NEUBER'S RULE APPLIED TO FATIGUE
OF NOTCHED SPECIMENS

by
T. H. Topper, R. M. Wetzel, J. Morrow
Department of Theoretical and Applied Mechanics
University of Illinois, Urbana
Contract No. N156-46083

Distribution of this document
is unlimited
NOTICE

Reproduction of this document in any form by other than naval activities is not authorized except by special approval of the Secretary of the Navy or the Chief of Naval Operations as appropriate.

The following Espionage notice can be disregarded unless this document is plainly marked CONFIDENTIAL or SECRET.

This document contains information affecting the national defense of the United States within the meaning of the Espionage Laws, Title 18, U.S.C., Sections 793 and 794. The transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law.
FOREWORD

This investigation was conducted in the H. F. Moore Fracture Research Laboratories of the Department of Theoretical and Applied Mechanics, University of Illinois, in cooperation with the Aeronautical Structures Laboratory of the Naval Air Engineering Center.

This report covers work performed during the period 1 February 1966 through 30 April 1967, and together with report No. NAEC-ASL-1115 constitutes the final report on Item 2 of Contract N156-46083. Messrs. M. S. Rosenfeld and R. E. Vining acted as technical liaison for the Navy and Professor T. J. Dolan, Head of Theoretical and Applied Mechanics, furnished administrative and technical guidance.
SUMMARY

A method is presented for predicting the fatigue life of notched members from smooth specimen fatigue data. Inelastic behavior of the material at the notch root is treated using Neuber's rule which states that the theoretical stress concentration factor is equal to the geometric mean of the actual stress and strain concentration factors. This provides indices of equal fatigue damage for notched and unnotched members.

Experimental results for notched aluminum alloy plates subjected to one or two levels of completely reversed loading are compared with predictions based on these indices. Measured notched fatigue lives and lives predicted from smooth specimens agree within a factor of two.
TABLE OF CONTENTS

I. INTRODUCTION 1
II. ANALYSIS 1
III. DISCUSSION 2
IV. COMPARISON WITH EXPERIMENTAL RESULTS 4
V. CONCLUSIONS 5
VI. REFERENCES 6

Tables
1-a  FATIGUE DAMAGE AT FAILURE FOR NOTCHED 2024-T351 PLATES 8
1-b  FATIGUE DAMAGE AT FAILURE FOR NOTCHED 7075-T651 PLATES 9

Figures
1a  Smooth Specimen Fatigue Data in a Form Suitable for Predicting Lives of Notched Members 10
1b  Notched Fatigue Data Compared to the Life Curve Predicted from Smooth Specimen Data (2024-T3) 11
1c  Notched Fatigue Data Compared to the Life Curve Predicted from Smooth Specimen Data (7075-T6) 12
2   Cyclic Stress-Strain Curves 13
LIST OF SYMBOLS

E  Modulus of elasticity
S  Nominal stress on a notched member; axial load divided by net area
e  Nominal strain; strain which would occur in a smooth specimen subjected to S; equal to S/E when the nominal strain is elastic
σ  Actual stress at a point, frequently at a notch root
ε  Actual strain at a point, frequently at a notch root
ΔS, Δe, Δσ, Δε  Peak to peak change in the above quantities during one reversal
Kₜ  Theoretical stress concentration factor
Kₐ  Stress concentration factor, Δσ divided by ΔS
Kₑ  Strain concentration factor, Δε divided by Δe
K₇  Fatigue strength reduction factor or effective "fatigue stress concentration factor"
a  Material constant (see Eq. 1)
r  Notch root radius
I. INTRODUCTION

Stowell (1) and Neuber (2) have developed analyses which help describe the nonlinear stress-strain behavior of notches. Their work has recently been applied to the notch fatigue problem by a number of authors (3-6). These authors relate the cyclic load range on a notched member to the actual stress or strain range at the notch root and then estimate the life of the notched member from stress vs life or strain vs life plots obtained from smooth specimens.

An alternate approach is presented here which makes it unnecessary to solve for the actual stress or strain at the notch root. Instead, Neuber's rule is used to convert the smooth specimen data for a given metal into a master life plot which can be used to estimate the fatigue life of any notched member made of that particular metal.

II. ANALYSIS

The theoretical stress concentration factor, $K_t$, only applies when the material at the notch root remains elastic. Neuber (2) has proposed a rule which may be applied even when the material at the notch root is strained into the inelastic region. He states that the theoretical stress concentration factor is equal to the geometric mean of the actual stress and strain concentration factors.

$$K_t = \left( K_a K_e \right)^{1/2}$$

Neuber's Rule

That the product of $K_a$ and $K_e$ might be constant is intuitively reasonable because $K_a$ decreases and $K_e$ increases as yielding occurs.

It is well known that small notches have less effect in fatigue than is indicated by $K_t$. Several authors have suggested theoretical or empirical expressions for evaluating a "fatigue stress concentration factor," $K_f$, which corrects for size effect. In this paper we employ $K_f$ factors based on Peterson's approach (7)

$$K_f = 1 + \frac{K_e - 1}{1 + \frac{a}{r}}$$

(1)

where $r$ is the root radius and "a" is a material constant determined from long life fatigue data for sharply notched specimens. For notches with large radii $K_f$ is nearly equal to $K_t$. For sharp notches, however, $K_t$ is unnecessarily conservative and $K_f$ should be used in preference to $K_t$. 

1
To apply Neuber's rule to the notch fatigue problem, $K_f$ will be used in place of $K_t$ and $K_\sigma$ and $K_\epsilon$ are written in terms of ranges of stress and strain.

$$K_f = \left( \frac{\Delta \sigma}{\Delta S} \frac{\Delta \epsilon}{\Delta \epsilon} \right)^{1/2}$$

It is convenient to write the above equation in the following form:

$$K_f (\Delta S \Delta \epsilon E)^{1/2} = (\Delta \sigma \Delta \epsilon E)^{1/2} \quad \cdots \cdots \quad (2)$$

where $\Delta S$ and $\Delta \epsilon$ are the nominal stress and strain ranges applied to a notched member, $\Delta \sigma$ and $\Delta \epsilon$ are the local stress and strain ranges at the notch root, and $E$ is the elastic modulus.

Note that Eq. (2) reduces to the following simple form if the nominal stress and strain are limited to the elastic region.

$$K_f \Delta S = (\Delta \sigma \Delta \epsilon E)^{1/2} \quad \cdots \cdots \quad (2a)$$

This special case is important because it covers many problems of engineering interest.

At even longer lives and lower values of $\Delta S$, the notch root remains essentially elastic and Eq. (2) reduces to the familiar form

$$K_f \Delta S = \Delta \sigma \quad \cdots \cdots \quad (2b)$$

This is the equation which is frequently misused at shorter lives when the material near the notch behaves inelastically.

III. DISCUSSION

Equation (2) relates the nominal stress-strain behavior of a notched member to the actual stress-strain behavior at the critical location. It can also be interpreted as furnishing indices of equal fatigue damage in notched and unnotched specimens. In completely reversed, constant amplitude tests, a notched specimen and a smooth specimen will form detectable cracks at the same life provided $K_f (\Delta S \Delta \epsilon E)^{1/2}$ for the notched specimen is equal to $(\Delta \sigma \Delta \epsilon E)^{1/2}$ for the smooth specimen. This means that life data from notched and unnotched specimens can be plotted on the same graph or that smooth specimen results can be used to produce master life plots for estimating the fatigue life of notched members.
Figure 1a is an example of such a master plot of the quantity 
$(\Delta \sigma \Delta e E)^{1/2}$ vs life for two aluminum alloys using data reported by Endo and Morrow (8). Points represent failure of smooth specimens for which the value of $(\Delta \sigma \Delta e E)^{1/2}$ was calculated from steady-state stress and strain ranges. It is well documented (9) that the stress and strain ranges of unnotched specimens approach a steady-state value after a small percentage of life and Blatherwick and Olsen (10), and Crew and Hardrath (4) have shown that the strain range at a notch root rapidly stabilizes. Recent results from our laboratory (11) using the same metals shown in Fig. 1a, indicate that rapid stabilization of the hysteresis loop occurs following a step change in strain amplitude.

The life of a notched member can be predicted by entering the value of $K_f(\Delta S \Delta e E)^{1/2}$ on the ordinate of smooth specimen curves of the type shown in Fig. 1a. In the low life region, the loads may be large enough to cause yielding throughout the specimen. If this happens $\Delta e$ must be determined by entering $\Delta S$ on a cyclic stress-strain curve (Fig. 2). At longer lives there is no need for the cyclic stress-strain curve since the nominal strains are essentially elastic. In this case, the quantity $K_f(\Delta S \Delta e E)^{1/2}$ reduces to $K_f \Delta S$.

Some of the limitations on the above approach to the notch fatigue problem will now be discussed.

Crack Initiation and Propagation: The above method is limited to predicting crack initiation or final failure where the crack propagation stage is negligible. This is usually the case for small unnotched specimens of the type used to obtain fatigue life data.

In service applications, crack propagation may occupy a widely varying portion of the useful life of notched members and structures. Weight critical applications represent one extreme. The tendency is to surround notches with a minimum of elastic material and to select a high strength and therefore relatively brittle metal. In this case crack propagation may be a small part of the total life. On the other hand, heavy structures made of ductile metal may have relatively large flaws present from the beginning and will occupy their entire life in propagating a crack to failure.

Effect of Mean and Residual Stress: The reader is reminded that the mean stress at the notch root has been assumed to be zero. Thus, the present approach is inadequate for predicting the effect of mean loads on the fatigue life of notched members. Even if the loading is completely reversed, but the level is changed during the test, the creation and relaxation of mean stress at notch roots may complicate the notch problem. Large tensile loads tend to induce compressive mean stresses for subsequent smaller
cycles while large compressive loads induce tensile mean stresses. The ensuing fatigue life may be greatly altered. The problem is further complicated by the fact that mean stresses at the notch root will tend to relax toward zero in the presence of sufficient cyclic plastic strain (11).

Using Eq. (2) with the restrictions and limitations discussed above, it is possible to predict the lives of many types of notched specimens from readily available smooth specimen fatigue data. It should be noted that curves of $(\Delta \sigma \Delta \epsilon)^{1/2}$ vs life can be easily derived from any two of the following curves: stress vs life, total strain vs life, plastic strain vs life, and cyclic stress vs cyclic strain.

IV. COMPARISON WITH EXPERIMENTAL RESULTS

Two metals are considered, 2024 and 7075 aluminum alloys. Due to the nearly identical fatigue properties of the T3, T351 and T4 conditions of 2024 and T6 and T651 conditions of 7075, no distinction needs to be made between these various conditions over the life region of interest here.

The smooth curves in Figs. 1b and c are transferred from Fig. 1a. They represent the predicted lives of notched members of these metals. Points are from Illg's data for notched plates with $K_t$ values of 2.0 and 4.0 (12). Loading was completely reversed and therefore did not introduce significant mean stress.

Values of $K_r$ calculated from Eq. (1) are used in preference to $K_t$. The value of "$a" for use in Eq. (1) was determined in the following manner: A value of $K_r$ for Illg's sharply notched specimen was found directly by comparison of long life data for the sharply notched specimen with data for unnotched specimens. The $K_r$ thus determined is 3.0 for both materials; the value of $K_t$ is 4.0, and the root radius, $r$, is 0.057 in. These values of $K_r$, $K_t$, and $r$ were substituted into Eq. (1) and "a" was determined for use in calculating $K_r$ for notches of other geometries. The value of "a" for both 7075 and 2024 was found to be approximately 0.028 in.

Agreement between life data and predictions is seen to be good for 2024 and excellent for 7075. The relationship should be checked for other materials, particularly those with a yield point.

Step Tests: The curves in Fig. 1 were also used to perform a linear damage summation for notched specimens subjected to two levels of reversed loading as a part of this investigation. Damage is defined as the number of reversals which occur at a given load level divided by the reversals to failure predicted from Fig. 1. The results of these tests are given in Table 1.
Specimens are similar to those used by Blatherwick and Olson (10). The radii of the notches are 0.25 in. or greater, so that there is no significant difference in $K_t$ and $K_f$.

Although only two amplitudes of loading were used in each test, the amplitude was frequently changed from one level to the other. Tests were planned so that nearly equal damage was done at each level. About 20 changes in level were made in each test. Visible cracks were never observed until the last 20% of life and usually not until the last 10%. The total damage summations in Table 1 are remarkably close to 1.0.

Even though the loading was completely reversed, there is a possibility of a mean stress effect depending upon how the amplitude is changed from the large to the small level. If the last peak reached at the higher amplitude is tensile, a beneficial compressive mean stress may be present for subsequent cycles at the lower amplitude. The effect may be detrimental if the last peaks at the higher level are compressive. Only two specimens were tested in a manner which could create compressive mean stresses and the results are inconclusive. However, for more severely notched specimens subjected to a few large load cycles followed by many smaller ones the mean stress effect can be significant.

**IV. CONCLUSION**

The equation

$$K_f (\Delta S \Delta \epsilon E)^{1/2} = (\Delta \sigma \Delta \epsilon E)^{1/2}$$

or for the case where nominal strains are essentially elastic.

$$K_f \Delta S = (\Delta \sigma \Delta \epsilon E)^{1/2}.$$

relates the behavior of notched specimens to readily available smooth specimen data. Master plots of $(\Delta \sigma \Delta \epsilon E)^{1/2}$ vs life based on smooth specimen fatigue results may be used to accurately predict fatigue of notched aluminum alloy plates subjected to completely reversed loading.
VI. REFERENCES


<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>4-2</th>
<th>4-3</th>
<th>4-5</th>
<th>4-8</th>
<th>4-9</th>
<th>4-10</th>
<th>4-39</th>
<th>4-40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_t$</td>
<td>1.84</td>
<td>1.84</td>
<td>1.84</td>
<td>2.43</td>
<td>2.43</td>
<td>2.43</td>
<td>1.84</td>
<td>1.84</td>
</tr>
<tr>
<td>1st level $K_t \Delta S$ (ksi)</td>
<td>97.5</td>
<td>136</td>
<td>136</td>
<td>183</td>
<td>164</td>
<td>143</td>
<td>98</td>
<td>70</td>
</tr>
<tr>
<td>2nd level $K_t \Delta S$ (ksi)</td>
<td>139</td>
<td>69.2</td>
<td>94.8</td>
<td>91.3</td>
<td>73.1</td>
<td>180</td>
<td>141</td>
<td>139</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Sigma n_1/N_{11}$</th>
<th>10,500</th>
<th>640</th>
<th>740</th>
<th>290</th>
<th>200</th>
<th>750</th>
<th>6,100</th>
<th>52,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma n_2/N_{12}$</td>
<td>12,000</td>
<td>1,600</td>
<td>1,600</td>
<td>350</td>
<td>600</td>
<td>1,200</td>
<td>11,000</td>
<td>75,000</td>
</tr>
<tr>
<td>Total Damage</td>
<td>1.23*</td>
<td>1.06</td>
<td>0.98</td>
<td>1.22</td>
<td>1.00</td>
<td>1.30**</td>
<td>1.04</td>
<td>1.17</td>
</tr>
</tbody>
</table>

* Buckled due to a machine failure before a visible fatigue crack had formed.
** Last peaks at higher level were tensile tending to cause compressive mean stress. Last peaks were compressive in all other cases.
<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>5-1</th>
<th>5-3</th>
<th>5-5</th>
<th>5-6</th>
<th>5-7</th>
<th>5-8</th>
<th>5-9</th>
<th>5-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_t$</td>
<td>2.43</td>
<td>1.84</td>
<td>1.84</td>
<td>2.43</td>
<td>1.84</td>
<td>1.84</td>
<td>1.84</td>
<td>1.84</td>
</tr>
<tr>
<td>1st level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_t \Delta S$ (ksi)</td>
<td>203</td>
<td>89.1</td>
<td>149</td>
<td>88.4</td>
<td>84.3</td>
<td>82.3</td>
<td>152</td>
<td>109</td>
</tr>
<tr>
<td>2nd level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_t \Delta S$ (ksi)</td>
<td>92.1</td>
<td>130</td>
<td>68.3</td>
<td>199</td>
<td>155</td>
<td>123</td>
<td>83.0</td>
<td>152</td>
</tr>
<tr>
<td>Damage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma n_1/N_{f1}$</td>
<td>180</td>
<td>16,500</td>
<td>550</td>
<td>10,500</td>
<td>39,400</td>
<td>15,000</td>
<td>580</td>
<td>3,850</td>
</tr>
<tr>
<td>$\Sigma n_2/N_{f2}$</td>
<td>7,000</td>
<td>1,080</td>
<td>82,500</td>
<td>250</td>
<td>350</td>
<td>990</td>
<td>25,300</td>
<td>680</td>
</tr>
<tr>
<td>Total Damage</td>
<td>0.88</td>
<td>0.89</td>
<td>0.97</td>
<td>1.07**</td>
<td>1.21</td>
<td>0.54</td>
<td>1.02</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* Last peaks at higher level were tensile tending to cause compressive mean stress. Last peaks were compressive in all other cases.
Fig. la Smooth Specimen Fatigue Data in a Form Suitable for Predicting Lives of Notched Members

2Nf, Reversals to Failure

Pts. are Ave. of Several Tests

Endo and Morrow

2024 - T4

7075 - T6 Illo

1000 500 200 100 50

\( \sigma \) vs. \( \log N_f \)

\( \sigma \) in ksi

10^2 10^3 10^4 10^5 10^6

10
Fig. 1b Notched Fatigue Data Compared to the Life Curve Predicted from Smooth Specimen Data
Open Symbols Indicate $\Delta S = \Delta e$ such that $K_f = \Delta e$ and $\Delta S$

Thus, $K_f (\Delta e E)^{1/2} = K_f \Delta S$

$2N_f$, Reversals to Failure

Fig. 1c Notched Fatigue Data Compared to the Life Curve Predicted from Smooth Specimen Data
Fig. 2. Cyclic Stress - Strain Curves

\( \Delta \sigma / 2 \), Stress Amplitude, kL

\( \Delta \varepsilon / 2 \), Strain Amplitude
A method is presented for predicting the fatigue life of notched members from smooth specimen fatigue data. Inelastic behavior of the material at the notch root is treated using Neuber's rule which states that the theoretical stress concentration factor is equal to the geometric mean of the actual stress and strain concentration factors. This provides indices of equal fatigue damage for notched and unnotched members.

Experimental results for notched aluminum alloy plates subjected to one or two levels of completely reversed loading are compared with predictions based on these indices. Measured notched fatigue lives and lives predicted from smooth specimens agree within a factor of two.
### INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. Give the inclusive dates when a specific reporting period is covered.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** Enter the total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

   1. "Qualified requesters may obtain copies of this report from DDC."
   2. "Foreign announcement and dissemination of this report by DDC is not authorized."
   3. "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through..."
   4. "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through..."
   5. "All distribution of this report is controlled. Qualified DDC users shall request through..."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 153 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.