FOREIGN TECHNOLOGY DIVISION

STABILITY OF ORTHOTROPIC VISCOELASTIC SHELLS

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Abstract

(U) The authors review the basic principles of the closed quasilinear quadratic theory of viscoelasticity of physically nonlinear media as proposed by A. A. Il'yushin and P. M. Ogibalov. A system of nonlinear equations of bending and stability is proposed for flexible (i.e., with regard to geometric nonlinearity) shallow plates and shells made from orthotropic materials with linear properties (fiberglass-reinforced plastics). It is assumed that the hypothesis of straight normals is applicable to these plates and shells. It is also assumed that stresses normal to the middle surface are insignificantly small compared to the other components and that the shells and plates remain orthotropic throughout the entire deformation process. A method of solving the proposed equations is outlined and illustrated by analysis of the stability of a rectangular orthotropic plate of slightly curved panel of fiberglass-reinforced plastic with given stress relaxation curves. The results agree satisfactorily with experimental data on creep in a square plate hinged at the edges. Methods are also given for determining the upper and lower critical loads as related to the loading conditions and the critical time. A viscoelastic solution is found by the proposed method for the problem of stability of a compressed cylindrical shell and compared with an elastic solution found by the Ritz method. Greek, art, war, %, formula.

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Prikladnaya Mekhanika (Russian)
STABILITY OF ORTHOTROPIC VISCOELASTIC SHELLS

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(Moscow)

This article presents the fundamental concepts of heredity theories. For orthotropic materials with linear properties the article presents a system of nonlinear equations of flexure and stability for plates and shells, as well as a method for constructing solutions. Methods are presented for determining the upper and lower critical loads as a function of the loading regime and of the critical time.

The ever-increasing use of plastics, fiberglass and other synthetic materials requires providing new methods for solving engineering problems which take into account the rheology of their properties.

The elastic aftereffect phenomenon, discovered in 1834 by L. Visic and then by K. Weber, was subjected in the middle of the past century to the scrutiny of such major scientists as F. Kohlrausch, R. Clausius, D. Maxwell, S. Thomson, L. Boltzmann, and others. Bringing his theory in line with experimental results L. Boltzmann put forward two hypotheses: 1) the stress depends not only on the strain prevailing at the given time, but also on the preceding strains, whose effect is the weaker, the farther they are removed in time from the present; 2) the law of independent force action is applicable to elastic forces.

On the basis of these hypotheses we shall represent the relationships between stresses and strains in time suggested by L. Boltzmann in the following form

\[ \sigma(t) = \sigma(t) + \int \left[ K \left( \Gamma(t) - \Gamma(t) \right) \right] dt \]

For the beginning of the present century, V. Volterra developed a theory of integral equations from which, in particular, follows a relationship between the kernel \( K \) and the resolvent \( \Gamma \).

\[ K(t) = \int K(t - \tau) \left[ I(t) - \Gamma(t) \right] d\tau. \]

Some nonlinear effects of creep for a noncompressible medium can be described by the equations

\[ \sigma(t) = \sigma(t) + \int K \left( \sigma(t) - \sigma(t - \tau) \right) d\tau, \quad \sigma(t) = \sigma(t) + \int K \left( \sigma(t - \tau) - \sigma(t - \tau) \right) d\tau. \]

where function \( \psi(t) \) is constructed from similarity of the creep curves, while function \( \phi(t) \) is constructed from similarity of theocrotonous curves, characterizing the strain on rapid loading.

Reference [1] recommends to represent the general relationship \( \sigma \sim \varepsilon \sim t \) between the stresses, strains and time by means of a linear tensor operator. References [2]-[4] suggest a closed quasi-linear quadratic theory of viscoelasticity of media which have a physical nonlinearity. Here we already established all the general principal one-to-one relationships between tensors \( \varepsilon_{ij} \) and \( \sigma \) and, relations are established between the secondary kernels of creep and relaxation, as well as between them and the corresponding kernels of the linear theory.

Let us consider the main concepts of the theory being suggested. Let \( S_{ij}(t) \), \( S_{ij} (t), \ldots, S_{ij} (t) \) be the stresses at times \( t, \ldots, t \) of the interval \( [t, t] \) at the time at which it is required to determine the strain in the body, acting during a short time interval \( \Delta t \). Let us assume that it is required to study the function of process \( Z_{ij} \) at time \( t \). For example, the strain which is produced by the entire ensemble of stress pulses \( S_{ij}(t) \). The sum of contribution of individual stress pulses with the corresponding influence function will give a representation of \( Z_{ij} \) in the form of a linear operator of \( S_{ij} \) of the type of (1), which is the first approximation. The following approximation will be the continued action of two preceding stress pulses \( S_{ij} \) at times \( t \) and \( t \). This effect will be proportional to the product of these pulses during the applicable times, multiplied by the joint effect function of this pair. The following approximation is the effect of three stress pulses, acting during three different times, with the joint effect function of three stress pulses, etc. For deviations of the stress and deformation tensors the relationship \( \sigma \sim \varepsilon \sim t \) should be odd, since for a nonvanishing value of \( m = \varepsilon_{ij} \), a reversal in the direction of shear should change the sign of \( S_{ij} \).

According to the isotropy postulate, the relationship \( \sigma \sim \varepsilon \sim t \) should contain a triple product, which yields the quasi-linear tensor \( S_{ij}(t, t, t) \). As a result, for a process \( Z_{ij} \) (strains, for example) we get

\[ Z_{ij}(t) = \int K_{ij} \left( \sigma_{ij}(t) - \sigma_{ij}(t - \tau) \right) d\tau \]

The solution of this system of integral equations for stresses \( S_{ij} \)

\[ S_{ij}(t) = \int K_{ij} \left( \sigma_{ij}(t) - \sigma_{ij}(t - \tau) \right) d\tau \]

Equations (4) and (5) express the property of reciprocity; kernels \( K_{ij} \), \( K_{ij} \) and \( K_{ij} \) should be interconnected by integral equations which are not a function of \( Z \) and \( S \).
properties of resolvent $\Gamma$ should be analogous to those of kernel $K$, in particular, $\Gamma$ should be symmetrical with respect to arguments $x$ and $y$.

The integral equation relating kernels $K$ and $\Gamma$, and resolvents $\Gamma$ and $\Gamma$, reduces to the form

$$
\int K(x, t) \Gamma(t, y; x, u) dt = \int \Gamma(x, t) \Gamma(t, y; x, u) dt
$$

(6)

The solution of this equation is expressed as

$$
-\Gamma(u, y, t, x) = \int_{0}^{1} \Gamma(x, t) \Gamma(t, y; x, u) dt
g(x, u, t, x)
$$

(7)

Due to symmetry of kernels $K$ and $\Gamma$ it is possible to write an expression for $K$ in terms of $\Gamma$

$$
K(u, y, t, x) = \int_{0}^{1} \Gamma(x, t) \Gamma(t, y; x, u) dt
$$

(8)

It is clear from this that, the closer times $t_{h}$ and $t_{f}$, to time $t$, the greater is the effect on the process at hand of the values of $S_{k} \frac{dS}{dx}$ and, consequently, this property shows that the expressions for $K$, $\Gamma$, $K$, and $\Gamma$ contain Dirac's $\delta$-functions

$$
\bar{R}(x, t) = \delta(t - x) + K(x, t)
$$

(9)

where $K$ and $\Gamma$ are regular kernels (for example, such as $\Gamma(x,t) = e^{-xt}$).

The general form of singular kernel $R$ (which also means of $\bar{R}$ and $\bar{\Gamma}$), which is a scalar, has the form

$$
R(x) = R(x) + R_{x}\delta(x) + R_{x}\delta(x) + R_{x}\delta(x) + R_{x}\delta(x) + R_{x}\delta(x)
$$

Here

$$
R_{x} = \delta(x), \quad \delta(x) : = \cdot
$$

the quantity $R$ with its subscripts forms a set of three "tensors" of second, fourth and sixth-order regular kernels, which are a function of $x$. Expressions of $K$, $\Gamma$, $K$, and $\Gamma$ in terms of $\delta$-functions and regular kernels are more complex and are not presented in this paper (refer to them in [5]).

In constructing a theory of plates and shells made of material with rheonomic properties, it is first necessary to assume relationships in which the physical law is the first approximation of the general theory, given for the Boltzmann-Volterra linear process in the form of (1) and (2). Here kernel $K$ may be selected either with a singularity at $t = 0$, or as a sum of some Dirac kernel [6], which reflects the start of the process, and a regular $K$ in such a manner that the relationship $s = t$ will be

written in the form

$$
\alpha(t) = E \Gamma + \frac{1}{\gamma} \left[ \left. D \Gamma \right|_{\gamma} = \frac{s}{s + \gamma} \right] \Gamma_{\gamma} + \frac{1}{\gamma} \left[ \left. D \Gamma \right|_{\gamma} = \frac{s}{s + \gamma} \right] \Gamma_{\gamma}
$$

If the temperature scalar function $\Gamma_{\gamma}(T) = \frac{T}{\gamma}$, where $t$ is the time of observation and $\gamma$ is some reduced or "local" time, for the material under study is known, then the principal relationships of the heredity theories, which are valid for any temperatures from the range of operation of temperature-time analogy, will be written by replacing time $t$ by "time" $\gamma$ [7, 8]. For example, in the linear case we will have

$$
E(t) = E_{\gamma}(T) \Gamma_{\gamma}(T)
$$

Let us now pass on to the construction of equations of viscoelastic shells, understanding that $R$ and $K$ denote either a sum in the form $iD + K$, or kernels with a singularity in $t = 0$.

Since plates and shells are made quite extensively from fiberglass, whose properties in the majority of cases are orthotropic, we shall construct the equations of the theory of shells for flexible (i.e., with consideration of geometric nonlinearity) orthotropic shallow shells, whose material displays linear heredity. We assume that the hypothesis of undeformable normal is applicable to fiberglass plates and shells. It is assumed in addition that the stresses acting normal to the shell surface are negligible in comparison with other components and that orthoelastic is retained during the entire deformation process.

We select a coordinate system $(x, y, z)$ lining up axes $x$ and $y$ along the base and wall (principle) system, while axis $z$ is directed normal to the $(x, y)$ coordinate plane. We have the following expressions for the deformation of the middle layer of the shell and for the curvatures in terms of the displacement of the middle layer:

$$
\epsilon_{x} = \frac{\partial u}{\partial x} + \frac{1}{2} \frac{\partial v}{\partial y}, \quad \epsilon_{y} = \frac{\partial v}{\partial x}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}, \quad \epsilon_{z} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z}
$$

(10)

By virtue of the Kirchhoff theorem, the displacements and strains in a layer situated at distance $z$ from the middle surface are

$$
\epsilon_{x} = \epsilon_{x} - \frac{\partial u}{\partial z}, \quad \epsilon_{y} = \epsilon_{x} - \frac{\partial v}{\partial z}, \quad \gamma_{xy} = \epsilon_{x} - \frac{\partial u}{\partial z}, \quad \epsilon_{z} = \epsilon_{x} - \frac{\partial u}{\partial z}
$$

(11)

The relationship between stresses and strains for orthotropic materials is written in the form

$$
\sigma_{x} = \epsilon_{x} + \frac{1}{2} \gamma_{xy}, \quad \sigma_{y} = \epsilon_{y} + \frac{1}{2} \gamma_{xy}, \quad \sigma_{xy} = \gamma_{xy}
$$

$$
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$$
\[ a_n = B_n \rho_n \cdot \quad - \int r_n(s) \, ds + B_e^m = - \int \left( R_n \, t - \mu a_n(s) \right) \, ds. \]

\[ a_m = B_m \cdot - \int R_m(s) \, ds + B_e^m = - \int R_m(s) \, ds + \int R_m(t) \, dt. \]  

\[ a_m = 2B_m \cdot - \int R_m(t) \, dt + \int R_m(t) \, dt. \]  

Here

\[ B_m = \frac{E_m}{1 - \nu_m}; \quad B_n = B_m - \frac{\nu_m E_n}{1 - \nu_m}; \quad B_n = B_m = \frac{E_n}{1 - \nu_m}; \]

\[ 2B = G = \frac{E_m}{2(1 + \nu_m)}; \quad \nu_m - \nu_n = \frac{\nu_m}{\nu_n}; \quad \nu_n = \nu_n = \frac{\nu_n}{\nu_n}. \]  

are the components of the tensor of the moduli of elastic anisotropy and the Poisson ratio for tension along the axis and along the width:

\[ R_m = B_m R_m(t); \quad R_m = B_m R_m(t); \quad R_n = B_m R_m(t); \quad K = BR \]  

are the components of the tensor of the relaxation kernels.

The forces acting per unit width of shell-element cross section are

\[ T_n = \int \sigma_n d\alpha; \quad M_n = \int \sigma_n d\alpha; \quad \tilde{\sigma} = \tilde{\sigma}; \quad \tilde{\sigma} = \int \sigma_n d\alpha. \]

\[ H = H_n = \int \sigma_n d\alpha; \]

while stresses \( \sigma_{ms} \) are given by Eqs. (12). Substituting Eqs. (10), (12) and (14) into the expression for forces, we get

\[ T_n = h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) + h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right); \quad T_n = h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) + h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right); \]

\[ \tilde{\sigma} = 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right). \]  

where

\[ J = \int r_n(s) \, ds \]

After a Laplace transformation, Eqs. (15) take on the form

\[ T_n = h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) + h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right); \quad T_n = h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) + h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right); \]

\[ \tilde{\sigma} = 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right). \]  

Here

\[ \sigma_n = B_n (1 - R_m); \quad R_m = \frac{1}{K} R_m(t) \, dr. \]

From Eqs. (16) we find

\[ \sigma_n = \frac{1}{K} R_m(t) \, dr; \quad \sigma_n = \frac{1}{K} R_m(t) \, dr; \quad \sigma_n = \frac{1}{K} R_m(t) \, dr. \]

We set up the identity

\[ \frac{\partial}{\partial \alpha} (2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right)) \quad - \frac{\partial}{\partial \alpha} (2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right)) \quad + \frac{\partial}{\partial \alpha} (2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right)) \quad + \frac{\partial}{\partial \alpha} (2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right)) \quad - \frac{\partial}{\partial \alpha} (2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right)) \]

\[ = 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right); \quad \sigma_n = \frac{1}{K} R_m(t) \, dr. \]

into which we substitute expressions for \( \sigma_n \) from the preceding expressions, we simplify and introduce the stress function \( v \) using the formulas

\[ \frac{\partial^2 v}{\partial \alpha^2} = \frac{1}{h} \frac{\partial^2 v}{\partial \alpha^2} = \frac{1}{h} \frac{\partial^2 v}{\partial \alpha^2} \]

and also apply the Laplace transform

\[ T_n = h \frac{\partial^2 v}{\partial \alpha^2}; \quad T_n = h \frac{\partial^2 v}{\partial \alpha^2}; \quad \tilde{\sigma} = h \frac{\partial^2 v}{\partial \alpha^2}. \]

As a result we get a continuity equation (7) for an orthotropic elastic shell, whose material has linear heredity properties. In the operator form

\[ B_m = \frac{\partial^2}{\partial \alpha^2} \left( 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) \right) \quad + \frac{\partial^2}{\partial \alpha^2} \left( 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) \right) \quad + \frac{\partial^2}{\partial \alpha^2} \left( 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) \right) \quad + \frac{\partial^2}{\partial \alpha^2} \left( 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) \right) \quad + \frac{\partial^2}{\partial \alpha^2} \left( 2h \delta_n \sigma_n \left( \frac{d x}{d \alpha} \right) \right) \]

The right-hand side of the above expression has nonlinear terms \( \left( \frac{\partial^2 v}{\partial \alpha^2} \right) \)

It will be subsequently assumed that it is possible to represent the deflection in the form \( x(s, t) = \phi(s, t) \), \( \psi(s, t) \) (1), where function \( \phi(s, t) \), its derivatives and their squares allow the Laplace transform.

With the aid of (12), the expressions for the moments have the form

\[ M_n = \int \sigma_n d\alpha = - \int \sigma_n \frac{d^2 x}{d\alpha^2} \right) + \frac{1}{12} \int \left( R_n \, t - \mu a_n(s) \right) \, ds \]

\[ + \sigma_n \left( \frac{d^2 x}{d\alpha^2} \right). \]  

-5-
\[ M_s = -\frac{h}{12} \left[ B_0 \left( \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \right) + B_0' \left( \frac{\partial^2 \sigma_j}{\partial x^2} - \frac{\partial^2 \sigma_j}{\partial y^2} \right) \right]. \]

Introducing the values of \( M_s, M_n, H \) and of \( T_x, T_y, \bar{T} \) into the equilibrium equation of the shell

\[ \frac{\partial M_s}{\partial x} + 2 \frac{\partial H}{\partial y} + \frac{\partial M_n}{\partial y} + T_x \left( \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \right) + T_y \left( \frac{\partial^2 \sigma_j}{\partial x^2} - \frac{\partial^2 \sigma_j}{\partial y^2} \right) + 2 \frac{\partial^2 \sigma_j}{\partial x \partial y} + \varepsilon = 0 \]

and applying the Laplace transform, we find the equilibrium equation for an elastic orthotropic shell from a material with linear heredity properties

\[ B_0 \frac{\partial^2 \sigma_j}{\partial x^2} + 2 (B_0 + 4B_0') \frac{\partial^2 \sigma_j}{\partial x \partial y} + B_0' \frac{\partial^2 \sigma_j}{\partial y^2} - 12 \left( \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \right) - 12 \left( \frac{\partial^2 \sigma_j}{\partial x \partial y} \right) - 12 \left( \frac{\partial^2 \sigma_j}{\partial x \partial y} \right) = 0. \]

Here \( B_0 = B_0(1 - R^2) \) is a known function of \( p \).

In intermediate calculations we have obtained the expressions

\[ M'_s = -\frac{h}{12} \left[ B_0 \left( \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \right) \right], \]
\[ M'_n = -\frac{h}{12} \left[ B_0 \left( \frac{\partial^2 \sigma_j}{\partial x^2} - \frac{\partial^2 \sigma_j}{\partial y^2} \right) \right], \]
\[ H = -\frac{h}{12} \left[ \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \right]. \]

We thus have two remaining equations (19) and (20), for the unknown stress \( \sigma_j, \tau, \) and deflection wix, y, t functions. We note that these two equations contain terms of two types: such as \( \frac{\partial^2 \sigma_j}{\partial x^2} \), in which coefficients \( B_0 \) are known function of the complex parameter p, while the transform of the unknown functions combination, \( \frac{\partial^2 \sigma_j}{\partial x^2} \), where, by virtue of the method being presented, one of the functions for example, \( \sigma_j \) contained in parentheses will be considered as a known function of position and time, specified with unknown constant \( C \) (see equation (10)). Consequently, with respect to unknowns not specified \( N \) functions (for example, \( \sigma_j \), the above product will be linear.

To clarify this method we now present one of the possible approximate solutions of the system of equations (19) and (20). We shall consider an orthotropic plate or weakly bent plate, rectangular in the plan, with the region, for which the stress relaxation curves in specimens under tension are known and are representable, for example, in the form

\[ \sigma(t) = \sigma(0) e^{-\frac{t}{\tau}} \]

Obviously, the kernel of relaxation will be

\[ \rho = \frac{\sigma_0}{\tau} e^{-\frac{t}{\tau}}, \quad 0 < \tau < 1. \]

where \( \sigma_0 \) and \( \tau \) depend on the orientation of the specimen.

The transform of the kernel has the form

\[ K = \frac{\sigma_0}{\tau} \cdot \tau \]

We assume henceforth for simplicity that all the

\[ \rho = \sigma_0(1 - e^{-\tau}) \]

and that \( w \), the deflection of the specimen, is given in the form

\[ w(x, y, t) = \sigma_0 e^{-\tau} \theta(x, y, t), \]

where \( w(x, y, t) \) is the elastic solution, while \( w(t) \) is to be determined within unknown parameters \( \lambda \) and \( \mu \) in the form

\[ \theta(x, y, t) = \lambda - \mu e^{-\tau}, \quad \lambda = \mu = 1. \]

Stress function \( \sigma(t) \) is sought in the form

\[ \sigma(x, y, t) = \sigma_0 e^{-\tau} \theta(x, y, t), \]

here \( \theta(x, y, t) \) is the elastic solution, while \( \theta(t) \) for a given form of function \( \theta(t) \) is to be determined.

Laplace transform of \( \theta(t) \) yields

\[ \theta(p) = \frac{\lambda}{p - \rho - \omega}. \]

Substituting Eqs. (23), (26) and (28) into continuity equation (10), we find

\[ \theta = \frac{\lambda}{p - \rho - \omega}, \theta = \frac{\lambda}{p - \rho - \omega}. \]

Here

\[ n_t = \frac{\sigma_j}{\tau^2} = \frac{K}{B_0} \left( \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \right) \]

\[ \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} = \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \]

\[ C = \frac{1}{2b} (4B_0 - 2B_0 K), \quad K = B_0(1 - R^2) \]

\[ n_t = \frac{\partial^2 \sigma_j}{\partial x^2} + \frac{\partial^2 \sigma_j}{\partial y^2} \]
Substituting Eqs. (24) and (29) into Eq. (30) and having reference to the fact that
\[
\frac{1}{(p + m_0)\sqrt[3]{(p + m_0)}} - \frac{1}{(m - x)\sqrt[3]{(p + m_0)}} + \frac{1}{(m - x)\sqrt[3]{(p + m_0)}}
\]
we get
\[
q_1 = \frac{A_1}{\rho} + \frac{A_2}{\rho + \alpha} + \frac{A_3}{\rho + \beta} + \frac{A_4}{(p + \alpha)^2}.
\]

(31)

Inverting (31) we find
\[
q_1 = A_1 + A_2\rho^{2\alpha} - A_3\rho^{2\beta} + A_4\rho^{2\alpha}.
\]

(32)

where
\[
A_1 = \delta (1 - \gamma)(\kappa_0 + \lambda_n y), \quad A_2 = \eta_0 \delta (\gamma \lambda_0^2 - 2\mu - \gamma\lambda) - \kappa_0 \delta (\mu - \lambda y); \\
A_3 = \eta_0 \delta \gamma (\kappa_0 + 2\gamma \lambda).
\]

To determine the sought parameters \( \lambda \) and \( \mu \) we substitute Eqs. (26) and (28), with reference to (27) and (32), into equilibrium equation (20). Then
\[
\frac{K_1}{\rho} + \frac{K_2}{\rho + \alpha} + \frac{K_3}{\rho + \beta} + \frac{K_4}{(p + \alpha)^2} + \frac{K_5}{(p + \beta)^2} = 0.
\]

which, after inversion, yields an equation relating the load and the deflection in the form
\[
K_1 + K_2\rho^{2\alpha} + K_3\rho^{2\beta} + K_4\rho^{2\alpha} + K_5\rho^{2\beta} = 0.
\]

(34)

Here
\[
K_1 = \nu^2\nu_0\nu_1(1 - \gamma) - \frac{12}{\delta} \nu_0\nu_1\lambda_0^2 - \frac{12}{\delta} M\lambda_0^2 - \frac{12}{\delta} M\lambda_0^2 = 0;
\]
\[
K_2 = -\nu^2\nu_0\nu_1(\mu - \lambda y) - \frac{12}{\delta} \nu_0\nu_1\lambda_0^2 + \frac{12}{\delta} M(\kappa_0 - \mu \lambda_0);
\]
\[
K_3 = -\nu^2\nu_0\nu_1(\mu - \lambda y) - \frac{12}{\delta} \nu_0\nu_1\lambda_0^2 - \frac{12}{\delta} M(\kappa_0 - \mu \lambda_0);
\]
\[
K_4 = \nu^2\nu_0\nu_1\delta - \frac{12}{\delta} \nu_0\nu_1\lambda_0^2 - \frac{12}{\delta} M\lambda_0^2 - \frac{12}{\delta} M\lambda_0^2 = 0.
\]

(35)

From Eq. (34) we get
\[
K_1 + K_2 + K_3 + K_4 = 0 \quad \text{when} \quad t = 0;
\]
\[
K_1 = 0 \quad \text{when} \quad t = \infty.
\]

To determine \( \lambda \) we substitute the value of \( A_1 \) from Eq. (33b) into (35). A result we get, for example, for \( q = q_1 = \text{const} \)

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with the elastic values. Thus, = 0.558, = 0.483, = 0.171, = 0.137. We note that the values of the "upper" and "lower" critical loads for viscoelastic shells depends appreciably on the loading rate (with an increase in this rate the "upper" critical load becomes greater).

We note an important circumstance which can be successfully used, namely: if the temperature range for which the temperature-time analogy T -> t holds is known, i.e., the temperature shear function _temp(T) = t/loc for the given material has been constructed, then for any temperature T_k from this range the solutions are obtained by simple replacement of the time axis t by the "local" time _temp(T) = t/loc, where T_k is the shell's "functioning" temperature.

REFERENCES


Moscow State University Submitted 20 August 1966

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