A THEORETICAL INVESTIGATION OF THE STABILITY OF THE M151 ½-TON MILITARY TRUCK

by

M. Peter Jurkat

Prepared for
United States Army Tank-Automotive Command
38111 Van Dyke Avenue
Warren, Michigan 48090

under
Contract DAAE07-69-C-0356
(Project Themis)

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Approved,

I. Robert Ehrlich, Manager
Transportation Research Group
ABSTRACT

The stability of the M151 ½-ton military truck was analyzed to study the effect of variations in many of its design parameters. A linear, three-degree-of-freedom frequency-domain model was used to calculate the pole locus of the transfer function which related the yaw rate to the front wheel angle. It was concluded that the jeep in its nominal design configuration was stable at highway speeds up to 60 mph, but could become unstable below those speeds when operated under off-design conditions which could result from poor maintenance or loading.

Keywords

Motor Vehicle
(Handling, Stability, Steering Control)
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INTRODUCTION

This report describes a theoretical study to isolate those parameters of the M151 ½-ton military truck (jeep) whose variation has a gross effect on the stability of the vehicle. The effort was conducted as part of the over-all effort under THEMIS to study the conflicts between good off-road mobility and good on-road stability.

The study restricted itself to the linear regime of straight, smooth, and steady (S³) testing; that is, forward motion in a straight line on level ground at a constant speed (speed, however, was varied in four discreet increments). A frequency-domain analysis was employed. Specifically, the lateral equations of motion were linearized and their Laplace transforms were calculated. These transforms were then used to derive the transfer function which related the front wheel angle to the yaw rate of the vehicle. The location of the poles of this function (the roots of the characteristic equation of the system) served as the indicators of stability or instability.
NOMENCLATURE

$S^o$  shorthand for "straight, smooth, and steady"

$U$  forward speed, ft/sec

$M$  mass of vehicle, slugs

$M_s$  sprung mass of vehicle, slugs

$I_z$  yaw moment of inertia, slug-ft$^2$

$I_x$  roll moment of inertia, slug-ft$^2$

$I_{xz}$  yaw-roll cross product of inertia, slug-ft$^2$

$\beta, B$  sideslip of vehicle (rad) and its Laplace transform

$r, R$  yaw velocity of vehicle (rad/sec) and its Laplace transform

$\varphi, \dot{\varphi}$  roll angle of vehicle (rad) and its Laplace transform

$p$  roll velocity of vehicle (rad/sec), $\dot{\varphi}$

$C_1, C_2$  tire cornering coefficients, lb/rad, 2 tires

$a$  distance front axle to CG, ft

$b$  distance rear axle to CG, ft

$\varepsilon_1, \varepsilon_2$  suspension roll-steer coefficients, rad/rad

$\frac{\partial Y}{\partial \gamma}$  change in tire lateral force due to change in camber angle, lb/rad

$\frac{\partial Y}{\partial \delta_1}, \frac{\partial Y}{\partial \delta_0}$  tire camber change due to vehicle roll, rad/rad

$AT_1, AT_2$  tire self-aligning torque, ft-lb/rad

$\alpha_X/\alpha_Z$  change in tire resistance due to load, lb/lb

$Z_1, Z_2$  roll center heights, ft

$K_{\varphi}$  roll stiffness, lb/rad

$D_{\varphi}$  roll damping, lb/rad/sec
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>h</td>
<td>height of CG above &quot;roll axis,&quot; ft</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity, ft/sec²</td>
</tr>
<tr>
<td>s</td>
<td>Laplace transform variable</td>
</tr>
<tr>
<td>δ,Δ</td>
<td>front wheel angle (rad) and its Laplace transform</td>
</tr>
<tr>
<td>( )₁</td>
<td>subscript indicating front</td>
</tr>
<tr>
<td>( )₂</td>
<td>subscript indicating rear</td>
</tr>
</tbody>
</table>
THEORETICAL MODEL

The motion of a vehicle about its center of gravity is usually described by six equations. For control and handling-stability analysis, this six-degree-of-freedom system has been decoupled by Segel into equations for lateral motion and other motion. The lateral equations of motion retain three degrees of freedom: sideslip, yaw rate, and roll. Upon linearizing the applied forces, these equations have the following form:

\[ M \dot{\beta} + M_s \dot{\varphi} = Y_\beta \beta + Y_r r + Y_\varphi \varphi + Y_\delta \delta \]

\[ l_z \dot{r} + l_{xz} \dot{p} = N_p \beta + N_r r + N_\varphi \varphi + N_p p + N_\delta \delta \]

\[ l_x \dot{p} + M_s h U(\beta + r) + l_{xz} \dot{r} = L_p \beta + L_\varphi \varphi \]

where

\[ Y_\beta = C_1 + C_2 \]

\[ Y_r = \frac{a}{1U} - \frac{b}{2U} \]

\[ Y_\varphi = -\varepsilon \frac{C}{11} - \frac{C}{22} + \frac{\partial Y}{\partial Y} \left[ \frac{\partial Y}{\partial \varphi_1} + \frac{\partial Y}{\partial \varphi_2} \right] \]

\[ Y_\delta = -C_1 \]

\[ N_\beta = aC_1 - bC_1 + AT_1 + AT_\delta + \frac{\partial X}{\partial Z} \left[ C_1 \varphi_1 + C_2 \varphi_2 \right] \]

\[ N_r = C_1 \frac{a^2}{1U} + C_2 \frac{b^2}{2U} + AT_1 \frac{a}{1U} - AT_2 \frac{b}{2U} + \frac{\partial X}{\partial Z} \left[ C_1 \varphi_1 - C_2 \varphi_2 \right] \]
\[ N_\phi = c_1 \left[ -aC_1 - AT \right] + c_2 \left[ bC_2 - AT \right] + \frac{\partial Y}{\partial \phi} \left[ Z \frac{\partial Y}{\partial \phi_1} + Z \frac{\partial Y}{\partial \phi_2} \right] \]

\[ - \frac{\partial X}{\partial Z} \left\{ Z \frac{\partial C_1}{\partial Z} + Z \frac{\partial C_1}{\partial Z} - K_\phi - \frac{\partial Y}{\partial \phi} \left[ Z \frac{\partial Y}{\partial \phi_1} + Z \frac{\partial Y}{\partial \phi_2} \right] \right\} \]

\[ N_\phi = -aC_1 - AT - \frac{\partial X}{\partial Z} Z \frac{\partial C_1}{\partial Z} \]

\[ L_\phi = D \phi \]

\[ L_\phi = H_s g h + K_\phi \]

Note that the design parameters in these equations include "linearized" tire-cornering forces, self-aligning torques, and camber thrust, as well as front and rear roll-steer and roll-camber coefficients. The spring and damper (shock absorber) forces are used in the model only as they affect total roll stiffness and damping.

The report which first described the prototype of this model also included data which validated it. The frequency response curve as calculated from the equations gave, when applied to parameters describing a 1953 Buick Sedan, curves which agree very closely with data derived from this vehicle in actual road tests. This model, and extensions of it to include steering-system compliances, has been used extensively since then, most recently in a report by Kohno, Tsuchiya, and Koda.

If \( \beta \), \( r \), and \( \phi \) are considered functions of time, with initial values of zero in \( s^0 \) tests, the Laplace transformation may be applied to these equations, resulting, after collection of terms and dividing through by \( \Delta(s) \), in

\[(M\dot{u} + Y_\beta) \frac{B(s)}{\Delta(s)} + (M\dot{u} - Y_\delta) \frac{R(s)}{\Delta(s)} + (M \dot{s} h s^2 - Y_\phi) \frac{2(s)}{\Delta(s)} = Y_\delta\]
The characteristic equation of the system can now be read as

\[ -N_{\beta} \frac{B(s)}{\Delta(s)} + (sN_r + N_{s}) \frac{R(s)}{\Delta(s)} + (s^2 + N_p + N_{s}) \frac{q(s)}{\Delta(s)} = N_{\beta} \]

\[ M_{s} hU_{s} \frac{B(s)}{\Delta(s)} + (M_{s} hU + xz) \frac{R(s)}{\Delta(s)} + (s^2 - L_p - L_{s}) \frac{q(s)}{\Delta(s)} = 0 \]

When expanded, this characteristic equation is a fourth-degree polynomial whose roots are the poles of the transfer functions indicated by the ratios

\[
\begin{vmatrix}
M_{s} hU_{s} & M_{s} hU + xz & M_{s} hU_{s}^2 - Y_{\varphi} \\
N_{\beta} & x_{\beta} - N_{r} & x_{\beta}^2 + N_{p} + N_{s} \\
-M_{s} hU_{s} & M_{s} hU + xz & x_{s}^2 - L_{p} - L_{s}
\end{vmatrix} = 0
\]

These roots are, in general, two pairs of complex conjugates, although in some special cases there appear two real and one complex conjugate pairs of roots.

When the real parts of the complex roots or the real roots are positive, the vehicle is divergently unstable; when the real parts or real roots are zero, the system is oscillatory. Neither of these conditions is acceptable in any controlled element of a servo-system such as the driver-vehicle system.
VEHICLE DESCRIPTION

The basic description of the M151 used in this study came from Parquette and Kraemer. It contained some of the vehicle parameters that were needed here. The rest were obtained by personal communications with personnel at the Land Locomotion Laboratory and the Ford Motor Company. The data for the standard military tires were obtained from work previously performed at Stevens, during stability and control studies of a four-vehicle train of jeeps.

The "nominal" jeep used in this study had the following parameters:

Tire Characteristics
\[
\begin{align*}
C_1 &= 12,800 \text{ lb/rad (20 psi, front tire)} \\
C_2 &= 13,800 \text{ lb/rad (24 psi, front tire)} \\
AT_1 &= 1,400 \text{ ft-lb/rad} \\
AT_2 &= 1,400 \text{ ft-lb/rad} \\
\frac{\partial \gamma}{\partial \gamma} &= 2,866 \text{ lb/rad} \\
\frac{\partial \alpha}{\partial \alpha} &= 0.012 \text{ lb/lb}
\end{align*}
\]

(Estimates for military tires based on data from other tires)

Dimensions
\[
\begin{align*}
a &= 3.125 \text{ ft (53.1\% of weight on front)} \\
b &= 3.542 \text{ ft (42.9\% of weight on rear)} \\
h &= 1.008 \text{ ft} \\
Z_1 &= 0.670 \text{ ft ("Roll axis" inclined at a 5.35\% angle)} \\
Z_2 &= 1.292 \text{ ft}
\end{align*}
\]
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Inertial Characteristics

\[ M = 74.5 \text{ slugs (Weight = 2,400 lb)} \]
\[ M_s = 61.8 \text{ slugs (Sprung weight = 1,990 lb)} \]
\[ I_x = 937.5 \text{ slug-ft}^2 \]
\[ I_z = 1046.2 \text{ slug-ft}^2 \]
\[ I_{xz} = 79.2 \text{ slug-ft}^2 \]

Suspension Characteristics

\[ K_\phi = -35,460 \text{ ft-lb/rad (Front stiffness = 47,706 ft-lb/rad)} \]
\[ (\text{Rear stiffness = 47,763 ft-lb/rad)} \]
\[ D_\phi = -6,942 \text{ ft-lb/rad/sec} \]
\[ \epsilon_1 = -0.12 \text{ rad/rad} \]
\[ \epsilon_2 = 0.094 \text{ rad/rad} \]
\[ \frac{\partial y}{\partial \phi_1} = 0.9 \text{ rad/rad} \]
\[ \frac{\partial y}{\partial \phi_2} = 1.2 \text{ rad/rad} \]

All these parameters, except \( \frac{\partial x}{\partial z} \), were initially varied through a range of 50 percent to 200 percent, to determine the sensitivity of the roots to each. Subsequently, some parameters were varied through different ranges.

*Previous studies indicated that the \( \frac{\partial x}{\partial z} \) effect is not important in the linear regime.
RESULTS

The results of this study are presented in the figures at the end of the report. The figures display the location of the roots of the characteristic equation on the complex plane. Since the roots are either real or complex conjugates, only the roots with non-negative imaginary parts are shown. The others are at the mirror image of the upper half-plane reflected in the real axis.

NOMINAL CONFIGURATION

The location of the roots of the nominal jeep is shown in Fig. 1. Note that the roots, even at 88 fps, are away from the imaginary axis, indicating stability at all speeds used. The natural frequency of the yaw-rate response to steering ranges from 2.43 cycles per second at 22 fps to 0.509 cycles per second at 88 fps. The reported difficulty in driving the jeep near 88 fps (60 mph) is probably due to this low natural frequency, which compares poorly with a natural frequency of 0.851 cycles per second at 88 fps for a 1965 Ford sedan.

WEIGHT VARIATION

The weight of the vehicle was varied in two ways: total weight was increased by 33 percent and the center-of-gravity location was moved forward and rearward.

Figure 2 shows the pole-root locus of the vehicle configuration with 33-percent greater total weight distributed between front and rear as in the nominal vehicle. Note that there is little variation in loci, although a slight tendency toward diminished stability at all speeds.

Figure 3 shows the effect of maintaining the weight of the vehicle at its nominal level but shifting the weight forward and rearward so that
75 percent of the weight is on one or the other axle. Note the instability beginning between 44 fps and 66 fps when 75 percent of the vehicle weight is on the rear axle. This weight distribution can be nearly achieved by a 100-percent overload on the rear axle. In contrast, note the stabilizing influence of a forward CG.

ROLL-STIFFNESS VARIATION

Although difficult to vary in the field, roll stiffness is a parameter under the control of the vehicle designer. Its effect on stability is presented in Fig. 4.

Note the destabilizing influence of soft roll stiffness, here 20 percent of nominal. Relative stability is not greatly affected by roll stiffness from 60 percent to 200 percent of nominal, but the variation in the natural frequency of the yaw rate/front wheel response is striking. The table below gives the values.

### Natural Frequency of Yaw Rate/Front Wheel Response as a Function of Roll Stiffness

<table>
<thead>
<tr>
<th>Roll Stiffness</th>
<th>22 fps</th>
<th>44 fps</th>
<th>66 fps</th>
<th>88 fps</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% nominal</td>
<td>0.091*(a)</td>
<td>0.022*(a)</td>
<td>0.0438*(a)</td>
<td>Unstable</td>
</tr>
<tr>
<td>60% nominal</td>
<td>0.691</td>
<td>0.503*(a)</td>
<td>0.366*(a)</td>
<td>0.337</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.936</td>
<td>0.923</td>
<td>1.26</td>
<td>1.28</td>
</tr>
<tr>
<td>200% nominal</td>
<td>1.361</td>
<td>1.401</td>
<td>1.466</td>
<td>1.512</td>
</tr>
</tbody>
</table>

(a) Mode 2 had real poles in these cases. Cutoff frequency is given instead of natural frequency.

Human-controller-response range is below 1.0 cps, so that natural frequencies within this range would present the driver with ever more difficult control tasks, the lower the frequency. Note that this includes
both cases of lower than nominal roll stiffness.

ROLL-DAMPING VARIATION

Roll-damping variation may be achieved in practice by shock absorber deterioration. This affect on stability is shown in Fig. 5.

Note the destabilizing influence of lower roll damping. The mode 2 poles approach the positive imaginary axis as the roll damping is decreased to 10 percent of nominal. This deterioration is particularly difficult to counteract, since the mode 2 natural frequencies remain near nominal levels, around 1 cps. A plant with very low damping (represented by the positive parts of the roots), and natural frequencies near those of the controller, is very difficult to control.

VARIATION IN REAR-TIRE-CORNERING COEFFICIENT

The relationship between tire pressure and cornering coefficient for military tires was known from earlier studies. Figure 6 compares the stability of the jeep with its rear tires inflated to about 7 psi with its stability when these tires are normally inflated (24 psi).

Note the destabilizing influence of lower pressures in the rear tires. At 7 psi the vehicle is unstable from some speed between 44 fps and 66 fps (30 mph and 45 mph). In contrast, the high rear-tire pressures enhance stability greatly.

VARIATION OF OTHER PARAMETERS

All other parameters in the model were varied from 50 percent to 200 percent of their nominal values. None of them resulted in significant reduction of stability, while some enhanced stability.

Noteworthy is the finding that variation in the roll-camber and roll-steer parameters did not effect stability to any large extent for the S maneuver under consideration. This means that the swing-axle rear
suspension is not detrimental to straight-line motion, with constant speed. Other analysis has shown that it may have a detrimental effect near terminal cornering maneuvers.

Some have blamed the high rear roll center for the jeep's poor behavior; but for the $S^3$ maneuver, "roll axis" inclination did not have a great effect on stability.
CONCLUSIONS

On the basis of the results of analysis restricted to the linear regime about straight, smooth, and steady driving, it may be concluded that the M151 ⅓-ton military truck (jeep) is a stable vehicle in its nominal configuration but that variations within normal operating procedures can make it unstable within normal driving speeds.

Three phases of operation are destabilizing:

(1) Overloading the rear axle to 100-percent overload, without compensating changes in tire pressures (a common procedure)

(2) Deterioration of shock absorbers, to 10 percent or less of their damping rate when new (time alone will do this)

(3) Underpressuring the rear tires to 30 percent of their recommended value (sometimes done when traversing soft ground)

A fourth parameter, low roll stiffness, was shown to be destabilizing; but this cannot be easily varied in field operation. Initial design can control this parameter.

Design values for the suspension were found to be adequate for stability in the S3 maneuver. It is possible that this conclusion may not apply beyond the linear regime of this analysis. Cornering on cambered or rough roads, in particular, is not included.

It is felt that the above conditions (2) and (3) are extremely important. Condition (2) can easily result from worn or damaged shock absorbers; condition (3) can easily result from the lowering of tire pressure for off-road operations.
RECOMMENDATIONS

On the basis of the conclusions it is recommended --

(1) That full-scale verification of results be obtained, since conclusions herein were reached by mathematical modeling and computer calculation.

(2) That a full-scale extension of the results into non-linear and trim-and-transient maneuvers be undertaken, since the state of the art in road-vehicle theoretical analysis is restricted to linear regimes.

(3) That if the conclusions hold after full-scale experiments are performed, the operating procedures of field personnel, with regard to the jeep, be reviewed.
ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Mr. Karl Schinke for the design and construction of the computer program used to calculate the results; and to TACOM and Ford for data relating to the jeep.
REFERENCES


FIG. 1. POLE-ROOT LOCUS PLOT OF THE YAW-RATE/FRONT-WHEEL TRANSFER FUNCTION (ALL PARAMETERS AT NOMINAL VALUES)
FIG. 2. POLE/ROOT LOCUS PLOT OF YAW-RATE/Front-Wheel
TRANSFER FUNCTION* WITH 33% GREATER THAN NOMINAL
TOTAL WEIGHT (NOMINAL WEIGHT GIVEN BY DASHED LINES)
FIG. 3. POLE LOcus of yaw-rate/frOnt-wheEl transfer function, with center of gravity variation 75% forward and 75% rearward (nominal CG given by dashed line)
FIG. 5. POLE LOCUS OF YAW-RATE/FROnt-WHEEL TRANSFER FUNCTION, WITH ROLL-DAMPING VARIATION FROM NOMINAL TO 10% NOMINAL (NOMINAL ROLL DAMPING GIVEN IN DASHED LINES)
FIG. 6. POLE LOCUS OF YAW-RATE/FRONT-WHEEL TRANSFER FUNCTION, WITH REAR-TIRE-CORNERING COEFFICIENT FROM 50% NOMINAL TO 200% NOMINAL (NOMINAL CORNERING COEFFICIENT GIVEN IN DASHED LINES)
A THEORETICAL INVESTIGATION OF THE STABILITY OF THE M151 ½-TON MILITARY TRUCK

The stability of the M151 ½-ton military truck was analyzed to study the effect of variations in many of its design parameters. A linear, three-degree-of-freedom frequency-domain model was used to calculate the pole locus of the transfer function which related the yaw rate to the front wheel angle. It was concluded that the jeep in its nominal design configuration was stable at highway speeds up to 60 mph, but could become unstable below those speeds when operated under off-design conditions which could result from poor maintenance or loading.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
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<tr>
<td>Motor Vehicle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Handling, Stability, Steering Control)</td>
<td></td>
<td></td>
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</tbody>
</table>
Errata Sheet for

A THEORETICAL INVESTIGATION OF THE
STABILITY OF THE M151 1/4-TON MILITARY TRUCK

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page 5 - The first two equations should read

\[-N \frac{B(s)}{A(s)} + (1 \frac{s-N_r}{s}) \frac{R(s)}{A(s)} + (1 \frac{s^2-N_p}{s} \frac{\phi(s)}{A(s)} \right) = N_\delta \]

\[M \frac{hUs}{s} \frac{B(s)}{A(s)} + (M \frac{hU}{s} + I \frac{s-N_r}{s}) \frac{R(s)}{A(s)} + (I \frac{s^2-L_p-L_s}{s}) \frac{\phi(s)}{A(s)} = 0 \]

The determinant should read

\[
\begin{vmatrix}
MUs - Y & MU - Y_r & M_s h s^2 - Y_p \\
-N_B & 1 \frac{s-N_r}{s} & I \frac{s^2-N_p-N_s}{s} \\
M_s hUs & M_s hU - I \frac{s-N_r}{s} & I \frac{s^2-L_p-L_s}{s} \\
\end{vmatrix} = 0
\]