COMPUTER ANALYSIS OF THREE-FACTOR INTERACTIONS IN CONTINGENCY TABLES

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THE UNIVERSITY OF WISCONSIN
madison, wisconsin
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ABSTRACT

Numerous methods have been proposed for testing the hypothesis of zero three-factor interaction in contingency tables. These methods are incorporated into a single computer program which is designed to calculate the test statistic by any or all of the proposed techniques, and which also provides estimators of the interactions and corresponding two-sided confidence intervals. The program is written in FORTRAN 63 for the CDC-3600 computer, and will analyze interactions in contingency tables of dimensions $2 \leq r \leq 5$, $2 \leq s \leq 5$, $2 \leq t \leq 16$, for $r \leq s \leq t$. 
COMPUTER ANALYSIS OF THREE-FACTOR INTERACTIONS
IN CONTINGENCY TABLES

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1. Introduction

The analysis of three-factor interactions in contingency tables has been the subject of numerous recent papers culminating with the admirably lucid treatment and summary given by Goodman [15]. This series of papers begins with one by Kastenbaum and Lamphiear [8] in which the authors present an iterative technique for solving \((r-1)(s-1)(t-1)\) simultaneous fourth-degree equations. These equations result from an extension Bartlett's [2] test of zero interaction in a \(2 \times 2 \times 2\) table to the general three-way \((r \times s \times t)\) table as presented by Roy and Kastenbaum [6]. Subsequent authors [10, 11, 12, 13, 14, 15] have proposed alternative techniques of analysis, each progressively simpler than the preceding one, but all testing the same hypothesis. In every case; the test statistic is distributed asymptotically as chi-square with \((r-1)(s-1)(t-1)\) degrees of freedom.

It is the purpose of this paper to describe a computer program which has been designed to calculate the test statistic by any or all of the proposed techniques. This program also provides the estimates of interaction and corresponding simultaneous confidence intervals given by Goodman [15]. The program was written in FORTRAN 63 for the CDC-3600 computer, and will do analyses on contingency tables of dimensions \(2 \leq r \leq 5, 2 \leq s \leq 5, 2 \leq t \leq 16, \) for \(r \leq s \leq t\). [Appendix].

2. Tests of Hypotheses

Let \(n_{ijk}\) be the number of observations in the \(i\)th row, \(j\)th column, and

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kth layer of a three-way contingency such that \( \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} n_{ijk} = n \), the total sample size. Denote by \( 0 < \Pi_{ijk} < 1 \) the corresponding probability where \( \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} \Pi_{ijk} = 1 \). Also define the following relationships:

\[
\sum_{i=1}^{r} n_{ijk} = n_{i.k}, \quad \sum_{j=1}^{s} n_{ijk} = n_{i.k}, \quad \sum_{k=1}^{t} n_{ijk} = n_{ij},
\]

with similar relationships for the \( \Pi_{ijk} \)'s. The hypothesis of zero three-factor interaction in an \( r \times s \times t \) contingency table is given by:

\[
H_0: \quad \frac{\Pi_{rsk} \Pi_{ijk}}{\Pi_{lsk} \Pi_{rjk}} = \frac{\Pi_{rst} \Pi_{ijt}}{\Pi_{lst} \Pi_{rjt}} \tag{2.1}
\]

for all integers \( i, j, k \) such that \( 1 < i < r, \quad 1 < j < s, \quad 1 < k < t \).

2.1. Kastenbaum and Lamphiear [8]

For the \( r \times s \times t \) table, Kastenbaum and Lamphiear generalized an iterative technique proposed by Norton [4] for handling the simultaneous fourth-degree equations which arise in the estimation process. Under the null hypothesis (2.1), estimates of the parameters may be achieved by first solving for all \( x_{ijk} \) in the following systems of equations:

\[
\begin{align*}
\left( n_{rsk} - \sum_{i=1}^{r-1} \sum_{j=1}^{s-1} x_{ijk} \right) \left( n_{ijk} - x_{ijk} \right) &= \left( n_{rst} - \sum_{i=1}^{r-1} \sum_{j=1}^{s-1} x_{ijt} \right) \left( n_{ijt} - x_{ijt} \right), \\
\left( n_{lsk} + \sum_{j=1}^{s-1} x_{ijk} \right) \left( n_{rjk} + \sum_{i=1}^{r-1} x_{ijk} \right) &= \left( n_{lst} + \sum_{j=1}^{s-1} x_{ijt} \right) \left( n_{rjt} + \sum_{i=1}^{r-1} x_{ijt} \right),
\end{align*}
\]

for all positive integers \( i < r, \quad j < s, \quad \text{and} \quad k < t \), and subject to the constraints.
t-1
\sum_{k=1}^{t} x_{ijk} = -x_{ijt} \quad \text{for all positive integers } i \leq r \text{ and } j \leq s . \text{ Let } \nu_{ijk} \text{ be the values of } x_{ijk} \text{ which satisfy (2.1.1) for all integers } 1 \leq i \leq r, i \leq j \leq s , \text{ and } 1 \leq k \leq t . \text{ Then}
\[ x^2 = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} \nu_{ijk}^2 = \frac{(n_{ijk} - \nu_{ijk})}{(2.1.2)} \]
is distributed asymptotically as chi-square with \((r-1)(s-1)(t-1)\) degrees of freedom.

The program solves equations (2.1.1) by applying Newton's method of functional iteration to a technique proposed by Norton [4], and computes the value of the test statistic (2.1.2). It then prints out the number of iterations, the number of degrees of freedom, the value of the test statistic, and the observed and expected cell frequencies, with identification for all the cells in the contingency table. A sample print-out is given in Table 1.

2.2. Darroch [10]

An alternative solution to equations (2.1.1) is given by Darroch. This method involves the iterative solution for \( \delta_{jk} \), \( \phi_{ki} \), and \( \psi_{ij} \) in the following \((rs + rt + st)\) simultaneous non-linear equations:

\[ \frac{n_{ijk}}{n} = \delta_{jk} \sum_{i=1}^{r} \phi_{ki} \psi_{ij} \quad (2.2.1) \]

\[ \frac{n_{i,k}}{n} = \phi_{ki} \sum_{i=1}^{s} \psi_{ij} \delta_{jk} \quad (2.2.2) \]

\[ \frac{n_{i,j}}{n} = \psi_{ij} \sum_{k=1}^{t} \delta_{jk} \phi_{ki} \quad (2.2.3) \]

The iteration begins by setting \( \phi_{ki} = \phi_{ki}^{(1)} = \frac{n_{i,k}}{n} \) and \( \psi_{ij} = \psi_{ij}^{(1)} = \frac{n_{ij}}{n} \) in equation (2.2.1) and solving for \( \delta_{jk}^{(2)} \), then in equation (2.2.2) let
\[ \psi_{ij} = \psi^{(1)}_{ij} \text{ and } \delta_{jk} = \delta^{(2)}_{jk} \] and solve for \( \phi_{ki} = \phi^{(2)}_{ki} \). In equation (2.2.3), let \( \delta_{jk} = \delta^{(2)}_{jk} \text{ and } \varphi_{ki} = \varphi^{(2)}_{ki} \) and solve for \( \psi_{ij} = \psi^{(2)}_{ij} \). Then return to equation (2.2.1), letting \( \varphi_{ki} = \varphi^{(2)}_{ki} \text{ and } \psi_{ij} = \psi^{(2)}_{ij} \), and solve for \( \delta_{jk} = \delta^{(3)}_{jk} \). Continue the iteration in this way until \( \psi_{ij}^{(m)} = \psi_{ij}^{(m-1)}, \delta_{jk}^{(m)} = \delta_{jk}^{(m-1)}, \) and \( \varphi_{ki}^{(m)} = \varphi_{ki}^{(m-1)} \) to five decimal places for all positive integers \( i \leq r, j \leq s, k \leq t \).

At this point calculate

\[
X^2 = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} \left[ n_{ijk} - n\delta_{jk}^{(m)} \varphi_{ki}^{(m)} \psi_{ij}^{(m)} \right]^2 / n\delta_{jk}^{(m)} \varphi_{ki}^{(m)} \psi_{ij}^{(m)} \quad (2.2.4)
\]

Darroch has shown that this test statistic is distributed asymptotically as chi-square with \( (r-1)(s-1)(t-1) \) degrees of freedom. Moreover (2.2.4) and (2.1.2) are algebraically identical.

The program solves equations (2.2.1), (2.2.2), and (2.2.3), and computes the value of the test statistic (2.2.4). The print-out is identical to that of Table 1, except that the number of iterations is generally smaller. In this sense, the Darroch procedure is superior to the Kastenbaum-Lampheir procedure.

2.3. Plackett [11]

Let \( \theta_{ijk} = \Pi_{ijk} / \Pi_{..k} \) be the conditional probability that an observation will fall in the \( i \)th row and \( j \)th column, given that it is in the \( k \)th layer. It follows that the null hypothesis (2.1) may be rewritten as

\[
H_0: \Delta_{ijk} = \Delta_{ijt} \quad (2.3.1)
\]

for all integers \( 1 \leq i < r, 1 \leq k < t \), where \( \Delta_{ijk} = \theta_{rsk} \theta_{ijk} / \theta_{is k} \theta_{rjk} \). Using a criterion suggested by Woolf [5], Plackett presents a procedure for testing a null hypothesis equivalent to (2.3.1), namely

\[
H_0: \Gamma_{ijk} = \Gamma_{ijt} \quad (2.3.2)
\]
for all integers $1 \leq i < r$, $1 \leq j < s$, $1 \leq k < t$, where $\Gamma_{ijk} = \log \Delta_{ijk}$. This procedure is based on the fact that the maximum likelihood estimator of $\log \Delta_{ijk}$ is $\log d_{ijk}$, where $d_{ijk} = n_{ijk}^{rsk} n_{isk}^{rjk}$ for all integers $1 \leq i < r$, $1 \leq j < s$, $1 \leq k < t$. Moreover, the variance of $\log d_{ijk}$ may be estimated consistently by

$$ u_{ijk} = \frac{1}{n_{ijk}} + \frac{1}{n_{rsk}} + \frac{1}{n_{isk}} + \frac{1}{n_{rjk}}. $$

If $R$ and $S$ are two matrices of order $(r-1) \times r$ and $(s-1) \times s$ respectively with rows orthogonal to each other and to the unit vector, then the direct product, $[R*S]$, is a matrix with $(r-1)(s-1)$ rows and $rs$ columns. The elements of each row of this matrix provide the coefficients of a linear combination of the logarithm of the frequencies in the $k$th layer of the contingency table. More specifically, the matrices $R$ and $S$ are formed as follows:

$$ R = \{r_{\alpha i}\} = \begin{cases} 1 & \text{for} \ 1 \leq i \leq \alpha, \\ -\alpha & \text{for} \ i = \alpha + 1, \\ 0 & \text{for} \ i > \alpha + 1, \end{cases} \text{where} \ 1 \leq i < r, 1 \leq \alpha < r, \text{and} $$

$$ S = \{s_{\beta j}\} = \begin{cases} 1 & \text{for} \ 1 \leq j \leq \beta, \\ -\beta & \text{for} \ j = \beta + 1, \\ 0 & \text{for} \ j > \beta + 1, \end{cases} \text{where} \ 1 \leq j < s, 1 \leq \beta < s. $$

Then for each positive integer $k \leq t$, a column vector $z_k$ is generated from the product of the $(r-1)(s-1) \times rs$ matrix $[R*S]$ with the $rs \times 1$ vector $\{\log n_{ijk}\}$.

The elements of $z_k$ are

$$ z_{k\alpha\beta} = \sum_{i=1}^{r} \sum_{j=1}^{s} r_{\alpha i} s_{\beta j} \log n_{ijk}. \quad (2.3.3.) $$

The asymptotic distribution of $z_k = \{z_{k\alpha\beta}\}$ is multivariate normal, with dispersion matrix

$$ V_k = [R*S] D^{-1}_{ijk} [R*S]'. $$
where $D^{-1}_{n_{ijk}}$ is a square matrix of order $rs$ with elements $\{\frac{1}{n_{ijk}}\}$ on the diagonal and zeros elsewhere. The elements of $V_k$ are

$$\text{Cov}[z_{k\alpha'\beta'k\alpha'\beta'}] = \sum_{i=1}^{r} \sum_{j=1}^{s} \rho_{\alpha'i} \rho_{\alpha'j} \sigma_{\beta'j} / n_{ijk},$$

(2.3.4)

for all positive integers $\alpha, \alpha' < r$ and $\beta, \beta' < s$. If $z_k'$ denotes the transpose of $z_k$, and $V_k^{-1}$ the inverse of $V_k$, then on the hypothesis of zero three-factor interaction, (2.3.2), the statistic

$$Y^2 = \sum_{k=1}^{t} z_k' V_k^{-1} z_k - \left[ \sum_{k=1}^{t} z_k' V_k^{-1} \right] \left[ \sum_{k=1}^{t} V_k^{-1} \right]^{-1} \left[ \sum_{k=1}^{t} V_k^{-1} z_k \right]$$

(2.3.5)

is distributed asymptotically as chi-square with $(r-1)(s-1)(t-1)$ degrees of freedom.

This portion of the program computes the elements of all the vectors $z_k$ and of their associated dispersion matrices $V_k$ from formulas (2.3.3) and (2.3.4) for all integers $1 < k < t$. To evaluate the test statistic, (2.3.5), $(t+1)$ square matrices of order $(r-1)(s-1)$ are inverted. The print-out, Table 2, displays all the vectors $z_k$ and their associated dispersion matrices $V_k$ for all integers $1 < k < t$; and $\sum_{k=1}^{t} z_k' z_k, \sum_{k=1}^{t} V_k^{-1}, F = \left[ \sum_{k=1}^{t} V_k^{-1} \right]^{-1}, \sum_{k=1}^{t} z_k' V_k^{-1}$, as well as the value of the test statistic, (2.3.5), and the number of degrees of freedom.

2.4. Goodman [14, 15]

The approach suggested by Goodman modifies Plackett’s method for testing the null hypothesis (2.3.2) by redefining the two matrices $R$ and $S$ as follows:

$$R = \{\rho_{\alpha}\} = \begin{cases} 1 & \text{for } i = \alpha, \\ -1 & \text{for } i = r, \\ 0 & \text{elsewhere, where } 1 \leq i \leq r, 1 \leq \alpha < r, \end{cases}$$

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\[ S = \{ \sigma_j \} = \begin{cases} 1 & \text{for } j = \beta \\ -1 & \text{for } j = s \\ 0 & \text{elsewhere}, \text{ where } 1 \leq j \leq s, 1 \leq \beta < s . \end{cases} \]

As in Plackett's method, column vectors \( g_k = \{ g_{k\alpha\beta} \} \) are generated, where

\[ g_{k\alpha\beta} = \sum_{i=1}^{r} \sum_{j=1}^{s} \rho_{\alpha i} \sigma_j \log n_{ij} . \tag{2.4.1} \]

The asymptotic distribution of each \( g_k \) is multivariate normal with dispersion matrix \( U_k \), whose elements are

\[ \text{Cov}[g_{k\alpha\beta}, g_{k\alpha'\beta'}] = \sum_{i=1}^{r} \sum_{j=1}^{s} \rho_{\alpha i} \rho_{\alpha' i} \sigma_j \sigma_{\beta' j} n_{ij} \tag{2.4.2} \]

for all positive integers \( \alpha, \alpha' < s \). If \( U_k^{-1} = M_k \), \( Q = [\sum_{k=1}^{t} M_k]^{-1} \), and \( \tilde{g} = \sum_{k=1}^{t} M_k g_k \), then the statistic

\[ Y^2 = \sum_{k=1}^{t} g_k^t M_k g_k - \tilde{g}^t Q \tilde{g} \tag{2.4.3} \]

is distributed asymptotically as chi-square with \( (r-1)(s-1)(t-1) \) degrees of freedom. Moreover, expressions (2.35) and (2.4.3) are identical.

Except for the new definitions of the matrices \( R \) and \( S \), the computational procedure using either Plackett's or Goodman's methods appear to be identical. In both methods it is necessary to invert \( (t+1) \) matrices of order \( (r-1)(s-1) \). However, Goodman makes a significant contribution, at this point, by demonstrating that only one \( (r-1)(s-1) \) matrix and \( t \) matrices of order \( (r-1) \), for \( r \leq s \), need to be inverted in calculating the test statistic (2.4.3). The reduction in computing time which results from this modified procedure may be appreciable. The calculations are carried out as follows:
(i) For every integer $k$, $1 \leq k \leq t$, define the $(r-1) \times (r-1)$ matrices

$$B^{(k)} = D_{n_{isk}} - \frac{1}{n_{sk}} \{n_{isk}'\} \{n_{isk}\}',$$

$$B^{(k)}_j = D_{n_{ijk}} - \frac{1}{n_{ijk}} \{n_{ijk}'\} \{n_{ijk}\}',$$

where $D_{n_{isk}}$ and $D_{n_{ijk}}$ are $(r-1) \times (r-1)$ diagonal matrices with elements $n_{isk}$ and $n_{ijk}$ respectively, and where $\{n_{isk}\}'$ and $\{n_{ijk}\}'$ are the respective transposes of the $(r-1) \times 1$ column vectors $\{n_{isk}\}$ and $\{n_{ijk}\}$, for integers $1 \leq i < r$, and $1 \leq j < s$.

(ii) Evaluate $C^{(k)} = B^{(k)} + \sum_{j=1}^{s-1} B^{(k)}_j$, an $(r-1) \times (r-1)$ matrix, and its inverse $G^{(k)}$, $1 \leq k \leq t$.

(iii) Construct the submatrices

$$M^{(k)}_{jj'} = \begin{cases} B^{(k)}_j - B^{(k)}_j G^{(k)} B^{(k)}_j & \text{for } j = j' = 1, 2, \ldots, s-1, \\ - B^{(k)}_j G^{(k)} B^{(k)}_j & \text{for } j \neq j'. \end{cases}$$

(iv) For every $k$, $1 \leq k \leq t$, form the square matrix $M_k$ of order $(r-1)(s-1)$ in equation (2.4.3) from the $(s-1)^2$ submatrices $M^{(k)}_{jj'}$ each of order $(r-1) \times (r-1)$.

(v) Evaluate the test statistic using equation (2.4.3).

The print-out (Table 3), for this section of the program displays all the vectors $g_k'$, their associated dispersion matrices $U_k$, and

$$\sum_{k=1}^{t} H_k^2 = \sum_{k=1}^{t} g_k' M_k g_k', \quad Q, \quad G'$$

as well as the value of the test statistic, (2.4.3), and the number of degrees of freedom.
2.5. Goodman [15]: The $2 \times 2 \times t$ Contingency Table

For the three-way contingency table with $r = 2$ rows, $s = 2$ columns, and $t \geq 2$ layers, Goodman proposes three alternative test statistics. Two of these are based on an analysis of the cell frequencies, and the third, which is a special case of the procedure discussed in section 2.4, is based on an analysis of the log-frequencies.

(i) Define $d_k = \frac{n_{11k} n_{22k}}{n_{12k} n_{21k}}$, $u_k = \frac{1}{n_{11k}} + \frac{1}{n_{22k}} + \frac{1}{n_{12k}} + \frac{1}{n_{21k}}$, $v_k = d_k^2 u_k$, and $w_k = 1/v_k$, $1 \leq k \leq t$. Then

$$X^2 = \sum_{k=1}^{t} d_k^2 w_k - \left[ \sum_{k=1}^{t} d_k w_k \right]^2 / \sum_{k=1}^{t} w_k$$

(2.5.1)

is distributed asymptotically as chi-square with $(t-1)$ degrees of freedom.

Equation (2.5.1) may be used to test the hypothesis of zero three-factor interaction which, in the $2 \times 2 \times t$ table may be specified as

$$H_0: \frac{\Pi_{11k} \Pi_{22k}}{\Pi_{12k} \Pi_{21k}} = \frac{\Pi_{11k} \Pi_{22k}}{\Pi_{12k} \Pi_{21k}} \quad 2 \leq k \leq t.$$  

(2.5.2)

Equation (2.5.2) may be rewritten as

$$H_0: \Delta_k = \Delta, \quad 2 \leq k \leq t,$$

(2.5.3)

where

$$\Delta_k = \frac{\Pi_{11k} \Pi_{22k}}{\Pi_{12k} \Pi_{21k}} = \frac{\theta_{11k} \theta_{22k}}{\theta_{12k} \theta_{21k}}$$

is a measure of the two factor interaction in the $k$th layer of the table. The maximum likelihood estimator of $\Delta_k$ is $d_k$, and its variance can be estimated consistently by $v_k$.

Moreover, Goodman points out that this null hypothesis may be partitioned into the following $(t-1)$ sub-hypotheses:
\(H_1: \Delta_1 = \Delta_2\)

\(H_2: \Delta_1 = \Delta_3 \quad (\text{given that } \Delta_1 = \Delta_2)\)

\(H_3: \Delta_1 = \Delta_4 \quad (\text{given that } \Delta_1 = \Delta_2 = \Delta_3)\)  

\(\vdots\)

\(H_{t-1}: \Delta_1 = \Delta_t \quad (\text{given that } \Delta_1 = \Delta_2 = \ldots = \Delta_{t-1})\).

Each of these sub-hypotheses \(H_k \ (1 \leq k < t)\) can be tested using a single degree of freedom as follows:

Define \(d_{\gamma} = \frac{n_{1\gamma}n_{2\gamma}}{n_{1\gamma}n_{21\gamma}} \quad 1 \leq \gamma < k\),

and corresponding quantities \(u_{\gamma}, v_{\gamma}, w_{\gamma}\). Let \(d_k^* = \sum_{\gamma=1}^{k} d_{\gamma} w_{\gamma} / \sum_{\gamma=1}^{k} w_{\gamma}\)

and \(v_k^* = 1 / \sum_{\gamma=1}^{k} w_{\gamma}\). Then to test \(H_k, \ 1 \leq k < t\), in equation (2.5.4) compute

\[ X_k^2 = \frac{(d_{k+1} - d_k^*)^2}{[v_{k+1} + v_k^*]} \ . \quad (2.5.5) \]

The sum, \(\sum_{k=1}^{t-1} X_k^2\) is equal to \(X^2\) given by (2.5.1).

When \(H_k\) is true, (2.5.5) is distributed asymptotically as chi-square with one degree of freedom. Also when \(H_0\) is true, each of the \((k-1)\) statistics \(X_k^2\) will have asymptotically independent chi-square distributions each with one degree of freedom.

(ii) Let \(b_k = \frac{1}{d_k}, \ a_k = b_k^2 u_k, \ h_k = \frac{1}{a_k}, \ 1 \leq k \leq t,\)

\(b_{\gamma} = \frac{1}{d_{\gamma}}, \ a_{\gamma} = b_{\gamma}^2 u_{\gamma}, \ h_{\gamma} = \frac{1}{a_{\gamma}}, \ 1 \leq \gamma \leq k,\)

\(b_k^* = \sum_{\gamma=1}^{k} b_{\gamma} h_{\gamma} / \sum_{\gamma=1}^{k} h_{\gamma}\), and \(a_k^* = 1 / \sum_{\gamma=1}^{k} h_{\gamma}\).  

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Here $b_k$ is the maximum likelihood estimate of $1/\Delta_k$, and its variance can be estimated consistently by $a_k$. When the null hypothesis (2.5.3) is true

$$Z^2 = \sum_{k=1}^{t} b_k^2 h_k - \left( \sum_{k=1}^{t} b_k h_k \right)^2 / \sum_{k=1}^{t} h_k$$

(2.5.6)

is distributed asymptotically as chi-square with $(t-1)$ degrees of freedom. Tests of $H_k$ ($1 \leq k < t$) in equation (2.5.4) are given by

$$Z_k^2 = \left[ b_{k+1} - \bar{b}_k \right]^2 / \left[ a_{k+1} + \bar{a}_k \right]$$

(2.5.7)

which, when $H_k$ is true, is distributed asymptotically as chi-square with one degree of freedom. Also $\sum_{k=1}^{t-1} Z_k^2 = Z^2$.

(iii) Let

$$g_k = \log d_k, \quad m_k = 1/u_k, \quad 1 \leq k \leq t,$$

$$g_y = \log d_y, \quad m_y = 1/u_y, \quad 1 \leq y \leq k,$$

$$g_k^* = \sum_{y=1}^{k} g_y m_y / \sum_{y=1}^{k} m_y,$$

and $u_k^* = 1/\sum_{y=1}^{k} m_y$.

Here $g_k$ is the maximum likelihood estimate of $\Gamma_k = \log \Delta_k$, and its variance can be estimated consistently by $u_k$. The null hypothesis, (2.5.3), becomes

$$H_0: \Gamma_1 = \Gamma_k \quad \text{for} \quad 2 \leq k \leq t \quad .$$

(2.5.8)

When (2.5.8) is true

$$\gamma^2 = \sum_{k=1}^{t} g_k^2 m_k - \left( \sum_{k=1}^{t} g_k m_k \right)^2 / \sum_{k=1}^{t} m_k$$

(2.5.9)

is distributed asymptotically as chi-square with $(t-1)$ degrees of freedom. This statistic, (2.5.9), is the special form which equation (2.4.3) takes when $r = 2$ and $s = 2$. Tests of hypotheses analogous to (2.5.4) involving the log-frequencies are given by

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for every integer $k$, $1 \leq k < t$, which, when $H_k$ is true, is distributed asymptotically as chi-square with one degree of freedom. Also, $\sum_{k=1}^{t} Y_k^2 = Y^2$. Moreover, when $H_k$ is true, $Y_k^2$ and $Z_k^2$ are asymptotically equivalent to $X_k^2$.

This portion of the program computes all the test statistics given by equations (2.5.1), (2.5.5), (2.5.6), (2.5.7), (2.5.9) and (2.5.10). The print out, displayed in Table 4, gives values of $d_k$, $v_k$, $b_k$, $a_k$, $g_k$, and $u_k$ for $1 \leq k \leq t$, and $X_k^2$, $Z_k^2$, and $Y_k^2$ for $1 \leq k < t$. In addition, the program computes and displays the values $X^2$, $Z^2$, $Y^2$, $\sum_{k=1}^{t-1} X_k^2$, $\sum_{k=1}^{t-1} Z_k^2$, $\sum_{k=1}^{t-1} Y_k^2$, and the number of degrees of freedom.

2.6. Kullback, Kupperman, and Ku [12,13]

The procedures outlined by Plackett [11] and Goodman [14,15] for testing the hypothesis of zero three-factor interaction, (2.1), are primarily generalizations of a method proposed by Woolf [5] in which the test criterion is based on a logit transformation of the data. This test criterion is one alternative to the Pearson [1] chi-square test of goodness of fit. Another alternative is the likelihood-ratio test criterion proposed by Wilks [3], and investigated by Woolf [7] and Kullback, Kupperman, and Ku [12,13].

Kullback, Kupperman, and Ku resort to an information theory approach and define a minimum discrimination information statistic [M.D.I.S.]. This statistic is distributed asymptotically as chi-square under the null hypothesis, and as noncentral chi-square under the alternative hypothesis, with appropriate degrees
of freedom and noncentrality parameter. It has additive properties in the sense that it can be analyzed into several additive components for a hypothesis which is equivalent to the combination of several hypotheses of interest. Each such component of the M.D.I.S. is itself an M.D.I.S., and is distributed asymptotic-ally as chi-square with appropriate degrees of freedom. Moreover, the M.D.I.S. has the convexity property which is useful in finding other M.D.I.S. under certain restrictions and groupings. In particular, for nonnegative real numbers \( a_i \) and \( b_i \), this property yields

\[
\sum_{i=1}^{n} a_i \ln(a_i/b_i) \geq \left( \sum_{i=1}^{n} a_i \right) \ln \left( \sum_{i=1}^{n} a_i / \sum_{i=1}^{n} b_i \right). \tag{2.6.1}
\]

Equality holds if and only if \( a_1/b_1 = a_2/b_2 = \ldots = a_n/b_n \).

In the \( r \times s \times t \) table, the criterion for testing \( H_0 \) (2.1), is given by

\[
2\mathbf{f} = 2 \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} n_{i,j,k} \ln \left( \frac{n_{i,j,k} n_{i,j,k}}{n_i \cdot n_j \cdot n_k} \right). \tag{2.6.2}
\]

This statistic is distributed asymptotically as chi-square with \((r-1)(s-1)(t-1)\) degrees of freedom. In any specific example the convexity property may not hold.

This may result in a negative value for (2.6.2), in which case Kullback, Kupperman and Ku [12, pp. 225-226] recommend one of the following alternative analyses of the data:
row-layer interaction with column

\[
2f = 2 \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} n_{ijk} \log \left( \frac{n_{ijk} n_{.jk} \cdot y_{i,k}}{n_{ij} \cdot n_{jk} n_{.k}} \right),
\]

where \( y_{i,k} = \sum_{j=1}^{s} n_{ij} \cdot n_{jk}/n_{.k} \).

column-layer interaction with row

\[
2f = 2 \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} n_{ijk} \log \left( \frac{n_{ijk} \cdot y_{.jk}}{n_{ij} \cdot n_{jk} n_{.k}} \right)
\]

where \( y_{.jk} = \sum_{i=1}^{r} n_{ij} \cdot n_{jk}/n_{.k} \).

row-column interaction with layer

\[
2f = 2 \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} n_{ijk} \log \left( \frac{n_{ijk} \cdot y_{ij,k}}{n_{ij} \cdot n_{jk} n_{.k}} \right)
\]

where \( y_{ij,k} = \sum_{k=1}^{t} n_{ij} \cdot n_{jk}/n_{.k} \).

The program computes the M.D.I.S. test statistics using equations (2.6.2), (2.6.3), (2.6.4), and (2.6.5), and prints out the corresponding values of \( 2f \) as well as the number of degrees of freedom.

3. Confidence Intervals

Of all the papers on the subject of three-factor interaction in contingency tables, only Goodman's [15] deals with the problem of estimation. Indeed, the final section of this paper is devoted exclusively to a discussion of methods for estimating the magnitude of the three-factor interaction, and for obtaining confidence limits for it.
Let
\[ A_{ibjckd} = \log \left\{ \frac{\Pi_{ijk} \Pi_{bck}}{\Pi_{bjk} \Pi_{ick}} / \frac{\Pi_{ijd} \Pi_{bcd}}{\Pi_{bjd} \Pi_{icd}} \right\} \] (3.1)

be the measure of a particular three-factor interaction for all integers \( 1 \leq i < b < r \), \( 1 \leq j < c < s \), and \( 1 \leq k < d < t \). Depending on the values of \( b \), \( c \), and \( d \), there can be as many as \( \binom{r}{2} \binom{s}{2} \binom{t}{2} \) such three-factor interactions in an \( r \times s \times t \) contingency table. In particular, for \( b = r \), \( c = s \), \( d = t \), there are \( (r-1)(s-1)(t-1) \) interactions of the form

\[ A_{irjskt} = \Gamma_{ijk} - \Gamma_{ijt} = \log(\Delta_{ijk}/\Delta_{ijt}) \]

\[ = \log \left\{ \frac{\Pi_{ijk} \Pi_{rsk}}{\Pi_{rjk} \Pi_{isk}} / \frac{\Pi_{ijt} \Pi_{rst}}{\Pi_{rjt} \Pi_{ist}} \right\} . \] (3.2)

The maximum likelihood estimator of \( A_{ibjckd} \) is

\[ a_{ibjckd} = \log \left\{ \frac{n_{ijk} n_{bck}}{n_{bjk} n_{ick}} / \frac{n_{ijd} n_{bcd}}{n_{bjd} n_{icd}} \right\} , \] (3.3)

and its variance is estimated consistently by

\[ S^2_{ibjckd} = u_{ibjck} + u_{ibjcd} , \] (3.4)

where \( u_{ibjck} = \frac{1}{n_{ijk}} + \frac{1}{n_{bck}} + \frac{1}{n_{ick}} + \frac{1}{n_{bjk}} \) . (3.5)

Using these definitions, Goodman proposes three alternative sets of approximate two-sided confidence intervals for \( A_{ibjckd} \):

\[ a_{ibjckd} \pm \chi^2_{1-P} S_{ibjckd} , \] (3.6)
where $\chi^2_T(P)$ is the $\left\lfloor P \times 100 \right\rfloor$th percentile of the chi-square distribution with 
T = (r-1)(s-1)(t-1) degrees of freedom;

$$a_{ibjckd} \pm \chi_1(P)S_{ibjckd} \quad ,$$

(3.7)

where $\chi_1(P)$ is the $\left\lfloor \frac{1+P}{2} \times 100 \right\rfloor$th percentile of the standardized normal distribution; and for a specified subset of interactions, $A_{(w)}$,

$$a_{(w)} \pm \Phi_W(P)S_{(w)} \quad ,$$

(3.8)

where, for any $(b, c, d)$, $w = (b-1)(c-1)(d-1)$ is the number of elements in the subset $(w) = (ibjckd)$; and

$$W = W(B, C, D) = \sum_{b \in B} \sum_{c \in C} \sum_{d \in D} (b-1)(c-1)(d-1)$$

where $B, C$, and $D$ are subsets of the sets of integers $I_r, I_s, I_t$ such that

$$B \subseteq I_r = \{2, 3, \ldots, r\}$$

$$C \subseteq I_s = \{2, 3, \ldots, s\}$$

$$D \subseteq I_t = \{2, 3, \ldots, t\}$$

and $\Phi_W(P)$ is the $\left\lfloor \frac{2W-1+P}{2W} \times 100 \right\rfloor$ percentile of the standardized normal distribution. It follows that $1 \leq w \leq W \leq \binom{r}{2}\binom{s}{2}\binom{t}{2}$. In particular, for the subset $(w) = (irjskt)$, $w = W = T = (r-1)(s-1)(t-1)$, and the intervals (3.8) become

$$a_{irjskt} \pm \Phi_T(P)S_{irjskt} \quad ,$$

(3.9)
where $\Phi_T(P)$ is the \frac{2T-1+P}{2T} \times 100$th percentile of the standardized normal distribution.

In the final paragraph of his paper, Goodman [15] discusses the relative merits of the three alternative sets of confidence intervals (3.6), (3.7), and (3.8). His remarks may be summarized as follows:

(i) If any of the intervals (3.6) do not include zero, the null hypothesis, (2.3.2), will be rejected at a significance level $1 - P$, when tested using (2.4.3).

(ii) When a specific set of $W$ three-factor interactions is of interest, $W \leq \binom{r}{2}(\binom{s}{2})^{t_2}, \quad T \geq 1$, and when the usual values of $P$, $[P = .95$ or $99]$, are used, (3.8) will yield smaller confidence intervals than (3.6).

(iii) The probability is (approximately) at least $P$ that all the $\binom{r}{2}(\binom{s}{2})^{t_2}$ intervals (3.6) include the corresponding true values. That is to say, the probability is (approximately) at least $P$ that all the $\binom{r}{2}(\binom{s}{2})^{t_2}$ confidence statements associated with (3.6) are correct.

(iv) The length of the confidence interval will be reduced if (3.7) is used in place of (3.6). However, the probability that all the confidence statements are correct will also be reduced. The expected proportion of correct confidence statements of type (3.7) will be approximately $P$.

(v) To insure that all confidence statements are correct, with probability (approximately) at least $P$, use form (3.6). If it suffices to insure that the expected proportion of correct confidence statements is $P$, then (3.7) should replace (3.6).
(vi) The same consequences will result when intervals (3.7) are used in place of intervals (3.8).

For a prescribed set of integers \((b, c, d)\), and for all \((i, j, k)\) in the range \(1 \leq i < b \leq r, 1 \leq j < c \leq s, 1 \leq k < d \leq t\), this portion of the program calculates the estimates of interaction and their estimated variances using equations (3.3) and (3.4). Moreover, it evaluates the three alternative sets of approximate two-sided confidence intervals given by equations (3.6), (3.7), and (3.8), for values of \(P = 0.95\) and \(0.99\). Values of \(\chi^2_T(P)\) in (3.6) are calculated using the Fisher-Cornish [9] approximation.

Finally, the program is designed to transform the estimates of interaction and the corresponding confidence intervals to their original scale by taking the antilogarithm of each of the computed values. The results of these calculations are displayed in logarithmic units in Table 5.
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PROGRAM HYPONIT
COMMON/ALL/UBS(400),IDENT(10),IR,JS,KT,NDEGREES,INPUT,OUTPUT
COMMON/FLAGS/OPTION(10),LOGFLAG
TYPE INTEGER OPTION
TYPE INTEGER OUTPUT
INPUT=60
OUTPUT=61
10 CALL READIN
   IF(OPTION(1).EQ.0) GO TO 20
   CALL KASTENBM
20 IF(OPTION(2)+OPTION(3).EQ.0) GO TO 30
   CALL DARROCH
30 IF(OPTION(4).EQ.0) GO TO 40
   CALL PLACKETT
40 IF(OPTION(5).EQ.0) GO TO 50
   CALL GOODMAN1
50 IF(OPTION(6).EQ.0) GO TO 60
   CALL GOODMAN2
60 IF(OPTION(7).EQ.0) GO TO 70
   CALL Tau3Ytau
70 IF(OPTION(8).EQ.0) GO TO 80
   CALL KKK
80 IF(OPTION(9).EQ.0) GO TO 90
   LOGFLAG=1
   CALL INTERVALS
90 IF(OPTION(10).EQ.0) GO TO 100
   LOGFLAG=0
   CALL INTERVALS
100 GO TO 10
END
SUBROUTINE KASTENBM
COMMON/ALL/OBS(400),IDENT(10),IR,JS,KT,NDEGREES,INPUT,OUTPUT
DIMENSION EXP(400),BIJ(16),CIJ(16)
TYPE INTEGER OUTPUT
2000 FORMAT(1H1,10A8)
2001 FORMAT(32HUKASTEN3AUK-LAMPHIEAR PROCEDURE )
2002 FORMAT(///12H ITERATION */8X*18H DEGREES OF FREEDOM */8X*
* 14H (CHI-SQUARE*/15X*/2H */8X*/2H */8X*/2H */T)
2003 FORMAT(//////8X*15H CELL */8X*18H OBSERVED FREQUENCY */8X*
* 18H EXPECTED FREQUENCY */1H )
2004 FORMAT(///3X*14*,18X*15,15X,F15.6,F15.6,F15.6,F15.6)
2005 FORMAT(6X*3,2X*13),8X,F15.6,11X,F15.6)
2006 FORMAT(25H0 ITERATION COUNT EXCEEDS 13)
WRITE OUTPUT TAPE OUTPUT 2000 IDENT
WRITE OUTPUT TAPE OUTPUT 2001
DELX=1.0E-5
ITERSTOP=100
NRST=IR*JS*KT
DO 10 N=1,NRST
EXP(N)=OBS(N)
10 CONTINUE
IR1=IR-1
JS1=JS-1
NX=IR1*JS1*KT
ITERS=0
20 NXZERO=0
ITERS=ITERS+1
DO 60 I=1,IR1
DO 50 J=1,JS1
SUMC=0.0
SUMBC=0.0
DO 30 K=1,KT
KPART=IR*(JS-K-JS-1)
IJSK=KPART+IR*JS+1
EXPIJK=EXP(IJSK)
IRJK=KPART+IR*JS+1
EXPRJK=EXP(IRJK)
IRJSDK=KPART+IR*JS+1
EXPRJK=EXP(IRJSDK)
IJK=KPART+IR*JS+1
EXPIJK=EXP(IJK)
CIJK=1.0/EXPIJK+1.0/EXPRJK+1.0/EXPRSK+1.0/EXPIJK
CIJK=1.0/CIJK
BIJK=EXPIJK*EXPRJK/EXPRSK/EXPIJK
SUMC = SUMC + CIJK
SUMBC = SUMBC + CIJK * 81JK
CIJK(K) = CIJK
SIJK(K) = BIJK
30 CONTINUE
HIJK = SUMC / SUMBC
DO 40 K = 1 * KT
KPART = INT(JS * K - KS - 1)
X1JK = CIJK(K) * (1.0 - HIJK * 81JK(K))
IF (X1JK <= 0.07) NXZERO = NXZERO + 1
IRJSK = KPART + IRC*JS + ID
EXP(IRJSK) = EXP(IRJSK) - X1JK
IJK = KPART + IR*JS + I
EXP(IJK) = EXP(IJK) - X1JK
IRJS = KPART + IR*JS + I
EXP(IRJSK) = EXP(IRJSK) + X1JK
IRJK = KPART + IR*JS + IR
EXP(IRJK) = EXP(IRJK) + X1JK
40 CONTINUE
50 CONTINUE
60 CONTINUE
IF (ITERS.GE.ITERSTOP) GO TO 200
IF (NXZERO.LT.NX) GO TO 20
CHISO = 0.0
DO 70 N = 1 * NRST
EXPN = EXP(N)
CHISO = CHISO + OBS(N) - EXPN) * (OBS(N) - EXPN) / EXPN
70 CONTINUE
WRITE OUTPUT TAPE OUTPUT , 2003
IJK = 0
DO 80 K = 1 * KT
DO 80 J = 1 * JS
DO 80 I = 1 * IR
IJK = IJK + 1
WRITE OUTPUT TAPE OUTPUT , 2005 , I , JS , OBS(IJK) , EXP(IJK)
80 CONTINUE
WRITE OUTPUT TAPE OUTPUT , 2002
WRITE OUTPUT TAPE OUTPUT , 2004 , ITERS , DEGREES , CHISO , IR , JS , KT
GO TO 300
200 WRITE OUTPUT TAPE OUTPUT , 2006 , ITERSTOP
300 END
SUBROUTINE ANROCH
COMMON/ALL, OBS(400), IDENT(10), IR*JS*KT*NDegrees, INPUT, OUTPUT
COMMON/SUMS/OBSR(80), OBSSS(80), OBST(25), OBSRS(16), ORSTR(5),
* OBSST(5), OBSST
COMMON/FLAG/OPTION(10), LOGFLAG
DIMENSION EXP(400), Theta(80), PHI(60), PSI(25)
TYPE INTEGER OPTION
TYPE INTEGER OUTPUT
2000 FORMAT(1HM1,1IA0)
2001 FORMAT(19HANROCH PROCEDURE)
2002 FORMAT(/12H ITERATION, 8X, 1d0 DEGREES OF FREEDOM, 8X,
* 14H CHI-SQUARE, 13X, 2H R, 8X, 2H S, 8X, 2H T)
2003 FORMAT(/6X, 13H CELL, 8X, 18H OBSERVED FREQUENCY, 8X,
* 18HEXPECTED FREQUENCY, 7H )
2004 FORMAT(3X, I4, 18X, I5, 15X, F15.6, 7X, 3(2X, I2))
2005 FORMAT(6X, 3(2X, I3), 8X, F15.6, 11X, F15.6)
2006 FORMAT(2SHUERATION COUNT EXCEEDS 13)
WRITE OUTPUT TAPE OUTPUT, 2000, IDENT
WRITE OUTPUT TAPE OUTPUT, 2001
ITERSTOP=100
TOLERSW=0.00005*0.00005
CALL GETSUMS
C
DO 140 I=1,IR
OBSSTI=OBSST(I)
DO 120 J=1,JS
IJ=I+IR*(J-1)
PSI(IJ)=OBST(IJ)/OBSSTI
120 CONTINUE
DO 130 K=1,KT
KI=K+KT*(I-1)
PHI(KI)=OBSR(KI)/OBSTR(K)
130 CONTINUE
140 CONTINUE
C
ITER=0
200 1STOP=1
ITER=ITERS+1
C
DO 230 K=1,KT
DO 220 J=1,JS
JK=J+JS*(K-1)
SUMI=0.0
DO 210 I=1,IR
KI=K+KT*(I-1)
IJ=I+IR*(J-1)
SUMI=SUMI+PHI(KI)*PSI(IJ)
210 CONTINUE
THETAJK=OBSTR(JK)/OBSTRS/SUMI
IF( (THETAJK-THETA(JK))**2.0 GE TOLERSW ) 1STOP=0
THETA(JK)=THETAJK
220 CONTINUE
230 CONTINUE
C
DO 260 I=1*IR
DO 250 K=1*KT
KI=K+KT*(I-1)
SUMJ=0*0
DO 240 J=1*JS
IJ=I+IR*(J-1)
JK=J+JS*(K-1)
SUMJ=SUMJ+PSI(IJ)*THETA(JK)
240 CONTINUE
PHIKI=OBST(I)/OBSRST/SUMK
IF((PHIKI-PHI(KI))*2*GE.TOLERSQ) ISTOP=0
PHI(KI)=PHIKI
250 CONTINUE

C
DO 290 J=1*JS
DO 280 I=1*IR
IJ=I+IR*(J-1)
SUMK=0*0
DO 270 K=1*KT
JK=J+JS*(K-1)
KI=K+KT*(I-1)
SUMK=SUMK+THETA(JK)*PHI(KI)
270 CONTINUE
PSIJJ=OBST(J)/OBSRST/SUMK
IF((PSIJJ-PSI(IJ))*2*GE.TOLERSQ) ISTOP=0
PSI(IJ)=PSIJJ
280 CONTINUE

C
DO 290 J=1*JS
DO 290 K=1*KT
DO 300 I=1*IR
IJ=I+IR*(J-1)
KI=K+KT*(I-1)
EXPIJK=THETA(JK)*PSI(KI)*PSI(IJ)*OBSRST
IJK=I+IR*(J-1)+JS*(K-1)
OBSIJK=OBST(IJK)
CHISQ=CHISQ+(OBSIJK-EXPIJK)*(OBSIJK-EXPIJK)/EXPIJK
IF(OPTION(4)*EO+0) GO TO 293
WRITE OUTPUT TAPE OUTPUT + 2003
293 CHISQ=0*0
DO 300 K=1*KT
DO 300 J=1*JS
JK=J+JS*(K-1)
DO 300 I=1*IR
IJ=I+IR*(J-1)
KI=K+KT*(I-1)
EXPIJK=THETA(JK)*PSI(KI)*PSI(IJ)*OBSRST
IJK=I+IR*(J-1)+JS*(K-1)
OBSIJK=OBST(IJK)
CHISQ=CHISQ+(OBSIJK-EXPIJK)*(OBSIJK-EXPIJK)/EXPIJK
IF(OPTION(4)*EO+0) GO TO 290
WRITE OUTPUT TAPE OUTPUT + 2005
WRITE OUTPUT TAPE OUTPUT + 2006
299 CONTINUE
WRITE OUTPUT TAPE OUTPUT + 2002
WRITE OUTPUT TAPE OUTPUT + 2004
WRITE OUTPUT TAPE OUTPUT + 2007
GO TO 500

C
400 WRITE OUTPUT TAPE OUTPUT + 2006
500 END
SUBROUTINE PLACKETT
COMMON ALL/OBS(400),IDENT(10),IR,JS,KT,INDEGREES,INPUT,OUTPUT
DIMENSION V(256),VINV(256),Z(16),ZINV(16)
TYPE INTEGER OUTPUT
2000 FORMAT (1H1, 10A8)
2001 FORMAT (20H0, PLACKETT, PROCEDURE )
2002 FORMAT (15X, DEGREES OF FREEDOM, 6X, 14H, CHI-SQUARE, 17X, 
* 10H SUM(L*L), 15X, R, 8X, 2H, S, 8X, 2H, T )
2003 FORMAT (6X, 15X, F15.6, 11X, F15.6, 8X, 3(8X, 12))
2004 FORMAT (3H0K = 12)
2005 FORMAT (12H CONTRASTS )
2006 FORMAT (/20H DISPERSION MATRIX )
2007 FORMAT (//11H Z-VECTOR )
2008 FORMAT (/11H P-MATRIX )
WRITE OUTPUT TAPE OUTPUT,2000, IDENT
WRITE OUTPUT TAPE OUTPUT,2001
IR1 = IR - 1
JS1 = JS - 1
IR1JS1 = IR1 * JS1
ZVINV2 = 0.0
DO 30 NAB = 1, IR1JS1
   ZVINV(NAB) = 0.0
30 CONTINUE
DO 300 K = 1, KT
   DO 40 NAP = 1, IR1JS1
   Z(NAP) = 0.0
40 CONTINUE
   DO 170 J = 1, JS
   DO 170 I = 1, IR
   IJK = I + IR*(J-1 + JS*(K-1))
   OBSIJK = OBS(IJK)
   OBSLOG = LOGF(OBSIJK)
   DO 160 NB = 1, JS1
   SIGMA = 1.0
   IF (J + NB - JS - 1) 60, 50, 170
50 SIGMA = -FLOATF(JS - NB)
60 DO 150 NA = 1, IR1
   RHO = 1.0
   IF (I + NA - IR1 - 1) 80, 70, 160
70 RHO = -FLOATF(IR - NA)
80 NAB = IR - NA + IR1*(JS1 - NB)
   Z(NAB) = Z(NA3) + RHO*SIGMA*OBSLOG
   DO 140 ND = 1, JS1
   OMEGA = 1.0
140 CONTINUE
IF(J+ND-JS-I) 100, 20, 150
90 OMEGA=FLOAT(JS-ND)
100 DO 130 NC=1, IR1
   TAU=1.0
   IF(I+NC-IR1) 120, 110, 140
110 TAU=FLOAT(IR-NC)
120 NCD=IR-NC+IR1*(JS1-ND)
   NABNCD=NA+IR1*JS1*(NCD-1)
   V(NABNCD)=VIN(NABNCD)+R*SIGMA*TAU*OMEGA/UBS1JK
130 CONTINUE
140 CONTINUE
150 CONTINUE
160 CONTINUE
170 CONTINUE
   WRITE OUTPUT TAPE OUTPUT, 2004, K
   WRITE OUTPUT TAPE OUTPUT, 2005
   CALL NPRINT(Z*IR1JS1, 1)
   WRITE OUTPUT TAPE OUTPUT, 2006
   CALL NPRINT(V*IR1JS1, IR1JS1)
   CALL MATINV(V, IR1JS1, V*U, D, IR1JS1)
   DO 190 NCD=1, IR1JS1
   ZVINVCD=ZVINV(NCD)
   ZCD=Z(NCD)
   DO 180 NAB=1, IR1JS1
   ZAB=Z(NAB)
   NABNCD=NAB+IR1*JS1*(NCD-1)
   VABCD=V(NABNCD)
   ZVINVZ=ZVINVZ+ZAB*VABCD*ZCD
   VINV(NABNCD)=VIN(NABNCD)+VABCD
   ZVINVCD=ZVINVCD+ZAB*VABCD
180 CONTINUE
   ZVINV(NCD)=ZVINVCD
190 CONTINUE
200 CONTINUE
   CALL MATINV(V*VINV, IR1JS1, V*U, D, IR1JS1)
   CHISO=ZVINVZ
   DO 210 NCD=1, IR1JS1
   ZVINVCD=ZVINV(NCD)
   DO 210 NAB=1, IR1JS1
   NABNCD=NAB+IR1JS1*(NCD-1)
   CHISO=CHISO-ZVINV(NAB)*VINV(NABNCD)*ZVINVCD
210 CONTINUE
   WRITE OUTPUT TAPE OUTPUT, 2007
   CALL NPRINT(ZVINV, IR1JS1, 1)
   WRITE OUTPUT TAPE OUTPUT, 2008
   CALL NPRINT(VINV, IR1JS1, IR1JS1)
   WRITE OUTPUT TAPE OUTPUT, 2009
   WRITE OUTPUT TAPE OUTPUT, 2003
   DEGREES, CHISO, ZVINVZ, IR, JS, K
300 END
SUBROUTINE GOODMAN1
COMMON/ALL/OBS(40), IDENT(10), IR*JS*KT, NDEGREES, INPUT, OUTPUT
DIMENSION V(256), VINV(256), Z(16), ZINV(16)
TYPE INTEGER OUTPUT
2000 FORMAT(1H1*10A8)
2001 FORMAT(19H GO GOODMAN PROCEDURE )
2002 FORMAT//(///19H DEGREES OF FREEDOM *8X*14H CHI-SQUARE, 17X,
* 10H SUM(H*H) *15X*2H R*8X*2H S,8X*2H T)
2003 FORMAT(/6X*15*15X*F15.6*11X*F15.6*8X*3(8X*12))
2004 FORMAT(3H0K= 12)
2005 FORMAT(12H CONTRASTS )
2006 FORMAT(/2H DISPERSION MATRIX )
2007 FORMAT//(///11H G-VECTOR )
2008 FORMAT(/11H Q-MATRIX )
WRITE OUTPUT TAPE OUTPUT,2000, IDENT
WRITE OUTPUT TAPE OUTPUT,2001
IR1=IR-1
JS1=JS-1
IR1JS1=IR1*JS1
ZINVZ=0.0
DO 30 NAB=1*IR1JS1
ZINV(V(NAB))=0.0
DO 30 NCD=1*IR1JS1
NABNCD=NAB+IR1JS1*(NCD-1)
VINV(NABNCD)=0.0
30 CONTINUE
DO 200 K=1*KT
DO 40 NAB=1*IR1JS1
Z(NAB)=0.0
DO 40 NCD=1*IR1JS1
NABNCD=NAB+IR1JS1*(NCD-1)
V(NABNCD)=0.0
40 CONTINUE
DO 170 J=1*JS
DO 170 I=1*IR
IJK=I+IR*(J-1+JS*(K-1))
OBSTJK=OBS(IJK)
OBSTLOG=LOGF(OBSTJK)
DO 160 NB=1*JS1
SIGMA=-1.0
IF(J.EQ.JS) GO TO 60
IF(NS*NE*J) GO TO 160
SIGMA=1.0
60 DO 150 NA=1*IR1
RH0=-1.0
IF(I.EQ.IR) GO TO 80
IF(NA*NE*I) GO TO 150
RH0=1.0
80 NAB=NA+IR1*(NB-1)
Z(NAB)=Z(NAB)+RH0*SIGMA*OBSTLOG
DO 140 ND=1*JS1
OMEGA=-1*0
IF(J.EQ.JS) GO TO 100
IF(ND.NE.J) GO TO 140
OMEGA=1*0
100 DO 130 NC=1*IR1
TAU=-1*0
IF(I.EQ.IR) GO TO 120
IF(NC.NE.I) GO TO 130
TAU=1*0
120 NCD=NC+IR1*(ND-1)
NABNCD=NAB+IR1JS1*(NCD-1)
V(NABNCD)=V(NABNCD)+ RHO*SIGMA*TAU*OMEGA/CBS1JK
130 CONTINUE
140 CONTINUE
150 CONTINUE
160 CONTINUE
170 CONTINUE
WRITE OUTPUT TAPE OUTPUT,2004
WRITE OUTPUT TAPE OUTPUT,2005
CALL NMPRINT(Z,IR1JS1,1)
WRITE OUTPUT TAPE OUTPUT,2006
CALL NMPRINT(V,IR1JS1,IR1JS1)
CALL MATIVE(V,IR1JS1,V,U,D,IR1JS1)
DO 190 NCD=1*IR1JS1
ZVINVC=ZVINVNCD
ZCD=Z(NCD)
DO 190 MAR=1*IR1JS1
ZAB=Z(NAB)
NABNCD=NAB+IR1JS1*(NCD-1)
VABCD=V(NABNCD)
ZVINV=ZVINV+ZAB*VABCD*ZCD
VINV(NABNCD)=VINV(NABNCD)+VABCD
ZVINVC=ZVINVC+ZAB*VABCD
190 CONTINUE
ZVINV(NCD)=ZVINVC
190 CONTINUE
200 CONTINUE
CALL MATIVE(VINV,IR1JS1,V,U,D,IR1JS1)
CHISQ=ZVINV
DO 210 NCD=1*IR1JS1
ZVINVC=ZVINVNCD
DO 210 MAR=1*IR1JS1
NABNCD=NAB+IR1JS1*(NCD-1)
CHISQ=CHISQ+ZVINV(NAB)*VINV(NABNCD)*ZVINVC
210 CONTINUE
WRITE OUTPUT TAPE OUTPUT,2007
CALL NMPRINT(ZVINV,IR1JS1,1)
WRITE OUTPUT TAPE OUTPUT,2008
CALL NMPRINT(VINV,IR1JS1,IR1JS1)
WRITE OUTPUT TAPE OUTPUT,2002
WRITE OUTPUT TAPE OUTPUT,2003
300 END
SUBROUTINE GOODMAN2
COMMON/ALL/OBS(400),IDENT(10),IR,JS,KT,NDEGREES,INPUT,OUTPUT
DIMENSION P(256),Q(256),B(80),C(16),G(16),PG(16)
TYPE INTEGER OUTPUT
2000 FORMAT(1H1,10A8)
2001 FORMAT(28HMODIFIED GOODMAN PROCEDURE *////1H)
2002 FORMAT(/12H CONTRASTS )
2003 FORMAT(*8X,14H DEGREES OF FREEDOM *8X,14H CHI-SQUARE,15X,2H F *)
2004 FORMAT(/6X,15*15X,F15.6,7X,3(6X,12))
WRITE OUTPUT TAPE OUTPUT,2004 IDENT
WRITE OUTPUT TAPE OUTPUT,2001
IR1=IR-1
JS1=JS-1
IR1IR1=IR1*IR1
IR1JS1=IR1*JS1
CHISQ=0.0
DO 15 NAB=1,IR1JS1
PG(NAB)=0.0
DO 10 NCD=1,IR1JS1
NABNCD=NAB+IR1JS1*(NCD-1)
Q(NABNCD)=0.0
10 CONTINUE
15 CONTINUE
DO 200 K=1,KT
DO 25 NAB=1,IR1JS1
G(NAB)=0.0
DO 20 NCD=1,IR1JS1
NABNCD=NAB+IR1JS1*(NCD-1)
P(NABNCD)=0.0
20 CONTINUE
25 CONTINUE
DO 35 II=1,IR1
DO 30 III=1,IR1
IIII=II+IR1*(II-1)
C(IIIIII)=0.0
30 CONTINUE
35 CONTINUE
DO 100 J=1,JS
JK=IR*(J-1+JS*(K-1))
OBSJK=0.0
DO 70 I=1,IR
IJK=I+JK
OBSIJK=OBS(IJK)
OBSLOG=LOGF(OBSIJK)
DO 60 NB=1,JS1
SIGMA=-1.0
IF(J.EQ.JS) GO TO 40
IF(NB.NE.J) GO TO 60
SIGMA=1.0
70 CONTINUE
100 CONTINUE
30 CONTINUE
IF(NB.NE.J) GO TO 60
SIGMA=1.0
40 DO 50 NA=1*IR1
   RHO=-1.0
   IF(I.EQ.IR) GO TO 45
   IF(NA.NE.1) GO TO 50
   RHO=1.0
45 NAB=NA+IR1*(NB-1)
   G(NAB)=G(NAB)+RHO*SIGMA*OBSLOG
50 CONTINUE
60 CONTINUE
   OBSJK=OBSJK+OBSIJK
70 CONTINUE
   DO 90 II=1*IR1
      IIJK=II+JK
   DO 80 III=1*IR1
      IIIJK=III+JK
      IIIJK=III+IR1*(III+I-1)
      IIIJK=III+IR1IR1*(I-1)
      IIIJK=-OBS*(IIJK)*OBS*(IIIJK)/OBSJK
   IF(I.IE.111) IIIJK=IIIJK+OBS*(IIJK)
   C(IIIJK)=C(IIIJK)+IIIJK
80 CONTINUE
90 CONTINUE
100 CONTINUE
WRITE OUTPUT TAPE OUTPUT,2002
CALL NMPRINT(G,IR1JS1,1)
CALL MATINV(C,IR1,C,0,U,IR1)
DO 170 NB=1*JS1
DO 160 ND=1*JS1
DO 150 NA=1*IR1
   NAB=NA+IR1*(NB-1)
   DO 140 NC=1*IR1
      NCD=NC+IR1*(ND-1)
      NABNCD=NAB+IR1JS1*(NCD-1)
   DO 120 II=1*IR1
      IIINCND=II+IR1*(NC-1+IR1*(ND-1))
   DO 110 II=1*IR1
      NI=NA+IR1*(III-1+IR1*(NB-1))
      III+IR1*(II-1)
      P(NABNCD)=P(NABNCD)+8(NA)*C(III)+8(III)
110 CONTINUE
120 CONTINUE
   IF(NB.NE.ND) GO TO 130
   NANCND=NA+IR1*(NC-1+IR1*(ND-1))
   P(NANCND)=P(NANCND)+8(NANCND)
130 P(NABNCD)=P(NABNCD)+P(NABNCD)*G(NCD)
140 CONTINUE
150 CONTINUE
160 CONTINUE
170 CONTINUE
    DO 190 NAB=1,IR1JS1
    DO 180 NCD=1,IR1JS1
    NABNCD=NAB+IR1JS1*(NCD-1)
    CHISQ=CHISQ+G(NAB)*P(NABNCD)*G(NCD)
180 CONTINUE
190 CONTINUE
200 CONTINUE
    CALL MATINV(Q,IR1JS1,Q,0,D,IR1JS1)
    DO 220,NAB=1,IR1JS1
    DO 210 NCD=1,IR1JS1
    NABNCD=NAB+IR1JS1*(NCD-1)
    CHISQ=CHISQ-PG(NAB)*Q(NABNCD)*PG(NCD)
210 CONTINUE
220 CONTINUE
    WRITE OUTPUT TAPE OUTPUT,2003
    WRITE OUTPUT TAPE OUTPUT,2004,NDEGREES,CHISQ,IR,JS,KT
300 END
SUBROUTINE TWOBYTWO
COMMON ALL/OBS(40),IDENT(10),IR,JS,KT,DEGREES,INPUT,OUTPUT
DIMENSION U(16),V(16),G(16),O(16),A(16),DSTAR(16),VSTAR(16),
* GSTAR(16),USTAR(16),3STAR(16),ASTAR(16)
TYPE INTEGER
2000 FORMAT(1H1,10A6)
2001 FORMAT(26J0,GO,GOODMANS 2X2XT PROCEDURE ,/1H )
2002 FORMAT(3H0 K,8X,3H D,10X,3H V,1X,3HX*X,1X,3H B,1X,3H A,1X,
* 3HX*X,10X,3H G,10X,3H U,10X,3HY*Y,1/1H )
2003 FORMAT(I3,9F13.5)
2004 FORMAT( 13,9F13.5,13X))
2005 FORMAT /////14M DEGREES OF FREEDOM .11X,3HXX*X,X,15X,3HXX*X,15X,3HY*Y,
* 15X,2H R,8X,2H J,8X,2H T,1H )
2006 FORMAT(6X,15,6X,3F18.3,5X,3F18.12)
2007 FORMAT(3X,3(26X,F13.5))
WRITE OUTPUT TAPE OUTPUT 2000, IDENT
WRITE OUTPUT TAPE OUTPUT 2001
IF(IR.NE.2) GO TO 100
IF(JS.NE.2) GO TO 100
WRITE OUTPUT TAPE OUTPUT 2002
SUM1=SUM2=SUM3=SUM4=SUM5=SUM6=SUM7=SUM8=SUM9=0.0
DO 20 K=1,KT
KPART=IR*JS*(K-1)
OBS11K=OBS(1+KPART)
OBS21K=OBS(2+KPART)
OBS12K=OBS(3+KPART)
OBS22K=OBS(4+KPART)
DK=OBS11K*OBS22K/OBS12K/OBS21K
UK=1.0/OBS11K+1.0/OBS22K+1.0/OBS12K+1.0/OBS21K
VK=DK*DK*UK
K=1.0/VK
GK=LOGF(DK)
PK=1.0/VK
BK=1.0/DK
AK=BLK*BLK*UK
HK=1.0/AK
SUM1=SUM1+DK*DK*UK
SUM2=SUM2+DK*UK
SUM3=SUM3+UK
SUM4=SUM4+GK*GK*PK
SUM5=SUM5+GK*PK
SUM6=SUM6+PK
SUM7=SUM7+BLK*BLK*HK
SUM8=SUM8+BLK*HK
SUM9=SUM9+HK
DSTAR(K)=SUM2/SUM3
VSTAR(K)=1.0/SUM3
GSTAR(K) = SUM5 / SUM6
USTAR(K) = U1.0 / SUM6
BSTAR(K) = SUM8 / SUM9
ASTAR(K) = 1.0 / SUM9
D(K) = DK
\( V(K) = VK \)
G(K) = GK
U(K) = UK
\( B(K) = BK \)
A(K) = AK

20 CONTINUE
SUMXSQ = SUMYSQ = SUMZSQ = 0.0
KT1 = KT - 1
DO 30 K = 1, KT1
XSQK = (D(K+1) - DSTAR(K)) * (D(K+1) - DSTAR(K)) / (V(K+1) + VSTAR(K))
YSQK = (G(K+1) - GSTAR(K)) * (G(K+1) - GSTAR(K)) / (U(K+1) + USTAR(K))
ZSQK = (B(K+1) - BSTAR(K)) * (B(K+1) - BSTAR(K)) / (A(K+1) + ASTAR(K))
WRITE OUTPUT TAPE OUTPUT, 2003, K, D(K), V(K), XSQK, B(K), A(K), ZSQK,
* G(K), U(K), YSQK
SUMXSQ = SUMXSQ + XSQK
SUMYSQ = SUMYSQ + YSQK
SUMZSQ = SUMZSQ + ZSQK
30 CONTINUE
WRITE OUTPUT TAPE OUTPUT, 2004, KT, D(KT), V(KT), B(KT), A(KT), G(KT),
* U(KT)
WRITE OUTPUT TAPE OUTPUT, 2007, SUMXSQ, SUMZSQ, SUMYSQ
XXSQ = SUM1 - SUM2 * SUM2 / SUM3
YYSQ = SUM4 - SUM5 * SUM5 / SUM6
ZZSQ = SUM7 - SUM8 * SUM8 / SUM9
WRITE OUTPUT TAPE OUTPUT, 2005
WRITE OUTPUT TAPE OUTPUT, 2006, NDEGREES, XXSQ, ZZSQ, YYSQ, IR, JS, KT
100 END
SUBROUTINE KKK
COMMON/ALL/OBS(140)*, IDENT(10)*, IR*J5, KT, NDEGREES, INPUT, OUTPUT
COMMON/SUMS/OBSR(80)*, OBSS(80)*, OBST(25)*, OBSTRS(16)*, OPSTR(5),
  * OBSS(5)*, OBSTRS
TYPE INTEGER OUTPUT
2000 FORMAT(1H1,10A8)
2001 FORMAT(19H DEGREES OF FREEDOM, 11X, 3H2, 11X, 4H2, 8X, 2H5, 8X, 2H1)
2002 FORMAT(/6X, 15, 6X, F18.5, 7X, 3(8X, I2))
WRITE OUTPUT TAPE OUTPUT, 200U, IDENT
WRITE OUTPUT TAPE OUTPUT, 2001
CALL GETSUMS
TWOI=0.0
DO 140 K=1, KT
  OBSR(K)=OBSR(K)
  DO 130 J=1, JS
    OBSR(K)=OBSR(K)
    J=J+JS*(K-1)
    OBSR(J)=OBSR(J)
  130 CONTINUE
  I=I+IR
  OBSST(I)=OBSST(I)
  I=I+IR*(J-1)
  OBSST(I)=OBSST(I)
  K=K+KT*(I-1)
  OBSS(I)=OBSS(I)
  I=I+IR*(J-1+JS*(K-1))
  OBSR(J)=OBSR(J)
  TWOI=TWOI+OBSIJK*LOGF(OBSIJK*OBSS(I)*OBST(I)*OBSR(K)/OBSS(I)*OBST(I))
120 CONTINUE
130 CONTINUE
140 CONTINUE
TWOI=2.0*TWOI
WRITE OUTPUT TAPE OUTPUT, 2002
WRITE OUTPUT TAPE OUTPUT, 2003, NDEGREES, TWOI, IR, JS, KT
400 END
SUBROUTINE INTERVALS
COMMON/BCD/IBVECTOR(20),JCVECTOR(20),KDVECTOR(20),
COMMON/ALL/OBS(400),IDENT(10),IR,JS,KT,NDEGREES,INPUT,OUTPUT
COMMON/FLAGS/OPTION(10),LOGFLAG
DIMENSION Q(6),CHIT(2),CHI1(2),PHIW(2),P(2)
TYPE INTEGER OUTPUT
TYPE INTEGER OPTION
DATA(P=0.95,0.99)
2000 FORMAT(1H1,10A8)
2001 FORMAT(61HOINTERACTION ESTIMATES AND CONFIDENCE INTERVALS IN LOG U
*UNITS *)
2002 FORMAT(63HOINTERACTION ESTIMATES AND CONFIDENCE INTERVALS IN BASIC
*UNITS *)
2003 FORMAT(/44H B C D )
2004 FORMAT(/9X*12*14X*12*14X*12)
2005 FORMAT(34H P CHI(T=15,43H)
* CHI(1) PH1(W=15,1H)/1H )
2006 FORMAT(/9X*F4.2*3(9X*F15.4))
2007 FORMAT(/106H0 I B J C K D P ESTIMATE <
*STANDARD 
ERROR CHI(T)
* PH1(W) /1H )
2008 FORMAT(613*F9.2*2E15.*3*4X*3*4X*2E10.*2))
2009 FORMAT(/106H0 I B J C K D P ESTIMATE
* CHI(1)
2010 FORMAT(613*F9.2*2E15.*3*15X*4X*3*4X*2E10.*2))
WRITE OUTPUT TAPE OUTPUT,2000,IDENT
IF(LOGFLAG.EQ.0) GO TO 5
WRITE OUTPUT TAPE OUTPUT,2001
GO TO 10
5 WRITE OUTPUT TAPE OUTPUT,2002
10 WRITE OUTPUT TAPE OUTPUT,2003
NW=0
NBCD=0
DO 15 1=1,20
IB=IBVECTOR(1)
IF(IB.EQ.0) GO TO 15
JC=JCVECTOR(1)
KD=KDVECTOR(1)
NBCD=NBCD+1
NW=NW+(IB-1)*(JC-1)*(KD-1)
WRITE OUTPUT TAPE OUTPUT,2004,IB,JC,KD
15 CONTINUE
WRITE OUTPUT TAPE OUTPUT,2005,NDEGREES,NW
W=FLOATF(NW)
T=FLOATF(NDEGREES)
DC 20 L=1,2
PL=P(L)
CHITL=CHI(T,PL)
CHI1L = CHI(1.0*PL)
PHIW(L) = PHI(W*PL)
WRITE OUTPUT TAPE OUTPUT, 2006, PL, CHIT, CHI1L, PHIWL
CHIT(L) = CHITL
CHI1(L) = CHI1L
PHIWL(L) = PHIWL
20 CONTINUE
IF (LOGFLAG.EQ.0) GO TO 25
WRITE OUTPUT TAPE OUTPUT, 2007
GO TO 30
25 WRITE OUTPUT TAPE OUTPUT, 2009
30 DO 100 N = 1, NBCD
KD = KDVECTOR(N)
KD1 = KD - 1
JC = JCVECTOR(N)
JC1 = JC - 1
IB = IBVECTOR(N)
IB1 = IB - 1
DO 70 K = 1, KD1
DO 60 J = 1, JC1
DO 50 I = 1, IB1
IJK = I + IR*(J-1 + JS*(K-1))
ORSIJK = ORS(IJK)
IBJCK = IB + IR*(JC-1 + JS*(K-1))
ORSJCK = ORS(IBJCK)
IBJCK = IB + IR*(JC-1 + JS*(K-1))
ORSIJK = ORS(IBJCK)
IJJK = I + IR*(J-1 + JS*(K-1))
ORSJJK = ORS(IJJK)
IBJCJK = IB + IR*(JC-1 + JS*(K-1))
ORSIJK = ORS(IBJCJK)
IBJCKD = IB + IR*(JC-1 + JS*(K-1))
ORSJCD = ORS(IBJCKD)
IBJCKD = IB + IR*(JC-1 + JS*(K-1))
ORSJCD = ORS(IBJCKD)
IBJCKD = LOGF(ORSIJK, ORSBCK, ORSBJD, ORSICD, ORSJCD, ORSBJK, ORSICK)
* 
45 L = 1, 2
PL = P(L)
W(1) = AIJBCKD - S1BJCKD*CHIT(L)
W(2) = 2.0*AIJBCKD - W(1)
Q(3) = AIJBCKD - S1BJCKD*CHI1(L)
Q(4) = 2.0*AIJBCKD - Q(3)
Q(5) = AIJBCKD - S1BJCKD*PHIWL(L)
Q(6) = 2.0 * AIBJCKD - Q(5)
IF (LOGFLAG .EQ. 0) GO TO 35
WRITE OUTPUT TAPE OUTPUT * 2008 * I * IB * J * JC * K * KD * PL * AIBJCKD * SIBJCKD * Q
GO TO 45
35 AIBJCKD = EXPF(AIBJCKD)
DO 40 M = 1, 6
Q(M) = EXPF(Q(M))
40 CONTINUE
WRITE OUTPUT TAPE OUTPUT * 2016 * I * IB * J * JC * K * KD * PL * AIBJCKD * Q
45 CONTINUE
50 CONTINUE
60 CONTINUE
70 CONTINUE
100 CONTINUE
END
SUBROUTINE READIN
COMMON/ALL/OBS(400), IDENT(10), IR, JS, KT, NDEGREES, INPUT, OUTPUT
COMMON/BCD/JVECTOR(20), JCDVECTOR(20), KDVECTOR(20)
COMMON/FLAGS/OPTION(10), LOGFLG
TYPE INTEGER OPTION
TYPE INTEGER OUTPUT
DIMENSION INMAT(10)
1000 FORMAT(10A8)
1001 FORMAT(2U14)
1002 FORMAT(101I)
READ INPUT TAPE INPUT*1000, IDENT
IF (IDENT(1) .EQ. 8) THEN ALL) GO TO 400
READ INPUT TAPE INPUT*1002, OPTION
READ INPUT TAPE INPUT*1001, IR, JS, KT
READ INPUT TAPE INPUT*1000, INMAT
NRST = IR * JS * KT
READ INPUT TAPE INPUT*INMAT*OBS(N)*N=1, NRST)
READ INPUT TAPE INPUT*1001, JVECTOR
READ INPUT TAPE INPUT*1001, JCDVECTOR
READ INPUT TAPE INPUT*1001, KDVECTOR
NDEGREES = (IR-1)*(JS-1)*(KT-1)
RETURN
400 STOP
END
SUBROUTINE GETSUMS
COMMON/ALL/OBS(400),IDENT(10),IR,JS,KT,NDEGREES,INPUT,OUTPUT
COMMON/SUMS/OBSR(80),OBS(80),OBSJ(25),OBRS(16),OBSR(5),
*  OBS(5),OBSRST
OBSRST=0.0
DO 30 K=1,KT
OBSR(K)=0.0
DO 20 J=1,JS
OBSR(J)=0.0
DO 10 I=1,IR
IJ=I+IR*(J-1+JS*(K-1))
OBSR(J)=OBSR(J)+OBSR(I)
OBSR(K)=OBSR(K)+OBSR(I)
OBSRST=OBSRST+OBSR(I)
10 CONTINUE
JK=J+JS*(K-1)
OBSR(JK)=OBSR(JK)
20 CONTINUE
OBSR(K)=OBSR(K)
30 CONTINUE
C
DO 60 J=1,JS
OBSR(J)=0.0
DO 50 I=1,IR
OBSR(J)=0.0
DO 40 K=1,KT
IK=I+IR*(J-1+JS*(K-1))
OBSR(J)=OBSR(J)+OBSR(I)
OBSR(K)=OBSR(K)+OBSR(I)
OBSR(J)=OBSR(J)
40 CONTINUE
IJ=I+IR*(J-1)
OBSR(J)=OBSR(J)
50 CONTINUE
OBSR(J)=OBSR(J)
60 CONTINUE
C
DO 90 I=1,IR
OBSS(I)=0.0
DO 80 K=1,KT
OBSK(I)=0.0
DO 70 J=1,JS
IJ=I+IR*(J-1+JS*(K-1))
OBSK(I)=OBSK(I)+OBSK(I)
OBSK(I)=OBSK(I)+OBSK(I)
OBSK(I)=OBSK(I)
70 CONTINUE
KI=K+KT*(I-1)
OBSK(KI)=OBSK(KI)
80 CONTINUE
OBSST(I)=OBSST(I)
90 CONTINUE
END
SUBROUTINE NMPRINT(X,N,M)
COMMON/ALL/VOL(400), IDENT(10), IR, JS, KTN, DEGREES, INPUT, OUTPUT
DIMENSION X(256), XNORM(256)
TYPE INTEGER OUTPUT
2000 FORMAT(/9X,32HALL NUMBERS HAVE BEEN DIVIDED BY 11.11 1H )
2001 FORMAT(8X,16F7.3)
N=M*N
XMAX=X(1)
DO 10 K=2,NM
IF(XMAX*XMAX.GT.X(K)*X(K)) GO TO 10
XMAX=X(K)
10 CONTINUE
XMAX=ABS(XMAX)
XMAXLOG=LOGF(XMAX)/2.3025850930
MAXLOG=XMAXLOG
IF(XMAX.LT.1.0) MAXLOG=MAXLOG-1
XMAX=10.0**MAXLOG
DO 20 K=1,NM
XNORM(K)=X(K)/XMAX
20 CONTINUE
WRITE OUTPUT TAPE OUTPUT*2000,XMAX
DO 30 I=1,M
J2=N*I
J1=J2-N+1
WRITE OUTPUT TAPE OUTPUT*2001,(XNORM(J), J=J1,J2)
30 CONTINUE
END
FUNCTION PHI(w,pl)
AREA=1.0+(pl-1.0)/2.0/w
TEST=0.
EP = .0000001
IF(AREA-.5)3,4,5
4 XORD=0.
GOTO6
3 AREA1=1.-AREA
C = -1.
GOTO7
5 AREA1=ARFA
C = 1.
7 X = (AREA1 -.5) * 2.5
8 DIV = 1.0 + .3275911 * X
E = 1.0/ DIV
S = ((((.940646070*E)-1.287822453)*E+1.259695130)*E-.252128668)*E
X+.225836846
AA = EXPF ( X **2 ) * .88622692
FX = (AREA1 - 1.0) * AA * DIV * 2.0 + S
FXP = - SD * .3275911 - S * 2.0 * X
XNEW = X - FX / FXP
IF(ABSF(XNEW-X) -EP ) 6, 6, 14
14 TEST=TEST+1.
IF(TEST=40.15*6,6
15 X = XNEW
GO TO 8
6 XORD = C * XNEW * 1.414213562
16 PHI=XORD
END
FUNCTION CHI(T,PL)
DIMENSION X(13+2)

DATA (X=3.6415599197.6159488116.7041.0002.32638131.21120)

* -0.524675991-0.123011776.0.792398674-0.19061767840.172451898
* 6.63555216113461327715.0561.0003.289460752.94515494
* -0.2995185440.244632590.411341788-0.3423156135.203111981

L=1
IF (PL.GT.0.575) L=2
IF (T.GT.5.5) GO TO 10
IT=T
CHI=X(IT+L)
GO TO 30

10 TRT=SQRTF(T)
CHI=X(13+L)
DO 20 K=1,7
CHI=CHI/TRT+X(13-K+L)
20 CONTINUE
CHI=T*CHI

30 CHI=SQRTF(CHI)
END
**Table 1 - Data from Kastenbaum and Lamphiear (1959) - 2 x 3 x 5**

**Kastenbaum-Lamphiear Procedure**

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**Iteration** | **Degrees of Freedom** | **Chi-Square**
---|---|---
26 | 8 | 3.159045

R S T
2 3 5


| TABLE 2 - DATA FROM KASTENRAHM AND LAMPHIFAR (1950) - 2 X 3 X 5 |

PLACKETT PROCEDURE

K = 1

**CONTRASTS**

All numbers have been divided by 1.000

2.095 -1.304

**DISPERSION MATRIX**

All numbers have been divided by 1.000

0.174 -0.113
-0.113 1.544

K = 2

**CONTRASTS**

All numbers have been divided by 1.000

0.256 -0.091

**DISPERSION MATRIX**

All numbers have been divided by 1.000

0.143 -0.093
-0.093 1.268

K = 3

**CONTRASTS**

All numbers have been divided by 1.000

-0.129 -1.322

**DISPERSION MATRIX**

All numbers have been divided by 1.000

1.535 -0.485
-0.485 1.202
**TABLE 2 (Continued)**

**K = 4**

**CONTRASTS**

All numbers have been divided by 1.0*000

-0.429 -1.117

**DISPERSION MATRIX**

All numbers have been divided by 1.0*001

2.147 -0.301
-0.301 7.036

**K = 5**

**CONTRASTS**

All numbers have been divided by 1.0*000

0.000 -1.954

**DISPERSION MATRIX**

All numbers have been divided by 1.0*000

0.600 0.300
0.300 1.335

**7-VECTOR**

All numbers have been divided by 1.0*000

-1.248 -5.760

**F-MATRIX**

All numbers have been divided by 1.0*001

0.336 -0.088
-0.088 1.951

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<th>SUM (L*1)</th>
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**Table 3 - Data from Kastenbaum and Lamphifar (1959) - 2 x 3 x 5**

**GOODMAN PROCEDURE**

**K = 1**

**CONTRASTS**

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**DISPERSION MATRIX**

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**K = 2**

**CONTRASTS**

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**DISPERSION MATRIX**

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**K = 3**

**CONTRASTS**

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**DISPERSION MATRIX**

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TABLE 3 (Continued)

\( K = 4 \)

**CONTRASTS**

All numbers have been divided by 1.0-001

\[-7.732 \quad -3.438\]

**DISPERSION MATRIX**

All numbers have been divided by 1.0-001

\[
\begin{pmatrix}
2.145 & 1.222 \\
1.222 & 2.446
\end{pmatrix}
\]

\( K = 5 \)

**CONTRASTS**

All numbers have been divided by 1.0-001

\[-9.769 \quad -9.769\]

**DISPERSION MATRIX**

All numbers have been divided by 1.0-001

\[
\begin{pmatrix}
6.338 & 1.838 \\
1.838 & 3.338
\end{pmatrix}
\]

**G-VECTOR**

All numbers have been divided by 1.0-000

\[-7.009 \quad -4.511\]

**Q-MATRIX**

All numbers have been divided by 1.0-002

\[
\begin{pmatrix}
5.426 & 3.986 \\
3.986 & 6.309
\end{pmatrix}
\]

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<tr>
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<th>SUM(H*H)</th>
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<td>D</td>
<td>V</td>
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DEGREES OF FREEDOM

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α 5  12

#636
**TABLE 5** - DATA FROM KASTENBAUM AND LAMPHIFAR (1959) - 2 x 3 x 5

**INTERACTION ESTIMATES AND CONFIDENCE INTERVALS IN LOG UNITS**

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TABLE 5 (Continued)

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