USAAVLABS TECHNICAL REPORT 68-61

DESCRIPTION OF A HELICOPTER ROTOR NOISE COMPUTER PROGRAM

By

J. B. Ollerhead
R. B. Taylor

January 1969

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

CONTRACT DAAJ02-67-C-0023
WYLE LABORATORIES
HUNTSVILLE, ALABAMA

This document has been approved for public release and sale; its distribution is unlimited.
Disclaimers

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission, to manufacture, use, or sell any patented invention that may in any way be related thereto.

Disposition Instructions

Destroy this report when no longer needed. Do not return it to the originator.
This report has been reviewed by the U. S. Army Aviation Materiel Laboratories and is considered to be technically sound. The scientific opinions expressed in the report are those of the authors and do not necessarily reflect the position of the U. S. Army Aviation Materiel Laboratories. The report is published for the dissemination of information and the stimulation of thought.
DESCRIPTION OF A HELICOPTER ROTOR
NOISE COMPUTER PROGRAM

Final Report
Wyle Research Staff Report WR 68-10

By
J. B. Ollerhead and R. B. Taylor

Prepared by
Wyle Laboratories
Huntsville, Alabama

for
U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

This document has been approved for public release and sale; its distribution is unlimited.
SUMMARY

This report contains a comprehensive description of a computer program developed for the numerical evaluation of the helicopter noise equations derived in USAAVLABS TR 68-60, "Studies of Helicopter Rotor Noise." It is completely self-contained in that the program details are described, starting from two basic acoustic equations and covering methods by which these equations are applied to the rotor noise problem. Program flow diagrams and a complete listing are presented together with input instructions and sample inputs and outputs. The program is written in FORTRAN IV for the CDC 3300 Computer.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 COORDINATE SYSTEMS AND TRANSFORMATIONS</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Rotor Axes x', y', z</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Flight Path Axes x, y, z</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Fixed Axes X, Y, Z</td>
<td>8</td>
</tr>
<tr>
<td>2.4 Blade Element Displacements</td>
<td>9</td>
</tr>
<tr>
<td>2.5 Blade Element Velocities and Accelerations</td>
<td>11</td>
</tr>
<tr>
<td>2.6 Aerodynamic Forces and Their Derivatives</td>
<td>13</td>
</tr>
<tr>
<td>3.0 METHOD OF SOLUTION BY DIGITAL COMPUTER</td>
<td>15</td>
</tr>
<tr>
<td>3.1 Phase I Initial Data Processing</td>
<td>15</td>
</tr>
<tr>
<td>3.2 Phase II Calculation of the Sound Field</td>
<td>15</td>
</tr>
<tr>
<td>4.0 PROGRAM FLOW CHARTS, EQUATIONS AND LISTING</td>
<td>23</td>
</tr>
<tr>
<td>4.1 Detailed Flow Chart for Program HERON 1</td>
<td>24</td>
</tr>
<tr>
<td>4.2 List of Equations Computed by Program HERON 1</td>
<td>30</td>
</tr>
<tr>
<td>4.3 Program Listing</td>
<td>37</td>
</tr>
<tr>
<td>4.4 Major FORTRAN Symbols used in Program HERON 1</td>
<td>49</td>
</tr>
<tr>
<td>5.0 PROGRAM INPUT/OUTPUT</td>
<td>56</td>
</tr>
<tr>
<td>5.1 Program HERON 1. Data Input</td>
<td>57</td>
</tr>
<tr>
<td>5.2 Sample Input</td>
<td>63</td>
</tr>
<tr>
<td>5.3 Sample Output</td>
<td>68</td>
</tr>
<tr>
<td>DISTRIBUTION</td>
<td>72</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Discrete Representation of Aerodynamic Loading</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Shaft Axes (x', y', z') and Flight Path Coordinates (x, y, z)</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Pitch and Roll Rotations</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>Shaft Inclinations</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>Fixed Axes (Plan View)</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Blade Element Displacements in Rotor Axes</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>Block Schematic Flow Diagram for HERON 1</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Velocity Diagram for Sound Radiation by Moving Aircraft</td>
<td>19</td>
</tr>
</tbody>
</table>
1.0 INTRODUCTION

The rotor sound field is calculated according to the general acoustic equation for aerodynamic forces in motion which is derived in Section 3.3 of USAAVLABS TR 68-60:

\[ p(t) - p_0 = \left[ \frac{(x_i - y_i)}{4\pi (1 - M_s^2) c s^2} \left( \frac{\partial F_i}{\partial t} + \frac{F_i}{1 - M_s^2} \frac{\partial M_s}{\partial t} \right) \right] \]  

(1)

This equation is written in tensor notation, where the \( i \) denotes summation over the three component directions, and the brackets denote evaluation at the appropriate retarded time \( t' = t - s/c \) which is the time at which the source generated the sound reaching the observer at time \( t \).

- \( p(t) - p \) is the instantaneous acoustic pressure.
- \( x_i \) are the coordinates of the observer.
- \( y_i \) are the coordinates of the source.
- \( M_s \) is the component of the source Mach number in the direction of the observer.
- \( F_i \) are the components of the aerodynamic force.
- \( c \) is the speed of sound.

In addition, the near field pressure fluctuations are calculated according to Lowson\(^1\) as follows:

\[ p'(t) - p_0 = \left[ \frac{1}{4\pi (1 - M_s^2) s^2} \left( \frac{F_i (x_i - y_i)}{s} \frac{(1 - M_s^2)}{(1 - M_s^2) - F_i M_s} \right) \right] \]  

(2)

where \( M \) is the source Mach number \( \sqrt{\frac{y_1^2 + y_2^2 + y_3^2}{c}} \)

and \( M_i \) is the component of \( M \) in the \( i \)-direction.

In all cases the sum of Equations (1) and (2) is calculated, but as shown in Section 6 of USAAVLABS TR 68-60 the component due to (2) is negligible at any significant distance from the rotor.

Equations (1) and (2) are solved by a numerical method which has been programmed for digital computation. This solution is exact to the extent that no approximations are made. The accuracy of the solution is only limited by that with which the real loads and motions experienced by the rotor dynamic system can be represented by the model used. The sound field for the simulated system is calculated accurately at any point in the near or far field.

The sound generated by a rotor blade in motion is the result of the distributed aerodynamic pressure acting over its entire surface. Rotational noise, which is the subject of this study, is defined as that component of the sound field which is directly attributable to the lift and drag forces acting on the blade. Strictly, the entire spanwise and chordwise distributions of these components should be taken into account, but for the sake of numerical expediency it is necessary to simulate the actual distributions by a discrete set of point loads. The implications of doing so are fully discussed in Section 5 of USAAVLABS TR 68-60. The spanwise loading distributions are divided into a number of segments, each of which is represented by two single force components, lift and drag. This model is illustrated in Figure 1. The point of application of each force pair then becomes an acoustic source which generates sound according to Equations (1) and (2). The \( F_i \) in those equations are the forces acting upon the air and are therefore opposed to the lift and drag forces acting upon the blade.

If an arbitrary set of orthogonal axes \( X, Y, Z \) are defined, Equations (1) and (2) can be combined. Using also a more convenient notation,

![Figure 1. Discrete Representation of Aerodynamic Loading.](image-url)
\[
\Delta p(t) = \frac{-1}{4\pi(1-M_\infty)^2 c s^2} \left[ -x \left\{ \hat{F}_X + F_X \left( \frac{\hat{M}_s + c s(1 - M^2)}{1 - M_s} - \frac{x}{s} \right) \right\} \\
+ y \left\{ \hat{F}_Y + F_Y \left( \frac{\hat{M}_s + c s(1 - M^2)}{1 - M_s} - \frac{y}{s} \right) \right\} \\
+ z \left\{ \hat{F}_Z + F_Z \left( \frac{\hat{M}_s + c s(1 - M^2)}{1 - M_s} - \frac{z}{s} \right) \right\} \right]
\]

\(\Delta p(t)\) is the observed sound pressure at time \(t\) due to the aerodynamic forces acting at any particular point on the blade whose components in the \(X\), \(Y\) and \(Z\) directions, at the retarded time \(t - \frac{s}{c}\), were \(F_X\), \(F_Y\) and \(F_Z\). The dot denotes differentiation with respect to time. The quantities \(x\), \(y\) and \(z\) are the coordinates of the observer, relative to the source in the \(X\), \(Y\) and \(Z\) directions. The negative sign accounts for the use of the blade loads which are equal and opposite to the forces which act on the air.

It can be seen from this equation that the force generates sound in two ways: through its fluctuation in time and its fluctuations in position which cause its accelerations toward the observer \((\hat{M}_s)\). The term \((1 - M_\infty)\) in the various denominators is essentially a Doppler effect which amplifies the sound radiated in the direction of motion. Dipole sound (the term in \(\hat{F}\)) is amplified by the factor \((1 - M_\infty)^{-2}\) and quadrupole sound (the accelerative term in \(F \cdot \hat{M}_s\)) by \((1 - M_\infty)^{-3}\). Thus the second term becomes increasingly important as the velocity in the direction of the observer increases.

The elements of the method of computing the sound field of a complete rotor are as follows:

1. Define the geometry, blade loading and resulting blade motions of the rotor as a function of time.
Define the position, attitude and velocity of the rotor with respect to the observer at the time $t$.

Calculate the orientation and magnitude of each of the elemental blade airloads at its appropriate retarded time.

Calculate and integrate the observed sound pressures due to all the sources.

Repeat operations (2) through (4) for a series of successive time intervals to construct a time history of the acoustic pressure amplitude and harmonically analyze this to obtain its frequency spectrum.

A computer program has been written to perform these operations taking account of the following variables:

(a) Number of rotors.
(b) Number of blades.
(c) Rotor diameter.
(d) Helicopter position in space with regard to a fixed observer including:
   1. Rotor hub vertical displacement.
   2. Rotor hub horizontal displacement.
   3. Rotor shaft inclination.
(e) Rotor hub motion including:
   1. Linear velocity.
   2. Roll angular velocity.
   3. Pitch angular velocity.
(f) Rotor blade tip speed.
(g) Articulated blade motions.
(h) Blade motion resulting from flapping bending, edgewise bending, and twist about the elastic axis.
(i) Phase relationships between rotor angular positions for multirotor vehicles.

(j) Time-dependent blade loading for at least 20 radial blade segments. The blade loading includes loading normal to the hub plane, radial loading in the hub plane, and tangential loading in the hub plane. These loadings are treated as loads at a chordwise point on the blade element with direction determined by orientation of the blade element in space. Variation in disc loading is considered as a function of variation in blade loading with the number of blades and blade chord held constant.

This program, code-named HERON 1, is described in detail in Sections 4 and 5. The axis transformations and outlines of the computational steps are discussed in the following sections.
2.0 COORDINATE SYSTEMS AND TRANSFORMATIONS

2.1 Rotor Axes \( x', y', z' \)

Since blade motions are measured relative to the rotor shaft it is desirable, for the direct utilization of experimental data, to specify all blade responses, including flapping, lagging and elastic deformations with respect of a set of "shaft axes".

Accordingly, a set of right-handed orthogonal axes \( x', y', z' \) are chosen with the \( z' \) axis coincident with the rotor shaft and positive downward (Figure 2). The longitudinal axis \( x' \) lies in the vertical plane through the aircraft velocity vector \( V \).

2.2 Flight Path Axes \( x, y, z \)

This system is introduced merely to simplify the transformations between the \( x', y', z' \) and \( X, Y, Z \) systems. The origin of this system is coincident with that of the rotor axes. However, the \( x, y \) plane is horizontal with the \( x \) axis in the vertical \( Y-x' \) plane. The aircraft rotation is specified with respect to this system in the following sequence (Figure 3).
Figure 3. Pitch and Roll Rotations.

(1) The pitch angle $\theta'$ is measured clockwise about the positive $y$ axis, rotating $x$ to $x'$.

(2) The roll angle $\phi'$ is then measured clockwise about the $x'$ axis, rotating $y$ to $y'$ and $z$ to $z'$.

(Vehicle angular displacements are measured in pitch and roll only, since rotor yaw is simply a shift in azimuth reference.)

The shaft inclinations to the vertical $z$ axis, measured in the $xz$ and $yz$ planes, are $\theta'$ and $\phi'$ so that (Figure 4) $\theta = \theta'$ and $\tan \phi = \cos \theta \tan \phi'$.

Figure 4. Shaft Inclinations.
The transformations between shaft and flight path axes are:

\[
\begin{align*}
    x &= x' \cos \theta + y' \sin \phi \sin \theta + z' \cos \phi \sin \theta \\
    y &= y' \cos \phi - z' \sin \phi \\
    z &= x' \sin \theta + y' \sin \phi \cos \theta + z' \cos \phi \cos \theta.
\end{align*}
\]  

(4)

2.3 Fixed Axes X, Y, Z

This coordinate system is used to define the aircraft and observer positions in space. The axes are fixed in space with arbitrary origin, and XY is the horizontal ground plane. Z is measured positive vertically upwards.

The XY and x y planes are thus parallel, and the angle between the x and X axes is defined as \( \chi \).

![Figure 5. Fixed Axes (Plan View).](image)

Thus, if the rotor hub coordinates are \( X_H, Y_H, Z_H \), any point in the X, Y, Z system is defined by

\[
\begin{align*}
    X &= X_H + x \cos \chi + y \sin \chi \\
    Y &= Y_H + x \sin \chi - y \cos \chi \\
    Z &= Z_H - z
\end{align*}
\]  

(5)
The coordinates of an observer at \( X_0, Y_0, Z_0 \), with respect to the source at \( x, y, z \), are

\[
\begin{align*}
\bar{x} &= X_0 - X \\
\bar{y} &= Y_0 - Y \\
\bar{z} &= Z_0 - Z
\end{align*}
\] (6)

The distance \( s \) between source and observer is

\[
s = (\bar{x}^2 + \bar{y}^2 + \bar{z}^2)^{1/2}
\] (7)

2.4 Blade Element Displacements

The \( x', y', z' \) coordinates of each specified blade loading point are calculated as a function of azimuth angle, flapping and lagging angles (where applicable), and normal and in-plane elastic displacements. The rotor azimuth angle is measured from the negative \( x' \) axis, clockwise about the positive \( z' \) axis, \( \psi = \Omega t \).

**NOTE**

\( \epsilon_f \) may be \( > \) or \( < \) \( \epsilon \).

Figure 6. Blade Element Displacements in Rotor Axes.
Referring to Figure 6,

- $e_f$ is the flap hinge offset from the shaft centerline.
- $e_l$ is the lag hinge offset.
- $\xi$ is the lag angle projection on the $x'y'$ plane (positive in the direction of rotation).
- $\beta$ is the flap angle between the undeformed blade axis and the $x'y'$ plane, positive in the negative $z'$ direction.
- $\eta_T(r)$ is the elastic displacement of blade station $r$ parallel to the $x'y'$ plane, normal to the undeformed blade axis and positive in the direction of rotation.
- $\eta_N(r)$ is the elastic displacement normal to the undeformed blade axis, in the plane containing the $z'$ axis and positive in the negative $z'$ direction.

These coordinates thus take account of all possible motion of the blade axis upon which the point aerodynamic loads are assumed to act.

If the flapping hinge is outboard of the lagging hinge, the coordinates of the radial station $r$ are:

\[
x' = - e_f \cos \psi - \left( (r - e_f) \cos \xi + (e_f - e_l) \right) \cos (\psi + \xi) \\
+ \eta_T \sin (\psi + \xi) + \eta_N \sin \beta \cos (\psi + \xi)
\]

\[
y' = - e_f \sin \psi - \left( (r - e_f) \cos \beta + (e_f - e_l) \right) \sin (\psi + \xi) \\
+ \eta_T \cos (\psi + \xi) + \eta_N \sin \beta \sin (\psi + \xi)
\]

\[
z' = - (r - e_f) \sin \beta - \eta_N \cos \beta
\]

If the flapping hinge is inboard of the lagging hinge, we have

\[\text{(8a)}\]
\[ x' = -(e_f + e_f \cos \beta) \cos \phi - (r -\epsilon_f) \cos \beta \cos (\phi + 5) \]
\[ + \eta_T \sin (\phi + 5) + \eta_N \sin \beta \cos (\phi + 5) \]
\[ y' = -(e_f + e_f \cos \beta) \sin \phi - (r -\epsilon_f) \cos \beta \sin (\phi + 5) \]
\[ - \eta_T \cos (\phi + 5) + \eta_N \sin \beta \sin (\phi + 5) \]
\[ z' = -(r -\epsilon_f) \sin \beta - \eta_N \cos \beta \]

(8b)

It is now possible, using Equations (4), (5) and (8), to find the fixed coordinates \( X, Y, Z \) of any blade station, defined in convenient rotor coordinates as a function of time, and subsequently, using Equations (6) and (7), the displacement components \( \ddot{x}, \ddot{y}, \ddot{z} \) and \( s \) of the sound Equation (3).

2.5 Blade Element Velocities and Accelerations

We also require the velocity and acceleration components defined in the sound Equation (3) as \( \frac{\partial}{\partial t} \), \( \frac{\partial}{\partial s} \), and \( \frac{\partial^2}{\partial s^2} \), which are the source Mach number and rate of change of Mach number in the direction of the observer. Resolving the source Mach number into its three components we have

\[ M = \sqrt{M_X^2 + M_Y^2 + M_Z^2} \]  

(9)

\[ M_s = \frac{\dot{x}}{s} \dot{M}_X + \frac{\dot{y}}{s} \dot{M}_Y + \frac{\dot{z}}{s} \dot{M}_Z \]  

(10)

and

\[ \dot{M}_s = \frac{\ddot{x}}{s} \ddot{M}_X + \frac{\ddot{y}}{s} \ddot{M}_Y + \frac{\ddot{z}}{s} \ddot{M}_Z \]  

(11)

where \( M_X, M_Y, \) and \( M_Z \) are simply \( \frac{\dot{x}}{c}, \frac{\dot{y}}{c}, \) and \( \frac{\dot{z}}{c} \), respectively; that is, the velocity components of the source, relative to the stationary air, expressed as a fraction of the atmospheric speed of sound. To obtain the first and second time derivatives of \( X, Y, \) and \( Z \) it is necessary to differentiate Equations (5), and subsequently Equations (4) and (8). From Equation (5),
\[ \dot{X} = \dot{X}_H + \dot{x} \cos x - x \dot{x} \sin x + \dot{y} \sin x + y \dot{x} \cos x \]
\[ = \dot{X}_H + (\dot{x} + y \ddot{x}) \cos x + (\dot{y} - x \ddot{x}) \sin x \]
\[ \dot{Y} = \dot{Y}_H + \dot{x} \sin x + x \dot{x} \cos x - \dot{y} \cos x + y \dot{x} \sin x \]
\[ = \dot{Y}_H - (\dot{y} - x \ddot{x}) \cos x + (\dot{x} + y \ddot{x}) \sin x \]
\[ \dot{Z} = \dot{Z}_H - \ddot{z} \quad (12) \]
\[ \ddot{X} = \ddot{X}_H + (\ddot{x} + 2 \dot{y} \dot{x} - x \dot{x}^2 + y \ddot{x} \sin x \]
\[ \ddot{Y} = \ddot{Y}_H - (\ddot{y} - 2 \dot{x} \dot{y} \sin x - y \dot{x}^2 \cos x + (\dot{x} + 2 \dot{y} \dot{x} - x \dot{x}^2 + y \ddot{x}) \sin x \]
\[ \ddot{Z} = \ddot{Z}_H + \ddot{z} \quad (13) \]

and from Equation (3),
\[ \dot{x} = \dot{x}' \cos \theta - x' \dot{\theta} \sin \theta - (z'\dot{\phi} - \dot{y}') \sin \phi \sin \theta \]
\[ + (\dot{z}' + y' \dot{\phi}) \cos \phi \sin \theta + y' \dot{\theta} \sin \phi \cos \theta + z' \dot{\theta} \cos \phi \cos \theta \]
\[ \dot{y} = (\dot{y}' - z' \dot{\phi}) \cos \phi - (y' \dot{\theta} + \dot{z}) \sin \phi \]
\[ \dot{z} = z - \dot{x}' \sin \theta - x' \dot{\theta} \cos \theta + (\dot{y}' - z' \dot{\phi}) \sin \phi \cos \theta \]
\[ + (y' \dot{\phi} - \dot{z}') \cos \phi \cos \theta - y' \dot{\theta} \sin \phi \sin \theta - z' \dot{\theta} \cos \phi \sin \theta \quad (14) \]
\[ \ddot{x} = (\ddot{x}' - x' \dot{\theta}^2) \cos \theta - (2 \dot{x}' \dot{\theta} + x' \ddot{\theta}) \sin \theta \]
\[ + (2y' \dot{\phi} + 2 \dot{z}' \dot{\theta} + z' \ddot{\theta}) \cos \phi \cos \theta \]
\[ + (\ddot{y}' - y' \dot{\phi}^2 + \dot{\phi}^2) - 2 \dot{z}' \dot{\phi} - z' \ddot{\phi}) \sin \phi \sin \theta \]
\[ + (2 \ddot{y}' \dot{\phi} + y' \dot{\phi} + \dot{z}' - z' \dot{\phi}^2 + \dot{\phi}^2) \cos \phi \sin \theta \]
\[ + (2y' \dot{\theta} + y' \dot{\phi} - 2 z' \dot{\phi} \dot{\theta}) \sin \phi \cos \theta \]
\[
\dot{y} = (\dot{y}' + y'\dot{\phi}^2 - z'\dot{\phi}) \cos \phi - (\dot{z}' - z'\dot{\phi}^2 + y'\dot{\phi}) \sin \phi
\]

\[
\ddot{z} = - (2\dot{x}'\dot{\theta} + x'\ddot{\theta}) \cos \theta - (\dot{x}' - x'\dot{\theta}^2) \sin \theta
+ (2\dot{y}'\dot{\phi} + y'\ddot{\phi} + \ddot{z}' - z'(\dot{\phi}^2 + \dot{\theta}^2)) \cos \phi \sin \theta
- (2\dot{y}'\dot{\phi} + y'\ddot{\phi} - 2z'\dot{\phi}) \sin \phi \sin \theta
- (2\dot{y}'\dot{\phi} + y'\ddot{\phi} + 2\dot{z}' + 2\ddot{\theta}) \cos \phi \sin \theta
+ (\dot{y}' - y'(\dot{\phi}^2 + \dot{\theta}^2) - 2z'\dot{\phi} - z'\ddot{\phi} - z'\dot{\phi}) \sin \phi \cos \theta
\] (15)

Expressions for the derivatives of \( x' \), \( y' \) and \( z' \) are not derived explicitly for reasons which will be explained in Section 3.0.

2.6 Aerodynamic Forces and Their Derivatives

The aerodynamic force components in the sound Equation (3) are derived from the components \( F_N \) and \( F_T \) which act directly on the blade. \( F_N \) is defined as the component of the aerodynamic load on the blade which acts normal to the blade axis (in its deformed condition) in the plane containing the shaft axis \( z' \). It acts in the same sense as the lift on the blade. \( F_T \) is the tangential component, again acting normal to the blade axis but parallel to the \( x' y' \) plane. Its sense is opposite to the blade drag force. Referring to Figure 6, we see that the blade force components in the \( x', y' \) and \( z' \) directions are:

\[
F_{x'} = F_T \sin (\psi + \zeta') + F_N \sin \beta' \cos (\phi + \zeta')
\]

\[
F_{y'} = - F_T \cos (\phi + \zeta') + F_N \sin \beta' \sin (\phi + \zeta')
\]

\[
F_{z'} = - F_N \cos \beta'
\] (16)

where \( \beta' \) and \( \zeta' \) are the blade slopes relative to the \( x' y' \) plane and the radius at azimuth \( \psi \) respectively. That is,
\[
\beta' = \beta + \tan^{-1} \left( \frac{d\eta N}{dr} \right)
\]

\[
\xi' = \xi + \tan^{-1} \left( \frac{d\eta T}{dr} \right)
\]  \hspace{1cm} (17)

The transformations which are required to obtain the components \(F_X', F_Y'\) and \(F_Z'\) in the fixed coordinate system are identical to those used to convert \(x', y', z'\) to \(X, Y, Z\) (Equations (4) and (5)). The derivatives \(\dot{F}_X', \dot{F}_Y'\) and \(\dot{F}_Z'\) are also derived from the rotor axis values using similar transformations (Equations (12) through (15)), and again the derivatives in the rotor coordinate system \(\dot{F}_X', \dot{F}_Y'\) and \(\dot{F}_Z'\) are not derived explicitly for reasons outlined in the following section.
3.0 METHOD OF SOLUTION BY DIGITAL COMPUTER

The technique for calculating the helicopter rotor sound field is best explained by a description of the computational steps of the program HERON 1. Figure 7 is an illustration of these basic steps in flow diagram form. The program is divided into two phases. The first processes the input data into a convenient form for storage and the second computes the sound pressure level at a preselected number of field points.

3.1 Phase I  Initial Data Processing

For each rotor, the blade loading and motion data is read by the program either as a number of arrays, listing their values at a discrete number of points over the rotor disc, or as a series of Fourier coefficients, one set for each radial station. To reduce the volume of later computations, this initial phase of the program calculates and stores the displacements of each loading point with respect to the rotor axes \( x', y', z' \) according to Equation (7). The three force components \( F_{x'}, F_{y'}, F_{z'} \) are also calculated using Equation (16). These calculations are performed upon the input data in whichever form it comes, at a predetermined number of azimuth stations. The history of each of these six variables around the azimuth is then harmonically analyzed for every radial station, and the Fourier coefficients are stored. This operation is carried out for one blade only since it is assumed that all blades are loaded and respond in identical manners. From the stored coefficients it is possible for later elements of the program to interpolate for all required blade loading and motion variables at any arbitrary azimuth station.

3.2 Phase II  Calculation of the Sound Field

Phase II of the program is repeated in its entirety for each required observer position, \( X_0, Y_0, Z_0 \).

3.2.1 "Observer Time" \( t \)

The sound field observed at the position \( X_0, Y_0, Z_0 \) is calculated as a time history of the acoustic pressure \( \Delta p \) at the "observer time" intervals \( 0, \Delta t, 2\Delta t, \ldots, T \), where \( T \) is the fundamental period. The sound harmonic amplitudes are then obtained through a Fourier analysis of this time history.

The fundamental period \( T \), to an observer moving with the aircraft, is the blade passage period \( 2\pi/\Omega B \). However, for a stationary observer this shifts by the factor \( (1 - M_0)^{-1} \) where \( M_0 \) is the aircraft Mach number component in the direction of the observer.
Figure 7. Block Schematic Flow Diagram For HERON 1.
Increment "Observer" Time Instant \( t \)

Select Blade \( b \)

Increment Radial Station \( r \)

Calculate: Retarded Time \( t' \) for Station \( r \) of Blade \( b \), Azimuth Angle \( \psi' \) of Blade \( b \) at \( t' \); Position of Hub \( h \), \( X_H, Y_H, Z_H \) at \( t' \)

Synthesize \( x', y', z' \), \( F_x', F_y', F_z' \) and Their Derivatives for Blade Station \( r \) and Azimuth Station \( \psi' \) from Stored Fourier Coefficients

Calculate Sound Pressure Increment \( \Delta p \) Due to Forces Acting on Blade \( b \), Radial Station \( r \)

Form Total Sound Pressure (at Observer Time) \( p(t) \) by Summing \( \Delta p \) for \( B \) Blades and \( N \) Radial Stations

Fourier Analyze Sound Pressure History at Field Point \( j \) and Output Coefficients as Sound Pressure Levels

Combine Sound Field at Each Point for \( H \) Rotor

END
One approach is to calculate the observed period which can then be divided into an appropriate number of intervals. However, because of the aircraft motion, a slight error is introduced by this procedure which results in the computation of something slightly less than a complete period. This error increases with sound harmonic number (which is equivalent to a reduction of wavelength).

The alternative which has been adopted in the program is to calculate the time history at a point which "moves" with the same velocity as the rotor hub but which has the average position $X_0, Y_0, Z_0$ during the period $T$. The only modification required for the final result is to correct the observed frequency for the Doppler shift factor $(1 - M_0)^{-1}$. The quotation marks are used because at each time increment the sound pressure is still calculated at a stationary point which has simply moved the same distance as the rotor since the previous time increment. All relative velocity effects in the acoustic calculation are retained.

For each observer time $t$, the sound pressure $\Delta p$ is calculated at the effective observer positions:

$$X'_0 = X_0 + \dot{X}_H (t - \frac{T}{2})$$

$$Y'_0 = Y_0 + \dot{Y}_H (t - \frac{T}{2})$$

$$Z'_0 = Z_0 + \dot{Z}_H (t - \frac{T}{2})$$

To calculate the Mach number component $M_0$, and hence the Doppler shift correction, it is first necessary to calculate the sound propagation time $\tau_0$ as follows:

The position of the rotor hub at time $t$ is $X_H', Y_H', Z_H$ and the distance between the hub and the observer at $X_0, Y_0, Z_0$, according to Equation (6), is

Figure 8. Velocity Diagram for Sound Radiation by Moving Aircraft.
\[ s = \left( x_0^2 + y_0^2 + z_0^2 \right)^{\frac{1}{2}} \]

However, the sound arriving at the observer at time \( t \) was generated at the retarded time \( t - \tau_0 \) when the hub was at \( X_H', Y_H', Z_H' \). The corresponding velocity triangle is shown in Figure 8. The resultant aircraft velocity is

\[ V = \left( \dot{x}_H^2 + \dot{y}_H^2 + \dot{z}_H^2 \right)^{\frac{1}{2}} \] (19)

and the propagation time \( \tau_0 \) is given by

\[ \tau_0 = -\frac{\sqrt{(\ddot{x}_0 \dot{X}_H + \ddot{y}_0 \dot{Y}_H + \ddot{z}_0 \dot{Z}_H) + \sqrt{(\ddot{x}_0 \dot{X}_H + \ddot{y}_0 \dot{Y}_H + \ddot{z}_0 \dot{Z}_H)^2 + s^2 (c^2 - V^2)}}}{c^2 - V^2} \] (20)

From the propagation time the coordinates at the instant of sound emission can be calculated. The component hub to observer separations are \( \tilde{x}_0', \tilde{y}_0', \) and \( \tilde{z}_0' \) where

\[ \tilde{x}_0' = \bar{x}_0 + \dot{x}_H \tau_0 \]
\[ \tilde{y}_0' = \bar{y}_0 + \dot{y}_H \tau_0 \]
\[ \tilde{z}_0' = \bar{z}_0 + \dot{z}_H \tau_0 \] (21)

Hence, the required Mach number component is

\[ M_0 = \frac{\tilde{x}_0' \dot{x}_H + \tilde{y}_0' \dot{y}_H + \tilde{z}_0' \dot{z}_H}{c^2 \tau_0} \] (22)

3.2.2 Retarded Time

The retarded time of each blade loading point has to be calculated independently for each observer time instant. This requires solution of the transcendental equation
This equation can only be solved by iteration, starting with some realistically chosen value of \( \tau \) and successively substituting newly calculated values into the right-hand side until the solution converges. The right-hand side of Equation (23) is in fact a very lengthy expression which includes Equations (4), (5), (6) and (8). The following procedure is followed to minimize the computational time involved.

For the most inboard blade loading point, the rotor hub retarded time \((t - \tau_0)\) is used as a starting value. The RHS of Equation (23) is calculated using the first harmonic Fourier coefficients of \(x', y'\) and \(z'\). As an example,

\[
\psi = \psi (t - \tau)
\]

\[
x' = a_{x_0}(r) + a_{x_1}(r) \cos \phi + b_{x_1}(r) \sin \phi
\]

The values of \(y'\) and \(z'\) are calculated similarly and the necessary transformations applied to compute \(x, y\) and \(z\). These are substituted into Equation (23) to yield a second approximation to the retarded time. The iteration proceeds but a successively greater number of harmonics is admitted in the calculation of the \(x', y', z'\) coordinates in successive iterations. By the time convergence is reached, the predetermined limiting number of harmonics is admitted. It is found that this method considerably reduces the volume of computation (through avoiding a large number of Fourier summations) without increasing the number of iterative cycles to convergence.

The calculations for the next radial station use the final value of \(\tau\) from the first radial station, and so on. Experience shows that an average of four or five iterations is required for a convergent solution with an error of less than \(10^{-5}\) seconds.

### 3.2.3 Blade Loads and Motions

The Fourier representation of the load and motion distributions enables the required displacements, velocities, accelerations, forces and rate of change of forces to be computed with a minimum amount of effort. From the retarded time \(t - \tau\), the relevant blade azimuth angle \(\phi\) is obtained and the motion and load variables in the \(x', y', z'\) system are synthesized as shown in the following example:

\[
\psi = \psi (t - \tau)
\]
\[ x' = a_k x'_k + \sum_{k=1}^{K} \left( a_k \cos k \psi + b_k \sin k \psi \right) \]

\[ x' = -\omega \sum_{k=1}^{K} k \left( a_k \sin k \psi - b_k \cos k \psi \right) \]

\[ x' = -\omega^2 \sum_{k=1}^{K} k^2 \left( a_k \cos k \psi + b_k \sin k \psi \right) \]

The transformations listed in Section 2.1.2 are then applied to derive all the necessary components of the sound Equation (3).

### 3.2.4 The Sound Field

The observed sound pressure increment generated by each loading point is calculated according to Equations (1) and (7). The increments due to all loading points on all blades are then summed to give the total pressure increment at time \( t \). The entire process is repeated for successive time instants until a history is obtained for one complete (blade passage) period. The final step is to Fourier analyze this time history into its sine and cosine component harmonics \( (a_n, b_n) \). Each harmonic sound pressure level is then expressed in decibels by performing the transformation

\[ SPL_n = \left( 10 \log \sqrt{a_n^2 + b_n^2} + 124.6 \right) \text{dB re } 0.0002 \mu \text{Bar} \]

### 3.2.5 Multiple Rotors

In its present form the program computes the sound harmonics at each field point for each rotor and outputs the results independently. If the rotors have the same fundamental blade passage frequency then the sound harmonics due to the individual rotors reinforce each other and the in-phase and out-of-phase components are simply added together separately. If the rotors have different fundamental frequencies (for example the main and tail rotors of a "single rotor" helicopter), then the individual harmonics do not interfere and the sound spectra are merely superimposed. The final block in Figure 7 is drawn with a broken line since the program does not perform this function automatically. However, sufficient information is output (namely the frequency, cosine and sine harmonic amplitude in lb/ft² and sound pressure level of each harmonic) to enable the necessary summations to be performed rapidly by hand.
4.0 PROGRAM FLOW CHARTS, EQUATIONS AND LISTING

The computer program HERON 1 is written in FORTRAN IV for the Control Data Corporation's CDC 3300 utilizing the SCOPE operating system. The minimum hardware configuration required is as follows:

1. 3300 series computer with 32,000-word core storage.
2. Card reader.
3. Line printer.
4. One scratch tape unit (LUN2)

A complete description of the program is contained in this section which includes the following items:

4.1 A detailed flow chart for the program. The noted equation numbers correspond to list 4.2.
4.2 A complete list of programmed equations.
4.3 A program listing.
4.4 A list of program symbols and definitions (Table I).
List of Equations Computed by Program HERON

Equation No.

1  a) $x' = -e_y \cos \phi - (r - e_f) \cos \beta + (e_f - e_y) \cos (\varphi + \xi) + \eta_T \sin (\varphi + \xi) + \eta_N \sin \beta \cos (\varphi + \xi)$
    b) $y' = -e_y \sin \phi - (r - e_f) \cos \beta + (e_f - e_y) \sin (\varphi + \xi) - \eta_T \cos (\varphi + \xi) + \eta_N \sin \beta \sin (\varphi + \xi)$
    c) $z' = -(r - e_f) \sin \beta - \eta_N \cos \beta$

2  a) $x' = -(e_y + e_f \cos \beta) \cos \phi - (r + e_y) \cos \beta \cos (\varphi + \xi) + \eta_T \sin (\varphi + \xi) + \eta_N \sin \beta \cos (\varphi + \xi)$
    b) $y' = -(e_y + e_f \cos \beta) \sin \phi - (r + e_y) \cos \beta \sin (\varphi + \xi) - \eta_T \cos (\varphi + \xi) + \eta_N \sin \beta \sin (\varphi + \xi)$
    c) $z' = -(r - e_f) \sin \beta - \eta_N \cos \beta$

3  a) $\frac{d\eta_N}{dr} = \frac{(\eta_N(n+1) - \eta_N(n-1))/(r(n+1) - r(n-1))}{r(2) - r(1)}$
    b) $\frac{d\eta_N}{dr} = \frac{(\eta_N(2) - \eta_N(1))/(r(2) - r(1))}{r(2) - r(1)}$
    c) $\frac{d\eta_T}{dr} = \frac{(\eta_T(n+1) - \eta_T(n-1))/(r(n+1) - r(n-1))}{r(2) - r(1)}$
    d) $\frac{d\eta_T}{dr} = \frac{(\eta_T(2) - \eta_T(1))/(r(2) - r(1))}{r(2) - r(1)}$
    e) $\frac{d\eta_N}{dr} = \frac{(\eta_N(n) - \eta_N(n-1))/(r(n) - r(n-1))}{r(n) - r(n-1)}$
    f) $\frac{d\eta_T}{dr} = \frac{(\eta_T(n) - \eta_T(n-1))/(r(n) - r(n-1))}{r(n) - r(n-1)}$
4  a) $\beta' = \beta + \tan^{-1}(d\eta_N/dr)$

   b) $\xi' = \xi + \tan^{-1}(d\eta_T/dr)$

5  a) $F_{x'} = T \sin(\psi + \xi') + F_N \sin\beta' \cos(\psi + \xi')$

   b) $F_{y'} = -F_T \cos(\psi + \xi') + F_N \sin\beta' \sin(\psi + \xi')$

   c) $F_{z'} = -F_N \cos\beta'$

6  a) $a_{0n}(\omega) = \frac{1}{M} \sum_{m=1}^{M} \omega$

   b) $a_{kn}(\omega) = \frac{2}{M} \sum \omega \cos((m-1) \times 2\pi k/M)$

   c) $b_{kn}(\omega) = \frac{2}{M} \sum \omega \sin((m-1) \times 2\pi k/M)$

7  a) $a_{0i}(F_i) = \frac{1}{M} \sum_{m=1}^{M} F_i$

   b) $a_{ki}(F_i) = \frac{2}{M} \sum F_i \cos((m-1) \times 2\pi k/M)$

   c) $b_{ki}(F_i) = \frac{2}{M} \sum F_i \sin((m-1) \times 2\pi k/M)$

8  a) $\tilde{x}_H = X_H - X_0'$

   b) $\tilde{y}_H = Y_H - Y_0'$

   c) $\tilde{z}_H = Z_H - Z_0'$

9  $S_H = \sqrt{\tilde{x}_H^2 + \tilde{y}_H^2 + \tilde{z}_H^2}$
\[ V = \sqrt{\frac{x'^2_H + y'^2_H + z'^2_H}{2}} \]

\[ \tau'_0 = \left\{ -(x_H \dot{x}_H + y_H \dot{y}_H + z_H \dot{z}_H) \cdot \left( (x_H x'^2_H + y_H y'^2_H + z_H z'^2_H)^2 \right) \right\} / \left( \sigma_0^2 - \nu^2 \right) \]

12.

a) \[ \ddot{x}'_H = \ddot{x}_H + \dot{x}_H \tau'_0 \]

b) \[ \ddot{y}'_H = \ddot{y}_H + \dot{y}_H \tau'_0 \]

c) \[ \ddot{z}'_H = \ddot{z}_H + \dot{z}_H \tau'_0 \]

13.

\[ M_0 = (\dddot{x}'_H x'_H + \dddot{y}'_H y'_H + \dddot{z}'_H z'_H) / \sigma_0^2 \tau'_0 \]

14.

a) \[ x'_H = x_H - \dot{x}_H \tau'_0 \]

b) \[ y'_H = y_H - \dot{y}_H \tau'_0 \]

c) \[ z'_H = z_H - \dot{z}_H \tau'_0 \]

15.

a) \[ T = 2\pi / \Omega B \]

b) \[ \Delta T = 2\pi / \omega M \]

16.

a) \[ x_0 = x'_0(j) + \dot{x}_H \left( t - \frac{T}{2} \right) \quad t = 0 (\Delta T) \quad T - \Delta T \]

b) \[ y_0 = y'_0(j) + \dot{y}_H \left( t - \frac{T}{2} \right) \]

c) \[ z_0 = z'_0(j) + \dot{z}_H \left( t - \frac{T}{2} \right) \]

17.

\[ \phi_0 = \psi_0 + \frac{2\pi}{B} \cdot b \]

b = 1 (1) B
18  a) \( \phi = \phi_0 + \Omega t' \)

b) \( X_H = X_{H_0} + \dot{X}_H t' \)

c) \( Y_H = Y_{H_0} + \dot{Y}_H t' \)

d) \( Z_H = Z_{H_0} + \dot{Y}_H t' \)

e) \( \theta = \theta_0 + \dot{\theta} t' \)

f) \( \phi = \phi_0 + \dot{\phi} t' \)

g) \( \phi^{-1} = \tan^{-1} (\cos \theta \tan \phi) \)

h) \( \dot{\phi}' = \cos^2 \phi' (\dot{\phi} \cos \theta \sec^2 \phi - \dot{\theta} \sin \theta \tan \phi) \)

19  a) \( \omega' = \omega + a_k (\omega) \cos k \psi + b_k (\omega) \sin k \psi \)

b) \( \dot{\omega}' = \dot{\omega}' - k s \left[ a_k (\omega) \sin k \psi - b_k (\omega) \cos k \psi \right] \)

c) \( \ddot{\omega}' = \ddot{\omega}' - k^2 \omega^2 \left[ a_k (\omega) \cos k \psi + b_k (\omega) \sin k \psi \right] \)

20  a) \( x' = x' \cos \theta + y' \sin \phi' \sin \theta + z' \cos \phi' \sin \theta \)

b) \( y' = y' \cos \phi' - z' \cos \phi' \)

c) \( z' = -x' \sin \theta + y' \sin \phi' \cos \theta + z' \cos \phi' \cos \theta \)

21  a) \( \hat{z} = \hat{z}' \cos \theta - x' \hat{\theta} \sin \theta - (z' \hat{\phi}' - y') \sin \phi' \sin \theta \)

+ \( (\dot{z}' + y' \hat{\phi}') \cos \phi' \sin \theta + y' \dot{\theta} \sin \phi' \cos \theta \)

+ \( z' \hat{\theta} \cos \phi' \cos \theta \)

b) \( \dot{y}' = (y' - z' \hat{\phi}') \cos \phi' - (y' \hat{\phi'} + \dot{z}') \sin \phi' \)
c) \[ \dot{z} = - \dot{x}' \sin \theta - x' \dot{\theta} \cos \theta + (\dot{y}' - z' \dot{\phi}) \sin \phi' \cos \theta \]
+ (y' \dot{\phi}' + z') \cos \phi' \cos \theta - y' \dot{\theta} \sin \phi' \sin \theta - z' \dot{\phi} \cos \phi' \sin \theta

a) \[ \ddot{x} = (\ddot{x}' - x'' \dot{\theta}^2) \cos \theta - 2 \dot{x}' \dot{\theta} \sin \theta + (2 y' \dot{\phi}' \dot{\theta} + 2 z' \dot{\phi}) \cos \phi' \cos \theta \]
+ (\ddot{y}' - y' (\dot{\phi}'^2 + \dot{\theta}^2)) \sin \phi' \sin \theta + (2 \ddot{y}' \dot{\phi}' + \ddot{z}' - z' (\dot{\phi}'^2 + \dot{\theta}^2)) \cos \phi' \sin \theta \]
+ (\ddot{z}' (\dot{\phi}'^2 + \dot{\theta}^2)) \cos \phi' \sin \theta - (2 \ddot{y}' \dot{\theta} - 2 z' \dot{\phi} \dot{\theta}) \sin \phi' \sin \theta
- (2 y' \dot{\phi}' \dot{\theta} + 2 z' \dot{\theta}) \cos \phi' \sin \theta + (\ddot{y}' - y' (\dot{\phi}'^2 + \dot{\theta}^2)) \cos \phi' \sin \theta
- 2 z' \dot{\theta} \sin \phi' \cos \phi \sin \theta.

b) \[ \ddot{y} = (\ddot{y}' \dot{\phi}'^2) \cos \phi' - (\ddot{z}' - z' \dot{\phi}'^2) \sin \phi' \]
c) \[ \ddot{z} = - 2 \ddot{x}' \dot{\theta} \cos \theta - (\ddot{x}' - x'' \dot{\theta}^2) \sin \theta + (2 \ddot{y}' \dot{\phi}' + \ddot{z}') \cos \phi' \cos \theta \]
- z' (\dot{\phi}'^2 + \dot{\theta}^2)) \cos \phi' \sin \theta - (2 \ddot{y}' \dot{\theta} - 2 z' \dot{\phi} \dot{\theta}) \sin \phi' \sin \theta
- (2 y' \dot{\phi}' \dot{\theta} + 2 z' \dot{\theta}) \cos \phi' \sin \theta + (\ddot{y}' - y' (\dot{\phi}'^2 + \dot{\theta}^2)) \cos \phi' \sin \theta
- 2 z' \dot{\theta} \sin \phi' \cos \phi \sin \theta.

23

a) \[ \ddot{x} = X_0 - X_H - x \cos \theta - y \sin \theta \]
b) \[ \ddot{y} = Y_0 - Y_H - x \sin \theta - y \cos \theta \]
c) \[ \ddot{z} = Z_0 - Z_H + z. \]

24

a) \[ \dot{X} = \dot{X}_H + \dot{x} \cos \theta + \dot{y} \sin \theta \]
b) \[ \dot{Y} = \dot{Y}_H + \dot{x} \sin \theta - \dot{y} \cos \theta \]
c) \[ \dot{Z} = \dot{Z}_H - \dot{z}. \]

25

a) \[ \ddot{x} = \ddot{x} \cos X + \ddot{y} \sin X \]
b) \[ \ddot{y} = - \ddot{y} \cos X + \ddot{x} \sin X \]
c) \[ \ddot{z} = \ddot{z}. \]

26

S = \[ \sqrt{x^2 + y^2 + z^2} \]
\[ M_s = (\ddot{x}X + \ddot{y}Y + \ddot{z}Z) / S \alpha_0 \]

\[ \dot{M}_s = (\ddot{x}X + \ddot{y}Y + \ddot{z}Z) / S \alpha_0 \]

29 a) \[ F_i = a_i (F_i) + a_k (F_i) \cos k\psi + b_k (F_i) \sin k\psi \]

b) \[ F_i = -k \Omega \ a_i (F_i) \sin k\psi - b_k (F_i) \cos k\psi \]

30 a) \[ F_x = F_x \cos \theta + F_y \sin \phi' \sin \theta + F_z \cos \phi' \sin \theta \]

b) \[ F_y = F_y \cos \phi' - F_z \sin \phi' \]

c) \[ F_z = -F_x \sin \theta + F_y \sin \phi' \cos \theta + F_z \cos \phi' \cos \theta \]

31 a) \[ \dot{F}_x = \dot{F}_x \cos \theta - F_x \hat{\theta} \sin \theta - (F_z \hat{\phi'} - \dot{F}_y) \sin \phi' \sin \theta \\
\quad + (\dot{F}_z + F_y \hat{\phi'}) \cos \phi' \sin \theta + F_z \hat{\theta} \sin \phi' \cos \theta + F_x \hat{\theta} \cos \phi' \cos \theta \]

b) \[ \dot{F}_y = (\dot{F}_y - F_z \hat{\phi'}) \cos \phi' - (F_y \hat{\phi'} + \dot{F}_z) \sin \phi' \]

c) \[ \dot{F}_z = -\dot{F}_x \sin \theta - F_x \hat{\theta} \cos \theta + (\dot{F}_y - F_z \hat{\phi'}) \sin \phi' \cos \theta \\
\quad + (F_y \hat{\phi'} + \dot{F}_z) \cos \phi' \cos \theta - F_y \hat{\theta} \sin \phi' \sin \theta - F_z \hat{\theta} \cos \phi' \sin \theta \]

32 a) \[ F_x = F_x \cos \theta + F_y \sin \theta \]

b) \[ F_y = F_x \sin \theta - F_y \cos \theta \]

c) \[ F_z = -F_z \]

33 a) \[ \dot{F}_x = \dot{F}_x \cos \theta + \dot{F}_y \sin \theta \]

b) \[ \dot{F}_y = \dot{F}_x \sin \theta - \dot{F}_y \cos \theta \]

c) \[ \dot{F}_z = -\dot{F}_z \]
34 \[ X_T = \dot{X}_s + a_0 (1 - M^2)/s \]

35 a) \[ X_{M_s} = X_T - \dot{X}/\hat{X} \]

b) \[ X_{M_s} = X_T - \dot{Y}/\hat{Y} \]

c) \[ Z_{M_s} = X_T - \dot{Z}/\hat{Z} \]

36 \[ \Delta p = -\sqrt{4 \pi \frac{1 - M_s^2}{a_0 s^2}} \left( \dot{X}_s + F_X \cdot X_{M_s} + \dot{Y} (F_Y + Y_{M_s}) + \dot{Z} (F_Z + Z_{M_s}) \right) \]

37 \[ p_j(t) = p_j(t) + \Delta p \]

38 a) \[ C_{P_{jk}} = 28 \left( \sum_{m=1}^{M/B} p_j(m) \cos (m - 1) \right) \sqrt{2\pi B/M} \]

b) \[ S_{P_{jk}} = 28 \left( \sum_{m=1}^{M/B} p_j(m) \sin (m - 1) \right) \sqrt{2\pi B/M} \]

c) \[ r_{P_{jk}} = \sqrt{C_{P_{jk}}^2 + S_{P_{jk}}^2} \]

39 \[ p_{jk} = 20 \log_{10} p_{jk} + 124.58 \]

40 \[ f_k = k \omega B / 2\pi \]
SEQUENCE OPERATOR

PROGRAM HERON 1

COMMON K1,K2,KSF,KSX,M,S1,M,N
COMMON BET(72),ZET(72),FN(72,12),FT(72,12),DMD(72),D(22)
COMMON PSIO,K,CDM,AAM
COMMON XIO,YIO,ZIO,XDH,YDH,ZDM
COMMON XD(72),YD(72),ZD(72)
COMMON WDP(3)
COMMON BETP(72,12),ZETP(72,12)
DIMENSION CARD(30),RN(12)
DIMENSION FF(72),CPJ(72),PJ(72),ALPHAJ(72)

C, PJ, K(72)

EQUIVALENCE (TAUO,TAUO),(XD,CXD),(YD, CYD),(ZD,CZD), (DF(1),RN(1))
EQUIVALENCE (CYDH,YDH),(ZDH,ZDH),(BET,F),(ZET,CPJ)

PI=3.1415926536
WRITE(61,1)
READ(60,2)CAR
WRITE(61,3)CAR
READ(60,1027)H,JCQN,C,RHO
DO 200 I=1,1H
READ(60,1029)OMN,R,PH,EPL,PSIO
OMN=6,283185307/60,
REAC(I,1000)M,N
IND, KSF, KSX, K1, K2,
XF=KSF-1
AM=X=FLOAT(M)
ZX=2./XM
DT =6,283185307/(OM*AM)
TT =6,283185307/(OM*BB)
DPSI=6,283185307/XM
TT=TT+DT
MFT=M+1
DMD(I)M=0
DSDM=360./XM
DO 405 J=1,MPT
405 DMD(J=1)=DMD(I+J)=DSDM(I+J)
READ(60,1028) (DI(1),I=1,N)
DO 10 I=1,N
10 RN(I)=DI(1)
READ(60,1028)XM,YM,ZM,TH,PH1,XUM,YUM,ZUM,THM,PHID
IF(XM=XI,AND,DI,YM,AND,OMM,AND,DI,ZM,AND,OMM,AND,DI,1083,1084)
1083 V=0
GO TO 1085
1084 W=SCRT(XM*XM+YM*YM+OMM*OMM)
1085 CONTINUE
IF(I88)
WRITE(61,4)I9,R,OMM,EPF,EPF,PSIO,TH,PH1,XUM,YUM,ZUM,V
WRITE(61,5)XM,YM,ZM
WRITE(61,6)
SINF=SINF(PHID)
COSD=COSD(PHID)
XM=XM
YM=YM
ZM=ZM
TH=TH
PW1=PH1
N is number of radial stations
M is the number of azimuth stations
X, Y, Z is the position of observer

BET is the flapping angle.
ZET is the lag angle.

Beta and zeta are input in degrees.

GO TO (16, 18) IND
16 CONTINUE
WRITE(61, 7)
REAC(60, 1028)(BET(I), I=1, M)
REAC(60, 1028)(ZET(I), I=1, M)
WRITE(61, 67)
WRITE(61, 68)(DMD(I), BET(I), ZET(I), I=1, M)
GO TO 20
18 CONTINUE
WRITE(61, 8)
CALL INPUT (1)
20 CONTINUE
DO 25 J=1, N
BET(I)=BET(I)+0.01745329
ZET(I)=ZET(I)+0.01745329
REWIND 2
GO TO (30, 40) IND
30 DO 35 J=1, N
THE FOLLOWING STATEMENTS INPUT THE ETA AND ETA T ARRAYS INTO THE FN AND FT STORAGE LOCATIONS, THESE SAME LOCATIONS WILL LATER BE USED FOR THE FN AND FT ARRAYS,
REAC (60, 1028) (FN(I,J), I=1, M)
REAC (60, 1028) (FT(I,J), I=1, M)
WRITE(61, 69)
WRITE(61, 63)
WRITE(61, 64)(RN(I), I=1, N)
DO 111 J=1, M
WRITE (61, 651)((DMD(J), (FT(J,I), I=1, N))
111 WRITE (61, 1091)
WRITE(61, 71)
WRITE(61, 63)
WRITE(61, 64)(RN(I), I=1, N)
DO 112 J=1, M
WRITE (61, 655)((DMD(J), (FN(J,I), I=1, N))
112 WRITE (61, 1091)
GO TO 42
40 CALL INPUT (3)
42 DO 70 J=1, N
PSI*I=0.0
DO 60 J=1, M
60 CONTINUE
COSB*COS(BET(I4))
COSPZ*COS(P S1*ZET(I4))
SINPZ>SIN(P S1*ZET(I4))
COSP-SIN(P S1)
SINPZ=SIN(P S1)

RELATIVE COORDINATE OF BLADE STATION,

IF (EPF.0,0,ETP,15.58)
54 T=(RN(15)*EPF+COSB*EPF=EPL
X(D(I4)=EPL*COSP*T=COSPZ*FN(14,15)*SINB*COSPZ
C-FT(I4,15)=SINPZ
Y(D(I4)=EPF+T=SINPZ*FT(I4,15)*COSPZ*FN(14,15)*SINB*SINPZ
Z(D(I4)=(RN(15)*EPF)*SINB*FN(14,15)*COSP
GO TO 60
58 X(D(I4)=(EPF*EPL*COSB)*COSP(RN(15)*EPL)*COSB*EPF*FT(I4,15):
SINPZ*FN(14,15)*SINB*COSPZ
Y(D(I4)=(EPF*EPL*COSB)*SINPZ*EPF*FN(14,15)*SINB*FN
Z(D(I4)=(RN(15)*EPF)*SINB*FN(14,15)*COSP
60 PS1*PS1*EPS
WRITE (2,1003) (X(D(I4),14*1,M)
WRITE (2,1003) (Y(D(I4),14*1,M)
WRITE (2,1003) (Z(D(I4),14*1,M)
DO 93 15=1,N
DO 93 14=1,M
AERODYNAMICS FORCE COMPONENT,

IF (N,E0.1) 91,83
83 IF (15,E0.1) 84,86
84 DNDR=(FN(I4,2)=FN(14,1))/RN(2)=RN(1))
DTOR=FT(14,2)-FT(14,1)/RN(2)=RN(1))
GO TO 92
88 IF (15,E0.1) 89,90
89 DNDR=(FN(I4,15)*FN(I4,15-1))/RN(15)*RN(15-1))
DTOR=FT(14,15)-FT(14,15-1)/RN(15)*RN(15-1))
GO TO 92
90 DNDR=(FN(I4,15)*FN(I4,15-1))/RN(15)*RN(15-1))
DTOR=FT(14,15-1)-FT(14,15)/RN(15)*RN(15-1))
GO TO 92
91 DNDR=DTOR=0,
92 BETP(I4,15)=BET(4)*ATAN(DNDR)
93 ZETP(I4,15)=ZET(4)*ATAN(DTOR)
GO TO (94,96)
94 DO 95 J=1,N
95 REAC (60,1028) (FN(I,J),1=1,M)
WRITE(61,42)
WRITE(61,43)
WRITE(61,44)(RN(I),1=1,N)
DO112 J=1,N
WRITE(61,1091)
912 WRITE (61,65) (DNDB(J),(FN(J),1=1,N))
WRITE(61,66)
WRITE(61,63)
WRITE(61,64)(RN(I),1=1,N)
DO111 J=1,N

39
```
1111 WRITE(61,1001)
1112 WRITE(61,65)((GMD(J), (JT(J), l=1,n)) )
1113 GO TO 97
96 CALL INPUT (2)
97 DO 160 15=1,n
PSI=0,
DO 100 14=1,m
ZE=ZETP(14,15)
XQ(14)*FT(14,15)*SIN(PST*ZE )+FN(14,15)*SIN(BE )+COS(PST*ZE )
YQ(14)*FT(14,15)*COS(PST*ZE )+FN(14,15)*SIN(BE )+SIN(PST*ZE )
ZQ(14)=FN(14,15)*COS(BE )
100 PSI=PSI+DPS
100 WRITE(2,1003)/(XQ(14),14=1,m)
100 WRITE(2,1003)/(YQ(14),14=1,m)
100 WRITE(2,1003)/(ZQ(14),14=1,m)
150 CONTINUE
END FILE 2
REWNDC 2
G
G = CALCULATE HARMONICS
G
17=0
DO 190 12=1,2
GO TO (165,166) 12
166 KSP=XX
GO TO 167
167 IF (KSP,LE,0) 165,169
168 KSP=1
169 DO 190 13=1,n
DO 190 14=1,3
17=17+1
REAC (2,1003) (REAT)(15,15=1,m)
SUM1=0,
DO 172 19=1,m
172 SUM1=SUM1+REAT(19)
AD(17)=SUM1/XM
DO 190 16=1,KSP
XX=PSI+FLOAT(16)
SUM2=SUM3*X=0,
DO 180 18=1,m
SUM2=SUM2+REAT(18)*COS(X*XX)
SUM3=SUM3+REAT(18)*SIN(X*XX)
180 XX=1,
AK(16,17)=SUM2+ZZ
BK(16,17)=SUM3+ZZ
190 CONTINUE
G
****** .......... BEGIN PART 2 *. * CALC. OF SOUND FIELD ************
G
13=1FIX(88)
WRITE(61,78)
DO 700 11=1,1CON
7=0
REAC (60,1016)KOP,YOP,ZOP
SBW=XXOP*XHO
SYBH= YOP*YM 0
SBW=ZOP*ZH0
IF(SBWH,EG,SYBH,AND,SYWH,EG,SBWH,AND,SBWH,EG,0,1073,1072)
8073 SN=0.
```
1072 GO TO 1075
1075 S = SQRT(SXH*SBH + SYH*SBH + SZH*SZBH)
1076 Q = SQRT(SXH*SBH + YDH*SZBH + SZH*SZBH)
1077 CONTINUE
1078 TAUP = SQRT(OS)/QT
1080 CONTINUE
1083 XP = X(283153) - TAUP
1084 YP = Y(283153) - TAUP
1085 ZP = Z(283153) - TAUP
1086 X0 = X0 + XP
1087 Y0 = Y0 + YP
1088 Z0 = Z0 + ZP
1089 P(I) = O
1090 DO 350 I2 = 1, I3
1091 TPEU = PSID*283153 - 1000000
1092 TAU1 = TAU0
1093 TAUS = TAUP
1094 I = 0
1095 DO 360 I4 = 1, N
1096 READ RETARDED TIME
1097 CONTINUE
210 TAU1 = TAU0
211 TPA = TAU0
212 TAUS = SF (TP, KK, I4, PNEU, CM1) / C
213 I = 1
220 IF (I, GT, 50) 260, 220
221 IF (ER, GT, 0.01) 205, 226
225 KK = 1
226 IF (ER, GT, 0.001) 250, 220
227 MET = 1
228 IF (ER, GT, 0.0001) 230, 232
230 KK = 3
231 IF (ER, GT, 0.00001) 230, 232
236 KK = 4
238 IF (ER, GT, 0.000001) 236, 242
242 IF (KK, EQ, XX) 250, 254
246 CONTINUE
BEGIN CALCULATION OF VELOCITIES AND ACCELERATIONS.

PSI=PM+OM*TP
X=AM*HC+DK*TP
Y=AM*HC+YM*TP
Z=2H0+2DP*TP
TH=TH+TND*TP
PH=PH+P*PHI*TP
COST=CO(F1M)
SINT=SIN(T11)
TANP=TAN(P11)
PHI=TAN(COST*TANP)
SINPHI=SIN(PHI)
COSPHI=COS(PHI)

FORMT(X, 6PSI*X8*, 12E10, 5)
DO 265 I5=1,3
I=I+1
WP(I5)=AD(11)
XK=I
WD(P(5)=WD(DP(15)=0
DO 263 K=1,XX
ARG=XX*PSI
COST=CO(ARG)
SINP=SIN(ARG)
WP(I5)=WP(I5)*AK(KL, 11)*COSK+R(KL, 11)*COSK
WD(DP(15)=WD(DP(15)=XX*OM*(AK(KL, 11)*SINK+R(KL, 11)*SINK)
263 CONTINUE

TH02=THD+TND
P2=PH1DP+PH1DP
X=WP(I5)*COST+WP(2)*SINPHI*WP(3)*COSPHI*SINT
Y=WP(2)*COSPHI+WP(3)*SINPHI
Z=WP(3)*SINT+WP(2)*SINPHI+WP(3)*COST+WP(3)*COSPHI+WP(3)

X=UP(I5)*COST+UP(1)*THD+SINT*UP(3)*PHI+DP+WD(P(2))=SINPHI*UP(3)*SINT
Y=UP(3)*COST+UP(2)*THD+SINT*UP(3)*COST+UP(2)*SINT
Z=UP(2)*UP(1)*COST+UP(2)*THD+UP(2)*SINT

X=UP(2)*UP(3)*COST+UP(2)*PHI+DP+WD(P(3))=SINPHI
Y=UP(2)*UP(3)*COST+UP(2)*THD+SINT+UP(2)*COSPHI+UP(3)*COST
Z=UP(2)*UP(3)*THD+SINT+UP(2)*THD+UP(2)*SINT

XR=0*X=HC+COSCHI+Y*SINCH!
YR=0*Y=HC+SINCH+COSCH!

42
BEGIN CALCULATION OF FORCES AND DERIVATIVES,

DO 275 15=1,3
JJ=1
F(15)=AO(JJ)
FD(15)=0,
XX=1,
DO 270 16=1,KF
COSK=COSF(XX*PSI)
SINK=SINF(XX*PSI)
F(15)=F(15)*AK(16,JJ)*COSK(16,JJ)*SINK
FD(15)=FD(15)+OM*XX*(AK(16,JJ)*SINK+8K(16,JJ)*COSK)
270 XX=XX+1
275 CONTINUE

FX=F(1)+COST*F(2)+SINPHI*F(3)+COSPHI*SINT
FY=F(2)+COSPHI*F(3)+SINPHI*INT
FZ=F(3)+INT

FDX=FD(1)+COST*FD(2)+SINPHIF(3)+COSPHIF(3)+INT
1(FD(3)+F(2)+PHIDP)=FD(2)*SINPHI*INT
2(COST*FD(3)+COSPHI*INT)*THD
FDY=FD(2)+COSPHI*FD(3)+SINPHI*PHIDP
1(FD(1)+SINT*FD(2)+THD*PHIDP)=FD(2)*F(3)*PHIDP*SINPHI*INT
2(SINT)*COST
FDZ=FD(1)+COSPHI*FD(3)+SINPHI*PHIDP
1(FD(2)+PHIDP)=FD(2)*F(3)*COSPHI*PHIDP

BEGIN SOUND PRESSURE CALCULATIONS,

XT=XMC+S(1,1,1)*XMAS/SS
XM5=XT*XD/XBAR
XMYS=XT*YD/YBAR
XMZS=XT*ZD/ZBAR

43
Pj(I7)=Pj(I7)+Dp
330 continue
390 continue
10-Ti=DT
400 continue
6
HARmonic ANALYSIS OF SOUND FIELD,

F1=2.*BR/XM
F2=2.,*M/XM
F3=CM.*BR/(PI+P1)
F4=1.5.*4.3429449
XM=XM/HB
WM=XM/2.
BM2=FLOAT(MH/2)
IF (BM2,GT,BM2) 605,610
605 KMMax=(12*FIX(BB))
GO TO 615
610 KMMax=(12*FIX(HB))=1
615 XL=0,
DO 680 L1=1,KMAX
XL=XL+1,
SUM=SUM2*X=0,
DO 680 XM=1,MB
SUM1=SUM1+Pj(MM)*COS(X*F2*XL)
SUM2=SUM2+Pj(MM)*SIN(X*XL+F2)
664 FCHKMAT(2,212,(1J,X,F10,9))
660 X=1,
CPJ(L1)=F1*SUM1
SPJ(L1)=F1*SUM2
ALP=Aj(L1)=TAN(SPJ(L1)/CPJ(L1))
IF(CPJ(L1),EQ.0.)661,662
661 ALP=Aj(L1)=PI/2,
662 SPJ(L1)=ABS(SPJ(L1))
CPJ(L1)=ABS(CPJ(L1))
PJK(L1)=SQRT(CPJ(L1)+SPJ(L1)+SPJ(L1))
PJK(L1)=F4*ALOG(PJK(L1))/124.9B
FF(L1)=XL*F3 /(1.2*BMD)
680 continue
WRITE (61,1030)
WRITE (61,1032) L1,XP,YP,ZP
WRITE (61,1029) XPH,YPH,ZPH
WRITE (61,1030)
WRITE (61,1033)
WRITE (61,79)
DO 700 L1=1,KMAX
700 WRITE (61,1034) L1,FF(L1),CPJ(L1),SPJ(L1),PJK(L1)
WRITE (61,1030)
800 continue
WRITE (59,1002)
WRITE (61,1002)
WRITE (59,1002)
1 FORMAT('SHL,40X,16HPROGRAM ,HERON 1)
2 FORMAT('SHL,10A)
3 FORMAT('SHL,10A))
6 FORMAT('SHL,28X,16HBLADE LOADING AND MOTION DATA (PARALLEL AND NORMAL 14L TO THE SHAFT//)
4 FORMAT('SHL,///,28X,22H GEOMETRY OF ROTOR W ///,110 ,11X
1 SLABLADES,1,4,1.50X,E10,3.3X,3MFT,,4X,6,H,RADIUS,1/1,1W ,50X,E10,3.3X,
26XH,P,M,1/1,1W ,50X,E10,3.3X,3MFT,,4X,17HFLAP HINGE OFFSET,1/1,1W ,50X,
3510,0,3X,3MFT,3,17MDRAG HINGE OFFSET,1/1,1W,50X,E10,3.3X,1M DEGREES,27M REARWARD SHAFT 1

44
5 INCLINATION / 1 H, 50X E10, 3.2 X DEGREES, 3.1 H SHAFT INCLINATION TO STAR ANGULAR / 50X E10, 3.2 X, 4 MFT / SEC, 25H VELOCITY IN X-DIRECTION / 1 H, 50X E10, 3.2 X, 80 MFT / SEC, 25H VELOCITY IN Z-DIRECTION / 1 H, 50X E10, 3.2 X, 6 MFT / SEC, 25H R0 W RESULTANT VELOCITY / 1 H,
2 SUB ORDINATE:
9 FORAT(IH, 28X, 27H NUMBER OF LOADING STATIONS, 16 /)
REAL (60,1028) (CFN(NN,J),J=1,KSTP)
XJ=1,
74 FORMAT (1H,32X,1H4,12,7X, E10,4)  
76 FORMAT (1H,28X,3,4WBLADE FLAPPING HARMONICS (DEGREES)) WRITE(61,76)  
WRITE(61,74) (J,CBET(J),J=1,K1)  
REAL (60,1028) (SBET(KK),KK=1,K1)  
84 FORMAT (1H,32X,1HB,12,7X, E10,3) WRITE(61,84) (J,SBET(J),J=1,K1)  
XJ=2, 
X=1,  
DO 60 1X1,M  
X*X=1,  
PSI*XP*YPS!  
BET(1)=CBET(1)  
Y=0,  
DO 60 12X1,K1  
Y=Y+1,  
YK=*PSI  
60 BET(1)=BET(1)+CBET(12)*COS(YK)+SBET(12)*SIN(YK)  
GO  
COMPLETES COMPUTATION OF BETA ARRAY,  
GO  
75 FORMAT (1H,28X,31WBLADE DRAG LOAD HARMONICS (LBS))///  
85 FORMAT (1H,28X,33WBLADE THRUST LOAD HARMONICS (LBS))///  
REAL (60,1028) (CRE(T,J),J=1,K2)  
XJ=3,  
WRITE(61,77)  
WRITE(61,74) (J,CBET(J),J=1,K2)  
REAL (60,1028) (SBET(J),J=1,K2)  
XJ=4,  
WRITE(61,84) (J,SBET(J),J=1,K2)  
X=1,  
DO 70 13X1,M  
X*X=1,  
PSI*XP*YPS!  
ZET(13)=CBET(1)  
Y=0,  
DO 70 17X1,K1  
Y=Y+1,  
YK=*PSI  
70 ZET(13)=ZET(13)+CBET(KK)*COS(YK)+SBET(KK)*SIN(YK)  
79 FORMAT (1H,12X,1HB,12,7X, E10,4)  
79 FORMAT (1H,12X,1HB,12,7X, E10,3)  
72 FORMAT (1H,28X,14WHAUTAL STATION,F10,3)  
GO  
COMPLETES COMPUTATION OF ZETA ARRAY,  
GO TO 1200  
110 KSTF=KSF  
KF=1  
GO TO 125  
120 KSTF=KSF  
KF=2  
125 DO 130 1X1,N  
HEAC (60,1028) (CFN(NN,J),J=1,KSTP)  
XJ=5,  
HEAC (60,1028) (SFN(NN,J),J=1,KSTP)  
XJ=6,  
HEAC (60,1028) (CFN(NN,J),J=1,KSTP)  
XJ=7,
FUNCTION SF(TP, KK, NUM, PNEW, CM1)

COMMON X1, K2, KSF, KXY, DPSI, M, N
COMMON HET(72), ZET(72), FNM(72,12), FT(72,12), DMU(72), D(22)
COMMON PSI1, K, J, C, OM, A
COMMON XM, Ym, Zm, TM, THM, TMH
COMMON AK(12), AK(36,72), RK(36,72), F(3), FD(3)
DIMENSION W(9)

CALCULATED SOUND SOURCE TO OBSERVER DISTANCE.
0  
PS1*PKF=OM*TP
XM=XS+XUM*TP
YM=YDH+YDH*TP
ZM=ZDH+ZDH*TP
TH=THFH+THF*TP
PH1=PH10+PH10*TP
PHIF=ATAN(COSF(TH)*TANF(PH1))
DO 100 11=1,3
11=2*(NUM-1)+1
WP=40(I11)
XK=1,
DO 10 12=1,KK
900 FORAT (5X,A8,DATA,5X,7E14.6)
WP=P+AP((12,11)*COSF(XK*PS1)+A(12,11)*SIN(XK*PS1)
XK=NXK+1,
100  W11=WP
COST=COSF(TH)
SINFHP*SINF(PHIP)
SINT*SINF(TH)
COSFHP*COS(PHIP)
SINF*SFCHI1)
COSF#COSF(CH1)
XW1=1*COSF+2*SFHP*SINT*(3)*COSFHP*SINT
YW2=2*COSFHP+2*SFHP+2*SINT*(3)*COSFHP*COS
x3=3*XO=XW1*XCOS+Y*SINP
w2=3+YW1*X*SINP+Y*COS
w3=2+YW1*X*SINP+Y*COS
SF=QT(F11*W11*W2*W3*W(2)*W(3))
RTLNK
END
TABLE 1. MAJOR FORTRAN SYMBOLS USED IN PROGRAM HERON 1

(Other Symbols Which Appear are Generally Used for Local Calculations and have Self-Evident Meanings)

<table>
<thead>
<tr>
<th>PROGRAM SYMBOLS</th>
<th>ALGEBRAIC SYMBOLS</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>IH</td>
<td>H</td>
<td>Number of rotors</td>
</tr>
<tr>
<td>JCON</td>
<td>J</td>
<td>Number of observer positions</td>
</tr>
<tr>
<td>C</td>
<td>( c, a_0 )</td>
<td>Atmospheric speed of sound (ft/sec)</td>
</tr>
<tr>
<td>RHO</td>
<td>( \rho_0 )</td>
<td>Atmospheric density (slugs/ft³)</td>
</tr>
<tr>
<td>OHM</td>
<td>( \omega N )</td>
<td>Rotor speed (rpm)</td>
</tr>
<tr>
<td>R</td>
<td>R</td>
<td>Rotor radius (ft)</td>
</tr>
<tr>
<td>BB</td>
<td>B</td>
<td>Number of blades</td>
</tr>
<tr>
<td>EPF</td>
<td>( e_f )</td>
<td>Flapping hinge offset (ft)</td>
</tr>
<tr>
<td>EPL</td>
<td>( e_f )</td>
<td>Lagging hinge offset (ft)</td>
</tr>
<tr>
<td>PSIO</td>
<td>( \psi_0 )</td>
<td>Azimuth reference angle (radians)</td>
</tr>
<tr>
<td>OM</td>
<td>( \Omega )</td>
<td>Angular velocity of rotor (radians/sec)</td>
</tr>
<tr>
<td>M</td>
<td>M</td>
<td>Number of azimuth stations</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>Number of radial stations</td>
</tr>
<tr>
<td>IND</td>
<td>IND</td>
<td>Indicator: If IND = 1, loading/motion read as spanwise/azimuthwise distributions</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>If IND = 2, loading/motion read as Fourier coefficients</td>
</tr>
<tr>
<td>KSF</td>
<td>( K_f^+ )</td>
<td>Number of force harmonics read</td>
</tr>
<tr>
<td>KSX</td>
<td>( K_x^+ )</td>
<td>Number of displacement harmonics read</td>
</tr>
</tbody>
</table>

49
<table>
<thead>
<tr>
<th>PROGRAM SYMBOLS</th>
<th>ALGEBRAIC SYMBOLS</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>K_1</td>
<td>Number of flapping angle harmonics read</td>
</tr>
<tr>
<td>K2</td>
<td>K_2</td>
<td>Number of lagging angle harmonics read</td>
</tr>
<tr>
<td>KX</td>
<td>K_x</td>
<td>Number of displacement harmonics calculated</td>
</tr>
<tr>
<td>KF</td>
<td>K_f</td>
<td>Number of force harmonics calculated</td>
</tr>
<tr>
<td>DT</td>
<td>Δt</td>
<td>Time increment between sound pressure values in final time history (sec)</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>Fundamental blade passage period (sec)</td>
</tr>
<tr>
<td>DPji</td>
<td>Δψ</td>
<td>Azimuth increment (radians)</td>
</tr>
<tr>
<td>D(I)</td>
<td>x(n)</td>
<td>i\textsuperscript{th} radial station (nondimensional)</td>
</tr>
<tr>
<td>R(I)</td>
<td>r(n)</td>
<td>Radial distance of i\textsuperscript{th} station - R \cdot D(I)</td>
</tr>
<tr>
<td>XHO</td>
<td>X_{H_0}</td>
<td>Hub coordinates relative to fixed ground axes (ft) at time ( t = 0 )</td>
</tr>
<tr>
<td>YHO</td>
<td>Y_{H_0}</td>
<td>{ }</td>
</tr>
<tr>
<td>ZHO</td>
<td>Z_{H_0}</td>
<td>}</td>
</tr>
<tr>
<td>THO</td>
<td>θ₀</td>
<td>Pitch angle relative to aircraft trajectory coordinates (radians) at time ( t = 0 )</td>
</tr>
<tr>
<td>PHIO</td>
<td>φ₀</td>
<td>Roll angle relative to aircraft trajectory coordinates (radians) at time ( t = L )</td>
</tr>
<tr>
<td>XDH</td>
<td>\dot{X}_H</td>
<td>Hub velocity components in fixed coordinate directions (ft/sec)</td>
</tr>
<tr>
<td>YDH</td>
<td>\dot{Y}_H</td>
<td>}</td>
</tr>
<tr>
<td>ZDH</td>
<td>\dot{Z}_H</td>
<td>}</td>
</tr>
<tr>
<td>PROGRAM SYMBOLS</td>
<td>ALGEBRAIC SYMBOLS</td>
<td>DEFINITION</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------</td>
<td>------------</td>
</tr>
<tr>
<td>THD</td>
<td>( \dot{\theta} )</td>
<td>Pitch and roll rates (radians/sec)</td>
</tr>
<tr>
<td>PHID</td>
<td>( \dot{\phi} )</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>( V )</td>
<td>Resultant hub velocity (ft/sec)</td>
</tr>
<tr>
<td>CHI</td>
<td>( \chi )</td>
<td>Azimuth angle between ( x ) and ( X ) axes (radians)</td>
</tr>
<tr>
<td>XH</td>
<td>( X_H )</td>
<td></td>
</tr>
<tr>
<td>YH</td>
<td>( Y_H )</td>
<td>Rotor hub coordinates at retarded time ( t' ) (ft)</td>
</tr>
<tr>
<td>ZH</td>
<td>( Z_H )</td>
<td></td>
</tr>
<tr>
<td>BET</td>
<td>( \beta )</td>
<td>Flap angle (radians)</td>
</tr>
<tr>
<td>ZET</td>
<td>( \zeta )</td>
<td>Lag angle (radians)</td>
</tr>
<tr>
<td>FN</td>
<td>( F_N ) or ( n_N )</td>
<td>Normal section loading (lb) or elastic displacement (ft)</td>
</tr>
<tr>
<td>FT</td>
<td>( F_T ) or ( n_T )</td>
<td>In-plane section loading (lb) or elastic displacement (ft)</td>
</tr>
<tr>
<td>XD</td>
<td>( x' )</td>
<td>Coordinates in shaft axis system (ft). [Also used for blade section force components in same system (lb)]</td>
</tr>
<tr>
<td>YD</td>
<td>( y' )</td>
<td></td>
</tr>
<tr>
<td>ZD</td>
<td>( z' )</td>
<td></td>
</tr>
<tr>
<td>DNDTR</td>
<td>( dn_N/dr )</td>
<td>Local blade slope with respect to plane of rotation (radians)</td>
</tr>
<tr>
<td>DTDTR</td>
<td>( dn_T/dr )</td>
<td>Local blade slope with respect to nominal blade azimuth (radians)</td>
</tr>
<tr>
<td>AO</td>
<td>( a_0 )</td>
<td></td>
</tr>
<tr>
<td>AK</td>
<td>( a_k )</td>
<td></td>
</tr>
<tr>
<td>BK</td>
<td>( b_k )</td>
<td>Fourier coefficients of blade section force and displacement with respect to rotor axes (ft)</td>
</tr>
<tr>
<td>PROGRAM SYMBOLS</td>
<td>ALGEBRAIC SYMBOLS</td>
<td>DEFINITION</td>
</tr>
<tr>
<td>----------------</td>
<td>-------------------</td>
<td>------------</td>
</tr>
<tr>
<td>XOP</td>
<td>X'_0 (j)</td>
<td>Nominal observer position in fixed coordinate system (ft)</td>
</tr>
<tr>
<td>YOP</td>
<td>Y'_0 (j)</td>
<td></td>
</tr>
<tr>
<td>ZOP</td>
<td>Z'_0 (j)</td>
<td></td>
</tr>
<tr>
<td>SXBH</td>
<td>\bar{x}_H</td>
<td>Position of rotor hub relative to observer at time ( t ) in fixed coordinate system (ft)</td>
</tr>
<tr>
<td>SYBH</td>
<td>\bar{y}_H</td>
<td></td>
</tr>
<tr>
<td>SZBH</td>
<td>\bar{z}_H</td>
<td></td>
</tr>
<tr>
<td>SH</td>
<td>S_H</td>
<td>Distance between hub and observer at time ( t ) (ft)</td>
</tr>
<tr>
<td>TAUOP</td>
<td>\tau'_0</td>
<td>Time of propagation of sound from the rotor hub which reaches observer at time ( t ) (sec)</td>
</tr>
<tr>
<td>XPH</td>
<td>X'_H</td>
<td>Hub coordinates in fixed axis system at retarded time ( t' ) (ft)</td>
</tr>
<tr>
<td>YPH</td>
<td>Y'_H</td>
<td></td>
</tr>
<tr>
<td>ZPH</td>
<td>Z'_H</td>
<td></td>
</tr>
<tr>
<td>SXBPH</td>
<td>\bar{x}'_H</td>
<td>Position of hub relative to observer at retarded time ( t' ) (ft)</td>
</tr>
<tr>
<td>SYBPH</td>
<td>\bar{y}'_H</td>
<td></td>
</tr>
<tr>
<td>SZBPH</td>
<td>\bar{z}'_H</td>
<td></td>
</tr>
<tr>
<td>BMO</td>
<td>M_0</td>
<td>Hub Mach number component in direction of observer</td>
</tr>
<tr>
<td>T</td>
<td>t</td>
<td>&quot;Observer&quot; time ( t )</td>
</tr>
<tr>
<td>XO</td>
<td>X_0</td>
<td>Coordinates of &quot;moving observer&quot; at time ( t )</td>
</tr>
<tr>
<td>YO</td>
<td>Y_0</td>
<td></td>
</tr>
<tr>
<td>ZO</td>
<td>Z_0</td>
<td></td>
</tr>
<tr>
<td>PROGRAM SYMBOLS</td>
<td>ALGEBRAIC SYMBOLS</td>
<td>DEFINITION</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------</td>
<td>------------</td>
</tr>
<tr>
<td>PNEW</td>
<td>$\Phi_0$</td>
<td>Reference azimuth angle for particular blade</td>
</tr>
<tr>
<td>TAU</td>
<td>$\tau_1$</td>
<td>Sound propagation time</td>
</tr>
<tr>
<td>TP</td>
<td>$t'$</td>
<td>Retarded time $t' = t - \tau$</td>
</tr>
<tr>
<td>PSI</td>
<td>$\Phi$</td>
<td>Blade azimuth angle (nominal) at retarded time $t'$</td>
</tr>
<tr>
<td>XH</td>
<td>$X_H$</td>
<td>Hub coordinates (in fixed axes) at retarded time $t'$</td>
</tr>
<tr>
<td>YH</td>
<td>$Y_H$</td>
<td>}</td>
</tr>
<tr>
<td>ZH</td>
<td>$Z_H$</td>
<td>}</td>
</tr>
<tr>
<td>TH</td>
<td>$\Theta$</td>
<td>Pitch angle at retarded time (radians)</td>
</tr>
<tr>
<td>PHI</td>
<td>$\phi$</td>
<td>Roll angle at retarded time (radians)</td>
</tr>
<tr>
<td>PHIP</td>
<td>$\phi'$</td>
<td>Roll angle at retarded time (relative to vehicle) (radians)</td>
</tr>
<tr>
<td>PHIDP</td>
<td>$\dot{\phi}'$</td>
<td>Roll rate at retarded time (relative to vehicle) (radians/sec)</td>
</tr>
<tr>
<td>WP(I)</td>
<td>$w'$</td>
<td>Coordinates of blade station (rotor axes) (ft) ($w = x, y$ or $z$)</td>
</tr>
<tr>
<td>WDP</td>
<td>$\dot{w}'$</td>
<td>Blade element velocity components (ft/sec)</td>
</tr>
<tr>
<td>WDDP</td>
<td>$\ddot{w}'$</td>
<td>Blade element acceleration components (ft/sec$^2$)</td>
</tr>
<tr>
<td>X</td>
<td>$x$</td>
<td>{ Aircraft flight path coordinates ($x$ is flight azimuth direction) (ft)</td>
</tr>
<tr>
<td>Y</td>
<td>$y$</td>
<td>}</td>
</tr>
<tr>
<td>Z</td>
<td>$z$</td>
<td>}</td>
</tr>
<tr>
<td>XD</td>
<td>$\dot{x}$</td>
<td>Aircraft flight path velocity components (ft/sec)</td>
</tr>
<tr>
<td>YD</td>
<td>$\dot{y}$</td>
<td>}</td>
</tr>
<tr>
<td>PROGRAM SYMBOLS</td>
<td>ALGEBRAIC SYMBOLS</td>
<td>DEFINITION</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------</td>
<td>------------</td>
</tr>
<tr>
<td>ZD</td>
<td>$\dot{z}$</td>
<td>Aircraft flight path velocity component (ft/sec)</td>
</tr>
<tr>
<td>XDD</td>
<td>$\ddot{x}$</td>
<td></td>
</tr>
<tr>
<td>YDD</td>
<td>$\ddot{y}$</td>
<td>Aircraft flight path acceleration components (ft/sec^2)</td>
</tr>
<tr>
<td>ZDD</td>
<td>$\ddot{z}$</td>
<td></td>
</tr>
<tr>
<td>XBAR</td>
<td>$\bar{x}$</td>
<td>Coordinates of blade element relative to observer at retarded time (ft)</td>
</tr>
<tr>
<td>YBAR</td>
<td>$\bar{y}$</td>
<td></td>
</tr>
<tr>
<td>ZBAR</td>
<td>$\bar{z}$</td>
<td></td>
</tr>
<tr>
<td>CXD</td>
<td>$\dot{X}$</td>
<td></td>
</tr>
<tr>
<td>CYD</td>
<td>$\dot{Y}$</td>
<td>Velocity components of blade element at retarded time (fixed axes)(ft/sec)</td>
</tr>
<tr>
<td>CZD</td>
<td>$\dot{Z}$</td>
<td></td>
</tr>
<tr>
<td>CXDD</td>
<td>$\ddot{X}$</td>
<td></td>
</tr>
<tr>
<td>CYDD</td>
<td>$\ddot{Y}$</td>
<td>Acceleration components of blade element at retarded time (ft/sec^2)</td>
</tr>
<tr>
<td>CZDD</td>
<td>$\ddot{Z}$</td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>$S$</td>
<td>Distance traveled by sound (ft)</td>
</tr>
<tr>
<td>XMAS</td>
<td>$M$</td>
<td>Square of absolute Mach number of blade element</td>
</tr>
<tr>
<td>XMS</td>
<td>$M_s$</td>
<td>Mach number of element toward observer</td>
</tr>
<tr>
<td>XMDS</td>
<td>$\dot{M}_s$</td>
<td>Rate of change of Mach number toward observer</td>
</tr>
<tr>
<td>F(i)</td>
<td>$F_i$</td>
<td>Aerodynamic force component relative to rotor axes (lb)</td>
</tr>
<tr>
<td>FD(i)</td>
<td>$\dot{F}_i$</td>
<td>Rate of change of aerodynamic force component relative to rotor axes (lb/sec)</td>
</tr>
<tr>
<td>PROGRAM SYMBOLS</td>
<td>ALGEBRAIC SYMBOLS</td>
<td>DEFINITION</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------</td>
<td>------------</td>
</tr>
<tr>
<td>FX</td>
<td>( F_x )</td>
<td>Aerodynamic force components relative to aircraft flight path axes (lb)</td>
</tr>
<tr>
<td>FY</td>
<td>( F_y )</td>
<td></td>
</tr>
<tr>
<td>FZ</td>
<td>( F_z )</td>
<td></td>
</tr>
<tr>
<td>FDX</td>
<td>( \dot{F}_x )</td>
<td>Rate of change force components relative to aircraft flight path axes (lb/sec)</td>
</tr>
<tr>
<td>FDY</td>
<td>( \dot{F}_y )</td>
<td></td>
</tr>
<tr>
<td>FDZ</td>
<td>( \dot{F}_z )</td>
<td></td>
</tr>
<tr>
<td>FX</td>
<td>( F_X )</td>
<td></td>
</tr>
<tr>
<td>FY</td>
<td>( F_Y )</td>
<td></td>
</tr>
<tr>
<td>FZ</td>
<td>( F_Z )</td>
<td>Forces and rate of change of forces relative to fixed axes (lb, lb/sec)</td>
</tr>
<tr>
<td>FDX</td>
<td>( \dot{F}_X )</td>
<td></td>
</tr>
<tr>
<td>FDY</td>
<td>( \dot{F}_Y )</td>
<td></td>
</tr>
<tr>
<td>FDZ</td>
<td>( \dot{F}_Z )</td>
<td></td>
</tr>
<tr>
<td>DP</td>
<td>( \Delta P )</td>
<td>Acoustic pressure increment due to forces and motions of blade element (lb/ft²)</td>
</tr>
<tr>
<td>PJ(I)</td>
<td>( p_j(t) )</td>
<td>Total acoustic pressure at time ( t ) (lb/ft²)</td>
</tr>
<tr>
<td>CPJ(I)</td>
<td>( C_{pjk} )</td>
<td>In-phase harmonic pressure component (lb/ft²)</td>
</tr>
<tr>
<td>SPJ(I)</td>
<td>( S_{pjk} )</td>
<td>Out-of-phase harmonic pressure component (lb/ft²)</td>
</tr>
<tr>
<td>PJ( k )</td>
<td>( p_{jk} )</td>
<td>Harmonic pressure amplitude (lb/ft²)</td>
</tr>
<tr>
<td>( f_k )</td>
<td></td>
<td>Frequency (Hz)</td>
</tr>
</tbody>
</table>

55
5.0 PROGRAM INPUT/OUTPUT

Table II describes the preparation of the input data cards for HERON 1. This is followed by an example comprising a complete set of data written on coding forms. In conclusion, the computer output obtained for the example is presented.
<table>
<thead>
<tr>
<th>CARD NO.*</th>
<th>DESCRIPTION</th>
<th>SYMBOL</th>
<th>UNITS</th>
<th>FORMAT</th>
<th>CARD COL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>i, ii, iii</td>
<td>TITLE CARDS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>No. of rotors</td>
<td>H</td>
<td>-</td>
<td>I4</td>
<td>1-4</td>
</tr>
<tr>
<td>1</td>
<td>No. of field points</td>
<td>J</td>
<td>-</td>
<td>I4</td>
<td>4-8</td>
</tr>
<tr>
<td>1</td>
<td>Speed of sound</td>
<td>C</td>
<td>ft/sec</td>
<td>F10.0</td>
<td>11-20</td>
</tr>
<tr>
<td>1</td>
<td>Atmospheric density</td>
<td>Ψ0</td>
<td>slugs/ft³</td>
<td>F10.0</td>
<td>21-30</td>
</tr>
<tr>
<td>2</td>
<td>Rotor rpm****</td>
<td>ΩN</td>
<td>rpm</td>
<td>F10.0</td>
<td>1-10</td>
</tr>
<tr>
<td>2</td>
<td>Rotor radians</td>
<td>R</td>
<td>ft</td>
<td>F10.0</td>
<td>11-20</td>
</tr>
<tr>
<td>2</td>
<td>No. of blades</td>
<td>B</td>
<td>-</td>
<td>F10.0</td>
<td>21-30</td>
</tr>
<tr>
<td>2</td>
<td>Flapping hinge offset</td>
<td>eF</td>
<td>ft</td>
<td>F10.0</td>
<td>31-40</td>
</tr>
<tr>
<td>2</td>
<td>Lagging hinge offset</td>
<td>eLθ</td>
<td>ft</td>
<td>F10.0</td>
<td>41-50</td>
</tr>
<tr>
<td>2</td>
<td>Reference azimuth</td>
<td>Ψθ</td>
<td>rad</td>
<td>F10.0</td>
<td>51-60</td>
</tr>
<tr>
<td>3</td>
<td>No. of azimuth stations****</td>
<td>M</td>
<td>-</td>
<td>I4</td>
<td>1-4</td>
</tr>
<tr>
<td>3</td>
<td>No. of radial stations****</td>
<td>N</td>
<td>-</td>
<td>I4</td>
<td>5-8</td>
</tr>
<tr>
<td>3</td>
<td>Input format indicator***</td>
<td>IND</td>
<td>-</td>
<td>I4</td>
<td>9-12</td>
</tr>
<tr>
<td>3</td>
<td>No. of loading harmonics</td>
<td>Kf</td>
<td>-</td>
<td>I4</td>
<td>13-16</td>
</tr>
<tr>
<td>3</td>
<td>No. of displacement</td>
<td>Kx</td>
<td>-</td>
<td>I4</td>
<td>17-20</td>
</tr>
<tr>
<td>harmonics</td>
<td>No. of flapping harmonics</td>
<td>K₁</td>
<td>-</td>
<td>I4</td>
<td>21-24</td>
</tr>
<tr>
<td>3</td>
<td>No. of lagging harmonics</td>
<td>K₂</td>
<td>-</td>
<td>I4</td>
<td>25-28</td>
</tr>
<tr>
<td>CARD NO.*</td>
<td>DESCRIPTION</td>
<td>SYMBOL</td>
<td>UNITS</td>
<td>FORMAT</td>
<td>CARD COL.</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------------------------------</td>
<td>--------</td>
<td>-----------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>4+</td>
<td>Radial stations (r/R)</td>
<td>x(n)</td>
<td>-</td>
<td>F10.0</td>
<td>1-80</td>
</tr>
<tr>
<td>5</td>
<td>Hub coordinates ****</td>
<td>$X_H$</td>
<td></td>
<td>F10.0</td>
<td>1-10</td>
</tr>
<tr>
<td>5</td>
<td>At time t = 0</td>
<td>$Y_H$</td>
<td>ft</td>
<td>F10.0</td>
<td>11-20</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$Z_H$</td>
<td></td>
<td>F10.0</td>
<td>21-30</td>
</tr>
<tr>
<td>5</td>
<td>Shaft **** longitudinal</td>
<td>$\theta_0$</td>
<td>rad</td>
<td>F10.0</td>
<td>31-40</td>
</tr>
<tr>
<td>5</td>
<td>Inclination lateral</td>
<td>$\phi_0$</td>
<td></td>
<td>F10.0</td>
<td>41-50</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$X_H$</td>
<td></td>
<td>F10.0</td>
<td>51-60</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$Y_H$</td>
<td>ft</td>
<td>F10.0</td>
<td>61-70</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$Z_H$</td>
<td></td>
<td>F10.0</td>
<td>71-80</td>
</tr>
<tr>
<td>6</td>
<td>Pitch rate****</td>
<td>$\theta$</td>
<td>rad/sec</td>
<td>F10.0</td>
<td>1-10</td>
</tr>
<tr>
<td>6</td>
<td>Roll rate****</td>
<td>$\phi$</td>
<td></td>
<td>F10.0</td>
<td>11-20</td>
</tr>
</tbody>
</table>

**IF IND = 1 - Time History Input**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7+</td>
<td>Flapping angle at successive azimuth stations (m = 1(i)M)</td>
<td>$\beta(m)$</td>
<td>deg</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>8+</td>
<td>Lagging angle at successive azimuth stations (n = 1(i)M)</td>
<td>$\zeta(m)$</td>
<td>deg</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>CARD NO.*</td>
<td>DESCRIPTION</td>
<td>SYMBOL</td>
<td>UNITS</td>
<td>FORMAT</td>
<td>CARD COL.</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------</td>
<td>-------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>9+</td>
<td>Normal elastic displacement at successive radial stations ( n = 1(1)N )</td>
<td>( \eta_N(m,n) )</td>
<td>ft</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td></td>
<td>and successive azimuth stations ( m = 1(1)M )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td>Tangential elastic displacement at successive radial stations ( n = 1(1)N )</td>
<td>( \eta_T(m,n) )</td>
<td>ft</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td></td>
<td>and successive azimuth stations ( m = 1(1)M )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11+</td>
<td>Normal blade load at successive radial stations ( n = 1(1)N )</td>
<td>( F_N(m,n) )</td>
<td>lb</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td></td>
<td>and successive azimuth stations ( m = 1(1)M )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12+</td>
<td>Tangential blade load at successive radial stations ( n = 1(1)N )</td>
<td>( F_T(m,n) )</td>
<td>lb</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td></td>
<td>and successive azimuth stations ( m = 1(1)M )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IF IND = 2</td>
<td>Harmonic Input</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7a+</td>
<td>Flapping angle cosine harmonics for ( k = 0(1)K_1 )</td>
<td>( a(k) )</td>
<td>deg</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>7b+</td>
<td>Flapping angle sine harmonics for ( k = 1(1)K_1 )</td>
<td>( b(k) )</td>
<td>deg</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>CARD NO.*</td>
<td>DESCRIPTION</td>
<td>SYMBOL</td>
<td>UNITS</td>
<td>FORMAT</td>
<td>CARD COL.</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>--------</td>
<td>-------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>8a+</td>
<td>Lagging angle cosine harmonics for ( k = 0(1)K_1 )</td>
<td>( a(k) )</td>
<td>deg</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>8b+</td>
<td>Lagging angle sine harmonics for ( k = 1(1)K_1 )</td>
<td>( b(k) )</td>
<td>deg</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>9a+***</td>
<td>Normal elastic displacement cosine harmonics for ( k = 0(1)K_x )</td>
<td>( a(n,k) )</td>
<td>ft</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td></td>
<td>Repeated for successive radial stations ( n = 1(1)N )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9b+</td>
<td>Normal elastic displacement sine harmonics for ( k = 1(1)K_x )</td>
<td>( b(n,k) )</td>
<td>ft</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>9c+</td>
<td>Tangential elastic displacement cosine harmonics for ( k = 0(1)K_x )</td>
<td>( a(n,k) )</td>
<td>ft</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>9c+</td>
<td>Repeated for successive radial stations ( n = 1(1)N )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tangential elastic displacement sine harmonics for ( k = 1(1)K_x )</td>
<td>( b(n,k) )</td>
<td>ft</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>CARD NO *</td>
<td>DESCRIPTION</td>
<td>SYMBOL</td>
<td>UNITS</td>
<td>FORMAT</td>
<td>CARD COL.</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>--------</td>
<td>-------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>10a+***</td>
<td>Normal blade force cosine harmonics for $k = 0(1)K_F$ Repeated for successive radial stations</td>
<td>$a(n,k)$</td>
<td>lb</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>10b+</td>
<td>Normal blade force sine harmonics for $k = 1(1)K_F$</td>
<td>$b(n,k)$</td>
<td>lb</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>10c+</td>
<td>Tangential blade force cosine harmonics for $k = 0(1)K_F$ Repeated for successive radial stations</td>
<td>$a(n,k)$</td>
<td>lb</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>10d+</td>
<td>Tangential blade force sine harmonics for $k = 1(1)K_F$</td>
<td>$b(n,k)$</td>
<td>lb</td>
<td>F10.0</td>
<td>1-10 etc.</td>
</tr>
<tr>
<td>11</td>
<td>Observer coordinates for $j = 1(1)J$</td>
<td>$X_0(j)$, $Y_0(j)$, $Z_0(j)$</td>
<td>ft, ft, ft</td>
<td>F10.0, F10.0, F10.0</td>
<td>1-10, 11-20, 21-30</td>
</tr>
</tbody>
</table>

* Where a card number is followed by a + sign, additional cards are permissible if required to accommodate total data.

** When the number of harmonics ($K$) is specified as zero, there are no sine components and relevant cards are omitted.
TABLE II - Continued

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>** * * **</td>
<td>For ( n = 0 ), all necessary (a) cards are punched, followed by all (b), (c) and (d) cards, for ( n = 1 ). These are followed by complete sets for ( n = 2, 3 \ldots N ).</td>
</tr>
<tr>
<td>** * * * **</td>
<td>These parameters are determined as follows: -</td>
</tr>
<tr>
<td>( M )</td>
<td>Number of azimuth intervals. This controls the number of sound harmonics which will be calculated (( = ) integral part of ( M/2B ) if ( M/2B ) is nonintegral, or ( = (M/2B) - 1 ) if ( M/2B ) is integral). With the present storage limits, ( M ) must not exceed 72. When the loading and motion data is read in time history form, ( M ) defines the number of azimuthal stations for which values must be specified.</td>
</tr>
<tr>
<td>( N )</td>
<td>Number of radial stations must not exceed 12.</td>
</tr>
<tr>
<td>( \text{IND} )</td>
<td>Input format indicator. When ( \text{IND} = 1 ), all loading and motion data must be specified as azimuthal/radial distributions. When ( \text{IND} = 2 ), data is input as a set of harmonic coefficients for each radial station.</td>
</tr>
<tr>
<td>( X_H, Y_H, Z_H )</td>
<td>In determining hub and observer coordinates, it should be remembered that the positive direction of rotor rotation ( (\Omega_N \text{ positive}) ) is clockwise (viewed from above) that the ( Z ) axis is vertically upwards, and that the ( Y ) axis is rotated 90° anticlockwise from the ( X ) axis.</td>
</tr>
<tr>
<td>( X_0, Y_0, Z_0 )</td>
<td></td>
</tr>
<tr>
<td>( \Omega_N )</td>
<td></td>
</tr>
<tr>
<td>( \theta, \dot{\theta} )</td>
<td>The shaft longitudinal inclination and rate of pitch are positive in the nose-up sense.</td>
</tr>
<tr>
<td>( \phi, \dot{\phi} )</td>
<td>The shaft lateral inclination and rate of roll are positive in a starboard-down sense.</td>
</tr>
</tbody>
</table>
**Computer Coding Form**

**Case H1.51**

<table>
<thead>
<tr>
<th>N.</th>
<th>T. IN.</th>
<th>Level Flight At.</th>
<th>Kts.</th>
<th>Data From NASA TM X-952 Table 8a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.50</td>
<td>2.00</td>
<td>-0.157</td>
<td>-0.157</td>
</tr>
<tr>
<td>1</td>
<td>3.16</td>
<td>0.14</td>
<td>0.16</td>
<td>0.077</td>
</tr>
<tr>
<td>2</td>
<td>-0.26</td>
<td>-0.066</td>
<td>-0.18</td>
<td>-0.026</td>
</tr>
<tr>
<td>3</td>
<td>-0.87</td>
<td>-0.065</td>
<td>-0.55</td>
<td>-0.497</td>
</tr>
<tr>
<td>4</td>
<td>-0.04</td>
<td>-0.002</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NAME</td>
<td>CUSTOMER</td>
<td>JOB NO.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DATE</td>
<td>PAGE 3</td>
<td>OF 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>566.0</td>
<td>26.52</td>
<td>9.55</td>
<td>31.04</td>
<td>7.87</td>
<td>0.8715</td>
<td>1.768</td>
<td>12.00</td>
</tr>
<tr>
<td></td>
<td>-4.82</td>
<td>-15.70</td>
<td>10.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.08.5</td>
<td>10.17</td>
<td>1.350</td>
<td>-12.88</td>
<td>-15.82</td>
<td>-16.82</td>
<td>-9.958</td>
<td>14.528</td>
</tr>
<tr>
<td></td>
<td>-5.232</td>
<td>-9.250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>508.3</td>
<td>13.81</td>
<td>124.3</td>
<td>66.37</td>
<td>10.58</td>
<td>-24.82</td>
<td>17.74</td>
<td>12.70</td>
</tr>
<tr>
<td></td>
<td>-19.25</td>
<td>-3.038</td>
<td>15.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.846</td>
<td>-35!20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SAMPLE OUTPUT

The following pages give the program output for the example case presented previously. This output is fairly self-explanatory, although the following points are worthy of mention.

The results for the first two field points only (11 were requested) are included for the sake of brevity.

It should be remembered that the program calculates the sound field observed when the helicopter rotor is positioned at the nominal position $X_H, Y_H, Z_H$. This sound was actually generated by the rotor when it was in some other position (at the retarded time). This position is denoted in each case by the “retarded hub coordinates.”

Four sound harmonics are calculated since $M$ was specified as 36 so that $(M/2B) = 4.5$. The Doppler shift effect can be noted in the slightly different frequencies observed at the two positions.
PROGRAM, WEKON 1
CASE M.1.91,
*14 IN LEVEL FLIGHT AT 40 KTS.
DATA FROM NASA TN R-952 TABLE 8.

GEOMETRY OF MOTION
1 4 BLADES
    2,400E-01 FT.
    2,170E-02 RPM,
    1,000E 00 FT.
    1,000E 00 FT.
    0 DEGREES AZIMUTH PHASE
    -1,570E-02 DEGREES NEAR AND SHAFT INCLINATION
    -1,790E-02 DEGREES SHAFT INCLINATION TO STARBOARD
    6,790E 01 FT/SEC VELOCITY IN X-DIRECTION
    0 FT/SEC VELOCITY IN Y-DIRECTION
    0 FT/SEC VELOCITY IN Z-DIRECTION
    0.0000 FT
    0 FT.
    2,000E 09 FT.

BLADE LOADING AND MOTION DATA (PARALLEL AND NORMAL TO THE SHAFT)

NUMBER OF LOADING STATIONS: 7

NUMBER OF AZIMUTH INTEGRATION POINTS: 36

FORM OF INPUT: HARMONIC

BLADE FLAPPING HARMONICS (DEGREES)
\[ a_0 = 1.807e00 \]
\[ a_1 = 3.106e01 \]
\[ a_2 = 1.400e02 \]
\[ a_3 = 1.550e01 \]
\[ a_4 = 7.790e02 \]
\[ a_7 = 6.000e03 \]
\[ a_6 = 7.800e02 \]
\[ a_9 = 1.400e02 \]
\[ a_{10} = 7.000e02 \]
\[ a_1 = 4.720e01 \]
\[ a_2 = 8.790e02 \]
\[ a_3 = 8.990e02 \]
\[ a_4 = 5.500e02 \]
\[ a_5 = 4.400e02 \]
\[ a_6 = 2.400e02 \]
\[ a_7 = 6.000e03 \]
\[ a_8 = 0 \]
\[ a_9 = 4.000e02 \]
\[ a_{10} = 2.600e03 \]

BLADE LAGGING HARMONICS (DEGREES)
\[ a_0 = -4.790e00 \]

69
**BLADE ELEMENT ELASTIC DISPLACEMENTS NORMAL TO THE SHAFT**

<table>
<thead>
<tr>
<th>Radial Station</th>
<th>7,000</th>
<th>11,200</th>
<th>15,400</th>
<th>21,000</th>
<th>23,400</th>
<th>25,200</th>
<th>26,400</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**BLADE ELEMENT ELASTIC DISPLACEMENTS PARALLEL TO THE SHAFT**

<table>
<thead>
<tr>
<th>Radial Station</th>
<th>7,000</th>
<th>11,200</th>
<th>15,400</th>
<th>21,000</th>
<th>23,400</th>
<th>25,200</th>
<th>26,400</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**BLADE THRU LOAD HARMONICS (LVS)**

<table>
<thead>
<tr>
<th>Radial Station</th>
<th>7,000</th>
<th>11,200</th>
<th>15,400</th>
<th>21,000</th>
<th>23,400</th>
<th>25,200</th>
<th>26,400</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**BLADE DRAG LOAD HARMONICS (LVS)**

<table>
<thead>
<tr>
<th>Radial Station</th>
<th>7,000</th>
<th>11,200</th>
<th>15,400</th>
<th>21,000</th>
<th>23,400</th>
<th>25,200</th>
<th>26,400</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

[Table data continues]
COMPUTED SOUND FIELD

<table>
<thead>
<tr>
<th>K</th>
<th>HARMONIC NO.</th>
<th>F(K) FREQUENCY</th>
<th>CPJ(K) IN PHASE PRES. (P.S.F.)</th>
<th>SPJ(K) OUT OF PHASE PRES. (P.S.F.)</th>
<th>PJ(K) SOUND PRESSURE LEVEL (DB. RE., 0002 DYNES/SQ.CM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1,51038167E+01</td>
<td>2,42557947E+03</td>
<td>1,04885516E+02</td>
<td>8,52205780E+01</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3,02076334E+01</td>
<td>7,25207500E+03</td>
<td>2,87507291E+03</td>
<td>8,24322233E+01</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4,53145010E+01</td>
<td>1,25524132E+03</td>
<td>1,00197493E+03</td>
<td>6,86954979E+01</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>6,04152669E+01</td>
<td>1,08763223E+03</td>
<td>8,97429345E+04</td>
<td>6,75648703E+01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>K</th>
<th>HARMONIC NO.</th>
<th>F(K) FREQUENCY</th>
<th>CPJ(K) IN PHASE PRES. (P.S.F.)</th>
<th>SPJ(K) OUT OF PHASE PRES. (P.S.F.)</th>
<th>PJ(K) SOUND PRESSURE LEVEL (DB. RE., 0002 DYNES/SQ.CM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1,50170596E+01</td>
<td>1,17483472E+02</td>
<td>7,00402784E+03</td>
<td>8,73002754E+01</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3,00341192E+01</td>
<td>4,77944402E+03</td>
<td>7,01783579E+03</td>
<td>8,31586838E+01</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4,50511787E+01</td>
<td>5,58292364E+05</td>
<td>8,15007666E+04</td>
<td>6,28012229E+01</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>6,00662383E+01</td>
<td>4,91189606E+04</td>
<td>1,62179060E+05</td>
<td>6,91609781E+01</td>
</tr>
</tbody>
</table>

OBSERVER CO-ORDINATES J= 1 XO= -2,50000000E 02 YO= 2,50000000E 02 ZO= 0

RETARD MUB CO-ORDINATES XPM= -2,36923922E 01 YPM= 0 ZPM= 2,00000000E 02
This report contains a comprehensive description of a computer program developed for the numerical evaluation of the helicopter noise equations derived in USAAVLABS TR 68-60, "Studies of Helicopter Rotor Noise." It is completely self-contained in that the program details are described, starting from two basic acoustic equations and covering methods by which these equations are applied to the rotor noise problem. Program flow diagrams and a complete listing are presented together with input instructions and sample inputs and outputs. The program is written in FORTRAN IV for the CDC 3300 Computer.