TR-1359

ELECTRONICALLY TUNABLE
MICROWAVE BANDPASS FILTER

by

Ting Hei Mak
Frank Reggia

October 1967
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An equivalent circuit of two orthogonal striplines coupled by a ferrite (yttrium iron garnet) sphere is derived, using the scattering matrix method. This derivation shows that a nonreciprocal phase shifter can be represented by gyrator networks. Based on this analysis, a YIG bandpass filter, electronically tunable over a very broad frequency range (4:1), is designed by coupling two broadband 180-deg phase shift circuits with a small single crystal YIG sphere.

This report also reviews some basic principles pertaining to filter considerations, analyzes the filter circuit of two orthogonal striplines coupled by a ferrite sphere, and synthesizes the equivalent filter circuit by a scattering matrix method. These results describe the operation of the filter circuit under various terminal excitations.

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ABSTRACT

An equivalent circuit of two orthogonal striplines coupled by a ferrite (yttrium iron garnet) sphere is derived, using the scattering matrix method. This derivation shows that a non-reciprocal phase shifter can be represented by gyrator networks. Based on this analysis, a YIG bandpass filter, electronically tunable over a very broad frequency range (4:1), is designed by coupling two broadband 180-deg phase shift circuits with a small single crystal YIG sphere.

This report also reviews some basic principles pertaining to filter considerations, analyzes the filter circuit of two orthogonal striplines coupled by a ferrite sphere, and synthesizes the equivalent filter circuit by a scattering matrix method. These results describe the operation of the filter circuit under various terminal excitations.

1. INTRODUCTION

The phenomenon of ferrimagnetic resonance has been investigated extensively (ref 1). The application of this phenomenon to the design of microwave narrow-band filters using ferrite (YIG) spheres was first suggested by DeGrasse (ref 2). This original model was later modified to various design configurations electronically tunable over a relatively narrow frequency band (ref 3, 4).

The particular terminal excitation chosen to design a bandpass filter tunable over a very wide frequency range is shown in figure 1. The ferrite sphere is placed in the crossover region 0 of striplines 3-4 and 1-2, and magnetized to ferrimagnetic resonance at a desired operating frequency. Stripline ends 1,2 and 3,4 are then connected to external terminals T₁ and T₂ as shown in figure 1.

The input signal at one terminal is divided into two signals. These two signals, after passing through two different paths, interact with each other to give a null in the electric field at the crossover region 0. To obtain this electric null over a very wide frequency band, one of the two paths must include a broadband 180-deg phase shifter. This electric null is equivalent to a magnetic field maximum, which is necessary for efficient coupling between the ferrite sphere and the rf signal.

If the input signal has the same frequency as the resonance frequency of the magnetized ferrite sphere, it will be coupled to the other stripline circuit by the ferrite and appear at the output.
terminal. This report describes the design and evaluation of such a ferrite bandpass filter that is electronically tunable over a very wide frequency range, beginning first with a review of the basic principles of the filter design.

2. BASIC PRINCIPLES

This section describes the theory of operation, electrical properties of ferrite materials, and performance characteristics of an electronically tunable YIG bandpass filter, using the ferrimagnetic resonance phenomena (ref 1). To understand the operation of this YIG filter, it is first necessary to consider the motion of a magnetic moment $M$ in the presence of a constant magnetic field $H_0$ (fig. 2). When a large static magnetic field is applied to a ferrite material, a torque is exerted on the dipole $M$, given by

$$T = M \times H_0,$$
resulting in a precession of this electronic dipole moment about \( \vec{H}_0 \) at a frequency

\[ \omega = \gamma \vec{H}_0 \]

where \( \gamma \) is a constant (gyromagnetic ratio of the electron). Due to the dissipation of rf energy in the form of heat and re-radiation, the electronic dipoles will in time align themselves with the applied magnetic field \( \vec{H}_0 \), and the precession will stop.

If, in addition to the large static field \( \vec{H}_0 \), an rf magnetic field \( \vec{h} \) (perpendicular to \( \vec{H}_0 \) and with an amplitude much less than that of \( \vec{H}_0 \)) is applied to the dipole, the dipole will precess around the direction of \( \vec{H}_0 + \vec{h} \). If \( \vec{h} \) is circularly polarized, the vector \( \vec{H}_0 + \vec{h} \) will describe a cone around the direction OZ, carrying with it the dipole moment with a precessional motion superimposed on it. For the special case when \( \vec{h} \) rotates around \( \vec{H}_0 \) in the same sense and has the same frequency as that of the dipole rotation, ferrimagnetic resonance occurs. This causes the amplitude of the precession angle to increase until the energy supplied by the rf field balances the losses exactly, at which time the motion of the dipole is maintained in equilibrium at the expense of energy absorbed from the rf field. If the rf magnetic field \( \vec{h} \) rotates in the opposite sense—even at the same frequency as that of the rotating dipole moment—resonance will not occur. Therefore, at ferrimagnetic resonance, the energy absorbed by the uniform precessional motion will be reradiated,
most of which can be coupled to an external circuit if the dissipation-
losses are small.

When the above-described resonance phenomenon is applied to the
design of tunable bandpass filters, a number of important factors
must be considered. It is desirable to have a constant resonance
response, no secondary resonances, very narrow resonance peaks,
small heat loss, tight coupling between input and output terminals
at all operating frequencies, and minimum coupling during off-reso-
nance. Thus, a good tunable filter depends primarily on the ferri-
magnetic material used and the external coupling circuit. Suitable
single crystalline materials having low loss and very narrow reso-
nance line width include YIG, GaYIG (gallium-substituted yttrium
iron garnet), and lithium ferrite.

Thus far, the YIG material has proved to be the best material
for overall performance. Important electromagnetic properties of
the YIG materials that affect the filter performance include satu-
ration magnetization ($4\pi M_S$), line width ($\Delta H$), anisotropy field ($H_a$),
and Curie temperature ($T_C$). These properties are generally frequency
and/or temperature sensitive; their significance is implied in the
following resonance frequency equation (ref 1).

\[ 2\pi f_0 = \gamma \left[ H_o \pm H_a^{\text{eff}} + (N_t - N_z) 4\pi M_S \right] \]

where

- $f_o$ = resonant frequency in MHz,
- $\gamma$ = gyromagnetic ratio for the electron
  \[ (17.6 \times 10^8 \frac{\text{rad}}{\text{Oe/sec}}), \]
- $H_o$ = applied static magnetic field for
tuning (oersteds),
- $H_a^{\text{eff}}$ = effective crystalline anisotropy field
  (Oe), depending on the orientation of $H_o$,
- $N_t$ = transverse demagnetizing factor, and
- $N_z$ = demagnetizing factor in the direction
  of $H_o$.

For a sphere $N_t = N_z = 1/3$, the equation of resonance is simply

\[ 2\pi f_0 = \gamma H_o \pm H_a^{\text{eff}}. \]

The gyromagnetic ratio $\gamma$ of YIG is both temperature and frequency
independent. The anisotropy field $H_a^{\text{eff}}$ is temperature dependent.
and is due to the preferential alignment of the magnetic dipoles along certain crystalline directions. This directional dependence enables the applied field $H_0$ to be adjusted with respect to a certain preferential crystalline direction so that the effective anisotropy field is zero.

Another limitation on the resonance frequency is its lower limit. For resonance to be effectual, the material must be magnetically saturated. Therefore, for low-frequency operation, the saturation magnetization must be small. The last parameter—the line width—is both frequency and temperature sensitive and is broadened by poor surface polish, impurities, and porosity of the material.

Besides the electromagnetic properties mentioned above, the shape of the YIG will affect the resonance frequency, whereas the size of the sample will determine the spurious responses in addition to the desired resonance response. These spurious responses are due to the higher order spin-wave resonances caused by a nonuniform rf field, propagation through the sample, and the behavior of the sample as a dielectric resonator. The size effect can be minimized by making the sample small compared with the wavelength.

As to the external circuit, the important parameters to be considered are minimum coupling between the input and output ports, maximum coupling during resonance, and provision for mounting the sample so that the metallic-wall effect will not influence the performance. Various circuits have been considered by a number of researchers (ref 1, 2, 3). Their problems have included the requirement of more than one sample, complication of tuning when using more samples, limitation on the tunable bandwidth, and spurious responses.

3. THEORETICAL ANALYSIS

This section analyses the circuit properties of the two orthogonal strip-transmission lines coupled by a magnetized ferrite sphere, using the scattering matrix method. An equivalent circuit is obtained from the scattering matrix, and one excitation condition of the terminals is shown to be applicable to the design of a very broadband tunable bandpass filter.

The above circuit consists of two striplines, the center conductors of which cross each other at right angles as shown in figure 3. The YIG sphere is located symmetrically in the crossover region between the two center conductors, the size and position of which are chosen such that the rf field will not be disturbed appreciably at off-resonance frequencies. The coordinates and terminal designations are shown in this figure.
With the applied static magnetic field $\vec{H}_0$ in the X-direction and the YIG biased at magnetic saturation, the susceptibility tensor is given by (ref 1)

$$
\chi = \begin{bmatrix}
0 & 0 & 0 \\
0 & \chi_{22} & \chi_{23} \\
0 & -\chi_{32} & \chi_{33}
\end{bmatrix}
$$

where for negligible loss,

$$
\chi_{33} = \chi_{22} = \frac{\omega \mu_0 m}{\omega^2 - \omega_m^2} \quad \chi_{23} = -j \frac{\omega \mu_0 m}{\omega^2 - \omega_m^2}
$$

$$
\omega_m = 4\pi \gamma M_s \text{ (rad/sec)}
$$

$$
\omega_o = \gamma H_o + H_{eff}^a \text{ (rad/sec)}
$$

$$
\omega = \text{rf frequency} \text{ (rad/sec)}.
$$

Since the stripline propagates a TEM wave, the magnetic component of the rf field at the YIG sphere is in the Y-direction for strip 1-2 and in the Z-direction for strip 3-4. The positive directions of propagation in the two strips are the same as the directions of
the coordinate axes Y and Z. An rf field is assumed incident at terminal 1, and the other terminals are terminated in matched impedances. When the YIG sphere is in an rf magnetic field, a precessing magnetic dipole moment is induced in it. This dipole moment radiates an rf field which is guided by the external medium. Therefore, the resultant magnetic field at the YIG sphere is not the incident magnetic field only.

Collin has shown (ref 5) that to find the field radiated by an arbitrary magnetic moment \( \mathbf{M} \) in a cylindrical waveguide is to expand the radiated field in terms of a suitable set of normal modes, and to determine the amplitude coefficients in this expansion by an application of the Lorentz reciprocity theorem. His results show that when the field radiated in the forward direction is represented by

\[
\mathbf{E}^+ = \sum_n a_n \mathbf{E}_n^+ \\
\mathbf{h}^+ = \sum_n a_n \mathbf{h}_n^+
\]

and the field radiated in the backward direction is given by

\[
\mathbf{E}^- = \sum_n b_n \mathbf{E}_n^- \\
\mathbf{h}^- = \sum_n b_n \mathbf{h}_n^-
\]

where \( a_n \) and \( b_n \) are amplitude coefficients and \( \mathbf{E}_n^-, \mathbf{h}_n^-; \mathbf{E}_n^+, \mathbf{h}_n^+ \) are mode functions in the forward and backward directions, respectively.

Then

\[
2b_n = j\omega \mu \mathbf{h}_n^+ \cdot \mathbf{M} \\
2a_n = j\omega \mu \mathbf{h}_n^- \cdot \mathbf{M},
\]

where \( \mu \) is the permeability of the medium. These results are also valid for coaxial transmission lines.

In the case of strip-transmission lines, only the TEM mode is assumed to propagate and its amplitude is negligibly small at a short distance from the center conductor. Therefore, the stripline can be approximated by a coaxial transmission line and the above analysis can be used. Except for a change of coordinates and their respective vectorial directions, the TEM mode has the same magnetic functional form \( \mathbf{h} \) for all terminals. They also have the same magnitude at the origin of the coordinate system, because the sphere is symmetrically located with respect to the two striplines.
The YIG sphere—under the influence of the incident field and reradiating field—can be replaced by two arbitrary orthogonal magnetic dipole moments, \( M_z \) and \( M_y \). However, because of the tight coupling which exists between these orthogonal dipole moments in the YIG, it is necessary to impose the condition that the dipole moments induced by \( M_z \) and \( M_y \), given by the higher order coupling terms in the equation of motion, cancel each other. For this condition, the effective susceptibility tensor of the YIG sphere which is responsible for creating the moments \( M_z \) and \( M_y \) must be different when radiation occurs under the excitation of the incident wave.

Let the incident magnetic field at terminal 1 be \( \vec{A}' \vec{h}_y \), where \( \vec{y} \) is a unit vector in the \( Y \)-direction. The resultant magnetic field at the YIG sphere that induces the moments \( M_z \) and \( M_y \) will be solely determined by the incident wave and the radiation field of \( M_y \), and therefore in the \( Y \)-direction. Also, let the effective susceptibility tensor of the YIG sphere be

\[
\chi_e = \begin{pmatrix}
0 & 0 & 0 \\
0 & \chi_{22} & \chi_{23} \\
0 & -B_{23} & D_{23}
\end{pmatrix}
\]

and the resultant magnetic field be \( \vec{h}' \vec{y} \), where \( B, C, \) and \( D \) are to be determined. The magnetization of the YIG sphere is then given by

\[
\vec{m} = \chi_e \cdot \vec{h}' \vec{y} = \begin{pmatrix}
\chi_{22} \vec{h}' \\
-\vec{B}_{23} \vec{h}'
\end{pmatrix}
\]

If the YIG sphere has a volume \( V \) and the applied static and rf magnetic field are uniform inside the ferrite sphere, the total magnetic moment is \( \vec{M} = \vec{V} \vec{m} \). This moment \( \vec{M} \) is considered as a point magnetic dipole moment with components \( M_z \) and \( M_y \), situated at the origin \( 0 \) and reradiating rf energy into the external circuit.

From Collin's results, the waves coming from the terminals are given by

- terminal 1 \( \vec{h}_{11} = b_1 \vec{h}_1 = j \frac{2}{\mu} \chi_{22} V \vec{h}' \vec{h}_1 \)
- terminal 2 \( \vec{h}_{21} = a_1 \vec{h}_2 = -j \frac{2}{\mu} \chi_{22} V \vec{h}' \vec{h}_2 \)
- terminal 3 \( \vec{h}_{31} = b_1 \vec{h}_3 = j \frac{2}{\mu} \chi_{23} V \vec{h}' \vec{h}_3 \)
- terminal 4 \( \vec{h}_{41} = b_1 \vec{h}_4 = -j \frac{2}{\mu} \chi_{23} V \vec{h}' \vec{h}_4 \)
Since it has been assumed that the dipole moments induced by $M_z$ and $M_x$ in the YIG sphere will cancel each other, the resultant $Y$-component of these induced dipole moments is

$$
\chi_{22} \nu \left( j \omega \mu \chi_{22} V h^2 h' \right) + B \chi_{23} \nu \left( j \omega \mu \chi_{23} h^2 B h' V \right) = 0,
$$

where the first term is the $Y$-component of the dipole induced by the waves in the stripline 1-2, and the second term is the $Y$-component of the dipole induced by the waves in the stripline 3-4. $B$ is found from this equation to be

$$
B = \frac{\omega}{\omega_0}.
$$

Since there are no secondary induced dipole moments radiating additional fields, the effective magnetic field at the YIG is

$$
h' y = \left[ A' h - j \frac{\omega}{\omega_0} \nu \chi_{22} V h^2 h' \right] \left( \frac{v}{1 + j \omega_0 \chi_{22} V h^2} \right) y,
$$

where the first term is the magnitude of the incident wave, the second term is the magnitude of the reflected wave, the third term is the magnitude of the transmitted wave, and their respective vectorial directions have been included in the signs before each term.

Solving for $h'$,

$$
h' = A' h / (1 + j \omega_0 \chi_{22} V h^2),
$$

where $A' h$ is the amplitude of the incident magnetic field.

Finally, taking the reference plane at the YIG sphere, the waves coupled to the four terminals for an incident rf field at terminal 1 are obtained from Collin's results:

**Incident:** $\gamma A' h e^{-\beta y}$

**Reflected:** $h_{11} = y \left[ j \frac{\omega}{\omega_0} \frac{\chi_{22} V h^2}{1 + j \omega_0 \chi_{22} V h^2} \right] (A' h) e^{-\beta y}, z \leq 0$

**Transmitted:** $h_{21} = y \left[ 1 - j \frac{\omega}{\omega_0} \frac{\chi_{22} V h^2}{1 + j \omega_0 \chi_{22} V h^2} \right] (A' h) e^{-\beta y}, z \geq 0$

**Terminal 3:** $h_{31} = z \left[ \frac{\omega_0}{\omega} \frac{j \omega}{\chi_{22} V h^2} \right] (A' h) e^{\beta y}, y \leq 0$

**Terminal 4:** $h_{41} = z \left[ - j \frac{\omega}{\omega_0} \frac{\chi_{22} V h^2}{1 + j \omega_0 \chi_{22} V h^2} \left( \frac{\omega_0}{\omega} \right) \right] (A' h) e^{\beta y}, y \geq 0
where \( \vec{x}, \vec{y}, \vec{z} \) denote the unit vectors in the X-, Y-, Z-directions, respectively. When a similar analysis is applied to the other terminals and letting

\[
s_1 = \frac{j \frac{\omega}{2} \mu \omega \chi_{23} \text{Vh}^2}{1 + j \omega \mu \chi_{22} \text{Vh}^2},
\]

\[
s_2 = \left( \frac{\omega}{\text{Vh}} \right) \frac{j \frac{\omega}{2} \mu \omega \chi_{33} \text{Vh}^2}{1 + j \omega \mu \chi_{22} \text{Vh}^2},
\]

\( a_1, a_2, a_3, a_4 \) = the magnitudes of the incident waves at the respective terminals, and

\( b_1, b_2, b_3, b_4 \) = the magnitudes of the waves travelling away from the YIG sphere to the respective terminals; the input and output are related by the scattering matrix

\[
\begin{array}{c|cccc}
| & s_1 & l-s_1 & -s_2 & s_2 \\
\hline
b_1 & 1-s_1 & s_1 & s_2 & -s_2 \\
b_2 & s_2 & -s_2 & s_1 & l-s_1 \\
b_3 & s_2 & s_1 & l-s_1 & s_1 \\
b_4 & l-s_1 & s_1 & s_2 & -s_2 \\
\end{array}
\]

Substituting the expressions for \( \chi_{22} \) and \( \chi_{33} \) to obtain \( s_1 \) and \( s_2 \),

\[
s_1 = \frac{j \frac{\omega}{2} \mu \omega \omega \text{mVh}^2}{1 + j \omega \mu \frac{\omega \text{mVh}^2}{\omega \text{O} - \omega ^2}}
\]

\[
s_1 \big| j \omega = p = \frac{A}{2} \frac{p}{p^2 + Ap + \omega \text{O}}
\]

where \( A = \mu \omega \text{O} \omega \text{mVh}^2 \)

and \( s_2 = \frac{j \frac{\omega}{2} \mu (-J \chi_{22}) \text{Vh}^2}{1 + j \omega \mu \chi_{22} \text{Vh}^2} = -js_1 \)
The magnitudes of the elements for the above scattering matrix are less than unity. To realize the scattering matrix with passive circuit elements, \( 1 - \tilde{S} S \geq 0 \), where \( S \) denotes the conjugate transposition of \( S \).

Thus,

\[
\tilde{S} S = \begin{bmatrix}
  s_1^* & 1-s_1^* & j s_1^* & -j s_1^* \\
 1-s_1^* & s_1^* & -j s_1^* & j s_1^* \\
-j s_1^* & j s_1^* & s_1^* & 1-s_1^* \\
 j s_1^* & -j s_1^* & 1-s_1^* & s_1^*
\end{bmatrix} \begin{bmatrix}
  s_1 & 1-s_1 & j s_1 & -j s_1 \\
 1-s_1 & s_1 & -j s_1 & j s_1 \\
-j s_1 & j s_1 & s_1 & 1-s_1 \\
 j s_1 & -j s_1 & 1-s_1 & s_1
\end{bmatrix}
\]

where \( s^* \) denotes the conjugate of \( s \). Multiplying the two matrices together and simplifying, it is found that

\[
\tilde{S} S = \begin{bmatrix}
  1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{bmatrix}
\]

4. **SYNTHESIS OF EQUIVALENT FILTER CIRCUIT**

When attempting to realize the scattering matrix \( S \) derived above, it is found that both the admittance and impedance matrices do not exist and, therefore, \( S \) cannot be realized by the admittance or impedance method. To synthesize by the scattering matrix method, the following procedure is used (ref 6,7). The augmented admittance matrix \( Y_A \) of the scattering matrix \( S \) is given by \( Y_A = \frac{1}{2}(E - S) \); that is,

\[
Y_A = \frac{1}{2} \begin{bmatrix}
  1-s_1 & -(1-s_1) & -j s_1 & j s_1 \\
 -(1-s_1) & 1-s_1 & j s_1 & -j s_1 \\
 j s_1 & -j s_1 & 1-s_1 & -(1-s_1) \\
 -j s_1 & j s_1 & -(1-s_1) & 1-s_1
\end{bmatrix}
\]

where \( E \) is the identity matrix. This augmented admittance matrix can be diagonalized by a unitary transformation given by

\[
u = \frac{1}{2} \begin{bmatrix}
  1 & 1 & 1 & 1 \\
 -1 & 1 & 1 & -1 \\
 j & 1 & -1 & -j \\
 -j & 1 & -1 & j
\end{bmatrix}
\]

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Thus,
\[
\tilde{U} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2s) \end{bmatrix} U = Y_A U
\]

Since \( S = E - 2Y_A \) and \( \tilde{S} = \tilde{U}SU = E - 2\tilde{U}Y_AU \),

\[
\tilde{S} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (4s_1 - 1) \end{bmatrix}
\]

This diagonal matrix \( \tilde{S} \) represents a four-port network where the input reflection coefficient of each port is given by the value of the respective diagonal element. For the fourth port, the reflection coefficient \( \Gamma \) is \((4s_1 - 1)\). Therefore, the input impedance of this port is

\[
Z = \frac{1 + \Gamma}{1 - \Gamma} = \frac{2s_1}{1 - 2s_1}
\]

Since \( s_1 = \frac{A}{2} \frac{p}{p^2 + Ap + \omega_0^2} \), where \( p = j\omega \),

then,

\[
Z = \frac{Ap}{p^2 + \omega_0^2},
\]

which represents a parallel-resonant circuit as shown in figure 4.

\[\text{Figure 4. Equivalent parallel-resonant circuit.}\]
The second and third ports of $\hat{S}$ are open circuited, whereas the first port is a short circuit.

To convert the circuit for $\hat{S}$ to the network represented by $S$, it is seen that $\hat{USU} = \hat{S}$. If $\hat{V}, \hat{I}$ represent the voltage and current, respectively, for network $\hat{S}$, then

$$(\hat{V} - \hat{I}) = (\hat{USU})(\hat{V} + \hat{I}) = \hat{S}(\hat{V} + \hat{I})$$

and

$$U(\hat{V} - \hat{I}) = S[U(\hat{V} + \hat{I})].$$

Also, $(V - I) = S(V + I)$

where $V$, $I$ are the normalized voltage and current for network $S$.

Therefore,

$$V = \frac{1}{2}[(V + I) + (V - I)] = \frac{1}{2}U[(\hat{V} + \hat{I}) + (\hat{V} - \hat{I})] = UV$$

$$I = \frac{1}{2}[(\hat{V} + \hat{I}) - (\hat{V} - \hat{I})] = \frac{1}{2}\hat{U}[(V + I) - (V - I)] = \hat{U}I$$

Thus, the network for $S$ is obtained by connecting a multiport ideal transformer $U$ to $\hat{S}$.

It is observed in $U$ that an ideal transformer with complex turns ratio (shown in top of figure 5) is required. This transformer can be decomposed into the configuration shown in bottom of figure 5, where the complex transformer at the center represents a nonreciprocal lossless phase shifter. The proof of this is shown by defining a generalized gyrator

$$V_2 = \alpha I_1$$

$$V_1 = -\alpha^* I_2$$

where $\alpha$ is complex and equals $e^{j\theta}$. If the magnitude of $\alpha$ is not unity, real transformers of appropriate turns ratio can be extracted so that $\alpha$ is reduced to the form of $e^{j\theta}$.

The impedance matrix $Z$ for the generalized gyrator is

$$Z = \begin{bmatrix} 0 & -\alpha^* \\ \alpha & 0 \end{bmatrix}.$$
while its scattering matrix is

$$S = (Z - E)(Z + E)^{-1}$$

$$= \frac{1}{\bar{a}} \begin{vmatrix} 0 & -2\alpha^* \\ 2\alpha & 0 \end{vmatrix} = \begin{vmatrix} 0 & -e^{-j\theta} \\ e^{j\theta} & 0 \end{vmatrix}$$

---

Figure 5. Complex ideal transformer circuit.

This shows that the rf wave in going from port 1 to 2 has a phase shift of \(-\theta + 180\) deg while the wave from port 2 to 1 has a phase shift of \(\theta\) deg. This generalized gyrator is lossless and passive because

$$\hat{SS} = \begin{vmatrix} 0 & e^{-j\theta} \\ e^{j\theta} & 0 \end{vmatrix} \begin{vmatrix} 0 & -e^{-j\theta} \\ e^{j\theta} & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$
The complex ideal transformer can now be synthesized by extracting generalized gyrators so that the final network has the same terminal characteristic as that of the ideal complex transformer, resulting in the gyrator network shown in figure 5. The terminal characteristics of such a gyrator network are given by

\[ I_a = e^{j\theta} V_1 = e^{j\theta} I_b = \frac{I_b}{(e^{j\theta})^*} \]

\[ V_a = e^{j\theta} I_1 = e^{j\theta} V_b, \]

which are the same as that for the ideal complex transformer. Thus, when an incident wave travels from port b to port a, its phase changes by \( \theta \) deg; whereas, for an incident wave travelling in the opposite direction, the phase change is \((-\theta + 180 + 180)\) or \(-\theta\) deg. Therefore, an ideal complex transformer with a complex turns ratio of \( e^{j\theta}:1 \) is a nonreciprocal phase shifter, having a differential phase shift of \(2\theta\) deg.

With an ideal complex transformer now defined, the multiport ideal transformer \( U \) or the YIG filter scattering matrix \( S \) can be found. The voltage relation connecting the network of \( \hat{S} \) and the network of \( S \) is given by \( V = \hat{U}V \). Expanding this relationship results in

\[ V_1 = \frac{1}{2}(\hat{V}_1 + \hat{V}_2 + \hat{V}_3 + \hat{V}_4) \]

\[ V_2 = \frac{1}{2}(-\hat{V}_1 + \hat{V}_2 + \hat{V}_3 - \hat{V}_4) \]

\[ V_3 = \frac{1}{2}(j\hat{V}_1 + \hat{V}_2 - \hat{V}_3 - j\hat{V}_4) \]

\[ V_4 = \frac{1}{2}(-j\hat{V}_1 + \hat{V}_2 - \hat{V}_3 + j\hat{V}_4) \]

which is represented by the equivalent circuit of the cross-coupled tunable YIG filter shown in figures 9 and 10 of reference 8.

5. DESIGN OF 180-DEGREE PHASE SHIFTERS

The analysis given in section 4 shows that if equal and out-of-phase signals are fed into the opposite terminals of two orthogonal striplines coupled by a YIG sphere, equal and out-of-phase signals will appear at the other two terminals when the YIG sphere is magnetized to ferrimagnetic resonance. To provide 180-deg phase difference between the signals over a wide bandwidth, broadband phase shifters are required.

A circuit consisting of two paths differing by a multiple of half-wave-lengths is inherently narrowband. However, the design of a broadband 90-deg phase shifter, given by Schiffman (ref 9) and later
extended by Cristal and Shelton (ref 10, 11), is applicable to the solution of the above problem. The basic configuration of this broad-band phase shifter consists of a section of coupled lines as shown at the top of figure 6.

The image impedance $Z_1$ and the phase shift $\phi$ of the Shiffman model—called the phase section—are given by

$$Z_1 = \sqrt{Z_{00}Z_{oe}}$$

$$\cos \phi = \frac{Z_{oe} - \tan^2 \theta}{Z_{oe} + \tan^2 \theta} \frac{Z_{oe}}{Z_{00}}$$

where $Z_{oe}$ is the characteristic impedance of one line to ground when equal in-phase currents flow in both lines, $Z_{00}$ is the characteristic impedance of one line to ground when equal out-of-phase currents flow in both lines, and $\theta = \beta L$ is the electrical length of a uniform line of length $L$ and phase constant $\beta$.

The phase shift $\phi$, with $\rho = Z_{oe}/Z_{00}$ as a function of $\theta$ is given at the bottom of figure 6. The phase of a uniform line of length $3L$ and a coupled line of impedance ratio $\rho = 3$ are compared in figure 7. The difference in this phase $\phi$ is given in figure 8. These results show that by choosing the length of the uniform line and the impedance ratio $\rho$ of the coupled section, the difference in phase between the two lines can be nearly constant over a broad frequency band.

The deviation of the phase difference from the constant value in the operating band can be further minimized by connecting error-correcting networks or additional coupled lines to the terminals of the phase section (figure 9). The extent to which the variation of phase difference can be minimized and operating band can be broadened is limited by the mechanical tolerance of this phase section, the transmission-line losses, and the junction effects.

From mechanical and bandwidth considerations, a 180-deg phase shifter was fabricated by cascading four 45-deg phase shifters with a 5:1 bandwidth and 1.0046-deg ripple, the design parameters of which are given in references 10 and 11. The arrangement of the 180-deg phase shifter designed for 1.5- to 7.5-GHz frequency range, using teflon fiberglass as the dielectric medium, is shown in figure 10. The center conductor, represented by the solid lines, is etched on one side of a 1/64-in. thick teflon-fiberglass board and is connected electrically at the ends to the center conductor (represented by the dotted lines) on the opposite side of the board. This board is then placed between two 1/16-in. boards (center of fig. 10) where the two
outer solid lines represent the ground plane, and the two other lines represent the center conductors of the phase section. The phase difference between the paths along the two arms ACO and ABO is designed for 180 ± 4 deg over the desired operating bandwidth.

Figure 6. Phase shift versus electrical line length.
Figure 7. Characteristics of a 90-deg phase shifter.
Figure 8. Phase shift versus electrical line length.

Figure 9. Design characteristic of error-correcting network.
Figure 10. A 180-deg phase-shift circuit.
6. EXPERIMENTAL EVALUATION

The performance of the YIG bandpass filter depends primarily on the accuracy of the 180-deg phase shifter circuit to obtain an electrical null at the overlapping point 0 as mentioned in section 1. To evaluate the phase shifter circuit as designed in section 4, the experimental setup shown in figure 11 is used. As seen in this figure, the input power is divided into two equal signals, one passing through the 180-deg phase shifter and interacting with the signal in the other arm to give an electric null at point 0. To determine the position of this null, a tapered coplanar stripline probe is used to sample the rf power, the tip of which is a distance from the stripline center conductor at least equal to its width. The axes of the probe and center conductor are perpendicular to each other, the probe being able to slide a small distance parallel to the axis of this stripline center conductor. The stub tuner is used for impedance matching the probe to the detector for maximum sensitivity of the measuring system, and the isolator is used to minimize the reactive loading of the phase shifter on the rf generator.

Denoting the probe position as + and − with respect to the point 0 (shown in fig. 11), the experimental results of the two 180-deg phase shifters are given in figure 12. The deviation, in degrees from the desired 180-deg phase difference at the point 0, is given in millimeters from the reference null point 0. The 45- and 90-deg deviations are also shown in this figure as dotted lines symmetrical about the new reference null position for comparison. The results of these data show that the reference null should be −5 mm from the original position. Possible causes of the large deviations are probably due to the reactive effect of the bends and junctions, and inaccuracy in the etching process of the phase-shift networks.

To assemble a YIG bandpass filter, the two 180-deg phase shifters are cross coupled so that the new reference points 0 of each circuit overlap. A suitable hole is then made between these two points to accommodate the YIG sphere, oriented with its easy axis perpendicular to the plane of the phase circuit by a strong magnetic field while it is loosely held in this position.

Results of the insertion-loss measurements (using substitution method) made on the above bandpass filter at near resonance frequencies are given in figure 13. Although the insertion loss of this first test model was relatively high and its bandwidth varied over the design frequency range, these first results do show that the above phase shifter-YIG bandpass filter is capable of operating effectively over a very wide frequency range. Figure 14 shows the model used to obtain these results.
Figure 11. Experimental setup for evaluation of 180-deg phase shifter.
Figure 12. Performance of 180-deg phase shifter.
Assuming that a 45-deg deviation from 180-deg phase difference exists at the new reference null position, an insertion loss of only 1.5 dB is calculated from the scattering matrix of the filter at resonance. Also assuming the copper loss, reflection loss of the phase shifters, and the dissipation loss of the YIG sphere together equal 1.5 dB, the remaining losses are probably due to insufficient coupling and the generation of higher order modes caused by non-uniform fields in the YIG sphere region. An Airtron 112-225 YIG
sphere (0.053 in. diam and a resonance line width of 0.5 Oe) was used for the above measurements.

Design modifications of the above tunable bandpass filter have been completed for minimizing the phase errors caused by the reactive effects of the junctions and bends, and the inaccuracies of the etching process used to fabricate the phase-shift network. This later design model, using 1/8-in. ground plane spacing, was fabricated and evaluated over the operating frequency range of 2 to 8 GHz. The results obtained include an insertion loss of 4 ± 1 dB at all resonant frequencies and an isolation of greater than 30 dB at the off-resonant frequency over the entire operating bandwidth. Further improvements in the performance of this tunable bandpass filter are expected in a final design model (being fabricated) by more carefully aligning the stripline circuitry associated with the 180-deg phase shifter.

7. CONCLUSIONS

The theoretical analysis and experimental results evolving from this investigation indicate the feasibility of designing a YIG bandpass filter, magnetically tunable over more than a 4:1 frequency range. The limit of the tunable bandwidth is determined by the practical difficulties in the fabrication of the 180-deg phase shifters and the magnetic properties of the YIG sphere, including its size. The upper frequency limit is set by the reactive discontinuities and mechanical tolerances of the phase-shift circuits, whereas the lower frequency limit is determined by the saturation magnetization of the YIG sphere.

8. REFERENCES


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An equivalent circuit of two orthogonal striplines coupled by a ferrite (yttrium iron garnet) sphere is derived, using the scattering matrix method. This derivation shows that a nonreciprocal phase shifter can be represented by gyrator networks. Based on this analysis, a YIG bandpass filter, electronically tunable over a very broad frequency range (4:1), is designed by coupling two broadband 180-deg phase shift circuits with a small single crystal YIG sphere.

This report also reviews some basic principles pertaining to filter considerations, analyzes the filter circuit of two orthogonal striplines coupled by a ferrite sphere, and synthesizes the equivalent filter circuit by a scattering matrix method. These results describe the operation of the filter circuit under various terminal excitations.
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1. Ferrites
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