</td>
<td>1.39</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Tepic</td>
<td>346,595</td>
<td>565,295</td>
<td>4,600</td>
<td>3.88</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 Durango</td>
<td>346,595</td>
<td>565,295</td>
<td>4,600</td>
<td>3.88</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 Irapuato</td>
<td>346,595</td>
<td>565,295</td>
<td>4,600</td>
<td>3.88</td>
<td>C</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 San Nicolás de los Caballeros</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Nuevo León</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 Ciudad Madero</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 Santa Fe</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 México</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 Cuernavaca</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 Toluca</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17 Baja California</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 Tampico</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 Chihuahua</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 Monterrey</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 Ciudad de los Ángeles</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27 Guadalajara</td>
<td>211,808</td>
<td>368,743</td>
<td>15,935</td>
<td>28.71</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 1. Continued

<table>
<thead>
<tr>
<th>Município</th>
<th>Space Potential</th>
<th>Acre</th>
<th>% Local</th>
<th>#</th>
<th>Line</th>
<th>Dist</th>
<th>Diff</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>41 Ciudad de los Caballeros</td>
<td>45,066</td>
<td>25,030</td>
<td>10,034</td>
<td>22.30</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>42 Ciudad de los Caballeros</td>
<td>45,066</td>
<td>25,030</td>
<td>10,034</td>
<td>22.30</td>
<td>M</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43 Arandas</td>
<td>43,875</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44 Tamaulipas</td>
<td>41,900</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45 Villahermosa</td>
<td>39,375</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46 Coahuila</td>
<td>37,875</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47 Altamira</td>
<td>36,375</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48 Tampico</td>
<td>34,875</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49 Tamaulipas</td>
<td>33,375</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 Saltillo</td>
<td>31,875</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51 Zapopan</td>
<td>30,375</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52 Saltillo</td>
<td>30,375</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>53 Chihuahua</td>
<td>28,875</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>54 Juárez</td>
<td>27,375</td>
<td>25,012</td>
<td>12,763</td>
<td>17.90</td>
<td>H</td>
<td>E</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Key to Last Column: First letter: C-Crossroad, T-Terminal, M-Middle of Route
Second letter: E-East, W-West within 50 kilometers of transportation met 8-hour.*
### TABLE 2. — ARRAY OF INCREASE IN POTENTIAL CALCULATED WITH STRAIGHT LINE DISTANCE COMPARED TO POTENTIAL CALCULATED WITH ACTUAL ROAD DISTANCE, EXPRESSED AS A PERCENTAGE OF STRAIGHT LINE DISTANCE POTENTIAL.

<table>
<thead>
<tr>
<th>Municipio</th>
<th>Percent Change</th>
<th>Municipio</th>
<th>Percent Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahualulco</td>
<td>40.21</td>
<td>Doctor Arroyo</td>
<td>12.73</td>
</tr>
<tr>
<td>Tamaulip</td>
<td>33.71</td>
<td>Gonzalez</td>
<td>12.42</td>
</tr>
<tr>
<td>Villa Reyes</td>
<td>33.19</td>
<td>Camargo</td>
<td>12.14</td>
</tr>
<tr>
<td>Moctezuma</td>
<td>32.22</td>
<td>Santiago</td>
<td>12.05</td>
</tr>
<tr>
<td>Mexquitic</td>
<td>31.58</td>
<td>Matehuaca</td>
<td>11.65</td>
</tr>
<tr>
<td>Salinas</td>
<td>30.80</td>
<td>Villa Frontera</td>
<td>11.38</td>
</tr>
<tr>
<td>Ciudad Hadero</td>
<td>28.75</td>
<td>San Fernando</td>
<td>10.82</td>
</tr>
<tr>
<td>San Nicolas</td>
<td>28.71</td>
<td>Monclova</td>
<td>10.79</td>
</tr>
<tr>
<td>Ramos</td>
<td>28.70</td>
<td>Rioverde</td>
<td>10.27</td>
</tr>
<tr>
<td>Charcas</td>
<td>28.37</td>
<td>Altamura</td>
<td>10.27</td>
</tr>
<tr>
<td>Villa Juarez</td>
<td>26.41</td>
<td>Parras</td>
<td>10.16</td>
</tr>
<tr>
<td>Cadral</td>
<td>24.43</td>
<td>Miguel-Alemán</td>
<td>10.04</td>
</tr>
<tr>
<td>Ocampo</td>
<td>24.16</td>
<td>Anahuac</td>
<td>9.67</td>
</tr>
<tr>
<td>Santa Caterina</td>
<td>23.73</td>
<td>Monterrey</td>
<td>9.55</td>
</tr>
<tr>
<td>Guadalupe</td>
<td>22.79</td>
<td>Sabina</td>
<td>9.43</td>
</tr>
<tr>
<td>Ciudad Fernandez</td>
<td>22.30</td>
<td>Sabina Hidalgo</td>
<td>9.36</td>
</tr>
<tr>
<td>Cerritos</td>
<td>21.55</td>
<td>Temascal</td>
<td>8.77</td>
</tr>
<tr>
<td>Cardenas</td>
<td>21.55</td>
<td>General Terán</td>
<td>8.57</td>
</tr>
<tr>
<td>San Martin</td>
<td>21.29</td>
<td>Garza García</td>
<td>8.33</td>
</tr>
<tr>
<td>Alfredo Terrazas</td>
<td>20.58</td>
<td>Hidalgo</td>
<td>8.16</td>
</tr>
<tr>
<td>Villa Arriaga</td>
<td>20.11</td>
<td>Hidalgo</td>
<td>7.97</td>
</tr>
<tr>
<td>Zaragoza</td>
<td>18.99</td>
<td>San Juan de Sabina</td>
<td>7.89</td>
</tr>
<tr>
<td>Xicotencatl</td>
<td>18.89</td>
<td>Valle Naranjo</td>
<td>7.39</td>
</tr>
<tr>
<td>Tula</td>
<td>18.76</td>
<td>Tecatezchul</td>
<td>7.25</td>
</tr>
<tr>
<td>Hidalgo</td>
<td>18.26</td>
<td>Ciudad de Valles</td>
<td>7.17</td>
</tr>
<tr>
<td>Jumave</td>
<td>18.14</td>
<td>Kante</td>
<td>6.89</td>
</tr>
<tr>
<td>Arteaga</td>
<td>17.98</td>
<td>Tocotonic</td>
<td>6.78</td>
</tr>
<tr>
<td>Ramos Arizpe</td>
<td>17.29</td>
<td>Linares</td>
<td>5.98</td>
</tr>
<tr>
<td>Llano</td>
<td>15.72</td>
<td>Cárdenas Jímenez</td>
<td>5.87</td>
</tr>
<tr>
<td>Aquiscom</td>
<td>15.42</td>
<td>Francisco Nadero</td>
<td>5.53</td>
</tr>
<tr>
<td>Guadalcázar</td>
<td>15.30</td>
<td>Acuna</td>
<td>5.49</td>
</tr>
<tr>
<td>Ciudad Hidalgo</td>
<td>15.26</td>
<td>Piedras Negras</td>
<td>5.09</td>
</tr>
<tr>
<td>Rayón</td>
<td>15.02</td>
<td>Victoria</td>
<td>4.66</td>
</tr>
<tr>
<td>Viesca</td>
<td>14.82</td>
<td>San Pedro</td>
<td>4.52</td>
</tr>
<tr>
<td>General Copada</td>
<td>14.63</td>
<td>Saltillo</td>
<td>3.88</td>
</tr>
<tr>
<td>Galana</td>
<td>14.60</td>
<td>San Luis Potosi</td>
<td>2.85</td>
</tr>
<tr>
<td>Tamuin</td>
<td>14.25</td>
<td>Montecoreles</td>
<td>2.53</td>
</tr>
<tr>
<td>Santa María</td>
<td>13.23</td>
<td>Nuevo Laredo</td>
<td>1.80</td>
</tr>
<tr>
<td>Soledad</td>
<td>13.15</td>
<td>Rayon</td>
<td>1.50</td>
</tr>
<tr>
<td>Xilitla</td>
<td>13.13</td>
<td>Matamoros</td>
<td>1.39</td>
</tr>
</tbody>
</table>

1. Very Inefficient

2. Average

3. Average

4. Efficient

5. Very Efficient
<table>
<thead>
<tr>
<th>Municipio</th>
<th>Potent.</th>
<th>% Incr</th>
<th>Rank</th>
<th>Z - ab</th>
<th>Quint</th>
<th>Posit</th>
<th>Municipio</th>
<th>Potent.</th>
<th>% Incr</th>
<th>Rank</th>
<th>Z - ab</th>
<th>Quint</th>
<th>Posit</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Nicolas</td>
<td>34,138</td>
<td>4</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Camargo</td>
<td>5,836</td>
<td>43</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guadalupe</td>
<td>26,968</td>
<td>15</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Saltillo</td>
<td>5,863</td>
<td>75</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Héctor</td>
<td>25,827</td>
<td>7</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Xicotencatl</td>
<td>5,720</td>
<td>23</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. Cortina</td>
<td>17,230</td>
<td>14</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Jamapa</td>
<td>5,719</td>
<td>26</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexquitic</td>
<td>16,644</td>
<td>5</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Tamuin</td>
<td>5,336</td>
<td>37</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Hidalgo</td>
<td>14,484</td>
<td>1</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Aquismon</td>
<td>5,638</td>
<td>30</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tampico</td>
<td>22,841</td>
<td>57</td>
<td>4 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Llera</td>
<td>5,151</td>
<td>29</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Villa Reyes</td>
<td>11,706</td>
<td>3</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Rayon</td>
<td>5,127</td>
<td>33</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tamaulipas</td>
<td>10,908</td>
<td>2</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Parras</td>
<td>5,024</td>
<td>53</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charcas</td>
<td>10,338</td>
<td>10</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Gen. Cap.</td>
<td>4,875</td>
<td>35</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noctesuma</td>
<td>10,313</td>
<td>4</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Viesca</td>
<td>4,833</td>
<td>34</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T. Fernandez</td>
<td>10,034</td>
<td>16</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Tamazun.</td>
<td>4,713</td>
<td>64</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selina</td>
<td>9,472</td>
<td>6</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Malete</td>
<td>4,512</td>
<td>66</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramos</td>
<td>9,004</td>
<td>9</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>Monclova</td>
<td>4,459</td>
<td>48</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V. Juarez</td>
<td>8,447</td>
<td>11</td>
<td>1 - 1</td>
<td></td>
<td></td>
<td></td>
<td>V. Berroso</td>
<td>4,303</td>
<td>53</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramos</td>
<td>8,313</td>
<td>28</td>
<td>2 - 1</td>
<td></td>
<td></td>
<td></td>
<td>C. Valles</td>
<td>4,267</td>
<td>63</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arizpe</td>
<td>8,221</td>
<td>17</td>
<td>2 - 1</td>
<td></td>
<td></td>
<td></td>
<td>San Pedro</td>
<td>4,031</td>
<td>74</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cuauhtémoc</td>
<td>7,863</td>
<td>36</td>
<td>3 - 2</td>
<td></td>
<td></td>
<td></td>
<td>S. J. Sabina</td>
<td>4,033</td>
<td>62</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Galeana</td>
<td>7,674</td>
<td>12</td>
<td>1 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Altamira</td>
<td>3,995</td>
<td>50</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cedral</td>
<td>7,557</td>
<td>18</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>V. Frontera</td>
<td>3,910</td>
<td>46</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cardenas</td>
<td>7,528</td>
<td>27</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>S. Fernando</td>
<td>3,835</td>
<td>47</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arteaga</td>
<td>7,460</td>
<td>13</td>
<td>1 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Musquiz</td>
<td>3,809</td>
<td>60</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ocampo</td>
<td>7,350</td>
<td>49</td>
<td>3 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Sabina</td>
<td>3,693</td>
<td>55</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Río Verde</td>
<td>7,196</td>
<td>25</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Victoria</td>
<td>3,528</td>
<td>73</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V. Hidalgo</td>
<td>7,146</td>
<td>32</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>General</td>
<td>3,520</td>
<td>62</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ciudad V.</td>
<td>7,096</td>
<td>22</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>S. J. Sabina</td>
<td>4,033</td>
<td>62</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zacatecas</td>
<td>6,916</td>
<td>31</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Gen. Toran</td>
<td>3,413</td>
<td>53</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tula</td>
<td>6,845</td>
<td>24</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>V. Berroso</td>
<td>3,433</td>
<td>70</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Terraza</td>
<td>6,823</td>
<td>20</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Anahuac</td>
<td>3,242</td>
<td>53</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Santiago</td>
<td>6,554</td>
<td>44</td>
<td>3 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Linares</td>
<td>3,240</td>
<td>68</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matamoros</td>
<td>6,536</td>
<td>45</td>
<td>3 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Sabina</td>
<td>3,240</td>
<td>68</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R. Arroyo</td>
<td>6,500</td>
<td>41</td>
<td>3 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Hidalgo</td>
<td>3,091</td>
<td>56</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soledad</td>
<td>6,269</td>
<td>39</td>
<td>3 - 3</td>
<td></td>
<td></td>
<td></td>
<td>M. Aleman</td>
<td>2,954</td>
<td>52</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>San Martin</td>
<td>6,211</td>
<td>19</td>
<td>2 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Pied. Negros</td>
<td>2,842</td>
<td>72</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Garcia Garcia</td>
<td>6,100</td>
<td>27</td>
<td>4 - 3</td>
<td></td>
<td></td>
<td></td>
<td>Torreon</td>
<td>2,737</td>
<td>67</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V. Arreaga</td>
<td>6,145</td>
<td>21</td>
<td>3 - 2</td>
<td></td>
<td></td>
<td></td>
<td>Reynosa</td>
<td>2,227</td>
<td>79</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>San Luis</td>
<td>5,963</td>
<td>54</td>
<td>4 - 3</td>
<td></td>
<td></td>
<td></td>
<td>Matamoros</td>
<td>2,157</td>
<td>60</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potosi</td>
<td>6,011</td>
<td>76</td>
<td>5 - 3</td>
<td></td>
<td></td>
<td></td>
<td>M. Laredo</td>
<td>1,144</td>
<td>78</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xilitla</td>
<td>5,976</td>
<td>40</td>
<td>3 - 3</td>
<td></td>
<td></td>
<td></td>
<td>Acuna</td>
<td>1,673</td>
<td>71</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monterrey</td>
<td>5,963</td>
<td>54</td>
<td>4 - 3</td>
<td></td>
<td></td>
<td></td>
<td>Monclo.</td>
<td>1,350</td>
<td>77</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Santa Maria</td>
<td>5,932</td>
<td>30</td>
<td>3 - 3</td>
<td></td>
<td></td>
<td></td>
<td>Tamaulipas</td>
<td>5,024</td>
<td>53</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 4. —MUNICIPIO TRANSPORTATION CHARACTERISTICS: NUMBER OF MUNICIPIOS IN EACH CATEGORY

**Percentage Calculation**

<table>
<thead>
<tr>
<th>Categories</th>
<th>(T) Terminus</th>
<th>(C) Crossroad</th>
<th>(M) Middle of Route</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E² I</td>
<td>E I</td>
<td>E I</td>
</tr>
<tr>
<td>1. Very Inefficient</td>
<td>4 1</td>
<td>1 1</td>
<td>6 3</td>
</tr>
<tr>
<td>2. Inefficient</td>
<td>4 2</td>
<td>0 2</td>
<td>6 3</td>
</tr>
<tr>
<td>3. Average</td>
<td>6 1</td>
<td>4 0</td>
<td>7 1</td>
</tr>
<tr>
<td>4. Efficient</td>
<td>4 1</td>
<td>4 3</td>
<td>1 2</td>
</tr>
<tr>
<td>5. Very Efficient</td>
<td>4 0</td>
<td>4 4</td>
<td>1 0</td>
</tr>
</tbody>
</table>

**Absolute Calculation**

<table>
<thead>
<tr>
<th>Categories</th>
<th>(T) Terminus</th>
<th>(C) Crossroad</th>
<th>(M) Middle of Route</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E I</td>
<td>E I</td>
<td>E I</td>
</tr>
<tr>
<td>1. Very Inefficient</td>
<td>4 1</td>
<td>1 1</td>
<td>7 3</td>
</tr>
<tr>
<td>2. Inefficient</td>
<td>4 2</td>
<td>1 2</td>
<td>4 2</td>
</tr>
<tr>
<td>3. Average</td>
<td>5 2</td>
<td>3 1</td>
<td>4 2</td>
</tr>
<tr>
<td>4. Efficient</td>
<td>5 0</td>
<td>5 2</td>
<td>4 0</td>
</tr>
<tr>
<td>5. Very Efficient</td>
<td>5 0</td>
<td>3 4</td>
<td>1 2</td>
</tr>
</tbody>
</table>

²I = Location in interior of transportation network; E = Location within 50 kilometers of the border of the transportation network
Table 1. Continued

<table>
<thead>
<tr>
<th>Participant</th>
<th>Metric</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: Metric data includes various measurements such as speed, distance, and time.*
APPENDIX B

COMPUTER MAPPING
The map of Linton Center is an example of a computer produced map, this example resulting from the Harvard Computer Graphics SYMAP program. In the program, input forms are used in various "data packages" to specify such things as shape of study area, marginal information, number of data categories, location of data control points, printing on the map, treatment of the data, and symbols to be indicated on the map. With the input, the computer can produce a crude isarithmic map with intensity shadings for the several categories portrayed. Another "option" in the SYMAP program is to produce a map with a non-continuous variable, in the format of discrete regions without contour lines. These two formats handle many different types of problems that geographers commonly deal with. Note that the productions of regional shadings are handled by various combinations of typewriter key characters overprinted to achieve a gradation of grey tones. In the future there undoubtedly will be improvements in printing capabilities and in the options available for producing dot maps and other useful types.

One major contribution of the computer produced map is that the computer does the laborious task of map compilation and rendering of results. These two related functions are really technical rather than professional in nature. The computer thus provides the invaluable service of freeing the professionals for their major role, the design of
data inputs and the interpretation of results. Thus computer and scientist are joined in an information-interpretation system best suited to the present capabilities of each. Speed and ready recalculation of stored information when new data becomes available are other important computer advantages. When efficient regional data storage banks are available, it is possible that continuous updating of information on places will permit "almost-up-to-date" maps at the touch of a proper button. One major bottleneck in the SYMAP program is the time required to feed the information inputs to the computer. This problem is currently under study and perhaps voice, machine scanning by a television-like camera, or some other suitable input procedure will soon replace the input forms that must now be completed by hand.
This report summarizes the results of an intensive effort to search geographic and related literature and to elicit information about new research methods from professional geographers. The study seeks to report the "essence" of change in geographic methodology of recent years, in order to provide users of geographic methodology with an understanding of the types of insights which modern geography can supply. The report discusses the development of geographic thought, discusses a selection of new techniques in geography, identifies some especially promising techniques, and contains an annotated bibliography.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geographic Methodology</td>
<td>ROLE</td>
<td>WT</td>
<td>ROLE</td>
</tr>
<tr>
<td>Techniques of Geographic Analysis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantitative Techniques</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geographic Thought</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modern Approaches to Geography</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>