DETERMINATION OF THE NUMBER OF ANODES FOR CATHODIC PROTECTION
(Raschet kolichestva anodov antikorrozionnoy katodnoy zashchity)

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DETERMINATION OF THE NUMBER OF ANODES
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The anticorrosion cathodic protection is one of the methods of combating corrosion. The principle of its operation consists in the fact that from an external source of current to the object being protected, i.e. the metal hull of a ship, we deliver direct electrical current through the anodes. The minimal current density of the cathodic protection is established experimentally, proceeding from the conditions of the creation of a protective potential \( U_{\text{max}} \) in any point of the surface which is being subjected to protection. Its value will range from 0.03 a/m² for a freshly-painted hull to 0.1 a/m² for a hull with a disrupted paint-and-varnish coating. Since the current is supplied to the hull through the anodes, which can be considered point sources, the distribution of current density in the hull occurs unevenly, which in turn causes considerable deviations in the distribution of the potential \( U_{\text{max}} \). It was established [1] that the deviation in the potential \( \Delta U \) in the direction of higher values should not exceed \( U_{\text{max}} \), i.e.

\[
\Delta U = U_{\text{max}} - U_{\text{met}} = 0.15 \text{ a},
\]

where \( U_{\text{max}} \) = the maximal value of the protective potential of the metal (equalling 1 volt); and

\( U_{\text{met}} \) = the protective potential of steel at which the corrosion stops (≈ 0.85 v).
The variation in the potential in the direction of values higher than $U_{3aU_{\text{max}}}$, in case of considerable current densities leads to a rapid breakdown of the paint-and-varnish coating owing to the intensive release of hydrogen. However, at a low current density, the protective potential $U_{3aU_{r}}$ is not reached, i.e. the corrosion is partially reduced.

Let us examine the question of the analytical determination of the number of the anodes for cathodic protection. In conformity with the report [2], the value $\Delta U$ will be determined for an infinite plane in the form:

$$\Delta U = \frac{K}{f}.$$  \hspace{1cm} (2)

where $j = \text{the current density, a/m}^2$;

$\gamma = \text{the conductance of sea water, 1/ohms} \cdot \text{m;}$

$c = \text{distance in meters between the anodes; and}$

$f = \text{the coefficient taking into account the relationship of the dimensions of the shield and the value of the protective zone at a given current density, j, and a given number of anodes which establish the c-value.}$

The coefficient $f$ is a complex logarithmic function of the geometric characteristics $c$ and $b$, where $d$ equals the size of shield and is expressed by the relationship shown in Fig. 1. The experimental verification conducted by Greenblat [3] at $\gamma = 4.2 \cdot 1/\text{ohms} \cdot \text{m}$, confirmed the theoretical results [2].

An analysis of Eq. (2) indicates that at a constant current density, the value (size) of the protective zone $c$ changes in dependence on $\gamma$ and $f$ (Table 1).
Fig. 1. Dependence of $\log f = \varphi \log d/c$.

From Table 1, it follows: in order to maintain $\Delta U = \text{const}$ at a variation in $\gamma$, it is necessary to alter the value $c$ ($f = 0.3$ corresponds to the maximum size of shield, $f = 1.2$ corresponds to its least dimension). An exemplary calculation indicates that the number of the protective zones (i.e., the number of anodes) can fluctuate within very broad limits: from hundreds to units, which is established by the conditions of solving Eq. (2) at $j = \text{const}$.

Table 1

The $c$-values at $\Delta U = 15$ and $j = 0.1 \text{ a/m}^2$

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Conductance $f^{1/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>0.9</td>
<td>0.17</td>
</tr>
<tr>
<td>1.2</td>
<td>0.125</td>
</tr>
</tbody>
</table>

The more strict solution is obtained in the case when the value $\gamma/j$ is considered to be assigned and constant, i.e. the current will change according the law of variation in the conductance.
of sea water (in this case, \( c = \text{const} \)). In this manner, in the calculation based on Eq. (2), series difficulties develop, since there is required a knowledge of the regularity of the fluctuations in current density during the variation in the conductance of sea water. Therefore, the determination of the basic parameters of cathodic protection (the number of anodes and the dimension of the insulating shields) can best be conducted proceeding from the condition of the distribution of current density, assuring the creation of a protective potential in a point remote from the anode, under the prescribed geometric dimensions of the object which is being protected. Since for most of the transport ships, \( B \) can be assumed to equal \( 2T \) (Fig.2), having replaced the ship hull with a half-cylinder with a constant frame section, and conducting the unfolding (evolvement) of the surface of the half-cylinder onto a plane, we obtain the dimensions of the protected object, expressed by the length and width of the ship. The possibility of utilizing the formulas for determining the current density on the surface of the ship hull, with allowance for the form of the frame section and on the plane can easily be established under a comparison of the appropriate equations [4].

Fig. 2. Diagram of the cross section of a ship hull.

The possibility of utilizing the formulas for determining the current density on the surface of the ship hull, with allowance for the form of the frame section and on the plane can easily be established under a comparison of the appropriate equations [4].
this same report, the authors have examined the effect of the
dimensions of the shield upon the distribution of current density
for the disc-type and band-type anodes, and their dimensions were
also determined. Therefore, here we should dwell only on the
analytical determination of the number of anodes for cathodic pro-
tection.

The number of the anodes should be chosen, proceeding from
the given extent of the dimensionality of distribution of the
potential in the hull, while the distance between them should be
established according to the prescribed value of the protective
zone of an individual anode; by this zone, we connote the surface
on which the condition (1) is provided (maintained).

![Fig. 3. Diagram for determining the current density
in the remote point, D.](image)

It is natural that the maximum dimensionality in the dis-
tribution of the potential will create an extended band-type anode
(Fig.3). However, since the actual anodes are point sources of
current, in the distribution of the potential, we will find an
error, the value of which (at the values assumed in [1]), should
not exceed 15%. In accordance with [4], the value of current
density which is being developed by an infinite anode along the
line C-C¹, will acquire a form:

\[
j \sim \frac{I_{a\gamma}}{\gamma K(a) \sqrt{(x^2-a_1^2)(x^2-a_2^2)}}
\]  

(3)
where \( I \) is the current flowing from a unit of length of the anode, \( \text{a/m} \);
\( a_2 \) is width of shield, \( \text{m} \);
\( a_1 \) is width of anode, \( \text{m} \);
\( x \) is distance from the anode to the point of observation, \( \text{m} \); and
\( K(k') \) is the elliptical integral with the modulus \( k' = a_1/a_2 \).

Having replaced the linear anode by a disc-type anode within the limits of the sector under review (the square with the side equalling the width of the ship), let us determine the current density in the point D [4]:

\[
I_{\text{disc}} = \frac{I \sqrt{a_2^2 - a_1^2}}{\pi^2 (r^2 - a_1^2) \sqrt{r^2 - a_2^2}}.
\]

where \( a_1 \) is the anode radius, \( \text{m} \);
\( a_2 \) is the shield radius, \( \text{m} \);
\( r \) is the distance in meters from the source to the observation point.

The point D is the farthest from the anode and therefore the fulfillment for it of condition (1) automatically leads to its fulfillment in any other point of the protected zone which is under study. Since \( r = x \sqrt{2} \), we will obtain

\[
I_{\text{disc}} = \frac{I \sqrt{a_2^2 - a_1^2}}{\pi^2 (2x^2 - a_1^2) \sqrt{2x^2 - a_2^2}}.
\]

The condition of the equality of current density in the point D at replacement of the infinitely long anode by a disc-type
### Table 2

**Number of Anodes of Various Cathodic Protection Systems**

and Results of Calculation Using Eq. (12)

<table>
<thead>
<tr>
<th>Name of ships</th>
<th>Displacement, t</th>
<th>Length, $l$, meters</th>
<th>With, $b$, meters</th>
<th>Draft, $d$, meters</th>
<th>$R_{an}$, meters</th>
<th>$R_{sh}$, meters</th>
<th>Established number of anodes, $n$</th>
<th>$n_{cal}$, calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Svetilya&quot;</td>
<td>3000</td>
<td>94</td>
<td>14.5</td>
<td>3.6</td>
<td>0.06</td>
<td>1.0</td>
<td>20</td>
<td>17, 11</td>
</tr>
<tr>
<td>&quot;Ossetiya&quot;</td>
<td>3000</td>
<td>94</td>
<td>14.6</td>
<td>3.6</td>
<td>0.06</td>
<td>1.2*</td>
<td>4</td>
<td>17, 11</td>
</tr>
<tr>
<td>&quot;Kolchida&quot;</td>
<td>3000</td>
<td>94</td>
<td>14.6</td>
<td>3.6</td>
<td>0.06</td>
<td>0.6*</td>
<td>14</td>
<td>17, 11</td>
</tr>
<tr>
<td>&quot;Uzbekistan&quot;</td>
<td>3000</td>
<td>94</td>
<td>14.5</td>
<td>3.6</td>
<td>0.2*</td>
<td>0.2*</td>
<td>20</td>
<td>21, 15</td>
</tr>
<tr>
<td>&quot;P. Vinogradov&quot;</td>
<td>9420</td>
<td>114</td>
<td>16.1</td>
<td>7</td>
<td>0.2*</td>
<td>0.2*</td>
<td>20</td>
<td>21, 15</td>
</tr>
<tr>
<td>&quot;Mazem'les&quot;</td>
<td>4000</td>
<td>102</td>
<td>15</td>
<td>6.3</td>
<td>0.2*</td>
<td>0.2*</td>
<td>10</td>
<td>19, 13</td>
</tr>
<tr>
<td>&quot;Proletarsk&quot;</td>
<td>12740</td>
<td>130</td>
<td>17.7</td>
<td>7.8</td>
<td>0.1*</td>
<td>1.2*</td>
<td>24</td>
<td>17, 17</td>
</tr>
<tr>
<td>&quot;Arkhangel'sk&quot;</td>
<td>7400</td>
<td>109</td>
<td>16.4</td>
<td>6.1</td>
<td>0.1*</td>
<td>0.7*</td>
<td>6</td>
<td>20, 14</td>
</tr>
</tbody>
</table>

* The radius of a circle, the area of which equals the area of the rectangular anode or shield.
anode is observed at \( I = \text{const} \) in the case:

\[
\frac{J}{D_{\text{long}}(\text{length})} = \frac{J}{J_{\text{disc + shield}}}.
\]

(5)

**Fig. 4. Dependence \( n = \frac{(L/a_2)}{t} \).**

\[
1 - K(k) = 4.4; \quad k = \frac{0.1}{2};
\]

\[
2 - K(k) = 4.1; \quad k = \frac{0.1}{1.5};
\]

\[
3 - K(k) = 3.7; \quad k = \frac{0.1}{1}.
\]

where \( k \) is the coefficient taking into account the effect of the shield upon the distribution of current density in case of replacement of the infinitely long anode by a disc-type anode.

From Eq. (5), let us find \( k_3 \), in the connection taking into account that \( x = B/2 \):

\[
k_3 = \frac{a_2^2 (R^2 - 2a_1^2) \sqrt{B^2 - 2a_1^2}}{2 \sqrt{2} \sqrt{(B^2 - 4a_1^2)(B^2 - 4a_2^2)(a_2^2 - a_1^2) k(k)}} > 1.
\]

(6)

The coefficient \( k_3 \) was obtained, proceeding from the condition of the equality of current density \( j \) in the remote point, which can be achieved by way of increasing the sizes of the disc-type anode's shield \((k_3 > 1)\). However, the equality of a current density in the remote point can be achieved owing to an increase in the current flowing from the disc-type anode, where-in the dimensions of the shield are reduced. The current flowing
from a unit of the length of the infinitely long anode equals [4]:

\[ I_w = 2\Gamma(U_0 - U_{c1}) \frac{K(k')}{K(k)} \]  

(7)

where \( U_{c2} - U_{c1} \) = the difference in volts of the electrode potentials of the anode and cathode;

\( K(k), K(k') \) = the elliptical integrals with the moduli \( k' = a_1/a_2; \ k = \sqrt{1 - (k')^2} \).

The current flowing from the disc-type anode equals:

\[ I_{\text{disc}} = 4\Gamma(U_0 - U_{\text{c2}}) a_2 \]  

(8)

Then

\[ k_T = \frac{I_{\text{disc}}}{I_w} = \frac{K(k)}{2a_2K(k')} \]  

(9)

where \( k_T \) = the coefficient taking into account the fluctuations in the current of the disc-type anode at a variation in the shield's dimensions.

The conditions examined above pertained to one square sector with the side \( B \). The number of sectors can be calculated, if the value of the wetted surface of the ship hull is divided by \( B^2 \). In a first approximation, the number of sectors will be

\[ k_n = \frac{2l_{\text{wet}}}{2l - \frac{2l}{B}} \]

As the precise calculations indicate, the ratio of the actual value of the wetted surface to \( B^2 \):

\[ k_n' = \frac{L_{\text{wet}}(1.35 + 1.13B)}{\mu L} \]
where \( \delta = \frac{D}{BLT} \) (\( D \) = the displacement of ship) is less by 35\% than the designed value, i.e.

\[
k_n = \frac{2L}{B} \cdot 0.65, \tag{10}
\]

\( k_n \) = the coefficient taking into account the actual area of the wetted surface of the ship hull.

Utilizing the principle of superposition for the distribution of the current density in the point \( D \), let us find the number of anodes as follows:

\[
n = k_n \cdot k_1 \cdot k_2 = \frac{Z_2}{J} \cdot \frac{(B^2 - 2a_1^2) \sqrt{B^2 - 2a_1^2} L \cdot 0.65}{4 \left( B^2 - 4a_1^2 \right) \left( B^2 - 4a_2^2 \right) \lambda \cdot (a_2^2 - a_1^2) K(k') B}. \tag{11}
\]

Assuming in Eq. (11) that \( a_1 = 0 \) (in other words, we disregard the dimensions of the anode; the error of such an assumption comprises less than 2\%), we will obtain:

\[
n = \frac{\pi \sqrt{B^2 - 2a_2^2} L \cdot 0.65}{4 \sqrt{2} a_2 J \cdot K(k') B}. \tag{12}
\]

As follows from Table 2, the number of anodes installed on the operating ships is quite diversified, since their selection was conducted arbitrarily, without consideration for the distribution of the current density.

The formulas presented take into account the size of shield, the dimensions of the ship and the distribution of current density in the hull. A calculation based on Eq. (12) indicates that in most instances, the number of the anodes was underestimated. In Fig. 4, we have shown the relationship of the number of anodes to the value \( L/a_2 \). In this manner, with the proper selection of
the dimensions of the shield and anode, we can achieve a reduction in the number of anodes to a value stipulated by the efficient engineering and operational conditions.

REFERENCES


