A PROGRAM FOR COMPUTING TRANSMISSION OF
PLANE ELECTROMAGNETIC WAVES THROUGH
AN INHOMOGENEOUS PLASMA SLAB

Prepared by
D. C. Pridmore-Brown
Plasma Research Laboratory

July 1968

Laboratory Operations
AEROSPACE CORPORATION

Prepared for
SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

This document has been prepared for public release
and sale; its distribution is unlimited.
FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04701-68-C-0200.

This report, which documents research carried out from February 1968 through March 1968, was submitted on 8 October 1968 to Lieutenant Gregory Mayforth, SMTTA, for review and approval.

Approved

R. X. Meyer, Director
Plasma Research Laboratory

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Gregory S. Mayforth
Lieutenant, United States Air Force
Project Officer
ABSTRACT

A computer program for calculating plane wave transmission through an inhomogeneous cold plasma slab is described. The program accepts arbitrary profiles (including jumps) in number density and collision frequency. It prints out the electric and magnetic field profiles for the angles of incidence, direction of incidence, and polarizations of the wave that are called for. It also gives the reflection and transmission coefficients in each case.
I. INTRODUCTION

In estimating the attenuation and reflection of plane electromagnetic waves by a plasma layer it is often adequate to replace the layer by an equivalent one with uniform properties, for which the transmission loss is given by a simple formula (Ref. 1). On the other hand, in cases where the plasma properties vary significantly in the space of a wavelength, the uniform slab approximation is not justified. Furthermore, in studying breakdown under intense electromagnetic fields one is interested in the electric field profile in the vicinity of points where the dielectric constant goes to zero, and then it is clearly necessary to take account of the variation in the dielectric constant. Finally it is useful to have a systematic way for comparing existing programs (Refs. 2 through 4) and for checking estimates based on equivalent homogeneous slabs against exact calculations.

For these reasons it was decided to write a computer program for the inhomogeneous case that would be as flexible and easy to use as possible. The present report describes the program and gives a listing of it in the hope that it may also be of value to others.

The program applies to a cold plasma that is characterized by its electron number density and electron-neutral collision frequency profiles. It is designed to be as flexible as possible in the following respects. The user is free to input any arbitrary profiles of number density and collision frequency (including discontinuities) by listing the values of these quantities over any arbitrary set of corresponding abscissa values. He can call for the calculations to be performed for any number of frequencies and for any uniformly spaced sequence of angles of incidence, and for either or both polarizations of the incident wave. Finally he can have the wave be incident on the slab from either side. The computer output lists the collision frequency and number density profiles that were input. It then tabulates the transverse electric and magnetic field profiles, the dielectric constant profile, and the profile of the absolute value of the total electric field through the slab for
each case (angle of incidence, frequency, and polarization) that was called for. Finally it gives the reflection and transmission coefficients.

The next section shows some of the details of the formulation. The following one gives the exact format in which the data must be input and shows a sample of the output in an illustrative case. Finally the program itself is listed in the Appendix.
II. FORMULATION

The plasma slab is considered to be contained in the region 0 ≤ x ≤ d with the xy plane being the plane of incidence. For a plane wave incident on the slab at an angle θ with respect to the x-axis, Maxwell's equations take the form

\[ \frac{\partial E_z}{\partial x} = -i\omega B_y \]

\[ -i\omega \frac{\partial B_y}{\partial x} = k_0^2 (\beta^2 - K)E_z \]

\[ B_x = (\beta/c)E_z \]

for the TE mode (for which \(E_x, E_y,\) and \(B_z\) are zero) and

\[ \frac{\partial B_z}{\partial x} = \frac{i\omega}{c^2} KF_y \]

\[ \frac{i\omega}{c^2} K \frac{\partial E_y}{\partial x} = k_0^2 (\beta^2 - K)B_z \]

\[ E_x = -(\beta c/K)B_z \]

for the TM mode (for which \(B_x, B_y,\) and \(E_z\) are zero). Here \(\beta = \sin \theta\) and \(k_0 = \omega/c\), where \(\omega/2\pi\) is the frequency. The dielectric constant \(K\) for a cold plasma is given by

\[ K = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu_c)} \]
where \( \nu_c \) is the electron-neutral collision frequency and where \( \omega_p^2 \), the square of the plasma frequency, is proportional to the electron number density \( n \). If \( n \) is measured in electrons/cm\(^3\) then

\[
\omega_p^2 = 3.186 \times 10^9 n
\]

For the TE mode the computer gives the profiles of \( E_z, B_y, \) and \( |E_z| \). For the TM mode it gives the profiles of \( E_y, B_z, \) and \( \sqrt{|E_x|^2 + |E_y|^2} \). The electric field components \( E_z \) or \( E_y \) are always normalized to unity on the incident side. The K profile is given in both cases.

Outside the plasma slab the waves are taken to be of the form

\[
\begin{align*}
(1-R) E &= \exp(ik_0 \cos \theta x) - R \exp(-ik_0 \cos \theta x) \quad \text{for } x \leq 0 \\
(1-R) E &= T \exp(ik_0 \cos \theta x) \quad \text{for } x \geq d
\end{align*}
\]

where \( E \) is any component of the electric field. Solving for \( R \) and \( T \) we find

\[
R = \frac{E_1' - ik_0 \cos \theta E_1}{E_1' + ik_0 \cos \theta E_1}
\]

\[
T = (1-R)(E_z'/E_1) \exp(-ik_0 \cos \theta d)
\]

in terms of \( E_1 \) and \( E_z \), which are the values of \( E \) at \( x = 0 \) and \( x = d \), respectively. (The prime denotes the \( x \) derivative.) The computer furnishes these values together with their decibel equivalents (defined as

\[
20 \log_{10} |R| \quad \text{and} \quad 20 \log_{10} |T| , \quad \text{respectively}.
\]

-4-
The data are input on regular 80-column Fortran cards. At least 7 cards are required for each case. A particular case consists of a plasma slab defined by its thickness and its profiles of number density and collision frequency together with a set of plane waves having 1 to 10 different frequencies and any number of angles of incidence. Any number of cases can be run by stacking the corresponding groups of cards. The cards required to represent one case are as follows:

CARD 1 contains 6 two-digit numbers placed in first 12 columns as follows: (A one-digit number is preceded by 0, e.g., 3 becomes 03.)

Columns 1,2: Number of stations at which number density is given. (See CARD 2.) Maximum allowed is 30.

Columns 3,4: Number of stations at which collision frequency is given. (See CARD 3.) Maximum allowed is 30.

Columns 5,6: Number of frequencies. Maximum allowed is 10.

Columns 7,8: Number of angles of incidence. (See CARD 6.)

Columns 9,10: Mode: 01 for TE, 02 for TM and 00 (or blank) for both. (Only one mode is computed at normal incidence.)

Columns 11,12: Direction of x-component of incident wave: 00 (or blank) if to the right (i.e., the direction in which the data on CARDS 2 - 5 is entered), 01 if to the left.

CARD 2 lists stations (values of x/d) at which number densities are to be given. These values must be listed in increasing order starting with 0 (representing the front of the slab) and ending with 1 (representing the back). Each one
(including the last) requires a decimal point and occupies 5 columns, so the total number of columns is 5 times the first number entered on CARD 1. If more than 80 columns are required, two cards must be used, and the second then counts as the continuation of CARD 2. If at any station two values of the number density are to be given (corresponding to a discontinuity in the profile) then the corresponding station must appear twice. Thus a homogeneous slab requires 4 stations viz., 0., 0., 1., 1. 

CARD 3 lists stations at which collision frequencies are to be given. Remarks under CARD 2 apply.

CARD 4 lists number densities in multiples of $10^9$/cm$^3$ corresponding to the stations listed on CARD 2. The first and last values correspond to the media carrying the incident and transmitted waves respectively, and will normally be 0 (for vacuum). Each value occupies 5 columns, so the total number of columns used is the same as on CARD 2. The smallest possible number density is $10^5$/cm$^3$ (entered as .0001) and the largest is $10^{14}$/cm$^3$ (entered as 99999). Values between $10^{13}$/cm$^3$ and $10^{14}$/cm$^3$ are entered as 5 digit numbers without a decimal point.

CARD 5 lists collision frequencies in multiples of $10^9$ sec$^{-1}$ corresponding to the stations listed on CARD 3. Remarks under CARD 4 apply.

CARD 6 contains 3 numbers, each occupying 5 columns, including the decimal point.

Columns 1-5: Slab thickness in centimeters.

Columns 6-10: First angle of incidence to be run; 0 (or blank) corresponds to normal incidence. Thereafter this value is incremented by the number appearing in Columns 11-15 of this card until a total number of angles of incidence has been run equal to the number appearing in Columns 7, 8 of CARD 1.
Columns 11-15: Interval in degrees between successive angles of incidence.

CARD 7 lists wave frequencies in GHz. Each value occupies 5 columns, including the decimal point. The total number of frequencies listed must not be less than the number appearing in Columns 5 and 6 of CARD 1.

As a particular (rather artificial) example we consider a plasma slab 1.50 cm thick and having an electron number density which starts at $10^{12}/\text{cm}^3$ at $x/d = 0$, falls linearly to $10^{11}/\text{cm}^3$ at $x/d = .63$, at which point it jumps to $10^{11}/\text{cm}^3$, and then falls off linearly to zero at $x/d = 1$. We take the collision frequency to follow a triangular distribution, 0 at $x/d = 0$, 1 and $10^6\ \text{sec}^{-1}$ at $x/d = .5$. We call for one angle of incidence 37.5 deg, and one frequency, 3 GHz. We ask for the wave to be incident from the right and polarized in the TM mode. The input data as it would be entered on the seven cards is shown in Fig. 1. The corresponding output is shown in Fig. 2. The results (in this case the real and imaginary parts of $E_y$, $B_z$, and $K$, and the value of $\sqrt{|E_x|^2 + |E_y|^2}$) are printed out at intervals of $x/d = .1$ except at jumps where they are printed out twice, to the left and to the right of the jump. The electric field is always normalized to unity on the incident side. The fact that in the printout it is unity at $x/d = 1$ serves as a reminder that the wave was required to be incident on this side of the slab. On the transmitted side we have a single plane wave in free space for which $E_y/cB_z = -\cos(37.5^\circ)$. 

-7-
Figure 1. An Example of Input Data

<table>
<thead>
<tr>
<th>CARD</th>
<th>COLUMN 1</th>
<th>COLUMN 2</th>
<th>COLUMN 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.6</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>3.7</td>
<td>5.3</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>3.2</td>
<td>5.4</td>
<td>3</td>
</tr>
</tbody>
</table>
TRANSMISSION THROUGH PLASMA SLAB

NUMBER DENSITY AND COLLISION FREQUENCY PROFILES (SLAB THICKNESS D = 1.500 CM)

<table>
<thead>
<tr>
<th>x/d</th>
<th>n</th>
<th>x/d</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.03</td>
<td>1.00E+12</td>
<td>0.00</td>
<td>1.98E+06</td>
</tr>
<tr>
<td>0.03</td>
<td>1.00E+10</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

TRANSVERSE MAGNETIC MODE (FREQUENCY = 3.00E+09 Hz, ANGLE OF INCIDENCE = 37.0 DEG)

<table>
<thead>
<tr>
<th>x/d</th>
<th>e</th>
<th>b</th>
<th>k</th>
<th>abs(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>3.0656E-01</td>
<td>1.9876E-01</td>
<td>-1.6241E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.10</td>
<td>3.8056E-01</td>
<td>1.8076E-01</td>
<td>-1.5241E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.20</td>
<td>4.5824E-01</td>
<td>1.4314E-01</td>
<td>-1.2031E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.30</td>
<td>5.7785E-01</td>
<td>1.2124E-01</td>
<td>-1.6426E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.40</td>
<td>7.0885E-01</td>
<td>1.0099E-01</td>
<td>-3.3131E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.50</td>
<td>8.5085E-01</td>
<td>1.8188E-01</td>
<td>-6.2031E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.60</td>
<td>1.0045E-01</td>
<td>1.6399E-01</td>
<td>-1.0531E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.70</td>
<td>1.2658E-01</td>
<td>1.3485E-01</td>
<td>+1.3531E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.80</td>
<td>1.5378E-01</td>
<td>1.1639E-01</td>
<td>-6.4531E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>0.90</td>
<td>1.8258E-01</td>
<td>1.0026E-01</td>
<td>-8.9531E-09</td>
<td>+7.9338E-10</td>
</tr>
<tr>
<td>1.00</td>
<td>2.000E+00</td>
<td>1.000E+00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

REFLEXION COEFFICIENT = (-0.0956; -0.0052) = -1.0 DB
TRANSMISSION COEFFICIENT = (0.4936; -0.3147) = -4.7 DB

Figure 2. An Example of Output Data
REFERENCES


APPENDIX

THE PROGRAM
PROGRAM PLASMA(INPUT=OUTPUT=TAPE1=INPUT=TAPE2=OUTPUT)
COMMON BETA,C(30),EPS,E(I),E(I)=1.0,IMAX,JX,JMAX,K,F,K,M,II(130),X(30)
* CNXII(30),NCX(30),NCX,NDP,CF1(30),F1(30),F2(30),F3(30)
DIMENSION E(40),MHD(2),S(3+40),TEMPS(90),X(40)
COMPLEX EPS,PHASE,R,E,T
REAL X,W
DATA DP,C1,C2,TP1/.0111,.01E+0,-0.944E+0/5.071E+0/.28319/
DATA MODE,G(5)4.25,E(5,5),E(4,5),/H(5),MNI,/0,0,0,0/9
EXTERNAL DERIV
*INPUT
00012 READ(I,200)AN(1),MAX,MIN,MINF,MII,INC
00031 II = IMAX + 1
00033 READ(I,201)XN(I),I=1,II
00045 JJ = JMAX + 1
00047 READ(I,201)XC(J),I=1,JJ
00061 READ(I,201)H(I),I=1,IMAX
00073 READ(I,201)C(J),J=1,JMAX
00095 READ(I,201)XN(I),I=1,II
00105 READ(I,201)DTHETA1,DTHETA
00116 READ(I,201)F(K),K=1,JMAX
00130 WRITE(I,205)XN(I),XC(I),C(I),H(I),IMAX
00152 IF(1,200)WRITE(I,200)AN(I),I=1,II
00171 IF(1,200)GO TO 300
*INTEGRATION
00017 DO 150 KA = 1,10
00174 THETA = THETA1 + DTHETA*(KA-1)
00201 DO 150 KF = 1,M
00207 IF(M,EQ.0 AND THETA+ME.GE.0) SENSE LIGHT 1
00212 IF(M.EQ.0) M = 1
00216 IF(M.EQ.0) M = M1
00217 GO TO 5
00217 4 M = 2
00220 9 R2 = THETA1,017453
00222 BETA = SIN(R2)
00225 KO = C1*F(KF)
00230 XP = 0.
008234  00 & K = 1+50
008235  01 TEMP(x) = 0,
008236  Y(1) = 0,
008237  Y(2) = 1,
008238  Y(3) = 0,
008239  Y(4) = 0,
008240  Y(5) = -K
008241  V(1) = 0X
008242  EPS = (1.0,0,,)
008243  I = 0
008244  J = 0
008245  L = 0
008246  10 I = 1
008247  IF(X(I),=E0.X(X(I+1))) GO TO 100
008248  IF(X(I+1),=E0 GO TO 80
008249  20 J = J+1
008250  IF(X(J),=E0.X(X(J+1)) GO TO 170
008251  IF(J,=L+1 GO TO 90
008252  30 L = L+1
008253  IP = 3P+DP
008254  X(L) = Y(1)
008255  GO TO(60,45,6)
008256  40 S[1,=L = CMPLX(Y(2),Y(3))
008257  S[2,=L = CMPLX(Y(4),Y(5))
008258  S[3,=L = CMPLX(Y(2),Y(3))+(O,,1,=E0) #G/[3]
008259  50 S[3,=L = EPS
008260  IF(L,=L+1) GO TO 100
008261  CALL DERIV
008262  60 CALL FAMW(K,YP,DERIV,E0,E0,E0,EL,EMAX,MIN TEMP)
008263  70 IF(Y(1),= (86(R)+29041) 60,10,10
008264  80 IF(Y(1),= (86(R)+29041) 60,20,20
008265  90 IF(Y(1),= (86(R)+29041) 60,30,30
008266  100 IF(Y(1),= (1,=E0) 60,100,100
008267  110 PHASE = CMPLX(COS(K)),(SIN(K))
008268  R = (CMPLX(YP(2R)+YP(2R+1)) + K#(B,=I,)+S(1,=L))/((CMPLX(YP(2R)+
008269  *YP(2R+1)) - K#(B,=I,)+S(1,=L))
008270  T = (1,-R)*PHASE/(S[1,L])+(86+1,)=K#(N=1)+3=M

07/11/60 PLASMA
07/11/68 PLASMA

#0547  NAH = 20*ALOG10(CABS(N))
#0554  DBT = 20*ALOG10(CABS(T))
#0562  S(I+L+1) = L/B(I+L)
#0575  DO 120 K=1,L
#0576  S(I+K) = S(I+K)*S(I+L+1)
#0610  S(2*K) = S(2*K)*S(1+L+1)*(1.0+2.*INC)
#0627  GO TO (115,116) M
#0636  113 E(K) = CABS(S(I+K))
#0644  GO TO 120
#0645  116 E(K) = SQRT(REAL(*E16)*BETA*BETA*S(2*K)/S(3*K)*CONJG(S(2*K)/S(3*K)  
 #) + S(I+K)*CONJG(S(I+K))))
#0720  120 CONTINUE
#0723  S(I+L) = (1.0+0.)
#0727  IF(INC) 135=135+135
#0750  130 DO 131 K = 1+L
#0752  131 X(K) = 1.0 - X(K)
#0737  WRITE(2,206) NODE(M),F(KF),THETA,THETA+(K*L+1-J),S(K+L+1-J),K=1,3
#0773  135 WRITE(2,206) NODE(M),F(KF),THETA,THETA+(K*L+1-J),S(K+L+1-J),J=1,3
#0827  140 WRITE(2,207) R=DBR,T,DBT
#0842  IF(SENSE LIGHT 1) 4+150
#0945  150 CONTINUE
#0952  GO TO 1

*JUMPS
#0103  160 I = I+1
#0105  IF(Y(I)) GE X(J=1=2*DX)) J = J+1
#0106  IF(XC(J=1)) LE X(J=1)) J = J+1
#0107  GO TO 100
#0109  170 J = J+1
#0107  180 DO 190 X = 1.+2
#0107  L = L+1
#0107  IF(K.LE.2) CALL DERIV
#0107  X(L) = Y(I)
#0104  GO TO (185,186) M
#0113  185 S(1+L) = CMPLX(Y(2)+Y(3))
#0114  S(2+L) = CMPLX(Y(4)+Y(5))
#0115  186 S(1+L) = CMPLX(Y(6)+Y(5))
#0115  S(2+L) = CMPLX(Y(2)+Y(3))*(0.+1.E-8)/3.*K0/3.
SUBROUTINE DERIV
COMMON BETA,C(30),EPS,Y(10),I+IMAX,INC,J,JMAX,KF,KM,N(30),X(30)
*+YK(30)+Y(5)+Y(5)
COMPLEX EPS,Y,W
REAL K0,K0XK

DATA C2,YP15.071E00,6.2519/

NX = N(I) + (N(I+1)-N(I))*Y(1-I)/X(KI+1)-X(I)
CJ = C(J) + (C(J+1)-C(J))*Y(1-J)/X(J+1)-X(J)
EPS = 1.0 - C2*XK/(X(KF)+CMPLX(YP15(KF)+CX))

V = CMPLX(Y(4)+Y(5))
W = K0*K0*(BETA*BETA - EPS)*CMPLX(Y(2)+Y(3))

GO TO 2+1) N

1 V = EPS*V
2 W = V/EPS
3 IF DIVIDE CHECK 10+2
4 YP(2) = REAL(V)
5 YP(3) = AIMAG(V)
6 YP(4) = REAL(V)
7 YP(5) = AIMAG(V)

RETURN
10 Y(1) = 1.0
RETURN
END
A computer program for calculating plane wave transmission through an inhomogeneous cold plasma slab is described. The program accepts arbitrary profiles (including jumps) in number density and collision frequency. It prints out the electric and magnetic field profiles for the angles of incidence, direction of incidence, and polarizations of the wave that are called for. It also gives the reflection and transmission coefficients in each case.
Electromagnetic wave transmission
Inhomogeneous plasma slab
Plane wave transmission

Abstract (Continued)