BOUNDARY LAYER PROFILE MEASUREMENTS
IN HYPERSONIC NOZZLES

NORMAN E. SCAGGS
FLUID DYNAMICS FACILITIES RESEARCH LABORATORY

PROJECT 7065

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JULY 1966

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FOREWORD

This technical report was prepared by Norman E. Scaggs, Fluid Dynamics Facilities Research Laboratory of the Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio. The work reported was accomplished under Project 7065, "Aerospace Simulation Techniques Research," and in partial fulfillment of the requirements for the degree of master of science.

The author wishes to express his gratitude to Dr. John. D. Lee of the Ohio State University who conceived this investigation and whose direction and helpful advice made possible the completion of this task. The author also wishes to thank the staff of the Aerodynamic Laboratory of the Ohio State University for their cooperation and assistance in the conduction of the experiments.
ABSTRACT

An experimental investigation made to determine the flow parameter profiles across the boundary layer on contoured, axisymmetric hypersonic nozzles is described. The pitot pressure and total temperature profiles measured across the boundary layers on nozzles of Mach number seven and twelve are shown in graphical form. The static temperature and velocity profiles, calculated with the assumption of constant static pressure across the boundary layer, are given. A correlation is shown to exist between the exponent of the velocity profile power law and the product of the ratios of wall temperature to free stream total temperature and axial distance to momentum thickness. The static temperature profiles, calculated from the measured data is compared with Crocco's relationship for the static temperature in terms of the velocity profile.
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SYMBOLS

A \quad \text{area}

V \quad \text{velocity}

T \quad \text{temperature}

P \quad \text{pressure}

M \quad \text{Mach number}

\rho \quad \text{density}

\dot{m} \quad \text{mass flow rate}

a \quad \text{speed of sound}

\gamma \quad \text{ratio of specific heats}

R \quad \text{gas constant for air}

K \quad \text{constant in mass flow equation (B-4)}

K_3 \quad \text{constant in the total temperature equation (B-9)}

y \quad \text{distance from nozzle wall}

x \quad \text{distance from nozzle throat along the nozzle centerline}

n \quad \text{reciprocal of the velocity profile power law exponent}

\theta \quad \text{momentum thickness}

\delta^* \quad \text{displacement thickness}

\delta \quad \text{boundary layer thickness}

Pr \quad \text{Prandtl number}

H \quad \text{shape factor}

\text{Re}_x \quad \text{Reynolds number at the edge of the boundary layer based on distance from nozzle throat}

\text{Re}_\theta \quad \text{Reynolds number at the edge of the boundary layer based on the momentum thickness}

\text{Re}_{\delta^*} \quad \text{Reynolds number at the edge of the boundary layer based on the displacement thickness}
\( \text{Re}_\delta \)  \hspace{1em} \text{Reynolds number at the edge of the boundary layer based on the boundary layer thickness}

\( D^* \)  \hspace{1em} \text{diameter of nozzle throat}

**SUBSCRIPTS**

\( \infty \)  \hspace{1em} \text{free stream condition}

\( \text{stagnation condition} \)

\( 1 \)  \hspace{1em} \text{condition just upstream of first throat}

\( 2 \)  \hspace{1em} \text{condition just upstream of second throat}

\( w \)  \hspace{1em} \text{condition at nozzle wall}

\( \text{(none)} \)  \hspace{1em} \text{static condition within the boundary layer}

**SUPERSCRIPyTS**

\( * \)  \hspace{1em} \text{at the critical cross section (the choked throats of the sonic-pneumatic probe system and the nozzle throats) — except for \( \delta^* \)
I. INTRODUCTION

The boundary layer growth on hypersonic nozzles is extremely important because it greatly affects the flow field produced by any physical wall contour. In order to design a nozzle to produce a given set of free stream conditions, a correction to the inviscid core coordinates must be made to allow for the boundary layer effects. In general the design of the nozzle is carried out by first calculating the inviscid core and then adding a displacement thickness correction at each station to obtain the final physical wall coordinate. The main difficulty in nozzle design is the accurate calculation of this displacement thickness correction.

There are several methods for calculating the displacement thickness growth, either by empirical methods or analytical methods, Reference (1-5). The analytical methods require assumptions about the flow parameter profiles (V, T, p, M) across the boundary layer. If the Mach number and two of the other parameters are known, then all of the flow parameters in the boundary layer can be calculated including the displacement thickness.

The purpose of this investigation was to make actual measurements of the flow parameters in the boundary layer on hypersonic nozzles in an effort to answer some of the questions about the variation of the parameters across the boundary layer. The experiments were carried out in two nozzles, one delivered a nominal Mach number of seven, the other delivered a nominal Mach number of twelve. The pitot pressure distribution and the total temperature distribution were measured across the boundary layer. The static pressure was assumed
to be constant across the boundary layer. These measured parameters were then used to calculate all the other flow parameters as well as displacement and momentum thicknesses.

One of the novel aspects of this investigation was the use of a sonic-pneumatic probe in conjunction with a pitot probe to measure the total temperature distribution. This type temperature probe has been used by other authors to measure exhaust temperatures from jet engines, Reference (8), and the boundary layer measurements on models, Reference (6) and (7), but as far as the author knows this is the first attempt to use such a probe on hypersonic nozzles where the boundary layers are very thick (of the order of one to two inches) and the possibility of probe interactions with the flow field is minimized.
II. EXPERIMENTS

A. General

This investigation was carried out in the twelve-inch hypersonic wind tunnel of the Aerodynamic Laboratory of The Ohio State University. The facility is described in detail in Reference (9). Two different nozzles were used for the experiments, one producing a nominal Mach number of seven, the other producing a nominal Mach number of twelve. The nozzles were contoured, axisymmetric type. The details of the nozzles are shown in Figs. 5 and 6.

Four surveys of the boundary layer were made in each nozzle. In the nominal Mach seven nozzle, two surveys were made 3.5 inches upstream of the nozzle exit at Reynolds numbers, \( \text{Re}_x \) of \( 0.71 \times 10^6 \) and \( 2.24 \times 10^6 \), and two surveys were made 10.5 inches upstream of the nozzle exit at Reynolds numbers, \( \text{Re}_x \) of \( 0.66 \times 10^6 \) and \( 2.05 \times 10^6 \). In the nominal Mach twelve nozzle, two surveys were made 4.5 inches upstream of the nozzle exit at Reynolds numbers, \( \text{Re}_x \) of \( 1.28 \times 10^6 \) and \( 2.82 \times 10^6 \), and two surveys were made 12 inches upstream of the nozzle exit at Reynolds numbers, \( \text{Re}_x \) of \( 1.18 \times 10^6 \) and \( 2.68 \times 10^6 \).

In all the tests, the pitot pressure distribution was measured, the total temperature distribution was measured, the wall temperature and wall static pressure were measured. The tunnel stagnation pressure and temperature were also measured.

B. Measurement Techniques

A combination pitot pressure and sonic-pneumatic probe was used to measure the pitot pressure distribution and the total temperature
distribution. A schematic of the combination probe is shown in Fig. 1. The design of the sonic-pneumatic probe is based on the results of Reference (10). It was shown in Reference (10) that it is possible to measure the total temperature in a hypersonic stream within ±3% of the true value within the temperature range of these tests.

The principle of the sonic-pneumatic probe is based on the application of the perfect gas law, the law of the conservation of mass, and Bernoulli's Theorem. Its operation is based on the principle of equal mass flow through two sonic orifices in series. The mass flow rate through a sonic orifice is given by:

\[ \dot{m} = \rho^* a^* A^* \]  \hspace{1cm} (B-1)

Now by the perfect gas law:

\[ \rho^* = \frac{\rho^*}{\rho_o} \rho_o = \frac{\rho^*}{\rho_o} \frac{P_o}{RT_o} \]  \hspace{1cm} (B-2)

and:

\[ a^* = \frac{\gamma^* RT^*}{\rho_o} \]  \hspace{1cm} (B-3)

Putting (B-2) and (B-3) into (B-1) gives:

\[ \dot{m} = \frac{\rho^*}{\rho_o} \frac{P_o}{RT_o} \sqrt{\gamma^* RT^*} A^* = \frac{\rho^*}{\rho_o} \sqrt{\frac{T^*}{T_o}} \sqrt{\frac{\gamma^*}{R}} \frac{P_o}{T_o} \frac{A^*}{A^*} = K \frac{P_o}{\sqrt{T_o}} A^* \]  \hspace{1cm} (B-4)

If two orifices are in series as shown in Fig. 3 and sufficient pressure ratio is maintained across each to ensure that both are choked, then the mass flow rate through the first orifice is:
\[ \dot{m}_1 = K_1 \frac{P_{O_1}}{\sqrt{T_{O_1}}} A_1^* \quad (B-5) \]

and the mass flow rate through the second orifice is:

\[ \dot{m} = K_2 \frac{P_{O_2}}{\sqrt{T_{O_2}}} A_2^* \quad (B-6) \]

Now by the principle of the conservation of mass, the mass flow rate through the first orifice is equal to the mass flow rate through the second orifice.

Thus:

\[ \dot{m}_1 = \dot{m}_2 \quad (B-7) \]

or equating equations (B-5) and (B-6)

\[ K_1 \frac{P_{O_1}}{\sqrt{T_{O_1}}} A_1^* = K_2 \frac{P_{O_2}}{\sqrt{T_{O_2}}} A_2^* \quad (B-8) \]

Now solving for \( T_{O_1} \):

\[ T_{O_1} = \left( \frac{K_1}{K_2} \right)^2 \left( \frac{P_{O_1}}{P_{O_2}} \right)^2 \left( \frac{A_2}{A_1^*} \right)^{-2} T_{O_2} = K_3 \left( \frac{P_{O_1}}{P_{O_2}} \right)^2 \left( \frac{A_2}{A_1^*} \right)^{-2} T_{O_2} \quad (B-9) \]

A plot of \( K_3 \) versus total temperature from Reference (11) is shown in Fig. 4. For this series of tests, \( K_3 \) varies from .966 to 1.000 and since an iteration process would be required if these corrections were made, a value of \( K_3 = 1.00 \) was assumed throughout the data reduction. So the final equation is:
\[ T_{O_1} = \left( \frac{P_{O_1}}{P_{O_2}} \right)^2 \left( \frac{A_2^*}{A_1^*} \right)^{-2} T_{O_2} \]  \hspace{1cm} (B-10)

In this series of tests, \( P_{O_1} \), \( P_{O_2} \), and \( T_{O_2} \) were measured at each point in the boundary layer, so the only thing left undetermined was the ratio, \( \frac{A_2^*}{A_1^*} \). If \( \frac{A_2^*}{A_1^*} \) were simply the physical area ratio of the two orifices, then everything would be known; however, the effective area in each throat is decreased by the boundary layer established there by the flow through the sonic-pneumatic probe. The boundary layer thickness is a function of the flow conditions at each throat and, therefore, \( \frac{A_2^*}{A_1^*} \) must be determined by a calibration. The calibration must be performed where the total temperature, \( T_{O_1} \), upstream of the first throat is known.

Before this series of tests were conducted, a calibration of the probe was made in the same nozzles as the experiments were made but the probe was kept in the isentropic core. In the isentropic core the stagnation temperature upstream of the first orifice is equal to the tunnel stagnation temperature. Thus, \( T_{O_1} \), \( T_{O_2} \), \( P_{O_1} \), and \( P_{O_2} \) are all known and \( \left( \frac{A_2^*}{A_1^*} \right)^{-2} \) is determined from the following equation:

\[ \left( \frac{A_2^*}{A_1^*} \right)^{-2} = \frac{T_{O_1}}{T_{O_2}} \left( \frac{P_{O_1}}{P_{O_2}} \right)^2 \]  \hspace{1cm} (B-11)
Equation (B-11) is a rearrangement of equation (B-10). The tunnel stagnation conditions were varied to obtain a range of upstream conditions for the first orifice and the results of the calibration were plotted versus the Reynolds number of the first orifice. Fig. 2 shows the results of the calibration. With the area ratio now determined as a function of the first orifice Reynolds number, $T_{O_1}$ can be determined at any point in the boundary layer. An iteration process is required since the exact Reynolds number of the first orifice is not known until the exact total temperature upstream of the first orifice is known. This does not require a great number of calculations if the determination of $T_{O_1}$ is started from the free stream edge of the boundary layer where $T_{O_1}$ is known and continued toward the nozzle wall. Three iterations were usually sufficient to determine $T_{O_1}$.

$P_{O_1}$ was sensed by a $0 - 2.0$ PSID variable reluctance transducer, $P_{O_2}$ was sensed by a $0 - 0.1$ PSID variable reluctance transducer. $T_{O_2}$ was sensed by a chromel–alumel bare wire thermocouple and recorded on a Brown Recorder. The pressures were recorded by the printer on the laboratory's analog computer described in Reference (9). The tunnel stagnation pressure was measured with a Laboratory Test Gage having an accuracy of $0.25$ per cent of the gage range. The tunnel stagnation temperature was measured with a Type S (platinum–platinum rhodium) thermocouple and recorded on a Brown self-balancing potentiometer. The nozzle wall temperatures were measured with Type T thermocouples and recorded on a Brown self-balancing potentiometer. The nozzle wall static pressures were measured on a
silicone manometer tilted to thirty degrees from the horizontal. The instrumentation for the laboratory and the wind tunnel is discussed in more detail in Reference (9).
III. RESULTS

One of the first requirements in the reduction and analyzation of the data was the determination of the actual boundary layer thickness. Probably the most logical method was to look at the total temperature or velocity profiles and choose the boundary layer thickness where these parameters reached some percentage, such as ninety-nine per cent of free stream value. But this method does not work very well in a hypersonic nozzle since both these parameters are very close to ninety-nine per cent of free stream values over a large portion of the boundary layer and thus a wide range of values could be chosen for the boundary layer thickness. A different method was used in this investigation to determine the boundary layer thickness. This method involved the pitot pressure profiles and the edge of the boundary layer was chosen as the point where the pitot pressure gradient, \( \frac{dp_{oi}}{dy} \), was equal to zero.

Another decision that had to be made before the data could be reduced was what value of the static pressure across the boundary layer to use in the calculation of the Mach number distribution and the density distribution. Measurements of the wall static pressure opposite the axial position of the probe were made for each test. These values were compared to the free stream static pressure calculated from the Mach number at the edge of the boundary layer and the tunnel stagnation conditions. In all cases the wall static pressure was approximately ten per cent higher than the calculated free stream value. However, since these values were of the order of 0.01 PSIA,
it was assumed that the free stream value was correct and that some deficiency in the measuring technique allowed the indicated higher values. The free stream static pressure at the edge of the boundary layer was used in the data reduction.

The pitot pressure distributions for the Mach seven nozzle are shown in Fig. 7 and for the Mach twelve nozzle in Fig. 8. The profiles do not seem to be affected by the free stream Reynolds number but do tend to become fuller as the Mach number and the boundary layer thickness decrease.

The Mach number profiles are shown in Fig. 9 for the Mach seven nozzle and in Fig. 10 for the Mach twelve nozzles. These profiles naturally exhibit the same trends as the pitot pressure profiles. The Mach number profiles were calculated by using the tables of Reference (11) for the ratio of static pressure to pitot pressure versus Mach number.

The total temperature profiles, shown in Figs. 11 and 12, seemed to be affected by both the free stream Reynolds number, \( \text{Re}_x \), and the temperature ratio, \( \frac{T_w}{T_{01\infty}} \). \( T_w \) is the wall temperature and \( T_{01\infty} \) is the total temperature of the free stream at the edge of the boundary layer. The profiles become fuller as both the temperature ratio and the free stream Reynolds number increase.

Even though data was taken all the way to the wall until the probe touched, some of the data nearest the wall was discarded. Two criteria were used to eliminate some of the data. One criterion was when the centerline of the probe was within one probe tip diameter.
(0.063 inches) of the wall. The other criterion was the pressure ratio, \( \frac{P_{o_1}}{P_{o_2}} \). When this ratio became less than five, there was suspicion that the first throat was unchoked thus invalidating the data. Using these two limits for the data, the profiles extend only to approximately \( \frac{V}{\delta} \) values of 0.2. To probe the flow nearer to the wall than this, a much smaller sonic-pneumatic probe would have to be used.

With the Mach number and total temperature profiles now available, the velocity profiles and the static temperature profiles were calculated. The static temperature at each location was calculated by using the tables of Reference (11) for the ratio of static temperature to total temperature for the measured Mach number and total temperature. The velocity at a given point was calculated using the equation:

\[
V = M \sqrt{\gamma RT}
\]

where \( T \) is the static temperature and \( M \) is the Mach number at a particular point. The velocity and static temperature profiles are shown in Figs. 13 through 20. The Mach number profiles are also included on these figures.

In order to correlate the velocity profiles with each other, an exponential curve of the form:

\[
\frac{V}{V_\infty} = \left( \frac{y}{\delta} \right)^\frac{1}{n}
\]

was fitted through the velocity profile points and an \( n \) determined. The values of \( n \) varied from 3.31 to 7.49 for all the test conditions.
A correlation was found to exist for the \( n \)'s of the form:

\[
n = k \left( \frac{T_w}{T_{01\infty}} \right) \left( \frac{x}{\theta} \right)
\]

where \( k \) is a constant, \( \frac{T_w}{T_{01\infty}} \) is the ratio of the wall temperature to the free stream total temperature, \( x \) is the distance from the nozzle throat along the axis of symmetry, and \( \theta \) is the momentum thickness. Fig. 21 illustrates this correlation. In Reference (3), a correlation of \( n \) versus \( \text{Re}_\theta \) is made for experimental data from several investigators. There is considerable scatter in the data with points deviating from a mean line drawn through the data by as much as plus or minus twenty per cent. The data from the present investigation falls within this range of variation, but it seemed reasonable that a better correlation could be made by using a parameter that includes the heat transfer effects. The parameter, \( \left( \frac{T_w}{T_{01\infty}} \right) \left( \frac{x}{\theta} \right) \), includes this effect and the correlation is much better as shown in Fig. 21 where the scatter is less than plus or minus ten per cent.

An attempt was also made to compare the static temperature profiles with the Crocco Relationship for the static temperature in terms of the velocity profile. Crocco's Relationship has the following form:

\[
\frac{T}{T_{01\infty}} = \frac{T_w}{T_{01\infty}} - \left( \frac{T_w - T_{aw}}{T_{01\infty}} \right) \left( \frac{V}{V_{\infty}} \right) - \left( \frac{T_{aw} - T_{\infty}}{T_{01\infty}} \right) \left( \frac{V}{V_{\infty}} \right)^2
\]
where

\[ T_{aw} = T_\infty + \left( Pr \right)^{\frac{1}{3}} \left( T_{O\infty} - T_\infty \right) \]

from Reference (4). This relationship predicts static temperatures that are much higher than the values calculated from the measured data. A comparison of the actual data to that predicted by Crocco is shown in Figs. 22 and 23 for two different Mach numbers. The experiments would not have shown the inflection point near the surface but the values away from the wall are also much different than the values calculated by Crocco's Relationship. It appears that the Crocco Relationship does not allow for a high enough energy loss to the nozzle wall.

In addition to the above mentioned parameters, the displacement thicknesses and the momentum thicknesses were calculated for each test condition. The displacement thickness is defined as:

\[ \frac{\delta^*}{\delta} = \int_0^1 \left( 1 - \frac{\rho}{\rho_\infty} \frac{V}{V_\infty} \right) \frac{d \frac{V}{\delta}}{ \delta } \]

and the momentum thickness is defined as:

\[ \frac{\theta}{\delta} = \int_0^1 \frac{\rho}{\rho_\infty} \frac{V}{V_\infty} \left( 1 - \frac{V}{V_\infty} \right) \frac{d \frac{V}{\delta}}{ \delta } \]

These values along with several other parameters including \( \delta \), \( Re_\theta \), \( Re_\delta \), \( Re_{\delta^*} \), \( H \), and others are listed in Table I.
IV. CONCLUSIONS

A. The pitot pressure and the total temperature profiles were measured across the boundary layer in two hypersonic nozzles at four different conditions each.

B. All of the other parameters across the boundary layer, including $\delta^*$, $\theta$, and $H$, were calculated for these conditions with the assumption that the static pressure was constant across the boundary layer.

C. A correlation was found to exist between the exponent, $n$, of the velocity profile power law and the product of the two ratios

$$\frac{T_w}{T_{0_{\infty}}} \quad \text{and} \quad \frac{X}{\theta}.$$ 

D. Crocco's Relationship for the static temperature in terms of the velocity profile predicts temperatures much higher than the calculated value from the measured data except at the wall and the edge of the boundary. These points are forced to match.

E. The velocity profiles exhibit the characteristic shape of turbulent boundary layer profiles shown in Reference (12). Based on this observation, all the boundary layers in this series of tests were assumed to be turbulent.

F. The sonic-pneumatic probe used in this investigation is not considered adequate for examining the flow in the region nearest the wall. It is felt that the results in the outer eighty per cent of the boundary layer are reasonably accurate and this particular probe is a useful tool in this region.
REFERENCES


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**TABLE I**

**COMPOSITE LISTING OF TEST CONDITIONS AND RESULTANT PARAMETERS**

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FIGURE 3

SCHEMATIC DIAGRAM OF A SONIC-PNEUMATIC PROBE SYSTEM
\[ K_3 = \frac{\left( \frac{\rho_1}{\rho_{o1}} \right)^2 \frac{T_1}{T_{o1}} \frac{\gamma_1}{\gamma_{o1}}}{\left( \frac{\rho_2}{\rho_{o2}} \right)^2 \frac{T_2}{T_{o2}} \frac{\gamma_2}{\gamma_{o2}}} \]

**Figure 4**

Caloric Imperfection Correction
FIGURE 5
MACH SEVEN NOZZLE

SCALE = 1" = 10" IN X DIRECTION
SCALE = 1" = 5" IN Y DIRECTION
\( M_\infty = 6.65 \), \( Re_x = 0.71 \times 10^6 \), \( Tw/T_{_0\infty} = 0.3118 \)
\( \delta = 0.127 \text{ FT.} \)

\( M_\infty = 6.72 \), \( Re_x = 2.24 \times 10^6 \), \( Tw/T_{_0\infty} = 0.4082 \)
\( \delta = 0.118 \text{ FT.} \)

\( M_\infty = 6.46 \), \( Re_x = 0.66 \times 10^6 \), \( Tw/T_{_0\infty} = 0.3167 \)
\( \delta = 0.108 \text{ FT.} \)

\( M_\infty = 6.54 \), \( Re_x = 2.05 \times 10^6 \), \( Tw/T_{_0\infty} = 0.4473 \)
\( \delta = 0.091 \text{ FT.} \)

\[ \text{PITOT PRESSURE PROFILES AT MACH 6.5} \]
\[ M_\infty = 11.52, \quad \text{Re}_X = 1.28 \times 10^6, \quad T_w/T_{\infty} = 0.2684 \]
\[ \delta = 0.201 \text{ FT.} \]
\[ \Delta M_\infty = 11.30, \quad \text{Re}_X = 1.18 \times 10^6, \quad T_w/T_{\infty} = 0.2752 \]
\[ \delta = 0.184 \text{ FT.} \]
\[ \bigcirc M_\infty = 11.66, \quad \text{Re}_X = 2.82 \times 10^6, \quad T_w/T_{\infty} = 0.2913 \]
\[ \delta = 0.159 \text{ FT.} \]
\[ \Delta M_\infty = 11.40, \quad \text{Re}_X = 2.68 \times 10^6, \quad T_w/T_{\infty} = 0.3010 \]
\[ \delta = 0.151 \text{ FT.} \]

Figure 9
PITOT PRESSURE PROFILES AT MACH 11.5
\( \bullet M_\infty = 6.65, \text{ Re}_x = 0.71 \times 10^6, \frac{T_w}{T_{\infty}} = 0.3118 \)

\( \triangle M_\infty = 6.72, \text{ Re}_x = 2.24 \times 10^6, \frac{T_w}{T_{\infty}} = 0.4082 \)

\( \square M_\infty = 6.46, \text{ Re}_x = 0.66 \times 10^6, \frac{T_w}{T_{\infty}} = 0.3167 \)

\( \triangle M_\infty = 6.54, \text{ Re}_x = 2.06 \times 10^6, \frac{T_w}{T_{\infty}} = 0.4473 \)

**Figure 9**

MACH NUMBER PROFILES AT MACH 6.5
\[ \begin{align*}
\bigcirc & \ M_\infty = 11.66, \ Re_x = 2.82 \times 10^6, \ Tw / T_{0,\infty} = 0.2913 \\
\triangle & \ M_\infty = 11.52, \ Re_x = 1.28 \times 10^6, \ Tw / T_{0,\infty} = 0.2684 \\
\square & \ M_\infty = 11.30, \ Re_x = 1.18 \times 10^6, \ Tw / T_{0,\infty} = 0.2752 \\
\triangle & \ M_\infty = 11.40, \ Re_x = 2.68 \times 10^6, \ Tw / T_{0,\infty} = 0.3010
\end{align*} \]
\( M_\infty = 6.65, \text{ Re}_x = 0.71 \times 10^6, \frac{T_w}{T_{w,\infty}} = 0.3118 \\delta = 0.127\ \text{FT.} \)

\( \blacktriangle M_\infty = 6.46, \text{ Re}_x = 0.61 \times 10^6, \frac{T_w}{T_{w,\infty}} = 0.3167 \\delta = 0.1080\ \text{FT.} \)

\( \square M_\infty = 6.72, \text{ Re}_x = 2.24 \times 10^6, \frac{T_w}{T_{w,\infty}} = 0.4082 \\delta = 0.1167\ \text{FT.} \)

\( \blacktriangle M_\infty = 6.54, \text{ Re}_x = 2.05 \times 10^6, \frac{T_w}{T_{w,\infty}} = 0.4473 \\delta = 0.0914\ \text{FT.} \)

![Graph showing total temperature profiles at Mach 6.5](image-url)

**Figure 11**

Total temperature profiles at Mach 6.5
\[ M_\infty = 11.30, \text{ Re}_x = 1.18 \times 10^6, \frac{T_w}{T_{o,\infty}} = 0.2752 \]
\[ \delta = 0.184 \text{ FT.} \]

\[ M_\infty = 11.52, \text{ Re}_x = 1.28 \times 10^6, \frac{T_w}{T_{o,\infty}} = 0.2684 \]
\[ \delta = 0.201 \text{ FT.} \]

\[ M_\infty = 11.40, \text{ Re}_x = 2.68 \times 10^6, \frac{T_w}{T_{o,\infty}} = 0.3010 \]
\[ \delta = 0.151 \text{ FT.} \]

\[ M_\infty = 11.66, \text{ Re}_x = 2.82 \times 10^6, \frac{T_w}{T_{o,\infty}} = 0.2913 \]
\[ \delta = 0.159 \text{ FT.} \]

**TOTAL TEMPERATURE PROFILES AT MACH 11.5**

**FIGURE 12**

\[ \frac{T_0}{T_{o,\infty}} \]
\( \odot \) \( V / V_\infty \)

\( \triangle \) \( M / M_\infty \)

\( \square \) \( T / T_{\infty} \)

--- \((Y/\delta)^{1/3.31}\)

\( M_\infty = 6.65 \), \( Re_x = 0.71 \times 10^6 \), \( Tw / T_{\infty} = 0.3118 \)

\( T_{\infty} = 1860^\circ R \), \( P_{o\infty} = 42 \text{ psia} \), \( P_\infty = 0.0134 \text{ psia} \)

\( T_\infty = 186^\circ R \), \( \delta = 0.127 \text{ FT} \), \( \delta^*/\delta = 0.2796 \)

\( \theta / \delta = 0.1240 \), \( H = 2.26 \), \( X = 4.01 \text{ FT} \).

**Figure 13**

Mach number, velocity, and static temp profiles.
\[ \begin{align*}
\bigcirc \quad V/V_\infty \\
\Delta \quad M/M_\infty \\
\Box \quad T/T_\infty \bigg( \frac{\gamma}{8} \bigg)^{1/4.34}
\end{align*} \]

\( M_\infty = 6.46, \quad Re_x = 0.66 \times 10^6, \quad Tw/T_\infty = 0.3167 \)

\( T_\infty = 1860^\circ R, \quad P_\infty = 42 \text{ PSIA}, \quad P_\infty = 0.0161 \text{ PSIA} \)

\( T_\infty = 199^\circ R, \quad 8 = 0.1080 \text{ FT}, \quad 8^*/8 = 0.2372 \)

\( \theta/8 = 0.1016, \quad H = 2.33, \quad X = 3.51 \text{ FT}. \)

**Figure 14**

MACH NUMBER, VELOCITY, AND STATIC TEMP. PROFILES
\( V/V_\infty \)

\( M/M_\infty \)

\( T/T_{\infty} \)\(^{1/4.85} \)

\[ (\gamma/\delta)^{1/4.85} \]

\[ M_\infty = 6.72, \ Re_x = 2.24 \times 10^6, \ Tw/T_{\infty} = 0.4082 \]

\[ T_{\infty} = 1460^\circ R, \ P_{\infty} = 95 \text{PSIA}, \ P_\infty = 0.0291 \text{PSIA} \]

\[ T_\infty = 145^\circ R, \ \delta = 0.1176 \text{ FT}, \ \delta/\delta = 0.2786 \]

\[ \theta/\delta = 0.0958, \ H = 2.91, \ X = 4.01 \text{ FT} \]

**Figure 15**

Mach number, velocity, and static temp. profiles
\( \bigcirc \) \( \frac{V}{V_\infty} \)
\( \bigtriangleup \) \( \frac{M}{M_\infty} \)
\( \surd \) \( \frac{T}{T_{\infty}} \) \( \left( \frac{Y}{\delta} \right)^{1.749} \)

\( M_\infty = 6.54 \), \( Re_X = 2.05 \times 10^6 \), \( \frac{T_w}{T_{\infty}} = 0.4473 \)
\( T_{\infty} = 1460^\circ R \), \( P_{\infty} = 94 \) PSIA, \( P_\infty = 0.0341 \) PSIA
\( T_\infty = 153^\circ R \), \( \delta = 0.0914 \) FT., \( \delta / \delta = 0.2435 \)
\( \theta / \delta = 0.0673 \), \( H = 3.62 \), \( X = 3.51 \) FT.

**Figure 16**

MACH NUMBER, VELOCITY, AND STATIC TEMP. PROFILES
$V/V_\infty$
$M/M_\infty$
$T/T_\infty$

$(y/\delta)^{1/3.69}$

$M_\infty = 11.30, \text{Re}_x = 1.18 \times 10^6, T_w/T_{\infty} = 0.2752$
$T_{\infty} = 2060^\circ \text{R}, P_{\infty} = 314 \text{ PSIA}, P_\infty = 0.0031 \text{ PSIA}$
$T_\infty = 78^\circ \text{R}, \delta = 0.1842 \text{ FT., } \delta^*/\delta = 0.3730$
$\theta/\delta = 0.0839, H = 4.44, X = 4.83 \text{ FT.}$

**Figure 17**
MACH NUMBER, VELOCITY, AND STATIC TEMP. PROFILES
○ $V/V_\infty$
△ $M/M_\infty$
□ $T/T_\infty$

$(Y/\delta)^{1/4.92}$

$M_\infty = 11.52$, $Re_x = 1.28 \times 10^6$, $T_w/T_\infty = 0.2684$

$T_\infty = 2060^\circ R$, $P_\infty = 314$ PSIA, $P_\infty = 0.0027$ PSIA

$T_\infty = 75^\circ R$, $\delta = 0.2010$ FT, $\delta^*/\delta = 0.4489$

$\theta/\delta = 0.0547$, $H = 8.21$, $X = 5.46$ FT.

**Figure 18**

Mach Number, Velocity, and Static Temp Profiles
\( \bigcirc \) \( V/V_\infty \)
\( \triangle \) \( M/M_\infty \)
\( \square \) \( T/T_\infty \)^{1/6.16}

\[ M_\infty = 11.40, \quad Re_x = 2.68 \times 10^6, \quad Tw/T_\infty = 0.3010 \]
\[ T_{\infty} = 2060^\circ R, \quad P_{\infty} = 714 \text{ PSIA}, \quad P_\infty = 0.0065 \text{ PSIA} \]
\[ T_\infty = 76^\circ R, \quad \delta = 0.1512 \text{ FT.}, \quad \delta/\delta = 0.3877 \]
\[ \theta/\delta = 0.0533, \quad H = 7.27 \text{ FT.}, \quad X = 4.83 \text{ FT.} \]

**Figure 19**
MACH NUMBER, VELOCITY, AND STATIC TEMP. PROFILES
\( V/V_\infty \)
\( \Delta M/M_\infty \)
\( \square T/T_{o_\infty} \)
\[ \frac{(Y/\delta)^{1/2}} {17} \]

\( M_\infty = 11.66, \quad Re_x = 2.82 \times 10^6, \quad T_w/T_{o_\infty} = 0.2913 \)
\( T_{o_\infty} = 2060^\circ R, \quad P_{o_\infty} = 714 \text{ PSIA}, \quad P_\infty = 0.0056 \text{ PSIA} \)
\( T_\infty = 73^\circ R, \quad \delta = 0.1587 \text{ FT}, \quad \delta/\delta = 0.3731 \)
\( \theta/\delta = 0.0477, \quad H = 7.83, \quad X = 5.46 \text{ FT} \)

**Figure 20**

MACH NUMBER, VELOCITY, AND STATIC TEMP. PROFILES
$M_\infty = 6.46, \quad Re_x = 0.66 \times 10^6, \quad Tw/T_{a_\infty} = 0.3167$
$8 = 0.108$ FT.

--- CALCULATED FROM CROCCO'S RELATIONSHIP
REFERENCE (4)

\[ T/T_{a_\infty} = 0.3167 + 0.5863 \frac{V}{V_\infty} - 0.7960 \left( \frac{V}{V_\infty} \right)^2. \]

**Figure 22**
COMPARISON OF STATIC TEMP PROFILE
AT MACH 6.5
○ $M_\infty = 11.66, \quad Re_x = 2.82 \times 10^6, \quad Tw/T_\infty = 0.2913$
$\delta = 0.159$ FT.

- CALCULATED FROM CROCCO'S RELATIONSHIP
  REFERENCE (4)
  
  \[ \frac{T}{T_\infty} = 0.2913 + 0.6087 \frac{V}{V_\infty} - 0.8660 \left( \frac{V}{V_\infty} \right)^2 \]

FIGURE 23
COMPARISON OF STATIC TEMP. PROFILE
AT MACH 11.5
An experimental investigation made to determine the flow parameter profiles across the boundary layer on contoured, axisymmetric hypersonic nozzles is described. The pitot pressure and total temperature profiles measured across the boundary layers on nozzles of Mach number seven and twelve are shown in graphical form. The static temperature and velocity profiles, calculated with the assumption of constant static pressure across the boundary layer, are given. A correlation is shown to exist between the exponent of the velocity profile power law and the product of the ratios of wall temperature to free stream total temperature and axial distance to momentum thickness. The static temperature profiles, calculated from the measured data is compared with Crocco's relationship for the static temperature in terms of the velocity profile.
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