Technical Note

Experimental Results of a $\frac{\sin x}{x}$ Tapped-Delay-Line Matched Filter

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ABSTRACT

A \( \frac{\sin x}{x} \) bandpass filter was constructed using tapped delay-line techniques. Measurements were taken of its performance as a matched-filter for a rectangular pulse-modulated input. The simplicity of its construction and absence of tuning adjustments along with providing a good approximation to an ideal filter are the main advantages of this technique.

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INTRODUCTION

A simple I.F. matched filter for a short, rectangular, constant frequency pulse was constructed and its performance measured. The approach used was to construct a tapped-delay-line approximation, which offered a relatively straightforward way of implementation for such a filter. The following is a brief summary of the work done.

DESCRIPTION OF FILTER

Figure 1 shows a block diagram of the type of filter that was implemented. The rectangular input pulse has a length of $T \mu s$, and the period of the carrier frequency is $t \mu s$. The time delay between each tap was chosen to be $\frac{t}{2} \mu s$. Consequently, alternate outputs have to undergo a $180^\circ$ phase change. Although it is shown on this diagram as accomplished with $180^\circ$ phase shifters, in practice the phase shift was done by reversing phase (in a transformer) of one of the 2 output summing points. It can be seen from the figure that if everything is delayed and added properly, an approximately triangular shaped output pulse will result. This is the expected output from a matched filter for a rectangular constant frequency input pulse.

Figure 2 shows the relationships between the desired and the tapped delay line approximation. The impulse response of the tapped delay line filter is a series of impulses, both positive and negative, whose positions correspond to the peaks of the waveform of the desired impulse response. Figures 2c and 2d show the frequency response of the desired results and the approximation. The main difference is that the tapped delay line filter response is repetitive at odd multiples of $\frac{1}{t}$, (since $\frac{t}{2}$ tap spacing is used). Rough filtering to eliminate the higher frequency responses would be done by adding a lumped-constant zonal filter in series with the delay-line filter. In the experiments done here the oscilloscope high frequency cutoff effectively acted like this zonal filter.
CONSTRUCTION

The parameters for the unit were as follows:

1. Center frequency: 40 MHz
2. Pulse length: 250 or 125 ns (20 or 10 taps—the 10-tap measurements were made by using only half the taps of the full unit)
3. Delay line constructed from Phelps Dodge CT-085-50 miniature solid coaxial cable having the following specifications:
   a. Characteristic impedance: 50 ohms
   b. Outer diameter: .085"
   c. Velocity of propagation: 70% (.69 ft per ns.)
   d. Attenuation at 40 MHz: 3 db per hundred feet
   e. Attenuation slope at 40 MHz: .04 db/MHz per hundred feet.

For the 10 tap (125 ns, 8 MHz B.W.) case the attenuation variation amounted to a total change of .275 db over 8 MHz. For the 20 tap (250 ns, 4 MHz B.W.) case it was also .275 db.

A schematic and mechanical layout is shown in Fig. 3 and 4 respectively. The unit was built using 1/4 watt resistors as coupling elements and no account was taken of the attenuation change with frequency encountered in the cable. The resistor values used were chosen to minimize loading of the delay line and to compensate for the attenuation down the line.

The transformer used to add together the positive and negative taps is a wideband unit covering the frequency range from .25 to 500 MHz. It has an amplitude unbalance of no more than 0.1 db at 40 MHz and a phase unbalance of no more than 0.1 degree at 40 MHz. An adjustable termination resistor was used at the end of the line to minimize reflections. Optimum results were obtained when it was set to a value slightly greater than the nominal 50 Ω. No great effort was made to package the unit compactly. The cable was bent as shown in Fig. 4 and the overall size of the filter came out to be around 1"x 3"x29". The filter for the 125 ns pulse was made by simply disconnecting 10 of the taps and moving the termination and the transformer. Experiments with variable trimmers resistors were made by inserting small (1/4" diameter)
trimmers in series with each tapping resistor.

TEST RESULTS

Figure 5 shows the test set-up used to measure the performance of the network. It is fairly straightforward. The set-up to measure frequency response, however, did not use a standard sweep generator and detector. Instead, a manually tuned C.W. oscillator and a spectrum analyzer were used. This enables one to examine the frequency response in some detail on a 60 db calibrated scale. The photographs were taken by opening the camera shutter and varying the C.W. input manually from about 20 to 60 MHz. (The brighter images shown 40-50 db down are spurious responses of the analyzer and should be ignored.)

The test results are contained in Figs. 6 through 11 and should be self explanatory. A few comments here will suffice. The input pulse as shown in Fig. 6a, 8a and 10a is not a perfectly shaped rectangular pulse but was the best that could be obtained at the time. It is felt, however, that a better shaped pulse would not have given any better test results, though this has not been proven.

It was found in all cases that driving port No. 2 (normally considered the output port) and observing the output on port No. 1 gave a better time sidelobe response than the other way around. In both cases the termination resistor was adjusted for minimum time sidelobes. No explanation is offered for this behavior; however the filter should be, to a first approximation, reciprocal. Most of the tests performed, consequently were done by driving port No. 2. The best sidelobe response obtained with the 20-tap filter was about 40 db, and is shown in Fig. 6c. The best response for the 10-tap filter was 32 db and is shown in Fig. 9c. The insertion loss of the filter was around 20 db in both cases. This could have been lowered, if necessary, by decreasing the value of the coupling resistors. However, the matched filter response would have been degraded at some point.

The frequency responses shown in Figs. 8c and 10 indicate an approximately \( \frac{\sin x}{x} \) response. The first sidelobes are around 12 to 13 db below the main lobe and the first nulls are anywhere from 25 to 35 db below the peak.
In order to see whether the coupling resistor values were critical or not, 2K Ω trim pots were added in series with the 1/4 watt resistors in the 10 tap line to obtain variation in resistance. They first were set to zero and the output of the filter looked at as in 11c. (This is identical with 9b.) A video pulse generator was then used as the input with a pulse width set to about 10 ns (approximately a triangular shaped pulse) which was the minimum pulse width for this generator. The output of the filter was observed as in 11a. This is an approximation of the impulse response of the filter. This test indicated which of the 10 trimming pots needed adjusting to bring the response more closely to the ideal equal output impulses. After adjustment, the "impulse" response was as shown in Fig. 11b. The test set up of 5a was then used again and the matched filter response measured. As far as one can tell there seems to be little significant difference between that and when no adjustments were used. This would indicate that using the closest RETMA resistor values was sufficient at least to the error level being measured.

CONCLUSIONS

The \( \frac{\sin x}{x} \) filter can be easily built using the tapped delay line approach. Construction is straightforward and no tuning is needed when it is finished. The response gives a good approximation to the desired one and, it is believed, can be easily repeated.

No more work is planned at the present on this type of filter, but more could be done in the area of optimizing parameters and construction techniques; though it is felt that the ones used are adequate for most purposes.
ACKNOWLEDGEMENTS

The author is indebted to Mr. J. Margolin for first suggesting the idea and Mr. B. Loesch for other helpful suggestions. More theory and experimental results for other filters may be found in "A Technique For Generating Signals and Their Matched Filters" By B. Loesch, E. M. Hofstetter and J. P. Perry. (To be published.)
Fig. 1. Tapped-delay-line filter.
Fig. 2. Time and frequency relationships.
Fig. 3. 40-MHz 250-ns \( \frac{\sin x}{x} \) filter.
Fig. 4. 40-MHz $\frac{\sin x}{x}$ filter mechanical details.
Fig. 5. Test setup. (a) Output time response. (b) Coherent output time response. (Pulse repetition rate integrally related to the carrier frequency.) (c) Frequency response.
(a) Input pulse.
0.1 μs/cm horizontal
0.5 V/cm vertical
(+ 9 dBm peak level)
≈250 ns (6 dB) width
[Switch with driver]

(b) Output of 20 tap filter.
0.1 μs/cm horizontal
(Port 2 driven)
0.05 V/cm vertical
(peak of pulse shown, 11 dBm)
≈500 ns to baseline
≈250 ns (6 dB)

(c) Output of 20 tap filter (expanded).
0.1 μs/cm horizontal
(Port 2 driven)
0.01 V/cm vertical
Sidelobe ≈ 39–40 dB below peak

Fig. 6. 40-MHz 20 tap delay line filter.
Fig. 7. 40-MHz 20 tap delay line filter.
(a) Input pulse 40-MHz synchronized.
   0.1 μs/cm horizontal
   0.1 V/cm vertical
   (switch)

(b) Output of 20 tap filter with synchronized input pulse.
    (Port 2 driven)
    0.1 μs/cm horizontal
    ≈ 0.01 V/cm vertical

(c) Frequency response of 20 tap filter.
    (Port 2 driven)
    10 dB/cm vertical
    ≈ 3–1/4 MHz/cm horizontal
    Center of peak response is at 40 MHz.

Fig. 8. 40-MHz 20 tap delay line filter.
Fig. 9. 40-MHz 10 tap (125 ns) filter (resistor valves).
Fig. 10. Frequency response of 10 tap filter.

10 dB/cm vertical  
≈ 4-1/2 MHz/cm horizontal  
(Port 2 driven)  
Center of peak response is at 40 MHz.
Fig. 11. 40-MHz 10 tap (125 ns) filter with value resistors plus 2KΩ trimmers (Port 2 driven).
A \( \frac{\sin x}{x} \) bandpass filter was constructed using tapped-delay-line techniques. Measurements were taken of its performance as a matched-filter for a rectangular pulse-modulated input. The simplicity of its construction and absence of tuning adjustments along with providing a good approximation to an ideal filter are the main advantages of this technique.