BUCKLING OF HEADS, CYLINDERS, AND RINGS

Kenneth P. Buchert
ARO, Inc.

December 1966

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BUCKLING OF HEADS, CYLINDERS, AND RINGS

Kenneth P. Buchert *
ARO, Inc.

*Participant in the Summer Facility Program, ARO, Inc., and Professor at the University of Missouri, Columbia, Missouri.

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FOREWORD

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This technical report has been reviewed and is approved.

F. N. Price
Lt Col, USAF
AF Representative, AEF
Directorate of Test

Leonard T. Glaser
Colonel, USAF
Director of Test
ABSTRACT

The purpose of this report is to briefly review and summarize the research that has been accomplished on the buckling of shells and to develop practical methods of stability analyses that are applicable to large fabricated shell construction. The analyses are applicable to doubly curved shells (i.e., spheres, ellipsoids, dished heads, etc.), to stiffened and unstiffened cylinders, and to rings. Although the methods presented were specifically developed in the analysis of the Mark I - Aerospace Environmental Facility, Arnold Engineering Development Center, Arnold Air Force Station, Tennessee, they may be used to analyze the general class of shells constructed by the "Heavy Plate Fabricators". This general type of shell is fabricated in steel or aluminum, is relatively thick, has a relatively large radius of curvature, and is fabricated generally in accordance with the ASME-Section VIII-Unfired Pressure Vessel Code. Although the loading is basically caused by external pressure, such loadings as temperature variations and concentrated loads and moments are considered. The important effects of fabrication tolerances are also considered in the analyses.
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SECTION I
INTRODUCTION

The sections of this report give the recommended methods for the stability analyses of doubly curved shells, stiffened cylindrical shells, and rings. The general procedure is as follows:

1. Determine the deviations from a geometrically perfect surface caused by construction methods, temperature variations, concentrated loads, and other effects.

2. Substitute the results of Step 1 above into the appropriate buckling equation.

3. Calculate the factor of safety by dividing the calculated or allowable pressure by the operating pressure.

The state-of-the-art of the analyses of the stability of shell structures leaves much to be desired. Based upon previous experience, it is recommended that, if at all possible, factors of safety of two or more should be provided for any given operating or emergency condition. It must be remembered that such factors as residual stresses do have an important effect on buckling but have not been included in the calculations.

SECTION II
GENERAL COMMENTS ON BUCKLING

Although shell structures subjected to external pressure-type loadings have been built for several thousand years, the present state-of-the-art of analysis leaves much to be desired. The stress distribution in shells can often be handled with confidence and reliability by using well developed equations and digital computers. Such is not the case for the buckling problem. Ample evidence is available (such as the failures of the NASA-Environmental Chamber at Houston, the 340-ft dome in Bucharest, the common bulkhead of the Saturn SIVB, and the 300-ft dome in Fargo) to illustrate the difficulties encountered by the engineer in designing an external pressure-type structure. The pioneers in the field of elastic-plastic stability are reviewing and continually changing the basic concepts of buckling, as well as the mathematical techniques, in an attempt to correlate theories and practice. Considerable progress has been made in the last five years in the development of new theories and in the explanation of previous seemingly inconsistent laboratory and field results. The work at Cal Tech, Harvard, Florida, Stanford, and NASA-Langley on cylinders and the work at Missouri, MIT, Cal Tech, the Navy-David Taylor Model Basin, and the Army-Watertown Materials
Research Agency on spheres have added to our knowledge of the physics of the buckling process. However, almost every investigator and engineer in the field agrees that there is not a reliable, prudent method of analysis available that will ensure the success of a design without the use of a relatively high factor of safety.

It is therefore recommended that only a reasonable amount of time and expense be devoted to the calculations of deflections, stresses, etc., for use in the buckling equations presented herein. Calculations accurate to, say ±25 percent are sufficient for the buckling analysis, since to spend the effort for greater accuracy is to neglect the crux of the problem. It should be noted that the ductility of the material has allowed many marginal designs, when stress is the important parameter, to function without visible signs of distress. Unfortunately, marginal designs when buckling is important are usually readily visible to all in that failure usually occurs.

The great majority of research in the shell stability or buckling field has been done on the spherical segment subjected to external pressure and on the cylinder subjected to axial compression (note: the practical application of the cylinder usually includes axial compression because of the end thrust on the vessel). The general comments enumerated above apply to both of these problems. It is quite interesting to note that the "classical-linear-solutions", developed about a half century ago, to both these problems are identical, i.e.,

\[
\sigma_{cr} = \frac{E}{\sqrt{3(1-\nu^2)}} \left( \frac{t}{R} \right)
\]  \hspace{1cm} (1)

Where

- \( \sigma_{cr} \) - is the critical buckling stress
- \( E \) - is the modulus of elasticity
- \( t \) - is the shell thickness
- \( R \) - is the radius of curvature
- \( \nu \) - is Poisson's ratio

Most experimental results and field failures for both spheres and cylinders indicate a buckling stress of about one-third to one-fourth of this theoretical value. The theory indicates in both cases that the buckle wavelength is relatively short. Experiments indicate that the buckle wavelength is considerably longer than that given by the theory (Ref. 1). This fact has been used in the design of stiffened cylindrical shells for a number of years.
The use of stiffeners to reduce the unstiffened shell wavelength of buckle and thus raise the buckling load has been used on the Mark I Chamber and, of course, has been used on the large ducting in most of the AEDC facilities. The use of rings to reduce the unstiffened shell buckle wavelength of cylinders is similar in many respects to reducing and effective length of a column and thus raising its buckling load. This same technique has been developed at the University of Missouri for spheres and other doubly curved surfaces. A number of full scale structures have been built using this technique.

Recently developed large deflection theories of buckling have brought stress or pressure closer together. Considerable research has been done on factors that significantly influence the buckling loads which do not appear in Eq. (1). These factors include the following:

1. **Imperfections**: Obviously, it is impossible to make a geometrically perfect shell. In fact, the ASME Code allows considerable variations from a perfect shell in order to minimize cost. Certain kinds of imperfections (deviations from a perfect surface) may reduce the buckling load. Some types of imperfections have little effect on the buckling load.

2. **Edge Conditions**: Recent tests and theoretical studies indicate that edge conditions greatly influence the buckling load. Unfavorable edge conditions have been shown experimentally and theoretically to influence the buckling load by factors of three or more. Although the qualitative effects are known, no detailed theoretical method is available to quantitatively evaluate the effect of edge conditions on all ASME Code-type shells. It should be pointed out that, given the freedom of choice, the designer can often with little effort design a shell whose edges are not critical in determining the buckling load.

3. **Yield Strength or Pseudo-Elastic Effects**: It is well known that yield stress is an important parameter in column buckling. The AISC specification uses the yield strength as one of the parameters in determining the load carrying capacity of a column.

   Since experiments on shells seem to confirm that buckling is a large deflection and a relatively large wavelength process, it would appear that yield strength of the material is a factor in shell buckling. As deflections become large, stresses increase and in many practical cases reach the yield strength of the material during buckling. Numerous investigators
have shown that yield strength is important in cylindrical shell buckling. Tests at the University of Missouri indicate that the same is true for spheres.

After a careful review of the details of the Mark I design and the available theory on cylinders and spheres, it is recommended that the critical buckling loads be determined on the basis of a deflection-allowable pressure analysis. The method recommended is as follows. The deviations from a perfect surface are determined. These deviations include the fabrication tolerances, the deflections caused by temperature variations, and the deflections caused by concentrated loads, edge conditions, etc. Based on the total deviations or deflections, the allowable external pressure loading is determined from available theory. It is recommended that Donnell's and Wan's (Ref. 1) theory be used on the cylindrical shells and the writer's theories (Ref. 2) be used on the heads. These theories are modified as indicated later in this report to take into account the various parameters indicated above and to provide the engineer with a usable tool.

The method of comparing the allowable buckling stress to the actual stresses that exist under all loading conditions was eliminated after careful consideration. There were many reasons for the decision. One example can be cited as to the very unconservative nature of this type of calculation. In the case of temperature effects on buckling, one can calculate reductions in stress caused by temperature decreases which would indicate an increase in the factor of safety under external pressure. However, the deflections caused by the temperature decrease could be high enough to significantly decrease the factor of safety. Similar effects would be present when considering erection tolerances and residual welding stresses and distortions.

SECTION III
HEAD BUCKLING EQUATIONS

The following equation is recommended for finding the critical buckling pressure for heads:

$$p_{cr} = 1.41 \cdot a \cdot \frac{A^{1/2}}{B} \cdot E \left( \frac{1}{r_1} \right)^2$$

(2)

Where $a$ is a factor determined from Fig. 1,
\[ A = \frac{1}{2} + (C_4-1) + (C_4-1)^2 + \frac{1}{2} (C_4-1)^3 + \frac{1}{10} (C_4-1)^4 + \frac{1}{2} \left( \frac{1}{C_4^2} - 1 \right) \]
\[ + \frac{4}{3} \left( \frac{1}{C_4^2} - 1 \right) (C_4-1) + \frac{3}{2} \left( \frac{1}{C_4^2} - 1 \right) (C_4-1)^2 + \frac{4}{5} \left( \frac{1}{C_4^2} - 1 \right) (C_4-1)^3 \]
\[ + \frac{1}{6} \left( \frac{1}{C_4^2} - 1 \right) (C_4-1)^4 + \frac{1}{6} \left( \frac{1}{C_4^2} - 1 \right)^2 + \frac{1}{2} \left( \frac{1}{C_4^2} - 1 \right)^2 (C_4-1) \]
\[ + \frac{3}{5} \left( \frac{1}{C_4^2} - 1 \right)^3 (C_4-1)^4 + \frac{1}{3} \left( \frac{1}{C_4^2} - 1 \right)^4 (C_4-1)^3 + \frac{1}{14} \left( \frac{1}{C_4^2} - 1 \right)^5 (C_4-1)^2 \]

\[ B = \frac{1}{2} + \frac{1}{2} (C_2-1) + \frac{1}{6} (C_2-1)^2 + \frac{3}{4} \left( \frac{1}{C_2^2} - 1 \right) + \left( \frac{1}{C_2^3} - 1 \right) (C_2-1) \]
\[ + \frac{3}{5} \left( \frac{1}{C_2^2} - 1 \right) (C_2-1)^2 + \frac{1}{2} \left( \frac{1}{C_2^2} - 1 \right)^2 + \frac{3}{4} \left( \frac{1}{C_2^2} - 1 \right)^3 (C_2-1) \]
\[ + \frac{3}{10} \left( \frac{1}{C_2^2} - 1 \right)^2 (C_2-1)^3 + \frac{1}{8} \left( \frac{1}{C_2^2} - 1 \right)^3 + \frac{1}{5} \left( \frac{1}{C_2^2} - 1 \right)^4 (C_2-1) \]
\[ + \frac{1}{12} \left( \frac{1}{C_2^2} - 1 \right)^4 (C_2-1)^2 \]

where \( R_i > R_s \)

and \( R_1 \) and \( R_2 \) = principal radii of curvature

\[ C = \frac{R_1}{R_2} \]
\[ C_2 = \frac{\beta_2}{\beta_1} \]

\( \Delta \) = deviation from a perfect surface

\( \beta_1, \beta_2 \) = half wave angles of buckle in the principal directions (wavelength divided by two times the radius of curvature)

For a given geometry, all of the quantities will be known except the coefficient \( C_2 \) (the ratio of the half wave angles of buckle).

The value of \( C_2 > 0 \) is varied until the minimum value of \( \rho_{cr} \) is found. This minimum value of \( \rho_{cr} \) is the buckling pressure.

For an ellipsoidal head

\[ R_a = \frac{a^2 b^3}{[a^2 \sin^2 \phi + b^2 \cos^2 \phi]^{3/2}} \]
\[ R_b = \frac{a^2}{[a^2 \sin^2 \phi + b^2 \cos^2 \phi]^{1/2}} \]
Where

\[ a = \text{length of semimajor axis} \]
\[ b = \text{length of semiminor axis} \]
\[ \phi = \text{angle between axis of symmetry of head and point at which curvature is being calculated.} \]

A computer program has been written to determine \( p_{cr} \) which greatly reduces the amount of time for the calculations.

If \( \Delta \) is small, Eq. (2) reduces to that given in Ref. 2. If \( R_1 = R_2 \), the shell is spherical and Eq. (2) reduces to the well known "von Karman-Tsien" sphere buckling equation. The \( \Delta \) correction was originally published by the writer in Ref. 3 in linear form for small \( \frac{\Delta}{t} \) values. It was later extended for larger values of \( \frac{\Delta}{t} \), as given in Fig. 1. It should be noted that Eq. (2) gives a conservative prediction of the buckling pressure. In calculating \( \frac{\Delta}{t} \), it is assumed that any kind of deviation from a perfect surface will cause a reduction in the buckling pressure. This is not the case as was demonstrated in Ref. 2. The research on the effects of imperfections has not advanced to the stage such that detailed discriminatory quantitative calculations can be made.

SECTION IV
CYLINDER BUCKLING EQUATIONS

The following equation is recommended for finding the critical buckling pressure for ring stiffened cylinders.

\[
p_{cr} = \frac{\beta \, k_p \, \sigma^2}{12(1-\nu^2)} \left( \frac{L}{L} \right)^2 \frac{t}{R}
\]

(3)

Where

\( \beta \) - is a factor determined from Fig. 2
\( k_p \) - is a factor determined from Fig. 3
\( \sigma = \frac{pR}{2t} \)
\( p \) = operating pressure
\( U = 0.62 \left( \frac{\Delta}{t} \right) \frac{n^2}{R} \)
\( n = 0.45 \, \sqrt{Rt} \)
\[ \sigma_y = \text{yield stress of the material} \]

\[ Z_{1r} = \frac{L^2 (1-t^2)^{\frac{1}{2}}}{rt} \]

\( L \) = distance between stiffeners

If the imperfections are negligible, Eq. (3) reduces to the equation given in Ref. 4. The reduction factor \( \beta \) was given in Ref. (1) for cylinders under axial compression. The cylinder subject to external pressure is subjected to axial compression and radial pressure. The experimental results for short cylinders under external pressure buckle at pressures considerably below the "classical linear theory" value similar to cylinders under axial compression. Based on the above, it is considered reasonable to use the results shown in Fig. 2. Here again, the results are probably conservative since all imperfections or deflections are considered as reducing the buckling load.

SECTION V
RING BUCKLING EQUATIONS

The following equation is recommended for finding the critical buckling pressure for stability rings:

\[ \sigma_y = \frac{\rho_{cr} LR}{A} + \left[ \frac{\rho_{cr} LAR}{1 - \frac{\rho_{cr} L R^3}{3E1}} + M_T \right] \frac{c}{1} \]  \hspace{1cm} (4)

Where

\( \Delta \) = is the deflection of the ring caused by imperfections, loads, temperature gradients, etc.

\( M_T \) = the moment caused by a temperature variation

\( A \) = the area of the ring

\( I \) = the moment of inertia of the ring

\( c \) = the distance from the neutral axis of the ring to the outer fiber.

This equation is an extension of the one given by Timoshenko in Ref. 5. The moment of inertia and area of the ring should be calculated assuming an effective area of shell acting with the ring that is physically attached to the shell. The usual assumption is that the effective length of shell is 1.56 \( \sqrt{Rt} \).
It should be emphasized that the primary purpose of the stability rings is to stiffen the cylindrical shell such that the wavelength of buckle of the cylinder will be approximately equal to the distance between stiffeners. Extreme care should be exercised in applying significant extraneous loads on the stability rings.

If the temperature variations are axisymmetric, they do not contribute to the value of \( \Delta \) in Eq. (4) because they are of a constant value. If the temperature variations are not axisymmetric, they do contribute to \( \Delta \).

SECTION VI
GENERAL COMMENTS ON DEFLECTIONS CAUSED BY CONCENTRATED LOADS

The least weight structure is never the least cost structure. As a result, the structure designed and built in general accordance with ASME Code is not the least weight structure. Designers attempt to arrive at structural details that will result in a minimum cost structure. Therefore, the fabrication tolerances are liberal to be consistent with low cost and competitive situations. Usually the part of the value or \( \Delta \) assigned to fabrication tolerances that will be used in the preceding formulas will be relatively large. Since most of the effects of \( \Delta \) on the buckling pressure are non-linear and decrease as \( \Delta \) increases, gross approximations in computing the deflections due to temperature and other load effects will have a relatively small effect on the calculated buckling pressure. This, coupled with the state-of-the-art of the knowledge of the buckling process, allows rather crude approximations for the calculated deflections caused by temperature and other effects.

Typical acceptable approximations might be as follows:

1. An ellipsoidal head may be assumed to be spherical.
2. Edge conditions on heads may be assumed to be simply supported or fixed.
3. Small openings and reinforcing may be neglected.
4. Ring deflection may be neglected compared to shell deflection.
5. Deflections caused by temperature variations may be assumed to be unrestrained. That is, the contribution to \( \Delta \) could be assumed as the coefficient of expansion, times the temperature change, times the length over which the temperature change takes place.
6. Such approximations as given in Roark (Ref. 6) may be used.

The above comments apply to the buckling analysis. Obviously, the approximations cannot be used when the detailed stress distribution is under investigation.

REFERENCES


2. Buchert, K. P. "Buckling of Doubly Curved Orthotropic Shells." Engineering Experiment Station, University of Missouri, Columbia, Missouri, November 1965.


BIBLIOGRAPHY

The following is a list of selected papers that will give the reader additional background in the subject of shell buckling.


Fig. 1 Coefficient $\alpha$ as a Function of $\frac{A}{t}$
Fig. 2 Coefficient $\beta$ as a Function of $\frac{UR}{t}$
Fig. 3 Coefficient $k_p$ as a Function of $Z_L$
The purpose of this report is to briefly review and summarize the research that has been accomplished on the buckling of shells and to develop practical methods of stability analyses that are applicable to large fabricated shell construction. The analyses are applicable to doubly curved shells (i.e., spheres, ellipsoids, dished heads, etc.), to stiffened and unstiffened cylinders, and to rings. Although the methods presented were specifically developed in the analysis of the Mark I - Aerospace Environmental Facility, they may be used to analyze the general class of shells constructed by the "Heavy Plate Fabricators". This general type of shell is fabricated in steel or aluminum, is relatively thick, has a relatively large radius of curvature, and is fabricated generally in accordance with the ASME-Section VIII-Unfired Pressure Vessel Code. Although the loading is basically caused by external pressure, such loadings as temperature variations and concentrated loads and moments are considered. The important effects of fabrication tolerances are also considered in the analyses.
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**KEY WORDS**

- buckling
- heads
- cylinders
- rings
- pressure
- temperature

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