THE HYPERSONIC WAKE PROGRAM
I. A FORTRAN PROGRAM
FOR THE INTEGRAL SOLUTION
by
James E. Wollrab
May 1966

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U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama
THE HYPERSONIC WAKE PROGRAM
I. A FORTRAN PROGRAM
FOR THE INTEGRAL SOLUTION

by

James E. Wollrab

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Aerodynamics Branch
Advanced Systems Laboratory
Research and Development Directorate
U.S. Army Missile Command
Redstone Arsenal, Alabama 35809
ABSTRACT

A FORTRAN program is explicitly listed for the basic integral solution of the wake of a hypersonic blunt or conical reentry body. The near wake pressure gradient, nonequilibrium chemistry, and laminar-turbulent transition are included.
CONTENTS

Abstract ................................................. ii

1. Introduction ........................................... 1

2. Definition of the Flow Field ........................... 1

3. Basic Conservation Equations, Transformations, and Boundary Conditions ...................................... 2

4. Reacting Air Mixture .................................... 8

5. Laminar-Turbulent Transition and the Pressure Gradient ........................................... 11

6. Sequence of Calculations ................................ 13

Literature Cited ........................................... 15

Appendix A. Fortran Program .............................. 17

Appendix B. Sample Calculations ........................... 39
### SYMBOLS

<table>
<thead>
<tr>
<th>Text</th>
<th>Program*</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td>Base area, feet^2, Equation (70).</td>
</tr>
<tr>
<td>A', A''</td>
<td></td>
<td>Constants, Equations (60) and (61).</td>
</tr>
<tr>
<td>ALT</td>
<td>CD</td>
<td>Altitude, feet</td>
</tr>
<tr>
<td>CD</td>
<td></td>
<td>Drag coefficient, Equation (70).</td>
</tr>
<tr>
<td>cp</td>
<td></td>
<td>Specific heat at constant pressure of air mixture, °K, Equation (15).</td>
</tr>
<tr>
<td>cp(i)</td>
<td></td>
<td>Specific heat of species i, °K, Equation (15).</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>Drag, pounds, Equation (69).</td>
</tr>
<tr>
<td>Dijave</td>
<td></td>
<td>Average binary diffusion coefficient between species i and j, Equation (8).</td>
</tr>
<tr>
<td>d</td>
<td>LD</td>
<td>Base diameter of body, feet.</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>Electronic energy, feet^2 per second^2, Equation (54).</td>
</tr>
<tr>
<td>Echem</td>
<td></td>
<td>Zero-point energy, feet^2 per second^2, Equation (54).</td>
</tr>
<tr>
<td>eint</td>
<td></td>
<td>Internal energy, feet^2 per second^2 per unit mass, Equation (4).</td>
</tr>
<tr>
<td>Evib(i)</td>
<td></td>
<td>Vibrational energy, feet^2 per second^2 (equal to Planck's constant times (\gamma^{(i)})).</td>
</tr>
<tr>
<td>g</td>
<td></td>
<td>Electronic degeneracy, Equation (54).</td>
</tr>
<tr>
<td>H</td>
<td></td>
<td>Total enthalpy, feet^2 per second^2, Equation (57).</td>
</tr>
</tbody>
</table>

*See footnote at end of symbols.*
<table>
<thead>
<tr>
<th>Text</th>
<th>Program*</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>LHXN ( h(x, n) )</td>
<td>Enthalpy, ( \text{feet}^2 \text{ per second}^2 ), Equation (53).</td>
</tr>
<tr>
<td>( Hn_0 )</td>
<td>HNNO</td>
<td>( (\partial^2 H/\partial n^2)_0 ), Equation (43).</td>
</tr>
<tr>
<td>( \bar{T} )</td>
<td></td>
<td>Unit tensor, Equation (6).</td>
</tr>
<tr>
<td>K</td>
<td></td>
<td>Thermal conductivity, Equation (7).</td>
</tr>
<tr>
<td>k</td>
<td></td>
<td>Boltzmann's constant, Equation (54).</td>
</tr>
<tr>
<td>( L_e )</td>
<td>LEO ( (L_e_0) )</td>
<td>Lewis number, Equation (10).</td>
</tr>
<tr>
<td>M</td>
<td>W</td>
<td>Molecular weight of atmosphere model, Equation (74).</td>
</tr>
<tr>
<td>( M_\infty )</td>
<td>MIN</td>
<td>Free-stream Mach number, Equation (64).</td>
</tr>
<tr>
<td>m</td>
<td></td>
<td>Transformation variable, Equation (23).</td>
</tr>
<tr>
<td>( M^{(i)} )</td>
<td>FM</td>
<td>Mass of ( i )th species, Equation (54).</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>Transformed radial coordinate, Equation (24).</td>
</tr>
<tr>
<td>p</td>
<td>PRESS</td>
<td>Pressure, ( \text{pounds per foot}^2 ), Equation (6).</td>
</tr>
<tr>
<td>Pr</td>
<td>PRO ( (Pr_0) )</td>
<td>Prandtl number, Equation (10).</td>
</tr>
<tr>
<td>q</td>
<td></td>
<td>Dynamic pressure, ( \text{pounds per foot}^2 ), Equation (70).</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td></td>
<td>Heat flux vector, Equation (4).</td>
</tr>
<tr>
<td>R</td>
<td></td>
<td>Radial distance to shock, feet, Equation (69).</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td>Radial coordinate, feet, Equation (25).</td>
</tr>
<tr>
<td>T</td>
<td>TEMP</td>
<td>Temperature (kinetic), °K, Equation (75).</td>
</tr>
<tr>
<td>( T_m )</td>
<td>TEMPM</td>
<td>Molecular-scale temperature, Equation (76).</td>
</tr>
</tbody>
</table>

*See footnote at end of symbols.
<table>
<thead>
<tr>
<th>Text</th>
<th>Program*</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>t</strong></td>
<td></td>
<td>Time, seconds.</td>
</tr>
<tr>
<td><strong>u</strong></td>
<td></td>
<td>Velocity in downstream (x) direction, feet per second, Equation (9).</td>
</tr>
<tr>
<td><strong>u_∞</strong></td>
<td>JIN</td>
<td>Free-stream velocity, feet per second.</td>
</tr>
<tr>
<td><strong>V</strong></td>
<td></td>
<td>Diffusion velocity, feet per second, Equation (5).</td>
</tr>
<tr>
<td><strong>v</strong></td>
<td></td>
<td>Velocity in the radial (r) direction, feet per second, Equation (9).</td>
</tr>
<tr>
<td><strong>v</strong></td>
<td></td>
<td>General fluid velocity, feet per second, vector, Equation (1).</td>
</tr>
<tr>
<td><strong>x</strong></td>
<td></td>
<td>Downstream coordinate, feet, Equation (9).</td>
</tr>
<tr>
<td><strong>x_τ</strong></td>
<td>XT</td>
<td>Laminar turbulent transition coordinate, feet, Equation (64).</td>
</tr>
<tr>
<td><strong>Δx</strong></td>
<td></td>
<td>Increment of x, feet.</td>
</tr>
<tr>
<td><strong>x_C</strong></td>
<td>START X</td>
<td>Initial x value, feet.</td>
</tr>
<tr>
<td><strong>STOP X</strong></td>
<td></td>
<td>Terminal x value, feet.</td>
</tr>
<tr>
<td><strong>X_i</strong></td>
<td></td>
<td>End-point altitudes, feet, Equation (76).</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td></td>
<td>Transition coordinate parameter, Equation (64).</td>
</tr>
<tr>
<td><strong>Y_i</strong></td>
<td></td>
<td>End-point temperatures, Equation (77).</td>
</tr>
<tr>
<td><strong>Z_0</strong></td>
<td>ZOE_1</td>
<td>Compressibility factor on the axis.</td>
</tr>
<tr>
<td><strong>A, B, C, D, E,</strong></td>
<td></td>
<td>Constants (Paragraph 4).</td>
</tr>
<tr>
<td><strong>F, K, L, N, P</strong></td>
<td></td>
<td>Atmosphere constants, Equations (77) through (81).</td>
</tr>
<tr>
<td><strong>B, C, D, E_j, H_j</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>ALPH</td>
<td>Species concentration, Equation (11).</td>
</tr>
</tbody>
</table>

*See footnote at end of symbols.
<table>
<thead>
<tr>
<th>Text</th>
<th>Program*</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>DEL</td>
<td>Radial distance in $x, r$ coordinate system, feet, Equation (72).</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>DELM2 ($\delta_m^2$)</td>
<td>Transformed distance to outer edge of wake, feet, Equation (25).</td>
</tr>
<tr>
<td>$\theta$</td>
<td>THETA</td>
<td>Momentum thickness, feet, Equation (17).</td>
</tr>
<tr>
<td>$\theta_E$</td>
<td></td>
<td>Energy thickness, feet, Equation (18).</td>
</tr>
<tr>
<td>$\theta^{(i)}$</td>
<td></td>
<td>Concentration thickness, feet, Equation (19).</td>
</tr>
<tr>
<td>$\Lambda_c$</td>
<td>DELTAC</td>
<td>Constant depending on initial conditions, Equation (44).</td>
</tr>
<tr>
<td>$\Lambda^{(i)}_e$</td>
<td></td>
<td>Defined by Equation (59).</td>
</tr>
<tr>
<td>$\mu$</td>
<td>MU</td>
<td>Viscosity coefficient, $\frac{lb \cdot sec}{ft^2}$, Equation (6).</td>
</tr>
<tr>
<td>$\gamma^{(i)}$</td>
<td></td>
<td>Vibrational frequency of $i^{th}$ molecular species, Equation (54).</td>
</tr>
<tr>
<td>$\rho$</td>
<td>DENSNG</td>
<td>Density, Equation (1), $\frac{lb \cdot sec^2}{ft^4}$</td>
</tr>
<tr>
<td>$\phi_N$</td>
<td>PHI</td>
<td>Cone half-angle, degrees.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>S2B</td>
<td>Shock half-angle, degrees, Equation (67).</td>
</tr>
<tr>
<td>$\phi$</td>
<td></td>
<td>Coordinate in initial cylindrical coordinate system (not used in equations).</td>
</tr>
</tbody>
</table>

**SUBSCRIPTS AND SUPERSCRIPTS**

- $i$ $I$: Index for eight chemical species.
- $c$ $C$: Initial $x$ coordinate for calculation.

*See footnote at end of symbols.*

vii
<table>
<thead>
<tr>
<th>Text</th>
<th>Program*</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>E</td>
<td>Values at ( n = 1 ).</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>Values at ( n = 0 ).</td>
</tr>
<tr>
<td>( \infty )</td>
<td>IN</td>
<td>Free-stream conditions.</td>
</tr>
</tbody>
</table>

*Only a few representative program symbols are listed; however, unlisted symbols can be deduced from the text equations. The units used in the program are feet, pounds, seconds, and degrees Kelvin. Corresponding units are assumed throughout the text unless noted otherwise.
1. Introduction

The analysis presented in this report is essentially a programming of the hypersonic axisymmetric wake problem as formulated by Bloom and Steiger and Ness and Fanucci. The object of this work is to use the fundamental wake analysis as a basis for an extension of the problem to include contributions from ablation and base mixing and to determine their effects on the initial wake conditions and concentrations of radiating species. The FORTRAN program is in Appendix A. To be complete and internally consistent, the program is described in detail beginning with the basic conservation equations. The solutions are obtained for an air model of eight species using nonequilibrium chemistry. The downstream pressure gradient in the wake and the laminar-turbulent transition are included. However, a number of important assumptions have been employed to make the analysis tenable. The units used in the program are feet, pounds, seconds, and degrees Kelvin. Corresponding units are assumed throughout the text, unless noted otherwise.

2. Definition of the Flow Field

The basic regions of the flow field accompanying a hypersonic body as it enters the atmosphere have been extensively studied both experimentally and theoretically. Air is heated irreversibly and compressed by the strong detached bow shock associated with a blunt body. The fluid entering the boundary layer overexpands around the rear edge of the body. In this base-flow region, the fluid is recompressed and turned back parallel to the axial direction producing a secondary shock which is weaker than the main bow shock. Fluid which is not captured by the recirculating base flow passes through a high pressure, minimum diameter neck and into the viscous wake. A conical body is characterized by an attached shock which produces much less heating in the inviscid portion of the wake and by a thicker boundary layer.

The inner viscous wake may be divided into two regions distinguished by the presence of an axial pressure gradient. The near wake extends from the vicinity of the neck to the downstream coordinate at which the pressure has essentially decayed to the ambient value. The far wake is then controlled primarily by diffusion processes. Large pressure gradients and high temperatures characterize the initial portions of the near wake. The outer inviscid wake passes through the bow shock and the recompression shock to be "swallowed" by the radial growth of the inner wake at some downstream coordinate determined by the flight...
characteristics and the laminar-turbulent transition point of the inner wake.

3. Basic Conservation Equations, Transformations, and Boundary Conditions

When both the chemical and fluid-mechanical properties of the flow are considered, conservation relations can be written for the momentum, energy, and chemical species. In addition, an equation expressing global continuity (conservation of mass) is valid. Using concise vector notation, the continuity equation is

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \]  

(1)

Because the calculations are done under the assumption of steady flow, all time derivatives vanish.

\[ \nabla \cdot (\rho \mathbf{v}) = 0 \]  

(2)

Similarly, the momentum, energy, and species equations are respectively

\[ \nabla \cdot (\rho \mathbf{v}\mathbf{v} + \mathbf{p}) = 0 \]  

(3)

\[ \nabla \cdot (\frac{1}{2} \rho |\mathbf{v}|^2 \mathbf{v} + \rho c_{int} \mathbf{v} \cdot \mathbf{v} + \mathbf{p} + \mathbf{q}) = 0 \]  

(4)

\[ \nabla \cdot \rho \mathbf{v} \mathbf{v} = \left( \frac{\partial \rho}{\partial t} \right)_{chem} \]  

(5)

\( \mathbf{p} \) is the pressure tensor, \( \mathbf{q} \) is the heat flux vector, and \( \mathbf{v}^{(i)} \) is the diffusion velocity.

\[ \mathbf{p} = \mathbf{p} - \mu \left[ \nabla \mathbf{v} + (\nabla \mathbf{v})^T - \frac{2}{3} \left( \nabla \cdot \mathbf{v} \right) \mathbf{I} \right] \]  

(6)

\[ \mathbf{q} = -K \nabla T + \sum_i \rho^{(i)} \mathbf{v}^{(i)} n^{(i)} \]  

(7)
\[ \nabla(i) = -D_{ijave} \nabla a(i)/a(i) \] (8)

Superscript \( i \) represents the \( i \)th chemical species of the reacting air mixture. \( D_{ijave} \) is the average binary diffusion coefficient between species \( i \) and \( j \). \( \Upsilon \) represents the unit tensor, \( K \) is the thermal conductivity, and \( \epsilon_{int} \) represents the internal energy per unit mass of the air mixture. The remaining notation can be obtained from the list of symbols.

The physical coordinate system consists of a downstream coordinate \( x \), a radial coordinate \( r \), and an angular coordinate \( \phi \). However, the cylindrical symmetry of the problem removes any \( \phi \) dependence. It is also possible to apply boundary layer approximations if the gradients normal to the flow direction are stronger than those along the flow direction.

a. **Momentum**

\[
\frac{\partial}{\partial x} (r u^2) + \frac{\partial}{\partial r} (r p u v) - \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{\partial}{\partial x} (r p) = 0
\] (9)

b. **Energy**

\[
\frac{\partial}{\partial x} (r p u H) + \frac{\partial}{\partial r} (r p v H) = \frac{\partial}{\partial r} \left( \frac{\mu}{Pr} r \frac{\partial H}{\partial r} \right)
\]

\[
+ \frac{\partial}{\partial r} \left[ \frac{\mu}{Pr (Pr-1)} r u \frac{\partial u}{\partial r} \right] + \frac{\partial}{\partial r} \left[ \frac{\mu}{Pr (Le-1)} r \sum_{i} h^{(i)} \frac{\partial a^{(i)}}{\partial r} \right]
\] (10)

c. **Chemical Species**

\[
\frac{\partial}{\partial x} (r p u a^{(i)}) + \frac{\partial}{\partial r} (r p v a^{(i)})
\]

\[
- \frac{\partial}{\partial r} \left( \frac{Le}{Pr} \frac{\partial a^{(i)}}{\partial r} \right) = \left( \frac{\partial p^{(i)}}{\partial t} \right)_{chem}
\] (11)
d. **Continuity**

\[
\frac{\partial}{\partial x} (rpu) + \frac{\partial}{\partial r} (rpv) = 0 \tag{12}
\]

Using the expression for the temperature gradient

\[
\nabla T = \frac{1}{\varepsilon_p} \nabla \left( H - \frac{u^2}{2} \right) - \frac{1}{\varepsilon_p} \sum_i h^{(i)} \nabla a^{(i)} \tag{13}
\]

and Equation (8) for the diffusion velocity, the heat flux vector may be written as

\[
\overline{q} = -\frac{\mu}{Pr} \nabla H + \frac{\mu}{Pr} \nabla \left( \frac{u^2}{2} \right) + \frac{\mu}{Pr} (1 - Le) \sum_i h^{(i)} \nabla a^{(i)} \tag{14}
\]

\[
\tau_p = \sum_1 \alpha^{(i)} \varepsilon_p (i). \tag{15}
\]

The conservation equations can now be integrated across the wake from \( r = 0 \) to \( r = \delta \). \( \delta(x) \) represents the wake thickness as a function of the downstream coordinate \( x \). Using Leibnitz's rule for the derivative of an integral, the continuity equation becomes

\[
(rp^v)_e = (rp^u)_e \frac{d\delta}{dx} - \frac{d}{dx} \int_0^\delta rpu dr. \tag{16}
\]

Further defining the integrals

\[
\theta = \int_0^\delta \frac{pu}{\rho_e u_e} \left( 1 - \frac{u}{u_e} \right) r dr \tag{17}
\]

\[
\theta E = \int_0^\delta \frac{pu}{\rho_e u_e} \left( 1 - \frac{H}{H_e} \right) r dr \tag{18}
\]

\[
\theta^{(i)} = \int_0^\delta \frac{pu}{\rho_e u_e} \left( a_e^{(i)} - a^{(i)} \right) r dr \tag{19}
\]
the integral form of the momentum, energy, and chemical species equations in the \( x, r \) system is

\[
\frac{d\theta}{dx} + \theta \frac{d}{dx} \left( \ln \rho_0 u_e^2 \right) - \left( \int_0^\delta \frac{\rho u}{\rho_0 u_e} \, rdr \right) \frac{d}{dx} \ln u_e \\
- \frac{\delta^2}{2 \rho_0 u_e^2} \frac{dp}{dx} = 0
\]

(20)

\[
\rho_0 u_e \theta_E = \text{constant}
\]

(21)

\[
\frac{d\theta^{(i)}}{dx} + \theta^{(i)} \frac{d}{dx} \left( \ln \rho_0 u_e \right) - \left( \int_0^\delta \frac{\rho u}{\rho_0 u_e} \, rdr \right) \frac{d\alpha_e^{(i)}}{dx} =
\\
- \frac{1}{\rho_0 u_e} \int_0^\delta \left( \frac{\partial p^{(i)}}{\partial t} \right)_{\text{chem}} \, rdr.
\]

(22)

The transformed radial coordinate \( n \) is now introduced by the Dorodnizten transformation through the relationships

\[
mdm = \frac{\rho}{\rho_0} \, rdr
\]

(23)

\[
m = n \delta_m
\]

(24)

which lead to

\[
\delta^2 \frac{m}{\rho_0} \frac{dn}{dx} = \rho \, rdr.
\]

(25)

The effect of this transformation is to remove the radial density dependence from the calculations by introducing the \( x, n \) coordinate system in place of the cylindrical \( x, r \) system.

Several sets of boundary conditions apply to the concentrations, velocity, and enthalpy at the axis (\( n = 0 \)) and on the outer edge (\( n = 1 \)).
n = 0:

\[ a^{(i)} = a_0^{(i)}(x) ; \left( \frac{\partial a^{(i)}}{\partial n} \right)_0 = 0 \]  
\[ u = u_0 (x) ; \left( \frac{\partial u}{\partial n} \right)_0 = 0 \]  
\[ H = H_0 (x) ; \left( \frac{\partial H}{\partial n} \right)_0 = 0 \]  

n = 1:

\[ a_e^{(i)} (x) ; \left( \frac{\partial a_e^{(i)}}{\partial n} \right)_e = \left( \frac{\partial^2 a_e^{(i)}}{\partial n^2} \right)_e = 0 \]  
\[ u = u_e (x) ; \left( \frac{\partial u}{\partial n} \right)_e = \left( \frac{\partial^2 u}{\partial n^2} \right)_e = 0 \]  
\[ H = H_e ; \left( \frac{\partial H}{\partial n} \right)_e = \left( \frac{\partial^2 H}{\partial n^2} \right)_e = 0 \]  

These boundary conditions follow from a consideration of the physical model of the symmetrical wake. From these conditions it is possible to formulate profile expressions of the proper form

\[ \frac{a^{(i)}(x,n) - a_0^{(i)}(x)}{a_e^{(i)}(x) - a_0^{(i)}(x)} = \frac{u(x,n) - u_0(x)}{u_e(x) - u_0(x)} = 6n^2 - 8n^3 + 3n^4. \]  

For the enthalpy, \( \left( \frac{\partial^2 H}{\partial n^2} \right)_0 \) is included by using a fifth-degree polynomial.

\[ H = H_0 + (H_e - H_0) (10n^3 - 15n^4 + 6n^5) \]
\[ + \frac{1}{2} H_{nn0} (n^2 - 3n^3 + 3n^4 - n^5) \]  

The momentum Equation (9), the energy Equation (10), and the chemical species Equation (11) are next specialized to \( n = 0 \) and \( n = 1 \) after transformation to the \( x,n \) system.
Because the stagnation enthalpy is assumed to be constant along the outer edge of the wake, the energy equation is automatically satisfied.

The transformed integral momentum equation is

\[
\frac{d\theta}{dx} = -\theta \frac{d\ln \rho_e}{dx} + \left[ 2\theta + \frac{\delta_m^2}{2} - \frac{\delta_m^2}{10} \left( 4 + \frac{u_0}{u_e} \right) \right] \frac{1}{\rho_e u_e^2} \frac{dp}{dx}
\]

where

\[
\delta_m^2 = \frac{210 \theta}{\left( 1 - \frac{u_0}{u_e} \right) \left( 10 + 11 \frac{u_0}{u_e} \right)}
\]

The integral form of the energy Equation (21) remains unchanged and the constant is given by the value of \( \rho_e u_e \theta_e \) at the initial coordinate of the calculation.
\[ \rho_e \mathbf{u}_e \mathbf{\theta}_E = (\rho_e \mathbf{u}_e \mathbf{\theta}_E)_c \]  

(41)

where

\[ \mathbf{\theta}_E = \frac{\nu_m}{22} \left[ \frac{1}{210} \left( 1 - \frac{H_0}{H_e} \right) (349 + 311 \frac{u_0}{u_e}) \right] \]  

\[ - \frac{H_{nn_0}}{840 H_e} \left( 43 + 23 \frac{u_0}{u_e} \right) \]  

(42)

The shape parameter \( H_{nn_0} \) is then

\[ H_{nn_0} = 4(H_e - H_0) \left[ \frac{349 u_e + 311 u_0}{43 u_e + 23 u_0} \right] - \frac{(\rho_e \nu_m)_c \Lambda_c}{\rho_e \nu_m (43 u_e + 23 u_0)} \]  

(43)

\( \Lambda_c \) depends on the initial conditions as indicated by the subscript \( c \).

\[ \Lambda_c = 4(H_e - H_{0c}) (349 u_e + 311 u_0)_c - (H_{nn_0})_c (43 u_e + 23 u_0)_c \]  

(44)

4. Reacting Air Mixture

A reacting air mixture containing eight chemical species is used to represent the fluid chemistry. The air species are \( N_2, O_2, N, O, NO^+, O_3^-, NO, \) and \( e^- \). The 22 allowed chemical reactions involving these eight species are

(R1 - R5) \[ O_2 + X \rightleftharpoons 2O + X \] (X = O, O_2, N, N_2, NO)

(R6 - R10) \[ N_2 + X \rightleftharpoons 2N + X \] (X = O, O_2, N, N_2, NO)

(R11 - R15) \[ NO + X \rightleftharpoons N + O + X \] (X = O, O_2, N, N_2, NO)

(R16) \[ NO + O \rightleftharpoons N + O_2 \]

(R17) \[ N_2 + O \rightleftharpoons NO + N \]

(R18) \[ N + O \rightleftharpoons NO^+ + e^- \]

(R19) \[ NO + e^- \rightleftharpoons NO^+ + 2e^- \]

(R20) \[ N_2 + O_2 \rightleftharpoons 2NO \]

(R21) \[ O_2 + e^- + O_2 \rightleftharpoons O_2^- + O_2 \]

(R22) \[ NO^+ + O_2^- \rightleftharpoons NO + O_2 \]

Constants involved in the chemical kinetics are given by Ness and Fanucci.²

The chemical kinetics enter the analysis through the chemical production terms \( \frac{\partial \rho^{(i)}_c}{\partial t} \) in the law of mass action. The partition
functions are derived by assuming equilibrium for all degrees of freedom. The resulting expressions are

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(O)}}{\partial t} \right)_{\text{chem}} = A + \frac{m^{(O)}}{m^{(NO)}} (C - D - E) - \frac{m^{(O)}}{m^{(NO^+)}} F \tag{45}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(N)}}{\partial t} \right)_{\text{chem}} = B + \frac{m^{(N)}}{m^{(NO)}} (C + D + E) - \frac{m^{(N)}}{m^{(NO^+)}} F \tag{46}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(O_2)}}{\partial t} \right)_{\text{chem}} = -A + \frac{m^{(O_2)}}{m^{(NO)}} (D - \frac{1}{2} L) - \frac{m^{(O_2)}}{m^{(O_2^+)}} (N - P') \tag{47}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(N_2)}}{\partial t} \right)_{\text{chem}} = -B - \frac{m^{(N_2)}}{m^{(NO)}} (E + \frac{1}{2} L) \tag{48}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(NO)}}{\partial t} \right)_{\text{chem}} = E + L - C - D - K + \frac{m^{(NO)}}{m^{(O_2^+)}} P' \tag{49}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(NO^+)}}{\partial t} \right)_{\text{chem}} = F + \frac{m^{(NO^+)}}{m^{(NO)}} K - \frac{m^{(NO^+)}}{m^{(O_2^+)}} P' \tag{50}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(e^-)}}{\partial t} \right)_{\text{chem}} = \frac{m^{(e^-)}}{m^{(NO^+)}} F + \frac{m^{(e^-)}}{m^{(NO)}} K - \frac{m^{(e^-)}}{m^{(O_2^-)}} N \tag{51}
\]

\[
\frac{1}{\rho} \left( \frac{\partial \rho^{(O_2^-)}}{\partial t} \right)_{\text{chem}} = N - P'. \tag{52}
\]

The constants A, B, C, D, E, F, K, L, N, and P' are given by Ness and Fanucci.²

The total static enthalpy \( h \) of the air mixture is

\[
h = \sum_{i} a^{(i)} h^{(i)} = H - \frac{u^2}{2} \tag{53}
\]

The individual species states enthalpy is a function of the static temperature and molecular and atomic constants only.
\[
h^{(i)} = \frac{7}{2} \frac{kT}{m^{(i)}} + \frac{E_{\text{vib}}^{(i)}}{m^{(i)}} \left( \frac{E_{\text{vib}}^{(i)}}{e^{kT}} - 1 \right) \\
+ \frac{1}{m^{(i)}} \left[ \sum_{\ell} \frac{g_{\ell}^{(i)} E_{\ell}^{(i)} e^{-E_{\ell}^{(i)}/kT}}{kT} - \frac{E_{\ell}^{(i)}}{kT} \right] + \frac{E_{\text{chem}}^{(i)}}{m^{(i)}}
\]

From the equation of state,

\[
p = \rho \ kT \sum_{i} \frac{a_{i}^{(i)}}{m^{(i)}}
\]

differentiation leads to

\[
\frac{d\ln \rho e}{dx} = \frac{dp}{dx} - \frac{d\ln T_{e}}{dx} \sum_{i} \frac{1}{m^{(i)}} \frac{da_{e}^{(i)}}{dx} - \sum_{i} \frac{a_{e}^{(i)}}{m^{(i)}}
\]

The temperature derivative can be obtained from

\[
H_{e} = \sum_{i} a_{e}^{(i)} h_{e}^{(i)} + \frac{u_{e}^{2}}{2}
\]

Substitution of the \( h_{e}^{(i)} \) values into Equation (57) and differentiation gives

\[
\frac{d\ln T_{e}}{dx} = - \frac{A^{t} + \frac{A^{tt}}{kT_{e}}}{\sum_{i} \frac{a_{e}^{(i)}}{m^{(i)}} \left[ \Lambda_{e}^{(i)} \right]^{2} + \Lambda_{e}^{(i)} \frac{E_{\text{vib}}^{(i)}}{kT_{e}} + \frac{7}{2} \sum_{i} \frac{a_{e}^{(i)}}{m^{(i)}}} \\
\Lambda_{e}^{(i)} = \frac{E_{\text{vib}}^{(i)}}{kT_{e}} \left( \frac{E_{\text{vib}}^{(i)}}{e^{kT_{e}}} - 1 \right)
\]

10
5. Laminar-Turbulent Transition and the Pressure Gradient

The laminar-turbulent transition is treated by changing the viscosity model. The laminar portion of the wake is governed by the Sutherland equation.

\[
\mu_0 = 3.05 \times 10^{-8} \frac{T_0^{3/2}}{T_0 + 110}
\]  

(62)

The turbulent viscosity is

\[
\mu_0 = 0.02 \delta_m \rho_e (u_e - u_0)
\]  

(63)

The transition coordinate is computed using the equation formulated by Zeiberg:

\[
x_t = \frac{\mu_0 \infty}{\rho_\infty u_\infty} \left[ \frac{5}{M_\infty^2} + 6 \left( \frac{1 + \frac{5}{M_\infty^2}}{M_\infty^2} \right) M_\infty^2 \sin^2 \phi + 5 \left( \frac{6}{7M_\infty^2 \sin^2 \phi - 1} \right) \right]^{5/7}
\]  

(64)

\(Y\) is defined by

\[M_\infty \leq 8 : \log_{10} Y = 4.744 + 0.124 M_\infty + 0.00976 M_\infty^2\]  

(65)

\[M_\infty > 8 : \log_{10} Y = 5.48 + 0.11 M_\infty\]  

(66)

A cubic can be written for the shock angle \(\phi\)

\[
\left( \sin^2 \phi \right)^3 - \left( \frac{M_\infty^2 + 2}{M_\infty^2} + 1.4 \sin^2 \phi_c \right) \left( \sin^2 \phi_N \right)^2 + \left( \frac{2M_\infty^2 + 1}{M_\infty^2} + \left( 1.44 + \frac{0.4}{M_\infty^2} \right) \sin^2 \phi_N \right) \sin^2 \phi - \frac{\cos^2 \phi N}{M_\infty^4} = 0
\]  

(67)

\(\phi_N\) is the cone half-angle.
For a blunt body (detached bow shock) \( \phi_N = \frac{\pi}{2} \) and \( \phi = \frac{\pi}{2} \). For a cone, \( \phi \) is obtained as the intermediate root of Equation (67) which has three positive roots for reasonable cone angles.

The pressure gradient in the near wake is expressed by the equation

\[
\frac{p}{p_\infty} = \frac{0.0665M_\infty^2}{x/d} + 1
\]

(68)

Initial calculations were begun at a coordinate downstream of the neck \((x > 0)\) to prevent the first term from becoming infinite.

From the momentum theorem the drag can be used to give an initial value for the momentum thickness.

\[
D = 2\pi \int_0^\infty (p_\infty - p) \, r \, dr + 2\pi \int_0^\infty \rho u (u_\infty - u) \, r \, dr
\]

(69)

\( R \) is the radial distance to the shock. It is assumed that the total drag occurs in the area up to \( R = 6 \) and that \( p = p_\infty \) for \( R > 6 \).

\[
D = c_D q a = c_D \left( \frac{p_\infty u_\infty^2}{2} \right) \frac{\pi d^2}{4}
\]

(70)

The initial momentum thickness is then

\[
\theta_c = \frac{\rho_\infty}{(p_e)_{c}} \frac{c_D d^2}{16} + \left( \frac{p_e}{p_\infty} \right) \left( \frac{p_e - p_\infty}{2(p_e_{c} u_e)} \right) \delta_c^2
\]

(71)

Since

\[
\delta = \sqrt{Z \delta_m \left[ \int_0^1 \frac{\rho_e(x)}{\rho(x,n)} \, n \, d \rho \right]^2},
\]

(72)

\[
\theta_c = \left[ 1 - \frac{210(p - p_\infty)}{\rho_e u_e^2 \left( 1 - \frac{u_0}{u_e} \right) \left( 10 + 11 \frac{u_0}{u_e} \right)} \right] \frac{1}{c} \int_0^{p_e} \frac{\rho_e \, n \, d \rho}
\]

(73)
6. **Sequence of Calculations**

A printout of the FORTRAN source program is given in Appendix A. In the right-hand margin references are made to the equations used in the calculations. Important program symbols are defined in the list of symbols and correlated with the symbols used in the text.

The input data include α values for \(O_2\), \(N_2\), \(O\), \(N\), \(NO\), \(O_2^-\), \(NO^+\), and \(e^-\) and \(\alpha_e\) values for \(O_2\) and \(N_2\). The remaining initial \(\alpha_e\) values have been set equal to zero internally. Initial \(u_0\), \(H_0\), \(T_0\), \(\phi_n\), \(Z_0\), \(H_{nn0}\), and \(x\) values as well as \(Pr\), \(Le_0\), \(c_D\), \(d\), \(u_{\infty}\), and final \(x\) values are used as input. The altitude must be provided to initiate the atmospheric subroutine for the calculation of free-stream temperature, pressure, density, viscosity, Reynolds number, and speed of sound. This routine was adopted from an IBM 1620 program for the calculation of atmospheric properties up to 2320 kilofeet. A polynomial function of temperature as a function of geopotential altitude is used up to approximately 295 kilofeet. Below this altitude it is assumed that air has a constant molecular weight of 28.9644. Geometric altitude is employed above 295 kilofeet. For these higher altitudes the molecular weight is given by the expression

\[
M = 32.154 + AKM \left[ -0.034513 + AKM \left( 2.0326 \times 10^{-6} \right)^2 \right] + 2.1032 \times 10^{-8} \text{AKM}
\]

where \(AKM\) is the altitude in kilometers. Molecular-scale temperature (°R), the rate of change of temperature with geopotential altitude, and the initial pressure values (lb-ft-²) are listed for 23 altitude values in the atmosphere table.

The kinetic temperature is obtained from the molecular-scale temperature through the equation

\[
T = \frac{T_m M}{28.9644}
\]

where the molecular-scale temperature can be obtained from the relation

\[
T_m (X) = D + BX + \sum_i E_i |X - X_i|
\]

The \(X_i\)'s are altitudes at the end points of straight line segments over which the variation of altitude and molecular-scale temperature is assumed linear. The constants \(D\), \(B\), and \(E_i\) are functions of the \(X_i\) and the end-point temperatures which are denoted as \(Y_i\).
Once the free-stream conditions are known, the laminar-turbulent transition coordinate is calculated as a function of the free-stream conditions and $M_0$ using Equation (64). The axial viscosity is obtained from the Sutherland Equation (62) for the laminar region and from Equation (63) in the turbulent zone. Constants $A$ through $P$ which appear in the chemical production terms are evaluated on the axis using $\rho_0$, $T_0$, and $a_0$, and at the edge of the wake using $\rho_e$, $T_e$, and $a_e$. Density species derivatives in Equations (45) through (52) are then calculated at the axis and on the outer edge. From the profiles for species concentrations, velocity, and enthalpy, $a(x,n)$, $u(x,n)$, and $H(x,n)$ can be evaluated. $T(x,n)$ is then obtained by iteration of Equation (53) using the individual species enthalpies given in Equation (54). Once the pressure is evaluated, from Equation (68), $p(x,n)$ is given by Equation (55).

At this point, $\theta_C$ is calculated from Equation (73). This is only done for the initial coordinate. Once $\delta_m$ is obtained from Equation (40), the derivatives of $a_0(i)$, $a_e(i)$, $p$, $u_0$, and $u_e$ in Equations (36), (38), (68), (34), and (37), respectively, allow these parameters to be calculated at the next downstream coordinate. Equations (58) and (56) give $\frac{d\ln T_e}{dx}$ and $\frac{d\ln p_e}{dx}$, and $\delta$ is obtained by integration of Equation (72). These lead to $\frac{d\theta}{dx}$ as Equation (39). $\Lambda_C$ and $H_{mn}$ from Equations (44) and (43) allow $\frac{dH_0}{dx}$ to be determined using Equation (35).

Finally, $T_0$ can be evaluated using the new values of $H_0$, $u_0$, and $a_0(i)$, and the calculations are then repeated for an incremented value of $x$. In this calculation, $x$ was incremented uniformly as $\Delta x = 0.1$. Better results may be obtained by using smaller increments for small values of $x$; however, the error was not large enough to justify a procedure of this type.
LITERATURE CITED


Appendix A

FORTRAN PROGRAM

The hypersonic WAKE program described in the text is given along with the required subroutines. Comments appearing in the right-hand margin refer to equations discussed in the text. Some important symbols are defined in the list of symbols; however, most of the quantities can be identified by reference to the sequence of calculation (Paragraph 6) and the original equations.
ALPH[71]=0.
Dx=1
FNN=1.
X=STARTX
MM=1
NN2=0
NN3=0
HH(1)=0.
DO 1002 KK=2,23
1002 HH(KK)=-5*(AA(KK,2)-AA(KK-1,2))/(AA(KK,1)-AA(KK-1,1))
BB=HH(2)-HH(23)
CC=(AA(1,2)+AA(23,21))/2.
DD=CC+HH(2)*AA+HH(23)*AA(23,1)
IF(ALT=295272)*11003+1003+1004
1003 ALTf=(20855490.4*ALT)/(20855490.4*ALT)
GO TO 1005
1004 ALTf=ALT
1005 FF=0.
DO 1006 KK=2,22
1006 FF=EE**((ALTf-AA(KK,1))**(ALTf-AA(KK,1)))**5*FF.
TEMP=DD+BB*ALTf+FF
WO=28.9644
IF(ALT=35114.4)*11007+11007+11008
1007 W=28.9644
GO TO 1009
1008 AKM=ALT/3280.8333
W=32.154+AKM*(-0.034513+AKM*(2.0326E-6+2.1032E-8*AKM))
1009 TEMP=TEMP+Wo
IF(ALT=295272)*11101+1010+1016
1010 DO 1011 KK=2,23
1011 CONTINUE
1012 KK=KK+1
DALT=ALTf-AA(KK,1)
IF(AA(KK,3))*11103+1014+1015
1013 PRESS=AA(KK,4)/(AA(KK,21)/TEMP)**(0.0187434/(1-AA(KK,3)))
1014 VI=(0.0187434/AA(KK,21))DALT
PRESS=AA(KK,4)*(1.0/EXP(VI))
GO TO 1019
1015 PRESS=AA(KK,4)/(AA(KK,21)/TEMP)**(0.0187434/AA(KK,3))
GO TO 1019
1016 DO 1017 KK=10,23
IF(ALT- AA(KK+1)) 1018 1017
1018  CONTINUE
KK=KK-1
DALT=ALT- AA(KK+1)
XXX=S/(S-AA(KK+2)/AA(KK+3)+AA(KK+1))
Y1=XXX*(ALOG(AA(KK+2)/AA(KK+3)+DALT)/(S+ALT))=ALOG(AA(KK+2)/
1AA(KK+3))/((S-AA(KK+1))*(S+ALT))
Y2=-S*DALT/((S-AA(KK+1))*(S+ALT))
Y3=XXX*(Y1+Y2)
PRESS=AA(KK+4)/EXP(WO/AA(KK+3)*Y3/R)
1019 DENS=PRESS/153.352*TFPM)
G=GA*(S/(S+ALT))*2
DENS=DENS/32.1714
IF(ALT-295272.) 1021 1021 1022
1021 CS=411*TEMP**.5
SS=19A+72
RR=7.3025 E-7
VS=1.3-B*TEMP**1.5/(TEMP+SS)
VSG=VIs/g
MIN=UIN/CS
RE=DENS*CS/VIS
TEMP=TEMP/1.8
GO TO 1023
1022 VSG=0.
MIN=0.
RE=0.
TEMP=0.
CS=0.
1023 PIN=PRESS
TIN=TEMP
RHIN=DENS
MUIN=VSG
UOE(1)=UIN
TOE(1)=TIN
ZOE(1)=1.
P=1.0665*MIN**2*LD/X+1.)*PIN
DO 10231 I=1,2
EQUATION 68
C --EQUATION OF STATE--
10231 RHOE(1)=15.2324*ZOE(1)*P/TOE(1)
WRITE(6,101)
101 FORMAT(1H1)
WRITE(6,102)
102 FORMAT(15H INITIAL CONDITIONS)
WRITE(6,106)
106 FORMAT (3HO X15X2HOUO14X2HUE14X2HTO14X2HTE14X2HIO14X2HHE)
WRITE (6,104)X,UE,TO,HO
103 FORMAT(11HO ALPHAO(0)7X9HALPHAON(7)7X10HALPHA0(O2)16X10HALPHA0(NO)6X
11HHALPHA0(NO+15X11HALPHA0(O2-1)5X10HALPHA0(E-)6X10HALPHA0(N2))
WRITE(6,104)ALPH
104 FORMAT(8E16.8)
WRITE(6,105)
105 FORMAT(11HO ALPHAE(0)7X9HALPHAEN(7)7X10HALPHAEO(O2)16X10HALPHAEO(NO)6X
11HHALPHAEO(NO+15X11HALPHAEO(O2-1)5X10HALPHAEO(E-)6X10HALPHAEO(N2))
WRITE(6,104)ALPHE
WRITE(6,107)
107 FORMAT(6HO M(0)12X4HM(N)12X5HM(02)11X5HM(NO)11X6HM(NO+)10X6HM(02-)
110X5HM(E-111X5HM(N2))
WRITE(6,104)FM
WRITE(6,108)
108 FORMAT(6HO EP1912X4HEP2012X4HEP2112X4HEP2212X2HCD14X2HLD14X4UIEF1
12X4HTINF)
WRITE(6,109)
109 FORMAT(6HO PIN12X6HRHO1N10X5HMINF11X4HMINF12X2HN114X2HN214X2HN3
114X2HN4)
WRITE(6,104)PIN,RHIN,MUIN,MN,FR
WRITE(6,110)
110 FORMAT(5HO PHI113X3HALT13X2HZE14X4HRHO012X4HRHOE12X4HNN012X
13HPRO)
WRITE(6,104)PHI,ALT,ZOERHO,HNNO,PRO
WRITE(6,1112)
1112 FORMAT(3HO W15X2HRE14X2HCS)
WRITE(6,104)W,RE,CS
WRITE(6,111)
111 FORMAT(5HO LE013X5HSTOPX11X2HDX)
WRITE(6,104)LED,STOPX,DX
WRITE(6,101)
F90=90.,57.295779.0001
PHI1=PHI/57.295779
IF(PHI,INF,90.)GO TO 121
S2B=1.
GO TO 122
121 COE(4)=-COS(PHI1)**2/MN**4
COE(3)=(2.*MN**2+1.)/MN**4*(1.E4+4./MN**2)*SIN(PHI1)**2
COE(2)=-(1.*MN**2+1.)/MN**2+1.4*SIN(PHI1)**2
COE(1)=1.
CALL RTSOLVE(COE,5,ROOT,R).ROOT1)
DO 112 I=1,3
IF(ABS(ROOT(I))*GT.0.000001)GO TO 120
SOLUTION OF EQUATION 67
112 CONTINUE
   GO TO 115
120  S2B=1.
   GO TO 122
115  DO 114 I=1,2
   DO 114 J=2,3
   IF(ROOR(I).LE.ROOR(J))GO TO 114
   TMP=ROOR(I)
   ROOR(I)=ROOR(J)
   ROOR(J)=TMP
114  CONTINUE
   S2B=ROOR(2)
122  IF(MIN+LE.8.)GO TO 116
   Y=EXP(2.30258509*(5.48+11*MIN))
   GO TO 123
116  Y=EXP(2.30258509*(4.74+MIN+1.124+0.0976*MIN))
   X=MUIN/RHIN/Y/UN*(-5./MIN**2+5.*/MIN**2+MIN**2*528/(MIN**
123  X=MUIN/RHIN/Y/UN*(-5./MIN**2+5.*/MIN**2+MIN**2*528/(MIN**
   100  IF(X*GF+XT1GO TO 117
   MUN=3.05E-8*(TOE/TOE+110.)
   GO TO 118
117  MUN=M2*ROTH(DELMP)*RHOE(2)*(UOE(2)-UOE)
118  DO 9 I=1,2
   GO TO (3,5,1)
3  DO 4 J=1,8
   ALPH(J)=ALPH(J)
   GO TO 7
5  DO 6 J=1,8
6  ALPH(J)=ALPH(J)
7  TMP1=5.*EXP(-228.*TOE(I)+EXP(-326.*TOE(I))
   TMP2=1.*EXP(-2745./TOE(I))
   TMP3=3.*EXP(-11390./TOE(I))
   TMP4=TOE(I)**(-1.5)
   TMP5=SOBT(10E(I))
   TMP6=1.*EXP(-3395./TOE(I))
   TMP7=1.*EXP(-2700./TOE(I))
   TMP8=1.*EXP(-1759./TOE(I))
   TMP9=1.*EXP(-11728./TOE(I))
   XP=59365./TOE(I)
   IF(XP+GE.-88.*AND.+XP.LE.+88.)GO TO 8
   IF(XP+10.10+11
10  XP=88.
   GO TO 8
11  XP=88.
8 A(1)=1.003E16*RHOE(1)*TMP4*(5.*ALPH(1)/8.+ALPH(3)/16.+ALPH(2)/21.+ 
 1*ALPH(8)/42.+ALPH(4)/45.)*((-2345*ALPH(3)*TMP5*EXP(XP)*TMP2/TMP3* 
 2TMP1-RHOE(1))**2)
XP=-113200.*/TOE(1)
IF(XP,GE,-88.*AND.XP,LF,88.)*GO TO 12
IF(XP)*13,13,14
13 XP=88.*
GO TO 12
14 XP=88.*

12 B(1)=1.147E19*RHOE(1)*TMP4*(5.*ALPH(2)/28.+ALPH(8)/56.+ALPH(1)/96. 
 1+ALPH(3)/128.+ALPH(4)/180.)*13.47*ALPH(8)*TMP5*EXP(XP)*TMP6-RHOE 
 21*ALPH(2)**2
XP=-75310.*/TOE(1)
IF(XP,GE,-88.*AND.XP,LF,88.)*GO TO 15
IF(XP)*16,16,17
16 XP=-88.*
GO TO 15
17 C(1)=2.15*E19*RHOE(1)*TMP4*(ALPH(4)/30.+ALPH(1)/16.+ALPH(2)/14.+ 
 1*ALPH(3)/32.+ALPH(8)/28.)*5.234*ALPH(4)*TMP5*EXP(XP)*TMP7/TMP8* 
 2TMP1-RHOE(1)*ALPH(2)*ALPH(1))
D(1)=3.45E10*RHOE(1)*TMP5*EXP(-3120.*/TOE(1))*(-.994*ALPH(4)*ALPH(1) 
 1*EXP(-15945.*/TOE(1))/TMP2*TMP7/TMP8*TMP3/TMP1-ALPH(2)*ALPH(3))
XP=-37890.*/TOE(1)
IF(XP,GE,-88.*AND.XP,LF,88.)*GO TO 18
IF(XP)*19,19,20
19 XP=-88.*
GO TO 18
20 XP=88.*

21 E(1)=4.78E11*RHOE(1)*(16.04*ALPH(1)*ALPH(8)*EXP(XP)*TMP7/TMP6/TMP1 
 1*TMP=ALPH(2)*ALPH(4))
XP=-32130.*/TOE(1)
IF(XP,GE,-88.*AND.XP,LF,88.)*GO TO 21
IF(XP)*22,22,23
22 XP=-88.*
GO TO 21
23 XP=88.*

25 XP=-88.*
GO TO 24

A OF PARAGRAPH 4

B OF PARAGRAPH 4

C OF PARAGRAPH 4

D OF PARAGRAPH 4

E OF PARAGRAPH 4

F OF PARAGRAPH 4
24 XP=88.
25 K(1)=1.765E30*RHOE(1)*TMP4*ALPH(7)*(1+88E-12*ALPH(4)*TOE(1)**1.5
1/TMP6*EXP(XP)*TMP7-RHOE(1)*ALPH(5)*ALPH(7))
XP=-430000/TOE(1)
IF(XP-GE-88.*AND.XP-LE.88.)GO TO 27
IF(XP)28,28,29
28 XP=-88.
GO TO 27
29 XP=88.
30 P(1)=3.44E13*RHOE(1)*TOE(1)**(-2.5)*EXP(XP)*(16.15*ALPH(8)*ALPH(3)
1*EXP(-21945./TOE(1))*TMP6/TMP3*TMP2/TMP8-ALPH(4)**2
N(1)=4.83E9*RHOE(1)*ALPH(3)*TOE(1)*(2.80E11*RHOE(1)*ALPH(3)*
1*ALPH(7)*TMP4/TMP9*TMP3-ALPH(6)*EXP(-51100./TOE(1))
XP=-1020000/TOE(1)
IF(XP-GE-88.*AND.XP-LE.88.)GO TO 30
IF(XP)31,31,32
31 XP=-88.
GO TO 30
32 XP=88.

24 K OF PARAGRAPH 4
25 L OF PARAGRAPH 4
26 N OF PARAGRAPH 4
27 P OF PARAGRAPH 4
28 EQUATION 45
29 EQUATION 46
30 EQUATION 47
31 EQUATION 48
32 EQUATION 49
33 EQUATION 50
34 EQUATION 51
35 EQUATION 52
36 EQUATION 53
37 EQUATION 54
EQUATION 36

\[
PPTSBO(4) = PPTNO(1) + 1 + \frac{PPTSBO(1)}{UOE} \left( \frac{ALPH(1)}{DELm2*PRO} \right) * (ALPH(1) - ALPH(1))
\]

EQUATION 38

\[
DALPHO(1) = 24.4 * RHoe(2) * MUO/UEO*LEO/(DELm2*PRO)* (ALPH(1) - ALPH(1)) + \frac{PPTSBO(1)}{UOE}
\]

DO 40 I=1,8
DALPHO(1) = 24.4 * RHoe(2) * MUO/UEO*LEO/(DELm2*PRO)* (ALPH(1) - ALPH(1)) + \frac{PPTSBO(1)}{UOE}
DO 40 I=1,8
YY = ALPH(1)
DO 42 I=1,4
FK(I) = DX * (24.4 * RHoe(2) * MUO/UEO*LEO/(DELm2*PRO)* (ALPH(1) - YY)) / PRO * PPTSBO(1) / UOE
GO TO (43, 43, 44, 42) * I
43 YY = ALPH(1)* FK(I) / 2
GO TO 42
44 YY = ALPH(1) * FK(I)
42 CONTINUE
41 ALPH(1) = ALPH(1) + \frac{FK(1) + 2 * FK(2) + 7 * FK(3) + FK(4)}{6}
ALPH(7) = FM(7)*ALPH(5)/FM(5) = FM(7)*ALPH(6)/FM(6)
SUM = 0
DO 45 I=1,7
SUM = SUM + ALPH(1)
ALPH(8) = 1 - SUM
DO 46 I=1,8
FK(1) = DX * PPTSBO(1) / UOE
46 ALPH(1) = ALPH(1) * FK(I)
ALPH(7) = FM(7)*ALPH(5)/FM(5) = FM(7)*ALPH(6)/FM(6)
SUM = 0
DO 47 I=1,7
SUM = SUM + ALPH(1)
ALPH(8) = 1 - SUM
DPDX = -0.065 * PIN*MIN**2*LD/X**2
YY = UOE
XX = X
DO 51 I=1,4
FK(I) = DX * (24.4 * RHoe(2) * MUO/YY*UEO(2) - YY) / DELm2 * 0.065 / XX**2 * PIN/1 RHoe*MIN**2/YY*LD)
GO TO (52*52+53*51)*I
52 XX=XX+DX/2.
   YY=UOE(1)/2.
   GO TO 51
53 XX=XX+DX
   YY=UOE(1)
51 CONTINUE
   DUXDX=F(1)/DX
   GO TO (51*51+51)*M
511 UOE1=UOE
512 UOE=UOE+(F(1)+2.*F(2)+2.*F(3)+F(4))/6.
   XX=XX
   YY=UOE(1)
   DO 54 I=1,4
      FK(1)=0.665/RH0E(2)**PIN/YY**MIN**2/XX**2*LD*DX
      GO TO (55*55+55*55)*I
55 XX=XX+DX/2.
   YY=UOE(1)+F(1)/2.
   GO TO 54
56 XX=XX+DX
   YY=UOE(1)+F(1)
54 CONTINUE
   DUXDX=F(1)/DX
   GO TO (54*54+54)*M
541 UOE1(2)=UOE(1)
   MNO1=MNO
   HOE1(2)=HOE(1)
542 UOE(2)=UOE(1)+(F(1)+2.*F(2)+2.*F(3)+F(4))/6.
   YY=TOE(1)
   DO 57 I=1,4
      TMP1=2274.*YY
      TMP2=2395.*YY
      TMP3=2340.*YY
      TMP4=1728.*YY
      TMP5=EXP(TMP1)-1.
      TMP6=EXP(TMP2)-1.
      TMP7=EXP(TMP3)-1.
      TMP8=EXP(TMP4)-1.
      FK(1)=DX*YY*(1.18E-5*UOE(2)+DUXDX(1)/2.*2+TMP1/TMP2/32)*
      10*DLPHE(3)*I(TMP2/2+TMP2/TMP3/28+DALPHE(3)+7./60+TMP3/30+TMP3/30.6++
      2366.)*YY*DLPHE(4)*I(TMP2/130.+3950.)*YY*DLPHE(5)*I
      315/32.*1853.*YY*DLPHE(1)*/7./64+TMP4/32.*TTPM1-1+3.*YY)*
      4*DLPHE(6)*I(15./28+6042.*YY)*DLPHE(7)/4550.*DLPHE(1)/({(TMP3/TMP3+TTPM1)/
      5*TPM5)**2+TTPM5+7./2.)*ALPHE(3)/32.*({(TMP3/TMP3)**2+TTPM3**2
      6/TMP7+7./2.)*ALPHE(4)/30.+({(TMP2/TMP2)**2+TTPM2**2+TTPM2+7./2.)*
      7*ALPHE(8)/28+ALPHE(5)/30.}+{(TMP4/TMP8)**2+TTPM4**2/TMP8+7./2.)}
28

8\textsc{ALPHE}(6)/32+5/32\star \textsc{ALPHE}(1)+5/28\star \textsc{ALPHE}(2)+4550\star \textsc{ALPHE}(7)

\textbf{GO TO} (58, 58, 59, 57, 1)

58 \textbf{YY}=\textsc{TOF}(2)+\textsc{FK}(1)/2

\textbf{GO TO} 57

59 \textbf{YY}=\textsc{TOF}(2)+\textsc{FK}(1)

\textbf{CONTINUE}

\textsc{DLTDX}=\textsc{FK}(1)/(\textsc{DX}\star \textsc{TOE}(2))

\textsc{TOE}(2)=\textsc{TOE}(2)+(\textsc{FK}(1)+2\star \textsc{FK}(2)+2\star \textsc{FK}(3)+\textsc{FK}(4))/6

\textsc{SUM}=0

\textsc{SUM1}=0

\textbf{DO} 60 \textbf{I}=1, 8

\textsc{SUM}=\textsc{SUM}\star \textsc{ALPHE}(1)/\textsc{FM}(1)

60 \textsc{SUM}=\textsc{SUM}\star \textsc{ALPHE}(1)/\textsc{FM}(1)

\textbf{YY}=\textsc{RHOE}(2)

\textbf{DO} 61 \textbf{I}=1, 4

\textsc{FK}(1)=\textsc{DX}\star \textsc{YY}\star (\textsc{DX}/\textsc{FM}\star \textsc{DLTDX}-\textsc{SUM}/\textsc{SUM1})

\textbf{GO TO} (62, 62, 63, 61, 1)

62 \textbf{YY}=\textsc{RHOE}(2)+\textsc{FK}(1)/2

\textbf{GO TO} 61

63 \textbf{YY}=\textsc{RHOE}(2)+\textsc{FK}(1)

\textbf{CONTINUE}

\textsc{DLREDX}=\textsc{FK}(1)/(\textsc{DX}\star \textsc{RHOE}(2))

\textbf{GO TO} (61, 61, 62, 75, 77, 5, 5)

61 \textsc{RHOE}(1)=\textsc{RHOE}(2)

\textsc{HOE}=\textsc{HOE}

62 \textsc{RHOE}(2)+\textsc{RHOE}(2)+(\textsc{FK}(1)+2\star \textsc{FK}(2)+2\star \textsc{FK}(3)+\textsc{FK}(4))/6

\textbf{CALL} \textsc{IGRAF}(0.1, +0.1, +0.1, \textsc{ANS}=1)

\textsc{DEL}=\textsc{SORT}(2\star \textsc{DEL}=2\star \textsc{ANS})

\textbf{YY}=\textsc{THETA}

\textbf{DO} 64 \textbf{I}=1, 4

\textsc{FK}(1)=\textsc{DX}\star \textsc{YY}\star (\textsc{DX}/\textsc{DLREDX}+2\star \textsc{YY}\star \textsc{DEL}=2/2+\textsc{DEL}=2/2\star \textsc{DEL}=2/2\star \textsc{UOE}3/\textsc{UOE}(2)/10)

\textbf{RHOE}(2)=\textsc{DX}/\textsc{UOE}(2)+2)

\textbf{GO TO} (65, 65, 66, 64, 1)

65 \textbf{YY}=\textsc{THETA}+\textsc{FK}(1)/2

\textbf{GO TO} 64

66 \textbf{YY}=\textsc{THETA}+\textsc{FK}(1)

\textbf{CONTINUE}

\textsc{DTHDX}=\textsc{FK}(1)/\textsc{DX}

\textsc{THETA}=\textsc{THETA}+(\textsc{FK}(1)+2\star \textsc{FK}(2)+2\star \textsc{FK}(3)+\textsc{FK}(4))/6

\textbf{GO TO} (67, 68, 75, 77, 5, 5)

67 \textsc{MM}=2

\textsc{DELTAC}=4\star (\textsc{HOE}(2)-\textsc{HOE}(1))\star (349\star \textsc{UOE}(2)+311\star \textsc{UOE}(1)-\textsc{HNN}(1, (43\star \textsc{UOE}(2)+23\star \textsc{UOE}(1)+121)+23\star \textsc{UOE}(1))

\textbf{EQUATION 43}

\textbf{EQUATION 44}

\textbf{EQUATION 58}

\textbf{EQUATION 72}
1RH0E1(2)/RH0E(2)*DEL21/DEL2*DELTAC/(43.*U0E(2)+23.*U0E)
SUM=0.
CALL HSUR(Toe*HSUBS)
DO 69 I=1,8
69 SUM=SUM+HSURS(I)*ALPHF(I)
YY=HOE
DO 70 I=1,4
FK(I)=2.*RH0E(2)*MU0/(U0E*DEL21)*(HNNO*PRO+12.*(PRO-1.)/PRO*U0E*
I*(U0E(2)-U0E)+12.*THEO-1.)/PRO*(SUM-YY+U0E**2/2.)*DX
GO TO (71,71,72+70)+1
71 YY=HOE+FK(I)/2.
GO TO 70
72 YY=HOE+FK(I)
70 CONTINUE
HOE=HOE+(FK(I)+2.*FK(2)+2.*FK(3)+FK(4))/6.
TBH=TOE
IM=1
75 TMP1=EXP(2274.*/TBH)-1.
TMP2=EXP(3995.*/TBH)-1.
TMP3=EXP(2740.*/TBH)-1.
TMP4=EXP(1728.*/TBH)-1.
T2=(HOE-5.*U0E**2-2800.)/TMP1*2274.*ALPHO(1)-3200./TMP2*3995.*
1ALPHO(8)-2985./TMP3*2740.*ALPHO(4)-2985./TMP2*3995.*ALPHO(5)-
22800./TMP4*1728.*ALPHO(6)+1.6638*ALPHO-3.6288*ALPHO(2)-3.28E7*
3ALPHO(4)+3.5368*ALPHO(5)+1.4297*ALPHO(6)/1.3512800.*ALPHO(3)+
4.5*3200.*ALPHO(8)+1.64*ALPHO+1.64*ALPHO(2)+3.512985.*ALPHO(4)+
5.5*2985.*ALPHO(5)+4.0748*ALPHO(7)+9.5*2800.*ALPHO(7)
TEST=ARS(T2-TBH)/ABS(T2)
IF(TEST<0.0001)74+74,741
741 GO TO (742,743)+IM
742 TBNM1=TOE
TBH=T2
T1=T2
IM=2
GO TO 75
743 SA=(T2-T1)/(TBH-TBNM1)
SD=SA/(SA-1.)
TBNP1=SD*TBN+(1.-SD)*T2
T1=T2
TBNM1=TBN
TBH=TBNP1
GO TO 75
74 TOE=T2
CALL HSUB(TOE(2),HSUBS)
SUM=0.
DO 76 I=1,8
    SUM=SUM+ALPHE(I)*HSUBS(I)
    HOE(2)=SUM*UOE(2)**2+SUM
    RHOE=2.29E-4/ZOE*P/TOF
    X=STARTX+FNN*DXX
    IF(EXMIT.EQ.0)GO TO 90
66 C
   -- INTERMEDIATE OUTPUT IF EXOUT IS NOT EQUAL TO ZERO --
   WRITE (6,119)
119 FORMAT(21H0,INTERMEDIATE RESULTS)
   WRITE(6,104) X, P, UOE, TOF, HOE
   WRITE(6,104) ALPHE
   WRITE(6,104) DALPHI
   WRITE(6,104) A, B, C, D
   WRITE(6,104) E, F, K, L
   WRITE(6,104) N, MOU, PP, PPT0
   WRITE(6,104) PPTM, PPT02M, PPTN0
   WRITE(6,104) PPTN, PPT02, PPTN2, PPTNO
   WRITE(6,104) DPDX, DUODX, DUEDX, DLTEDX, THETA, DELM2
   WRITE(6,104) DLREDX, DEL, DTM0X, DELTAC, S28, XT, RHOE
   WRITE(6,104) ROOTR1, ROOTI1, ROOTR2, ROOTI2, ROOTR3, ROOTI3
   WRITE(6,104) COE(1), COE(2), COE(3), COE(4), UXN
   WRITE(6,104) HXN, LHXN
   WRITE(6,104) ALPHXN
90 CONTINUE
   FNN=FNN+1.
   NN2=NN2+1
   IF(NN2.NE.10) GO TO 77
   NN2=0
   WRITE (6,1060)
1060 FORMAT(3HO X15X1HP15X2HOI4X2HUE14X2HT014X2HTE14X2HWO14X2HHE)
   WRITE(6,104) X, P, UOE, TOF, HOE
   WRITE(6,103) ALPHE
   WRITE(6,105) DALPHI
   WRITE(6,104) A, B, C, D
   WRITE(6,104) E, F, K, L
   WRITE(6,104) N, MOU, PP, PPT0
   WRITE(6,104) PPTM, PPT02M, PPTN0
   WRITE(6,104) PPTN, PPT02, PPTN2, PPTNO
   WRITE(6,104) DPDX, DUODX, DUEDX, DLTEDX, THETA, DELM2
   WRITE(6,104) DLREDX, DEL, DTM0X, DELTAC, S28, XT, RHOE
   WRITE(6,104) ROOTR1, ROOTI1, ROOTR2, ROOTI2, ROOTR3, ROOTI3
   WRITE(6,104) COE(1), COE(2), COE(3), COE(4), UXN
   WRITE(6,104) HXN, LHXN
   WRITE(6,104) ALPHXN
    77 IF(X+0.000001).*GE.*0.1)GO TO 1
   GO TO 100
END

END PROGRAM
SUBROUTINE A5
LIST
SUBROUTINE HSUB(T,HSUB)
C
--
DIMENSION HSUB(10)
XP=2274./T
X=T/2.
HSUB(3)=2000.*T*(X+XP/(EXP(XP)-1.))
XP=3395./T
HSUB(4)=3200.*T*(X+XP/(EXP(XP)-1.))
HSUB(5)=1.4E4*T+1.66E8
HSUB(6)=1.6E4*T+3.62E8
XP=2740./T
HSUB(7)=2983.*T*(X+XP/(EXP(XP)-1.))+3.18E7
XP=3395./T
HSUB(8)=2985.*T*(X+XP/(EXP(XP)-1.))+3.536E8
HSUB(9)=4.074E8*T
XP=1720./T
HSUB(10)=2000.*T*(X+XP/(EXP(XP)-1.))-1.429F7
RETURN
END

SUBROUTINE A4
LIST
SUBROUTINE INTEQS(X,Y,NOE0)
C
--
COMMON FN(4),RHOEXN(4),RHOE(2)
NOE0=NOE0
CALL INTERP(X,FN,RHOEXN,RHOE,ANS,NERR)
YY=RHOE(2)*XX/ANS
RETURN
END

SUBROUTINE A1
LIST
SUBROUTINE IGRAT(LIM,ULIM,DELTA,ANS,NOE0)
C
--
SUBROUTINE IGRAT
C
1 LLIM LOWER LIMIT OF INTEGRATION (FLOATING PT.)
2 ULIM UPPER LIMIT OF INTEGRATION
3 DELTA STEP SIZE FOR SOLUTION
4 ANS VALUE OF INTEGRAL
5 NOE0 ALL EQUATIONS TO BE INTEGRATED MUST BE

C
COMPILED IN A SUBROUTINE CALLED INTEQS.

C
NOE0 IS TO BE USED BY THIS SUBROUTINE

C
TO SELECT THE PROPER EQUATION FOR

C
INTEGRATION. IE. NOE0=1 WILL BE TO INTEGRATE

C
EQUATION 1, NOE0=2 TO INTEGRATE EQUATION

C
2, ETC. THE FORM OF INTEQS MUST BE...

C
INTEQS(X,Y,NOE0) WHERE

C
1 X INDEPENDENT VARIABLE
2 Y DEPENDENT VAR. IE. Y=F(X)
DIMENSION F(6), U(6), R(3)
DATA (U(I), I=1,6) / *11930959, *11930959, *33060469, *33060469, 
A *46623476, *46623476 /
DATA (R(I), I=1,3) / *23395697, *18098079, *85662246E-1
REAL LLIM
5 A = LLIM
10 B = DEL = DELTA
LAST = 1
10 A = A + 4.0*DEL
10 IF(R-ULIM)40,30,20
20 B = ULIM
30 LAST = 2
40 DO 50 I=1,6
X = (R-A)*U(I) + 5*(A+B)
50 CONTINUE
60 ANS = ANS + (B-A)*R(1)*F(1)+F(2) + R(2)*F(3)+F(4) + R(3)*
A (F(5)+F(6))
GO TO (70,80,LAST)
70 A = B
GO TO 10
80 RETURN
END $IBFIC A2 LIST
SUBROUTINE INTERP( ARG, XTAB, YTAB, NX, NP, ANS, NERR)
-- SUBROUTINE INTERP
  1* ARG THE INDEPENDENT VARIABLE FOR THE DESIRED UNKNOWN.
  2* XTAB TABLE OF INDEPENDENT VALUES. MUST BE IN INCREASING ORDER.
  3* YTAB TABLE OF DEPENDENT VALUES.
  4* NX NUMBER OF POINTS IN XTAB.
  5* NP NUMBER OF POINTS FOR INTERPOLATION.
  6* ANS THE DEPENDENT VALUE CORRESPONDING TO THE VALUE OF ARG.
  7* NERR WILL BE SET UNEQUAL ZERO IF ARG IS NOT ON XTAB. ROUTINE WILL EXTRAPOLATE.

3 NOEO SAME AS ABOVE. PROBABLY USED IN A COMPUTED GO TO.
10 SEPT 64
RALPH SELLERS
DIMENSION XTAB(NX), YTAR(NX)
NERR = 0
IM = NP/2
1 = 1
IF (XTAB(I) = ARG) 30, 20, 10
10 IM = 0
12 NERR = 01
GO TO 70
20 ANS = XTAB(I)
GO TO 999
30 I = NX
IF (XTAB(I) = ARG) 12, 20, 50
50 L = IM + 1
DO 60 I = L, NX
IF (XTAB(I) = ARG) 60, 20, 70
60 CONTINUE
70 K = I - IM
N = K + NP - 1
ANS = 0.0
IF (I = 0) 90, 90, 80
80 N = NX
K = NX - NP + 1
90 DO 120 J = K, N
P = 1.0
DO 110 I = K, N
IF (I = J) 110, 110, 100
100 P = P * (ARG - XTAB(I)) / (XTAB(J) - XTAB(I))
110 CONTINUE
120 ANS = ANS + XTAB(J) * P
999 RETURN
END
$IFTC A3
LIST SUBROUTINE RTSOLV (COE,N1,ROOTR,ROOTT)
C
DIMENSION COE(51), ROOTR(50), ROOTT(50)
N2=N1 + 1
CTEBB=COE(1)
DO 1 I = 1, N2
1 COE(I)=COE(I)/CTEBB
AN30D=ABS(COE(I))
DO 2 J = 2, N2
IF (ABS(COE(J)) * GT. AN30D) AN30D=ABS(COE(J))
FJD=FLOAT(J)
AKD=1.0/(FJD-1.0)
TMD=AN30D**AKD
2 CONTINUE
DO 3 K=1,N2
J=K-1
3 COE(K)=COE(K)/(TMD**J)
N4=0
I=N1+1
IF (COE(I))9,7,9
7 N4=N4+1
ROOTR(N4)=0
ROOTI(N4)=0
T=I-1
IF (N4-N1)19,37,19
9 CONTINUE
10 AXR=0.8
AXI=0.
L=1
N3=1
ALP1R=AXR
ALP1I=AXI
M=1
GO TO 99
11 BET1R=TEMR
BET1I=TEMI
AXR=0.35
ALP2R=AXR
ALP2I=AXI
M=2
GO TO 99
12 BET2R=TEMR
BET2I=TEMI
AXR=0.9
ALP3R=AXR
ALP3I=AXI
M=3
GO TO 99
13 BET3R=TEMR
BET3I=TEMI
14. TE1=ALP1R-ALP3R
TE2=ALP1I-ALP3I
TE5=ALP3R-ALP2R
TE6=ALP3I-ALP2I
TEM=TE5*TE5+TE6*TE6
TE3=[TE1*TE5+TE2*TE6]/TEM
TE4=(TE2*TE5-TE1*TE6)/TEM
TE7=TE3+1
TE9=TE3*TE3-TE4*TE4
TE10=2*TE3*TE4
DE15=TE7*GET3R-TE4*GET3I
DE16=TE7*GET9I+TE4*GET3R
TE11=TE3*GET2R-TE4*GET2I+GET1R-DE15
TE12=TE3*GET2I+TE4*GET2R+GET11-DE16
TE7=TE9-1
TE1=TE9*GET2R-GET10*GET2I
TE2=TE9*GET2I+GET10*GET2R
TE13=TE1-GET1R-TE7*GET3R+TE10*GET3I
TE14=TE2-GET1I-TE7*GET3I-TE10*GET3R
TE15=DE15*TE3-DE16*TE4
TE16=DE15*TE4+DE16*TE3
TE1=TE13*TE13-TE14*TE14-4*(TE11*TE15-TE12*TE16)
TE2=2*(TE13*TE14-4*(TE12*TE15+TE11*TE16)
TEM=SORT(TE1+TE1+TE2)
IF(TE1)113,113,112
113 TE4=SORT(+5*(TEM-TE1))
TE3=-5*TE2/TE4
GO TO 113
112 TE3= SORT(+5*(TEM+TE1))
IF(TE2)110+200,200
110 TE3=TE3
200 TE4=5*TE2/TF3
111 TE7=TE13+TE3
TE8=TE14+TE4
TE9=TE13+TE3
TE10=TE14-TE4
TE1=2*TE15
TE2=2*TE16
IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205
204 TE7=TF9
TE8=TE10
205 TEM=TE7+TE8*TE8
TE3=(TE1*TE7+TE2*TE8)/TEM
TE4=(TE2*TE7-TE1*TE8)/TEM
AXR=ALP5+TE3*TE5-TE4*TE6
AXI=ALP3+TE3*TE6+TE4*TE5
ALP4=AXR
ALP4=AXI
M=4
GO TO 99
15 N6=1
38 IF(ABS(BELL)+ABS(BELL1)-1.E-20)16,18,16
16 TE=ABS(ALP3R-AXR)+ABS(ALP3I-AXI)
IF(TE7/(ABS(AXR)+ABS(AXI))-1.E-7)18,18,17
17 N3=N3+1
   ALP1R=ALP2R
   ALP1I=ALP2I
   ALP2R=ALP3R
   ALP2I=ALP3I
   ALP3R=ALP4R
   ALP3I=ALP4I
   RETR=RET2R
   BET1I=RET2I
   RET2R=RET3R
   RET2I=RET3I
   BET3R=TEM
   BET3I=TEM
   IF(N3=100)14,18,18
18 N4=N4+1
   ROOT(R(N4)=ALP4R
   ROOT(I(N4)=ALP4I
   N3=0
41 IF(N4-N1)30,37,37
37 DO 5 L=1,N2
   ROOT(R(L)=ROOT(R(L)*TMD
5   ROOT(I(L)=ROOT(I(L)*TMD
   DO 6 M=1,N2
   J=M - 1
6   COE(M)=(COE(M)*TMD**J)*CTEBB
   RETURN
30 IF(ABS(ROOTI(N4))-1.E-5)10,10,31
31 GO TO(32,10)+L
32 AXR=ALP1R
   AXI=ALP1I
   ALP1R=ALP2R
   AXI=ALP2I
   ALP2R=ALP3R
   M=5
   GO TO 99
33 BET1R=TEM
   BET1I=TEM
   AXR=ALP2R
   AXI=ALP2I
   M=6
GO TO 99
34 RET2R=TEMR
BET21=TEM1
AXR=ALP3R
AXI=ALP3I
ALP31=X0L31
L=2
M=3
99 TEMR=COE(1)
TEM1=0.*0.
DO 100 I=1,N1
TE1=TEMR*AXR-TEM1*AXI
TEM1=TEM1*AXR+TEMR*AXI
100 TEMR=TE1+COE(I+1)
HELL=TEMR
PELL=TE1
42 IF(N4)102,103,102
102 DO 103 I=1,N4
TEM1=AXR-ROOTR(I)
TEM2=AXI-ROOTI(I)
TE1=TEM1*TEM1+TEM2*TEM2
TE2=(TEMR*TEM1+TEM1*TEM2)/TE1
TEM1=[TEM1*TEM1-TEMR*TEM2]/TE1
101 TEMR=TE2
103 GO TO(11,12,13,15,33,34),M
END
SAMPLE CALCULATIONS

Sample cases involving a blunt body \((\phi_N = 90\,\text{degrees})\) and a cone \((\phi_N = 20\,\text{degrees})\) were run at 150 kilofoots and \(M_{\infty} = 17.2\). The calculations began at \(x = 3\) and carried through \(x = 50\). As an example, the figure compares the axial temperature as a function of \(x\) for the blunt body and the cone. The transition from laminar to turbulent flow occurs sooner for the blunt body thereby initiating a rapid decay in axial temperature.

Input data for the cone are listed below. Velocities are in feet per second. Temperature is in degrees Kelvin. Enthalpies are in feet² per second². Altitude, diameter, and \(x\) are in feet. EXOUT determines the amount of output data. For minimum output, EXOUT = 0.0 or is omitted. If EXOUT \# 0.0, intermediate results at each \(\Delta x = 0.1\) are printed as listed preceding statement 90 in Appendix A.

<table>
<thead>
<tr>
<th>Card</th>
<th>1-10</th>
<th>11-20</th>
<th>21-30</th>
<th>31-40</th>
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<td>20.0</td>
<td>150,000.0</td>
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<td>(H_{\infty})</td>
<td>(P_0)</td>
<td>(L_{\infty})</td>
<td>(C_D)</td>
<td>(d)</td>
<td>(u_\infty)</td>
<td>(X_c)</td>
</tr>
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<td></td>
<td>1.217</td>
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<td>1.0</td>
<td>1.0</td>
<td>0.104</td>
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Figure. Axial Temperature Profile (°K)
## Initial Conditions

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<th>TC</th>
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<th>HC</th>
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<td>ALPHAO(D2)</td>
<td>ALPHAO(D3)</td>
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<td>ALPHAO(D5)</td>
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<td>ALPHAEO(D1)</td>
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<td>PINP10</td>
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<tr>
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<td>ZR</td>
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## Output Data for a 20-Degree Cone -- Partial Listing