UNSTEADY AERODYNAMICS FOR ADVANCED CONFIGURATIONS

PART VII — VELOCITY POTENTIALS IN NON-UNIFORM TRANSONIC FLOW OVER A THIN WING

L. V. ANDREW and T. E. STENTON
North American Rockwell Corporation

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FOREWORD

This report covers a portion of the research conducted by the Los Angeles Division of North American Rockwell Corporation, Los Angeles, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF33(615)-2896.

The work was performed to advance the state-of-the-art of flutter prediction for flight vehicles as part of the Air Force Systems Command exploratory development program. The research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicles", Task No. 137003 "Prediction and Prevention of Aero- thermoelastic Problems". Messrs. James J. Olsen and Samuel J. Pollock of the Aerospace Dynamics Branch were Project Engineers.

Mr. H. Hoge was the Program Manager for North American Rockwell. Mr. L. V. Andrew and Mr. T. E. Stenton were Principal Investigators. The basic approach was outlined by Dr. M. T. Landahl of the Massachusetts Institute of Technology. The calculus of variations approach was suggested by Mr. James Olsen.

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ABSTRACT

Two methods have been outlined in detail, and one of them has been mechanized, for calculating acoustic ray paths emanating from any point in a non-uniform transonic flow field surrounding a wing. It gives the ray path, and the time, for the minimum time of travel from the acoustic source point to the field point. The resulting velocity potential is also computed.

It was necessary to establish an accurate representation of the flow characteristics in the field surrounding the wing. Some ray lines travel over the planform and into the surrounding flow field. It was established that once off the planform they do not return.

Available methods predict phase lags based on the assumption that acoustic rays travel in straight lines. The results of this study show this to be a very poor approximation at transonic speeds. Therefore, it is recommended that the method presented in this report be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict these phase lags with reasonable accuracy, and the corresponding flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in the available technology.
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SYMBOLS

c chord
C speed of sound
g time of travel of an acoustic signal
M Mach number
r Slope in the y-direction, \( \frac{dx}{dy} \)
\( \delta R \) Increment in radius vector
s Distance along a ray path, span
t Time
U Free-stream velocity
V Velocity
\( x, y, z \) Location of a field point
\( x_o, y_o, z_o \) Location of a source or doublet point
\( X, Y \) \( \frac{x}{\beta s}, \frac{y}{s} \)
\( X^*, Y^* \) Linear transformation of coordinates \( X, Y \)
\( \hat{i}', \hat{j}', \hat{k}' \) Unit vectors along \( x', y', z' \) axes
\( \delta R \) Radius vector
\( \nabla \) Vector gradient operator, \( \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \)
\( \beta \) \( \sqrt{1-M'^2}, \sqrt{r^2 + 1-M^2} \)
\( \delta \) \( \sqrt{\gamma + M^2} \), an increment
\( \phi \) Velocity potential
\( \Lambda \) Ray Angle
\( \tau \) Thickness ratio

Subscripts
a Advancing
l Local, lower
SYMBOLS (Continued)

r  Receding
u  Upper
x, y  Partial derivatives with respect to x, y
τ  Sonic line
∞  Infinity

Superscripts

*  Derivative with respect to time
'  Derivative with respect to the independent variable
INTRODUCTION

When an airfoil travels through the air at speeds near the speed of sound, the local speed of flow varies from subsonic near the forward edges to supersonic near the trailing edge. These wide variations of speed from that of the free-stream characterize the non-uniform transonic flow. This non-uniformity of the flow field must be accounted for in accurate calculations of unsteady pressures and forces; particularly their phase lags.

In order to determine an unsteady transonic flow field one requires solutions for singularities immersed in a non-uniform steady flow, (Reference 1). Source solutions for a mean flow that varied in the x-direction only were given in the high-frequency limit by Landahl (Reference 2). Rodemich (Reference 3) presented a "box" solution, based on pulsating doublets, which assumes a uniform mean flow at Mach number 1.0. No exact solutions for the case of a mean flow with arbitrary spatial variations have been found, thus far, but Landahl proposed the basic form of a solution which removes most of the limitations and restrictions of these approximate solutions. The method focuses attention on the time of transmission of an acoustic signal from a pulsating sending source to a distant receiving point. The signal travels through a nearly sonic flow field where the Mach number varies in a prescribed manner.

This report contains a difference equation approach, and differential equation approach to computing the paths and the transmission times for acoustic signals. The independent variable in the latter approach is a spatial rather than a time variable. A procedure that could be used to calculate the velocity potentials and generalized forces on an oscillating surface is described.
POTENTIAL OF A UNIT SOURCE

The basic expressions proposed by Landahl for the velocity potential at the point \((x,y,z)\) due to a pulsating source at \((x_o,y_o,z_o)\) are:

(a) for a source in a locally subsonic flow region

\[
\phi = -\frac{1}{4\pi R} \exp \left[ i\omega (t-g(x,y,z,x_o,y_o,z_o)) \right]
\]

where

\[
R = \sqrt{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2}
\]

\[
M = \text{Local Mach Number}
\]

\(x_o,y_o,z_o\) = Location of source point

\(g(x,y,z,x_o,y_o,z_o)\) = Time required for a disturbance to travel from \((x_o,y_o,z_o)\) to \((x,y,z)\).

(b) for a source in a locally supersonic flow region

\[
\phi = -\frac{1}{4\pi R} \left[ \exp[ i\omega (t-g_a)] + \exp[ i\omega (t-g_r)] \right]
\]

where

\[
g_a, r = g_a, r(x,y,z,x_o,y_o,z_o) = \text{Time required for the advancing, receding wave to travel from } (x_o,y_o,z_o) \text{ to } (x,y,z)
\]

It is likely that good accuracy may be obtained with use of the value of \(g_a\) for uniform flow (in the supersonic case, and also for the advancing wave portion in the subsonic case). However, our purpose is to produce a general solution for \(g\) which applies to both the advancing and the receding portions of the wave and compare values with those for uniform flow.

Since the primary interest is in wing flows, we consider that both the source and receiver points lie in the \(x, y\)-plane, so that \(z = z_o = 0\). Furthermore, we consider that signals do not return to the plane once they leave. The problem is thus simplified to one in two spatial dimensions. Its solution should be applicable to a wide variety of nearly planar lifting surfaces.

Consider a signal emanating from a source at the point \((x_o,y_o)\) on a wing. A second point past which the signal travels is located an incremental distance \((dx, dy)\) away. There are two components of velocity of the signal, a radial component, \(C\), where \(C\) is the local speed of sound and an \(x\)-component, \(U\), where \(U\) is the local speed of flow over the wing. \(\lambda\) is the angle the radial component makes with the negative extension of the \(x\)-axis. The path of this wavefront point will be referred to as a "ray". The shape of any ray depends on the initial choice of \(\lambda\); for a given \(\lambda\), \(dx\) and \(dy\) are components of the first element of this particular ray emanating from \((x_o,y_o)\). The situation depicted is general in that it applies not only at the source, but at any point on the ray path. Thus, the
velocity at any point on the path is a function of three spatial parameters which vary with position, \( U, C, \) and \( A. \) From the sketch, it is clear that

\[
\begin{align*}
\frac{dx}{dt} &= [U(x, y) - C(x, y) \cos A] dt \\
\frac{dy}{dt} &= C(x, y) dt \sin A
\end{align*}
\] 

Equations were developed for two methods of tracing the ray path to establish the magnitude and the phase relationship at field points to a unit source. These methods are: (1) a difference equation method, and (2) a non-linear differential equation method.

**Difference Equation Method**

In this method, time is the independent variable. Equations (3) are two of the three equations needed to establish the variation of \( x, y, \) and \( \Lambda \) with time. The third equation is obtained by considering the acceleration of the ray in the non-uniform flow field (see Figure 1).

![Figure 1. Velocity Components of a Sonic Ray Line In A Moving Airstream](image)

In terms of components in the directions of the rotating unit vectors \( \hat{r}_1 \) and \( \hat{r}_2, \)

\[
\begin{align*}
\dot{\mathbf{R}}_1 &= (U \sin \Lambda) \hat{r}_1 + (C - U \cos \Lambda) \hat{r}_2 \\
\dot{\mathbf{R}}_2 &= (U \sin \Lambda + C \dot{\Lambda}) \hat{r}_1 + (C - U \cos \Lambda) \hat{r}_2
\end{align*}
\]

It is necessary to express the angular velocity \( \dot{\Lambda} \) in terms of space variables. To do this, consider that at time \( t \) a second ray point is located at \( \mathbf{R}_3 = \mathbf{R}_1 + \Delta \mathbf{R}, \) where \( \Delta \mathbf{R} \) is small, and it's direction of travel is \( \dot{\mathbf{R}}_3 = \dot{\mathbf{R}}_1 + \Delta \mathbf{R}. \) Let the superscripts \( (o) \) and \( (l) \) denote times \( t_o \) and \( t_1, \) respectively. Then at time \( t_1 \)
\[
R_i^{(*)} = \mathbf{R}_i^{(*)} + \mathbf{R}_i^{(*)} \Delta t
\]
and \[
R_j^{(*)} = \mathbf{R}_j^{(*)} + \mathbf{R}_j^{(*)} \Delta t
\]
Subtracting the first equation from the second
\[
\delta \mathbf{R}^{(*)} = \delta \mathbf{R}^{(*)} + \delta \mathbf{R}^{(*)} \Delta t
\]
where \[
\delta \mathbf{R} = \mathbf{R}_j - \mathbf{R}_i
\]
Recalling that the cross product of two vectors is a vector normal to the plane defined by the two vectors, and has a magnitude equal to the product of the two magnitudes times the sine of the angle between them, then
\[
\delta \mathbf{R}^{(*)} \times \delta \mathbf{R}^{(*)} = \mathbf{\hat{A}}' (-\delta \mathbf{R}^{(*)} \delta \mathbf{R}^{(*)} \sin \Delta \mathbf{A})
\]
which has the correct sense. When \(\Delta \mathbf{A}\) is small, and when Equation (5) is substituted into the left side of Equation (6), we get
\[
\delta \mathbf{R}^{(*)} \times \delta \mathbf{R}^{(*)} \Delta t = \mathbf{\hat{A}}' (-\delta \mathbf{R}^{(*)} \delta \mathbf{R}^{(*)} \Delta \mathbf{A})
\]
This may be rewritten as
\[
\frac{\Delta \mathbf{A}}{\Delta t} = -\frac{\delta (C-U \cos \mathbf{A})}{\delta \mathbf{R}^{(*)}}
\]
and in the limit as \(\Delta t \to 0\)
\[
\mathbf{\hat{A}} = \mathbf{\hat{A}}' \cdot \mathbf{\nabla} (C-U \cos \mathbf{A})
\]
where the operator \(\mathbf{\hat{A}}' \cdot \mathbf{\nabla}\) is
\[
\mathbf{\hat{A}}' \cdot \mathbf{\nabla} = (\sin \mathbf{A} \frac{\partial}{\partial u} + \cos \mathbf{A} \frac{\partial}{\partial v})
\]
and operates only on \(C\) and \(U\).

Equation (7) has a revealing physical interpretation. From Figure 4 we see that the gradient of the speed of sound \(C\), on forward portions of the wing, is a vector pointing forward and slightly outward from the centerline; whereas, from Figure 3 we see that the gradient of the local flow speed \(U\) is nearly in the opposite direction. Although it is not apparent from the figures because they are plotted to different scales, the magnitude of the gradient of \(U\) is about five times that of the gradient of \(C\). From the energy equation \(C^2 + \frac{U^2}{2} = \text{constant}, \mathbf{\nabla} U = -5.0 \mathbf{V}C\). The local Mach number is increasing in the downstream direction. Figure 2 shows that, under these conditions there are only two stable ray angles; those for which the gradient of \(C - U \cos \mathbf{A}\) is zero. As the ray propagates through the flow field it will always tend towards one of these two orientations.
We now write Equations (3) and (7) in difference form

\begin{align*}
\Delta x &= \left[U - C \cos \lambda\right] \Delta \theta \\
\Delta y &= \left[ C \sin \lambda \right] \Delta \theta \\
\Delta \lambda &= - \left[ \sin \lambda \left( \frac{\partial C}{\partial x} - \cos \lambda \frac{\partial U}{\partial x} \right) \right. \\
&\quad \left. + \cos \lambda \left( \frac{\partial C}{\partial y} - \cos \lambda \frac{\partial U}{\partial y} \right) \right] \Delta \xi
\end{align*}

where \( \Delta \theta \) represents an increment in disturbance travel time \( \theta \), defined previously. To determine \( \phi(x, y, 0, x_0, y_0, 0) \) it is necessary to know a steady state distribution of \( C(x, y), U(x, y) \), and their derivatives at any point in the flow field over the wing and in the surrounding flow field in the plane of the wing. A means for establishing these is given in Section 5. Assume they are known. Then the procedure used is as follows:

1. Select any source point, on or off the wing, \((x_0, y_0)\).
2. Select a series of initial ray angles, \( \theta_i, i = 1, 2, \ldots \).

3. Select an initial increment in disturbance travel time, \( \Delta t_0 \).

4. For each of the ray angles store \( x^{(1)}, y^{(1)}, \sin \theta^{(1)}, \cos \theta^{(1)}, \) and \( \Delta x^{(1)}, \Delta y^{(1)}, \Delta \theta^{(1)}, i = 1, 2, \ldots \).
   a. At \( x^{(1)}, y^{(1)} \) compute and store \( x^{(1)} = x^{(1)} + \Delta x^{(1)}/2 \) and \( y^{(1)} = y^{(1)} + \Delta y^{(1)/2} \), holding \( \theta \) constant.
   b. Iterate on \( x_2^{(1)} = x^{(1)} + \Delta x^{(1)}/2, y_2^{(1)} = y^{(1)} + \Delta y^{(1)/2}, \)
      and \( \Delta \theta(x_2^{(1)}, y_2^{(1)}) \) until they converge or exceed ten trials.
      In the latter case replace \( \Delta x^{(1)} \) by \( \Delta x^{(1)/2} \) and repeat the iteration. If they converge in three trials or less, replace \( \Delta x^{(1)} \) by \( 2\Delta x^{(1)} \).
   c. Replace \( x^{(1)} \) by \( x_2^{(1)} \), \( y^{(1)} \) by \( y_2^{(1)} \), and return to a.

The solutions presented above are believed to be good approximations to the exact solutions for the following reasons:

1. For the case of a uniform flow they reduce to the proper linearized expressions.

2. The phase of the disturbance will be exact, although the amplitude may be slightly in error.

3. In an inner region in the immediate neighborhood of the source location \((x_0, y_0, z_0)\) they approach the correct solution.

4. For a one-dimensional mean flow with \( M_\infty \) approaching unity they reduce to Landahl's earlier solution (Reference 2).

5. In the limit of steady flow \((\alpha = 0)\), the solutions give results equivalent to the local linearization method of Spreiter and Alksne (Reference 4). This has been demonstrated by Rubbert (Reference 5).

6. Inasmuch as the proposed approximation only affects the receding part of the solution, the proper limiting solution for high frequencies (Reference 1), should always be obtained since then receding-wave effects are largely cancelled out due to the rapid phase variations.

This method gives reasonable results, i.e., reasonable based on a comparison with results obtained from the differential equation method. However, the ray paths did not conclusively show the existence of the focal point that the second method revealed.
Non-Linear Differential Equation Method

From Equations (3) we may write the slope of the ray path
\[
\frac{dx}{dy} = \frac{M - \cos \Lambda}{\sin \Lambda}
\]  
and solving this equation for \( \cos \Lambda \), we get
\[
\cos \Lambda = \frac{M \pm r \sqrt{r^2 + 1 - M^2}}{1 + r^2}
\]  
where
\[
r = \frac{dz}{dy}
\]

The transmission time from source to receiving point is given by
\[
\tau = \int \frac{ds}{n}
\]  
where the integration is taken along the path and
\[
ds = \sqrt{1 + r^2} \ dy
\]  
The velocity along the path is obtained from the vector sum of the two velocity components
\[
V = C \sqrt{M^2 + 1 - 2M \cos \Lambda}
\]  

Substituting equations (12), (13), and (10) into equation (11) we have:
\[
\tau = \int \frac{\sqrt{1 + r^2} \ dy}{C \sqrt{M^2 + 1 - zM \left[ \frac{M \pm r \sqrt{r^2 + 1 - M^2}}{1 + r^2} \right]}}
\]  
which reduces to
\[
\tau = \int \frac{(1 + r^2) \ dy}{C \sqrt{M^2 r^2 + zMr \sqrt{r^2 + 1 - M^2} + r^4 + 1 - M^2}}
\]  
The radicand in the denominator is a perfect square. Thus,
\[
\tau = \int \frac{(1 + r^2) \ dy}{C \left[ Mr \mp r \sqrt{r^2 + 1 - M^2} \right]}
\]  
which reduces to
\[
\tau = \int \frac{Mr \mp r \sqrt{r^2 + 1 - M^2}}{C (M^2 - 1)} \ dy
\]
At this point we relate the local acoustic velocity, \( C = C(x, y) \), to the local Mach number by imposing the condition of conservation of energy. For non-viscous flow, the total temperature is conserved. It is easily verified, that under this condition

\[
\frac{C^2}{c_0^2} = \frac{\gamma + M^2}{\gamma + M_\infty^2} \tag{16}
\]

where \( \gamma = 1.4 \), for a diatomic gas, has been used. Substituting Equation (16) into Equation (15), we get

\[
g = \frac{1}{C_\infty \sqrt{\gamma + M_\infty^2}} \int \frac{\sqrt{\gamma + M^2} \left[ M \rho \pm \sqrt{r^2 + 1 - M^2} \right]}{(M^2 - 1)} \, dy \tag{17}
\]

where the upper sign applies to receding waves and the lower sign to advancing waves. Equation (17) contains all the elements for the solution. However, the integrand is a function of \( x, y, \) and \( dx/dy \). This equation may be written in symbolic form

\[
g = \int_{y_0}^{y_1} F(x, y, \frac{dx}{dy}) \, dy
\]

which suggests the use of Euler's equation to find the minimum time \( t \), for the disturbance to travel to a field point \((x_1, y_1)\)

\[
\frac{\partial F}{\partial y} \frac{\partial \mathcal{V}}{\partial r} - \frac{\partial F}{\partial x} = 0 \tag{18}
\]

In order to simplify the notation, we set

\[
F = \frac{\beta (M r \pm \beta)}{M^2 - 1}
\]

where

\[
\beta = \beta(x, y) = \sqrt{\gamma + M^2}
\]

\[
\beta = \beta(x, y, r) = \sqrt{r^2 + 1 - M^2}
\]

and \( r \) has been previously defined. We will need

\[
\frac{\partial F}{\partial x} = \frac{\beta}{M^2 - 1} \left[ \frac{M r \pm \beta}{\rho} \right] + \frac{M r \pm \beta}{(M^2 - 1)^2} \left[ \frac{M M \pm 2 \delta M}{\beta} \right]
\]

\[
\frac{d}{dy} \left( \frac{\partial F}{\partial r} \right) = \frac{\beta}{M^2 - 1} \left[ \frac{\partial M}{\partial y} + \frac{M}{\beta} \frac{d \rho}{dy} - \frac{r}{\beta^2} \frac{d \beta}{dy} \right]
\]

\[
+ (M \pm \frac{r}{\beta}) \left[ \frac{(M^2 - 1) \frac{d \beta}{dy} - 2 \delta M \frac{d M}{dy}}{(M^2 - 1)^2} \right]
\]
Then, making use of the relationships

\[
\frac{dM}{dy} = rM_x + M_y
\]
\[
\frac{d\phi}{dy} = \frac{1}{b} \left[ r \frac{dr}{dy} - rM_x - MM_y \right]
\]
\[
\frac{d\xi}{dy} = \frac{1}{b} \left[ rM_x + MM_y \right]
\]

solving for \(dr/dy\), and combining terms, we get

\[
\frac{dr}{dy} = \frac{1}{\delta^2(M^2 - 1)} \left\{ \left[ \frac{-N(N^2+1)}{M^2-1} \right] r^2 + \frac{\beta(7N^2+5)}{M^2-1} \right\} r^2
\]
\[
+ \left[ 2M(N+2) r \pm 3(7N^2+3) \right] M_y + \left[ \frac{2\beta}{M^2} (r^2 + \delta) \right] M_x
\]

Equation (19) is a second order, second degree differential equation of the form

\[
\frac{d^2\xi}{dy^2} = f(x, y, \frac{dx}{dy}, \frac{dy}{dx})
\]

It is second degree because \(\beta\) represents a radical. However, it can be solved numerically by any of the standard repetitive processes. We employed a fourth order Runge-Kutta procedure.

There are certain difficulties that arise in the numerical evaluation of Equation (19). These are first listed and interpreted and then equations used to surmount them are presented.

(1) Along some ray paths \(dx/dy\) becomes infinite even when the Mach number is not equal to one.

(2) Equation (19) is singular at Mach number = 1.0.

(3) In the supersonic region, signals sometimes become trapped on the local Mach line. This happens when \(\cos \Lambda = 1/M\). Signals tend to gravitate to this condition. Such trapped signals cannot then cross the sonic line. They approach the sonic line as a limit, and are cancelled out there.

To overcome the difficulty listed in Item (1), it is necessary to use \(x\) instead of \(y\) as the independent variable. This is done by applying the equation

\[
\frac{d^2y}{dx^2} = \left( \frac{dx}{dy} \right)^3 \frac{d^2x}{dy^2}
\]
It is convenient here to introduce some new notation. Re-write equation (19) in the form

$$\chi'' = \frac{1}{AB} \left\{ -\frac{M^3}{B} \left( \frac{1}{x'^2+1} \right) \chi' + 2M (M^3 + 3) x' + \frac{3}{B} \right\} M_x$$

$$+ \frac{M}{A} \left\{ x'^2 + 1 \right\} M_x$$

(21)

where the new notation, together with some other notation which will be used later, is defined as follows

$$\chi' = \frac{\partial \chi}{\partial t}$$

$$A = M^2 + 1$$

$$M_x = \frac{3M}{\partial x}$$

$$B = M^2 - 1$$

$$M_y = \frac{3M}{\partial y}$$

$$R_x = \sqrt{x'^2 - (M^2 - 1)}$$

$$R_y = \sqrt{1 - y'^2(M^2 - 1)}$$

$$\beta = \sqrt{B}$$

$$E = c_0 \sqrt{5 + M_x^2}$$

(22)

Substituting Equation (20) into Equation (21), we get

$$\chi'' = \frac{1}{AB} \left\{ \frac{M^3}{B} \left( M^3 x'^2 + x'^2 + \frac{3}{B} \right) \right\} M_x$$

$$- \frac{M}{A} \left( 6x'^2 + 1 \right) M_x$$

; \quad 0 \leq x'^2 \leq 1

(23-a)

$$\chi'' = \frac{1}{AB} \left\{ \frac{M^3}{B} \left( M^3 x'^2 + x'^2 + \frac{3}{B} \right) \right\} M_x$$

$$- \frac{M}{A} \left( 6x'^2 + 1 \right) M_x$$

; \quad y'^2 < 0

(23-b)

The limiting form of Equation (20) at \( M = 1 \) is:

$$\chi'' \bigg|_{M=1} = \frac{1}{2A} \left\{ 2x'^2 + x' + \frac{1}{x'^2 + 1} \right\} M_y + \frac{1}{A} \left( x'^2 + 1 \right) M_x$$

In the supersonic region, when the signal is trapped on the local Mach line, and

$$\cos \lambda = \frac{x'}{M} , \quad \sin \lambda = \sqrt{1 - \frac{x'^2}{M^2}} , \quad \text{and} \quad \sqrt{M^2 - 1} = \beta$$

equation (20) reduces to

$$\chi'' = M \left( \frac{M^2 - 1}{M} + M_x \right)$$
A complete set of equations, together with their areas of applicability, will now be outlined.

Complete Set of Equations where $Y$ is the Independent Variable

\[
\frac{d^2 x}{dy^2} = \frac{1}{A^2} \left\{ \frac{-M}{B} (M^3 + 11) \chi^3 + 2M (M^2 + 8) \chi' - (7M^2 + 5) \frac{F}{B} \right\} M_y
\]

\[
\frac{d t}{dy} = \frac{1}{E} \left\{ \frac{15 + M^2 (M^2 + 8)}{B} \right\}
\]

\[
\chi'' \bigg|_{M=1.0} = \frac{1}{2A} \left\{ \frac{10}{7} \chi^3 + \chi' + \frac{2}{7} \right\} M_y + \frac{M}{A} (\chi^2 + \omega)
\]

\[
\frac{d \chi'}{dy} \bigg|_{M=1.0} = \frac{10}{2E} (\chi' + \frac{1}{\chi'})
\]

\[
\chi'' \bigg|_{\chi'=\beta} = M \left( \frac{M_0}{\beta} + M_x \right)
\]

\[
\frac{d t}{dy} \bigg|_{\chi'=\beta} = \frac{M \sqrt{\frac{5 + M^2}{E}}}{{\chi'}}
\]

A complete set of equations were also developed using $X$ as the independent variable. However, for the sake of brevity, and since they are obtained by a simple change of variable, they will not be listed here. Equations (26) and (27) apply where an advancing ray path crosses the sonic line, and equations (28), (29) apply where a ray path, in the supersonic region, becomes trapped on the local Mach line. It remains to describe the regions of applicability of the upper and lower signs of equations (24) and (25). In what follows, "right branch" will be specified where $0 < \pi < 2\pi$ and left branch will be specified if $-\pi < \pi < 0$. Here $\pi$ is the local value along the ray path. The end points are not specified because for these points we use $X$ as the independent variable.

The upper sign is used for

(1) Subsonic, left branch

(2) Supersonic, receding, right branch

(3) Supersonic, advancing, left branch
The lower sign is used for

(1) Subsonic, right branch
(2) Supersonic, receding, left branch
(3) Supersonic, advancing, right branch
In the application of each of the methods contained in this report, it is necessary to know certain of the properties of the transonic flow field on, and in the neighborhood of, the wing.

Figures 3 and 4 show the distributions of local flow speeds and sonic speeds over a 65° delta wing model in a wind tunnel in which the Mach number was 1.04 (taken from Reference 6). Speeds were computed from steady state pressure data at 27 points on the wing. The figures are intended only to show the general characteristics of the flow, such as: (1) The local sonic line shifts aft with distance from the centerline but crosses the leading edge inboard of the tip, (2) Mach number variations in both the streamwise and spanwise directions must be considered and cannot be considered to be linear, and (3) Separated flow is indicated over the aft and inboard portion of the wing. To consider the last of these characteristics is beyond the scope of this study. However, the first two are amenable to analysis using available theories and techniques.
Figure 3: Local Flow Distribution on a 65° A at a Transonic Speed
Figure 4. Sonic Speed Distribution on a 65° Δ at a Transonic Speed
Mach number distributions over areas off the wing were computed from an approximate theoretical solution of the flow field that matched pressure distributions on the wing. In order to avoid a discontinuity at the juncture of the two regions, a small transition region was defined over which the two functions were joined by a numerical smoothing technique.

Let:

\[ M_L, \quad \Phi, \quad \gamma, \quad T/\theta(x,y) \]

\[ \begin{align*}
M_{L} & = M_{L}(x,y) = \text{Mach number} \\
\Phi & = \Phi(x,y) = \text{Perturbation potential} \\
\gamma & = T/\theta(x,y) = \text{Thickness ratio}
\end{align*} \]

For a steady-state, non-lifting flow

\[ (1 - M_{L}^2) \Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0 \] (30)

and

\[ \Phi_{z}(x,y, \sigma^+) = \pm \gamma \Phi_{x}(x,y) \] (31)

Where \( \Phi(x,y) \) is a function describing the variation of the surface from the mean.

Using parametric differentiation with respect to \( \gamma \), (Reference 5),

\[ \Phi = \Phi(x,y) = \frac{\partial \Phi}{\partial \gamma} \]

Equation (30) becomes:

\[ \frac{\partial}{\partial x} \left[ (1 - M_{L}^2) \Phi_{x} \right] + \Phi_{yy} + \Phi_{zz} = 0 \] (32)

\[ \Phi_{z}(x,y, \sigma^+) = \pm \Phi_{x}(x,y) \]

After having obtained the solution of equation (32), the local Mach number distribution is obtained by relating local Mach number to the coefficient of pressure, \( (C_p) \). Starting with the following basic relations:

Let

\[ u = \frac{U_{L} - U_{\infty}}{U_{\infty}} \]

then

\[ \kappa = \frac{1}{U_{\infty}} \frac{\partial U_{L}}{\partial \xi} = - \frac{C_p}{2} \] (33)

\[ \alpha^2 + \frac{1}{2} (\sigma - 1) \Phi^2 = \text{Constant} \] (34)

where

\[ q = U_{\infty} \text{ at infinity} \]

\[ q = U_{\infty} (1 + \kappa) \text{ elsewhere} \]

\[ a = \text{speed of sound} \]
We have:
\[ a^2_\infty + \frac{1}{3} (\gamma - 1) \frac{v^2_\infty}{A} = a^2 + \frac{1}{3} (\gamma - 1) v^2 (1 + M)^2 \]
\[ \frac{v^2_\infty}{A} (1 + M)^2 = v^2 (1 + 2 M) \]
\[ a^2_L \simeq a^2_\infty - (\gamma - 1) v^2 \]

using equation (33)
\[ a^2_L \simeq a^2_\infty \left[ 1 + \frac{1}{4} (\gamma - 1) M^2 \right] \]

The coefficient of pressure, \( C_D \), is of order \( \mathcal{O} \), and \( M \) is \( 0(1) \). Therefore, to sufficient accuracy,
\[ a^2_L \simeq a^2_\infty \left[ 1 + \frac{1}{4} (\gamma - 1) M^2 \right] \]
\[ \frac{v^2_\infty}{A} (1 + M) = v^2_\infty (1 - \frac{1}{4} C_D) \]

and from these relations:
\[ M = \frac{M_\infty (1 - \frac{1}{2} C_D)}{1 + \frac{1}{4} (\gamma - 1) M^2} \]

Noting again the order of \( M_\infty \) and \( C_D \), to sufficient accuracy,
\[ M = M_\infty \left[ 1 - \frac{1}{4} (\gamma - 1) M^2_\infty \right] \]
or
\[ M = M_\infty \left[ 1 - \frac{\gamma + 1}{4} C_D \right] \]

Equation (35) is the expression that was used to relate local Mach number to \( C_D \) on regions off the wing.

A solution of equation (35), using the results of equation (36), was worked out for a special configuration. The special wing configuration is depicted in figure (5).

![Typical Section](image)

**Fig. 5. A Thin Wing In Rectilinear Flight**

The solution is:
\[ L_\rho(x, y) = C_\rho(x, 0) = -2 \int \left[ |y - S|^{\ell +} + |y + S|^{\ell -} \right] \left[ y - S, |y - S|^{\ell +}, \|y + S|^{\ell -} \right] H(x - a) \]
\[ -2 \int \left[ |y - S|^{\ell +}, |y + S|^{\ell -} \right] H(x - b) \]

where \( H(x) \) is a step function.
After determining a distribution of $C_0$ and its derivatives from equations (36), (37), and (38), the Mach number distribution, with its derivatives, is computed from equation (35).
DESCRIPTION OF THE COMPUTER PROGRAM

The equations for the ray paths are solved in the following manner:
Let the independent variable be $y$ and

$$\begin{align*}
V_1 &= \frac{dx}{dy} \\
V_2 &= x \\
V_3 &= t
\end{align*}$$

Then

$$\begin{align*}
\frac{dV_1}{dy} &= f_1(V_1, V_2, y) \\
\frac{dV_2}{dy} &= V_1 \\
\frac{dV_3}{dy} &= f_2(V_1, V_2, y)
\end{align*}$$

These three simultaneous differential equations are solved in a step-by-step manner by use of a standard "SHARE" subroutine which is based on the Runge Kutta method. When $dx/dy$ becomes greater than one, a variable change takes place in the program, and $x$ becomes the independent variable.

A signal (in the supersonic region) is considered "trapped" on the local Mach line when

$$|x^2 - (M^{-1})| \leq E1$$

When, for this trapped signal, $(M-1) < E2$, the integration stops and a new ray line is started. This logical flow is shown in the chart on page 21.

The values of $A_0$ used in the program are determined by the parameter (NLA). If (NLA) is an odd integer, it will be rounded down in the program to an even integer. Values of $A_0$ vary from zero to $\pi$ and from zero to $-\pi$ in an arithmetic progression.

Computation of a ray path (other than for a "trapped signal") ceases under the following conditions:

$$\begin{align*}
\mathbf{Z} &\leq 0 \\
\mathbf{L} &\leq 0 \\
|y_{\text{MAX}}| &\leq 1 |y| \\
N_{\text{MAX}} &\leq N_{\text{CNT}}
\end{align*}$$

where $N_{\text{CNT}}$ is the number of points on the ray path already computed. This logical flow is shown in the chart on page 22.

Subroutine DERIV computes the appropriate derivatives.

Subroutine CNTRL accomplishes variable changes, stores local values in appropriate locations for later printing, and performs exit tests.

Subroutine FMACH computes the local Mach number and the partial derivatives of the Mach number.
Subroutine SOMK computes coordinates on the planform where $M = 1$.

Sample data sheets with numbers which have been used in a computer run are in Appendix II. The output sheets are included. The output format is self-explanatory, with the exceptions of certain test words that are printed out at the beginning of the plots for each ray-path. Definitions for these words can be found in the comment statements at the beginning of the listing in Appendix I. The values listed for these test words apply to the last point plotted for the ray-path.
MAIN PROGRAM

READ DATA
CALL SONK
COMPUTE $\lambda_{\theta_i}$
SET UP GRID LIMITS
YL, YR, XU, XL
CALL LIMIT

Next Source

SET UP GRID
CALL GRAPH
PLOT PLANFORM
CALL GRAPH
PLOT SONIC LINE
CALL FMACH

Next $\lambda_o$

SET IVAR
(INITIAL VALUE)
COMPUTE IBR
(Initial Value)
COMPUTE ISORS

SET INITIAL VALUES OF VARIABLES
x, y, dx/dy, time
CALL RK53
CALL POT

PRINT AND PLOT
VALUES FOR 1 PATH
NEXT $\lambda_o$ OR
NEXT SOURCE

Subroutine SONK Computes Sonic Line

Subroutine LIMITI Sets Plotting Grid Limits

Subroutine GRAPH Produces Cathode Ray Tube Plots

Subroutine FMACH Computes Mach No.
Determines Whether X or Y is Independent
Determines Left or Right Branch
Determines Type of Source

Runge Kutta Integrating Subroutine

Subroutine POT Computes Velocity Potential Along Path due to Source at (Xo, Yo)
SUBROUTINE RK3

Storage Allocation
Call RKINT
Return

SUBROUTINE RKINT

Storage Allocation
Initial Values of Variables
Call DERIV Compute Initial Values of Derivatives
Call CNTRL

Compute Variables and Derivatives at Two Half Steps
Compute Variables at End of Interval
Test for Fixed or Variable Interval
If interval is variable, compute error. If too large, decrease interval, repeat step. If too small, increase interval and accept.
Call CNTRL (NTRY)
If:
NTRY = 1, compute next step
NTRY = 2, exit to main program
NTRY = 3, repeat step
NTRY = 4, restart integration

Subroutine DERIV computes derivatives
Subroutine CNTRL executes variable changes, stores current values, executes exit tests
This loop calls DERIV 8 times
DISCUSSION OF RESULTS

This report contains two methods for calculating the velocity potential along sonic ray lines emanating from any point in a non-uniform flow field, i.e., one that varies from locally subsonic to supersonic speeds. Both methods apply to pulses emitted by sources or doublets. It has been demonstrated that both methods yield nearly identical ray paths and times of transmission. Those presented were obtained using the second method.

Figures 6 through 13 show ray paths of acoustic signals emanating from various points in a non-uniform transonic flow field. The reader may want to try his hand at tracing one of the ray paths in a region of interest such as near a leading edge. If so, it should be helpful to recall the discussion starting with Equation (7), through the difference equations of the path, Equation (8), and to the end of that section. An analysis of the differential equation of the path, Equation (24) should also be helpful. These show, for instance, that where the Mach number is constant the curvature of the ray path is zero; for a given Mach number and slope of ray path the curvature is proportional to rate of change of Mach number along the path. Figures 6, 7, 9, and 10 conclusively show that when the variation in Mach number is parabolic in the chordwise and spanwise directions focal points exist, both in subsonic and supersonic portions of the flow. None of the present theories accounts for the corresponding multiple crossings of the acoustic wave front. Figures 9 and 12 show acoustic signals traveling from regions of supersonic flow to regions of subsonic flow. This can occur, of course, only when the sonic line is swept downstream. Figures 9 and 12 also show rays that have been trapped on the Mach wave, travel outward to the sonic line where the spanwise slope of the ray path becomes zero, and are cancelled there. A study of the ray paths that cross the leading edge shows that in practical applications it is correct to assume they do not return.

These results permit the formulation of a numerical procedure. A box method is outlined in Appendix III. It establishes velocity potentials at all box centers on an aerodynamic surface and the corresponding generalized forces.
Figure 6. Ray Paths for a Source or Doublet at (0.18c, 0.0)
Figure 7. Ray Paths for a Source or Doublet at (0.26c, 0.0)
Figure 6. Ray Paths for a Source or Doublet at (0.6c, 0)
Figure 9. Ray Paths for a Source or Doublet at (3.42c, 0.0)
Figure 10. Acoustic paths for a source or doublet at (0.22c, 0.04c)
Figure 11. Ray Paths for a Source or Doublet at (0.34c, 0.14c)
Figure 12. Ray Paths for a Source or Doublet at \((0.54c, 0.16c)\)
Figure 13. Ray Paths for a Source or Doublet at (0.57c, 0.20c)
CONCLUSIONS AND RECOMMENDATIONS

Two methods have been outlined in detail, and one of them has been completely mechanised for calculating the velocity potentials along acoustic ray paths emanating from any point in a non-uniform transonic flow field over a lifting surface. The one mechanised gives the ray path and velocity potential for the minimum time of travel from the source point to the field point.

To calculate pressures over the planform and generalized forces, it will be necessary to develop a procedure for calculating the velocity potential at an arbitrary point due to a sheet of sources, covering the wing surface, and the flow field in the plane of the wing out to a distance of several wing spans in the y-direction, or due to a sheet of doublets covering the wing surface. The latter is recommended for economy reasons.

The computer program in this report may be used to refine the doublet box method of Rodemich (3) in such a way as to include the (possibly very important) influence of wing thickness distribution on transonic airloads. A doublet box method similar to the one Rodemich developed (Reference 3) is recommended. The procedure is heuristically described in Appendix III. For each of a selected set of points in a sending box, the distribution of velocity potentials along ray lines throughout the zone of influence can be determined. An interpolation scheme will yield from these the velocity potentials at box centers and a numerical integration procedure will yield a velocity potential influence coefficient for each of the box centers. It will be necessary to solve a set of simultaneous equations to establish the strengths of doublets required to satisfy the tangential flow condition in the subsonic flow region. The order of the set will be equal to the number of box centers in the subsonic region on the wing. In the supersonic region the doublet strengths can be established sequentially. The use of doublets to solve unsteady supersonic flow problems has been outlined by Ashley in Reference 7.

It is recommended that this method be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict, with reasonable accuracy, the flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in available technology.
REFERENCES


2. Lundahl, M. Approximate Solution for an Oscillating Source in a Non Uniform Transonic Stream. NAA, SID 63-1194 (August 1963)


6. Wind Tunnel Tests of Four Reflection Plane Mounted .024 Scale Models Simulating The YB-70 Wing to Investigate the Effect of Camber on the Chordwise Pressure Distribution at Mach Numbers From 0.4 to 3.0. NA-61-55, TWT-50, (Unpublished)


APPENDIX 1. Program Listings

```
$IBFCC MAIN 500
C FORTRAN PROGRAM TO COMPUTE (AND PLOT) THE PATHS OF ACOUSTIC SIG -
C NALS (AND TRANSMISSION TIMES) ON AN AIRFOIL IN A SONIC FLOW FIELD,
C ACCOUNTING FOR VARIATION IN LOCAL MACH NUMBER.
C CM = COEFFICIENTS OF MACH EQUATION. (SEE SUBROUTINE FMACH )
C PLX AND PLY ARE CONSTANTS DESCRIBING THE PLANFORM GEOMETRY.
C THE PROGRAM ALLOWS FOR EITHER X OR Y TO BE THE INDEPENDENT VARI-
C BLE, DEPENDING ON THE CURRENT VALUE OF X-PRIME, WHICH SETS IVAR. 
C IF IVAR = 1,
C YY = CURRENT VALUE OF X       YY = CURRENT VALUE OF Y
C DYY= CURRENT VALUE OF DX      DYY= CURRENT VALUE OF DY
C XX(1) = CURRENT VALUE OF Y-PRIME XX(1)= CURRENT VALUE OF X-PRIME
C XX(2) = CURRENT VALUE OF Y     XX(2) = CURRENT VALUE OF X
C XX(3) = CURRENT VALUE OF TIME  XX(3) = CURRENT VALUE OF TIME
C XX(4) = CURRENT VALUE OF R-BAR  XX(4) = CURRENT VALUE OF R-BAR
C DXX(1) = Y-DDOUBLE PRIME       DXX(1) = X-DDOUBLE PRIME
C DXX(2) = Curr. VALUE OF Y-PRIME DXX(2)= Curr. VALUE OF X-PRIME
C DXX(3) = Curr. VALUE OF DT/DX   DXX(3)= Curr. VALUE OF DT/DY
C DXX(4) = CURRENT VALUE OF DR/DX DXX(4)= CURRENT VALUE OF DR/DY
C IVAR IS ORIGINALLY SET IN MAIN PROGRAM, AND THEN RESET ON EACH
C PASS THROUGH SUBROUTINE CNTRL.
C WORK = WORKING AREA FOR SUBROUTINE RKS3 .
C IFVD = FALSE AND IDIF= TRUE FOR VARIABLE INTERVAL.
C IFVD = TRUE FOR FIXED INTERVAL.
C SX = VECTOR CONTAINING COMPUTED X- VALUES.
C SXP = VECTOR CONTAINING COMPUTED X-PRIME VALUES.
C SY CONTAINS COMPUTED Y VALUES
C SYP CONTAINS COMPUTED R-BAR VALUES
C TIM CONTAINS TRANSMISSION TIMES.
C FM = CURRENT MACH NUMBER
C ISORS = -1 DEFINES A SUPERSONIC SOURCE, RECEIVING PATH.
C ISROS = 0 DEFINES A SUPERSONIC SOURCE, ADVANCING PATH.
C ISORS = 1 DEFINES A SUBSONIC SOURCE.
C IDR = 1 FOR RIGHT BRANCH, 2 FOR LEFT
C NCNT IS THE COUNTER FOR THE VECTORS SX,SY,SXP,SYP,TIM. WHEN NCNT
C = NMAX, INTEGRATION STOPS, AND THE FLOW PASSES TO NEXT PATH
C ITRAP = 1 INDICATES SIGNAL IS TRAPPED ON THE LOCAL MACH CONE.
C D2 = INITIAL VALUE OF INCREMENT.
C CINF = REMOTE SPEED OF SOUND IN ROOT CHORDS PER SECOND.
C FMINF= REMOTE MACH NUMBER
C POTE = THE POTE MATRIX CONTAINS THE VELOCITY POTENTIALS ALONG A
C RAY PATH, NORMALIZED ON BD .
C FREQ =ASSUMED FREQUENCIES IN RADIANS PER SECOND.
C EXTERNAL DERIV, CNTRL
C COMMON
C */WORK/ WORK(50)
```

C

1000 FORMAT(2L12 )
1010 FORMAT(6E12.0 )
1020 FORMAT(6112 )
   3 READ (5,1020) NSORCE,NLA,NPL,NMAX,NF
   READ (5,1000) FVD,IBKP
   READ (5,1010) (XO(I),YO(I),I=1,NSORCE)
   READ (5,1010) (CH(I),I=1,6 )
   READ (5,1010) DZ,E1,E2,YMAX
   READ (5,1010) (ATABL(I),I=1,4), (RTABL(I),I=1,4 )
   READ (5,1010) (PLX(I),PLY(I),I=1,NPL )
   READ (5,1010) CINF,FHINF,TAU,TSAA
C
      TAU=MAX. (T/C), TSAA = TANGENT OF SEMI-APEX ANGLE
DIMENSION FREQ(10), POTE(101,2,10 )
   READ (5,1010) (FREQ(I),I=1,NF )
C
   DIMENSION XSO(40),YSO(40)
   ECM = CINF*SQRT(5.0*FHINF**2)
   ECM=1.0/ECM
   CALL SONK(40,NXY,YMAX,YSO,XSO,IER )
2000 FORMAT(49H0 ERROR IN SUBROUTINE SONIC. CHECK MACH CONSTANTS )
   GO TO (1,2), IER
2 WRITE (6,2000)
1 CONTINUE
C
      NVAR IS THE NUMBER OF VARIABLES
CH2(1) =0.3
CH2(2) =0.7
CH2(3) =ATAN(1./TSAA)
CH2(4) =TAU
CH2(5) =1.18*TSAA
CH2(6) =.04
CH2(7) =FHINF
C
DEVELOP LAMDAS
NL=2*(NLA/2)
C
THERE WILL ACTUALLY BE NL VALUES. IF NLA IS EVEN, NL=NLA. BUT NL=
C
NLA - 1 IF NLA IS ODD.
NL1=NL-1
NL2 = NL/2

XN = NL2*(NL2+1)

DG = 6.28318/XN

AL(1)=0.

DO 10 J=3,NL1,2

XJ=(J-1)/2

J1=J-1

AL(J)=AL(J-2)*XJ*DG

10 AL(J1)=-AL(J)

AL(NL)=3.14159

C SET UP GRID LIMITS

XU=0.

XL=1.

YL=-YMAX

YR=YMAX

CALL LIMIT1(YL, YR, XL, XU)

DO 600 NS=1,NSORCE

NS5=NS

CALL GRAPH(1,42,-NPL,PLY,PLX,2H,Y,2H X,15H ACOUSTIC PATHS)

XOF=XO(NS)

YOF=YO(NS)

CALL GRAPH(0,42,-NX,YSO,XSO)

NLLS=NL

DO 500 NLC=1,NL

NLCS=NLC

ITRAP=0

CALL FMACH(XOF,YOF,FMX,FMY)

TEST1 = FM - COS(AL(NLC))

TEST2 = SIN(AL(NLC))

IF (NLC .NE. NL) GO TO 11

IF (YOF .GT. 0.) GO TO 11

TEST2 = -TEST2

11 IF (NLC-1) 14,12,14

12 IVAR=1

GO TO 30

14 IF (NLC-NL) 10,12,10

18 IF (TEST1) 22,20,22

20 IVAR=2

GO TO 30

22 TEST = TEST1/TEST2

ART = ABS(TEST)

IF (ART .LT. 1.0) 20,12,12

30 CONTINUE

C SET IBR

FL=AL(NLC)

IF (NLC-1) 32,31,32

31 IF (YOF) 41,41,42

32 IF (NLC-NL) 36,34,36

34 IF (YOF) 42,41,41

36 IF (FL) 42,42,41
1080 FORMAT(1HO,27X, 617 )  SNIC1000
 WRITE (6,1070)  SNIC1005
 WRITE (6,1080) IVAR,NCNT,ISORS,IBR,ITRAP,NLCS  SNIC1010

C
1060 FORMAT(22H ERROR IN RK53, IERR = 14 )  SNIC1015
 IF (IERR) 103,140,103  SNIC1025
 103 WRITE (6,1060) IERR  SNIC1030
  GO TO 500  SNIC1035

1050 FORMAT(1H-,42X,4HKO = E16.8/ 43X,4HYO = E16.8/ 43X,1DDMACH NO. = ESNI1040
  116.8// 29X,31H ACOUSTIC RAY PATH FOR LAMBDAD = E16.8///17X,1HX,17X,SNIC1045
  11HY,14X,7HX-PRIME,11X,7HR-BAR ,12X,4HTIME// )  SNIC1050
 1040 FORMAT(1H7X,5E10.8)  SNIC1055
 140 WRITE (6,1050) X0(NS),Y0(NS),FM,FL  SNIC1060
 WRITE (6,1040) (SXI(I),SY(I),SXP(I),SYP(I),TIM(I),I=1,NCNT )  SNIC1065

C
  CALL GRAPH (0,NLC,NCNT,SX)  SNIC1070
  CALL POT CNFNC.E.POTC  )  SNIC1075

C
1100 FORMAT(1H1,25X,54H VELOCITY POTENTIALS ALONG A RAY PATH FOR A SOURSNIC1090
  ICE AT )  SNIC1095
 1110 FORMAT(1H-,42X,4HKO = E16.8//43X, 4HYO = E16.8/ 43X, 8MLAMDAD = E16SNIC1100
  1.0 ///39X,30HALTERNATING REAL AND IMAGINARY )  SNIC1105
 1120 FORMAT (1H-,6X,7HOMEGA =E16.8// )  SNIC1110
 1000 FORMAT(1H6X,6E16.6)  SNIC1115

C
  DO 300 N=1,NF  SNIC1120
    IF (N .NE. 1 ) GO TO 200  SNIC1125
 WRITE (6,1100)  SNIC1130
 WRITE (6,1110) X0(NS),Y0(NS),FL  SNIC1135
 200 WRITE (6,1120) FREQ(N)  SNIC1140
 WRITE (6,1030) (( POTE(I,K,N),K=1,2 ),I=1,NCNT )  SNIC1145
 300 CONTINUE  SNIC1150
 500 CONTINUE  SNIC1155
 600 CONTINUE  SNIC1160
 GO TO 3  SNIC1165
 END  SNIC1170

38
SUBROUTINE DERIV
COMMON ♦/XDX/ XX(4),DXX(4),YY,DYY,DZ
♦/CM/ CM(6)
♦/ICNf/ IVAR,NCNV,JSORS,IBR,ITRAP,NMAX
♦/EPS/ E1,E2,FH,YMAX
♦/NNN/ NSS,NLCS,NLLS
♦/ECH/ ECN
C
GO TO(10,50),IVAR
X IS THE INDEPENDENT VARIABLE
10 CALL FMACH (YY,XX(2),FN,FMX,FMY )
R=XX(1)
DXX(2)=R
B =FN*FM -1.0
TSI = 1.0-R*R*B
A =FN*FM + 5.0
SA = SQRT(A)
IF(B) 103,103,101
101 BETA = SQRT(B)
IF(ITRAP .EQ. 1) GO TO 104
IF(ISORS .EQ. 1) GO TO 103
IF(TSI .GT. E1) GO TO 103
ITRAP = 1
GO TO 104
103 IF(TSI .GE. 0.) GO TO 215
ITRAP = 2
TSI = 0.
215 RAD = SQRT (TSI)
DXX(4) = RAD
RAB= 1.0/(A*B)
TM1=FH*(FM*2 + 11.0)/B
TM2= 2.0*FM*(FM*2+8.0)*R**2
TM3=((RAD+3)/D)*(7.0*FM*2+5.0)
TM4=(FM/A)*R*(6.0*R**2 +1.0)
GO TO 105
104 RBB = 1.0/(B**2)
DXX(4) = 0,
105 IF(ISORS) 11,15,10
11 GO TO (12,13),IBR
12 IF(ITRAP) 91,91,7
13 IF(ITRAP) 92,92,0
15 GO TO(16,17),IBR
16 IF(ITRAP) 92,92,7
17 IF(ITRAP) 91,91,0
18 GO TO (92,91),IBR
91 IF(R) 4,3,3
92 IF(R) 3,3,4
C Y IS THE INDEPENDENT VARIABLE
50 CALL FMACH (XX(2),YY,FN,FX,FNY )
R = XX(1)
DXX(2) = R
B = FM*FN-1.0
TSI = R*R-B

C
A = 5.0*FM*FN
SA = SQRT(A)
IF (B .LT. 0.) GO TO 108

106 BETA = SQRT(B)
IF (ITRAP .GT. 1) GO TO 109
IF (ISORS .EQ. 1) GO TO 108
(TSI .GT. E1) GO TO 100
ITRAP = 1
GO TO 109

108 IF (TSI .GE. 0.) GO TO 107
TSI=0.
ITRAP = 2
107 RAD = SQRT(TSI)
DXX(4) = RAD
RAB = 1.0/(A*B)
TM1=(FM/B)*(FM**2+11.0)*R**3
TM2= 2.0*FM*(FM**2+8.0)*R
TM3= (RAB**3.0)*(7.0*FM**2+5.0)
TM4 = (FM/A)*(R**2+6.0)
GO TO 110

109 DXX(4) = 0.
110 IF (ISORS) 52,60,60
52 GO TO (54,56),IBR
54 IF (ITRAP ) 1,1,5
56 IF (ITRAP ) 2,2,6
60 GO TO (62,64),IBR
62 IF (ITRAP ) 2,2,5
64 IF (ITRAP ) 1,1,6
68 GO TO (2,1),IBR
C FORMULAS FOR THE SECOND DERIVS FOLLOW
C
1 IF (ABS(B) .LE. 1.E-03) GO TO 220
DXX(1)=RAD*(-TM1 + TM2 - TM3)*FNY + TM4 *FN
DXX(3)=(SA*ECM/B)*(FM*XX(1)+RAD)
GO TO 100

2 IF (ABS(B) .GT. 1.E-03 ) GO TO 209
220 DXX(1)=(.5/A)*(2.0*R**3+R+9./R)*FNY + (FM/A)*(R**2+6.0)*FN
DXX(3)=(1.22475*ECM)*(R+1./R)
GO TO 100

209 DXX(1)= RAD*(-TM1+TM2+TM3)*FNY+TM4*FN
DXX(3)=(SA*ECM/R)*(FM*XX(1)+RAD)
GO TO 100

3 IF (NLCS .LT. NLLS) GO TO 4
DXX(1)=RAB*(TM1-TM2+TM3)*FNY -TM4*FN
DXX(3) = (SA*ECM/B)*(FM + RAD) 
GO TO 100

4 IF(ABS(B) .GT. 1.E-03) GO TO 205
204 DXX(1) = (-3/A)*(9.0*FF1+R*2+2.)*FM - (P/A)*((6.0*R*2+1.)*R) 
DXX(3) = (1.22475*ECM)/(1. + FM) 
GO TO 100
205 DXX(1) = RAD*(TM1-TM2-TM3)*FM*TY - FM*TY 
DXX(3) = (SA*ECM/B)*(FM - RAD) 
GO TO 100
5 DXX(1) = FM*((FM/BETA) + FMX) 
DXX(3) = (SA*ECM/B)*FM*XX(1) 
GO TO 100
6 DXX(1) = FM*((-FM/BETA) + FMX) 
DXX(3) = (SA*ECM/B)*FM*XX(1) 
GO TO 100
7 DXX(1) = -(FM*RB)*((FM/BETA) + FMX) 
DXX(3) = (SA*ECM/B)*FM 
GO TO 100
8 DXX(1) = FM*RB*(-FM/BETA*FMX) 
DXX(3) = (SA*ECM/B)*FM 
GO TO 100
10 IF(DYY .LT. 0.) GO TO 31
DXX(3) = ABS(DXX(3)) 
GO TO 32
31 DXX(3) = -1.0*(ABS(DXX(3))) 
DXX(4) = -1.0*(ABS(DXX(4))) 
32 RETURN
END
SUBROUTINE CNTRKnTRY
COMMON /XYZ/ SX(1Ol),SXP(101),SY(101),SYp(101),AL(41),TIm(101)
/ICNT/ IVAR,NCnt,ISORS,IBR,ITRAP,NMAX
/EPS/ E1,E2,E3,YMAX
/NNN/ NSS,NLCS,NLLS
IF (NCNT .NE. 1 ) GO TO 6
NCO = 1
IF (NR .EQ. 1 ) GO TO 6
NR = 1
IF (ABS (DXX(1)*DYY ) .LE. .25) GO TO 6
4 DYY = .5*DYY
IF (ABS (DXX(1)*DYY ) .LE. .25) GO TO 7
GO TO 4
7 NTRY = 4
RETURN
6 IF (ABS (XX(1)) .LT. 1.0 ) GO TO 20
1 NTRY =4
GO TO (2,3),IVAR
2 IVAR=2
GO TO 5
3 IVAR=1
C SWITCH VARIABLES. SET NEW INITIAL CONDITIONS
5 SAV =YY
DYY = DYY*XX(1)
10 YY = XX(2)
XX(1)=1.0/XX(1)
XX(2)=SAV
RETURN
20 GO TO (25,35),IVAR
C STORE CURRENT VALUES WHERE X IS INDEPENDENT VARIABLE.
25 SX(NCNT) = YY
C CHANGE IDR WHEN Y-PRIM PASSES THROUGH ZERO
IF (ABS (XX(1)) .GT. 1.0 E-02) GO TO 15
IF (DXX(1)*DYY*XX(1)) .GE. 0.0) GO TO 15
IF (NCO .EQ. 2) GO TO 15
NCO = 2
XX(1)=-XX(1)
NTRY = 4
GO TO (11,12), IBR
11 IBR =2
GO TO 19
12 IBR = 1
GO TO 19
15 IF (NCO .NE. 2 ) GO TO 19
IF (ABS (XX(1)) .LT. 1.0 E-01) GO TO 19
NCO = 1
19 IF (XX(1) .NE. 0.0 ) GO TO 27
26 SX(NCNT) = UNDEF
     GO TO 28
27 SX(NCNT) = 1.0/XX(1)
28 SY(NCNT) = XX(2)
     SYP(NCNT) = XX(4)
     TIM(NCNT) = XX(3)
     GO TO 50
35 SX(NCNT) = XX(2)
     TIM(NCNT) = XX(3)
     SX(NCNT) = XX(1)
     SY(NCNT) = YY
     SYP(NCNT) = XX(4)
50 CONTINUE
C NOW TEST FOR EXIT CONDITIONS
   IF (ITRAP .NE. 2) GO TO 51
   ITRAP = 0
   NCNT = NCNT - 1
   GO TO 100
51 IF (ITRAP) 60,60,52
52 TEST = FM-.1.0
   IF (TEST) 100,100,53
53 IF (TEST-E2 ) 100,100,60
60 IF (SX(NCNT)) 100,70,70
70 IF (SX(NCNT) -1.0) 80,100,100
80 AY = ADS(SY(NCNT))
   IF (AY-YMAX) 105,100,100
105 IF (NCNT-NMAX) 110,100,100
100 NTRY = 2
     NR = 0
     RETURN
110 NCNT = NCNT + 1
     RETURN
END
$1087C MACH

C MASTER SUBR., M, MX, MY

SUBROUTINE FMACH (FX, FY, FMS, FMXS, FMYS)

COMMON

*/C4/CM2(7)

EQUIVALENCE (A, CM2(1)), (B, CM2(2)), (AL, CM2(3)), (TAU, CM2(4)), (AK, SNIC2260)

CM2 (5)), (R1, CM2 (6)), (FMINF, CM2 (7))

AY = ABS(FY)

AYY = ABS(AK*FX)

IF (AY .LE. AYY) GO TO 200

SK = 1./ (SQRT(1. + AK*AK))

T = (AY - AYY) * SK

100 CALL FMAC1 (FX, AYY, FMS, FMXS, FMYS)

CALL FMAC2 (FX, AY, A, B, AL, TAU, D1FM, D1MX, D1MY)

CALL FMAC2 (FX, AYY, A, B, AL, TAU, D2FM, D2MX, D2MY)

C

FMS = FMS - 0.6 * FMINF * (D1FM - D2FM)

FMXS = FMXS + FHY3 * AK - 0.6 * FMINF * (D1MX - D2MX - AK*D2MY)

FMYS = - 0.6 * FMINF * D1MY * (AY/FY)

IF (T .GE. R1) GO TO 300

120 CALL FMAC1 (FX, FY, SH, SMX, SMY)

ARG = 1.57079*T/R1

SI = SIN(ARG)

SMO = SI*S1

FMS = (FMS - SM) * SMO + SM

FMXS = (FMXS - SMX) * SMO + SMX

FMYS = (FMYS - SMY) * SMO + SMY

GO TO 300

200 CALL FMAC1 (FX, FY, FMS, FMXS, FMYS)

300 CONTINUE

RETURN

END
$105FTC MAC2 58D
SUBROUTINE FMAC2(X,Y,A,B,AL,TAU,DELCP,DDXCP,DDYCP)
C
SUBROUTINE COMPUTES DELTA CP
C
C$ = COS(AL)
C$1 = 1.0/((1.0-A)**2)**((C$**2))
C$2 = 1.0/((1.0-A)**2)**((C$**2))
TA = SIN(AL)/C$
TA1 = (1.0-A)*TA
TA2 = (1.0-B)*TA
EPS = TAU/(2.0*1.415927*A*C3)
EPS1 = EPS*C$1/C$1
EPS2 = EPS*A*C3/((1.0-B)*C3)
EDS = 1.0 - EPS
EDS1 = EPS1 + 1.0
EDS2 = EPS2 + 1.0
S = ABS(X/TA)
S1 = (X-A)/TA1
S2 = (X-B)/TA2
Q1 = ABS(Y-S)
Q2 = ABS(Y+S)
Q3 = ABS(Y-S1)
Q4 = ABS(Y+S1)
Q5 = ABS(Y-S2)
Q6 = ABS(Y+S2)
FAC = 2.0*C$/TA
FAC1 = 2.0*C$1/TA1
FAC2 = 2.0*C$2/TA2
DELC = -FAC*(Q1#EPS1+Q2#EPS2-2.0**EPS)
DDX = FAC*(-1.0/(Q1#EDS1)+1.0/(Q2+EDS1))-2.0/(S#EPS)
DDY = FAC*(1.0/(Q1#EDS1)+1.0/(Q2#EDS1))
IF (31) 10,10,5
10 DELCP = DEL
DDXCP = DDX
DDYCP = DDY
GO TO 30
5 DEL1 = FAC1*(1.0/(Q3#EPS1)+1.0/(Q4#EPS1)-2.0/(S1#EPS1))
DDX1 = FAC1*(-1.0/(Q3#EDS1)+1.0/(Q4#EDS1))-2.0/(S1#EDS1) * EPS1 /TA1SNIC2390
DDY1 = FAC1*(1.0/(Q3#EDS1)+1.0/(Q4#EDS1))
IF (32) 20,20,30
20 DELCP = DEL + DEL1
DDXCP = DDX + DDX1
DDYCP = DDY + DDY1
GO TO 30
30 DEL2 = FAC2*(1.0/(Q5#EPS2)+1.0/(Q6#EPS2)-2.0/(S2#EPS2))
DDX2 = FAC2*(-1.0/(Q5#EDS2)+1.0/(Q6#EDS2))-2.0/(S2#EDS2) * EPS2 /TA2SNIC2330
DDY2 = FAC2*(1.0/(Q5#EDS2)+1.0/(Q6#EDS2))
IF (33) 10,10,5
45
DDYCP = DDY \cdot DDY1 \cdot DDY2

50 RETURN

END
SUBROUTINE FMAC1 (FX, FY, FMS, FMXS, FMYS)

SUBROUTINE Computes Mach No., Mx, My.

FX = X
FY = Y
FMS = Mach No.
FMXS = Partial M w/resp to X
FMYS = Partial M w/resp to Y

Eq. for Mach is M = CM(2) * EXP (-CM(1) * Y**2 / X) * CM(4) * X**CM(5) * Y**CM(6) *

Eq. for Mach is M = CM(2) * EXP (-CM(1) * Y**2 / X) * CM(4) * X**CM(5) * Y**CM(6) *

COMMON ♦/CM/ CM(6)

FV = Y
FMXS = Partial M w/resp to X

EQUIVALENCE
1 ( C , CM(1)), ( FMO, CM(2)), ( A1, CM(3)), ( A2, CM(4))
2 ( A3, CM(5)), ( A4, CM(6))

IF (FX .EQ. 0.) GO TO 3
ARG1 = (-C*FY**2)/FX
ARG1 = - ABS(ARG1)
IF (ABS(ARG1) .GE. 50.) GO TO 5
ARG2 = A1*FX+A2*FX**2 + A3*F Y**2 + A4*FY**4

ARG3 = A1*F Y**2 + A2*FY**2
ARG4 = 2. * A3 * F Y**4 + A4 * F Y**3
EX = EXP(ARG1)
GO TO 10

5 FMS = FMO
FMXS = 0.
FMYS = 0.
RETURN

10 FMS = FMO + EX* ARG2
FMXS = EX* (-ARG1/FX)* ARG2 + ARG3
PAUL = -2.*C*FY/FX
FMYS = EX* (PAUL*ARG2 + ARG4)
RETURN
END
SUBROUTINE SONK(NM, NCR, YM, FY, FX, IER)

N4 = MAX NO OF X, Y ALLOWED. MUST EQUAL DIMENSION OF X, Y, IN MAIN SNIC2840
NCR = NO OF X, Y ACTUALLY COMPUTED SNIC2845
YM = MAX. ALLOWABLE VALUE OF Y SNIC2850
FX = X-VALUES SNIC2855
FY = Y-VALUES SNIC2860
IER = 1 IS NORMAL RETURN SNIC2865
IER = 2 INDICATES AN ERROR SNIC2870
CM = MACH CONSTANTS IN THE EQUATION M = EXP(-CM(1)*Y**2/X)*(CM(3)*X SNIC2875
*CM(4)*X*CM(5)*Y*CM(6)*Y**4) *CM(2) SNIC2880

THE SUBROUTINE COMPUTES A SET OF X AND Y VALUES ON THE WING WHERE SNIC2885
M = 1 SNIC2890

COMMON
*/CM/ CM(6)
DIMENSION FX(I), FY(I)
IER = 1 SNIC2895
C = CM(1) SNIC2900
FMO = CM(2) SNIC2905
A1 = CM(3) SNIC2910
A2 = CM(4) SNIC2915
A3 = CM(5) SNIC2920
A4 = CM(6) SNIC2925

FIRST COMPUTE X WHEN Y = 0
ARG = A1*Y**2 - 4*A2*(FMO - 1.) SNIC2930
IF(ARG .GE. 0.0) GO TO 2
1 IER = 2 SNIC2935
RETURN SNIC2940
2 FX(1) = (.5/A2)*(-A1-SQRT(ARG)) SNIC2945
FY(1) = 0. SNIC2950
IF(FX(1) .LT. 0.0) GO TO 1
IF(FX(1) .LT. 1.0) GO TO 4
FX(1) = (.5/A2)*(-A1-SQRT(ARG)) SNIC2955
IF(FX(1) .LT. 0.0) GO TO 1
IF(FX(1) .GE. 1.0) GO TO 1
4 NCR = 2 SNIC2960
10 NC1 = NCR - 1 SNIC2965
FX(NC1) = FX(NC1) + .01 SNIC2970
X = FX(NC1) + .01 SNIC2975
R = C/X SNIC2980
B = X*(A1*A2*X) SNIC2985
TO = FY(NC1)*X SNIC2990
TM1 = A3*R*B SNIC2995
TM2 = 2.*(A4-R*A3) SNIC3000
TM3 = R*A4 SNIC3005
TM4 = 2.*(A4+R*(R*B-2.*A3)) SNIC3010
TM5 = R*(R*A3-4.*A4) SNIC3015
48
IM6 = R*RA4
IMAX = 1

12 ET = EXP (-R*TO)
FT = ET (B*AT + TO + A4*TO*TO) + FM0 - 1.
FPT = ET (TP1 + TM2*TO - TM3*TO*TO)
FPTF = ET (TM4 + TM5*TO + TM6*TO*TO)
HO = -FT/FPT
IF (IFT < FM0) GT 0.0) GO TO 14
HO = .75*HO

14 TO = TO + HO
IMAX = IMAX + 1

1000 FORMAT (32H0 COMPETION FOR SONIC LINE WILL NOT CONVERGE, HO = E 116.8 )
IF (IMAX LT 10) GO TO 18
WRITE (6,1000) HO
GO TO 1
18 IF (HO LT .0001) GO TO 12
FY (NCR) = SQRT (TO)
IF (NCR GE NM) GO TO 20
IF (FY (NCR) GE YM) GO TO 20
IF (FX (NCR) GE 1.0) GO TO 20
NCR = NCR + 1
GO TO 10
20 RETURN
END
SUBROUTINE POT (NFR, FR, P)

COMMON /XRZ/ SX(101), SXP(101), SY(101), SYP(101), AL(41), TIM(101)

*/CM/ CM(6)

*/ICNT/ IVAR, NCNT, ISORS, IBR, ITRAP, NMAX

*/SOURCE/ XO(20), YO(20)

*/EPS/ E1, E2, FM, YMA

*/NNN/ NSS, NLC3, NLLS

DIMENSION FR(10), P(101,2,10)

CON = - .25/3.14159

XS = XO(NSS)
YS = YO(NSS)

DO 100 N = 1, NCNT

X = SX(N)

Y = SY(N)

T = TIM(N)

RBAR = SYP(N)

100 CONTINUE

10 IF(RBAR .LE. 1.E-9) GO TO 14

FACT = CON/RBAR

ARG = FR(NF) * T

CO = COS(ARG)

SI = SIN(ARG)

P(N,1,NF) = CO * FACT

P(N,2,NF) = - SI * FACT

CONTINUE

14 P(N,1,NF) = UNDEF

P(N,2,NF) = UNDEF

GO TO 10

16 IF(RBAR .LE. 1.E-9) GO TO 14

FACT = CON/RBAR

ARG = FR(NF) * T

CO = COS(ARG)

SI = SIN(ARG)

P(N,1,NF) = CO * FACT

P(N,2,NF) = - SI * FACT

CONTINUE

100 CONTINUE

RETURN

END
SUBROUTINE RK33 (DERIV, CNTRL, Y, ATABL, RTABL, WORK, X, DX, N, IFVO)
  EXTERNAL DERIV, CNTRL
  INTEGER N, NTRY, IERR
  LOGICAL IFVD, IBKP
  REAL Y, DY, ATABL, RTABL, X, DX
  DIMENSION Y(N), DY(N), ATABL(N), RTABL(N)
  DIMENSION WORK(1)
  CALL RKINT (DERIV, CNTRL, Y, ATABL, RTABL, WORK(1), WORK(3), WORK(5)
    , WORK(7), WORK(9), WORK(2*N+9), WORK(4*N+9), WORK(6*N+9)
    , WORK(7*N+9), WORK(8*N+9), X, DX, N, IFVD, IBKP, NTRY, IERR)
  RETURN
END

SUBROUTINE RKINT (DERIV, CNTRL, Y, ATABL, RTABL, WORK, X, DX, N, IFVO, IBKP, NTRY, IERR)
  EXTERNAL DERIV, CNTRL
  INTEGER N, NTRY, IERR
  LOGICAL IFVD, IBKP
  REAL REALY, Y2ERO, ATABL, RTABL, DELTAY, DYHALF, OYZERO, DELTAX, XHALF
  COUPLE PRECISION X, XHALF, XZERO, YZERO
  DIMENSION REALY(N), DY(N), ATABL(N), RTABL(N), Y(N), YHALF(N), YZERO(N)
  IERR = 0
10 DELTAX = DX
  Y = REALX
  GO TO 200
20 Y(I) = REALY(I)
  CALL DERIV
  GO TO 20
30 IF (DX .EQ. 0.) GO TO 230
  DELTAX = DX
  DX2 = DX/X2.
  DX4 = DX/X4.
  XZERO = X
  GO TO 40
40 DYZERO(I) = DY(I)
  DO 110 J = 1, 2
    XHALF = X
    X = X + DX4
    REALX = X
    GO TO 50
50 DELTAY(I) = DY(I) * DX4
    YHALF(I) = Y(I)
    Y(I) = Y(I) + DELTAY(I)
  110 DO I = 1, N
70 REALY(I) = Y(I)
CALL DERIV
DO 60 I=1,N
DELTA(Y(I)) = DELTA(Y(I)) + DY(I) * DX2
Y(I) = YHALF(I) + DY(I) * DX2
60 REALY(I) = Y(I)
CALL DERIV
X = XHALF + DX2
REALX = X
DO 70 I = 1,N
DELTA(Y(I)) = DELTA(Y(I)) + DY(I) * DX2
Y(I) = YHALF(I) + DELTA(Y(I))
70 REALY(I) = Y(I)
CALL DERIV
DO 80 I = 1,N
DELTA(Y(I)) = (DELTA(Y(I)) + DY(I) * DX2) / 3.
Y(I) = YHALF(I) + DELTA(Y(I))
80 REALY(I) = Y(I)
CALL DERIV
GO TO (90,110), J
90 DO 100 I = 1,N
100 DYNHALF(I) = DY(I)
110 CONTINUE
IF (IFVD) GO TO 200
ERRMAX = 0
DO 120 I = 1,N
ERR = ATACL(I) + ABS(RTABL(I) * REALY(I))
IF (ERR .EQ. 0.) GO TO 220
SR = (DYZERO(I) * 4. * DYNHALF(I) + CY(I)) / 3. * DX2
120 ERRMAX = AMAX1(ERRMAX, ABS(SR - (REALY(I) - SNGL(YZERO(I)))) / ERR)
IF (ERRMAX .LT. 1.) 130, 170, 190
IF (ERRMAX .LT. 0.75) 140, 200, 170
140 IF (ERRMAX .LT. 0.075) 150, 200, 200
150 DX = DX * 1.5849532
GO TO 200
160 DX = DX / 1.5849532
IF (.NOT. IEKP) GO TO 180
ERRMAX = ERRMAX / 10.
IF (ERRMAX .GT. 1.) GO TO 180
GO TO 180
180 X = XZERO
DO 190 I = 1,N
Y(I) = YZERO(I)
190 DY(I) = DYZERO(I)
GO TO 30
200 NTRY = 1
CALL CNTRL (NTRY)
GO TO (30, 210, 160, 10), NTRY
210 RETURN
220 IERR = 1
    RETURN
230 IERR = -1
    RETURN
END
## APPENDIX II. Sample Input and Output

<table>
<thead>
<tr>
<th>Deck No.</th>
<th>Programmer</th>
<th>Date</th>
<th>Page of</th>
<th>Job No.</th>
<th>Number</th>
<th>Identification</th>
<th>Description</th>
<th>Do Not Key Punch</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td></td>
<td>NSource (Number of source points, 20 maximum)</td>
<td></td>
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<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td>Nla (Number of A per source, 40 maximum)</td>
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</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td></td>
<td>Npl (Number of planform coordinates, 8 maximum)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td></td>
<td>Nmax (Limit number of points per plot, 100 max)</td>
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<td></td>
<td></td>
<td>3</td>
<td>73</td>
<td>Nf (Number of assumed frequencies, 10 maximum)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>IFVD Logical words - variable interval mode</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FALSE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IFVP if IFVD = FALSE and IFVP = TRUE. THIS CHOICE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TRUE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IS RECOMMENDED. FIXED INTERVAL IF IFVD = TRUE.</td>
<td></td>
</tr>
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</table>

Notes:
- IFVD = Variable Interval Mode
- IFVP = Fixed Interval Mode
- Nsource, Nla, Npl, Nmax, Nf: Integer values
- If IFVD = FALSE, IFVP = TRUE is recommended.

APPENDIX II, Sample Input and Output
<table>
<thead>
<tr>
<th>DECK NO.</th>
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<th>DESCRIPTION</th>
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<td>1</td>
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<td><strong>XO(1)</strong> COORDINATES OF SOURCE POINTS.</td>
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<tr>
<td>13</td>
<td>0</td>
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<td><strong>YO(1)</strong> (ALL GEOMETRY IS NORMALIZED ON b_o, THE)</td>
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<tr>
<td>25</td>
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<tr>
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<td><strong>YO(2)</strong> PORTION OF THE WING.)</td>
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<td>+ 0.0</td>
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<td></td>
<td></td>
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<td>6</td>
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<td><strong>XO(NSOURCE)</strong></td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>2</td>
<td>+ 0.0</td>
<td><strong>YO(NSOURCE)</strong></td>
<td></td>
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<td></td>
<td>73</td>
<td></td>
<td></td>
<td>80</td>
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**55**
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<tr>
<td>13</td>
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<td>CM(2) M = CM(2) + EXP [-CM(1)(Y^2/X)].</td>
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<td>CM(3)x + CM(4) x^2 + CM(5)y^2 + CM(6) y^4</td>
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<tr>
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<td>+ 0.0</td>
<td>THE COEFFICIENTS WERE DETERMINED BY A</td>
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<td>YMAX NORMALIZED SEMI-SPAN</td>
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Page 1 of 8
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<td>PLX(1) planform coordinates. List all corners</td>
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<td>73</td>
<td>PLY(1) starting from left along leading edge</td>
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<td>PLX(3) the right. All geometry is normalized</td>
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## FORTRAN FIXED 10 DIGIT DECIMAL DATA

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<tr>
<td>1</td>
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<td>FMINF REMOTE MACH NUMBER</td>
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<td>TAU MAXIMUM THICKNESS RATIO (T/C)</td>
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<td>FREQ(1) FREQUENCY, FOR COMPUTATION</td>
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<td>FREQ(2) OF POTENTIALS, (RADIANS/SEC)</td>
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<td>FREQ(HF)</td>
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<td>NCNT</td>
<td>ISORS</td>
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\[
\begin{align*}
xu &= 0.180000000E\ 00 \\
yu &= 0.000000000E\ -38 \\
mach\ no. &= 0.97311907E\ 00
\end{align*}
\]

**Acoustic Ray Path for \lambda = 0.57119817 E\ 00**

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<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)-Prime</th>
<th>(r)-Bar</th>
<th>Time</th>
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<td>0.00000000E-38</td>
<td>0.21141810E 00</td>
<td>0.00000000E-38</td>
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<td>0.18424259E 00</td>
<td>0.20000000E-01</td>
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<td>0.51691864E-01</td>
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<td>0.20135082E 00</td>
<td>0.10191058E 00</td>
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<td>0.19445186E 00</td>
<td>0.44702658E-01</td>
<td>0.23404072E-01</td>
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<tr>
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<td>0.21302884E 00</td>
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<tr>
<td>0.21700866E 00</td>
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<tr>
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<tr>
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<td>0.22699061E 00</td>
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<tr>
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<td>0.24104052E 00</td>
<td>0.16353054E 00</td>
<td>0.91917480E-01</td>
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</table>
VELOCITY POTENTIALS ALONG A RAY PATH FOR A SOURCE AT

\[ X_0 = 0.180000000E \ 00 \]
\[ Y_0 = 0.000000000E \ -38 \]
\[ \Lambda = 0.57119817E \ 00 \]

ALTERNATING REAL AND IMAGINARY

**OMEGA = 0.10000000E \ 02**

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<tr>
<th></th>
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<th>-0.109559E \ 02</th>
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<th>-0.421403E \ 01</th>
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<td>-0.173162E \ 01</td>
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<td>-0.147071E \ 01</td>
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<td>0.413568E \ 00</td>
<td>-0.112844E \ 01</td>
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<td>-0.733892E \ 00</td>
<td>0.416219E \ 00</td>
<td>-0.513086E \ 00</td>
<td>0.408919E \ 00</td>
<td>-0.295125E \ 00</td>
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**OMEGA = 0.200000000E \ 02**

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<td>-0.158867E \ 01</td>
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<td>0.792965E \ 00</td>
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<td>-0.915255E \ 00</td>
<td>0.780015E \ 00</td>
<td>-0.694583E \ 00</td>
<td>0.763032E \ 00</td>
</tr>
<tr>
<td>-0.433042E \ 00</td>
<td>0.724092E \ 00</td>
<td>-0.146383E \ CN</td>
<td>0.639566E \ 00</td>
<td>0.128649E \ 00</td>
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**OMEGA = 0.000000000E \ -38**

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</tr>
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APPENDIX III. Application to the Boundary Value Problem

A procedure that may be used to match the tangential flow condition on a wing surface is, in principle, the same as that employed by Rodemich in the box method for uniform sonic flow (Reference 3). The velocity potential at a field point \((x,y,z)\) due to a doublet sheet in its zone of influence, is

\[
\tilde{\phi}(x,y,z) = \frac{3}{2\pi} \int \frac{\Delta \phi\left(\eta, \gamma \right) \phi_0(x-\eta, y-\gamma, z) d\eta d\gamma}{S+W}
\]

where \(\Delta \phi\left(\eta, \gamma \right)\) is the velocity potential discontinuity through the doublet sheet over the region \(S+W\) (the surface and its wake), and

\[
\phi_0(x-\eta, y-\gamma, z) = \frac{-1}{2\pi R} \sum_{n=1}^{N} e^{-i\alpha_n n}
\]

where

\[
R = \sqrt{(x-\eta)^2 + [1-M^2(x,y,z)] [(y-\gamma)^2 + z^2]}
\]

and where \(N\) represents the number of times the wave front passes the field point. In uniform subsonic flow \(N\) equals one, in uniform supersonic flow it equals two, and in the limiting case of uniform sonic flow it equals one. As discussed previously, in uniform sonic flow the stationary portion of the perturbation wave front is not augmented by high frequency signals that follow it; instead, the pressure discontinuity is dissipated by them.

When the local flow in a non-uniform flow field is sonic the wave front gradually becomes stationary and is dissipated. Rays of this type are shown in Figures 9, 12, and 13. In certain regions of non-uniform flow a wave front may pass field points more than twice as shown in Figures 6, 7, 9, 10, and 12. These regions may be in the region of subsonic flow or in supersonic flow. Multiple crossings normally occur on receding portions of the wave front. Ray lines on advancing portions normally pass over the trailing edge before they cross. In these regions of multiple crossings of the wave front, care must be taken to establish an accurate value of \(N\), and of each of the corresponding \(\alpha_n\)'s, \(n = 1, 2, \ldots N\). A computer program that may be used to do this is contained herein. Figures 11 and 13 show that in some regions of both subsonic and supersonic flow even the receding ray lines do not cross. All of Figures 6 through 13 show that once a ray crosses the transition region at the edge of the planform it does not return to the wing region. This characteristic is important because when a doublet solution is employed a ray trace can be ignored once it reaches an edge that is not adjacent to the wake.
The next step in the procedure is to define a grid of square boxes over the region $S + W$, and assume that $\Delta \phi(x, \eta)$ is constant over the area of each box. For this to be a valid assumption as many as 50 boxes along the root chord may be required. The upwash adjacent to the upper surface may be written

$$
\bar{W}(x, y, 0+) = \lim_{z \to 0^+} \frac{\phi(x, y, z)}{z}
$$

or,

$$
\bar{W}(x, y, 0+) = \sum_{i,j} \psi(x_1, y_1 - \eta) \delta \xi \delta \eta
$$

(41)

i.e., the upwash at $(x, y)$ equals the summation (over all boxes $B_{i,j}$ that influence it), of products of the constant velocity potential discontinuities and their downwash influence coefficients. The latter are represented by the double integral of the kernel $\psi$ over the areas of the boxes. The limits of integration and $\Delta \phi$ of Equation (39) are not functions of $z$, so from Equation (40) we get

$$
\psi(x_1, y_1 - \eta) = \frac{-1}{2\pi} \lim_{z \to 0^+} \frac{1}{z} \frac{\partial}{\partial z} \sum \frac{e^{-i\kappa \eta}}{R}
$$

(42)

At this point it is theorized that for non-uniform flow around a nearly planar surface the variation in signal transmission time with distance normal to the surface is approximately equal to the variation in uniform flow, i.e.,

$$
\frac{\partial \delta n}{\partial z} = \frac{\partial}{\partial z} \frac{M(x, y) + R}{C(N^2 - 1)}
$$

or, performing the differentiation

$$
\frac{\partial \delta n}{\partial z} = \pm \frac{2}{C R}
$$

(43)

where the upper sign refers to the advancing portion of the wave front and the lower sign to the receding portion. $C$ is the speed of sound. Making use of equation (43) when taking the derivative in equation (42).

$$
\psi(x_1, y_1 - \eta) = \frac{-1}{2\pi} \frac{R^2 e^{i\kappa \eta}}{CR^3} \sum e^{-i\kappa \eta}
$$

(44)

The $g_n$'s are those obtained by tracing ray paths through the non-uniform flow field.
One way in which Equation (44) may be evaluated and integrated is as follows: Say for nine values of \((f,T)\) on each sending box, the values of the kernel at the center of the receiving box \((x_1,y_j)\) are evaluated. Since the ray paths are not known in advance, each of these values must be interpolated from values in its neighborhood. It is then necessary to evaluate the integral in Equation (41) given the values of the integrand at nine points in the region of integration.

The unknowns in Equation (41) are the \(\overline{\varphi}_i,j\)'s. When the center of a receiving box \((x_1,y_j)\) lies in the subsonic flow region it lies in the zone of influence of every other point in the subsonic region and may lie in the zone of influence of a small portion of the supersonic region (Figure 9). All velocity potentials in zones of mutual influence must be determined simultaneously. Once velocity potentials have been established that meet the tangential flow conditions on the surface and the zero pressure difference condition on the wake they may be fitted with analytical expressions that have the proper edge behavior. Using these expressions, local oscillatory pressures and generalized forces may be obtained in the way outlined in Reference 3.
Two methods have been outlined in detail, and one of them has been mechanized, for calculating acoustic ray paths emanating from any point in a non-uniform transonic flow field surrounding a wing. It gives the ray path, and the time, for the minimum time of travel from the acoustic source point to the field point. The resulting velocity potential is also computed.

It was necessary to establish an accurate representation of the flow characteristics in the field surrounding the wing. Some ray lines travel over the planform and into the surrounding flow field. It was established that once off the planform they do not return.

Available methods predict phase lags based on the assumption that acoustic rays travel in straight lines. The results of this study show this to be a very poor approximation at transonic speeds. Therefore, it is recommended that the method presented in this report be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict these phase lags with reasonable accuracy, and the corresponding flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in the available technology.
Unsteady Aerodynamics, Non-Uniform Transonic Flow, Velocity Potentials, Thickness Effects

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